

# **Unveiling Trends in Leading AI and ML Companies' Stock Price: A Time Series Analysis**

# **Abstract**

This study explores the potential of time series analysis in predicting significant trends within the historical stock prices of leading AI and ML companies: Alphabet, Meta, and Microsoft. The primary objective is to evaluate the effectiveness of various time series models in forecasting future stock prices, thereby providing useful insights for investors and companies by informing them about the best forecasting models and optimal adjustments to predict future stock prices.

To achieve this aim, the research addresses the following objectives: employing time series models to identify significant long-term and seasonal trends within the historical stock prices of Alphabet, Meta, and Microsoft; comparing the suitability of different time series models; analysing the similarities and differences in the observed trends; and evaluating the accuracy of the models in predicting future trends.

Addressing knowledge gaps in the literature, the research contrasts traditional statistical models like ARIMA with deep learning models such as LSTMs. Traditional models often struggle with non-linear relationships inherent in financial data, while deep learning approaches have shown promise in capturing long-term dependencies. However, discrepancies in comparative studies highlight the need for a comprehensive assessment of these models' effectiveness in the specific context of AI and ML companies' stock prices.

The methodology includes several steps to ensure robustness and accuracy. Initially, data preprocessing and outlier detection are performed to ensure data quality. Exploratory Data Analysis (EDA) follows, involving the use of ACF, PACF plots, and stationarity tests like the Augmented Dickey-Fuller (ADF) and Ljung-Box tests to uncover inherent patterns and ensure data stationarity. Data is then split into training and testing subsets for model evaluation.

Both statistical and deep learning models are employed, including ARIMA-GARCH, LSTM, and GRU. Model performance is evaluated using metrics such as RMSE and MAE. The best-performing models from each category are selected for forecasting mid-term future trends, with comparisons made against actual values to determine prediction accuracy.

Ethical considerations are meticulously addressed, acknowledging the secondary nature of the data sourced from Yahoo Finance and ensuring transparency in all analyses. An Ethos application has been completed to support the ethical conduct of this research.

The findings indicate that all three companies exhibited upward long-term trends, suggesting positive investor outlooks on AI/ML futures. No significant seasonal patterns were identified, pointing to consistent long-term influencing factors on stock prices. Machine learning models, particularly GRUs, outperformed traditional statistical models in capturing underlying trends and predicting mid-term future trends. The GRU model significantly outperformed the ARIMA-GARCH model across all three stocks. Overall, these findings highlight the potential of GRU models for investors and financial analysts seeking to make informed decisions within the AI/ML industry.

However, focusing solely on historical prices might overlook other influencing factors, potentially leading to inaccurate predictions. Future research directions include exploring techniques to mitigate overfitting and improving model generalisability to support high-risk decision-making.

In conclusion, this study demonstrates the superior performance of ML models in forecasting stock prices for leading AI and ML companies, providing useful insights for investors and companies navigating the dynamic AI/ML landscape.

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# Introduction

This chapter outlines the aim, objectives and justification of the research. It then delves to the research background to highlight significance of research questions.

## 1.1 Background

Stocks have long been considered a cornerstone asset for investors. They offer the potential for strong returns on investment over time and high liquidity. This is exemplified by the FTSE 100 on the London Stock Exchange, which delivered a total return of 645.2% (equivalent to an annualised return of 5.4%) between 1984 and 2022 (Bright, 2023). Publicly traded companies are required by law to disclose their financial status, providing a wealth of information for investors. Additionally, regulations governing the stock market help mitigate various financial risks.

However, maximising profits from stock investments demands a sound understanding of market dynamics and a well-defined strategy for buying and selling. This, in turn, hinges on accurate prediction of stock prices. Stock price prediction methods can be broadly categorized into two main approaches: fundamental analysis and technical analysis (Devadoss, 2013).

Fundamental analysis focuses on a company's intrinsic value, considering its future potential and financials. A stock trading below its intrinsic value might be a good buy, while an overvalued stock might be a sell candidate. Technical analysis focuses solely on the stock price itself, using charts and mathematical tools to identify trends and predict future price movements. This approach ignores company fundamentals and relies on historical price data (Raghunathan, 2007).

In response to these limitations, time series analysis is a statistical technique used to examine observations collected on a variable of interest over time. This technique



allows researchers to uncover patterns within historical datasets, such as seasonality (Montgomery et al., 2008). In the context of this research, time series analysis will be utilised to analyse the historical stock prices of leading Artificial Intelligence (AI) and Machine Learning (ML) companies.

Employing times series analysis, help us uncover long-term patterns within historical stock price data. This involves analysis techniques to identify trend or seasonality patterns within existing data (Shanaev et al., 2020). By analysing these trend components, we can assess the overall growth trajectory of AI/ML companies.

In a general sense, time series analysis will help us to understand if stock price movement in recent years has been impacted by technological advancements. While time series analysis mainly focuses on historical data, its power can be leverage to forecast future trends as well. Techniques such as ARIMA (Hyndman et al., 2021) or machine learning models like LSTM (Asokan, 2022) utilise historical patterns in data to predict future prices.

## **1.2 The Impact of AI and ML on the Stock Market**

The rapid advancements in AI and ML are driving a profound revolution across industries, fundamentally transforming their core operations (Rahman, 2023). Artificial Intelligence refers to the development of computer programs or intelligent machines that can mimic or even surpass human cognitive abilities in specific domains (Kaplan, 2016). For instance, AI-powered programs like AlphaGo have achieved superhuman performance in complex strategy games like GO by defeating the world grandmaster, Lee Sedol (Silver et al., 2016), while chess programs like Stockfish can defeat grandmasters consistently (Naroditsky, 2015). Similarly, advancements in Natural

Language Processing (NLP) enables machines to understand human-like languages and respond to them with increasing accuracy (Arkhangelskaya et al., 2023).

Machine Learning, on the other hand, empowers these intelligent machines to continually improve by learning from data without the need for explicit programming. Unlike traditional algorithms with fixed set of instructions, ML models can identify patterns and relationships within large datasets, allowing them to adapt and make predictions on new data (Kaplan, 2016). This self-learning capability has fuelled significant breakthroughs in robotics, where AI-powered robots are now performing complex tasks with greater intelligence and adaptability. These tasks include autonomous navigation in transportation systems like self-driving cars, object recognition for applications like inventory management and predictive maintenance to prevent equipment failures (Soori et al., 2023).

The utilisation of AI and ML technologies is significantly impacting the global economy and financial markets. A 2022 PwC report predicts that nearly 45% of total global economic profit by 2030 will be generated through product enhancements driven by AI and ML. This economic surge, driven by three key factors, will undoubtedly impact stock prices (PWC, 2022). AI-powered automation will streamline tasks across industries, boosting productivity. Secondly, AI will augment the workforce, empowering workers with intelligent tools that unlock greater human potential. Finally, AI's ability to analyse vast data sets will personalise consumer experiences, fuelling demand and economic growth across sectors.

Companies developing and utilising these technologies are experiencing substantial growth, attracting investor interest and influencing stock prices (Bughin et al., 2018). Understanding the current trends in how leading companies utilise AI and ML, and the

subsequent impact on their stock prices, can provide valuable insights for both companies and their investors. This knowledge can help them assess growth potential, identify related risks, and make more informed investment decisions.

### **1.3 Justifications, Research Aim and Objectives**

The economic potential of AI and ML has been extensively explored. Research by scholars such as Chia-Hui (2021) has highlighted their disruptive power, ability to drive innovation, and contribution to economic growth. However, there is a gap in understanding how these developments translate into the concrete stock market performance of leading AI and ML companies themselves. While the broader economic impact of AI and ML advancements is well-documented (Barclays, 2024), a gap exists in understanding how these advancements are reflected in the stock market performance of leading companies driving this revolution.

#### **1.3.1 Research Aim**

The primary objective of this research is to utilise time series analysis to evaluate the potential for predicting significant trends within historical stock prices of leading AI and ML companies. This analysis will involve uncovering long-term and seasonal patterns in the historical data, and subsequently assess the effectiveness of the chosen models for forecasting future trends.

By utilising time series analysis, this research aims to offer valuable insights within the AI and ML landscape for both investors and companies. Time series analysis can help investors make more informed decisions by identifying long-term growth trajectories and assessing the impact of AI and ML's disruptive power. For companies, this analysis can provide a better understanding of how their stock market performance aligns with

their technological advancements, allowing them to optimise future stock market strategies.

### **1.3.2 Research Objectives**

To achieve this aim, the research will address the following specific objectives:

1. Employ time series models to identify significant long-term and seasonal trends within the historical stock prices of Alphabet, Meta, and Microsoft.
2. Compare the suitability of different time series models for uncovering underlying trends in the stock prices of leading AI and ML companies.
3. Analyse the similarities and differences observed between the long-term and seasonal trends exhibited in the chosen companies' stock prices.
4. Evaluate the accuracy of the chosen time series models in predicting future trends in the stock prices of the leading AI and ML companies.

### **1.4 Research Questions**

In order to achieve the research aim and objectives outlined above, this research will address the following research questions:

1. How well can the chosen time series models identify significant long-term or seasonal trends within the historical stock prices of leading Artificial Intelligence (AI) and Machine Learning (ML) companies (Alphabet, Meta, and Microsoft)?
2. In comparison to each other, how suitable are the chosen time series models for uncovering underlying trends in the stock prices of leading AI and ML companies?
3. Are there any significant similarities or differences observed between the long-term and seasonal trends exhibited in the chosen companies' stock prices?
4. How accurately can the chosen time series models predict future trends in the stock prices of the leading AI and ML companies?

### **1.5 Research Approach**

This research will utilise secondary data sources for analysis. Historical stock prices will be collected from Yahoo Finance.

To ensure a robust analysis, the data selection process will consider established practices in financial time series analysis. The aim is to capture both long-term trends and potential seasonal patterns, while maintaining relevance to recent developments in the dynamic AI and ML sector. A specific timeframe will be chosen to balance these considerations.

Three leading global companies in the field of AI and ML have been selected for analysis: Alphabet (Google), Meta Platforms (Facebook), and Microsoft. These companies are recognised for their dominant market positions due to their market capitalisation and their well-established applications of AI and ML technologies.

## **1.6 Research Structure**

This study is structured into five chapters. Chapter 1 outlines the research aims, objectives, and justification. Chapter 2 explores relevant literature on applying time series analysis to stock market forecasting. Chapter 3 details the research methodology and data collection. Chapter 4 presents the research findings and analysis. Finally, Chapter 5 discusses the results, evaluates the study, and recommends future research directions.

# 1. Literature Review

## 2.1 Introduction

Stock price prediction is crucial for maximising investment returns. Time series analysis, analysing data sequences over time to identify patterns for forecasting, plays a vital role. Selecting the most effective model is essential, and finance boasts a rich history of developing specialised models for financial data.

This chapter delves into the core concepts of time series analysis. We will then explore some of the most commonly applied forecasting models. Finally, we will discuss a comparison and evaluation of these models in the context of real-world stock price prediction, drawing insights from various research studies and literature.

## 2.2 Time Series Analysis Core Concepts

Before diving into specific models, a brief introduction to the core time series concepts is appropriate.

- **Stationarity**

A stationary time series does not depend on the observation time and so will exhibit constant statistical properties (like mean and variance) in the long run. This contrasts with time series exhibiting seasonality or trends, where the time of observation affects the value. In this sense, cyclical behaviour can also be considered stationary because it doesn't have a fixed period within the data (Hyndman et al., 2021).

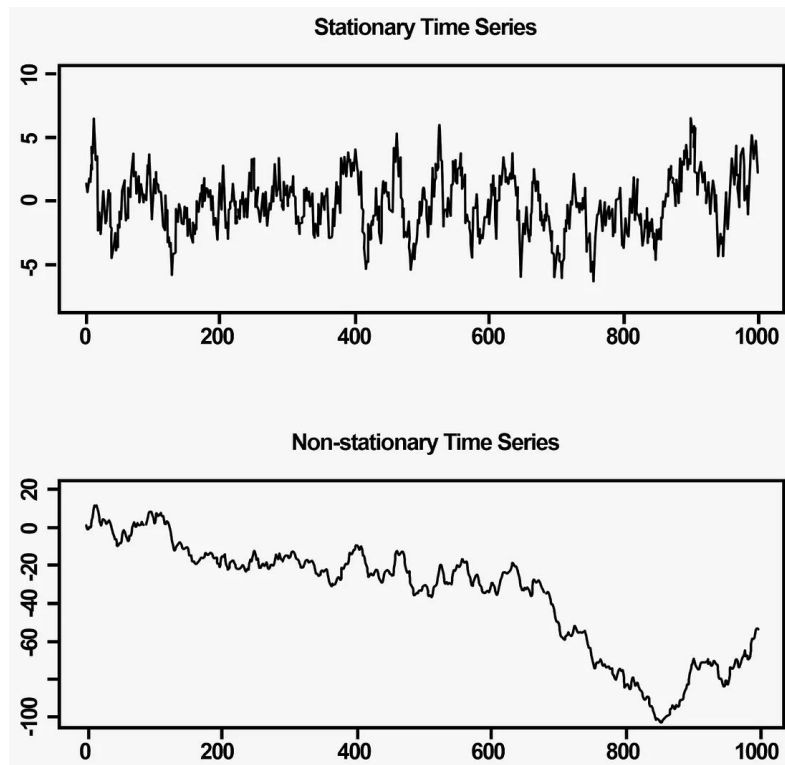


Figure 1: Stationary vs. Non-Stationary time series (source: Santra, 2023)

- **Differencing**

To transform a non-stationary time series into a stationary one, the difference between the current observation and its previous value (or a lagged value) is computed. This stabilises the variance of the time series. By removing the changes in the series, the mean becomes more stable, and any trends or seasonality patterns are removed (Hyndman et al., 2021).

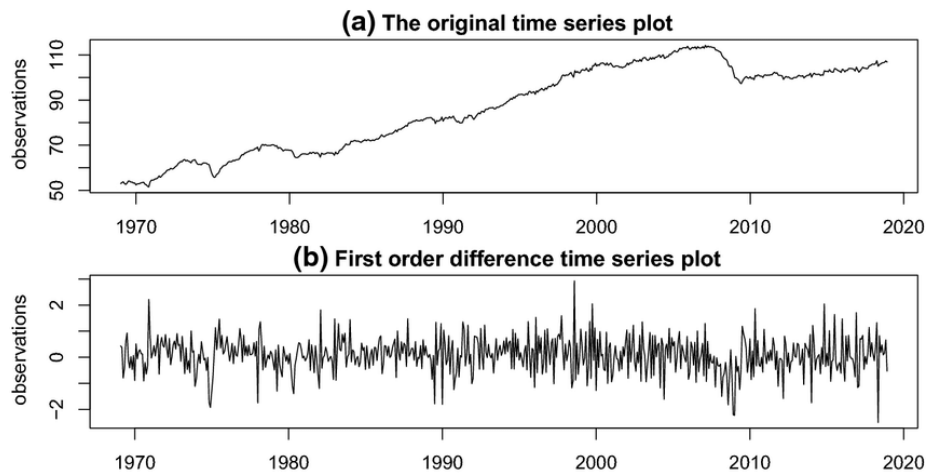


Figure 2: Differencing (source: Kai et al., 2022)

- **Trends**

A long-term decrease or increase in time series data, regardless of its linearity, suggests the presence of a trend. This trend might even change direction, for instance, from decreasing to increasing (Hyndman et al., 2021).

- **Seasonality**

Seasonality arises when a time series is influenced by recurring seasonal factors like day of the week or time of the year (Hyndman et al., 2021).

- **Cyclic**

A rise and fall in data without any constant frequency is termed a cycle. For instance, business cycles fluctuate due to changing economic conditions (Hyndman et al., 2021).

- **Irregular Variations**

Unpredictable fluctuations in a time series with no discernible pattern are termed irregular variations. Unlike seasonal or cyclical variations, they can occur at any time and often stem from unforeseen events (Valse, 2020), such as unexpected news announcements.



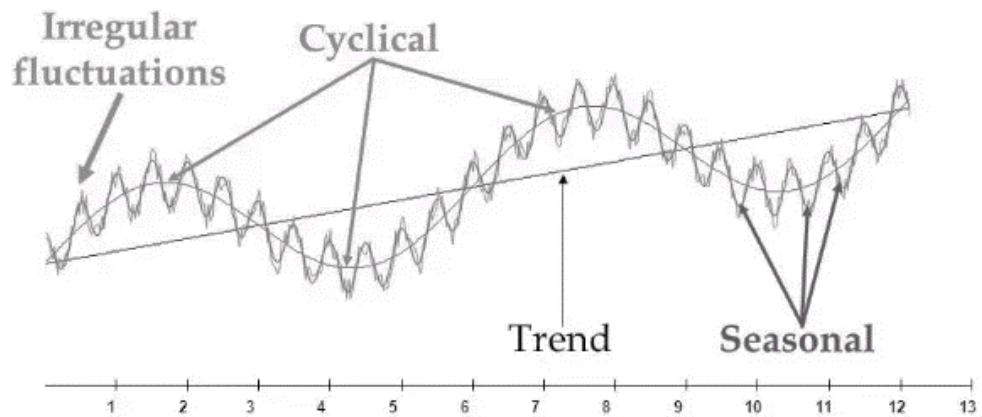


Figure 3: Time Series Components (source: Valse, 2020)

- **Autocorrelation Function (ACF)**

The ACF measures the correlation between a time series and its lagged observations (past values). Positive or negative values at early lags suggest trends, while spikes at specific lags indicate seasonality. Values close to zero imply little to no correlation. High ACF values at some lags, particularly for pronounced trends, can signal non-stationarity. However, interpreting the ACF for stationarity requires considering the overall pattern and employing additional tests like the Dickey-Fuller test (Hyndman et al., 2021; Brockwell, 2016).

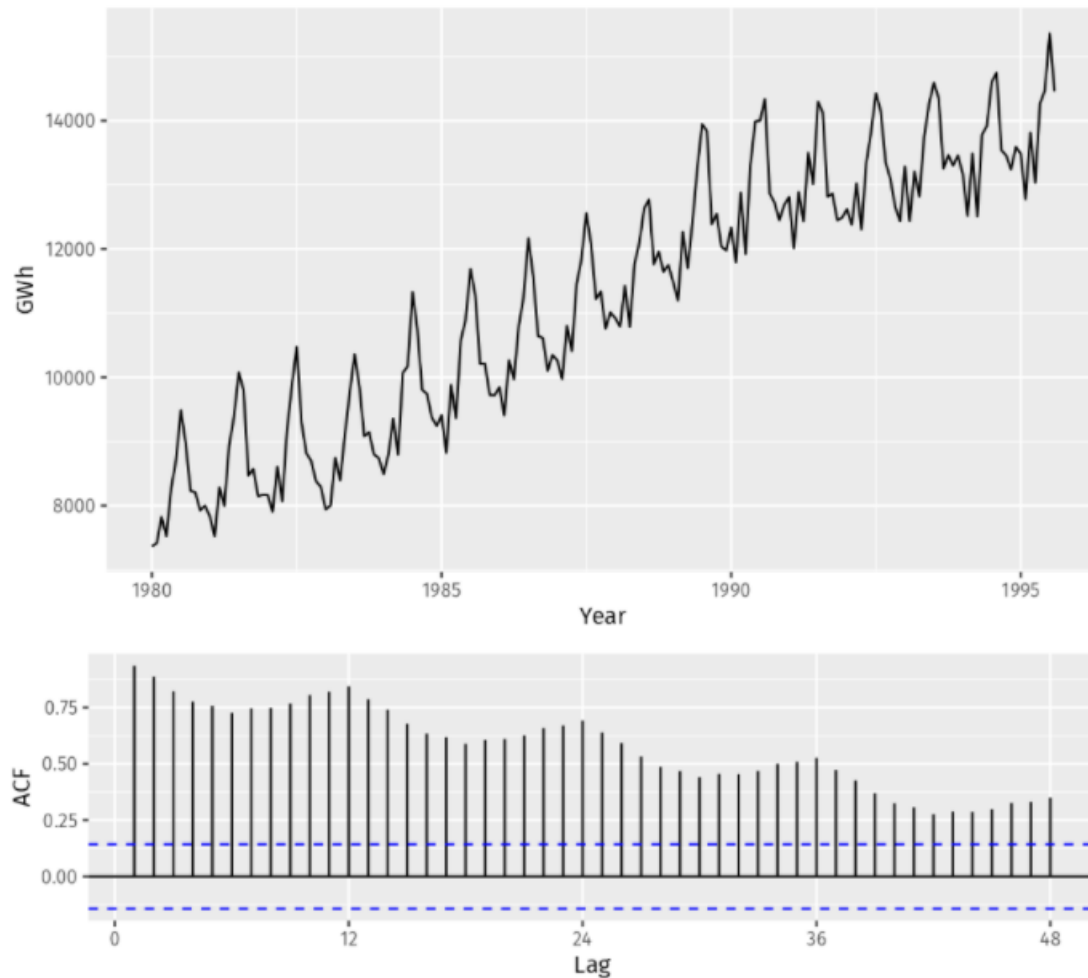


Figure 4: Autocorrelation Function (ACF) with both trend and seasonality (source: Hyndman et al., 2021)

- **Partial Autocorrelation Function (PACF):**

PACF is similar to the ACF with a difference that unlike ACF, it measures only the direct correlation of time series with its lagged observations excluding the influences of intermediate lags. A high PACF value suggest strong relationship between the current value and its lagged value. Similar to ACF, a value of zero or close to it indicates no direct correlation or a weak one at that lag (Dégerine et al., 2003).

## 2.3 Testing for Stationarity

### **2.3.1 Augmented Dickey-Fuller (ADF)**

This statistical test assesses whether a time series is stationary (constant mean, variance, and autocorrelation). The null hypothesis implies non-stationarity, while the alternative is stationarity.

A more negative test statistic indicates stronger evidence against non-stationarity. If the statistic falls below critical values at a chosen significance level (e.g., 5%), we fail to reject the null hypothesis, suggesting potential non-stationarity. Additionally, a p-value less than the significance level (e.g., 0.05) leads to rejection of the null hypothesis, implying stationarity (Dickey et al., 1979; Rizwan, 2011).

### **2.3.2 Ljung-Box**

In a well-specified model for a stationary time series, residuals should be independent and identically distributed (i.i.d.). This means they behave randomly and have no "memory" of past values. Non-random residuals (serial correlation) suggest issues with the model or data stationarity.

The Ljung-Box test's null hypothesis states no serial correlation in residuals (present residuals are unrelated to past ones). Non-stationary data (trends or seasonality) can lead to serial correlation, violating the null hypothesis. A low p-value from the Ljung-Box test on residuals indicates a potential stationarity issue (Ljung et al., 1978).

## **2.4 Statistical Time Series Forecasting Models**

### **2.4.1 Moving Average (MA)**

Moving averages (MAs) are a popular technical indicator used in financial trading to identify trends and potential support and resistance levels. Considering stock data as example, this approach assumes that recent prices hold more significance for predicting future prices compared to older data. For example, a 3-period moving

average calculates the average data over the last three periods (days, weeks, etc.) to predict the upcoming period's price. This implies that the price of future time periods is weighting by the patterns observed in the most recent data points. (Jain et al., 2005)

There are three main types of moving averages (Murphy, 1999):

- **Simple Moving Average (SMA)**

This basic MA takes the average of a chosen number of price points (e.g., closing prices). Recent data has equal weight as older data, making it more sensitive to recent price changes.

- **Linearly Weighted Moving Average (LWMA)**

This MA assigns higher weights to recent prices within the window, giving them more influence. While recent data is prioritized, some historical data is still considered.

- **Exponentially Weighted Moving Average (EMA)**

This MA assigns exponentially decreasing weights to past prices, with the most recent price having the most influence. This approach heavily emphasizes recent information while incorporating some historical context.

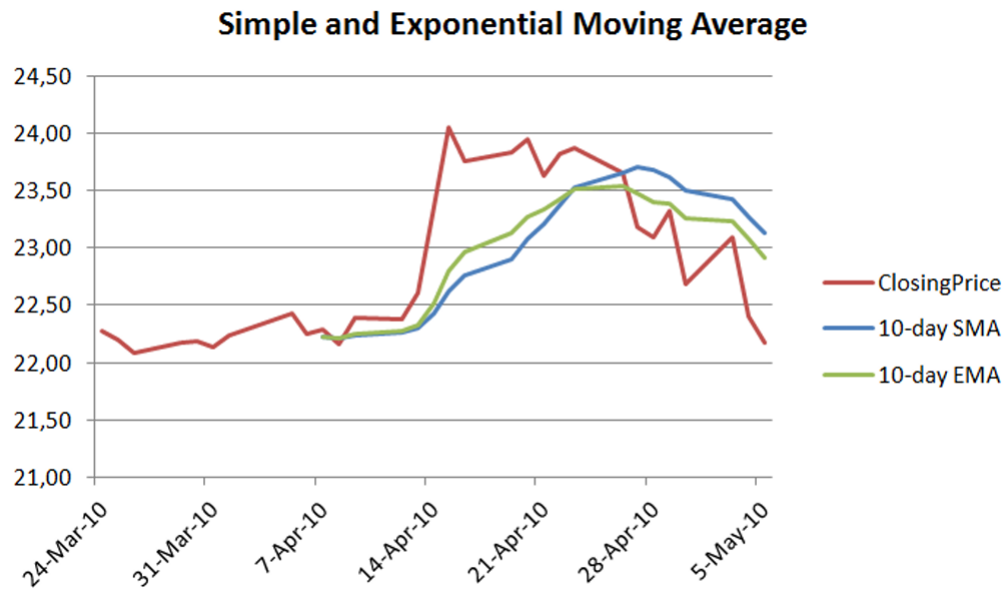


Figure 5: Comparison of Moving Averages (source: Kadam, 2012)

## 2.4.2 Exponential Smoothing

Exponential smoothing is a forecasting technique that assigns higher weights to recent data points compared to older ones. It emphasizes the most recent observations while progressively decreasing weight for older data. This approach offers quick and reliable forecasts (Nugus, 2009), making it a valuable tool across various industries since its introduction (Holt, 1957; Brown, 1959; Winters, 1960). Exponential smoothing comes in three main types: single, double, and triple. These methods can model time series data with varying degrees of complexity, accounting for trend level, trend slope, and even seasonal components (Yibin, 2019).

Holt's linear model excel at forecasting trends but it tends to over-predict if the trend is expected to level off. To address this limitation, researchers like Gardner & McKenzie (1985) introduced a parameter that dampens the trend, making more realistic forecast especially in situation which the trend eventually stabilise. This

optimisation has proven highly successful in improving forecasting accuracy (Hyndman et al., 2021).

Holt-Winters builds on exponential smoothing by capturing seasonal patterns with weights in a multiplicative model. Simpler and requiring less data than Box-Jenkins, its strength lies in "back-forecasting" for superior accuracy, though initially computationally expensive (Rumbe et al., 2024).

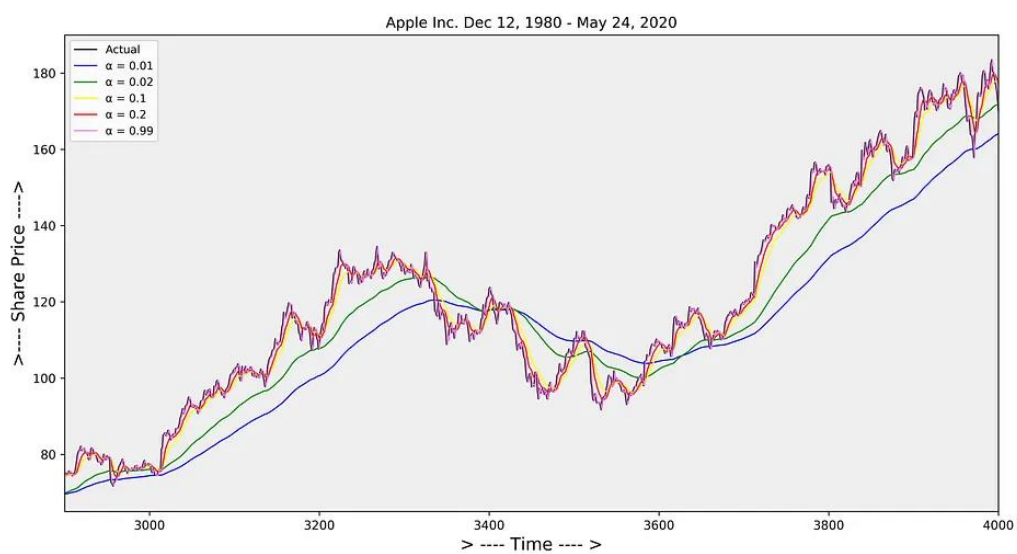


Figure 6: Exponential Smoothing (source: Dash, 2020)

### 2.4.3 Autoregressive Integrated Moving Average (ARIMA)

In the real world, observations of a variable often exhibit a dependence on past values, creating a serial dependency across time. For such data, exponential smoothing models become less effective in forecasting. To address this limitation, the ARMA (Auto Regressive Moving Average) model was developed by combining the strengths of Moving Average (MA) and Autoregressive (AR) approaches. Later, Box and Jenkins (Box et al., 1976) introduced the concept of stationarity, leading to the creation of more

robust ARIMA models. Finally, to handle seasonal patterns in data, SARIMA (Seasonal ARIMA) models were introduced (Shumway et al., 2015).

ARIMA models (p, d, q) are characterised by 3 key components (Shumway et al., 2015):

- **Autoregression (AR)**

This component leverages past observations of a time series to predict its current value. The parameter 'p' determines the number of past data points included in the model for forecasting.

- **Integrated (I)**

This step ensures the data is stationary. If trends or seasonality are present, differencing is applied 'd' times to achieve stationarity.

- **Moving Average (MA)**

MA incorporates past forecast errors -the difference between actual and predicted values. The parameter 'q' determines the number of past errors considered to smooth out fluctuations and potentially improve forecast accuracy.

ARIMA models require careful selection of p, d, and q parameters, often through statistical tests or validation. A basic example, ARIMA(0,1,0), is essentially a random walk model (d=1 for stationarity) that ignores past values (p=0) and errors (q=0) (Montgomery et al., 2008).

SARIMA (p, d, q) \* (P, D, Q)<sub>s</sub> models are an extension of ARIMA models, addressing a critical aspect missing in them: seasonality. They explicitly consider recurring patterns in the data based on time intervals. SARIMA introduces four additional parameters to capture seasonality. 'P' determines the number of past seasonal values; 'D' the number of times data needs to be seasonally differenced; 'Q' the number of

past seasonal forecast errors; and 's' defines the number of observations in a single seasonal period (example: monthly = 12 and quarterly= 4) (Malki et al., 2022).

#### **2.4.4 ARMA-GARCH (Generalized Autoregressive Conditional Heteroskedasticity)**

To capture volatility clustering of a time series which is variance fluctuation around the mean, GARCH models were introduced (Engle, 1982; Bollerslev, 1986). Since many time-series exhibits periods of high volatility followed by similar periods or vice versa, ARMA based models cannot capture these phenomena alone. However, combining ARMA models with GARCH models can recognise this volatility over time. This allows for better prediction by considering the past squared errors in calculating the variance of forecast errors (Kim, 2022).

GARCH models include two parameters of p and q. these parameters determine how past volatility influences the current conditional variance. Combination of two models, ARMA (p, d, q) – GARCH (p, q) works in two steps: First, ARMA model is applied to predict future values' mean based on the past values, differencing and forecast errors of past observations in time-series and then using GARCH component of past squared errors (volatility) to predict the future conditional variance. The obtained value explains how much data is likely to fluctuate around the mean (Lee, 2013).

### **2.5 Deep Learning Time Series Forecasting Models**

#### **2.5.1 Recurrent Neural Network (RNN)**

Artificial Neural Networks (ANNs) are a powerful computational approach inspired by the structure and function of the human brain. They consist of a vast network of interconnected processing units, often referred to as artificial neurons or processing elements (PEs). Unlike biological neurons with complex inner workings, these PEs are simpler computational units. Through a training process, ANNs can adjust the



connections (synapses) between these PEs and so capture knowledge and patterns from the data they are exposed to (Guresen et al., 2011)

Recurrent neural networks (RNNs) are a specific type of artificial neural network architecture designed to excel at processing temporal sequential data (Guo et al., 2012). Unlike traditional neural networks that process information in a single forward pass, RNNs are adept at handling sequences. They achieve this by employing a feedback loop mechanism that allows the output from one step to be fed back as input to the next step. This creates a form of internal memory within the network, enabling it to learn from past information and improve its understanding of future inputs in the context of the current observation within a sequence. In essence, RNNs overcome the limitation of traditional neural networks, which typically process information in a single forward pass (Ayodele et al., 2021).

RNNs operate by forecasting a sequence one element at a time. During each step, they combine the current input with the prediction from the previous step, retrieved from their internal memory. This combined information is then processed to generate a new output and update the internal memory in preparation for the next element in the sequence (Hewamalage et al., 2021).

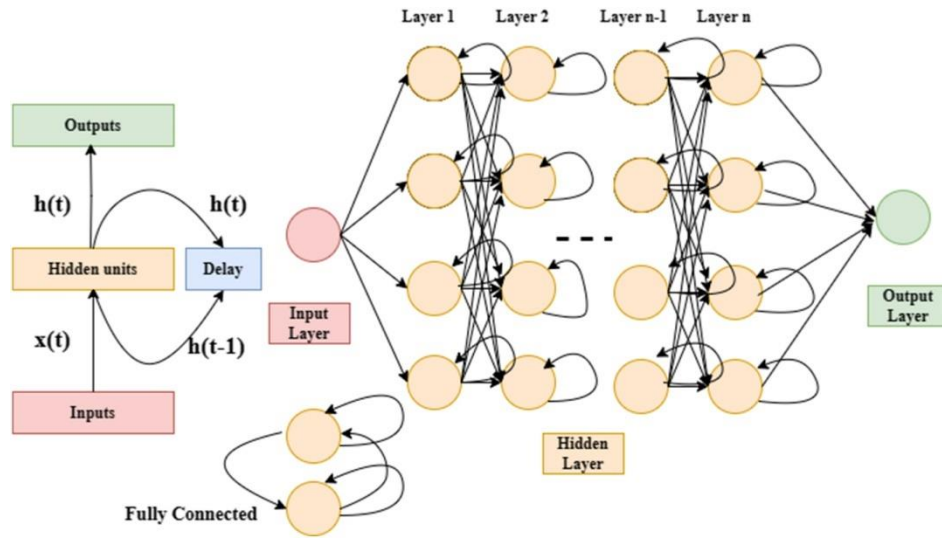


Figure 7: RNN Architecture (source: Beniwal et al., 2024)

However, RNNs face challenges when dealing with real-world applications involving long sequences. They can struggle to learn dependencies between elements that are far apart in the sequence, a phenomenon known as the vanishing gradient problem. Additionally, RNNs are limited by their internal memory capacity, which restricts their ability to effectively learn from very long sequences (Liu et al., 2023).

### 2.5.2 Long Short-Term Memory (LSTM)

To address the vanishing gradient problem that limits traditional Recurrent Neural Networks (RNNs), Long Short-Term Memory (LSTM) models were introduced (Hochreiter & Schmidhuber, 1997). LSTMs excel at learning long-term dependencies within sequences, allowing them to effectively analyse information even for sequences exceeding 1000 steps. They achieve this by employing a specially designed architecture that incorporates gates within their cell structure. These gates help manage the flow of information and overcome the vanishing gradient problem (Zhang et al., 2023).

An LSTM memory cell is the core component of an LSTM network and contains four major elements:

- Cell State: This acts as a long-term memory unit, storing, reading, writing and forgetting information relevant to the task. It does this by using three gates.
- Input Gate: This gate controls the flow of new information into the cell state.
- Output Gate: This gate determines what information from the cell state is used as output.
- Forget Gate: This gate decides what information to forget from the cell state, allowing the network to selectively retain or discard information over time.

Each gate utilises an activation function to compute a value between 0 and 1. The input gate controls the values entering the cell, the output gate controls the values used for output, and the forget gate controls the information retained within the cell (Gao et al., 2017).

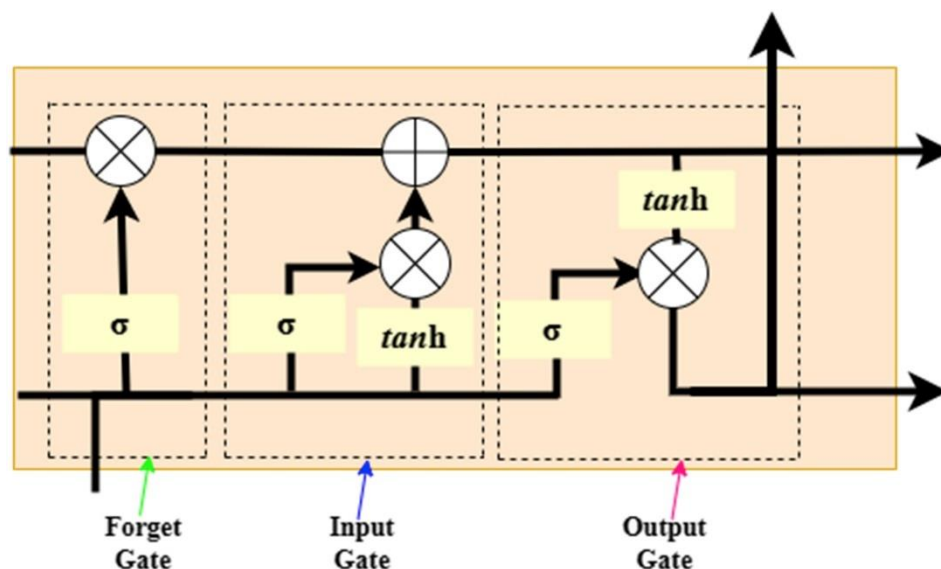


Figure 8: LSTM Architecture (source: Beniwal et al., 2024)

LSTMs excel at handling financial time series data due to their ability to capture long-term dependencies. However, non-stationary data and noise can challenge LSTMs, especially with sudden changes and extended prediction horizons, potentially reducing performance and forecast accuracy. To address these issues, researchers have developed LSTM-based models like LSTM-BN for financial predictions (Fang et al., 2023).

### **2.5.3 Gated Recurrent Unit (GRU)**

Gated Recurrent Units (GRUs), proposed by Cho et al. (2014) have emerged as a popular alternative to Long Short-Term Memory (LSTM) networks. While inspired by LSTMs, GRUs offer a more streamlined architecture, making them computationally efficient and well-suited for various applications (Pil-Soo et al., 2018).

One key difference lies in the gating mechanisms. GRUs combine the forget and input gates of LSTMs into a single update gate. This simplifies the information flow within the network. Unlike LSTMs with separate memory cells, GRUs modulate information flow directly through the hidden state, eliminating the need for dedicated memory units (ArunKumar et al., 2021).

This streamlined approach translates to faster training times and almost the same performance compared to LSTMs. The update gate acts as a controller, determining how much of the previous hidden state (information) should be retained and how much should be replaced with new information from the current input. This controlled flow of information allows GRUs to effectively learn long-term dependencies within sequences (Wang et al., 2019).

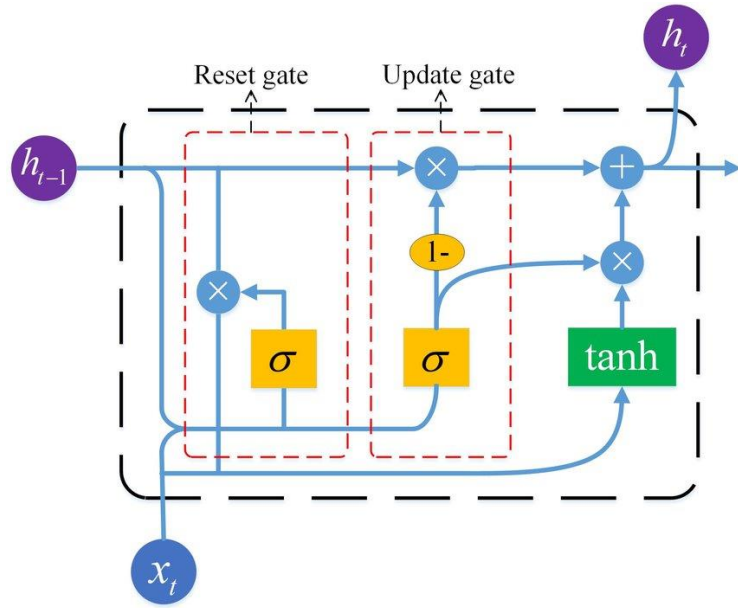


Figure 9: GRU Architecture (source: Jing et al., 2021)

However, a potential drawback of GRUs is their lack of a dedicated forget gate. While LSTMs can explicitly forget irrelevant information, GRUs rely solely on the update gate to manage the information flow. This could limit their ability to handle certain tasks where selective forgetting might be crucial (Srivatsavaya, 2023).

## 2.6 Comparison and Evaluation of Time-Series Models

The ARIMA model is widely used in finance for time series forecasting. Studies highlight its frequent use for predicting stock prices (Hu et al. 2020; Idrees et al. 2019). ARIMA excels in short-term predictions by leveraging past observations, requiring minimal parameters while balancing complexity and accuracy (Box et al., 1976; Ariyo et al., 2014; Wadi et al., 2018).

However, while ARIMA and similar models excel at capturing linear relationships within data, they can struggle with the complexities introduced by non-linear factors (Earnest et al., 2005) such as unexpected events or market sentiment. This is because ARIMA assumes stationarity, which might not always hold true for stock prices that

often exhibit trends and seasonality. The process of differencing data to achieve stationarity can also lead to information loss, potentially overlooking valuable insights (Hyndman et al., 2021).

In contrast, combining ARIMA with models like GARCH can address volatility, a common challenge in stock price data. This is highlighted in a study using GARCH to forecast volatility and return of the four major banks (Hu et al., 2020). However, optimizing these combined models can be computationally complex (Engle, 2001).

Other statistical models have also shown promise in dynamic environments like the Dhaka Stock Exchange. Kim (2022) explores the ARMA-GARCH model with positive results. Additionally, methods like exponential smoothing have been applied to detect share price movements and predict daily prices (Shahid et al., 2020).

Recent advancements in computational power have enabled the use of novel machine learning techniques for financial forecasting, such as GRUs, LSTMs, and RNNs (Selvin et al., 2017). For example, Selvin et al. (2017) successfully applied neural network models to capture the dynamic behaviour of data for predicting the stock prices of conglomerates listed on the National Stock Exchange of India (NSE). Furthermore, Chen et al. (2015) conducted similar research to predict stock price return on the China Stock Market, also achieving good result. A significant advantage of these models, particularly RNNs, is their ability to handle non-linear relationships within data. Unlike traditional time series models like ARIMA, they do not require the data to be stationary (have constant statistical properties over time) (Hyndman et al, 2021). Deep learning techniques based on RNNs, particularly Long Short-Term Memory (LSTM) networks, have shown promise in stock price forecasting due to their ability to learn and remember long-term dependencies within financial data (Sutskever et al., 2014). Studies like Beniwal et al. (2024) further support this, demonstrating that LSTMs

outperform other deep learning models in predicting the performance of various global stock indices.

Several comparative studies have been conducted to evaluate the performance of different models in stock price prediction. For instance, Siami-Namini et al. (2018) compared the performance of LSTM, a deep learning model, with ARIMA on 23 years of monthly data (1985-2018) for prominent indices like NASDAQ. Their findings suggest that LSTM outperformed ARIMA by a significant margin, with an error reduction rate of 84%-87% as measured by RMSE (Root Mean Squared Error). However, Kobiela et al. (2022) observed contrasting results when applying ARIMA and LSTM to predict NASDAQ stock exchange data. In their study, ARIMA achieved the best performance while using just one feature for modelling and forecasting multiple periods.

These contrasting outcomes highlight the importance of considering several factors when selecting a model for stock price prediction. Notably, the target population and the characteristics of the collected dataset can significantly impact model performance. Additionally, the prediction window, which refers to the timeframe for which forecasts are made, is another crucial factor. For example, Siami-Namini et al. (2018) focused on one-month predictions, whereas Chen et al. (2015) aimed for 30-day forecasts.

## **2.7 Knowledge Gaps and Research Questions**

Having reviewed various time series forecasting models, it becomes evident that while traditional statistical models like ARIMA excel in specific scenarios, they struggle with non-linear relationships and complexities inherent in financial data (Hu et al., 2020). The requirement for stationarity can also lead to information loss through differencing (Hyndman et al., 2021). Deep learning approaches, particularly LSTMs, have demonstrated promising results due to their ability to capture long-term dependencies

and handle non-linearity (Beniwal et al., 2024). However, comparative studies yield conflicting findings, with some highlighting LSTM's superiority over ARIMA (Siami-Namini et al., 2018) and others showcasing ARIMA's edge (Kobiela et al., 2022). These discrepancies can be attributed to variations in target data, prediction horizons (short-term vs. long-term), and feature selection.

Despite the advancements in computational power and novel machine learning techniques, there remains a lack of consensus on the most suitable models for predicting stock prices, particularly for leading AI and ML companies. This ongoing debate underscores the need for a comprehensive assessment that evaluates the suitability of various time series models for predicting trends in leading AI and ML company stock prices (Siami-Namini et al., 2018; Kobiela et al., 2022; Beniwal et al., 2024). While deep learning models hold promise due to their ability to capture complex relationships, a systematic comparison with traditional models is necessary to establish their relative effectiveness in this specific context.

Following the knowledge gaps described above, the main aim of this research is to employ a comprehensive evaluation framework to assess the effectiveness of various time series models, including traditional statistical models and deep learning models, in predicting long-term and seasonal trends in the stock prices of leading AI and ML companies.

Accordingly, the guiding research questions that arose from the literature review are the effectiveness of chosen models in identifying significant trends, their comparative suitability, the observed similarities and differences in trends among the companies, and the accuracy of these models in predicting future trends

## **2.8 Conclusion**



This chapter has comprehensively explored core time series concepts and delved into various time series forecasting models. The discussion highlighted the strengths and limitations of traditional statistical models and deep learning approaches in predicting stock prices. By outlining the knowledge gaps and identifying the research questions, this review has laid the groundwork for the methodological approach to be presented in the next chapter.

## **2. Methodology**

### **2.1. Introduction**

This chapter outlines the methodological approach used to forecast future trends in the stock prices of leading AI/ML companies: Alphabet, Meta, and Microsoft. It details the research process, including research design, approach, strategy, and data collection methods, ensuring the study's replicability and establishing the validity and reliability of its findings.

This research adopts a quantitative approach, focusing on analysing numerical data to generate knowledge and understanding (Burrell et al., 2017). Unlike qualitative research, quantitative research is well-suited for examining historical stock price data and identifying potential trends due to its emphasis on numerical precision and statistical analysis.

A deductive approach is employed, utilising established models like ARIMA and LSTM to validate their effectiveness in stock price prediction (Saunders et al., 2019). Additionally, a case study strategy is used, focusing on historical stock prices to provide in-depth insights (Yin, 2018).

The research relies on secondary data, using existing data collected by other sources. Historical stock price data will be obtained from Yahoo Finance, a reputable source of financial information. Unlike primary data, which can be time-consuming and expensive to collect, and may suffer from limitations in sample size or generalizability, secondary research offers access to extensive datasets that might be difficult or costly to collect independently, justified by the availability and comprehensiveness of the required historical stock price data (Johnston, 2014).

The collected data will be analysed using time series forecasting models, including ARIMA and LSTM. This chapter will provide a detailed explanation of the data collection process, data preprocessing steps, and the analytical techniques used to forecast future stock price trends.

The remainder of the chapter is structured into several key sections. Following this introduction, the literature search and a summary of the literature review is presented. The chapter then discusses the theoretical aspects of primary and secondary data, followed by an explanation of the research instruments. A detailed overview of the secondary data utilised for the research is provided, after which the data analysis section is presented. Ethical considerations are subsequently addressed. Finally, the chapter concludes with a summary of the key methodological aspects of the research.

## **2.2. Literature Search**

This chapter reviewed time series analysis for stock price prediction. We aimed to assess the effectiveness of various models, including traditional statistical and deep learning approaches, and identify their strengths and weaknesses. Research gaps were then pinpointed, and research questions formulated to address them.

Scholarly databases like ScienceDirect and IEEE Xplore were utilised for their collections in finance, computer science, and statistics. Google Scholar, with its extensive search capabilities, served as the primary search engine. Reviewing abstracts, introductions, and searching for keywords (time series analysis, stock price prediction, ARIMA, ARMA-GARCH, deep learning, RNN, LSTM, etc.) helped identify relevant articles (Appendix IV).

The search focused on recent, peer-reviewed journal articles and conference proceedings in English to ensure current and credible findings, incorporating the latest advancements in time series analysis and stock price prediction techniques.

### **2.3. Summary of Literature Review**

The reviewed literature was categorised according to time series concepts, stationarity tests, and time series models (statistical vs. deep learning) employed for stock price prediction. We analysed their strengths and weaknesses in capturing trends and forecasting. Themes emerged regarding the effectiveness of deep learning models (LSTMs) in certain scenarios and the utility of traditional statistical models (ARIMA) in others (Appendix III). This understanding informs our conceptual framework by emphasising that feature selection, prediction windows, and model adjustments are crucial when choosing a model for stock price prediction.

### **2.4. Primary data**

Primary data refers to information gathered directly by researchers through various methods like surveys, interviews, experiments, or observations. It is collected with a specific research question in mind, acting as a customized lens to explore a phenomenon. Unlike secondary data, which might be pre-existing or not perfectly aligned with the study's needs, primary data offers the distinct advantage of being tailored to the researcher's specific inquiry. This direct approach allows researchers to design and control the data-gathering process, ensuring its relevance in relation to the research objectives (Creswell, 2023).

Collecting primary data becomes necessary when existing information is scarce, outdated, or does not quite match with the research questions. By gathering primary data, researchers can directly test specific hypotheses and generate unique insights that might not be possible with secondary data alone (Fisher, 2010). This approach is particularly valuable in exploratory research, where understanding new or emerging

trends necessitates fresh data that captures the current context's nuances (Bell, 2022), which is not the main aim of the present study.

## **2.5. Secondary data**

The primary objective of utilising secondary data in research is to leverage existing information to address new research questions or validate findings without the need for primary data collection. This approach is particularly valuable in financial forecasting, where extensive historical datasets are often required. According to Bell et al. (2022), secondary data can provide a rich source of information that may otherwise be inaccessible due to constraints in time, cost, or logistical feasibility. By utilising secondary data, researchers can capitalise on previously gathered information to conduct comprehensive analyses, thereby enhancing the robustness and validity of their findings.

Secondary data can be classified into qualitative and quantitative types. Qualitative data is descriptive and often involves thematic analysis to uncover patterns and insights from non-numeric information such as interviews, texts, or visual media (Creswell et al., 2022). In contrast, quantitative data is numerical and is analysed using statistical methods to identify trends, correlations, or causal relationships. This research employs quantitative secondary data, specifically historical stock price data, as it aligns with the objective of forecasting future stock price trends of AI and ML companies. Quantitative data is advantageous for its precision and the ability to apply mathematical models to predict future outcomes (Fisher et al., 2010). The table below highlights the general strengths and limitations of using secondary data in research (Bell et al., 2022; Johnston, 2014).

Advantages	Disadvantages
Provides access to extensive datasets, facilitating comprehensive analysis.	May lack specific data tailored to the research needs.
Saves time and reduces costs compared to primary data collection.	Limited control over data collection process and potential biases inherent in original data
Enhances robustness and validity by leveraging previously gathered information	Potential issues with data completeness or gaps in historical data
Suitable for financial forecasting, where historical data is essential	Secondary data may be outdated or not align perfectly with the research timeline

*Table 1: Advantageous and disadvantageous of secondary data*

The type of secondary data utilised in this research includes historical stock price data. This data is sourced from Yahoo Finance. This platform provides extensive historical datasets for publicly traded companies, ensuring the reliability and accuracy required for financial forecasting. The selection of Yahoo Finance as the data source is justified by its extensive coverage and the granularity of the data it offers.

## **2.6. Research instrument design**

This study's research instrument aimed to collect historical stock price data for leading AI/ML companies (Alphabet, Meta Platforms, Microsoft). This data is essential for analysing and forecasting future stock prices.

Key design considerations included:

- **Relevance:** Ensuring data directly supports price analysis and forecasting.

- Accuracy: Selecting Yahoo Finance for its established reputation and extensive datasets.

The instrument focused on historical adjusted closing prices as numerical values for a defined date range. These variables were chosen for their relevance to financial forecasting and their impact on time series model accuracy.

Data was recorded in a standardised CSV format, ensuring consistency for analysis. Adjusted closing prices and dates were logged numerically and consistently.

## **2.7. Overview of secondary data used for analysis**

The dataset spans from 1st January 2015 to the end of May 2024, encompassing approximately nine years and five months, which translates to 2,368 working days. For the purposes of model evaluation and analysis, data from the beginning of the period until the end of February 2024 was utilised, resulting in 2,305 records. The remaining 63 working days, from 1st March 2024 to the end of May 2024, were reserved for validating the forecasting models.

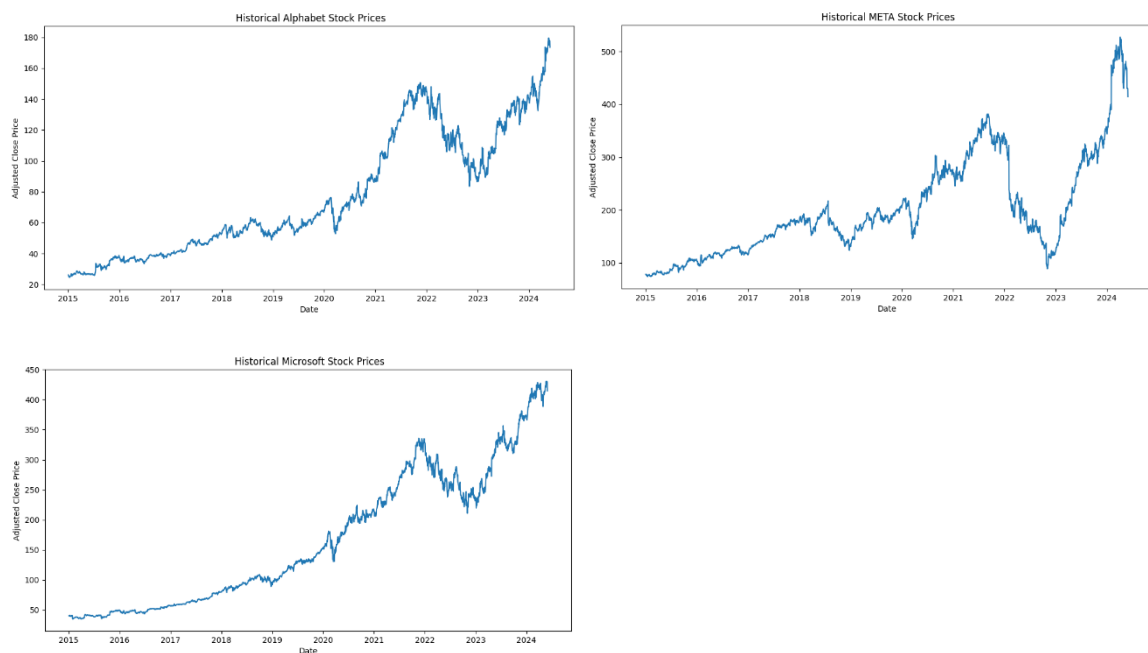
The primary variables extracted from the dataset are the date and the adjusted close price. The adjusted close price is chosen due to its adjustment for corporate actions, providing a more accurate reflection of the stock's value over time.

The data collection process was straightforward: the historical stock prices were downloaded directly from Yahoo Finance on the 1st of June 2024, following the last business day of May. This ensures that the dataset is up-to-date.

The data collection period begins in 2015, a strategic choice reflecting the significant developments in AI and deep learning during that time. Around 2015, Google's advancements in deep learning began to gain substantial attention, marking a period of

rapid innovation in the field (LeCun et al., 2015). Similarly, Facebook intensified its efforts in AI research, establishing the Facebook AI Research (FAIR) lab in 2013 and achieving notable progress by 2015 (BBC, 2015). This period is therefore pivotal for analysing the stock performance of companies heavily invested in AI and ML.

To provide a comprehensive understanding of the dataset, the following figures illustrate the trends in the adjusted close prices for Alphabet, Meta Platforms, and Microsoft from January 2015 to May 2024:



*Figure 10: Historical stock prices of Alphabet, META, and Microsoft*

## **2.8. Data analysis**

The data analysis process for this research involves several meticulous steps, each designed to ensure the robustness and accuracy of the findings. These steps are applied independently to the stock prices of Alphabet, Meta, and Microsoft.





Figure 11: Research Methodology

- **Data Preprocessing and Outlier Detection**

The initial stage focuses on data preprocessing and outlier detection. This stage ensures the quality and suitability of the data for subsequent analysis. Data is inspected for missing values and anomalies using descriptive statistics and visual inspection (Aggarwal, 2015). To detect outliers, statistical distribution methods were employed. This approach leverages the best-fitted distribution, according to statistical analysis to define data points deviating from the norm (Aggarwal, 2017).

- **Exploratory Data Analysis (EDA)**

During the EDA phase, we examine potential patterns and trends within the dataset. This includes preliminary tests using the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots to detect any inherent patterns (Hyndman et al., 2021; Brockwell, 2016; Dégerine et al., 2003). Stationarity tests, specifically the Augmented Dickey-Fuller (ADF) test (Dickey et al., 1979) and the Ljung-Box test (Ljung et al., 1978), are conducted to assess whether the time series data is stationary. If non-stationary, differencing is applied to achieve stationarity. After that, further analysis using ACF, PACF, and the Ljung-Box test is conducted to determine any seasonality within the dataset.

Finally, a simple moving average is overlaid on the original data to smooth out volatility and identify underlying trends. This helps in assessing trends, seasonality

and other temporal dependencies in the data (Box et al., 2015). Determining the presence of seasonal patterns is vital for deciding whether to use seasonal models for forecasting.

- **Data Splitting**

To train and evaluate models, datasets are commonly divided into training and testing subsets, usually in an 80:20 proportion. This method ensures that a significant portion of the data is used for training, while a separate part is kept for testing the models' predictive performance. This division facilitates a thorough assessment of how well the models perform on new, unseen data (Goodfellow et al., 2016).

- **Model Implementation**

To evaluate the performance of advanced forecasting models, simple and exponential moving averages are utilised as benchmark models. These baselines serve as reference points, allowing for the comparison of how effective the more complex models are in predicting outcomes (Makridakis et al., 2008).

Both statistical and deep learning models are employed for forecasting. These include:

- Statistical Models: Exponential Smoothing, ARIMA, and ARIMA-GARCH.
- Deep Learning Models: Recurrent Neural Networks (RNN), Long Short-Term Memory (LSTM), and Gated Recurrent Units (GRU).

The accuracy and reliability of each model are assessed through training and evaluation using metrics such as Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). These metrics serve as benchmarks to gauge how well the models perform in forecasting tasks (Chatfield et al., 2018).

To enhance model performance, a tuning algorithm is implemented to optimize the parameters of models. This step involves iterative adjustments to find the best configurations that minimize forecasting errors (Box et al., 2015).

- **Forecasting and Validation**

The best-performing models from each category (statistical and deep learning) are selected for mid-term future forecasting. Various time windows are analysed to assess the performance of the deep learning models, ensuring a comprehensive evaluation of their predictive capabilities (Wang et al., 2018).

The final step involves comparing the forecasted values against the actual, unseen values. This comparison, based on RMSE and MAE metrics, identifies the model with the best prediction accuracy. The results are then used to draw conclusions about the most effective model for stock price forecasting (Montgomery et al., 2015).

## **2.9. Ethics**

This research adheres to rigorous ethical guidelines to ensure its integrity and ethical conduct. As the study utilises secondary data, specifically historical stock prices from Yahoo Finance, there is no direct interaction with human participants. This minimises ethical risks associated with privacy and consent. However, it remains crucial to acknowledge the data source accurately and ensure its appropriate use within the research scope (Bell et al., 2022; Fisher, 2010). All analyses and interpretations are conducted with transparency and objectivity, upholding academic integrity and avoiding any data manipulation or misrepresentation.

## **2.10. Summary**

This chapter outlines the methodology for forecasting stock prices of Alphabet, Meta, and Microsoft. It details a quantitative approach using established models (ARIMA, LSTM) and a case study strategy with historical data (2015-May 2024) from Yahoo Finance. The analysis involves data preprocessing, exploratory data analysis, model implementation and finally forecasting and validation. Ethical considerations are addressed through an Ethos application (details in appendices). The next chapter dives into data analysis for insights and conclusions.

### **3. Data analysis and discussion**

#### **4.1 Introduction**

This chapter details the systematic approach taken to analyse the stock price data for Alphabet, Meta, and Microsoft. The analysis begins with data preprocessing and outlier detection, followed by exploratory data analysis to reveal inherent patterns. ADF and Ljung-Box tests assess data stationarity. ACF and PACF plots identify patterns, and simple moving averages smooth volatility to expose trends. The data, covering adjusted closing prices from January 1st, 2015, to April 30th, 2024, is then split for training and testing models.

Forecasting models, including simple and exponential moving averages as baselines, and advanced models (Exponential Smoothing, ARIMA, ARIMA-GARCH, RNN, LSTM, GRU), are implemented in Python (Appendix III). Model accuracy is evaluated using RMSE and MAE metrics, with parameter tuning to improve performance.

Finally, mid-term forecasting is conducted from March to May 2024, comparing forecasted values to actual prices to identify the most effective model for each company. These results guide conclusions on the best models for this research context.

#### **4.2 Data Preprocessing and Outlier Detection**

##### **4.2.1 Data preprocessing**

The data preprocessing stage plays an essential role in ensuring the quality of the data.

This phase involved a series of checks to confirm data integrity.

- **Missing Values**

An inspection of the data revealed no missing values (null or empty entries) within the adjusted closing stock price data for all three companies from January 1st, 2015, to April 30th, 2024.

- **Descriptive Statistics**

Descriptive statistics were calculated for each company's stock price data using excel. These initial observations did not reveal any significant anomalies or inconsistencies within the data.

Statistics	Alphabet	Meta	Microsoft
Mean	74.87707653	194.199443	160.5447804
Standard Error	0.780019228	1.704249905	2.184334189
Median	60.275002	177.761383	130.450317
Mode	26.834499	137.274353	56.675861
Standard Deviation	37.44904725	81.82174611	104.8707921
Sample Variance	1402.43114	6694.798137	10997.88303
Kurtosis	-1.057007205	0.125018762	-1.075224714
Skewness	0.5628959	0.80461222	0.486417161
Range	130.279926	416.158486	384.194637
Minimum	24.56007	73.971519	34.823277
Maximum	154.839996	490.130005	419.017914
Sum	172591.6614	447629.7161	370055.7188
Count	2305	2305	2305

*Table 2: Descriptive Statistics for stock prices of Alphabet, Meta, and Microsoft*

- **Date Inspection**

A visual inspection of the date data confirmed no inconsistencies or unexpected entries. This ensures the temporal integrity of the time series data.

## 4.2.2 Outlier Detection

Outliers were detected using the Python Fitter library, which evaluates over 16 statistical distributions (e.g., Normal, Exponential, Weibull\_Min) to find the best fit. Data points outside the threshold of the best-fit distribution were identified as outliers (Appendix I). Below are diagrams and results showing the best-fitting distributions, their statistics, and outliers for each stock:

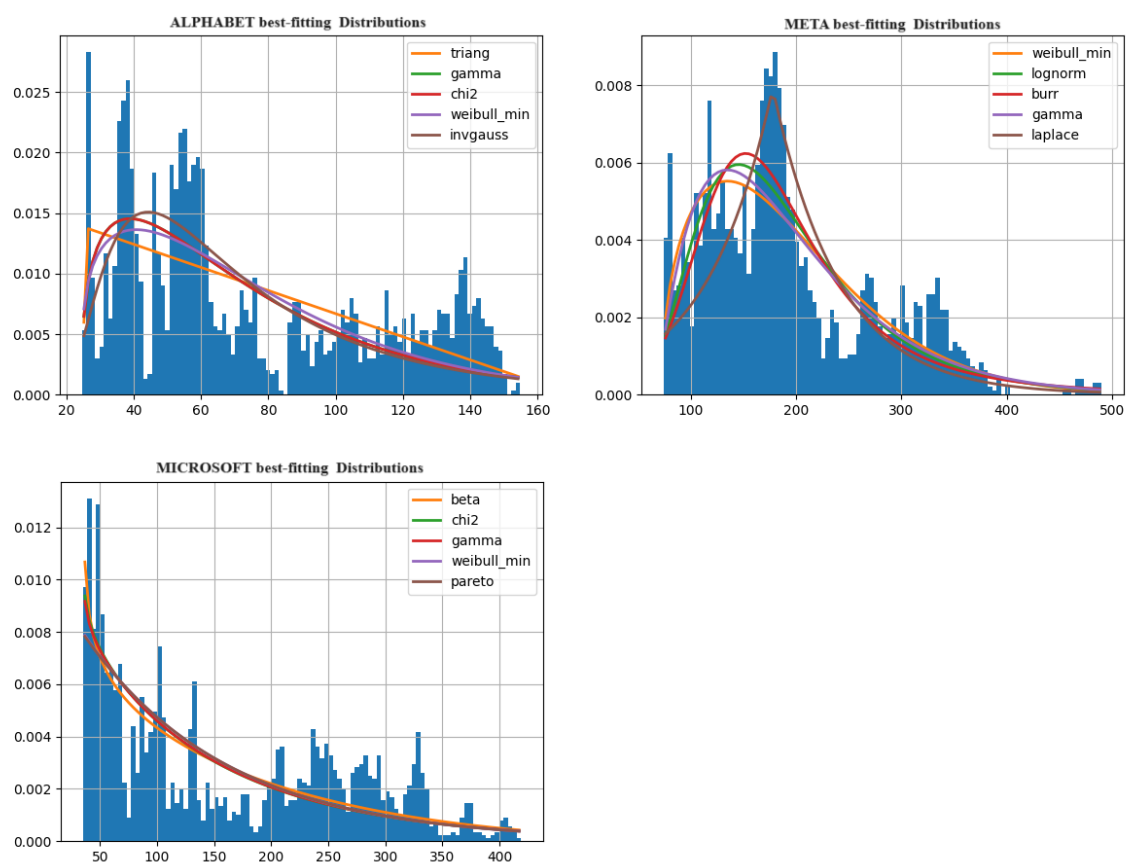


Figure 12: Best-fitting Distributions Plot of Alphabet, Meta, and Microsoft

Statistics	Alphabet	Meta	Microsoft
Best-Fitting Distribution	Triangular	Weibull_Min	Beta

Distribution Parameters	'c': 0.013965398369329092, 'loc': 24.325991499760093, 'scale': 145.6463625529612	'c': 1.476317889619243, 'loc': 72.3352781557903, 'scale': 34.36230849716202	'a': 0.8267846118714879, 'b': 4.442673066892139, 'loc': 34.823276999999999, 'scale': 782.2503021914882
Threshold Percentile	0.95	0.95	0.95
No of outliers	192	79	30
Min outliers	137.653503	355.073273	392.43512
Max outliers	154.839996	490.130005	419.017914

Table 3: Best-fitting distribution and outlier statistics for Alphabet, Meta, and Microsoft stocks

Upon inspection, Alphabet's outliers correspond to sharp growth from mid-2021 to early 2022 and from late 2023 onwards. Meta's outliers appear in Q3 2021 and late 2023 onwards. Microsoft's outliers are noticeable from early 2024.

Despite this, retaining outliers is justified due to the following factors:

- **Significant Events**

All three stocks showed significant growth with peaks in 2021-2022 and late 2023, likely influenced by major events. Retaining these outliers captures the market's behaviour accurately.

- **Market Volatility**

Stock prices are inherently volatile, and these fluctuations are essential for understanding stock behaviour.

- **Robust Modelling**

The models used are robust and can effectively handle the volatility introduced by outliers, capturing extreme values reliably.



Retaining outliers and using robust forecasting models provide a comprehensive view of stock prices, accounting for both routine fluctuations and significant market events.

### **4.3 Exploratory Data Analysis (EDA)**

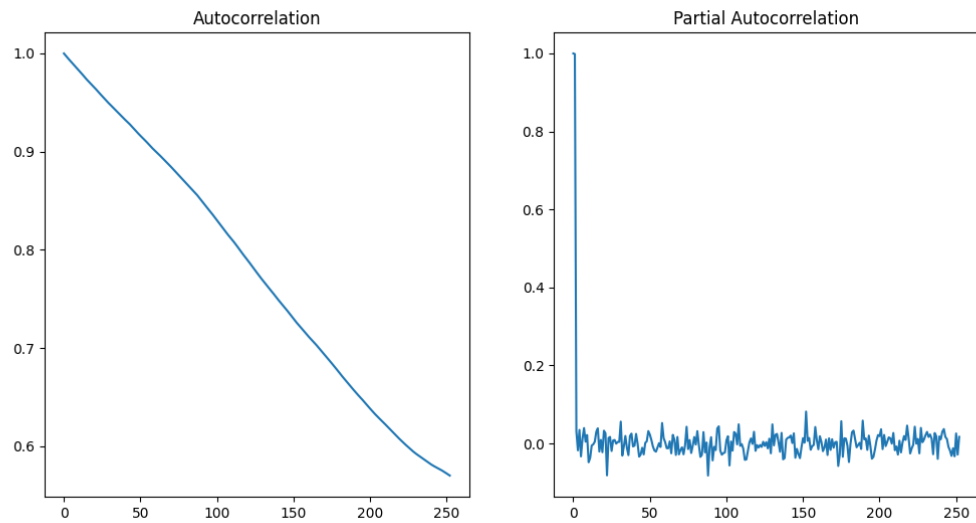
The exploratory data analysis (EDA) phase examined the data of all three stocks to identify patterns such as seasonality and trends. This was achieved using the Augmented Dickey-Fuller (ADF) and Ljung-Box tests, as well as ACF and PACF plots. To conduct these tests, we utilised Python libraries including ‘statsmodels.tsa.stattools’, ‘statsmodels.stats.diagnostic’, and ‘matplotlib.pyplot’. Finally, visual inspection of the original trends overlaid with a simple moving average confirmed the presence of patterns.

#### **4.3.1 EDA for Alphabet**

The ADF test results (statistic: -0.2321, p-value: 0.9346) indicated that the series was non-stationary, as the p-value was significantly higher than the usual significance level of 0.05.

The Ljung-Box test was performed to assess the presence of autocorrelation at various lags. The results consistently showed significant p-values ( $p < 0.05$ ) across multiple lags, suggesting that the data exhibited autocorrelation, further confirming non-stationarity.

To further validate these findings, ACF and PACF plots were examined. Our aim was to identify spikes at regular intervals. Based on our daily data, the lags were considered equal to 252.



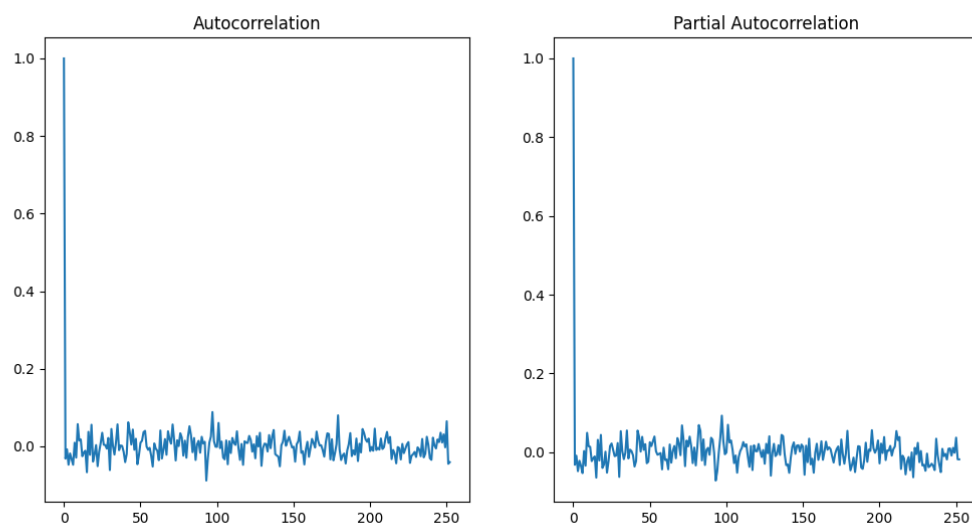
*Figure 13: Original ACF and PACF plots of Alphabet*

The ACF plot showed a slow decay, indicating non-stationarity and a strong long-range autocorrelation, but no clear seasonality. The PACF plot had a strong initial spike at lag 1, suggesting strong short-term correlation, with most subsequent lags falling within significance bounds, implying a weakened relationship after the first lag. Overall, no clear signs of seasonality were evident in the ACF and PACF plots.

Given these findings, differencing was applied to the data to unmask presence of seasonality. Subsequent analyses with ADF and Ljung-Box tests, ACF and PACF plots were conducted to identify any seasonality within the dataset.

The ADF test results indicated that after differencing, the time series was now stationary. The ADF statistic was significantly negative (-11.7524), and the p-value was extremely low (1.2003e-21), much lower than the usual significance level of 0.05. This confirmed that the null hypothesis of non-stationarity could be rejected, indicating that the differenced series was stationary.

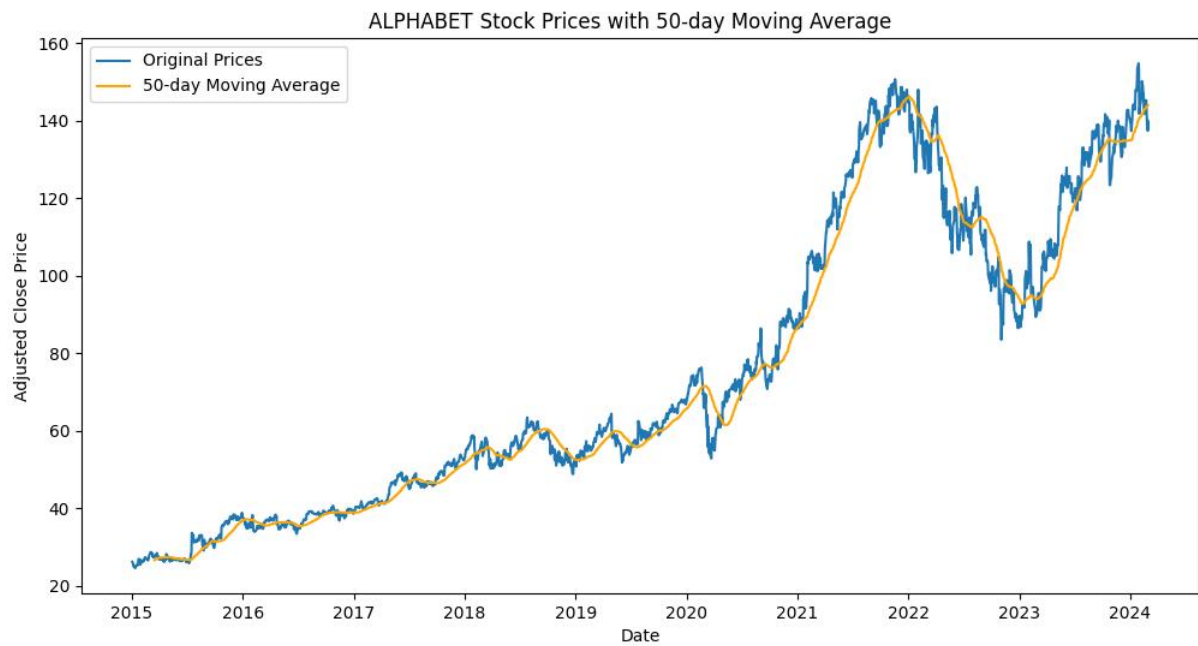
In the Ljung-Box test statistics, lower lags showed p-values higher than the significance level (0.05), indicating no significant autocorrelation at these lags. However, as the lags increased, the p-values became extremely low ( $p < 0.05$ ), indicating significant autocorrelation at these higher lags (Appendix II). This suggested that the differenced series still retained some long-term dependencies. Further analysis with ACF and PACF could fully address these dependencies.



*Figure 14: ACF and PACF plots of Alphabet after first-order differencing*

The ACF showed a strong initial spike at lag 1 due to first differencing, then dropped to near zero, indicating white noise. The PACF also showed no significant partial autocorrelation beyond the first lag. The ADF and Ljung-Box tests, along with the ACF and PACF plots, suggested that the series was now stationary. There were no visible periodic spikes or regular patterns, indicating a lack of strong seasonality in the Alphabet data.

Since there was no detectable seasonality, we focused on identifying trends using the original stock price overlaid with a simple moving average of 50 days (selected based on the best metric results of RMSE and MAE).

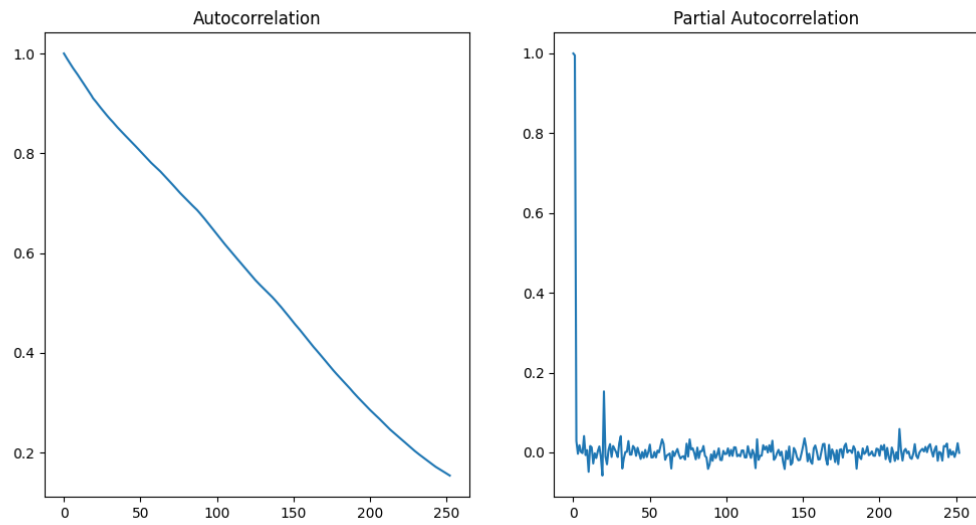


*Figure 15: Alphabet original trends overlaid by 50-day simple moving average*

Visual inspection of original and 50-day moving average trends revealed a clear upward trend in Alphabet's stock price from 2015 to 2024. This long-term trend indicates that the stock price has generally increased over this period.

#### **4.3.2 EDA for Meta**

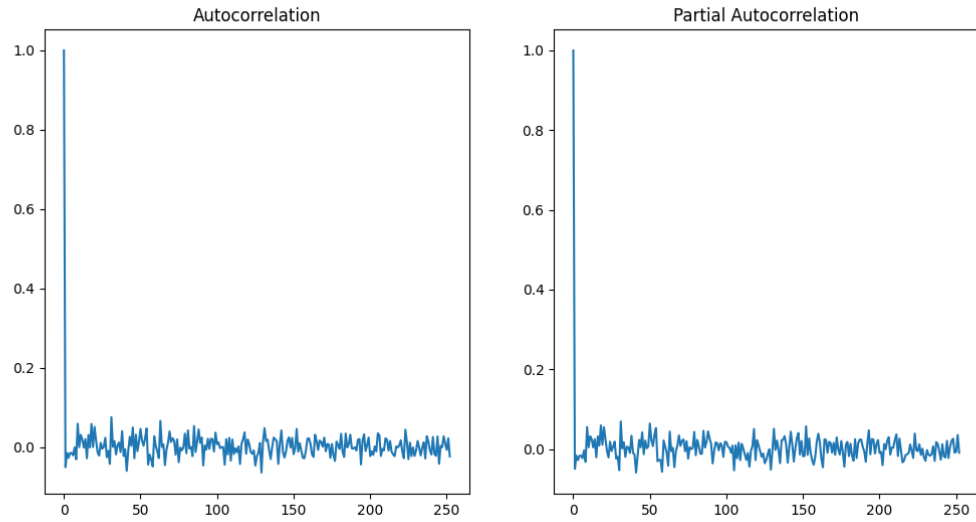
The ADF test (statistic: 0.0463, p-value: 0.9622) indicated that the series was non-stationary (a high p-value suggests this). The Ljung-Box test confirmed this non-stationarity, as it revealed statistically significant autocorrelation (p-values less than 0.05) across multiple lags. To strengthen the evidence for non-stationarity, ACF and PACF plots were examined.



*Figure 16: Original ACF and PACF plots of META*

META's plots suggest characteristics of non-stationarity. The ACF plot exhibits strong autocorrelation across most lags, indicating no recurring patterns that might suggest strong seasonality. Similarly, the PACF plot shows a pronounced spike at lag one, with most other lags close to zero, although a few outliers exist. Overall, there's no clear evidence of seasonality.

Based on these findings, first-order differencing was performed. The ADF test results (statistic: -8.6973, p-value: 3.8947e-14) strongly suggest that the data is now stationary. While the Ljung-Box test confirms this, it revealed some remaining autocorrelation at higher lags (Appendix II). To investigate these remaining dependencies in the differenced data, ACF and PACF plots were examined.



*Figure 17: ACF and PACF plots of META after first-order differencing*

The ACF and PACF plots confirmed stationarity, with the ACF showing a large initial spike due to differencing and subsequent values near zero and the PACF showing no significant autocorrelations beyond the first lag. No clear seasonal patterns were observed in the META data, and the lack of regular spikes or significant autocorrelation at specific lags further supports this. We therefore focused on trend detection using the original price data with a 210-day moving average (chosen based on RMSE and MAE metrics).

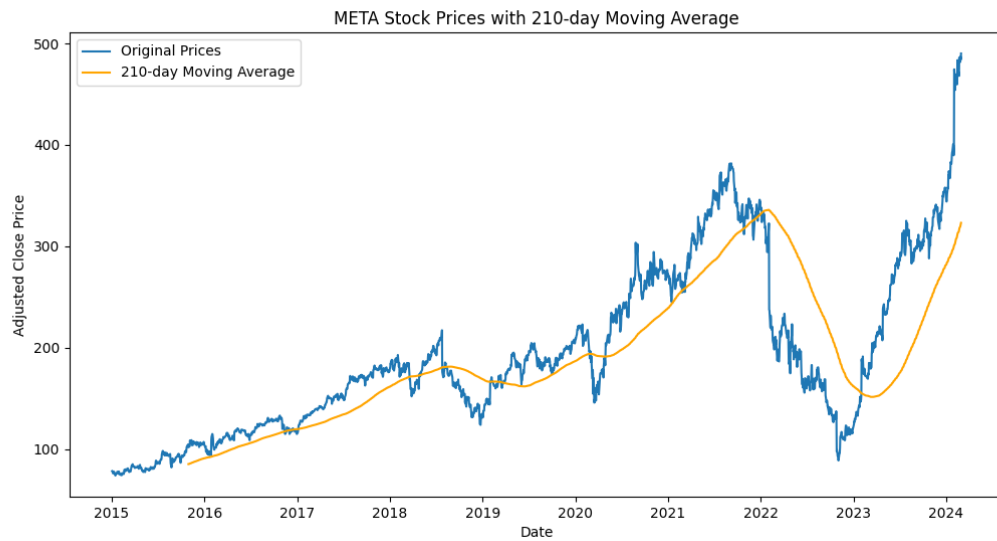
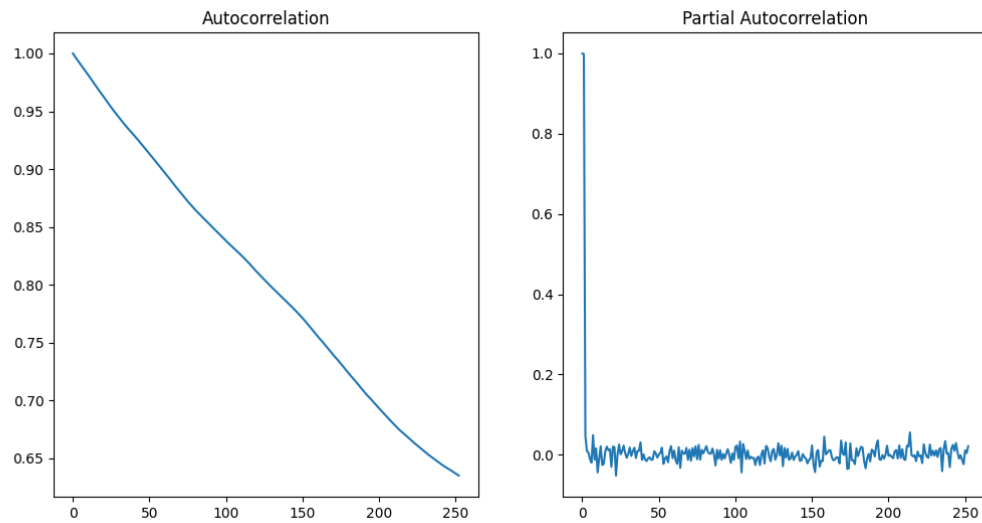


Figure 18: Alphabet original trends overlaid by 210-day simple moving average

Despite a sharp price drop in 2022, META's stock exhibits a clear upward trend from 2015 to 2024, confirmed by both the original price and the 210-day moving average. This long-term trend reflects a general increase over this period.

### 4.3.3 EDA for Microsoft

Analysis of stationarity in the data suggests it's likely non-stationary. The ADF test statistic (1.1992) with a high p-value (0.9959) indicates this. Furthermore, the Ljung-Box test results show statistically significant autocorrelation (p-values of 0 across all lags), reinforcing the evidence for non-stationarity. Visual inspection of ACF and PACF plots can further support the evidence for non-stationarity.

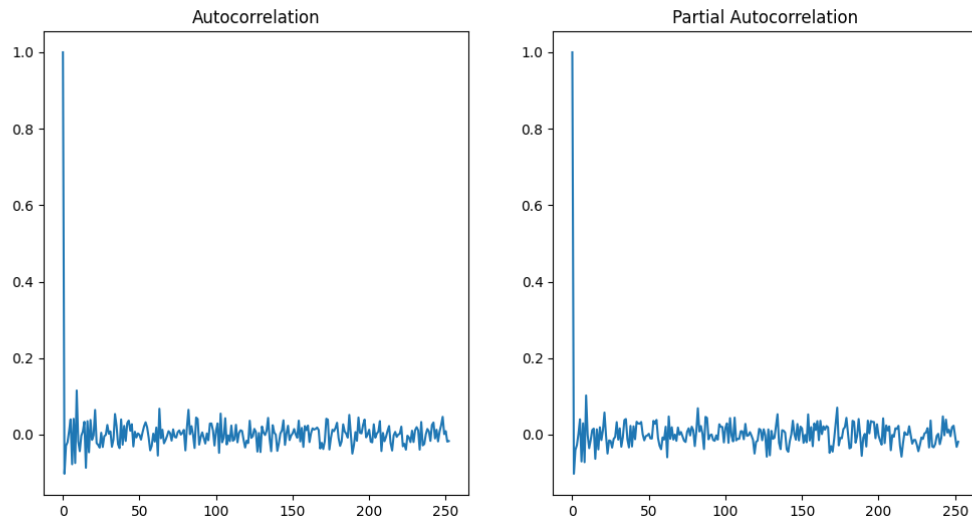


*Figure 19: Original ACF and PACF plots of Microsoft*

Examination of Microsoft's data reveals characteristics consistent with non-stationarity. The ACF plot exhibits strong autocorrelation across most lags, suggesting an absence of recurring patterns that would typically indicate seasonality. This observation is further supported by the PACF plot, which shows a pronounced spike at lag one but minimal autocorrelation at other lags, with a few exceptions.

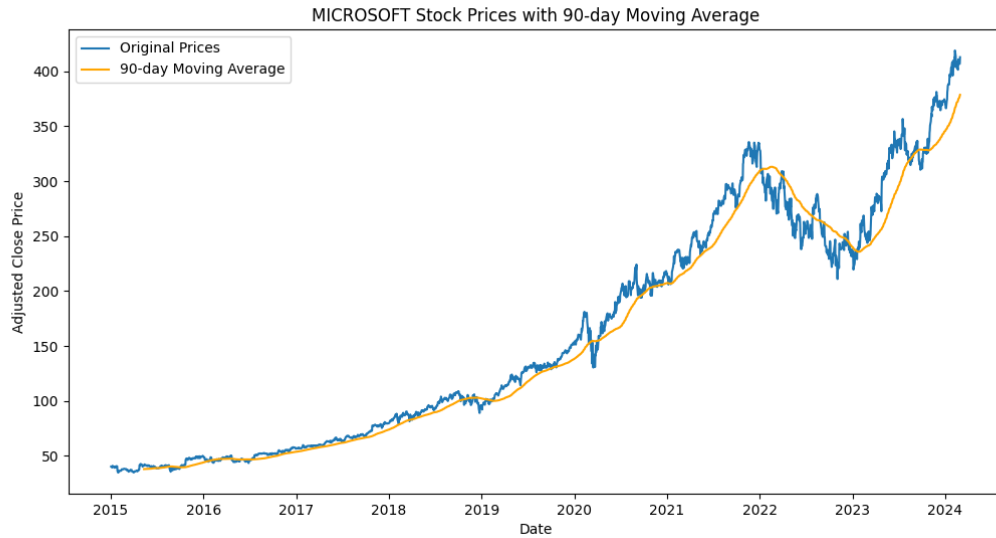
The ADF test statistic of -10.39 and a highly significant p-value ( $1.9279\text{e-}18$ ) strongly suggest the data is now stationary after first-order differencing. This is contradictory to the Ljung-Box test results which shows remaining autocorrelation (Appendix II). To further investigate potential seasonality, we examined the ACF and PACF plots.





*Figure 20: ACF and PACF plots of Microsoft after first-order differencing*

The ACF and PACF plots suggest that the first-order differenced series for Microsoft is stationary, as indicated by the initial spike at lag 1 followed by autocorrelations near zero. This implies that the data no longer exhibits significant autocorrelation or clear seasonal patterns. Consequently, we focused on trend detection using the original price data with a 90-day moving average (selected based on RMSE and MAE metrics).



*Figure 21: Microsoft original trends overlaid by 90-day simple moving average*

Examining the period from 2015 to 2024, Microsoft's stock price reveals a clear upward trajectory, despite a price dip in 2022. This long-term trend is evident in both the original price data and the 90-day moving average, indicating a general increase over time.

#### **4.4 Data Splitting**

To train and evaluate the performance of our forecasting models, we adopted an 80/20 train-test split. This means that 80% of the data is utilised for training the models, and the remaining 20% is reserved for testing and evaluating their performance.

The training data comprises 1,844 daily closing prices for each stock, spanning the period from 1st January 2015 to 28th April 2022. The test data encompasses daily closing prices from 29th April 2022 to 29th February 2024.

#### **4.5 Model Implementation**

This stage involves fitting a series of models to the data to determine which one best captures the trends in stock prices. First, we fit two simple statistical models as

baselines: Simple Moving Average (SMA) and Exponential Moving Average (EMA). These baselines are implemented using a naive approach. It means the last MA value from the training period is then used as a constant forecast for the entire test period. This naive approach assumes that the future stock prices will remain at the level of the last SMA or EMA value calculated from the training set.

Following the baseline models, we fit a range of more advanced models:

- Statistical Models:
  - Exponential Smoothing
  - ARIMA (Autoregressive Integrated Moving Average)
  - ARIMA-GARCH (combines ARIMA with GARCH for volatility modelling)
- Machine Learning Models:
  - Recurrent Neural Network (RNN)
  - Long Short-Term Memory (LSTM)
  - Gated Recurrent Unit (GRU)

### **Model Selection Considerations**

Our decisions regarding model selection are informed by the findings of the exploratory data analysis (EDA) stage. Notably, the absence of significant seasonality allows us to set the seasonality parameter to "false" during model training. Conversely, the observed presence of volatility in the stock prices, evidenced by the results of distribution fitting and identified outliers, justifies the inclusion of models equipped to handle this phenomenon, such as ARIMA-GARCH.

### **Model Tuning**

- Statistical Models

For these models, manual parameter tuning will be conducted across various scenarios to identify the configuration with the lowest Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). This process will involve trying different combinations of parameters and selecting the one that produces the best results.

- Machine Learning Models

To optimize these models, we will leverage an automated hyperparameter tuning function with 50 iterations. This function will automatically explore different combinations of hyperparameters and select the best set that minimizes the error metrics (MAE and RMSE).

### Python Libraries Used

We will utilise the following Python libraries to implement and forecast the models:

Main Library	Imported Class
'statsmodels.tsa.holtwinters'	'ExponentialSmoothing'
'statsmodels.tsa.arima.model'	'ARIMA'
'arch'	'arch_model'
'tensorflow.keras.models'	'Sequential'
'tensorflow.keras.layers'	'SimpleRNN', 'Dense', 'Dropout', 'LSTM', 'GRU'
'sklearn.preprocessing'	'MinMaxScaler'
'sklearn.model_selection'	'ParameterSampler'
'sklearn.metrics'	'mean_squared_error', 'mean_absolute_error'
'scipy.stats'	'randint', 'uniform'

Table 4: Python Libraries utilised for model implementation

### 4.5.1 Alphabet Stock Price Forecast

Before presenting the comparative results of the forecasting models, we first outline the hyperparameters obtained through the tuning process.

Forecasting Model	Tuned Parameters
SMA	50-day moving average
EMA	50-day moving average
Exponential Smoothing	Seasonality = False
ARIMA	order (2, 2, 2)
ARIMA-GARCH	ARIMA(2,2,2)-GARCH(2,2)
RNN	{'batch_size': 15, 'dropout': 0.25171866634056334, 'epochs': 48, 'learning_rate': 0.0030474457706320727, 'look_back': 1, 'return_sequences': False, 'units': 88}
LSTM	{'batch_size': 10, 'dropout': 0.1311217605210499, 'epochs': 35, 'learning_rate': 0.009105956549136991, 'look_back': 1, 'return_sequences': False, 'units': 48}
GRU	{'batch_size': 24, 'dropout': 0.0010595028648908156, 'epochs': 91, 'learning_rate': 0.0065272063812476955, 'look_back': 4, 'return_sequences': False, 'units': 34}

Table 5: Tuned Parameters of Forecasting Models for Alphabet Stock Price Prediction

The corresponding forecast charts are presented below:



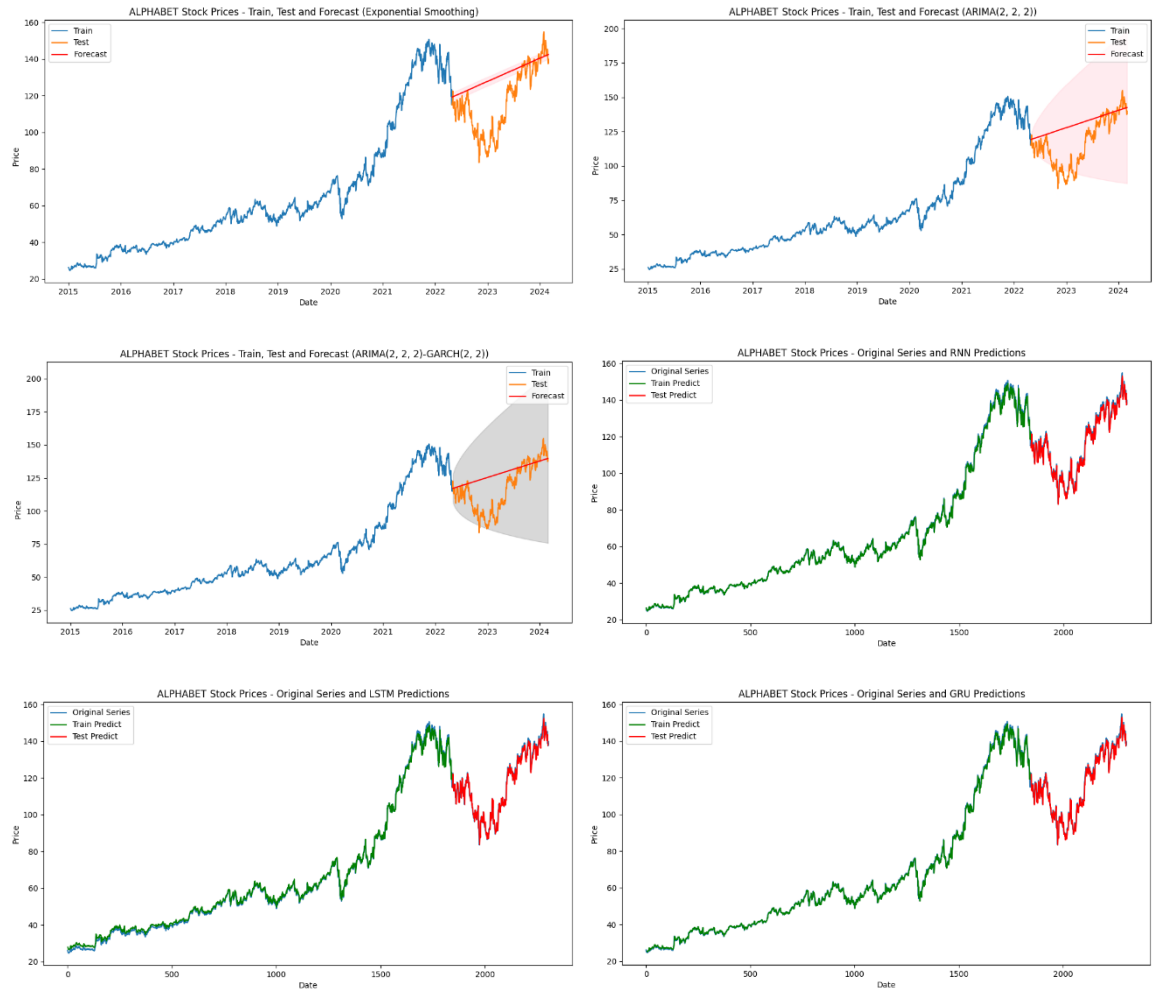


Figure 22: Alphabet Baseline and Forecasting Models' Charts

Based on the best-tuned hyperparameters, the following table presents a comparison of the forecasts for Alphabet stock prices:

Models	RMSE	MAE
SMA	23.3930	19.3182
EMA	22.9918	19.0076
Exponential Smoothing	19.3459	14.8952
ARIMA	19.3427	14.8919
ARIMA-GARCH	17.7210	13.4031
RNN	2.6674	2.0425

LSTM	2.5260	1.9194
GRU	2.5221	1.9002

Table 6: Comparative Results of Forecasting Models and Performance Metrics for Alphabet

### Interpretation of the Results

Among the statistical models, ARIMA-GARCH outperforms the SMA and EMA baseline models in forecasting Alphabet's stock price, as evidenced by its lower RMSE and MAE. The ARIMA-GARCH model also surpasses ARIMA due to its superior ability to capture volatility and temporal dynamics in the data.

However, advanced recurrent neural network models demonstrate substantially lower RMSE and MAE compared to statistical models, indicating higher accuracy in predicting the time series. This suggests that these models are better suited for capturing complex, non-linear relationships in the data.

Among these advanced models, the RNN achieved the lowest validation loss (5.9612 compared to 6.3194 for LSTM and 6.0293 for GRU). Nevertheless, the GRU model shows the most promising results in terms of RMSE and MAE, outperforming the baseline models and offering the best overall performance by effectively balancing complexity and predictive power.

#### 4.5.2 META Stock Price Forecast

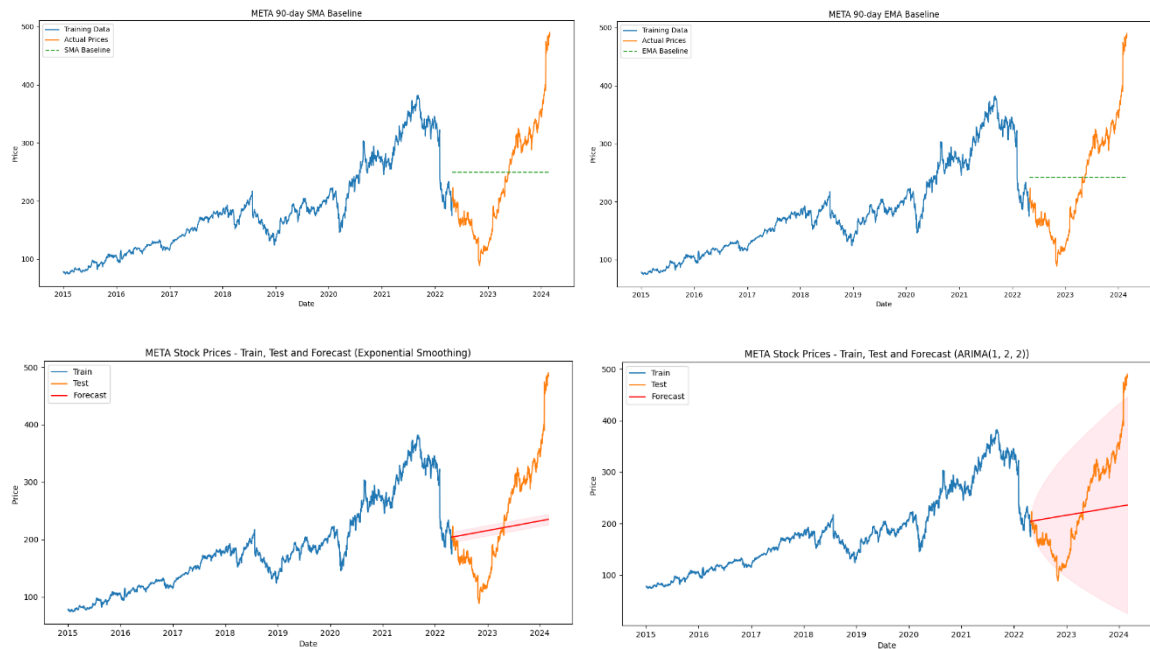
we first outline the hyperparameters obtained through the tuning process.

Forecasting Model	Tuned Parameters
SMA	90-day moving average
EMA	90-day moving average
Exponential Smoothing	Seasonality = False

ARIMA	order (1, 2, 2)
ARIMA-GARCH	ARIMA(1,2,2)-GARCH(1,1)
RNN	{'batch_size': 16, 'dropout': 0.05467804803565551, 'epochs': 23, 'learning_rate': 0.007659192886149137, 'look_back': 3, 'return_sequences': False, 'units': 38}
LSTM	{'batch_size': 19, 'dropout': 0.08034928178417022, 'epochs': 14, 'learning_rate': 0.00596315278232027, 'look_back': 1, 'return_sequences': False, 'units': 98}
GRU	{'batch_size': 24, 'dropout': 0.3997474663039183, 'epochs': 46, 'learning_rate': 0.010744806220539603, 'look_back': 5, 'return_sequences': False, 'units': 33}

Table 7: Tuned Parameters of Forecasting Models for META Stock Price Prediction

The corresponding forecast charts are presented below:





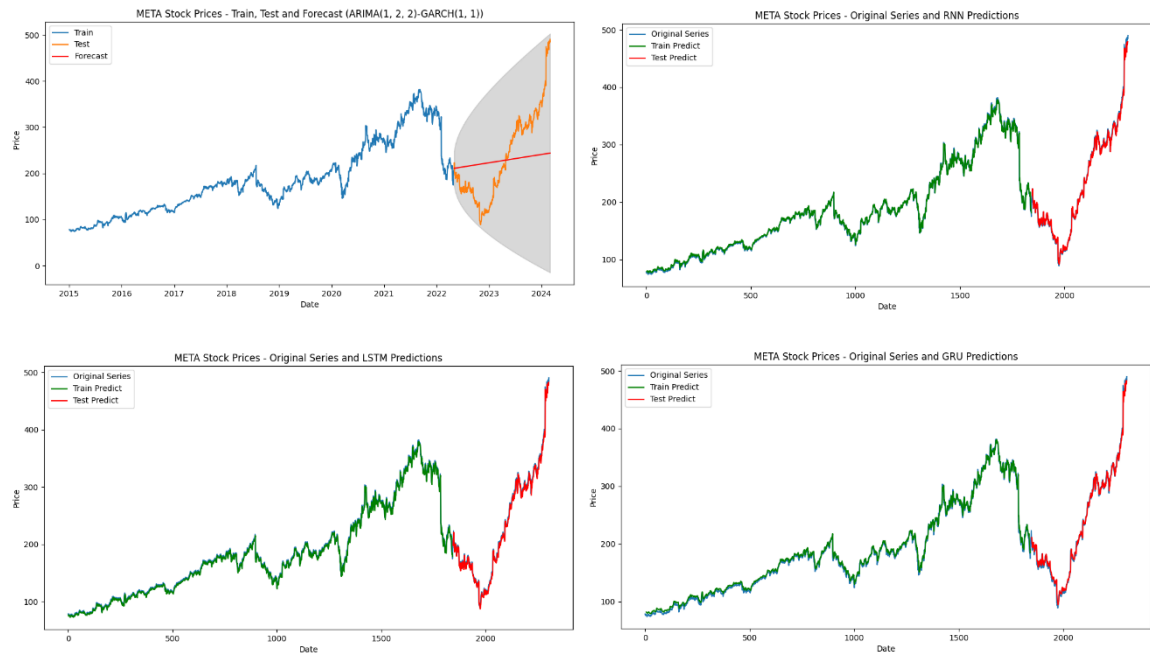


Figure 23: META Baseline and Forecasting Models' Charts

Based on the best-tuned hyperparameters, the following table presents a comparison of the forecasts for META stock prices:

Models	RMSE	MAE
SMA	94.7119	82.5677
EMA	93.7591	81.3003
Exponential Smoothing	87.3022	71.7266
ARIMA	87.0224	71.6127
ARIMA-GARCH	86.2684	72.3196
RNN	7.3929	4.7165
LSTM	7.5807	5.0083
GRU	7.3961	4.9242

Table 8: Comparative Results of Forecasting Models and Performance Metrics for META

## Interpretation of the Results

Among the statistical models, ARIMA-GARCH demonstrates the lowest RMSE and MAE compared to the baseline and other statistical models in forecasting META's stock price. However, advanced recurrent neural network models exhibit substantially lower RMSE and MAE compared to statistical models.

Among these neural network models, the RNN presented the lowest validation loss (48.8468 compared to 48.9812 for LSTM and 51.6435 for GRU), as well as the lowest RMSE and MAE, followed closely by the GRU. This indicates that RNN is the most effective model for predicting META's stock price, with GRU also performing well.

### 4.5.3 Microsoft Stock Price Forecast

we first outline the hyperparameters obtained through the tuning process.

Forecasting Model	Tuned Parameters
SMA	90-day moving average
EMA	90-day moving average
Exponential Smoothing	Seasonality = False
ARIMA	order (2, 2, 2)
ARIMA-GARCH	ARIMA(2,2,2)-GARCH(2,2)
RNN	{'batch_size': 23, 'dropout': 0.044295524080964044, 'epochs': 73, 'learning_rate': 0.010301134772215333, 'look_back': 5, 'return_sequences': False, 'units': 40}
LSTM	{'batch_size': 5, 'dropout': 0.04211393060968949, 'epochs': 69, 'learning_rate': 0.0016780074660399514, 'look_back': 9, 'return_sequences': False, 'units': 42}

GRU	{'batch_size': 16, 'dropout': 0.27228816701190384,  'epochs': 23, 'learning_rate': 0.001997670302453216,  'look_back': 1, 'return_sequences': False, 'units': 96}
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Table 9: Tuned Parameters of Forecasting Models for Microsoft Stock Price Prediction

The corresponding forecast charts are presented below:

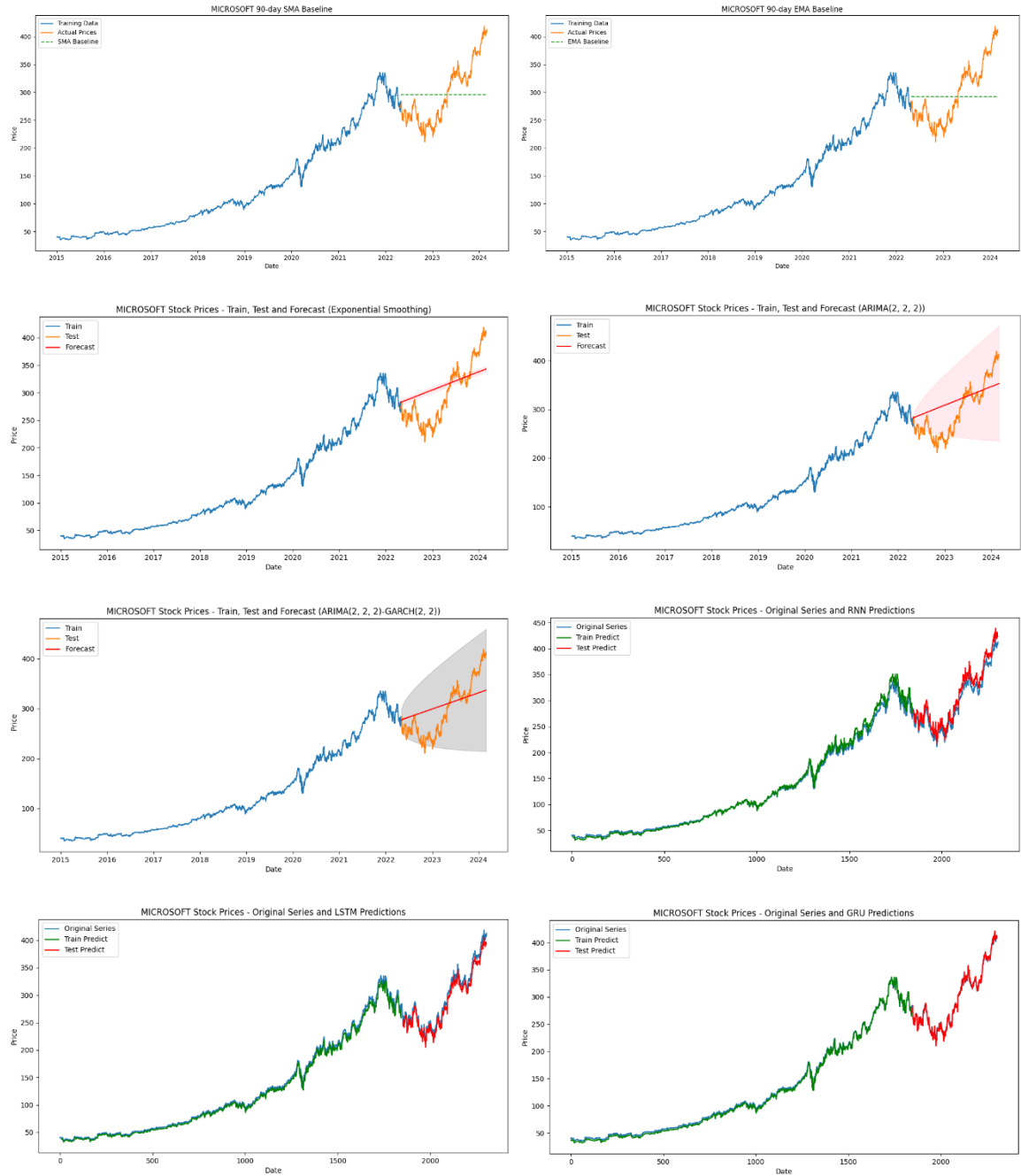


Figure 24: Microsoft Baseline and Forecasting Models' Charts

Based on the best-tuned hyperparameters, the following table presents a comparison of the forecasts for Microsoft stock prices:

<b>Models</b>	<b>RMSE</b>	<b>MAE</b>
SMA	53.3485	46.7654
EMA	53.5230	46.5023
Exponential Smoothing	42.2610	35.2332
ARIMA	42.5827	35.5366
ARIMA-GARCH	40.8237	34.1141
RNN	14.1964	12.9562
LSTM	10.0935	8.8062
GRU	5.1217	3.9073

*Table 10: Comparative Results of Forecasting Models and Performance Metrics for Microsoft*

### **Interpretation of the Results**

Among the statistical models, ARIMA-GARCH presented the lowest RMSE and MAE in forecasting Microsoft's stock price. Unlike the other two stocks, this time Exponential Smoothing outperformed ARIMA. However, advanced recurrent neural network models exhibited substantially lower RMSE and MAE compared to statistical models.

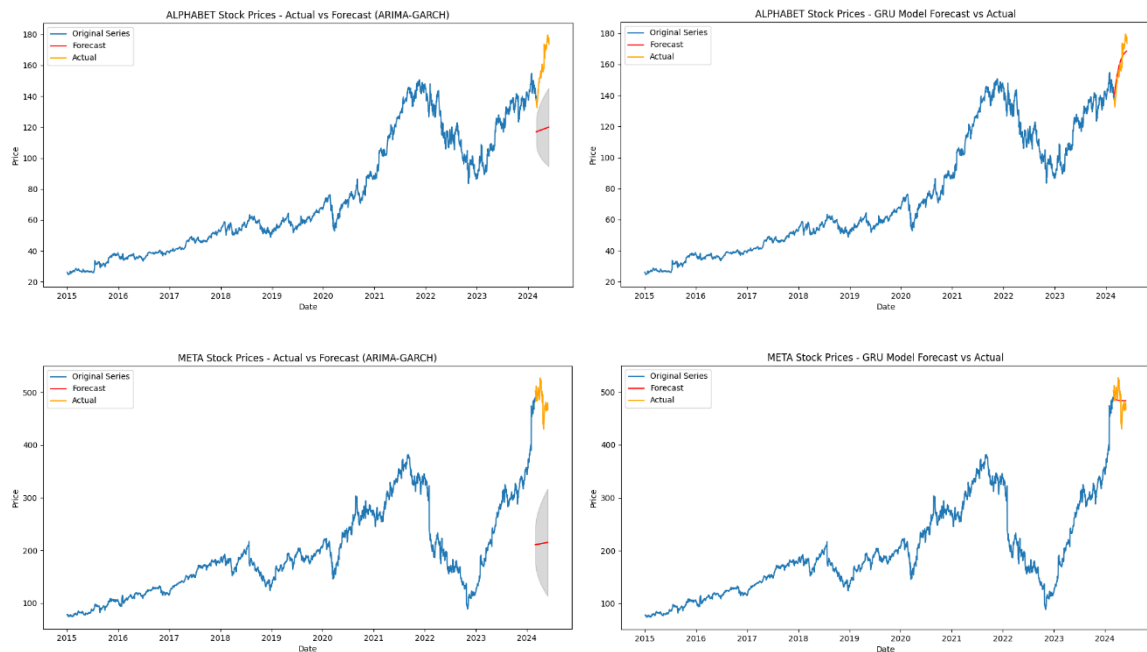
Among these neural network models, the GRU exhibited the lowest validation loss (27.8640 compared to 28.0490 for RNN and 27.8863 for LSTM), as well as the lowest RMSE and MAE. This indicates that GRU is the most effective model for predicting Microsoft's stock price.

## 4.6 Forecasting Performance for Mid-Term Unseen Data

To investigate discrepancies with findings from other studies, the final forecasts generated by the GRU model (representing deep learning models) are compared to those from the ARIMA-GARCH model (the best performer among statistical methods). This selection is based on the RMSE and MAE scores presented in the previous section. Notably, the GRU model's score for Meta was marginally lower than the RNN model, but the GRU was chosen due to the negligible difference.

The forecast horizon considered a mid-term timeframe, encompassing three calendar months – from the beginning of March to the end of May 2024 – which translates to 63 business days. For each model, the best-tuned parameters identified during the modelling phase were employed.

The visual representation of ARIMA-GARCH and GRU model forecasts for Alphabet, META, and Microsoft is as follows:



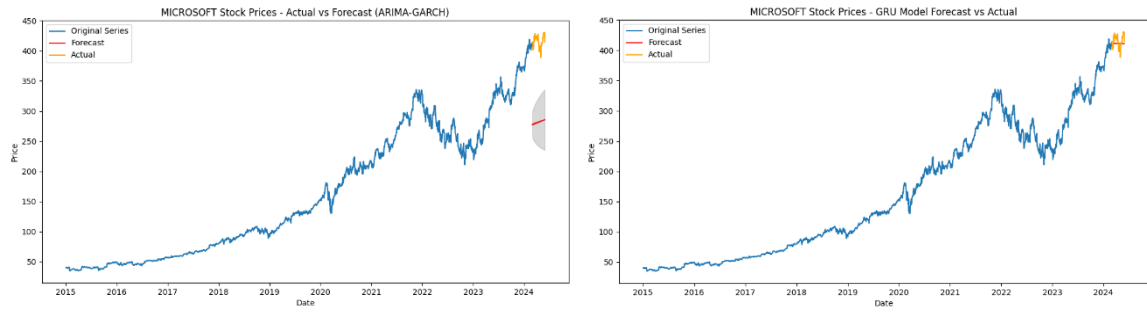


Figure 25: Mid-term Prediction Charts of Alphabet, META, and Microsoft

Appendix III provides the full prediction table, while the following tables summarize the key findings:

Statistics	Real Data	ARIMA-GARCH Forecast	GRU Forecast
Mean	158.8312698	118.5202297	159.4705051
Standard Error	1.677293441	0.110022838	1.029708699
Median	157.66	118.5305587	162.09752
Mode	151.94	#N/A	#N/A
Standard Deviation	13.31310396	0.873279204	8.173059424
Sample Variance	177.2387371	0.762616568	66.79890035
Kurtosis	-0.890379401	-1.19576964	-0.595870826
Skewness	-0.234767037	-0.003255059	-0.780712469
Range	46.98	2.98102	27.72407
Minimum	132.56	117.0109123	140.77133
Maximum	179.54	119.9919323	168.4954
Sum	10006.37	7466.77447	10046.64182
Count	63	63	63

Table 11: Summary of real and predicted values for Alphabet

Statistics	Real Data	ARIMA-GARCH Forecast	GRU Forecast
Mean	485.6965094	212.6107345	484.6987952

Standard Error	2.875910666	0.15982517	0.173658043
Median	491.350006	212.5984275	484.08276
Mode	#N/A	#N/A	#N/A
Standard Deviation	22.82683324	1.268572955	1.378367986
Sample Variance	521.0643159	1.609277343	1.899898306
Kurtosis	-0.079848557	-1.193884676	2.847212346
Skewness	-0.567019961	0.001294391	1.737867769
Range	97.170014	4.3322994	6.26981
Minimum	430.170013	210.4653215	483.61697
Maximum	527.340027	214.7976209	489.88678
Sum	30598.88009	13394.47627	30536.0241
Count	63	63	63

Table 12: Summary of real and predicted values for META

Statistics	Real Data	ARIMA-GARCH Forecast	GRU Forecast
Mean	415.2309	281.6476058	411.2951716
Standard Error	1.279222	0.288803756	0.040978789
Median	415.6703	281.6731341	411.18304
Mode	#N/A	#N/A	#N/A
Standard Deviation	10.15351	2.292308751	0.325259052
Sample Variance	103.0938	5.254679412	0.105793451
Kurtosis	-0.48916	-1.159064321	6.673300909
Skewness	-0.48231	-0.025395912	2.271451127
Range	41.89099	8.1259588	1.59491
Minimum	388.629	277.3374435	411.01602
Maximum	430.52	285.4634023	412.61093

Sum	26159.55	17743.79917	25911.59581
Count	63	63	63

Table 13: Summary of real and predicted values for Microsoft

The following table presents a prediction comparison for all three stock prices:

<b>Models</b>	<b>Alphabet</b> <b>(RMSE, MAE)</b>	<b>META</b> <b>(RMSE, MAE)</b>	<b>Microsoft</b> <b>(RMSE, MAE)</b>
ARIMA- GARCH	(41.6175, 39.7798)	(274.6141, 273.5990)	(133.7888, 133.4012)
GRU	(5.9912, 5.2584)	(22.3640, 18.0397)	(10.7544, 9.1580)

Table 14: Comparative Table for Mid-term Forecasting of Alphabet, META, and Microsoft

A visual inspection of the predicted values, alongside the RMSE and MAE results, clearly indicates that the GRU model significantly outperforms the ARIMA-GARCH model. This is evident from the lower values for both RMSE and MAE for GRU. This finding aligns well with the research of Siامي-Namini et al. (2018), who highlighted the superiority of LSTM (another recurrent neural network) compared to ARIMA models. This is further supported by the detailed statistics in Summary Tables, which demonstrate the GRU model's ability to predict values closer to the actual closing prices.

#### 4.7 Summary

In the EDA section, our examination of the stock price datasets for Alphabet, Meta, and Microsoft aimed to identify seasonality or trends. We utilised ADF and Ljung-Box tests, along with ACF and PACF plots. Additionally, a visual inspection of the original price data, overlaid with a simple moving average, helped to corroborate the presence of these patterns. None of the stock price series exhibited strong seasonality. However, all three companies displayed upward trends over the 2015-2024 period.



In the next step, to train and evaluate the performance of our forecasting models, we adopted an 80/20 train-test split. While the ARIMA-GARCH model provided a significant improvement over traditional statistical models, advanced recurrent neural networks, particularly GRU (with the exception of RNN for META), offered the highest accuracy for forecasting all three stock prices.

In the subsequent phase, the top performer from each category of models, i.e. ARIMA-GARCH and GRU was chosen to forecast a mid-term unseen data. A visual inspection of the predicted values, alongside the RMSE and MAE results, indicated that the GRU model significantly outperformed the ARIMA-GARCH model across all three stocks.

In the final chapter, "Conclusion, Limitations and Recommendations," we will discuss our research aims, objectives, and questions based on the findings of this chapter, and draw our final conclusions.

## **4. Conclusion, Limitations, and Recommendations**

### **4.1. Introduction**

This concluding chapter revisits the original research aims, objectives, and questions outlined in the first chapter. It then synthesizes the key findings from the data analysis presented in Chapter 4 to draw comprehensive conclusions about our investigation into stock price forecasting models. Finally, the chapter addresses the limitations of the current study and proposes valuable recommendations for future research directions in this field.

### **4.2. Research Conclusion**

#### **RQ1: Identifying Long-Term Trends**

This section investigates the effectiveness of time series models in identifying significant long-term trends within the historical stock prices of Alphabet, Meta, and Microsoft. The analysis utilised historical adjusted closing stock prices for each company, spanning the period from 1st January 2015 to 30th April 2024. Exploratory Data Analysis (EDA) revealed no strong evidence of seasonality within the data. However, upward trends were identified in all three stock price series.

Informed by the existing literature (Selvin et al., 2017; Siami-Namini et al., 2018; Hu et al. 2020; Hyndman et al., 2021; Kim, 2022; Kobiela et al., 2022), a variety of time series models were implemented to capture long-term trends. Traditional statistical models including ARIMA-GARCH, alongside deep learning models like LSTM. To ensure optimal performance, hyperparameter tuning was conducted for each model. Importantly, all models were compared to baseline models: Simple Moving Average (SMA) and Exponential Moving Average (EMA) to assess their effectiveness.

The chosen models successfully captured significant long-term trends within the historical stock prices. The best performing model varied for each company. Nevertheless, all models demonstrated the capacity to identify the overall direction in the stock price movements. The absence of significant seasonal trends suggests that the stock prices of these AI/ML companies are primarily influenced by long-term factors. These factors could include industry growth, company performance, or macroeconomic conditions (Wu et al., 2021). This research aligns with previous findings highlighting the strengths and limitations of traditional and deep learning models in time series forecasting (Hu et al., 2020; Siami-Namini et al., 2018; Kobiela et al., 2022).

## **RQ2: Comparative Suitability of Time Series Models for Trend Analysis in AI Stocks**

This research question investigates the relative effectiveness of different time series models in capturing underlying trends within the stock prices of leading AI/ML companies. The analysis revealed that, in general, machine learning models (RNNs, LSTMs, GRUs) demonstrated greater suitability compared to traditional statistical models (ARIMA, ARIMA-GARCH, Exponential Smoothing) for trend analysis. This can be attributed to the inherent limitations of statistical models.

- **Statistical Model Limitations**

These models often rely on assumptions about the data, such as stationarity. However, stock prices frequently exhibit trends and non-linear relationships, violating these assumptions (Hyndman et al., 2021). This can lead to suboptimal performance in capturing underlying trends. While ARIMA-GARCH addresses volatility, a common challenge in stock prices (Hu et al., 2020), its effectiveness can be limited, particularly when dealing with complex non-linearities. Exponential Smoothing can sometimes outperform ARIMA (Shahid et al., 2020), but its suitability depends heavily on the specific characteristics of the stock data.

- **Machine Learning Model Advantages**

In contrast, machine learning models, particularly RNNs and their variants (LSTMs, GRUs), excel at handling non-linear relationships and complex data structures (Selvin et al., 2017). This allows them to capture trends more effectively in financial data like stock prices, which often exhibit these characteristics (Beniwal et al., 2024).

Amongst statistical models, ARIMA-GARCH outperformed others due to its ability to handle volatility (Hu et al., 2020; Kim, 2022). Regarding deep learning models, while GRUs emerged as a strong performer across all companies, the most suitable advanced model differed for each company's stock data:

- Alphabet: GRU
- Meta: RNN (closely followed by GRU)
- Microsoft: GRU

In conclusion, the findings suggest that machine learning models, particularly RNNs and GRUs, are better at capturing underlying trends in the stock prices of leading AI/ML companies compared to traditional statistical models. This contradicts Kobiela et al. (2022) who found ARIMA to be superior, but aligns more closely with Siami-Namini et al. (2018). However, even among machine learning models, the optimal choice may vary depending on the specific company and the characteristics of its stock data. Unlike Beniwal et al. (2024), this study finds GRUs to be superior to LSTMs.

### **RQ3: Similarities and Differences in Long-Term and Seasonal Trends of AI Stock Prices**

This research question investigates the similarities and differences in long-term and seasonal trends within the stock prices of Alphabet, Meta, and Microsoft.

- **Similarities**

A key finding from the initial Exploratory Data Analysis (EDA) was the lack of significant seasonal patterns in the price movements for all three companies. This suggests that factors influencing stock prices, such as industry growth, company performance, or macroeconomic conditions (Wu et al., 2021), have a more consistent, long-term effect rather than exhibiting predictable seasonal fluctuations. Despite the absence of seasonality, all three companies exhibited a notable similarity: the presence of upward long-term trends in their historical stock prices. This finding suggests a sustained positive outlook from investors on the future prospects of these leading AI/ML players.

- **Differences**

While all companies displayed long-term growth, the specific model that best captured this trend differed for each:

- Alphabet: GRU emerged as the most effective model for capturing the long-term trend in Alphabet's stock price.
- Meta: RNN closely followed by GRU performed best in identifying the long-term trend for Meta's stock price.
- Microsoft: GRU again proved to be the most suitable model for capturing the long-term trend in Microsoft's stock price.

The AI/ML industry, unlike sectors with distinct seasonal patterns (e.g., retail with holiday shopping spikes) (Lili et al., 2023), experiences continuous development and adoption across various industries. This ongoing innovation likely contributes to a more consistent demand for AI-related services and products, reflected in the absence of significant seasonal variations in stock prices for all three companies.

In terms of variations in long-term trend model performance, even though all companies exhibited long-term growth, slight differences in their underlying data characteristics or market dynamics might have influenced the performance of specific time series models.

In conclusion, the analysis revealed a lack of significant seasonality in the stock prices, suggesting a focus on long-term factors influencing their value. While all companies displayed long-term upward trends, the optimal model for capturing these trends varied slightly between them. This underscores the importance of considering company-specific characteristics when selecting models for time series analysis in the AI/ML sector.

#### **RQ4: Accuracy of Time Series Models in Predicting AI Stock Price Trends**

This research question aimed to assess the effectiveness of various time series models in forecasting future trends within the stock prices of leading AI and ML companies. The analysis compared the performance of:

- **Statistical Models:** Represented by ARIMA-GARCH, which emerged as the most effective model amongst traditional statistical methods.
- **Machine Learning Models:** Represented by the GRU model, chosen due to its consistently strong performance.

The analysis focused on a mid-term forecast horizon, encompassing three months (March to May 2024). The analysis revealed that the GRU model significantly outperformed the ARIMA-GARCH model in predicting future trends for all three companies. This is evident from the consistently lower RMSE and MAE values for the GRU model compared to ARIMA-GARCH. This aligns with research highlighting the strengths of RNNs (similar to GRUs) in handling non-linear relationships within financial data (Siami-Namini et al., 2018). These findings support the notion that machine learning models, particularly RNN

variants like GRUs, are better suited for capturing complex non-linear relationships and patterns within financial data like stock prices (Hyndman et al., 2021).

In conclusion, this analysis indicates that, for mid-term forecasting of stock prices in leading AI and ML companies, machine learning models, specifically the GRU model in this case, offer superior accuracy compared to traditional statistical models. This finding supports the potential of machine learning approaches like GRUs in financial forecasting, particularly when dealing with data exhibiting non-linear relationships. By showcasing the effectiveness of GRUs, this research mitigates the risks associated with suboptimal decisions arising from traditional models' limitations with complex data. The benefits extend beyond academia, empowering investors to identify opportunities, manage risk, and ultimately enhance returns.

#### **4.3. Limitations**

This study identified three key limitations associated with the employed methodology:

- **Limitations of Univariate Analysis**

This research utilised historical stock prices to identify long-term trends. However, due to the inherent volatility of financial markets, past performance is not necessarily indicative of future results. Additionally, the univariate analysis employed in this study does not account for external influences, such as broader economic fluctuations, technological advancements, and other factors. This limitation restricts the analysis from capturing the complete picture of stock price movements, potentially leading to inaccurate predictions. Furthermore, univariate analysis cannot explain the "why" behind price changes and the underlying causes influencing them. Understanding these causes is crucial for making informed investment decisions (Narayan et al., 2010).

- **Challenges of Univariate Models**

Univariate models, such as the one employed in this study which relies solely on stock price data, are susceptible to overfitting. This means they might perform well on historical data but fail to generalise and accurately predict future, unseen data. Introducing additional variables relevant to stock price movements can help mitigate overfitting and enhance the model's ability to adapt to changing market conditions (Zhang et al., 2005).

- **Deep Learning Model Uncertainty**

Deep learning models present another challenge when dealing with prediction uncertainty. Each time they are run, they might provide different results due to random initialisation, nonlinearity, and internal randomness (Gal et al., 2016). For research purposes or non-critical analysis, running the model once or utilising ensemble methods is sufficient (Lakshminarayanan et al., 2017). However, for high-risk decisions a more conservative approach should be considered.

#### **4.4. Recommendations for future research**

While univariate analysis provided valuable insights in this study, future research could explore a multivariate approach that considers additional features relevant to stock price movements. These features could include broader market and economic, customer sentiment data, and technological advancements. A multivariate approach has the potential to improve forecasting accuracy by capturing a more comprehensive picture of the factors influencing stock prices. Additionally, it could help mitigate the overfitting issue observed in univariate models (Tsai et al., 2010).



For high-risk decisions, it is recommended to consider a more conservative approach by using confidence intervals. This provides a range of possible future outcomes and quantifies the uncertainty associated with the predictions. Additionally, integrating the results of the forecasting model with expert knowledge and analysis can combine quantitative and qualitative insights, leading to more informed and robust decision-making (Hyndman et al., 2021).

#### **4.5. Conclusion**

This chapter summarizes the analysis of stock price forecasting models for leading AI companies (Alphabet, Meta, Microsoft).

##### **Key Findings:**

- **Long-Term Trends:** All three companies exhibited upward long-term trends in their stock prices, suggesting a positive outlook from investors on the future of AI/ML.
- **Seasonal Trends:** No significant seasonal patterns were identified, indicating that factors influencing stock prices have a more consistent, long-term effect.
- **Model Comparison:** Machine learning models (RNNs, LSTMs, GRUs) were generally more effective than traditional statistical models (ARIMA, ARIMA-GARCH, Exponential Smoothing) in capturing underlying trends.
- **GRUs:** Amongst machine learning models, GRUs emerged as a strong performer for capturing long-term trends.
- **Forecasting Accuracy:** The GRU model significantly outperformed the ARIMA-GARCH model in predicting mid-term trends (3 months) for all three companies.

##### **Limitations:**

- Focusing solely on historical prices might miss key influencing factors and lead to inaccurate predictions.
- The model might be susceptible to overfitting and produce unreliable forecasts for unseen data.
- Deep learning models can exhibit variations in predictions.

**Future Directions:**

- Explore techniques to mitigate overfitting and improve model generalizability.
- For high-risk decisions, use confidence intervals and expert analysis alongside model predictions.

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# Appendices

## I. Outliers Data

Date	Adj. Close	Date	Adj. Close	Date	Adj. Close	Date	Adj. Close	Date	Adj. Close
7/23/2021	137.82	10/18/2021	142.96	12/14/2021	144.97	9/11/2023	137.74	1/12/2024	144.24
7/26/2021	139.64	10/19/2021	143.82	12/15/2021	147.37	9/14/2023	138.99	1/16/2024	144.08
8/9/2021	138.00	10/20/2021	142.41	12/16/2021	144.84	9/15/2023	138.30	1/17/2024	142.89
8/10/2021	138.10	10/21/2021	142.78	12/17/2021	142.80	9/18/2023	138.96	1/18/2024	144.99
8/11/2021	137.69	10/22/2021	138.63	12/20/2021	142.40	9/19/2023	138.83	1/19/2024	147.97
8/12/2021	138.39	10/25/2021	138.77	12/21/2021	144.22	10/6/2023	138.73	1/22/2024	147.71
8/13/2021	138.41	10/26/2021	139.67	12/22/2021	146.95	10/9/2023	139.50	1/23/2024	148.68
8/16/2021	138.92	10/27/2021	146.43	12/23/2021	147.14	10/10/2023	139.20	1/24/2024	150.35
8/20/2021	138.44	10/28/2021	146.13	12/27/2021	148.06	10/11/2023	141.70	1/25/2024	153.64
8/23/2021	141.10	10/29/2021	148.27	12/28/2021	146.45	10/12/2023	140.29	1/26/2024	153.79
8/24/2021	142.40	11/1/2021	143.77	12/29/2021	146.50	10/13/2023	138.58	1/29/2024	154.84
8/25/2021	142.95	11/2/2021	145.86	12/30/2021	146.00	10/16/2023	140.49	1/30/2024	153.05
8/26/2021	142.12	11/3/2021	146.79	12/31/2021	144.68	10/17/2023	140.99	1/31/2024	141.80
8/27/2021	144.55	11/4/2021	148.68	1/3/2022	145.07	10/18/2023	139.28	2/1/2024	142.71
8/30/2021	145.47	11/5/2021	149.24	1/4/2022	144.42	10/19/2023	138.98	2/2/2024	143.54
8/31/2021	145.46	11/8/2021	149.35	1/5/2022	137.65	10/23/2023	137.90	2/5/2024	144.93
9/1/2021	145.84	11/9/2021	149.25	1/10/2022	138.57	10/24/2023	140.12	2/6/2024	145.41
9/2/2021	144.22	11/10/2021	146.63	1/11/2022	140.02	11/16/2023	138.70	2/7/2024	146.68
9/3/2021	144.77	11/11/2021	146.75	1/12/2022	141.65	11/20/2023	137.92	2/8/2024	147.22
9/7/2021	145.52	11/12/2021	149.65	1/13/2022	139.13	11/21/2023	138.62	2/9/2024	150.22
9/8/2021	144.88	11/15/2021	149.39	1/14/2022	139.79	11/22/2023	140.02	2/12/2024	148.73
9/9/2021	144.91	11/16/2021	149.08	2/1/2022	137.88	11/24/2023	138.22	2/13/2024	146.37
9/10/2021	141.92	11/17/2021	149.06	2/2/2022	148.04	11/27/2023	138.05	2/14/2024	147.14
9/13/2021	143.46	11/18/2021	150.71	2/3/2022	142.65	11/28/2023	138.62	2/15/2024	143.94
9/14/2021	143.41	11/19/2021	149.95	2/4/2022	143.02	12/7/2023	138.45	2/16/2024	141.76
9/15/2021	145.21	11/22/2021	147.08	2/7/2022	138.94	12/19/2023	138.10	2/20/2024	142.20
9/16/2021	144.37	11/23/2021	146.76	2/8/2022	139.21	12/20/2023	139.66	2/21/2024	143.84
9/17/2021	141.46	11/24/2021	146.72	2/9/2022	141.45	12/21/2023	141.80	2/22/2024	145.32
9/20/2021	139.02	11/26/2021	142.81	2/10/2022	138.60	12/22/2023	142.72	2/23/2024	145.29
9/21/2021	139.65	11/29/2021	146.11	3/22/2022	140.28	12/26/2023	142.82	2/26/2024	138.75
9/22/2021	140.94	11/30/2021	142.45	3/23/2022	138.50	12/27/2023	141.44	2/27/2024	140.10
9/23/2021	141.83	12/1/2021	141.62	3/24/2022	141.31	12/28/2023	141.28	2/29/2024	139.78
9/24/2021	142.63	12/2/2021	143.78	3/25/2022	141.52	12/29/2023	140.93		
9/27/2021	141.50	12/3/2021	142.52	3/28/2022	141.95	1/2/2024	139.56		
10/7/2021	139.19	12/6/2021	143.80	3/29/2022	143.25	1/3/2024	140.36		
10/8/2021	140.06	12/7/2021	148.04	3/30/2022	142.64	1/4/2024	138.04		
10/11/2021	138.85	12/8/2021	148.72	3/31/2022	139.65	1/8/2024	140.53		
10/13/2021	137.90	12/9/2021	148.11	4/1/2022	140.70	1/9/2024	142.56		
10/14/2021	141.41	12/10/2021	148.68	4/4/2022	143.64	1/10/2024	143.80		
10/15/2021	141.68	12/13/2021	146.70	4/5/2022	141.06	1/11/2024	143.67		

Table 15: Outliers of Alphabet Stock Prices

Date	Adj. Close	Date	Adj. Close
6/28/2021	355.26	12/27/2023	357.45
7/23/2021	369.40	12/28/2023	357.94
7/26/2021	372.07	1/8/2024	358.28
7/27/2021	367.42	1/9/2024	357.05
7/28/2021	372.88	1/10/2024	370.08
7/29/2021	357.94	1/11/2024	369.28
7/30/2021	355.92	1/12/2024	374.09
8/4/2021	358.54	1/16/2024	367.07
8/5/2021	362.59	1/17/2024	367.98
8/6/2021	363.12	1/18/2024	375.73
8/9/2021	361.23	1/19/2024	383.04
8/10/2021	360.75	1/22/2024	381.38
8/11/2021	359.58	1/23/2024	384.79
8/12/2021	362.27	1/24/2024	390.29
8/13/2021	362.80	1/25/2024	392.76
8/16/2021	366.17	1/26/2024	393.72
8/17/2021	358.07	1/29/2024	400.59
8/18/2021	355.07	1/30/2024	399.64
8/20/2021	358.99	1/31/2024	389.73
8/23/2021	362.96	2/1/2024	394.36
8/24/2021	365.12	2/2/2024	474.49
8/25/2021	368.00	2/5/2024	458.92
8/26/2021	363.99	2/6/2024	454.24
8/27/2021	372.24	2/7/2024	469.09
8/30/2021	380.26	2/8/2024	469.50
8/31/2021	378.98	2/9/2024	467.61
9/1/2021	381.65	2/12/2024	468.40
9/2/2021	374.88	2/13/2024	459.63
9/3/2021	375.86	2/14/2024	472.78
9/7/2021	381.77	2/15/2024	483.52
9/8/2021	377.17	2/16/2024	472.82
9/9/2021	377.60	2/20/2024	471.25
9/10/2021	378.29	2/21/2024	468.03
9/13/2021	376.11	2/22/2024	486.13
9/14/2021	376.13	2/23/2024	484.03
9/15/2021	373.52	2/26/2024	481.74
9/16/2021	372.66	2/27/2024	487.05
9/17/2021	364.33	2/28/2024	484.02
9/20/2021	355.32	2/29/2024	490.13
9/21/2021	357.10		

Table 16: Outliers of META Stock Prices

Date	Adj. Close
1/18/2024	392.44

1/19/2024	397.22
1/22/2024	395.07
1/23/2024	397.45
1/24/2024	401.09
1/25/2024	403.40
1/26/2024	402.46
1/29/2024	408.23
1/30/2024	407.10
1/31/2024	396.13
2/1/2024	402.31
2/2/2024	409.72
2/5/2024	404.17
2/6/2024	404.01
2/7/2024	412.54
2/8/2024	412.60
2/9/2024	419.02
2/12/2024	413.75
2/13/2024	404.84
2/14/2024	408.75
2/15/2024	405.83
2/16/2024	403.33
2/20/2024	402.06
2/21/2024	401.46
2/22/2024	410.91
2/23/2024	409.60
2/26/2024	406.81
2/27/2024	406.75
2/28/2024	406.99
2/29/2024	412.90

Table 17: Outliers of Microsoft Stock Prices

## II. Ljung\_Box Test Results After First-Order Differencing

Lag	lb_stat	lb_pvalue	bp_stat	bp_pvalue	Lag	lb_stat	lb_pvalue	bp_stat	bp_pvalue
1	2.352157	0.12511	2.349097	0.125356	127	271.9014	1.44E-12	264.5476	1.05E-11
2	2.499779	0.286536	2.496463	0.287012	128	274.8216	9.62E-13	267.3032	7.36E-12
3	7.736351	0.051787	7.72168	0.052128	129	281.0644	2.56E-13	273.1914	2.21E-12
4	8.532401	0.073911	8.515659	0.074414	130	281.1281	3.75E-13	273.2514	3.19E-12
5	11.01618	0.05106	10.9919	0.051541	131	281.2371	5.39E-13	273.3541	4.54E-12
6	16.27747	0.01234	16.23494	0.012547	132	281.3105	7.81E-13	273.4233	6.50E-12
7	16.4674	0.021173	16.42412	0.021512	133	281.755	1.02E-12	273.8418	8.45E-12
8	18.45294	0.018076	18.40105	0.018413	134	283.7746	8.68E-13	275.7422	7.42E-12
9	25.9644	0.002071	25.87669	0.00214	135	283.8363	1.25E-12	275.8003	1.06E-11
10	26.46274	0.003165	26.37243	0.00327	136	285.4153	1.20E-12	277.2848	1.03E-11

11	27.1535	0.004356	27.05929	0.004501	137	289.6771	5.59E-13	281.2897	5.23E-12
12	28.69035	0.004379	28.58681	0.004536	138	290.6919	6.24E-13	282.2429	5.89E-12
13	29.32865	0.005878	29.22097	0.006089	139	292.095	6.27E-13	283.5602	6.02E-12
14	29.64135	0.008548	29.5315	0.008849	140	293.845	5.74E-13	285.2024	5.65E-12
15	40.08339	0.000441	39.89656	0.00047	141	300.4314	1.43E-13	291.3804	1.61E-12
16	43.27244	0.000254	43.06071	0.000274	142	300.447	2.09E-13	291.395	2.33E-12
17	44.41461	0.000297	44.19347	0.000321	143	300.7596	2.82E-13	291.6879	3.11E-12
18	51.53756	4.40E-05	51.25465	4.87E-05	144	304.7019	1.43E-13	295.3806	1.71E-12
19	55.18947	2.17E-05	54.8733	2.43E-05	145	304.7054	2.09E-13	295.3839	2.47E-12
20	56.17335	2.74E-05	55.84779	3.06E-05	146	305.1676	2.70E-13	295.8164	3.17E-12
21	56.1927	4.73E-05	55.86695	5.28E-05	147	306.6359	2.67E-13	297.1898	3.19E-12
22	62.3481	9.98E-06	61.95829	1.14E-05	148	306.8668	3.65E-13	297.4058	4.33E-12
23	62.82708	1.48E-05	62.43207	1.69E-05	149	306.8944	5.26E-13	297.4316	6.14E-12
24	62.98737	2.39E-05	62.59055	2.73E-05	150	306.8989	7.60E-13	297.4358	8.74E-12
25	65.78445	1.59E-05	65.35488	1.84E-05	151	310.7977	3.95E-13	301.0759	4.92E-12
26	65.8199	2.64E-05	65.38991	3.04E-05	152	310.86	5.62E-13	301.134	6.90E-12
27	65.83954	4.31E-05	65.40929	4.94E-05	153	311.2574	7.30E-13	301.5047	8.91E-12
28	65.92943	6.75E-05	65.49802	7.72E-05	154	314.7486	4.23E-13	304.7597	5.55E-12
29	66.99055	7.74E-05	66.54487	8.87E-05	155	315.462	5.06E-13	305.4245	6.65E-12
30	75.81169	7.76E-06	75.2436	9.32E-06	156	315.8749	6.54E-13	305.8091	8.54E-12
31	80.42654	2.88E-06	79.79241	3.54E-06	157	321.3387	2.27E-13	310.8962	3.35E-12
32	80.42743	4.77E-06	79.79329	5.84E-06	158	321.4737	3.16E-13	311.0219	4.60E-12
33	81.50368	5.52E-06	80.85321	6.79E-06	159	321.7953	4.18E-13	311.321	6.04E-12
34	81.7651	8.22E-06	81.11054	1.01E-05	160	323.625	3.73E-13	313.0221	5.56E-12
35	89.21472	1.27E-06	88.44063	1.62E-06	161	323.6879	5.26E-13	313.0806	7.73E-12
36	89.45776	1.92E-06	88.67967	2.45E-06	162	324.5682	6.00E-13	313.8983	8.87E-12
37	89.46673	3.09E-06	88.68848	3.93E-06	163	324.7489	8.17E-13	314.0661	1.19E-11
38	89.46805	4.91E-06	88.68979	6.23E-06	164	324.7788	1.15E-12	314.0938	1.66E-11
39	89.95717	6.66E-06	89.17021	8.44E-06	165	328.3232	6.66E-13	317.3816	1.03E-11
40	93.89195	3.18E-06	93.03332	4.13E-06	166	328.812	8.37E-13	317.8347	1.29E-11
41	94.8177	3.78E-06	93.94181	4.92E-06	167	328.8198	1.18E-12	317.842	1.80E-11
42	103.7699	3.85E-07	102.7232	5.33E-07	168	328.8345	1.67E-12	317.8556	2.49E-11
43	106.7935	2.41E-07	105.6878	3.41E-07	169	329.0947	2.20E-12	318.0965	3.26E-11
44	106.8402	3.82E-07	105.7336	5.38E-07	170	330.8857	1.98E-12	319.7539	3.03E-11
45	111.1733	1.60E-07	109.9783	2.32E-07	171	331.1606	2.61E-12	320.0082	3.94E-11
46	111.3658	2.42E-07	110.1668	3.49E-07	172	333.9233	1.84E-12	322.5624	2.95E-11
47	112.2263	2.95E-07	111.0091	4.27E-07	173	336.4281	1.39E-12	324.8771	2.34E-11
48	117.2854	9.94E-08	115.9584	1.50E-07	174	339.286	9.61E-13	327.5169	1.72E-11
49	118.5332	1.08E-07	117.1787	1.63E-07	175	340.087	1.11E-12	328.2564	1.99E-11
50	118.724	1.62E-07	117.3652	2.44E-07	176	343.3888	6.85E-13	331.3034	1.33E-11
51	119.1934	2.20E-07	117.8237	3.32E-07	177	344.1771	7.93E-13	332.0305	1.54E-11
52	122.3001	1.36E-07	120.8577	2.09E-07	178	344.2516	1.09E-12	332.0991	2.09E-11
53	126.0059	6.98E-08	124.4751	1.11E-07	179	360.1365	2.82E-14	346.7373	8.29E-13
54	126.008	1.09E-07	124.4772	1.73E-07	180	360.2997	3.86E-14	346.8876	1.12E-12
55	126.2081	1.60E-07	124.6724	2.52E-07	181	363.4726	2.45E-14	349.8087	7.63E-13
56	126.2503	2.44E-07	124.7135	3.82E-07	182	364.8754	2.44E-14	351.0996	7.78E-13

57	127.6699	2.48E-07	126.0967	3.90E-07	183	365.7647	2.77E-14	351.9175	8.90E-13
58	134.1603	5.69E-08	132.4182	9.52E-08	184	370.7296	1.11E-14	356.4819	4.05E-13
59	134.2858	8.48E-08	132.5404	1.41E-07	185	371.0314	1.47E-14	356.7592	5.30E-13
60	134.3786	1.27E-07	132.6307	2.10E-07	186	371.0983	2.06E-14	356.8207	7.29E-13
61	134.6943	1.76E-07	132.9378	2.90E-07	187	373.9732	1.41E-14	359.46	5.34E-13
62	137.7525	1.11E-07	135.9112	1.88E-07	188	375.6007	1.33E-14	360.9533	5.17E-13
63	141.6581	5.50E-08	139.7067	9.65E-08	189	375.8907	1.75E-14	361.2193	6.76E-13
64	144.0365	4.22E-08	142.017	7.56E-08	190	377.0696	1.85E-14	362.3001	7.23E-13
65	144.2545	6.05E-08	142.2286	1.08E-07	191	378.059	2.04E-14	363.2067	8.08E-13
66	145.0576	7.28E-08	143.008	1.30E-07	192	381.7312	1.14E-14	366.5699	4.96E-13
67	146.2077	7.93E-08	144.1238	1.42E-07	193	381.8348	1.58E-14	366.6648	6.74E-13
68	149.7378	4.40E-08	147.5467	8.17E-08	194	382.4774	1.90E-14	367.2528	8.12E-13
69	150.5118	5.32E-08	148.2969	9.92E-08	195	387.4299	7.73E-15	371.7821	3.76E-13
70	150.6164	7.75E-08	148.3982	1.44E-07	196	390.3828	5.19E-15	374.4815	2.72E-13
71	158.0951	1.41E-08	155.6401	2.83E-08	197	391.1792	6.03E-15	375.2092	3.17E-13
72	158.7204	1.79E-08	156.2454	3.59E-08	198	391.5065	7.87E-15	375.5081	4.10E-13
73	161.9323	1.08E-08	159.3529	2.25E-08	199	392.4962	8.68E-15	376.4116	4.57E-13
74	162.4934	1.40E-08	159.8954	2.90E-08	200	392.8816	1.11E-14	376.7632	5.81E-13
75	162.4937	2.10E-08	159.8957	4.32E-08	201	392.8916	1.57E-14	376.7723	8.01E-13
76	165.1606	1.49E-08	162.4724	3.15E-08	202	393.2318	2.03E-14	377.0825	1.03E-12
77	166.3454	1.60E-08	163.6166	3.41E-08	203	398.428	7.82E-15	381.8167	4.57E-13
78	167.765	1.61E-08	164.987	3.46E-08	204	398.665	1.04E-14	382.0325	6.00E-13
79	168.2531	2.09E-08	165.458	4.49E-08	205	398.7083	1.45E-14	382.0719	8.19E-13
80	170.3504	1.74E-08	167.4806	3.81E-08	206	398.8974	1.94E-14	382.244	1.08E-12
81	171.7406	1.76E-08	168.8208	3.90E-08	207	399.902	2.12E-14	383.1575	1.20E-12
82	178.0735	4.52E-09	174.9231	1.08E-08	208	400.0266	2.87E-14	383.2708	1.59E-12
83	179.9951	3.96E-09	176.7738	9.66E-09	209	400.0431	3.99E-14	383.2858	2.17E-12
84	180.5563	5.05E-09	177.3141	1.23E-08	210	401.1026	4.30E-14	384.2479	2.37E-12
85	181.6229	5.59E-09	178.3404	1.37E-08	211	404.5635	2.58E-14	387.3891	1.56E-12
86	184.6217	3.63E-09	181.2248	9.23E-09	212	404.7401	3.44E-14	387.5493	2.04E-12
87	184.6877	5.29E-09	181.2883	1.33E-08	213	406.1787	3.37E-14	388.8538	2.06E-12
88	185.1456	6.89E-09	181.7283	1.73E-08	214	408.979	2.38E-14	391.3918	1.56E-12
89	185.8169	8.44E-09	182.3731	2.13E-08	215	409.2152	3.12E-14	391.6057	2.02E-12
90	187.2257	8.46E-09	183.7257	2.16E-08	216	410.1517	3.46E-14	392.4537	2.25E-12
91	187.341	1.20E-08	183.8363	3.03E-08	217	415.098	1.45E-14	396.9303	1.09E-12
92	187.6536	1.60E-08	184.1362	4.03E-08	218	415.1101	2.01E-14	396.9412	1.48E-12
93	206.4954	1.35E-10	202.2018	4.51E-10	219	415.1715	2.75E-14	396.9967	1.98E-12
94	208.8459	1.05E-10	204.4545	3.60E-10	220	418.1401	1.86E-14	399.6795	1.45E-12
95	209.0652	1.49E-10	204.6645	5.07E-10	221	418.2286	2.53E-14	399.7595	1.94E-12
96	210.5718	1.46E-10	206.107	5.06E-10	222	419.0338	2.88E-14	400.4865	2.22E-12
97	229.2672	1.02E-12	223.9998	4.78E-12	223	419.1327	3.89E-14	400.5757	2.94E-12
98	229.6854	1.40E-12	224.3999	6.54E-12	224	419.1933	5.29E-14	400.6304	3.92E-12
99	229.6891	2.16E-12	224.4035	9.99E-12	225	419.4807	6.80E-14	400.8894	4.98E-12
100	229.6951	3.32E-12	224.4092	1.52E-11	226	424.1708	3.07E-14	405.1158	2.58E-12
101	238.4035	4.01E-13	232.7286	2.11E-12	227	425.7393	2.92E-14	406.5286	2.53E-12
102	238.462	6.12E-13	232.7845	3.19E-12	228	426.838	3.10E-14	407.5177	2.73E-12

103	238.807	8.54E-13	233.1138	4.43E-12	229	427.399	3.73E-14	408.0225	3.28E-12
104	240.8867	7.18E-13	235.0979	3.82E-12	230	429.3645	3.22E-14	409.7902	2.96E-12
105	243.6511	4.94E-13	237.734	2.73E-12	231	429.3857	4.41E-14	409.8093	3.96E-12
106	244.1843	6.51E-13	238.2422	3.59E-12	232	429.5617	5.80E-14	409.9675	5.13E-12
107	249.4155	2.19E-13	243.2262	1.31E-12	233	431.2607	5.33E-14	411.4933	4.89E-12
108	249.805	3.01E-13	243.5971	1.80E-12	234	432.1607	5.91E-14	412.3012	5.47E-12
109	250.5471	3.73E-13	244.3034	2.23E-12	235	434.2737	4.93E-14	414.197	4.81E-12
110	251.6512	4.15E-13	245.3539	2.51E-12	236	435.2288	5.40E-14	415.0536	5.32E-12
111	251.838	6.00E-13	245.5316	3.60E-12	237	436.7524	5.17E-14	416.4193	5.25E-12
112	251.8663	9.05E-13	245.5584	5.36E-12	238	436.7862	7.00E-14	416.4496	6.96E-12
113	255.4552	4.94E-13	248.9684	3.10E-12	239	439.243	5.40E-14	418.6496	5.72E-12
114	255.5258	7.35E-13	249.0354	4.55E-12	240	442.4038	3.53E-14	421.4787	4.09E-12
115	258.5373	4.73E-13	251.8941	3.07E-12	241	443.654	3.60E-14	422.5972	4.27E-12
116	258.6207	6.99E-13	251.9733	4.49E-12	242	443.6619	4.91E-14	422.6042	5.68E-12
117	264.2061	2.17E-13	257.2705	1.54E-12	243	443.747	6.55E-14	422.6802	7.43E-12
118	264.6025	2.94E-13	257.6463	2.07E-12	244	444.5233	7.45E-14	423.3738	8.48E-12
119	264.8	4.20E-13	257.8334	2.93E-12	245	444.8162	9.45E-14	423.6353	1.06E-11
120	264.9762	6.01E-13	258.0002	4.16E-12	246	447.9446	6.25E-14	426.4272	7.69E-12
121	266.6981	5.57E-13	259.6303	3.94E-12	247	448.1794	8.04E-14	426.6367	9.74E-12
122	267.3571	6.95E-13	260.2538	4.92E-12	248	450.5775	6.29E-14	428.7748	8.13E-12
123	267.8272	9.11E-13	260.6985	6.43E-12	249	450.5976	8.49E-14	428.7927	1.07E-11
124	267.8854	1.33E-12	260.7535	9.30E-12	250	461.3085	9.73E-15	438.3331	1.79E-12
125	270.2518	1.03E-12	262.9896	7.47E-12	251	466.621	3.83E-15	443.0627	8.39E-13
126	271.9002	9.75E-13	264.5465	7.20E-12	252	470.9896	1.88E-15	446.9502	4.74E-13

Table 18: Ljung\_Box Test Results of Alphabet

La g	lb_stat	lb_pvalue	bp_stat	bp_pvalue	La g	lb_stat	lb_pvalue	bp_stat	bp_pvalue
1	5.707707	0.016891	5.700281	0.016962	127	216.7782	1.20E-06	211.1518	3.99E-06
2	6.147515	0.046247	6.139327	0.046437	128	217.0748	1.49E-06	211.4317	4.92E-06
3	7.732063	0.051886	7.720439	0.052157	129	226.8914	2.27E-07	220.6906	8.99E-07
4	8.190353	0.084849	8.177536	0.085287	130	227.3517	2.75E-07	221.1245	1.08E-06
5	8.638018	0.124403	8.623843	0.125042	131	233.1498	9.99E-08	226.5883	4.36E-07
6	9.573394	0.143805	9.555974	0.14464	132	233.7969	1.16E-07	227.1978	5.08E-07
7	9.573394	0.214065	9.555974	0.215163	133	234.7202	1.27E-07	228.067	5.58E-07
8	11.64076	0.167966	11.61437	0.169257	134	234.8636	1.65E-07	228.2019	7.17E-07
9	19.88264	0.018651	19.81694	0.019076	135	236.9502	1.39E-07	230.1646	6.21E-07
10	19.88386	0.030371	19.81815	0.03102	136	236.9535	1.86E-07	230.1677	8.19E-07
11	22.21191	0.022782	22.13308	0.023362	137	238.4595	1.78E-07	231.5829	7.97E-07
12	23.12425	0.026689	23.03988	0.027389	138	239.4639	1.90E-07	232.5264	8.58E-07
13	23.12474	0.040203	23.04037	0.041194	139	240.0917	2.21E-07	233.1157	9.95E-07
14	24.12907	0.044204	24.03773	0.045344	140	244.3522	1.15E-07	237.1139	5.58E-07
15	25.8788	0.039316	25.77456	0.04046	141	244.5797	1.46E-07	237.3273	7.02E-07
16	28.16517	0.030213	28.04309	0.031247	142	249.081	7.17E-08	241.5475	3.75E-07
17	28.16529	0.043044	28.0432	0.044436	143	249.3615	9.01E-08	241.8104	4.67E-07
18	36.27189	0.006511	36.07949	0.006892	144	250.9819	8.41E-08	243.3282	4.45E-07

19	36.27435	0.009766	36.08193	0.010314	145	251.2719	1.05E-07	243.5997	5.52E-07
20	42.46617	0.002403	42.21468	0.002593	146	251.9991	1.19E-07	244.2802	6.27E-07
21	43.11694	0.003037	42.85896	0.003276	147	253.5729	1.12E-07	245.7523	6.03E-07
22	43.50763	0.004084	43.24558	0.004401	148	253.5731	1.49E-07	245.7525	7.88E-07
23	44.49934	0.004575	44.22654	0.004937	149	254.9698	1.46E-07	247.0577	7.83E-07
24	44.80449	0.006143	44.52825	0.00662	150	255.6852	1.65E-07	247.726	8.88E-07
25	44.83098	0.008741	44.55443	0.009395	151	255.6903	2.18E-07	247.7307	1.15E-06
26	44.83541	0.01227	44.5588	0.013152	152	261.0204	9.13E-08	252.7049	5.37E-07
27	46.2011	0.012103	45.90732	0.013018	153	261.2928	1.14E-07	252.959	6.64E-07
28	47.62032	0.01177	47.30808	0.012707	154	261.5709	1.42E-07	253.2183	8.18E-07
29	47.79237	0.015443	47.47781	0.016639	155	261.6997	1.82E-07	253.3383	1.04E-06
30	51.81969	0.007965	51.44925	0.008726	156	263.3951	1.67E-07	254.9175	9.72E-07
31	65.33814	0.000303	64.77424	0.000356	157	265.2599	1.47E-07	256.6537	8.83E-07
32	65.34353	0.000453	64.77955	0.000531	158	265.4373	1.86E-07	256.8189	1.10E-06
33	65.93063	0.00057	65.35774	0.000667	159	266.587	1.92E-07	257.8883	1.15E-06
34	66.65956	0.000684	66.07529	0.000801	160	267.8971	1.91E-07	259.1063	1.16E-06
35	66.72693	0.000972	66.14158	0.001134	161	268.104	2.39E-07	259.2987	1.44E-06
36	67.08683	0.001264	66.49555	0.001473	162	268.8455	2.67E-07	259.9873	1.61E-06
37	67.40924	0.001644	66.81251	0.001912	163	270.2487	2.60E-07	261.2901	1.59E-06
38	71.33745	0.000849	70.67257	0.001008	164	272.7724	2.01E-07	263.6322	1.29E-06
39	72.4333	0.000908	71.74895	0.001081	165	273.411	2.29E-07	264.2245	1.47E-06
40	72.48909	0.001259	71.80372	0.001493	166	273.5098	2.92E-07	264.3161	1.84E-06
41	80.71907	0.000212	79.88023	0.000265	167	274.254	3.25E-07	265.0058	2.05E-06
42	80.90276	0.000291	80.06042	0.000363	168	274.7093	3.84E-07	265.4275	2.41E-06
43	82.58284	0.000268	81.70771	0.000337	169	274.7184	4.95E-07	265.436	3.06E-06
44	82.62533	0.000378	81.74936	0.000473	170	276.2419	4.70E-07	266.8459	2.97E-06
45	88.5231	0.000116	87.52692	0.000151	171	276.4723	5.78E-07	267.059	3.61E-06
46	90.26483	0.000105	89.2324	0.000138	172	276.6674	7.14E-07	267.2393	4.41E-06
47	92.67547	8.02E-05	91.59181	0.000107	173	276.7933	8.93E-07	267.3557	5.43E-06
48	92.88258	0.000109	91.79443	0.000145	174	278.5469	8.09E-07	268.9755	5.06E-06
49	93.30982	0.00014	92.21222	0.000185	175	278.7258	9.99E-07	269.1406	6.17E-06
50	98.44486	5.23E-05	97.23147	7.20E-05	176	279.6824	1.06E-06	270.0233	6.58E-06
51	99.12545	6.29E-05	97.89642	8.66E-05	177	282.5843	7.67E-07	272.7	5.04E-06
52	99.16657	8.88E-05	97.93657	0.000122	178	283.1121	8.84E-07	273.1866	5.78E-06
53	100.3571	9.28E-05	99.0987	0.000128	179	283.2073	1.11E-06	273.2743	7.13E-06
54	105.7673	3.27E-05	104.3776	4.69E-05	180	283.2435	1.40E-06	273.3077	8.85E-06
55	110.1958	1.46E-05	108.6966	2.17E-05	181	286.1768	1.01E-06	276.0082	6.78E-06
56	110.9606	1.73E-05	109.4421	2.57E-05	182	286.5217	1.20E-06	276.3256	7.99E-06
57	113.1271	1.40E-05	111.5532	2.12E-05	183	287.9671	1.16E-06	277.6551	7.84E-06
58	118.5632	4.77E-06	116.8478	7.57E-06	184	291.0258	8.17E-07	280.467	5.89E-06
59	120.4369	4.20E-06	118.672	6.75E-06	185	291.0258	1.04E-06	280.467	7.35E-06
60	120.4694	6.07E-06	118.7036	9.68E-06	186	291.1895	1.27E-06	280.6173	8.90E-06
61	120.7059	8.24E-06	118.9336	1.31E-05	187	293.7628	9.90E-07	282.9798	7.26E-06
62	122.4345	7.53E-06	120.6143	1.21E-05	188	293.7775	1.25E-06	282.9932	9.01E-06
63	133.0299	6.33E-07	130.9111	1.13E-06	189	293.7995	1.57E-06	283.0135	1.11E-05
64	133.0327	9.35E-07	130.9137	1.66E-06	190	293.7998	1.98E-06	283.0137	1.38E-05

65	133.1583	1.33E-06	131.0356	2.34E-06	191	293.9505	2.42E-06	283.1518	1.66E-05
66	137.8715	5.41E-07	135.6099	1.00E-06	192	294.9086	2.54E-06	284.0293	1.76E-05
67	137.9454	7.80E-07	135.6816	1.43E-06	193	296.0166	2.60E-06	285.0436	1.82E-05
68	138.2744	1.04E-06	136.0006	1.91E-06	194	300.6846	1.38E-06	289.3149	1.07E-05
69	142.1301	5.40E-07	139.7375	1.02E-06	195	301.4462	1.51E-06	290.0114	1.17E-05
70	142.5706	7.00E-07	140.1644	1.33E-06	196	304.2013	1.14E-06	292.5299	9.35E-06
71	143.8262	7.28E-07	141.3802	1.39E-06	197	304.4167	1.37E-06	292.7267	1.12E-05
72	144.5247	8.76E-07	142.0563	1.67E-06	198	304.5028	1.70E-06	292.8053	1.36E-05
73	145.3304	1.02E-06	142.8358	1.95E-06	199	306.0671	1.60E-06	294.2333	1.31E-05
74	146.3016	1.14E-06	143.7749	2.19E-06	200	307.279	1.60E-06	295.339	1.33E-05
75	149.6092	6.90E-07	146.9721	1.37E-06	201	307.5187	1.92E-06	295.5577	1.57E-05
76	149.6096	9.90E-07	146.9725	1.95E-06	202	308.4238	2.04E-06	296.3827	1.68E-05
77	149.7393	1.36E-06	147.0978	2.67E-06	203	308.4408	2.54E-06	296.3982	2.05E-05
78	149.7407	1.93E-06	147.0991	3.74E-06	204	308.6908	3.02E-06	296.6258	2.41E-05
79	152.5855	1.33E-06	149.844	2.65E-06	205	311.897	2.12E-06	299.5443	1.81E-05
80	153.1039	1.65E-06	150.3439	3.28E-06	206	314.2303	1.73E-06	301.667	1.55E-05
81	157.4838	7.69E-07	154.5662	1.61E-06	207	315.3976	1.75E-06	302.7286	1.59E-05
82	157.5034	1.08E-06	154.5851	2.25E-06	208	315.4326	2.17E-06	302.7604	1.93E-05
83	157.5056	1.52E-06	154.5873	3.13E-06	209	315.5814	2.63E-06	302.8956	2.31E-05
84	158.6622	1.60E-06	155.7006	3.31E-06	210	316.9143	2.58E-06	304.106	2.30E-05
85	165.5828	3.97E-07	162.3602	8.95E-07	211	317.857	2.71E-06	304.9616	2.43E-05
86	165.8937	5.19E-07	162.6592	1.17E-06	212	318.019	3.26E-06	305.1085	2.89E-05
87	166.3824	6.47E-07	163.129	1.45E-06	213	318.6659	3.60E-06	305.6951	3.19E-05
88	171.2521	2.68E-07	167.8087	6.37E-07	214	318.9871	4.21E-06	305.9862	3.68E-05
89	171.572	3.49E-07	168.1159	8.27E-07	215	319.7696	4.54E-06	306.6951	3.98E-05
90	173.0427	3.40E-07	169.528	8.16E-07	216	321.01	4.52E-06	307.8182	4.02E-05
91	178.0121	1.37E-07	174.297	3.50E-07	217	321.0259	5.54E-06	307.8326	4.84E-05
92	178.0696	1.91E-07	174.3522	4.85E-07	218	321.0259	6.81E-06	307.8326	5.83E-05
93	178.1494	2.64E-07	174.4286	6.64E-07	219	321.0886	8.26E-06	307.8893	6.94E-05
94	179.2681	2.81E-07	175.5008	7.11E-07	220	321.9098	8.81E-06	308.6314	7.43E-05
95	179.3007	3.90E-07	175.532	9.77E-07	221	322.2348	1.02E-05	308.925	8.51E-05
96	180.4673	4.09E-07	176.649	1.03E-06	222	324.1239	9.10E-06	310.6307	7.85E-05
97	181.4541	4.47E-07	177.5935	1.13E-06	223	329.1936	4.75E-06	315.2056	4.65E-05
98	181.7086	5.82E-07	177.837	1.47E-06	224	329.6434	5.40E-06	315.6114	5.25E-05
99	185.0429	3.60E-07	181.0251	9.46E-07	225	331.8788	4.54E-06	317.6267	4.61E-05
100	185.2026	4.80E-07	181.1778	1.25E-06	226	332.24	5.24E-06	317.9522	5.27E-05
101	185.5739	6.05E-07	181.5326	1.57E-06	227	333.454	5.23E-06	319.0456	5.34E-05
102	185.5818	8.30E-07	181.5401	2.13E-06	228	333.454	6.40E-06	319.0456	6.40E-05
103	185.5832	1.13E-06	181.5414	2.88E-06	229	334.6395	6.42E-06	320.1124	6.51E-05
104	185.5849	1.54E-06	181.543	3.88E-06	230	334.7694	7.67E-06	320.2292	7.64E-05
105	190.1794	7.20E-07	185.9244	1.93E-06	231	335.4352	8.38E-06	320.8277	8.34E-05
106	191.2097	7.72E-07	186.9064	2.09E-06	232	335.4358	1.02E-05	320.8283	9.93E-05
107	191.9756	8.80E-07	187.6361	2.38E-06	233	336.5016	1.04E-05	321.7855	0.000103
108	193.3746	8.65E-07	188.9684	2.37E-06	234	336.5017	1.26E-05	321.7855	0.000122
109	196.0462	6.33E-07	191.5113	1.79E-06	235	336.9298	1.43E-05	322.1697	0.000137
110	197.004	6.89E-07	192.4227	1.96E-06	236	338.933	1.25E-05	323.9661	0.000125



111	197.6992	7.96E-07	193.0838	2.27E-06	237	340.9834	1.09E-05	325.804	0.000113
112	197.7122	1.07E-06	193.0962	3.03E-06	238	341.554	1.21E-05	326.3153	0.000124
113	198.0756	1.33E-06	193.4415	3.73E-06	239	341.6866	1.43E-05	326.434	0.000145
114	198.0932	1.78E-06	193.4581	4.93E-06	240	342.5333	1.51E-05	327.1918	0.000154
115	202.3632	9.13E-07	197.5115	2.70E-06	241	344.3038	1.38E-05	328.7757	0.000145
116	202.5771	1.17E-06	197.7145	3.43E-06	242	345.0589	1.48E-05	329.4509	0.000155
117	203.6111	1.24E-06	198.695	3.67E-06	243	346.2113	1.49E-05	330.4809	0.000158
118	207.109	7.61E-07	202.011	2.36E-06	244	348.1702	1.32E-05	332.2308	0.000145
119	207.735	8.87E-07	202.6041	2.75E-06	245	352.5729	7.88E-06	336.162	9.76E-05
120	208.7029	9.57E-07	203.5208	2.99E-06	246	352.6108	9.48E-06	336.1958	0.000115
121	208.894	1.22E-06	203.7017	3.79E-06	247	352.6112	1.15E-05	336.1962	0.000136
122	209.2311	1.51E-06	204.0207	4.66E-06	248	354.6125	1.01E-05	337.9805	0.000124
123	209.8462	1.76E-06	204.6025	5.40E-06	249	354.8431	1.17E-05	338.186	0.000142
124	209.9864	2.25E-06	204.735	6.86E-06	250	354.9461	1.39E-05	338.2777	0.000166
125	215.0592	9.90E-07	209.5284	3.27E-06	251	356.3102	1.35E-05	339.4921	0.000164
126	216.3641	9.89E-07	210.7608	3.31E-06	252	357.6496	1.32E-05	340.684	0.000164

Table 19: Ljung\_Box Test Results of META

Lag	lb_stat	lb_pvalue	bp_stat	bp_pvalue	Lag	lb_stat	lb_pvalue	bp_stat	bp_pvalue
1	24.19585	8.70E-07	24.16437	8.85E-07	127	296.9057	1.23E-15	291.0217	6.76E-15
2	26.14945	2.10E-06	26.11459	2.13E-06	128	296.9481	1.87E-15	291.0617	1.02E-14
3	27.06392	5.71E-06	27.02707	5.81E-06	129	302.0215	6.57E-16	295.8469	3.94E-15
4	27.22828	1.79E-05	27.191	1.82E-05	130	303.1276	7.35E-16	296.8897	4.46E-15
5	30.90355	9.79E-06	30.85512	1.00E-05	131	303.2008	1.11E-15	296.9586	6.66E-15
6	44.77961	5.18E-08	44.68304	5.41E-08	132	303.2037	1.70E-15	296.9613	1.01E-14
7	48.73272	2.56E-08	48.62072	2.69E-08	133	303.4642	2.40E-15	297.2066	1.42E-14
8	61.49423	2.37E-10	61.32688	2.56E-10	134	308.1647	9.51E-16	301.6299	6.16E-15
9	92.37844	5.42E-16	92.06377	6.27E-16	135	308.4982	1.32E-15	301.9436	8.52E-15
10	93.0907	1.30E-15	92.77233	1.51E-15	136	313.1161	5.35E-16	306.2851	3.77E-15
11	97.42624	5.77E-16	97.08342	6.74E-16	137	314.6017	5.33E-16	307.6812	3.84E-15
12	97.44384	1.76E-15	97.10092	2.05E-15	138	314.674	7.97E-16	307.7491	5.69E-15
13	97.5954	4.84E-15	97.25149	5.64E-15	139	314.6741	1.21E-15	307.7492	8.57E-15
14	100.2056	4.33E-15	99.8436	5.08E-15	140	319.09	5.23E-16	311.8931	4.03E-15
15	117.6739	5.34E-18	117.1831	6.65E-18	141	320.3882	5.50E-16	313.1109	4.31E-15
16	120.5532	4.30E-18	120.0399	5.40E-18	142	320.4102	8.29E-16	313.1315	6.43E-15
17	125.5023	1.37E-18	124.9482	1.75E-18	143	320.7012	1.16E-15	313.4042	8.92E-15
18	128.9216	8.52E-19	128.3379	1.10E-18	144	324.1422	6.60E-16	316.6273	5.44E-15
19	129.363	1.92E-18	128.7753	2.49E-18	145	325.9043	6.06E-16	318.2771	5.15E-15
20	129.37	5.12E-18	128.7822	6.59E-18	146	325.9347	9.08E-16	318.3056	7.62E-15
21	139.0683	2.05E-19	138.3837	2.75E-19	147	327.2722	9.39E-16	319.5566	8.03E-15
22	140.2105	3.29E-19	139.5141	4.44E-19	148	327.769	1.23E-15	320.0211	1.05E-14
23	141.9077	4.10E-19	141.1929	5.57E-19	149	327.7862	1.83E-15	320.0372	1.55E-14
24	144.605	3.31E-19	143.8598	4.54E-19	150	327.8131	2.72E-15	320.0624	2.27E-14
25	144.6605	8.08E-19	143.9146	1.11E-18	151	328.3443	3.49E-15	320.5582	2.92E-14
26	147.0902	7.15E-19	146.3149	9.89E-19	152	328.3451	5.19E-15	320.559	4.29E-14

27	147.1287	1.70E-18	146.3529	2.35E-18	153	328.5121	7.33E-15	320.7148	6.02E-14
28	147.13	4.04E-18	146.3542	5.57E-18	154	331.8091	4.41E-15	323.7888	3.87E-14
29	148.6254	5.10E-18	147.8295	7.07E-18	155	332.5918	5.27E-15	324.5182	4.65E-14
30	148.6387	1.17E-17	147.8425	1.62E-17	156	333.3561	6.32E-15	325.2301	5.61E-14
31	148.8059	2.47E-17	148.0074	3.41E-17	157	335.9028	4.67E-15	327.6012	4.35E-14
32	151.152	2.14E-17	150.3189	3.00E-17	158	337.1529	4.90E-15	328.7646	4.65E-14
33	151.5303	4.07E-17	150.6914	5.69E-17	159	337.4895	6.58E-15	329.0777	6.22E-14
34	158.3323	5.90E-18	157.3872	8.61E-18	160	338.8939	6.62E-15	330.3834	6.39E-14
35	159.5992	7.81E-18	158.6338	1.15E-17	161	339.833	7.54E-15	331.2561	7.36E-14
36	161.1248	9.26E-18	160.1343	1.37E-17	162	339.8644	1.09E-14	331.2852	1.06E-13
37	164.0431	6.30E-18	163.0032	9.49E-18	163	340.5344	1.34E-14	331.9073	1.29E-13
38	167.7455	3.14E-18	166.6413	4.85E-18	164	340.968	1.73E-14	332.3097	1.67E-13
39	169.3684	3.54E-18	168.2354	5.52E-18	165	341.531	2.17E-14	332.832	2.10E-13
40	170.5871	4.65E-18	169.4319	7.30E-18	166	342.3853	2.51E-14	333.624	2.45E-13
41	171.2306	7.59E-18	170.0634	1.19E-17	167	342.7499	3.30E-14	333.9619	3.21E-13
42	173.3428	6.96E-18	172.1353	1.11E-17	168	346.0442	2.01E-14	337.0133	2.10E-13
43	176.5012	4.25E-18	175.2321	6.93E-18	169	347.1833	2.16E-14	338.068	2.28E-13
44	176.7391	7.97E-18	175.4653	1.30E-17	170	350.7078	1.24E-14	341.3296	1.41E-13
45	178.4741	8.36E-18	177.1649	1.38E-17	171	351.4464	1.47E-14	342.0128	1.70E-13
46	180.7169	7.20E-18	179.361	1.21E-17	172	355.7482	6.89E-15	345.9899	8.74E-14
47	180.8512	1.37E-17	179.4925	2.29E-17	173	359.5732	3.64E-15	349.5247	5.03E-14
48	180.9803	2.60E-17	179.6187	4.33E-17	174	363.3421	1.95E-15	353.006	2.94E-14
49	181.0034	5.07E-17	179.6413	8.41E-17	175	363.5222	2.71E-15	353.1722	4.03E-14
50	181.0039	9.87E-17	179.6418	1.63E-16	176	363.9357	3.52E-15	353.5538	5.23E-14
51	181.4032	1.64E-16	180.0319	2.71E-16	177	364.1402	4.83E-15	353.7424	7.12E-14
52	181.4843	3.04E-16	180.1112	5.01E-16	178	364.9289	5.67E-15	354.4695	8.42E-14
53	182.6186	3.82E-16	181.2184	6.34E-16	179	367.3491	4.35E-15	356.6998	6.79E-14
54	185.0843	2.95E-16	183.6242	4.99E-16	180	367.5458	5.96E-15	356.881	9.22E-14
55	185.9584	4.03E-16	184.4767	6.87E-16	181	369.7067	4.89E-15	358.8704	7.91E-14
56	186.0854	7.17E-16	184.6006	1.22E-15	182	370.4761	5.76E-15	359.5784	9.38E-14
57	190.0934	3.18E-16	188.506	5.60E-16	183	372.2841	5.18E-15	361.2413	8.74E-14
58	192.1515	2.82E-16	190.5105	5.06E-16	184	372.4489	7.13E-15	361.3929	1.19E-13
59	192.167	5.17E-16	190.5256	9.24E-16	185	372.4611	1.02E-14	361.4041	1.67E-13
60	192.9514	7.17E-16	191.2889	1.29E-15	186	372.6188	1.40E-14	361.5489	2.27E-13
61	193.7638	9.79E-16	192.0791	1.76E-15	187	379.3023	3.58E-15	367.6846	6.92E-14
62	200.8647	1.47E-16	198.9829	2.85E-16	188	379.3027	5.14E-15	367.685	9.77E-14
63	211.7464	5.67E-18	209.5579	1.24E-17	189	385.4998	1.49E-15	373.3688	3.32E-14
64	211.9618	9.73E-18	209.7672	2.12E-17	190	387.4915	1.27E-15	375.1947	2.96E-14
65	212.3444	1.56E-17	210.1387	3.40E-17	191	387.6025	1.78E-15	375.2964	4.09E-14
66	213.5695	1.86E-17	211.3276	4.08E-17	192	388.2538	2.16E-15	375.8929	4.97E-14
67	214.0612	2.84E-17	211.8046	6.25E-17	193	393.2998	8.39E-16	380.5122	2.20E-14
68	214.0942	5.07E-17	211.8366	1.11E-16	194	393.3546	1.19E-15	380.5624	3.08E-14
69	214.2873	8.50E-17	212.0237	1.86E-16	195	393.3821	1.69E-15	380.5875	4.31E-14
70	214.3243	1.49E-16	212.0596	3.25E-16	196	395.043	1.58E-15	382.1058	4.16E-14
71	214.9044	2.17E-16	212.6214	4.72E-16	197	398.9672	8.23E-16	385.6913	2.40E-14
72	215.3483	3.28E-16	213.051	7.13E-16	198	399.0962	1.14E-15	385.8091	3.28E-14

73	215.432	5.56E-16	213.132	1.21E-15	199	399.099	1.63E-15	385.8117	4.60E-14
74	215.571	9.22E-16	213.2664	1.99E-15	200	399.514	2.09E-15	386.1904	5.87E-14
75	215.6716	1.54E-15	213.3637	3.31E-15	201	400.368	2.39E-15	386.9692	6.78E-14
76	215.9565	2.40E-15	213.6389	5.15E-15	202	401.3945	2.62E-15	387.9048	7.54E-14
77	215.9566	4.08E-15	213.639	8.72E-15	203	403.2355	2.33E-15	389.5822	6.98E-14
78	215.9997	6.81E-15	213.6806	1.45E-14	204	403.399	3.17E-15	389.7311	9.38E-14
79	216.3657	1.02E-14	214.0338	2.16E-14	205	403.4192	4.47E-15	389.7495	1.30E-13
80	220.4221	4.57E-15	217.9459	1.02E-14	206	403.5777	6.06E-15	389.8937	1.74E-13
81	221.2454	5.87E-15	218.7395	1.32E-14	207	406.7311	3.89E-15	392.7613	1.21E-13
82	231.3315	3.67E-16	228.4583	9.43E-16	208	411.4081	1.70E-15	397.0124	6.00E-14
83	231.3352	6.22E-16	228.4618	1.59E-15	209	411.4573	2.37E-15	397.0571	8.25E-14
84	232.5137	7.14E-16	229.5964	1.84E-15	210	412.2116	2.77E-15	397.742	9.70E-14
85	232.5139	1.20E-15	229.5966	3.08E-15	211	412.2962	3.83E-15	397.8188	1.32E-13
86	235.4376	7.79E-16	232.4087	2.07E-15	212	412.5475	5.06E-15	398.0468	1.73E-13
87	240.2104	2.78E-16	236.9973	7.87E-16	213	413.833	5.16E-15	399.2125	1.80E-13
88	244.2202	1.27E-16	240.8506	3.79E-16	214	415.0797	5.32E-15	400.3423	1.90E-13
89	245.8558	1.25E-16	242.4217	3.81E-16	215	419.4705	2.52E-15	404.32	1.01E-13
90	245.9585	2.03E-16	242.5202	6.15E-16	216	419.4798	3.53E-15	404.3284	1.40E-13
91	246.0388	3.30E-16	242.5973	9.95E-16	217	419.6089	4.80E-15	404.4452	1.87E-13
92	246.2143	5.18E-16	242.7657	1.55E-15	218	419.6953	6.57E-15	404.5234	2.52E-13
93	247.5271	5.64E-16	244.0244	1.71E-15	219	419.6954	9.17E-15	404.5235	3.46E-13
94	247.5507	9.21E-16	244.047	2.78E-15	220	419.7106	1.27E-14	404.5372	4.72E-13
95	248.0667	1.28E-15	244.5413	3.87E-15	221	420.825	1.35E-14	405.5439	5.09E-13
96	250.0793	1.11E-15	246.4684	3.44E-15	222	423.2022	1.05E-14	407.6901	4.22E-13
97	252.1174	9.56E-16	248.419	3.04E-15	223	424.6761	1.03E-14	409.0202	4.22E-13
98	252.4182	1.42E-15	248.7067	4.49E-15	224	428.6336	5.46E-15	412.5899	2.51E-13
99	254.3782	1.25E-15	250.5809	4.05E-15	225	428.6537	7.56E-15	412.608	3.41E-13
100	254.6338	1.86E-15	250.8252	6.03E-15	226	428.8087	1.01E-14	412.7476	4.50E-13
101	256.6998	1.58E-15	252.7989	5.27E-15	227	429.7927	1.11E-14	413.6339	4.99E-13
102	262.1651	4.69E-16	258.0177	1.70E-15	228	430.0615	1.43E-14	413.8759	6.41E-13
103	269.4822	7.72E-17	265.0016	3.14E-16	229	431.0142	1.58E-14	414.7332	7.15E-13
104	270.445	9.31E-17	265.9201	3.82E-16	230	431.353	2.01E-14	415.038	9.03E-13
105	270.6392	1.43E-16	266.1054	5.82E-16	231	432.3343	2.19E-14	415.9201	9.99E-13
106	275.0923	5.76E-17	270.3499	2.52E-16	232	432.662	2.79E-14	416.2145	1.26E-12
107	276.6928	5.68E-17	271.8748	2.54E-16	233	436.6174	1.50E-14	419.7668	7.60E-13
108	276.7011	9.18E-17	271.8826	4.07E-16	234	439.4193	1.06E-14	422.282	5.80E-13
109	277.4216	1.19E-16	272.5685	5.28E-16	235	441.4425	9.06E-15	424.0973	5.19E-13
110	278.7858	1.25E-16	273.8664	5.67E-16	236	442.993	8.63E-15	425.4877	5.11E-13
111	279.4126	1.66E-16	274.4625	7.52E-16	237	442.9944	1.19E-14	425.489	6.90E-13
112	279.7938	2.36E-16	274.8249	1.07E-15	238	443.5578	1.43E-14	425.9938	8.30E-13
113	281.3951	2.31E-16	276.3463	1.07E-15	239	443.7371	1.88E-14	426.1543	1.08E-12
114	282.3785	2.73E-16	277.2803	1.27E-15	240	444.4238	2.20E-14	426.769	1.26E-12
115	282.422	4.27E-16	277.3216	1.98E-15	241	446.0807	2.04E-14	428.2512	1.22E-12
116	282.7376	6.13E-16	277.621	2.83E-15	242	448.6711	1.52E-14	430.5675	9.73E-13
117	282.9539	9.04E-16	277.8261	4.15E-15	243	448.9236	1.97E-14	430.7932	1.24E-12
118	283.4194	1.23E-15	278.2674	5.67E-15	244	449.3451	2.44E-14	431.1698	1.53E-12

119	286.1861	8.48E-16	280.8889	4.07E-15	245	450.1526	2.76E-14	431.8907	1.74E-12
120	286.8005	1.10E-15	281.4708	5.31E-15	246	450.3237	3.61E-14	432.0434	2.25E-12
121	287.9838	1.21E-15	282.591	5.92E-15	247	451.8955	3.42E-14	433.4455	2.21E-12
122	291.2099	7.28E-16	285.6436	3.75E-15	248	457.502	1.27E-14	438.4442	9.76E-13
123	291.3851	1.08E-15	285.8093	5.51E-15	249	457.5065	1.73E-14	438.4482	1.30E-12
124	291.4301	1.65E-15	285.8519	8.36E-15	250	457.7225	2.24E-14	438.6406	1.67E-12
125	291.9784	2.17E-15	286.37	1.10E-14	251	458.5912	2.49E-14	439.414	1.88E-12
126	292.103	3.22E-15	286.4876	1.63E-14	252	459.3211	2.87E-14	440.0635	2.17E-12

Table 20: Ljung\_Box Test Results of Microsoft

### III. Mid-Term Stock Price Forecasts (Real vs. Predicted)

Date	Real Adj. Close	ARIMA-GARCH Forecast	GRU Forecast
3/1/2024	138.08	117.0109123	140.77133
3/4/2024	134.20	117.1148563	141.83636
3/5/2024	133.78	117.1190027	142.74832
3/6/2024	132.56	117.2080723	143.68272
3/7/2024	135.24	117.2153377	144.5996
3/8/2024	136.29	117.3022552	145.50374
3/11/2024	138.94	117.3115488	146.38942
3/12/2024	139.62	117.3964951	147.25711
3/13/2024	140.77	117.4077094	148.10616
3/14/2024	144.34	117.4907838	148.93617
3/15/2024	142.17	117.5038225	149.74673
3/18/2024	148.48	117.5851188	150.53743
3/19/2024	147.92	117.5998905	151.30797
3/20/2024	149.68	117.6794978	152.05807
3/21/2024	148.74	117.6959156	152.78758
3/22/2024	151.77	117.7739186	153.4963
3/25/2024	151.15	117.7918999	154.18423
3/26/2024	151.70	117.868379	154.8513
3/27/2024	151.94	117.8878456	155.49756
3/28/2024	152.26	117.9628772	156.12308
4/1/2024	156.50	117.9837545	156.72797
4/2/2024	155.87	118.0574112	157.31245
4/3/2024	156.37	118.0796285	157.87672
4/4/2024	151.94	118.1519792	158.42104
4/5/2024	153.94	118.1754693	158.94566
4/8/2024	156.14	118.2465796	159.45093
4/9/2024	158.14	118.2712787	159.9372
4/10/2024	157.66	118.3412106	160.40485
4/11/2024	160.79	118.3670581	160.85428
4/12/2024	159.19	118.4358708	161.2859
4/15/2024	156.33	118.4628091	161.70016
4/16/2024	156.00	118.5305587	162.09752

4/17/2024	156.88	118.5585331	162.4784
4/18/2024	157.46	118.6252729	162.8433
4/19/2024	155.72	118.6542315	163.19272
4/22/2024	157.95	118.720012	163.52707
4/23/2024	159.92	118.7499055	163.8469
4/24/2024	161.10	118.8147749	164.15265
4/25/2024	157.95	118.8455564	164.44481
4/26/2024	173.69	118.9095604	164.72385
4/29/2024	167.90	118.9411854	164.99023
4/30/2024	164.64	119.0043673	165.24446
5/1/2024	165.57	119.0367934	165.48695
5/2/2024	168.46	119.0991945	165.7182
5/3/2024	168.99	119.1323817	165.93864
5/6/2024	169.83	119.194041	166.14868
5/7/2024	172.98	119.2279511	166.34874
5/8/2024	171.16	119.2889059	166.53928
5/9/2024	171.58	119.3235026	166.72064
5/10/2024	170.29	119.3837882	166.89323
5/13/2024	170.90	119.4190371	167.05743
5/14/2024	171.93	119.4786871	167.21362
5/15/2024	173.88	119.5145555	167.36217
5/16/2024	175.43	119.5736017	167.50336
5/17/2024	177.29	119.6100585	167.63756
5/20/2024	178.46	119.6685312	167.7651
5/21/2024	179.54	119.705547	167.88628
5/22/2024	178.00	119.763475	168.00139
5/23/2024	175.06	119.8010217	168.11072
5/24/2024	176.33	119.8584322	168.21452
5/28/2024	178.02	119.8964832	168.31306
5/29/2024	177.40	119.9534022	168.4066
5/30/2024	173.56	119.9919323	168.4954

Table 21: Mid-Term Stock Price Forecasts (Real vs. Predicted) for Alphabet

Date	Real Adj. Close	ARIMA-GARCH Forecast	GRU Forecast
3/1/2024	502.299988	210.5180617	489.88678
3/4/2024	498.190002	210.4653215	488.42148
3/5/2024	490.220001	210.6770481	487.8196
3/6/2024	496.089996	210.6036823	487.45547
3/7/2024	512.190002	210.8142265	487.2603
3/8/2024	505.950012	210.7430348	487.05295
3/11/2024	483.589996	210.9513757	486.80322
3/12/2024	499.75	210.8823722	486.5621
3/13/2024	495.570007	211.0885419	486.34396
3/14/2024	491.829987	211.0216926	486.14682
3/15/2024	484.100006	211.225725	485.96536

3/18/2024	496.980011	211.1609963	485.79602
3/19/2024	496.23999	211.3629247	485.6382
3/20/2024	505.519989	211.3002835	485.49158
3/21/2024	507.76001	211.5001407	485.35544
3/22/2024	509.579987	211.4395544	485.22913
3/25/2024	503.019989	211.6373728	485.11182
3/26/2024	495.890015	211.5788094	485.00278
3/27/2024	493.859985	211.7746208	484.90146
3/28/2024	485.579987	211.7180487	484.8073
4/1/2024	491.350006	211.9118843	484.71988
4/2/2024	497.369995	211.8572726	484.63858
4/3/2024	506.73999	212.0491632	484.56302
4/4/2024	510.920013	211.9964812	484.49283
4/5/2024	527.340027	212.1864572	484.42758
4/8/2024	519.25	212.1356748	484.3669
4/9/2024	516.900024	212.3237661	484.31046
4/10/2024	519.830017	212.2748537	484.2581
4/11/2024	523.159973	212.4610896	484.20932
4/12/2024	511.899994	212.414018	484.164
4/15/2024	500.230011	212.5984275	484.1219
4/16/2024	499.76001	212.5531681	484.08276
4/17/2024	494.170013	212.7357796	484.04633
4/18/2024	501.799988	212.692304	484.01242
4/19/2024	481.070007	212.8731457	483.98096
4/22/2024	481.730011	212.8314261	483.95163
4/23/2024	496.100006	213.0105255	483.9244
4/24/2024	493.5	212.9705346	483.89905
4/25/2024	441.380005	213.1479188	483.87552
4/26/2024	443.290009	213.1096297	483.8536
4/29/2024	432.619995	213.2853255	483.83322
4/30/2024	430.170013	213.2487115	483.8142
5/1/2024	439.190002	213.4227453	483.7966
5/2/2024	441.679993	213.3877803	483.78018
5/3/2024	451.959991	213.560178	483.765
5/6/2024	465.679993	213.5268363	483.75082
5/7/2024	468.23999	213.6976233	483.7376
5/8/2024	472.600006	213.6658797	483.72537
5/9/2024	475.420013	213.8350813	483.71396
5/10/2024	476.200012	213.8049107	483.70328
5/13/2024	468.01001	213.9725515	483.69342
5/14/2024	471.850006	213.9439295	483.68423
5/15/2024	481.540009	214.1100338	483.6757
5/16/2024	473.230011	214.0829362	483.66776
5/17/2024	471.910004	214.2475281	483.66034
5/20/2024	468.839996	214.2219311	483.65347
5/21/2024	464.630005	214.3850341	483.64713

5/22/2024	467.779999	214.3609144	483.6411
5/23/2024	465.779999	214.5225517	483.63553
5/24/2024	478.220001	214.4998861	483.63037
5/28/2024	479.920013	214.6600807	483.62564
5/29/2024	474.359985	214.6388466	483.6212
5/30/2024	467.049988	214.7976209	483.61697

Table 22: Mid-Term Stock Price Forecasts (Real vs. Predicted) for META

Date	Real Adj. Close	ARIMA-GARCH Forecast	GRU Forecast
3/1/2024	414.751892	277.3374435	412.61093
3/4/2024	414.172974	278.2081936	412.5961
3/5/2024	401.925018	277.798426	411.83636
3/6/2024	401.366028	278.4038513	411.70245
3/7/2024	408.403381	278.068078	411.66315
3/8/2024	405.488617	278.6384933	411.70847
3/11/2024	403.791656	278.3293514	411.69836
3/12/2024	414.532288	278.8753534	411.6664
3/13/2024	414.352631	278.5896583	411.6245
3/14/2024	424.454407	279.1129143	411.58896
3/15/2024	415.670258	278.8493315	411.55884
3/18/2024	416.568634	279.3510821	411.53198
3/19/2024	420.651276	279.1084164	411.5061
3/20/2024	424.464386	279.5898216	411.48096
3/21/2024	428.596924	279.3669453	411.45663
3/22/2024	427.968048	279.8291019	411.43356
3/25/2024	422.098633	279.6249481	411.4116
3/26/2024	420.890808	280.068894	411.3907
3/27/2024	420.671204	279.8824532	411.3708
3/28/2024	419.962494	280.3091702	411.35178
4/1/2024	423.805573	280.1394874	411.33368
4/2/2024	420.681213	280.5499044	411.31647
4/3/2024	419.692993	280.3960761	411.30002
4/4/2024	417.127625	280.7910719	411.2844
4/5/2024	424.753845	280.6522434	411.26947
4/8/2024	423.825531	281.0326493	411.2553
4/9/2024	425.512482	280.9080118	411.24176
4/10/2024	422.497925	281.2746146	411.22885
4/11/2024	427.159515	281.1634031	411.21655
4/12/2024	421.140381	281.5169469	411.2048
4/15/2024	412.895264	281.4184374	411.19363
4/16/2024	413.833557	281.7596263	411.18304
4/17/2024	411.09848	281.6731341	411.1729
4/18/2024	403.542114	282.0026341	411.16327
4/19/2024	398.401398	281.9275113	411.15405
4/22/2024	400.238068	282.2459527	411.14526
4/23/2024	406.836182	282.1815862	411.13693

4/24/2024	408.323486	282.4895653	411.129
4/25/2024	398.321533	282.4353753	411.1214
4/26/2024	405.58844	282.7334559	411.11417
4/29/2024	401.525757	282.6888938	411.1072
4/30/2024	388.628998	282.9776098	411.10062
5/1/2024	394.228912	282.9421563	411.09436
5/2/2024	397.123688	283.2220125	411.0884
5/3/2024	405.927826	283.1951767	411.08273
5/6/2024	412.795441	283.4666508	411.07724
5/7/2024	408.602997	283.447968	411.0721
5/8/2024	409.800842	283.7115119	411.0672
5/9/2024	411.577637	283.7005426	411.0625
5/10/2024	413.993256	283.9565839	411.05804
5/13/2024	412.975098	283.9529121	411.0538
5/14/2024	415.809998	284.2018553	411.0497
5/15/2024	423.079987	284.2050876	411.04587
5/16/2024	420.98999	284.4473153	411.0422
5/17/2024	420.209991	284.4570796	411.03867
5/20/2024	425.339996	284.6929539	411.03534
5/21/2024	429.040009	284.708898	411.03217
5/22/2024	430.519989	284.9387614	411.02914
5/23/2024	427	284.960552	411.02628
5/24/2024	430.160004	285.1847287	411.02344
5/28/2024	430.320007	285.2120507	411.02087
5/29/2024	429.170013	285.4308472	411.01834
5/30/2024	414.670013	285.4634023	411.01602

Table 23: Mid-Term Stock Price Forecasts (Real vs. Predicted) for Microsoft

#### IV. Sample of Selected and Analysed Articles for Literature Review

Title	Year	AIM	Findings	Journal
Research on Stock Returns Forecast of the Four Major Banks Based on ARMA and GARCH Model	2020	Compare the effectiveness of ARMA and GARCH models in forecasting daily stock returns of four major banks	The GARCH model better captured historical volatility patterns of the banks' stock returns, but, the ARMA model offered slightly more accurate forecasts for future prices. This suggests past volatility might not always predict future stock movements.	J. Phys
A Prediction Approach for Stock Market Volatility Based on Time Series Data	2019	explore the use of ARIMA models, to predict stock market movements.	The ARIMA model provided a reasonably accurate forecast	IEEE



Stock Price Prediction Using the ARIMA Model	2014	the effectiveness of ARIMA models in predicting stock prices	The ARIMA model performed well, particularly for short-term predictions for both the NYSE and NSE. It could be a competitive technique in the field of stock price forecasting.	UKSim-AMSS 16th International Conference on Computer Modelling and Simulation
ARMA–GARCH model with fractional generalized hyperbolic innovations	2022	Develop a more sophisticated model to capture the complexities of high-frequency stock returns	Introduces a multivariate ARMA-GARCH model that incorporates fractional generalized hyperbolic (fGH) innovations. The fGH model more effectively captures the fat tails, volatility clustering, and long-range dependence properties compared to the standard model.	Financial Innovation
Stock price prediction using LSTM, RNN and CNN-sliding window model	2017	develop a model-independent approach using three deep learning architectures to predict short-term stock prices for companies listed on the NSE (National Stock Exchange of India)	Convolutional Neural Network (CNN) performed best among three deep learning architectures tested for short-term stock price prediction.	Proceedings of the 2017 International Conference on Advances in Computing, Communications and Informatics (ICACCI)
Forecasting multistep daily stock prices for long-term investment decisions: A study of deep learning models on global indices	2024	the ability of deep learning models to predict long-term (up to a year) daily prices of global stock indices	LSTM emerged as the best deep learning model for long-term (up to a year) stock price prediction of global indices, outperforming other models like CNN and RNN.	Engineering Applications of Artificial Intelligence
A LSTM-based method for stock returns prediction: A case study of China stock market	2015	the effectiveness of a Long Short-Term Memory (LSTM) network in predicting stock returns in the Chinese market	LSTM model improved the accuracy of stock return prediction in China from 14.3% (random guessing) to 27.2%, highlighting LSTM's potential for this task despite the market's inherent difficulty	IEEE International Conference on Big Data (Big Data)
ARIMA vs LSTM on NASDAQ stock exchange data	2022	compares the performance of ARIMA (statistical) and LSTM (deep	The study found that ARIMA outperformed LSTM for predicting NASDAQ stock prices	Procedia Computer Science

		learning) models for predicting stock prices on the NASDAQ exchange	when limited to one feature (historical price) and predicting multiple future periods (months). ARIMA's advantage increased with longer prediction windows (30 days to 9 months).	
A Comparison of ARIMA and LSTM in Forecasting Time Series	2018	whether deep learning algorithms like LSTM outperform traditional methods like ARIMA for forecasting time series data	The study shows that LSTM, a deep learning approach, significantly outperforms the traditional ARIMA model in forecasting time series data.	IEEE International Conference on Machine Learning and Applications (ICMLA)

Table 24: Sample of selected and analysed articles for literature review