HW4

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Var 1

1 Task 1

1.1 matrix 1

1. No strictly dominant strategy
2. No weakly dominant strategy
3. Maximax Strategy max payoff for Player 2:
(8, 7, 3), max here = 8 –> we choose strategy 1
For player 1:
(4, 5, 4) –> strategy 2
Minimax strategy max payoff for player 2:
(8, 7, 3) –> min here 3 –> strategy 3
for player 1:
(4, 5, 4) –> min 4 –> strategy 1 or 3
Maximin strategy min payoff for player 2:
(3, 1, 2)–> max here = 3 –> strategy 1
min payoff for player 1:
(3, 2, 1) –> strategy 1
4. $b_2(S_1) = \{(1,1),(2,2),(3,1)\}\, and\, b_1(S_2) = \{(1,1),(1,2),(2,3)\}$
$NE(G) = \{(1,1)\}$
$5.b_2(S_1) = \{(1,1),(2,2),(3,1)\}\, with\, u1 = 4,\ u1 = 7\, and\, u1 = 8$
$b_1(S_2) = \{(1,1),(1,2),(2,3)\}\, with\, u2 = 3,\ u2 = 4\, and\, u2 = 5$
$S1 = \{(2,3)\}\, and\, S2 = \{(3,1)\}$

1.2 matrix 2

1. No strictly dominant strategy
2. No weakly dominant strategy
3. Maximax for player 1 max payoff:
(3, 4, 4, 4, 4) –> strategy 2/3/4/5
for player 2:
(4, 3, 4, 3, 3, 4) –> strategy 1/3/6
Maximax strategy for player 1:
strategy 1
for player 2:

strategy 2/4/5
minimax strategy for player 1:
(0, 0, 0, 0, 1) –> strategy 6
for player 2:
(0, 1, 0, 0, 0, 1) –> strategy 2/6
4. $b_2(S_1) = \{(1,1),(1,3),(1,6),(2,4),(3,3),(3,4),(3,5),(4,6),(5,1),(5,2),(5,6)\} and$
$b_1(S_2) = \{(3,1),(2,2),(5,2),(2,3),(3,3),(4,3),(2,4),(4,4),(3,5),(5,5),(1,6),(2,6)\}$
$NE(G) = \{(5,2),(3,3),(2,4),(3,5),(1,6)\}$
$5.b_2(S_1) = \{(1,1),(1,3),(1,6),(2,4),(3,3),(3,4),(3,5),(4,6),(5,1),(5,2),(5,6)\}$
$with\, u1 = 4,\ u1 = 3\ ,\ u1 = 3,\ u1 = 4,\ u1 = 3$
$b_1(S_2) = \{(3,1),(2,2),(5,2),(2,3),(3,3),(4,3),(2,4),(4,4),(3,5),(5,5),(1,6),(2,6)\}$
$with\, u2 = 4,\ u2 = 2\ ,\ u2 = 4,\ u2 = 4,\ u2 = 4,\ u2 = 3$
$S1 = \{(3,1),(2,3),(3,3),(4,3),(2,4),(4,4),(3,5),(5,5)\}\ and$
$S2 = \{(1,1),(1,3),(1,6),(4,6)\}$

1.3 Mixed strategy

With setting the probabilities there exist no mixed strategy equilibrium because probabilities are not in range from 0 to 1 which means there must exist a pure equilibrium:
b2(S1) = {(1, 1), (2, 3), (3, 3)}
b1(S2) = {(1, 1), (3, 2), (1, 3)}
Nash equilibrium is strategy (1, 1) where the first player gets payoff 4 and second player gets 3

2 Task 2

2.1 Kurnot

$P = 16 - 2 * (x_1 + x_2)$
$C_1 = 7 * x_1$
$C_2 = 7 * x_2$
$Solution:$
$x_2 = b_2(x_1)$
$\prod_2 - - > max$
$\prod_2 = px_2 - C_2 = (p-7)x_2 = (9 - 2x_1 - 2x_2)x_2 - - > max$
$\frac{\partial \prod_2}{\partial x_2} = 9 - 2x_1 - 4x_2 = 0$
$x_2 = (9 - 2x_1)/4$
$x_1 = b_1(x_2)$
$\prod_1 - - > max$
$\prod_1 = px_1 - C_1 = (p-7)x_1 = (9 - 2x_1 - 2x_2)x_1 - - > max$
$\frac{\partial \prod_1}{\partial x_1} = 9 - 4x_1 - 2x_2 = 0$
$x_1 = (9 - 2x_2)/4$

$x_2 = (9 - 2x_1)/4, x_1 = (9 - 2x_2)/4$
$x_1 = (9 - 9/2 + x_1)/4) = 1.5$
$x_2 = 1.5$
$\prod_1^* = (9 - 2 * 1.5 - 2 * 1.5) * 1,5 = 4.5 = \prod_2^*$
$NE(G) = \{1.5, 1.5\}$

Stackelberg equilibrium for the first player
$\prod_1 -- > max$
$x_2 = b_2(x_1)$
$\prod_1 = (9 - 2x_1 - 2x_1)x_1$
$x_2 = (9 - 2x_1)/4$
$\prod_1 = (9 - 2x_1 - 2x_2)x_1$
$\frac{\partial \prod_1}{\partial x_1} = 9 - 4x_1 - 1/2(9 - 4x_1) = -2x_1 + 9/2 = 0$
$x_1^* = 2.25$
$x_2^* = 1.125$
$P^* = 16 - 2 * 3.375 = 8.25$
$\prod_1^* = 1.25 * 2.25 = 2.8125$
$\prod_2^* = 1.25 * 1.125 = 1.5$
$Stackelberg\, equilibrium\, for\, the\, second\, player$
$\prod_2 -- > max$
$x_1 = b_1(x_2)$
$\prod_2 = (9 - 2x_1 - 2x_2)x_2$
$x_1 = (9 - 2x_2)/4$
$\prod_2 = (9 - 2x_1 - 2x_2)x_2$
$\frac{\partial \prod_2}{\partial x_2} = 9 - 4x_2 - 1/2(9 - 4x_2) = -2x_2 + 9/2 = 0$
$x_2^* = 2.25$
$x_1^* = 1.125$
$P^* = 16 - 2 * 3.375 = 8.25$
$The\, same$

2.2 Bertrand

$x_1 = 7 - p_1 + p_2$
$x_2 = 8 - p_2 + 2p_1$
$C_1 = 4x_1$
$C_2 = 2x_2$

Nash equilibrium: For first player
$p_2 = b_2(p_1)$
$\prod_2 -- > max$
$\prod_2 = (p_2x_2 - C_2) = (p_2 - 2)(8 - p_2 + 2p_1)$
$\frac{\partial \prod_2}{\partial p_2} = 8 - 2p_2 + 2p_1 + 2 = 0$
$p_2 = 5 + p_1$

For second player

$p_1 = b_1(p_2)$
$\prod_1 --> max$
$\prod_1 = (p_1 x_1 - 4x_1) = (p_1 - 4)(7 - p_1 + p_2)$
$\frac{\partial \prod_1}{\partial p_1} = 11 - 2p_1 + p_2 = 0$
$p_1 = 11/2 + 1/2 p_2$

$p_1 = 3 + 0.5 p_1$
$p_1^* = 6$
$p_2^* = 11$
$NE(G) = \{(6, 11)\}$