Leviosa:
Linear optimization of solenoid-based EM levitation

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Contents

Abstract

This paper aims to investigate the use of mathematical optimization to achieve stable levitation through *attraction* in an electromagnetic system.

A system of interactions between magnetic objects will modelled for optimization; in particular, equations that govern electromagnetic systems and gravitational forces will be utilized. Fundamental concepts such as flux, divergence and gradient, will compose the core of this optimization experiment.

1 Introduction

Magnetic levitation has been the subject of many science fiction stories and future predictions. From floating cities to maglev trains, it is hard for us to imagine when defying gravity with magnets was considered normal.

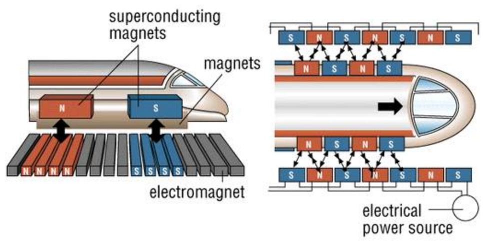

Figure 1: Interaction between electromagnets in Maglev trains [1]

To explore this phenomenon, we have utilized Linearity II principles to optimize for a permanent magnet's uniform levitation. Using PID control, gradients, and magnetism principles, we have successfully demonstrated the relationship between the current through a solenoid and the levitation of a permanent magnet (e.g. Neodymium).

2 Model

System Characterization

Our model examines the optimization of an electromagnetic levitation system with two main forces at play: (1) Gravitational force and (2) Magnetic force.

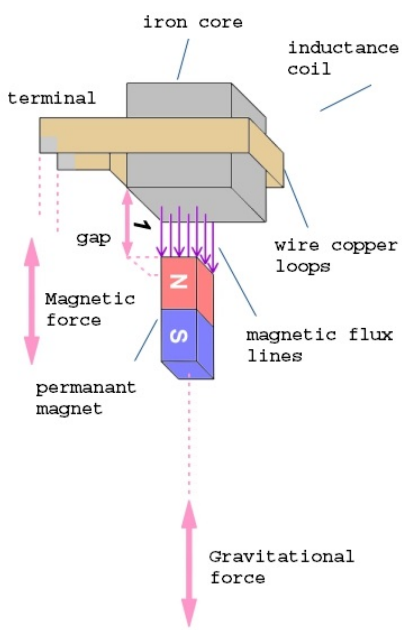

Figure 2: Pictorial representation of the magnetic system

Interacting Elements

The electromagnetic force induced by the solenoid on the permanent magnet can be characterized as:

$$U = -\int M(x) \cdot B(x) dV$$

$$\vec{F}_m = \nabla(M(x) \cdot B(x))$$

where M(x) is the *magnetic moment of the magnetic object being levitated*, and F is the *gradient of the field potential, or the interaction energy*, **U**.

The primary opposing vector is the **gravitational force**, which can be classically expressed as:

$$\vec{F}_g = m\vec{g}$$

where g is the gravitational acceleration of the Earth.

Stability is therefore defined as the *dynamic equilibrium* between the competing primary vectors - electromagnetic force and gravitational force.

$$\vec{F}_m + \vec{F}_g = 0$$

$$\nabla(M(x) \cdot B(x)) + m\vec{g} = 0$$

Solenoid

The solenoid will be fixed at our "origin" and will be considered as the reference point, so we don't have to consider the gravitation forces acting upon it. Magnetic forces prove to be more complex. The solenoid generates a nearly uniform magnetic field due to the turns and current, aptly summarized by the Biot-Savart Law:

$$\mathbf{B} = \frac{\mu_\mathbf{0}}{\mathbf{4}\pi} \int_\gamma \frac{\mathbf{Id}\ell \times \mathbf{\hat{r}}}{\mathbf{r^2}}$$

μ_0 represents the permittivity of free space,
I represents the current through the solenoid,
dl represents the small change in solenoid length,
r represents the distance from a point in question to the solenoid, and finally,
$\hat{r}$ represents the direction of the magnetic field at the point in question.

Thus, the magnetic flux density is computed by taking the path integral over the trajectory of current flow. The resulting B magnetic field from this calculation influences the permanent magnet. Another result of the solenoid is H, the magnetic field strength. The magnetic field strength is proportionally related to the B field:

$$H = B/\mu_0$$

This magnetic field strength plays an important role in obtaining the magnetization of the permanent magnet, which we'll discuss in the next section.

Levitron

A hard (i.e. *magnetized*) ferromagnet is selected as the object for modelling the attraction-based levitation upon.

Magnetic field strength is given by:

$$H = \frac{B}{\mu_0} - M$$

Correspondingly, the magnetic flux density is characterized as:

$$B = \mu H$$

Since there is a direct proportionality between *magnetic field strength* and *magnetic flux density*, H can be optimized by increasing flux density. This in turn, can be determined by material choice, primarily though μ_r, the *relative permeability of the material.*

It should be noted that permeability is related by :

$$\mu = \mu_r \mu_0$$

From the figure below, it can be shown that the relative permeability of ferromagnetic materials are *substantially higher* than that of other magnetic types. Consequently, ferromagnetic materials also enjoy considerably larger flux densities than other materials.

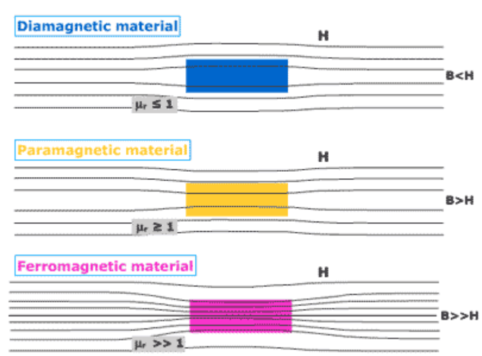

Figure 3: Permeability and field strengths of different magnetic materials [2]

Additionally, ferromagnets are subject to a phenomenon known as *hysteresis*. When exposed to an *externally* applied magnetic field H, the ferromagnet's dipoles exhibit some form of resistance to magnetization - also known as *coercivity*. In the absence of a external magnetic field, a magnetized ferromagnet would also retain some level of magnetization; this level is also known as the *magnetic remanance*.

Depending on the magnetic field strength (H) imposed upon the hard ferromagnet, its' magnetization (M) changes. We can then obtain the magnetic force required to levitate the magnet, based on the values of M.

Jiles-Atherton Model for Hysteresis

There are several mathematical models that characterizes the hysteresis effect upon a ferromagnetic levitron - for example, the Preisach model, created in 1935, was an early generalization of hysteresis in ferromagnets. It however, did not include the physical parameters of the magnetic material in its' modelling considerations.

The Jiles-Atherton model, introduced in 1984, solved that problem, enabling calculations of minor and major hysteresis loops to be made. Revisions by Szewczyk between 2006 to 2012 extended the model to apply to both anisotropic and isotropic materials.

The Jiles-Atherton model for ferromagnetism can be characterized by two sets of equations:

$$B = \mu_0 M$$
$$M = M_{rev} + M_{irr}$$

where B is the magnetic flux density, and M is the magnetization, which is composed of both reversible (M_{rev}) and irreversible (M_{irr}) components.

Both components correspond to the magnetic domain wall's bending and displacement, which accounts for the S-shaped portion of the magnetization curve, and the hysteresis portion respectively. M_{rev} is related to M_{irr} by:

$$M_{rev} = c(M_{an} - M_{irr})$$

where M_{an} represents the anhysteretic magnetization when $c = 1$, where $M = M_{an}$ effectively, allowing us to disregard M_{irr}. (M_{irr} does not contribute to M in calculating anhysteretic magnetization)

The simultaneous forms M_{an}, M_{irr}, and $\frac{dM_{irr}}{dH}$ are given by:

$$M_{an} = M_s[coth(\frac{H + \alpha M)}{A} - \frac{A}{H + \alpha M}]$$

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})}$$

The Langevin function $\mathscr{L}(x)$ can be used to discretize the quantity in the expression M_{an} for easy expression, where:

$$\mathscr{L}(x) = coth(x) - \frac{1}{x}$$

The function can be *approximated* to a piecewise function, containing 3 parts:

$$\mathscr{L}(x) = \begin{cases} \frac{x}{3} & |x| \ll 1 \\ 1 & x \gg 1 \\ -1 & x \ll -1 \end{cases}$$

δ represents the polarity of time rate of change of the applied magnetic field H, where:

$$\delta = \begin{cases} +1 & \frac{dH}{dt} > 0 \\ -1 & \frac{dH}{dt} < 0 \end{cases}$$

Finally, M_{rev} can be substituted in the equation with the use of the previous simultaneous forms, to simplify M as:

$$M = cM_{an} + (1 - c)M_{irr}$$

Other defined constants include α, the mean field parameter, c, the weighting between anhysteretic and irreversible components, k, the size of the hysteresis (A/m), and A, the scale for the magnetic field strength (A/m).

By using a polyfitted interpolation, we can find the value of M, the magnetization of the levitron, when exposed to an applied magnetic field, H, to a *floating point* degree.

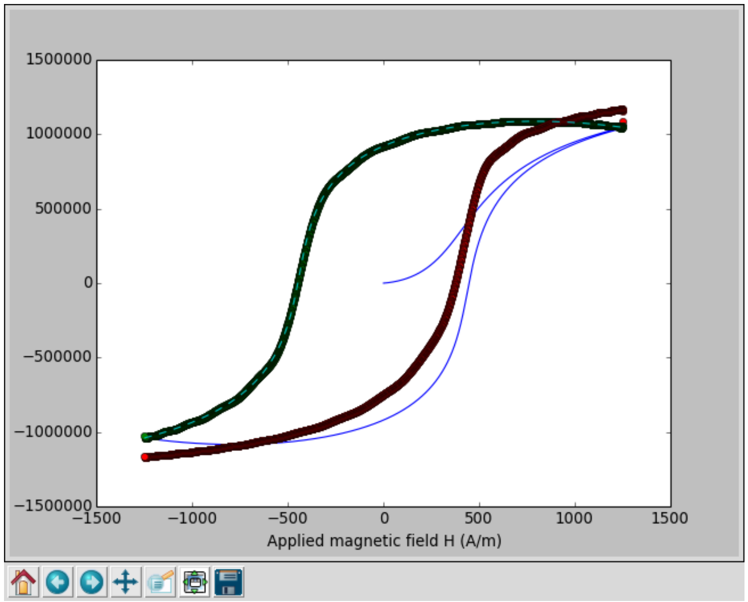

Figure 4: Using polyfitting to find fine-values of the simulated hysteresis curve

We can also isolate both anhysteric components of the S-curve, which reflects the magnetization response in *either* polarity, as shown below:

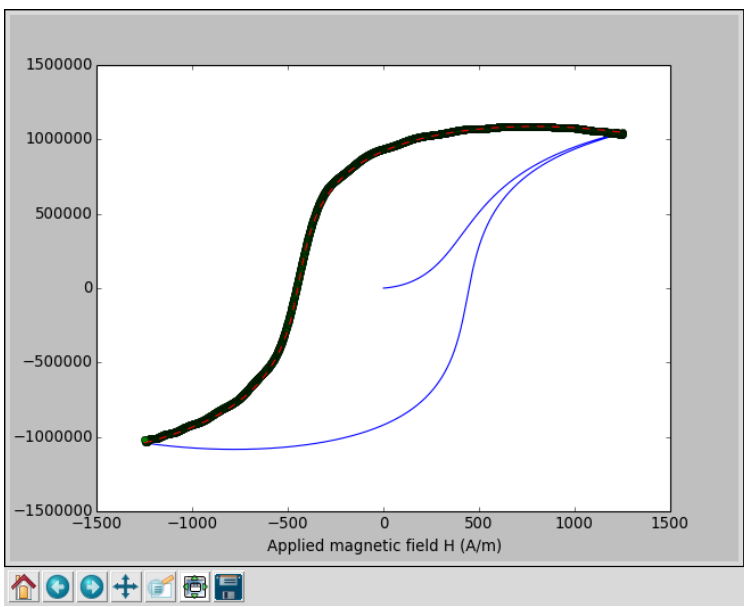

Figure 5: Modelling a polyfitted curve on the positive side of the S-curve

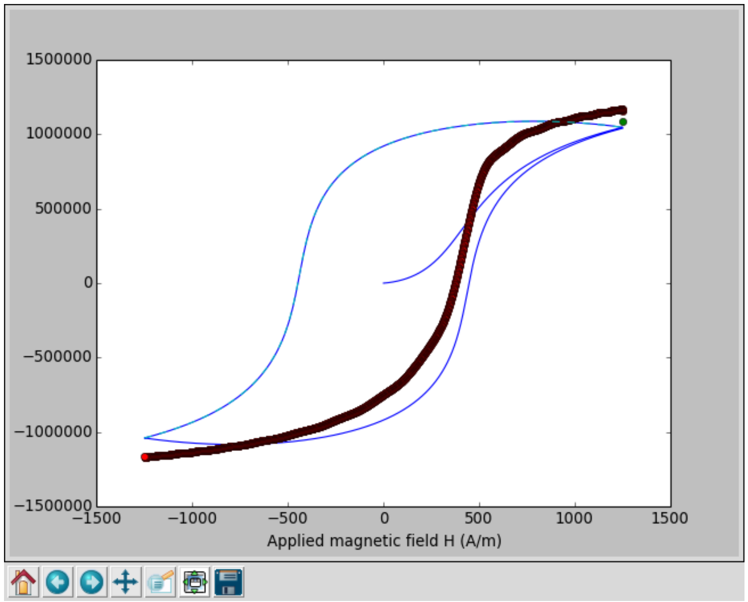

Figure 6: Modelling a polyfitted curve on the negative side of the S-curve

The magnetization values derived from the hysteresis curve are then used to derive the magnetic force induced upon the levitron, allowing a control algorithm to optimize its' value for levitation, using any given position in the z axis.

3 Control & Optimization

After we modelled the physical interactions that govern our system, we implemented a PID controller to control the position of the levitating magnet to a fixed point.

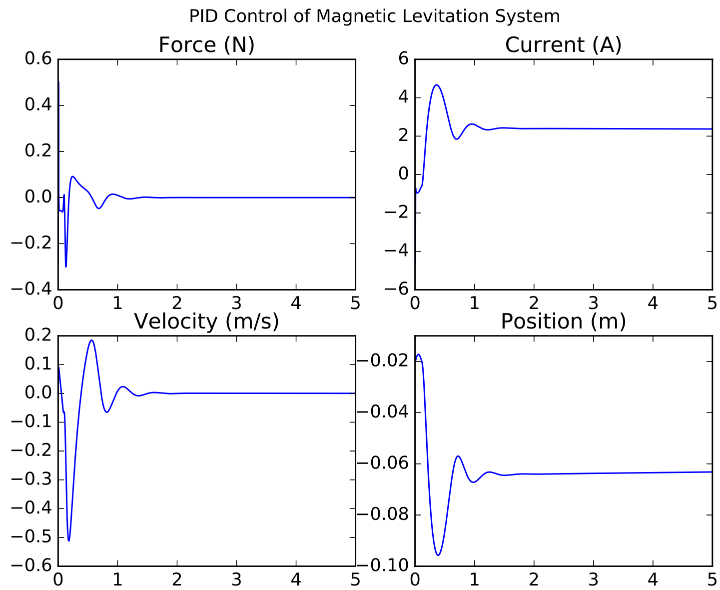

Figure 7: PID Control of the modelled system.

In controlling the system, initial conditions were tuned such that the system wouldn't undergo extreme disturbances. This is due to the fact that the strength of the magnetic force decreases in the 7th order of the distance between two magnets, so even a small change can manifest into a system-wide failure. The system was controlled as follows:

$$e(t) = p_t(t) - p(t)$$

$$c(t) = k_p e(t) + k_i \int_t e(t)dt + k_d \dot{e(t)}$$

where $k_p = 70$, $k_i = 0.3$, and $k_d = 2.0$ are empirically determined parameters.

4 Conclusion

We have grown from this project (like, really. Some of us grew beards from this). We learned tediously modeling a complex system with magnetic interactions between objects. Developing this project included skills like programming, abstracting, and analyzing to make sure our model was reasonable. We definitely struggled with certain aspects of this project, such as developing the piecewise hysteresis function or developing the control algorithm for the current, but we've all solidified our knowledge of electromagnetic forces.

As a next step, we would like to incorporate fluctuations in back-emf to complete a fully closed loop – i.e. no extrinsic sensing – as well as extending the model to be in higher-dimension spaces with more complex interactions, such as a complete three-dimensional suspension.

5 References

[1] Singh, Anuradha. *Indian Railways: Maglev Trains Or Floating Trains To Be Brought To India Within Next Three Years*, 1 Dec 2016. Accessed from: `http://topyaps.com/wp-content/uploads/2016/12/c01633.jpg`

[2] MarLed, Wikimedia Foundation. 2006. Permeabilitie magnetique. Accessed from: `https://commons.wikimedia.org/wiki/File:Magnetic_Permeability-no-caption.gif`