Problem description

Consider a stochastic optimization problem where an investor has paid $Q \in \mathbb{R}$ units of cash for the use of an electricity storage of maximum capacity $\bar{C}$ over time period $[0, T]$. The investor aims to use the storage optimally in order to maximize their terminal wealth by trading electricity at the prevailing market prices $(P_t)_{t=0}^T$. Any money generated over time will be invested in the money market at the prevailing money market rate $(r_t)_{t=0}^T$. We assume that if U_t units of energy is bought from the electricity market, the level of stored energy increases by $L_t(U_t)$, where L_t is a concave function on the real line. If U_t is negative, it means that $-U_t$ units of energy is sold in the electricity market.

Given a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $(\mathcal{F}_t)_{t=0}^T$ (an increasing sequence of sub-σ-algebras of $\mathcal{F}$), consider the problem

$$\begin{array}{ll} \text{maximize} & Eu(Z_T) \quad \text{over} \ \ (Z, C, U) \in \mathcal{N} \\ \text{subject to} & Z_0 = -Q, \\ & Z_t \leq (1 + r_t) Z_{t-1} - P_t U_t, \\ & C_t \leq (1 - l_t) C_{t-1} + L_t(U_t), \\ & C_t \in [0, \bar{C}], \end{array} \tag{1}$$

where Z_t is the amount of money invested in the money market, C_t is the amount of energy in the storage at time t and $\mathcal{N}$ denotes the linear space of three-dimensional stochastic processes adapted to the filtration $(\mathcal{F}_t)_{t=0}^T$. The parameter l_t is the storage loss over period $[t-1, t]$.