1 Problem modeling

In what follows we will state the stochastic optimization problem that we want to solve. An investor pays a price p_b at time 0. In exchange, he can use a battery with a power capacity $\bar{C}$ during T units of time. At each time step the investor looks at the electricity price for the specific time. Then, he can buy or sell electricity on the market. He can store the energy he has not sold. Is goal is to maximise is terminal wealth.

We introduce the following variables

- Z_t is the wealth of the investor at time t,
- C_t is the energy stored at time t,
- If U_t is positive, it is the energy bought on the market at time t. If it is negative, $-U_t$ is the energy sold to the market,
- P_t is the price of one unit of electricity at time t,
- r_t is the interest rate at time t,
- l_t is the proportion of electricity stored in the battery that is lost between t and t+1.

The electricity prices and the interest rates are both random. Instead of maximising the terminal wealth directly, we will maximise a convex increasing function of the terminal wealth. This function is called utility function according to classic economics theories. In what follows we will use the following utility function u defined by

$$u(z) = -\frac{\exp(-\rho z)}{\rho}$$

where ρ is a fixed parameter.

The problem can be expressed as follows

$$\begin{array}{lll}
\min\limits_{\pi \in \Pi} & E(-u(Z_T)) & \\
st. & Z_{t+1} = (1+r_t)Z_t - P_t U_t & \forall t \in \{0, ..., T-1\}, \\
& C_{t+1} \ = (1-l_t)C_t + U_t & \forall t \in \{0, ..., T-1\}, \\
& U_t = \pi_t(Z_t, C_t, P_t, r_t) & \forall t \in \{0, ..., T-1\}, \\
& C_t \in [0, \bar{C}] & \forall t \in \{1, ..., T\}, \\
& U_t \in [-C, \bar{C}] & \forall t \in \{0, ..., T-1\}, \\
& Z_0 = -p_b, & \\
& C_0 = 0. &
\end{array}$$

2 Price model

We chose the following price model:

$$\ln(P_t) = x_t + S_t$$

where (S_t) is a deterministic seasonality function, and (x_t) is a first order autoregressive process:

$$x_t = \phi x_{t-1} + \sigma \epsilon_t$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$.