7

Direct-Current Motors

7.1 INTRODUCTION

A motor is a machine that converts electrical energy into mechanical energy. The d.c. motor is very similar to a d.c. generator in construction. Generators are usually operated in more protected locations and therefore their construction is generally of the *open* type. On the other hand, motors are generally used in locations where they are exposed to dust, moisture, fumes and mechanical damage. Thus, motors require protective enclosures for example, drip-proof, fire-proof etc., according to the requirements.

7.2 MOTOR PRINCIPLE

When a conductor carrying current is put in a magnetic field, a force is produced on it. The effect of placing a current-carrying conductor in a magnetic field is shown in Fig. 7.1. Let us consider one such conductor placed in a slot of armature and suppose that it is acted upon by the magnetic field from a north pole of the motor. By applying left-hand rule it is found the conductor has a tendency to move to the left-hand side. Since the conductor is in a slot on the circumference of the rotor, the force F_C acts in a tangential direction to the rotor. Thus, a torque (turning effect) is developed on the rotor. Similar torques are produced on all the rotor conductors. Since the rotor is free to move, it starts rotating in the anticlockwise direction.

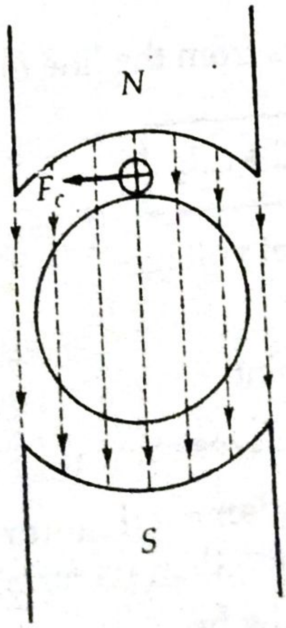


Fig. 7.1. *A current-carrying conductor placed in a magnetic field.*

The torque produced on the rotor is transferred to the shaft of the rotor and can be utilized to drive a mechanical load.

7.3 BACK E.M.F.

When the motor armature rotates, its conductors cut the magnetic flux. Therefore, the e.m.f. of rotation E_r is induced in them. In case of a motor, the e.m.f. of rotation is known as **back e.m.f.** or **counter e.m.f.** The back e.m.f. opposes the applied voltage. Since the back e.m.f. is induced due to generator action its magnitude is, therefore, given by the same expression as that for the generated e.m.f. in a d.c. generator. That is,

$$\boxed{E = \frac{N P \Phi Z}{60\, A}} \tag{7.3.1}$$

where the symbols have their usual meanings.

7.4 EQUIVALENT CIRCUIT OF A D.C. MOTOR ARMATURE

The armature of a d.c. motor can be represented by an equivalent circuit. It can be represented by three series-connected elements E, R_a and V_b. The element E is the back e.m.f., the element R_a is the armature resistance and V_b is the brush contact voltage drop. The equivalent circuit of the armature of a d.c. motor is shown in Fig. 7.2.

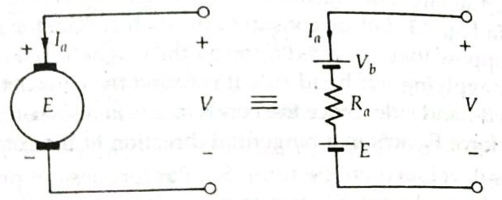


Fig. 7.2. *Equivalent circuit of the armature of a d.c. motor.*

In a motor, current flows from the line *into* the armature against the generated voltage. By KVL

$$\boxed{V = E + I_a R_a} \tag{7.4.1}$$

where V = motor terminal voltage

E = back e.m.f.

I_a = armature current

R_a = armature-circuit resistance

Equation (7.4.1) is the fundamental **motor equation**. It is seen that the back e.m.f. E of the motor is always less than its terminal voltage V.

If the voltage drop V_b in the brushes is also considered, then by KVL

$$V = E + I_a R_a + V_b \tag{7.4.2}$$

7.5 TORQUE OF A DC MACHINE

When a dc machine is loaded either as a motor or as a generator, the rotor conductors carry current. These conductors lie in the magnetic field of the air gap. Thus each conductor experiences a force. The conductors lie near the surface of the rotor at a common radius from its centre. Hence a torque is produced around the circumference of the rotor and the rotor starts rotating.

When the machine operates as a generator at constant speed, this torque is equal and opposite to that provided by the prime-mover. When the machine is operating as a motor the torque is transferred to the shaft of the rotor and drives the mechanical load.

The expression for the torque is the same for the generator and the motor. It can be deduced as follows :

The voltage equation of a d.c. motor is

$$V = E + I_a R_a \tag{7.5.1}$$

Multiplying both the sides of Eq. (7.5.1) by I_a we obtain

$$VI_a = EI_a + I_a^2 R_a \tag{7.5.2}$$

But VI_a = electrical power input to the armature

$I_a^2 R_a$ = copper loss in the armature

We also know that input = output + losses (7.5.3)

Comparison of Eqs. (7.5.2) and (7.5.3) shows that

EI_a = electrical equivalent of gross mechanical power developed by the armature (electromagnetic power)

Let τ_{av} = average electromagnetic torque developed by the armature in newton metres (Nm)

At this value of torque the electromechanical power conversion takes place. Mechanical power developed by the armature,

$$P_m = \omega \tau_{av} = 2\pi n \tau_{av}$$

Therefore

$$P_m = EI_a = \omega \tau_{av} = 2\pi n \tau_{av} \tag{7.5.4}$$

But

$$E = \frac{n P \Phi Z}{A}$$

Therefore

$$\frac{n P \Phi Z}{A} I_a = 2\pi n \tau_{av}$$

and

$$\boxed{\tau_{av} = \frac{PZ}{2\pi A} \Phi I_a} \tag{7.5.5}$$

Equation (7.5.5) is called the **torque equation** of d.c. motor.

For a given d.c. machine, P, Z and A are constant, therefore $\left(\frac{PZ}{2\pi A}\right)$ **is also a constant.**

Let $\frac{PZ}{2\pi A} = k$

$$\therefore \quad \tau_{av} = k\Phi I_a \tag{7.5.6}$$

$$\tau_{av} \propto \Phi I_a \tag{7.5.7}$$

(7.5.8)

Hence *the torque developed by a d.c. motor is directly proportional to the flux per pole and armature current.*

Alternative Proof

Consider a turn $a\,a'\,b'\,b$ whose two conductors aa' and bb' are placed under two adjacent poles as shown in Fig. 7.3.

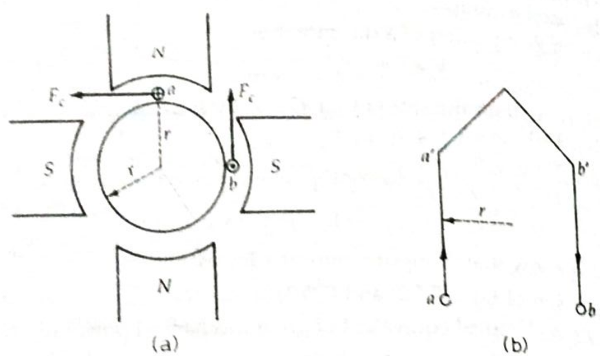


Fig. 7.3.

The force on a conductor of length l placed on the periphery of the conductor lying in a field of flux density B is

$$F_c = BI_c l = B\left(\frac{I_a}{A}\right)l$$

where I_c = current in the *conductor* of the armature winding

I_a = armature terminal current

A = number of parallel paths

In a motor the flux density B is not constant for all conductors. The average flux density is taken as B_{av} and l is taken as the length of the armature core.

The average torque developed by a conductor a

$$\tau_c = F_c r = B_{av}\left(\frac{I_a}{A}\right)lr$$

But, $B_{av} = \frac{\text{flux per pole}}{\text{area per pole}} = \frac{\varphi}{(2\pi rl/P)} = \frac{\Phi P}{2\pi rl}$

$$\therefore \quad \tau_{av} = \frac{\Phi P}{2\pi rl}\frac{I_a}{A}lr = \frac{\Phi P I_a}{2\pi A}$$

Since all the conductors in the armature winding develop torque in the same direction, therefore, the average torque developed by the armature will be the sum of all these torques. That is, the total electromagnetic torque developed is :

$$\tau_e = Z\tau_{av}$$

or

$$\boxed{\tau_e = \frac{PZ}{2\pi A}\Phi I_a} \tag{7.5.9}$$

7.6 TYPES OF D.C. MOTORS

Direct current motors are named according to the connection of the field winding with the armature. There are three types of d.c. motors.

1. Shunt wound or shunt motor.
2. Series wound or series motor.
3. Compound wound or compound motor.

7.6.1 Shunt motor

This is the most common type of d.c. motors. The field winding is connected in parallel with the armature, as shown in Fig. 7.4.

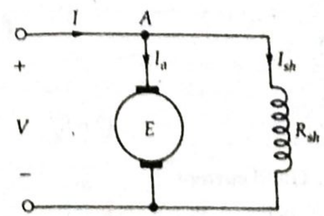


Fig. 7.4. *D.C. Shunt motor.*

The current, voltage and power equations for a shunt motor are written as follows :

Current equation

By KCL at junction A of Fig. 7.4,

Sum of the incoming currents at A

= sum of the outgoing currents at A

$$I = I_a + I_{sh} \tag{7.6.1}$$

where I = input line current ; I_a = armature current

I_{sh} = shunt field current

Voltage equations

The voltage equations are written by using Kirchhoff's voltage law (KVL).

For field-winding circuit

$$V = I_{sh} R_{sh} \tag{7.6.2}$$

For armature-winding circuit

$$V = E + I_a R_a \tag{7.6.3}$$

Power equations

Power input = mechanical power developed + losses in the armature + loss in the field

$$VI = P_m + I_a^2 R_a + I_{sh}^2 R_{sh}$$

$$= P_m + I_a^2 R_a + VI_{sh} \quad (7.6.4)$$

$$P_m = VI - VI_{sh} - I_a^2 Ra = V(I - I_{sh}) - I_a^2 R_a$$

$$= VI_a - I_a^2 R_a = (V - I_a R_a) I_a$$

$$\therefore \quad P_m = EI_a \quad (7.6.5)$$

Multiplying Eq. (7.6.3) by I_a we get

$$VI_a = EI_a + I_a^2 R_a$$

$$VI_a = P_m + I_a^2 R_a \quad (7.6.6)$$

where VI_a = electrical power supplied to the armature of the motor (7.6.7)

7.6.2 Series motor

In the series motor (Fig. 7.5), the field winding is connected in series with the armature.

Current equation

By KCL in Fig. 7.5

$$I = I_{se} = I_a \quad (7.6.8)$$

where I_{se} = series field current

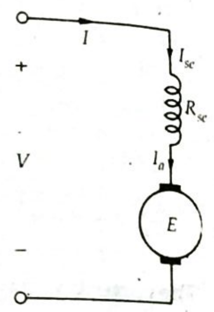


Fig. 7..5. DC series motor.

Voltage equation

By KVL in Fig. 7.5

$$V = E + I(R_a + R_{se}) \quad (7.6.9)$$

Power equations

Multiplying Eq. (7.6.9) by I we get

$$VI = EI + I^2(R_a + R_{se}) \quad (7.6.10)$$

Power input = mechanical power developed + losses in the armature + losses in the field

$$VI = P_m + I^2 R_a + I^2 R_{se} \quad (7.6.11)$$

Comparison of Eqs. (7.6.10) and (7.6.11) shows that

$$P_m = EI \quad (7.6.12)$$

7.6.3 Compound motor

A d.c. motor having both shunt and series field windings is called a compound motor. It may be a short-shunt compound motor or a long-shunt compound motor. A d.c. compound motor may be cumulatively compounded or differentially compounded as discussed in Sec. 6.5. The current relationships for a compound motor can be written by using KCL. The voltage relationships are written by using KVL.

7.7 ARMATURE REACTION IN A D.C. MOTOR AND INTERPOLES

The armature reaction is the effect of armature flux on the main flux. In case of a d.c. motor the resultant flux is *strengthened* at the *leading pole tips* and *weakened at the trailing pole tips.*

Armature reaction causes sparking at the brushes due to delay in commutation. Interpoles are placed in between the main poles in order to neutralize the effects of armature reaction in brush region and minimize sparking at brushes. Interpoles generate voltage necessary to neutralize the e.m.f. of self-induction in the armature coils undergoing commutation. Motor interpoles have a polarity opposite to that of the following main pole in the direction of rotation of armature. Since the interpoles are connected in series with the armature, the change in direction of current in the armature changes the polarity of the interpole. Thus, a d.c. machine that has correct interpole polarity when used as a generator will have the correct interpole polarity when used as a motor.

7.8 CHARACTERISTICS OF A SHUNT OR SEPARATELY EXCITED D.C. MOTOR

In both the cases of shunt and separately excited d.c. motors, the field is supplied from a constant voltage so that the field current is constant. The two motors, therefore, have similar characteristics. *Characteristic* is a graph between two dependent quantities.

7.8.1 Speed-armature current characteristics

In a shunt motor, $I_{sh} = V/R_{sh}$. If V is constant I_{sh} will also be a constant. Hence the flux is constant at no load. The flux decreases slightly due to armature reaction. If the effect of armature reaction is neglected, the flux Φ will remain constant. The motor speed is given by

$$N \propto \frac{V - I_a R_a}{\Phi} \quad (7.8.1)$$

If Φ is constant the speed can be written as

$$N \propto V - I_a R_a \quad (7.8.2)$$

Equation (7.8.2) is the equation of a straight line with a negative slope. That is, the speed N of the motor decreases linearly with the increase in armature current as shown in Fig. 7.6.

Since $I_a R_a$ at full load is very small compared to V, the drop in speed from no load to full load is very small. The decrease in N is partially neutralized by a reduction in Φ due to armature reaction. Hence for all practical purposes the shunt motor may be taken as a constant-speed motor.

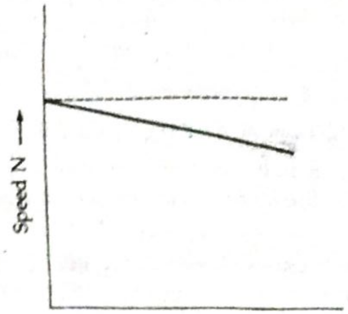


Fig. 7.6. *Speed-armature current (N/I_a) characteristic of a shunt or separately excited d.c. motor.*

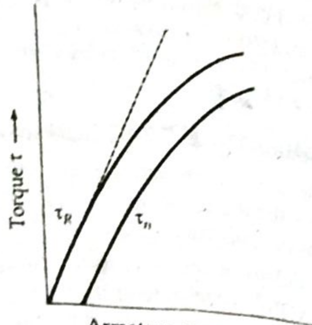


Fig. 7.7. *Torque/armature current characteristic of a shunt or separately excited d.c. motor*

7.8.2 Torque/armature current characteristic

From Eq. (7.5.8)

$$\tau_g \propto \Phi I_a \tag{7.8.3}$$

If the effect of armature reaction is neglected, Φ is nearly constant and

$$\tau_g \propto I_a \tag{7.8.4}$$

Equation (7.8.4) shows that the graph between τ_g and I_a is a straight line passing through the origin (Fig. 7.7).

If the effect of armature reaction is taken into account, the value of Φ decreases slightly with the increase in armature current. Hence at higher values of I_a the gross or total torque τ_g decreases slightly.

The relation between various torques is given by the relation

$$\tau_n = \tau_g - (\tau_f + \tau_w) \tag{7.8.5}$$

where τ_n = net torque or useful torque or load torque at the output shaft

τ_g = gross or total torque

τ_f = frictional torque

τ_w = windage torque

The graph showing the relationship between the net torque and the armature current is a curve parallel to the corresponding gross torque curve. It is slightly below it.

7.9 CHARACTERISTICS OF A D.C. SERIES MOTOR

7.9.1 Speed/armature current characteristic

The motor speed N is given by

$$N \propto \frac{V - I_a(R_a + R_{se})}{\Phi} \tag{7.9.1}$$

At low values of I_a, the voltage drop $I_a(R_a + R_{se})$ is negligibly small in comparison with V. Therefore

$$N \propto \frac{V}{\Phi} \tag{7.9.2}$$

Since V is constant

$$N \propto \frac{1}{\Phi} \tag{7.9.3}$$

In a series motor, the flux Φ is produced by the armature current flowing in the field winding so that $\Phi \propto I_a$. Hence the series motor is a variable flux machine. Equation (7.9.3) now becomes

$$N \propto \frac{1}{I_a} \tag{7.9.4}$$

Thus, for the series motor, the speed is inversely proportional to the armature (load) current. The speed-load characteristic is a rectangular hyperbola as shown in Fig. 7.8.

Equation (7.9.4) shows that when the load current is small, the speed will be very large. Therefore at no load or at light loads there is a possibility of dangerously high speeds, which may damage the motor due to large centrifugal forces. Hence *a series motor must never run unloaded.* It should always be coupled to a mechanical load either directly or through gearing. It should never be coupled by belt, which may break at any time. With the increase in armature current (which is also the field current) the flux also increases and therefore the speed is reduced.

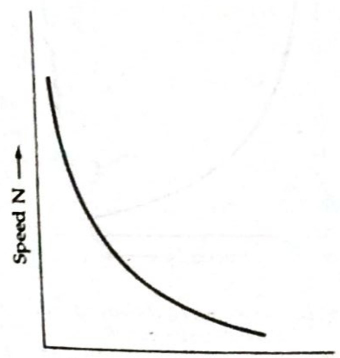


Fig. 7.8. *Speed-armature current characteristic of a d.c. series motor.*

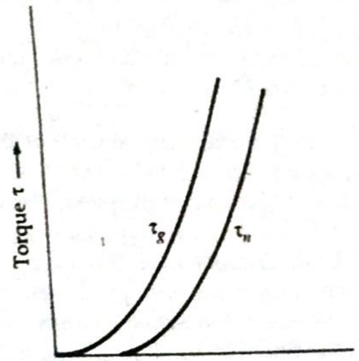


Fig. 7.9. *Torque/armature current characteristic of a d.c. series motor.*

7.9.2 Torque/Armature Current Characteristic

From Eq. (7.5.8)

$$\tau_g \propto \Phi I_a \tag{7.9.5}$$

Before saturation, $\Phi \propto I_a$ and hence at light loads

$$\tau_g \propto I_a^2 \tag{7.9.6}$$

Equation (7.9.6) shows that the torque/armatue current (τ/I_a) curve will be parabolic. When the iron becomes magnetically saturated, Φ becomes almost constant, so that at heavy loads

$$\tau_g \propto I_a \tag{7.9.7}$$

Equation (7.9.7) shows that the τ/I_a characteristic is a straight line. Thus, the torque/current characteristic of a d.c. series motor is initially parabolic and finally becomes linear when the load current becomes large. The characteristic changes smoothly from one curve to another. This characteristic is shown in Fig. 7.9.

The characteristic relating the net torque or useful torque τ_u to the armature current is parallel to the T_g/I_a characteristic, but is slightly below it (Fig. 7.9). The difference between the two curves is due to friction and windage losses.

7.9.3 Speed/torque characteristic

The speed/torque characteristic of a series motor can be derived from its speed-armature current (N/I_a) and torque-armature current (τ/I_a) characteristics as follows :

For a given value of I_a find τ from τ/I_a curve and N from N/I_a curve. This gives one point (τ, N) on speed-torque (N/τ) curve. Repeat this procedure for a number of values of I_a and find the corresponding values of speed and torque (τ_1, N_1), (τ_2, N_2) etc. These points are plotted to get the speed-torque characteristic of a d.c. series motor as shown in Fig. 7.10.

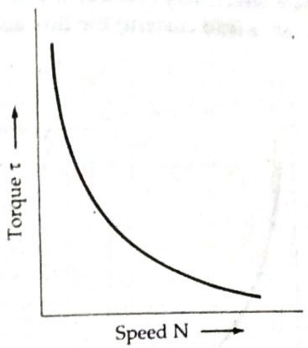


Fig. 7.10. *Speed-torque characteristic of a d.c. series motor.*

This characteristic shows that the d.c. series motor has a high torque at a low speed and a low torque at a high speed. Hence the speed of the d.c. series motor changes considerably with increasing load. It is a very useful characteristic for traction purposes, hoists and lifts where at low speeds a high starting torque is required to accelerate large masses.

7.10 CHARACTERISTICS OF A COMPOUND MOTOR

A compound motor has both shunt and series field windings, so its characteristics are intermediate between the shunt and series motors. The cumulative compound motor is generally used in practice. The speed-armature current characteristics are shown in Fig. 7.11.

The torque-armature current characteristics are shown in Fig. 7.12.

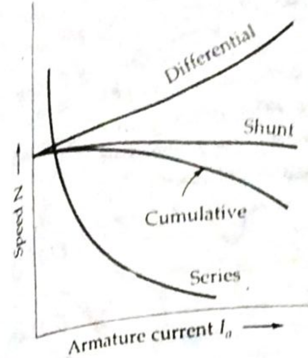


Fig. 7.11. *Speed-armature current characteristic of a d.c. motor.*

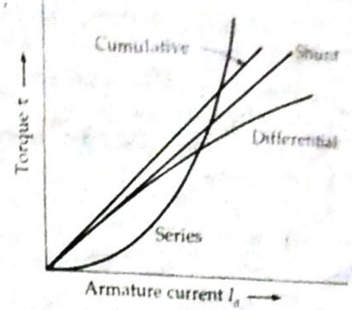


Fig. 7.12. *Torque/armature current characteristic of a d.c. motor.*

Figure 7.13 shows the speed-torque (N/τ) characteristic of a compound motor. It is found that a compound motor has a high starting torque together with a safe no-load speed. These factors make it suitable for use with heavy intermittent loads such as lifts, hoists etc.

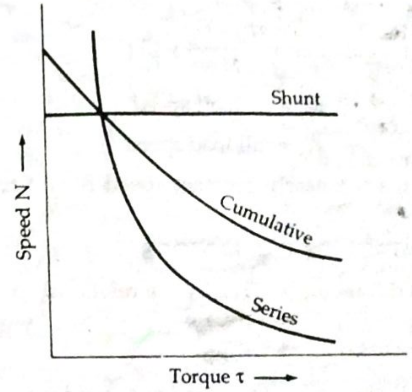


Fig. 7.13. *Speed-torque characteristic of a compound motor.*

7.11 SPEED OF A D.C. MACHINE

The e.m.f. equation of a d.c. machine is given by

$$E = \frac{NP\Phi Z}{60\,A}$$

Solving for N gives

$$N = \frac{60A}{PZ}\frac{E}{\Phi} \tag{7.11.1}$$

$$\therefore \quad N = \frac{E}{k\Phi}$$

where $$k = \frac{PZ}{60A}. \tag{7.11.2}$$

Equation (7.11.2) shows that the speed of a d.c. machine is directly proportional to the e.m.f. of rotation E and inversely proportional to flux per pole Φ.

Since the expression for e.m.f. of rotation applies equally to motors and generators, Eq. (7.11.1) gives the speed for both motors and generators.

If the suffixes 1 and 2 denote the initial and final values

$$N_1 = \frac{E_1}{k\Phi_1} \tag{7.11.3}$$

$$N_2 = \frac{E_2}{k\Phi_2} \tag{7.11.4}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} \tag{7.11.5}$$

7.11.1 Speed regulation

The **speed regulation** *is defined as the change in speed from no load to full load expressed as a fraction or a percentage of the full load speed.* It can be written as :

$$\text{Per unit speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}} \tag{7.11.6}$$

$$\text{Per cent speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100 \tag{7.11.7}$$

where N_{nl} = no-load speed

N_{fl} = full-load speed

A motor which has a nearly constant speed is said to have a good speed regulation.

7.12 SPEED CONTROL OF D.C. MOTORS

The speed of a d.c. motor is given by the relationship

$$N = \frac{V - I_a R_a}{k\Phi} \tag{7.12.1}$$

Equation (7.12.1) shows that the speed is dependent upon the supply voltage V, the armature circuit resistance R_a, and the field flux Φ, which is produced by the field current. In practice, the variation of these three factors is used for speed control. Thus, there are three general methods of speed control of d.c. motors.

1. Variation of resistance in the armature circuit.

   This method is called **armature resistance control.** (Rheostatic control)

2. Variation of field flux Φ

   This method is called **field flux control.**

3. Variation of applied voltage. (Armature Voltage Control)

7.12.1 Armature resistance control (Rheostatic Control)

In this method a variable series resistor R_e is put in the armature circuit. Fig. 7.14 shows the method of connection for a shunt motor. In this case, the field is directly connected across the supply and therefore the flux Φ is not affected by variation of R_e.

Figure 7.15 shows the method of connection of external resistance R_e in the armature circuit of a d.c. series motor.

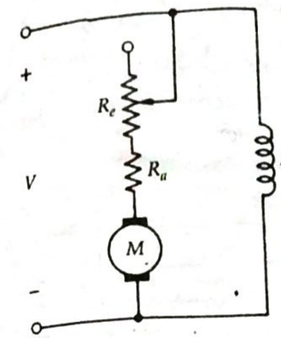


Fig. 7.14. *Speed control of a d.c. shunt motor by armature resistance control.*

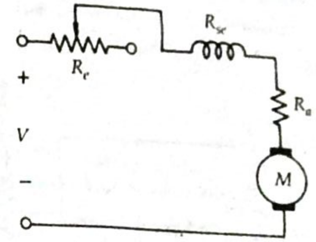


Fig. 7.15. *Speed control of a d.c. series motor by armature resistance control.*

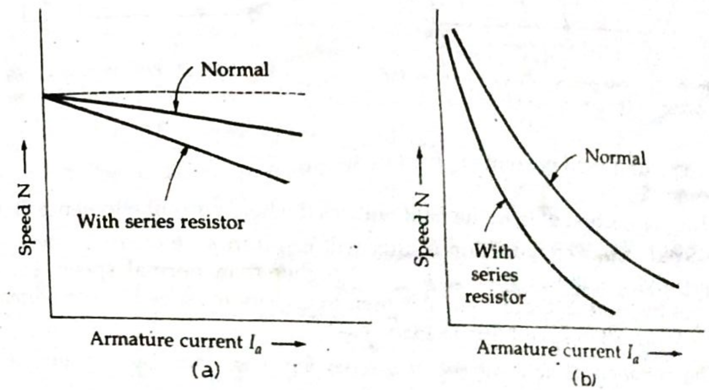


Fig. 7.16. *Speed/current characteristics (a) shunt motor (b) series motor.*

In this case the current and hence the flux are affected by the variation of the armature circuit resistance. The voltage drop in R_e reduces the voltage applied to the armature and therefore the speed is reduced. Figures 7.16 (*a*) and 7.16 (*b*) show typical speed/current characteristics for shunt and series motors respectively. In both the cases the motor runs at a lower speed as the value of R_e is increased. Since R_e carries full armature current, it must be designed to carry continuously the full armature current.

$$\therefore \quad N = \frac{E}{k\Phi} \tag{7.11.2}$$

where $\quad k = \frac{PZ}{60A}$.

Equation (7.11.2) shows that the speed of a d.c. machine is directly proportional to the e.m.f. of rotation E and inversely proportional to flux per pole Φ.

Since the expression for e.m.f. of rotation applies equally to motors and generators, Eq. (7.11.1) gives the speed for both motors and generators.

If the suffixes 1 and 2 denote the initial and final values

$$N_1 = \frac{E_1}{k\Phi_1} \tag{7.11.3}$$

$$N_2 = \frac{E_2}{k\Phi_2} \tag{7.11.4}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} \tag{7.11.5}$$

7.11.1 Speed regulation

The **speed regulation** *is defined as the change in speed from no load to full load expressed as a fraction or a percentage of the full load speed.* It can be written as :

$$\text{Per unit speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}} \tag{7.11.6}$$

$$\text{Per cent speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100 \tag{7.11.7}$$

where $\quad N_{nl}$ = no-load speed

N_{fl} = full load speed

A motor which has a nearly constant speed is said to have a good speed regulation.

7.12 SPEED CONTROL OF D.C. MOTORS

The speed of a d.c. motor is given by the relationship

$$N = \frac{V - I_a R_a}{k\Phi} \tag{7.12.1}$$

Equation (7.12.1) shows that the speed is dependent upon the supply voltage V, the armature circuit resistance R_a, and the field flux Φ, which is produced by the field current. In practice, the variation of these three factors is used for speed control. Thus, there are three general methods of speed control of d.c. motors.

1. Variation of resistance in the armature circuit.
   This method is called **armature resistance control.** (Rheostatic control)
2. Variation of field flux Φ
   This method is called **field flux control.**
3. Variation of applied voltage. (Armature Voltage Control)

7.12.1 Armature resistance control (Rheostatic Control)

In this method a variable series resistor R_e is put in the armature circuit. Fig. 7.14 shows the method of connection for a shunt motor. In this case, the field is directly connected across the supply and therefore the flux Φ is not affected by variation of R_e.

Figure 7.15 shows the method of connection of external resistance R_e in the armature circuit of a d.c. series motor.

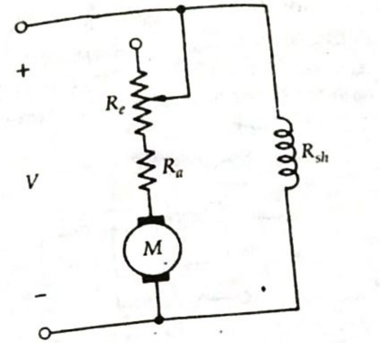


Fig. 7.14. *Speed control of a d.c. shunt motor by armature resistance control.*

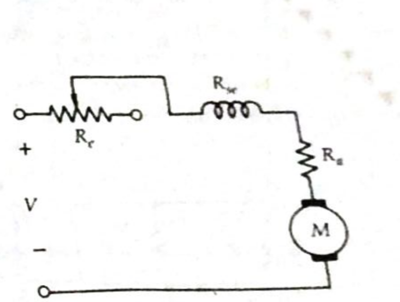


Fig. 7.15. *Speed control of a d.c. series motor by armature resistance control.*

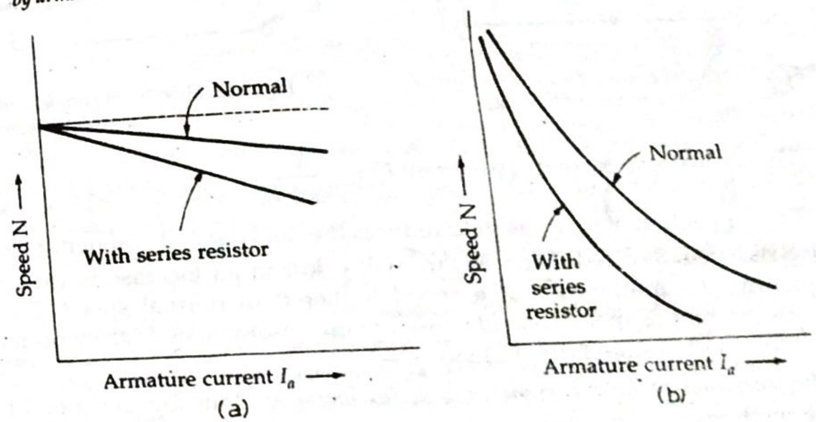


Fig. 7.16. *Speed/current characteristics (a) shunt motor (b) series motor.*

In this case the current and hence the flux are affected by the variation of the armature circuit resistance. The voltage drop in R_e reduces the voltage applied to the armature and therefore the speed is reduced. Figures 7.16 (*a*) and 7.16 (*b*) show typical speed/current characteristics for shunt and series motors respectively. In both the cases the motor runs at a lower speed as the value of R_e is increased. Since R_e carries full armature current, it must be designed to carry continuously the full armature current.

This method suffers from the following *drawbacks* :

(*i*) A large amount of power is wasted in the external resistance R_c.

(*ii*) Control is limited to give speeds below normal and increase of speed cannot be obtained by this method.

(*iii*) For a given value of R_c the speed reduction is not constant but varies with the motor load.

This method is only used for small motors.

7.12.2 Variation of field flux Φ (Field Flux Control)

Since the flux is produced by the field current, control of speed by this method is obtained by control of the field current. In the *shunt motor*, this is done by connecting a *variable resistor R_C in series with the shunt field winding* as shown in Fig. 7.17. The resistor R_c is called the *shunt field regulator*.

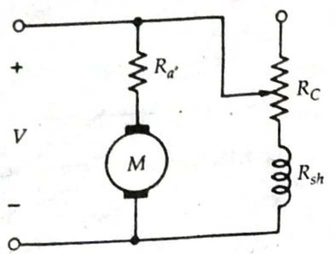


Fig. 7.17. *Speed control of a d.c. shunt motor by variation of field flux.*

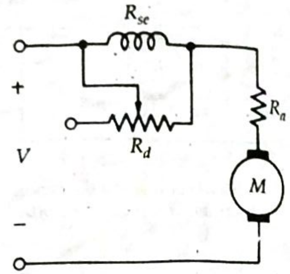


Fig. 7.18. *Diverter in parallel with the series of d.c.. motor.*

The shunt field current is given by $I_{sh} = \dfrac{V}{R_{sh} + R_c}$

The connection of R_c in the field reduces the field current and hence the flux Φ is also reduced. The reduction in flux will result in an increase in the speed. Consequently, the motor runs at a speed higher than normal speed. For this reason, this method of speed control is used to give motor speeds *above* normal or to correct for a fall in speed due to load.

The *variation of field current in a series motor* is done by any one of the following methods :

(*a*) A variable resistance R_d is connected in parallel with the series field winding as shown in Fig. 7.18. The parallel resistor is called the **diverter**. A portion of the main current is diverted through R_d.

Thus, the diverter reduces the current flowing through the field winding. This reduces the flux and *increases* the speed.

(*b*) The second method uses a **tapped field control** as shown in Fig. 7.19. Here the ampere-turns are varied by varying the number of field turns. This arrangement is used in electric traction.

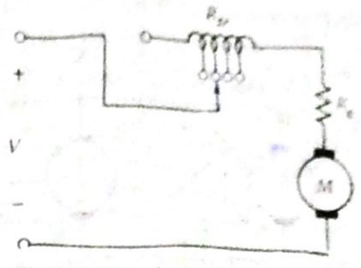


Fig. 7.19. *Tapped series field on d.c. motor*

Figures 7.20 (*a*) and 7.20 (*b*) show the typical speed/torque curves for shunt and series motors respectively, whose speeds are controlled by the variation of the field flux.

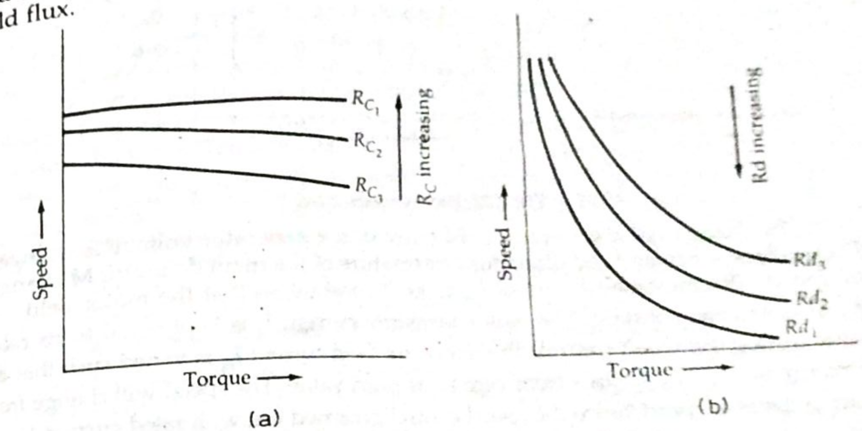


Fig. 7.20. *Typical speed/torque curves (a) shunt motor (b) series motor.*

The **advantages of field control** are as follows :

(*i*) This method is easy and convenient.

(*ii*) Since shunt field current I_{sh} is very small, the power loss in the shunt field is small.

The flux cannot usually be increased beyond its normal value because of saturation of the iron, so speed control by flux is limited to weakening, which gives an *increase* in speed. It is applicable over only a limited range, because if the field is weakened too much there is a loss of stability.

7.12.3 Armature Voltage Control

Speed control of dc motors can also be obtained by varying the applied voltage to the armature. **Ward-Leonard System** of speed control is based on this principle. This method was introduced in 1891. The schematic diagram of the Ward-Leonard method of speed control of a dc shunt motor is shown in Fig. 7.21. In this system M is the main dc motor whose speed is to be controlled, and G is a separately excited dc generator. The generator G is driven by a 3-phase driving motor which may be an induction motor or a synchronous motor. The combination of ac driving motor and the dc generator is called the motor-generator (M – G) set.

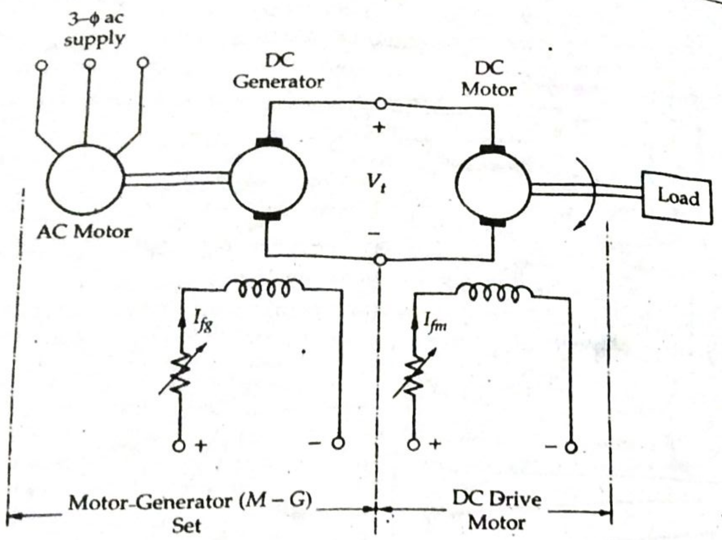


Fig. 7.21. *Ward-Leonard drive.*

By changing the generator field current, the generator voltage is changed. This voltage when applied direct to the armature of the main dc motor M changes its speed. The motor field current I_{fm} is kept constant so that the motor field flux Φ_m also remains constant. The motor armature current I_a is kept equal to its rated value during the speed control. The generator field current I_{fg} is varied such that the armature voltage V_t changes from zero to its rated value. The speed will change from zero to the base speed. Since the speed control is carried out with rated current I_a and with constant motor field flux Φ_m, a constant torque ($\propto \Phi_m I_a$) upto base (rated) speed is obtained. Since the power P (= torque × speed) is proportional to speed, it increases with speed. *Hence with armature voltage control method constant torque and variable power drive is obtained from speed below the base speed.* This is shown in Fig. 7.22.

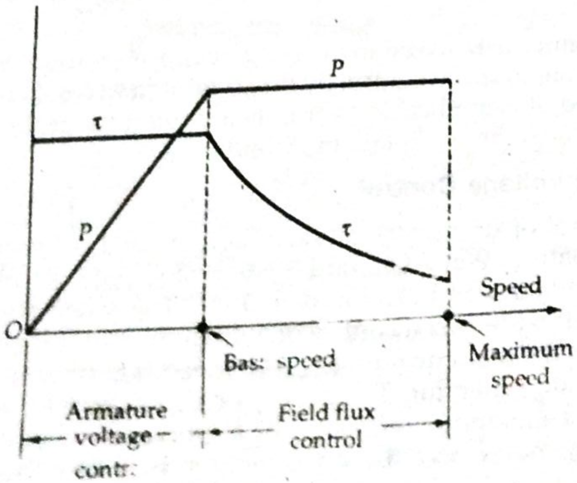


Fig. 7.22. *Torque and power characteristics is combined armature voltage and field control.*

For speed control above base speed field flux control is used. In this mode of operation, the armature current I_a is maintained constant at its rated value and the generator voltage V_t is kept constant. The motor field current I_{fm} is decreased and, therefore, the motor field flux Φ_m is decreased. That is, the field is weakened to obtain higher speeds. Since V_tI_a or EI_a remains constant, the electromagnetic torque τ ($\propto \Phi_m I_a$) decreases as the field flux Φ_m is decreased. Therefore, the torque τ decreases, as the speed increases. Thus, *in the field control mode, constant power and variable torque is obtained for speeds above base speed* as shown in Fig. 7.22.

When speed control over a wide range is required, combination of armature voltage control and field flux control is used. This combination permits the ratio of maximum to minimum available speeds to be 20 to 40. With closed loop control, this range can be extended upto 200.

As mentioned earlier the driving ac motor can be an induction motor or synchronous motor. An induction motor operates at a lagging power factor. The synchronous motor may be operated at a leading power factor by over-excitation of its field. Leading reactive power generated by over-excited synchronous motor compensates for the lagging reactive power taken by other inductive loads in the plant. Thus, the power factor of the plant is improved.

When the load is heavy and intermittent, a slip-ring induction motor is used as a prime mover. A flywheel is mounted on its shaft. This scheme is known as **Ward-Leonard-Ilgener Scheme.** It prevents heavy fluctuations in supply current.

When the driving ac motor is a synchronous motor supply current fluctuations cannot be reduced by mounting a flywheel on its shaft, because a synchronous motor operates only at a constant speed.

In another form of Ward-Leonard drive, non-electrical prime movers can also be used to drive the dc generator. For example, in diesel electric locomotive and ship populsion drives, the dc generator is driven by a diesel engine or a gas turbine. In this system regenerative braking is not possible because energy cannot flow in the reverse direction in the prime mover.

Advantages of Ward-Leonard Drives

The main advantages of the Ward-Leonard drive are as follows :

1. Smooth speed control of dc motors over a wide range in both directions is possible.
2. It has inherent regenerative braking capacity.
3. By using an overexcited synchronous motor as the drive for dc generator, the lagging reactive voltamperes of the plant are compensated. Therefore the overall power factor of the plant improves.
4. When the load is intermittent as in rolling mills, the drive motor used is an induction motor with a flywheel mounted on its shaft to smooth out the intermittent loading to a low value.

Drawbacks of Classical Ward-Leonard System

The classical Ward-Leonard system with rotating machines (M – G set) suffers from the following drawbacks :

1. Higher initial cost due to use of two additional machines (M – G set) of the same rating as the main dc motor.

2. Larger size and weight.
3. Requires more floor area and costly foundation.
4. Frequent maintenance is needed.
5. Lower efficiency due to higher losses.
6. The drive produces more noise.

7.13 SOLID-STATE CONTROL

Static Ward-Leonard drives are being used these days because of the drawbacks of the classical method. Rotating motor-generator sets have been replaced by solid-state converters to control the speed of dc motors. The converters used are controlled rectifiers or choppers.

In case of ac supply, controlled-rectifiers are used to convert fixed ac supply voltage into a variable ac supply voltage.

When the supply is dc, choppers are used to obtain variable dc voltage from the fixed-voltage dc supply.

Drawbacks of Static Ward-Leonard Drives

The main drawbacks of static Ward-Leonard drives are as follows :

1. They are not suitable for intermittent loads because load fluctuations produce large fluctuations of supply voltage and current. There is no provision of load equalisation in static Ward-Leonard system.
2. Harmonics are generated in the system which affect the quality of supply.
3. Such a system operates at low power factor particularly at low speeds.

In general static Ward-Leonard drives are used in most applications. However, conventional Ward-Leonard drives are used in large-size intermittent loads. In case of nonelectrical prime movers conventional Ward-Leonard system can only be used.

Applications of Ward-Leonard Drives

Ward-Leonard drives are used where a smooth speed control of dc motors over a wide range in both directions is required as in rolling mills, elevators, cranes, paper mills, diesel-electric locomotives, mine hoists etc.

7.14 STARTING D.C. MOTORS

A **starter** is a device to start and accelerate a motor. A **controller** is a device to start, control speed, reverse, stop and protect the motor.

7.14.1 Need for starters

The armature current of a motor is given by

$$I_a = \frac{V - E}{R_a} \tag{7.14.1}$$

Thus, I_a depends upon E and R_a if V is kept constant. When a motor is first switched on, the armature is stationary so the back e.m.f. E is zero. The initial starting armature current I_{as} is given by

$$I_{as} = \frac{V - 0}{R_a} = \frac{V}{R_a} \tag{7.14.2}$$

Since the armature resistance of a motor is very small, generally less than one ohm; therefore the starting armature current I_{as} would be very large. For example, if a motor with armature resistance of 0.5 ohm is connected directly to a 230-V supply, then

$$I_{as} = \frac{V}{R_a} = \frac{230}{0.5} = 460 \text{ A}$$

This large current would damage the brushes, commutator, or windings.

As the motor speed increases, the back e.m.f. increases and the difference $(V - E)$ goes on decreasing. This results in the gradual decrease of I_a until the motor attains its stable speed and the corresponding back e.m.f. Under this condition the armature current reaches its desired value. Thus, it is found that the back e.m.f. helps the armature resistance in limiting the current through the armature.

Since the starting current is very large, at the time of starting of all d.c. motors (except very small motors) an extra resistance must be connected in series with the armature. This would limit the initial current to a safe value until the motor has built up the stable speed and back e.m.f. E. The series resistance is divided into sections which are cut out one by one as the speed of the motor rises and the back e.m.f. builds up. When the speed of the motor builds up to its normal value, the extra resistance is completely cut out.

7.15 THREE-POINT D.C. SHUNT MOTOR STARTER

Figure 7.23 shows a three-point d.c. shunt motor starter. It consists of a graded resistance R to limit the starting current. Prior to starting, the handle H is kept in the OFF position by a spring S. For starting the motor, the handle H is moved manually and when it makes contact with the resistance stud 1 it is in the START position. In this position the field winding receives the full supply voltage, but the armature current is limited by the graded resistance $R(= R_1 + R_2 + R_3 + R_4)$. The starter handle is then gradually moved from stud to stud, allowing the speed of the motor to build up until it reaches the RUN position. In this position (*a*) the motor attains full speed, (*b*) the supply is directly across both the windings of the motor, and (*c*) the resistance R is completely cut out. The handle H is held in RUN position by an electromagnet energized by a no-volt trip coil NVC. The no-volt trip coil is connected in series with the field winding of the motor. In the event of switching off, or when the supply voltage falls below a predetermined value, or the complete failure of supply while the motor is running, NVC is deenergized. This results in release of the handle, which is then pulled back to OFF position by the action of the spring. The current to the motor is cut off, and the motor is not restarted without resistance R in the armature circuit. The NVC also provides protection against an open-circuit in the field winding. The NVC is called **no-volt or undervoltage protection** of the motor. Without this protection, the supply voltage might be restored with the handle in the RUN position. Consequently, full line voltage may be applied directly to the armature resulting in a very large current.

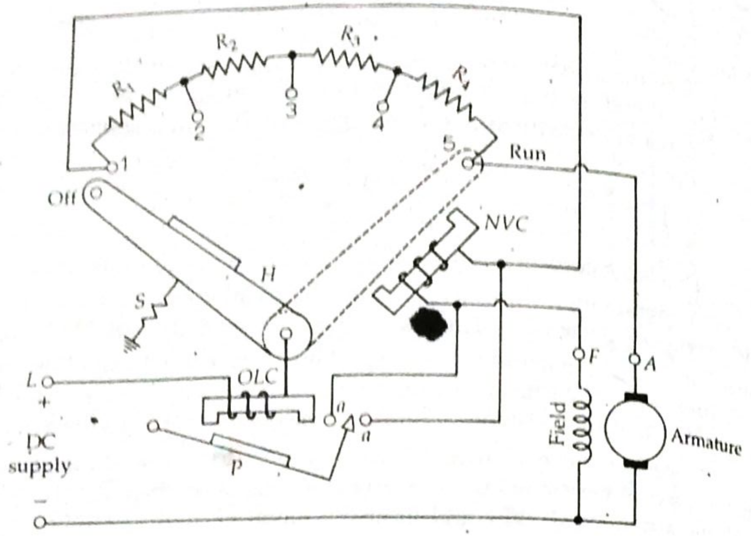


Fig. 7.23. *Three-point D.C. shunt motor starter.*

The other protective device incorporated in the starter is the **overload protection.** Overload protection is provided by the overload trip coil OLC and the NVC. The overload coil is a small electromagnet. It carries the armature current, and for normal values of armature current the magnetic pull of OLC is insufficient to attract the strip *P*. When the armature current exceeds the normal rated value (that is, when the motor is overloaded), *P* is attracted by the electromagnet of OLC and closes the contacts *aa*. Thus, NVC is short-circuited. This results in the release of the hundle *H*, which returns to the OFF position and the motor supply is cut off.

If the motor is to be stopped the main switch should be opened. To stop the motor, the starter handle should never be pulled back as this would result in burning the starter contacts.

7.16 DRAWBACKS OF A THREE-POINT STARTER

The three-point starter suffers from a serious drawback for motors with large variation of speed by adjustment of the field rheostat. To increase the speed of the motor the field resistance should be increased. Therefore the current through the shunt field is reduced. The field current may become very low because of the addition of high resistance to obtain a high speed. A very low field current will make the holding electromagnet too weak to overcome the force exerted by the spring. The holding magnet may release the arm of the starter during the normal operation of the motor and thus disconnect the motor from the line. This is not desirable. A four-point starter is used to overcome this difficulty.

7.17 FOUR-POINT STARTER

The schematic connection diagram of four-point starter is shown in Fig. 7.24. The basic difference in the circuit of a 4-point starter as compared to a 3-point starter is that, the holding coil is removed from the shunt field circuit and is connected directly across the line with a current limiting resistance R in series. Such an arrangement forms three parallel circuits :

1. Armature, starting resistance and overload release.
2. A variable resistance and shunt field winding.
3. Holding coil and current limiting resistance.

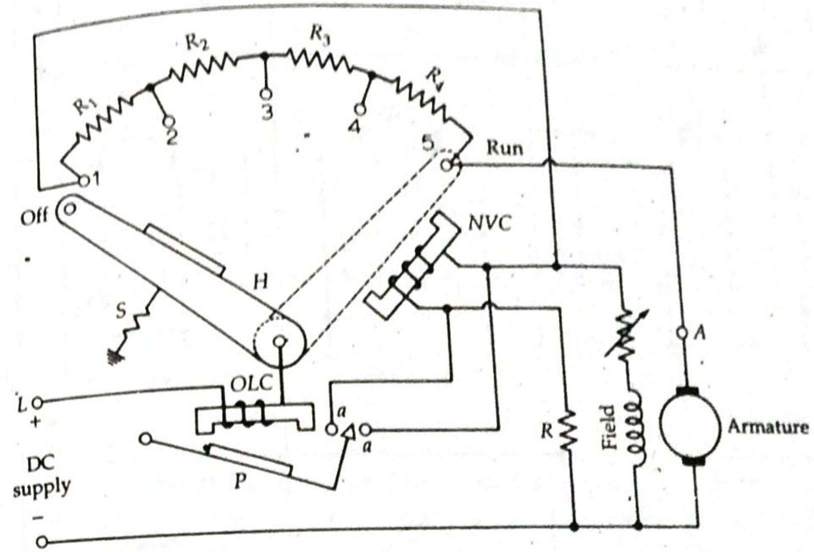


Fig. 7.24. *Four-point D.C. shunt motor starter.*

With this arrangement, a change in field current for variation of speed of the motor, does not affect the current through the holding coil, because the two circuits are independent of each other.

Nowadays automatic push button starters are also used. In such starters the ON push button is pressed to connect the current-limiting starting resistors in series with the armature circuit. These resistors are gradually disconnected by an automatic controlling arrangement until full line voltage is available to the armature circuit. With pressing the OFF button, the circuit is disconnected. Automatic starter circuits have been developed using electromagnetic contactors and time-delay relays. The automatic starters enable even an inexperienced operator to start and stop the motor without any difficulty.

7.18 REVERSAL OF ROTATION

The direction of rotation of a dc motor can be reversed by reversing the connections of *either* the field winding *or* the armature *but not both*. Figure 7.25 shows the alterations in connections required for the reversed rotation of series, shunt and compound motors.

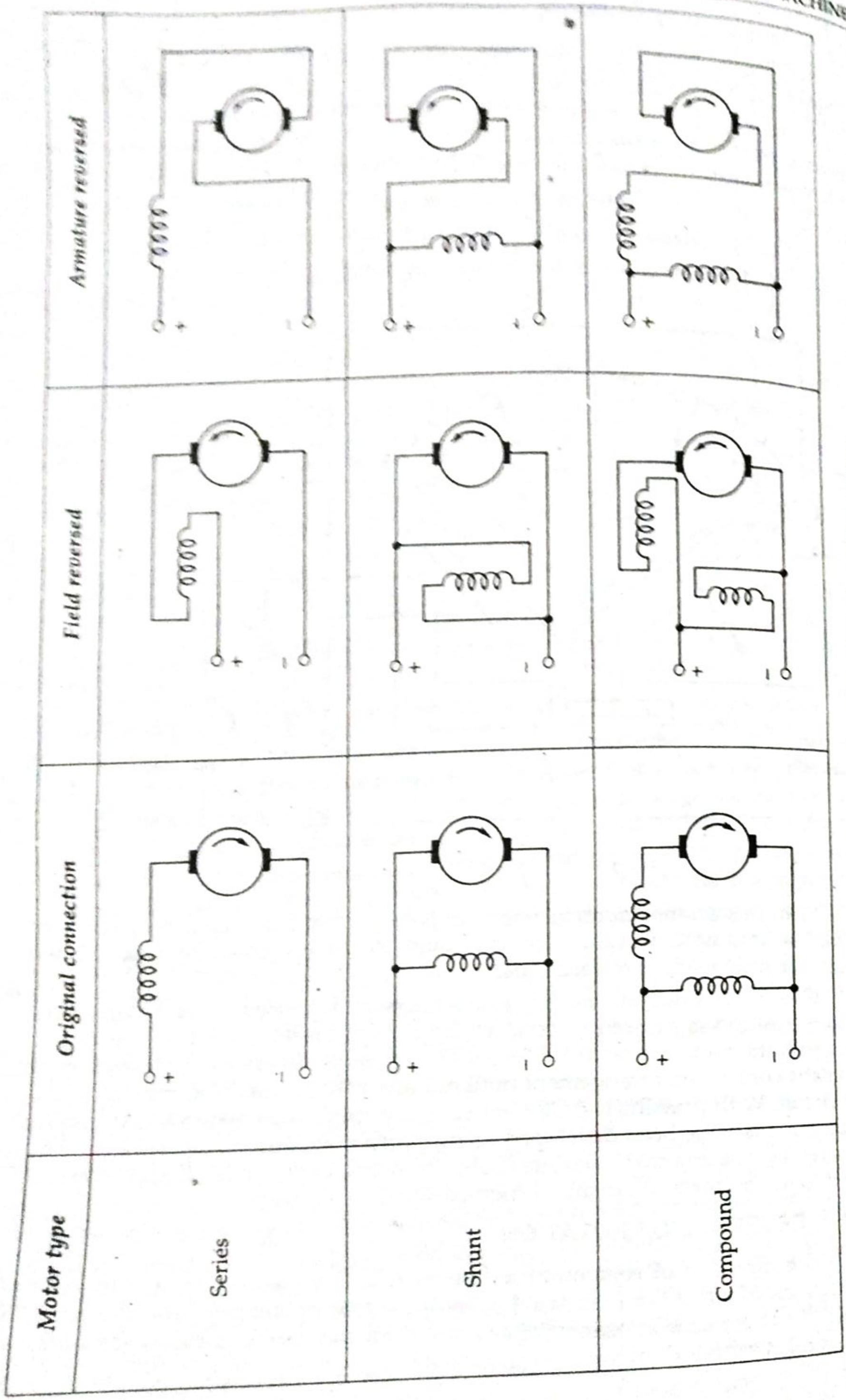


Fig. 7.25. Reversing direction of rotation of d.c. motors.

It is to be noted that in order to reverse the direction of rotation of a compound motor the reversal of the field connections involves both shunt and series windings.

7.19 LOSSES IN DC MACHINES

The losses that occur in dc machines can be divided into five basic categories :

1. Electrical or copper losses (I^2R losses)
2. Core losses or iron losses
3. Brush losses
4. Mechanical losses
5. Stray-load losses

7.19.1 Electrical or Copper Losses or Winding Losses

These losses are the winding losses. The copper losses are present because of the resistance of the windings. Currents flowing through these windings produce ohmic losses (that is, I^2R losses). The windings that may be present in addition to the armature winding are the field windings, interpole and compensating windings.

- Armature copper losses = $I_a^2R_a$ where I_a is armature current and R_a is armature resistance. There losses are about 30 per cent of total full-load losses.
- Copper loss in the shunt field of a shunt machine = $I_{sh}^2R_{sh}$ where I_{sh} is the current in the shunt field and R_{sh} is the resistance of the shunt field winding. The shunt regulating resistance is included in R_{sh}.
- Copper loss in the series field of a series machine = $I_{se}^2\, R_{se}$ where I_{se} is the current through the series field winding and R_{se} is the resistance of the series field winding.
- In a compound machine, both shunt and series field losses occur. These losses are about 20% of full load losses.
- Copper loss in interpole windings = $I_a^2R_i$ where R_i is the resistance of interpole windings.
- Copper loss in compensating winding if any = $I_a^2R_c$ where R_c is the resistance of compensating winding.

7.19.2 Magnetic Losses or Core Losses or Iron Losses

The core losses are the hysteresis losses and eddy-current losses. Since machines are usually operated at constant flux density and constant speed, these losses are almost constant. These losses are about 20 per cent of full-load losses.

7.19.3 Brush Losses

There is a power loss at the brush contacts between the copper commutator and the carbon brushes. In practice, thin loss depends upon the brush contact voltage drop and the armature current I_a. It is given by

$$P_{BD} = V_{BD} I_a$$

The voltage drop across a set of brushes is approximately constant over a large range of armature currents. Unless stated otherwise, the brush voltage drop is usually assumed to be about 2 V. The brush droploss is, therefore, taken as $2 I_a$.

7.19.4 Mechanical Losses

The losses associated with mechanical effects are called mechanical losses. They consist of *bearing friction loss* and *windage loss*. Windage losses are those associated with overcoming are friction between the moving parts of the machine and the air inside the machine for cooling purposes. These losses are usually very small.

7.19.5 Stray-Load Losses

Stray-load loss consists of all losses, not covered above. These are the miscellaneous losses that result from such factors as (*i*) the distortion of flux because of armature reaction, (*ii*) short circuit currents in the coil, undergoing commutation, etc. These losses are very difficult to determine. The indeterminate nature of the stray-load loss makes it necessary to assign it a reasonable value. For most machines stray losses are taken by convention to be one percent of the full-load output power. The term stray powerless should not be confused with stray load loss.

7.20 POWER-FLOW DIAGRAM

Power-flow diagram is used for determining the generator and motor efficiencies. A power-flow diagram for a d.c. generator is shown in Fig. 7.26.

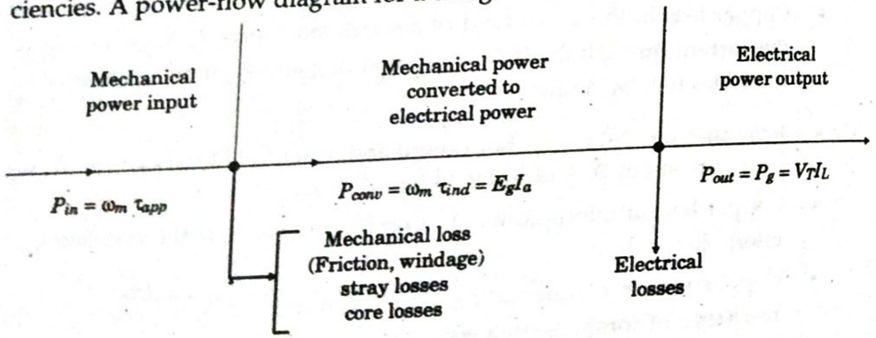


Fig. 7.26. *Power-flow diagram of a d.c. generator.*

In a d.c. generator, the input is the mechanical power input given by

$$P_{in} = \omega_m \tau_{app} \tag{7.20.1}$$

where ω_m = angular speed of armature in rad/s

τ_{app} = applied torque in Nm

The sum of stray losses, mechanical losses and core losses are subtracted from P_{in} to get the net mechanical power converted to electrical power by electro-mechanical conversion.

$$P_{conv} = P_i - \text{stray loss} - \text{mechanical loss} - \text{core losses}$$

$$= \omega_m \tau_{ind} = \omega_m \tau_e \tag{7.20.2}$$

where τ_e is the electromagnetic torque. The resulting electric power produced is given by

$$P_{conv} = E_g I_a \tag{7.20.3}$$

The net electrical power output is obtained by subtracting electrical I^2R losses and brush losses from P_{conv}.

$$P_{out} = P_{conv} - \text{electrical } I^2 R \text{ loss} - \text{brush losses} \tag{7.20.4}$$

$$P_{out} = V_T I_L \tag{7.20.5}$$

where V_T is the terminal voltage and I_L is the current delivered to the load.

The power-flow diagram for a dc motor motor is shown in Fig. 7.27.

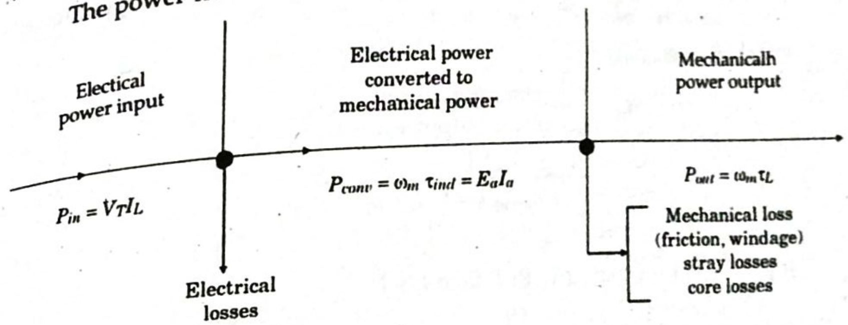


Fig. 7.27. *Power-flow diagram of a dc motor.*

In a dc motor, the input electrical power P_{in} is given by (7.20.6)

$$P_{in} = V_T I_L \tag{7.20.7}$$

$$P_{conv.} = P_i - \text{copper losses}$$

Power output (7.20.8)

$$P_{out} = \omega_m \tau_L$$

Also, $P_{out} = P_{conv.} -$ core losses – mechanical losses – stray losses (7.20.9)

where τ_L = load torque in N_m.

7.21 EFFICIENCY OF A D.C. MACHINE

(a) Generator

Let R = total resistance of the armature circuit (including the brush-contact resistance, at series winding resistance, interpole winding resistance, and compensating winding resistance, if any)

I = output current

I_{sh} = current through the shunt field

I_a = armature current = $I + I_{sh}$

V = terminal voltage

Total copper loss in the armature circuit = $I_a^2 R_{at}$

Power loss in the shunt circuit

= VI_{sh} (this includes the loss in the shunt regulating resistance)

Mechanical losses = friction loss at bearings + friction loss at commutator + windage loss

Core losses = hysteresis loss + eddy-current loss

Stray loss = mechanical loss + core loss

The sum of the shunt field copper loss and stray losses may be considered as a combined *fixed* (*constant*) loss that does not vary with the load current I.

∴ constant losses (in shunt and compound generators) = stray loss + shunt field copper losses

$$\text{Total losses} = I_a^2 R_{at} + p_k + V_{BD} I_a$$

Generator efficiency

$$\eta_G = \frac{\text{generator output}}{\text{generator output + losses}}$$

$$= \frac{VI}{VI + I_a^2 R_{at} + V_{BD} I_a + p_k}$$

$$I_a = I + I_{sh}$$

If I_{sh} is small compared with I, then $I_a = I$

$$\therefore \quad \eta_G = \frac{VI}{VI + I^2 R_{at} + V_{BD} I + p_k}$$

$$= \frac{1}{1 + \dfrac{I R_{at}}{V} + \dfrac{V_{BD}}{V} + \dfrac{p_K}{VI}}$$

The efficiency η_G will be a maximum when the denominator D_r is a minimum, where $D_r = 1 + \dfrac{IR_{at}}{V} + \dfrac{V_{BD}}{V} + \dfrac{p_k}{VI}$

D_r is a minimum when

$$\frac{dD_r}{dI} = 0 \quad \text{and} \quad \frac{d^2 D_r}{dI^2} > 0$$

$$\frac{dD_r}{dI} = \frac{d}{dI}\left(1 + \frac{IR_{at}}{V} + \frac{V_{BD}}{V} + \frac{p_k}{VI}\right)$$

$$0 = 0 + \frac{R_{at}}{V} + \frac{p_k}{V}\left(-\frac{1}{I^2}\right)$$

or $$\boxed{I^2 R_{at} = p_k} \tag{7.21.1}$$

Also, $$\frac{d^2 D_r}{dI^2} = \frac{d}{dI}\left(\frac{R_{at}}{V} - \frac{p_k}{VI^2}\right) = \frac{2p_k}{VI^3} > 0$$

Since $\dfrac{d^2 D_r}{dI^2}$ is positive, the expression given by Eq. (7.21.1) is a condition for the minimum value of D_r, and therefore the condition for maximum value of efficiency.

Equation (7.21.1) shows that *the efficiency of a dc generator is a maximum when those losses proportional to the square of the load current are equal to the constant losses of the dc generator.* This relation applies equally well to *all* rotating machines, *regardless* of type.

This relationship is sometimes *incorrectly* stated as "maximum efficiency occurs when the variable losses are equal to the constant losses".

Load Corresponding to Maximum Efficiency

Let I_{fl} = full-load current

I_M = current at maximum efficiency

For maximum efficiency

$$I_M^2 R_{at} = p_k$$

$$I_M^2 = \frac{p_k}{R_{at}}$$

$$= \frac{I_{fl}^2 p_k}{I_{fl}^2 R_{at}}$$

$$I_M = I_{fl}\sqrt{\frac{p_k}{I_{fl}^2 R_{at}}}$$

$$\therefore \quad \text{Current at maximum efficiency} = \text{full-load current} \times \sqrt{\frac{\text{constant loss}}{\text{f.l. copper loss}}} \tag{7.21.2}$$

7.22 TESTING OF DC MACHINES

Machines are tested for finding out losses, efficiency and temperature rise. Direct-loading tests may be performed on small machines. For large shunt machines, indirect methods are used. Swinburne's test and Hopkinson's test are mostly used in practice.

7.23 SWINBURNE'S TEST

It is an ***indirect method*** of testing dc machines. In this method the losses are measured separately, and the efficiency at any desired load is predetermined.

The machine is run as a motor at rated voltage and speed. Figure 7.28 shows the connection diagram for the test for dc shunt machine.

Let V = supply voltage

I_0 = no-load current

I_{sh} = shunt field current

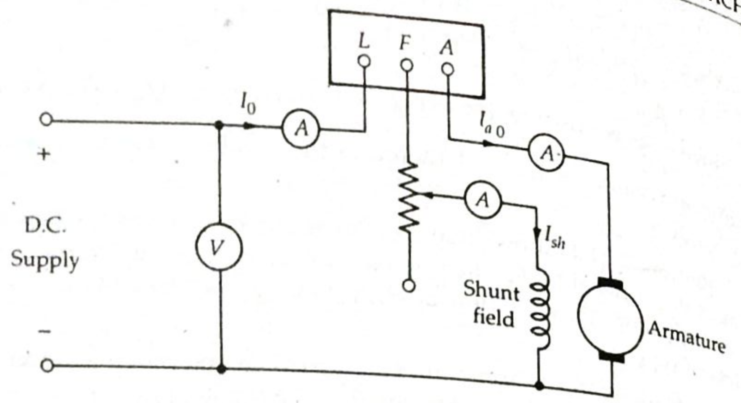


Fig. 7.28. *Swinburne's test.*

$\therefore$ no-load armature current

$$I_{a0} = I_0 - I_{sh}$$

No-load input $= VI_0$.

The no-load power input to the machine supplies the following :

(*i*) iron loss in the core,

(*ii*) friction losses at bearings and commutator,

(*iii*) windage loss

(*iv*) armature copper loss at no load.

When the machine is loaded, the temperature of the armature winding and field winding increases due to I^2R losses. In calculating the I^2R losses hot resistances should be used. A stationary measurement of resistances at room temperature of, say, $t°$ C is made by passing current through armature and then field from a low voltage dc supply. Then the hot resistance, allowing a temperature rise of say 50° C, is found as follows :

$$R_{t_1} = R_0\,(1 + \alpha_0 t_1)$$

$$R_{t_1+50°} = R_0\,[1 + \alpha_0\,(t_1 + 50°)]$$

where $\alpha_0 =$ temperature coefficient of resistance at 0° C

$$\therefore \quad R_{t_1+50°} = R_{t_1}\,\frac{1 + \alpha_0\,(t_1 + 50°)}{1 + \alpha_0 t_1}$$

Stray loss = iron loss + friction loss + windage loss = input at no load − field copper loss − no load armature copper loss

$$= VI_0 - p_f - p_{a0} = p_s \text{ (say)}$$

Also, constant losses

$p_c =$ no-load input − (no-load armature copper loss)

$$p_c = p_s + p_f$$

By knowing the constant losses of the machine, its efficiency at any other load can be determined as follows :

Let I be the load current at which efficiency is required.

Efficiency when running as motor

Motor input $= VI$

Armature copper loss $= I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses $= p_c$ (found above)

$\therefore$ total losses $= (I - I_{sh})^2 R_a + p_c$

Motor efficiency

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{VI - (I - I_{sh})^2 R_a - p_c}{VI}$$

Efficiency when running as generator

Armature current $I_a = I + I_{sh}$

Generator output $= VI$

Armature copper loss $= (I + I_{sh})^2 R_a$

Constant losses $= p_c$ (found above)

$\therefore$ total losses $= (I + I_{sh})^2 R_a + p_c$

Efficiency of generator

$$\eta_g = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + p_c}$$

Advantages of Swimburne's Test

The main advantages of Swinburne's test are :

1. It is a convenient and economical method of testing dc machines since power required to test a large machine is small.
2. The efficiency can be predetermined at any load because constant losses are known.

Main Disadvantages

1. No account is taken of the change in iron loss from no load to full load. At full load, due to armature reaction, flux is distorted which increases the iron losses.
2. As the test is on no load, it does not indicate whether the commutation on full load is satisfactory and whether the temperature rise would be within specified limits.

Limitations

1. Swinburne's test is applicable to those machines in which the flux is practically constant, that is shunt machines and level compound generators.
2. Series machines cannot be tested by this method as they cannot be run on light loads and secondly flux and speed vary greatly.

7.24 HOPKINSON'S TEST

This test is also called

(a) **regenerative test**
(b) **back-to-back test**
(c) **heat-run test.**

The test requires two *identical* shunt machines which are coupled mechanically and also connected electrically in parallel. One of them acts as a motor and the other as the generator. The motor takes its input from supply. The mechanical output of motor drives the generator and the electrical output of the generator is used in supplying the input to motor. Thus, output of each machine is fed as input to the other. When both machines are run on full load, the input from supply will be equal to the total losses of both the machines. Hence the power input from the supply is very small.

The circuit diagram for Hopkinson's test is shown in Fig. 7.29 machine M is started from the supply as motor with the help of a starter (not shown). The switch S is kept open. The field current of M is adjusted with the help of field rheostat R_M to enable the motor to run at rated speed. Machine G acts as a generator. Since G is mechanically coupled to M, it runs at the rated speed of M. The excitation of the generator G is so adjusted with the help of its field rheostat R_G that the voltage across the armature of G is slightly higher than the supply voltage. In actual practice, the terminal voltage of the generator is kept 1 or 2V higher than the supply busbar voltage. When this is achieved, that is, the voltage of the generator being equal and of the same polarity as the busbar voltage, the main switch S is closed and the generator is connected to the busbars. Thus, both the machines are now in parallel across the supply. Under this condition, the generator is said to float. That is, it is neither taking any current from nor giving any current to the supply. Any required load can now be thrown on the machines by adjusting the excitation of the machines with the help of field rheostats.

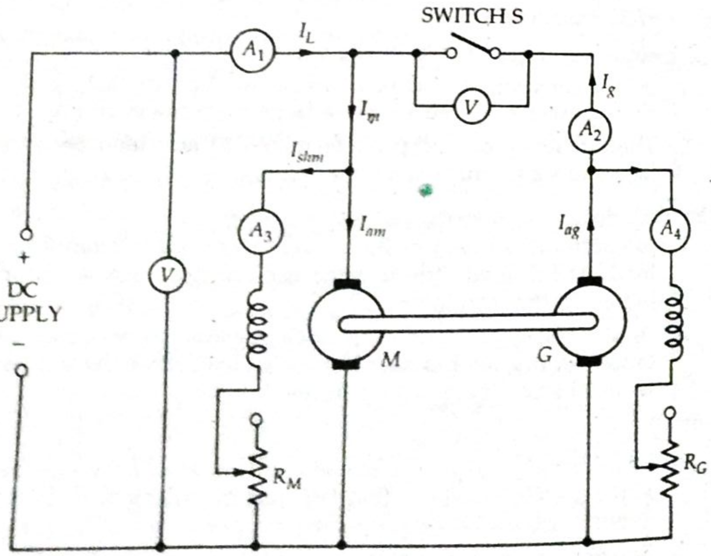


Fig. 7.29. *Hopkinson's test on two similar dc shunt machines.*

Let V = supply voltage
I_L = line current
I_m = input current to motor
I_g = output current from the generator
I_{am} = motor armature current
I_{ag} = generator armature current
I_{shm} = motor shunt field current
I_{shg} = generator shunt field current
R_a = armature resistance of each machine
R_{shm} = motor shunt field resistance
R_{shg} = generator shunt field resistance
E_g = generator induced voltage
E_m = motor induced voltage (back emf)

We have

$$E_g = V + I_{ag} R_a$$

$$E_m = V - I_{am} R_a$$

$\therefore$ $E_g > E_m$

But $E_g \propto \Phi_g N$

$E_m \propto \Phi_m N$

$\therefore$ $\Phi_g > \Phi_m$

Since $\Phi \propto I_f$ (I_f is field current and Φ is field flux)

$$I_{shg} > I_{shm}$$

Thus, the excitation of the generator shall always be greater than that of the motor. That is, *the machine with smaller excitation acts a a motor.*

The load on the machines can be adjusted as desired and readings taken. The efficiencies of the two machines can be determined as follows :

Power input from the supply = VI_L = total losses of both the machines

Armature copper loss of the motor $= I_{am}^2 R_a$

Field copper loss of the motor $= I_{shm}^2 R_{shm}$

Armature copper loss of the generator $= I_{ag}^2 R_a$

Field copper loss of the generator $= I_{shg}^2 R_{shg}$

For identical machines the constant losses P_c (iron, friction and windage osses) are assumed to be equal.

Constant losses of both the machines = power drawn from the supply − armature and shunt copper losses of both the machines

$$P_c = VI_L - (I_{am}^2 R_a + I_{shm}^2 R_{shm} + I_{ag}^2 R_a + I_{shg}^2 R_{shg})$$

Assuming that the constant losses (stray losses) are equal divided between the two machines,

$$\text{total stray loss per machine} = \frac{1}{2} P_c$$

The efficiencies of the two machines can be determined as follows :

Generator

$$\text{Output} = VI_{ag}$$

$$\text{Constant losses for generator} = \frac{P_c}{2}$$

$$\text{Armature copper loss} = I_{ag}^2 R_a$$

$$\text{Field copper loss} = I_{shg}^2 R_{shg}$$

Efficiency of the generator

$$\eta_g = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$= \frac{VI_{ag}}{VI_{ag} + I_{ag}^2 R_a + I_{shg}^2 R_{shg} + \frac{1}{2} P_c}$$

Motor

$$\text{Input} = VI_m = V(I_{am} + I_{shm})$$

$$\text{Constant losses for motor} = \frac{P_c}{2}.$$

$$\text{Armature copper loss} = I_{am}^2 R_a$$

$$\text{Field copper loss} = I_{shm}^2 R_{shm}$$

Efficiency of the motor

$$\eta_m = \frac{\text{Output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= \frac{V(I_{am} + I_{shm}) - \left(\frac{P_c}{2} + I_{am}^2 R_a + I_{shm}^2 R_{shm}\right)}{V(I_{am} + I_{shm})}$$

Merits of Hopkinson's Test

The main *advantages* of using Hopkinson's test for determination of efficiency are :

1. The total power taken from the supply is very low. Therefore this method is very economical.
2. The temperature rise and the commutation conditions can be checked under rated load conditions.
3. Stray losses are considered, as both the machines are operated under rated load conditions.

4. Large machines can be tested at rated load without consuming much power from the supply.
5. Efficiency at different loads can be determined.

Disadvantages

The main *disadvantage* of this method is the necessity of two practically identical machine to be available. Consequently, this test is suitable for manufacturers of large dc machines.

7.25 ELECTRIC BRAKING OF DC MOTORS

Electric braking is usually employed in applications to stop a unit driven by motors in an exact position or to have the speed of the driven unit suitably controlled during its deceleration. In applications requiring frequent, quick, accurate or rapid emergency stops, electric braking is used. For example, in suburban electric trains quick stops are required. Electric braking allows smooth stops without any inconvenience to passengers.

When a loaded hoist is lowered, electric braking keeps the speed within safe limits, otherwise, the drive speed will reach dangerous values.

When a train goes down a steep gradient, electric braking is employed to hold the train speed within safe limits. Similarly, in applications involving other active loads, electric braking is very commonly used.

The braking force can also be obtained by using mechanical brakes.

7.26 DISADVANTAGES OF MECHANICAL BRAKING

The following are the main disadvantages of mechanical braking :

1. It requires frequent maintenance and replacement of brake shoes.
2. Braking power is wasted as heat.

In spite of the disadvantages, mechanical braking is used along with electric braking to ensure reliable operation of the drive. Mechanical brakes are also used to hold the drive at standstill because many braking methods do not produce torque at standstill.

7.27 TYPES OF ELECTRIC BRAKING

There are three types of braking a dc motor :

1. Regenerative braking
2. Dynamic braking or rheostatic braking
3. Plugging or reverse current braking.

7.28 REGENERATIVE BRAKING

It is a form of braking in which the kinetic energy of the motor and its driven machinery is returned to the power supply system. This type of braking is possible when the driven load forces the motor to run at a speed higher than its no-load speed with a constant excitation. Under this condition, the motor back emf E_b is greater than the supply voltage V, which reverses the direction of motor armature current. The machine now begins to operate as a generator and the energy generated is supplied to the source.

Regenerative braking can also be carried out upto very low speeds if the motor is connected as a separately excited generator and excitation is increased as the speed is reduced, so that $E_b = \frac{nP\Phi Z}{A}$ and $V = E_b - I_a R_a$ are satisfied and the motor does not enter into saturation on increasing excitation.

Regeneration is possible with a shunt and separately excited motors and with compound motors with weak series compounding. Regenerative braking is used specially where more frequent braking or slowing of drives is required. It is most useful in holding a descending load of high potential energy at a constant speed. For example, regenerative braking is used to control the speed of motors driving loads such as electric locomotives, elevators, cranes and hoists. While descending, the load in this operation acts as the prime mover by virtue of its potential energy. The motor acts as a generator. The generated power is thus returned to the supply. The returned power is available for other devices operating from the same source of supply. Regenerative braking cannot be used for stopping the motor. It is used for controlling the speed above the no-load speed of the motor driving the descending loads.

The necessary condition for regeneration is that the back emf E_b should be greater than the supply voltage so that the armature current is reversed and the mode of operation changes from motoring to generating.

7.28.1 Regenerative Braking in dc Shunt Motors

Under normal operating conditions the armature current is given by

$$-I_a = \frac{V - E_b}{R_a}$$

When the load (such as lowering of load by a crane, hoist or lift) causes the motor speed to be greater than the no-load speed, the back emf E_b becomes greater than the supply voltage V. Consequently, armature current I_a becomes negative. The machine now begins to operate as a generator.

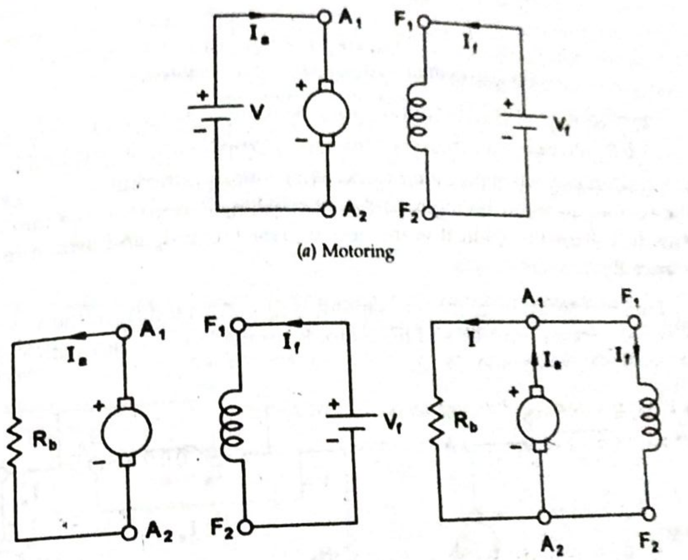
7.28.2 Regenerative Braking in dc Series Motors

In case of a dc series motor an increase in speed is followed by a decrease in the armature current and field flux. The back emf E_b cannot be greater than the supply voltage. Regeneration is not possible in a plain dc series motor since the field current cannot be made greater than armature current. However, in applications such as traction, elevators, hoists etc., where dc series motors are used extensively, regeneration may be required. For example, in an electrolocomotive moving down a gradient, a constant speed may be necessary, and in hoist drives the speed is to be limited whenever it becomes dangerously high. One commonly used method of regenerative braking of the dc series motor is to connect it as a shunt motor. Since the resistance of the field winding is low, a series resistance is connected in the field circuit to limit the current within the safe value.

7.29 DYNAMIC BRAKING OR RHEOSTATIC BRAKING

In dynamic braking, the dc motor is disconnected from the supply and a braking resistor R_b is immediately connected across the armature. The motor now works as a generator, producing the braking torque.

For the braking operation, the separately excited (or shunt) motor can be connected either as a separately excited generator, where the flux is kept constant, or it can be connected as a self-excited shunt generator, with the field winding in parallel with the armature. Fig. 7.30 shows the dynamic braking of separately excited dc motor. Fig. 7.31 shows the dynamic braking of a dc shunt motor.

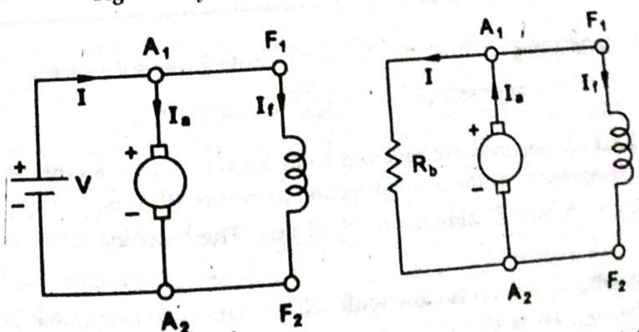


(a) Motoring

(b) Braking with separate excitation

(c) Braking with self excitation

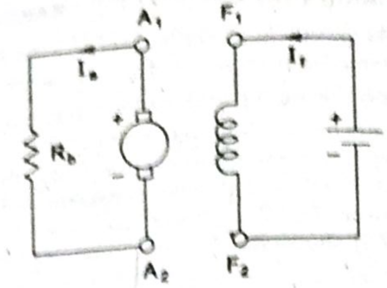
Fig. 7.30. *Dynamic braking of separately excited dc motor.*



(a) Motoring

(b) Braking with self excitation

Fig. 7.31. *Dynamic braking of dc shunt motor.*

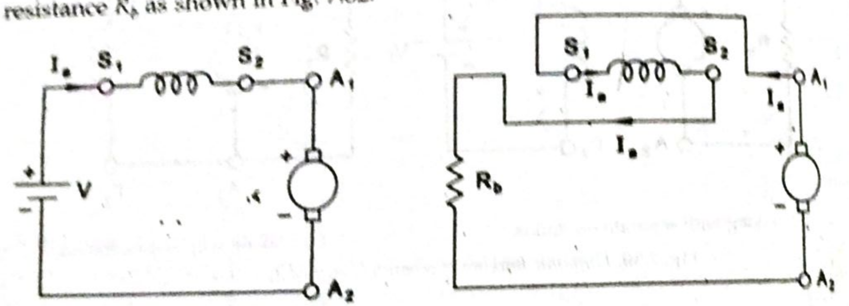


(c) Braking with separate excitation

Fig. 7.31. (contd.) *Dynamic braking of dc shunt motor.*

This method is also called ***rheostatic braking*** because an external braking resistance R_b is connected across the armature terminals for electric braking. During electric braking when the motor works as a generator, the kinetic energy stored in the rotating parts of the motor and connected load is converted into electric energy. It is dissipated as heat in the braking resistance R_b and armature circuit resistance R_a.

For dynamic braking, the series motor is disconnected from the supply, the field connections are reversed and the motor is connected in series with a variable resistance R_b as shown in Fig. 7.32.



(a) Motoring

(b) Braking with self excitation

Fig. 7.32. *Dynamic braking of a dc series motor.*

The field connections are reversed to make sure that the current through the field winding flows in the same direction as before (that is, from S_1 to S_2) in order that the back emf E_b aids the residual flux. The machine now works as a self-excited series generator.

The braking operation is slow with self excitation. When quick braking is required, the machine is connected for the separate excitation, and a suitable resistance is connected in series with the field to limit the current to a safe value.

Dynamic braking is an inefficient method of braking, because all the generated energy is dissipated as heat in resistances.

7.30 PLUGGING OR REVERSE CURRENT BRAKING

In this method the armature terminals (or supply polarity) of a separately excited (or shunt) motor when running are reversed. Therefore, the supply voltage V and the induced voltage E_b (back emf) will act in the same direction. Thus during braking the effective voltage across the armature will be $(V + E_b)$ which is almost twice the supply voltage. The armature current is reversed and a high braking torque is produced. In order to limit the armature current to a safe value, an external current-limiting resistor is connected in series with the armature.

For braking a series motor either the armature terminals or field terminals (but not both) are reversed. Reversing of both gives only normal working operation.

The braking torque is not zero at zero speed. When used for stopping a load, the motor must be disconnected from the supply at or near zero speed, otherwise, it will speed up in the reverse direction. Centrifugal switches are used to disconnect the supply.

Plugging is a highly inefficient method of braking because in addition to the power supplied by the load, power supplied by the source is wasted in resistances.

Plugging is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

7.31 FOUR-QUADRANT OPERATION OF DRIVES

A motor operates in two modes—motoring and braking. In motoring, it converts electrical energy to mechanical energy, which supports its motion. In braking, it works as a generator converting mechanical energy to electrical energy, and thus opposes the motion. Motor can provide motoring and braking operations for both forward and reverse directions. A motor drive capable of operating in both directions of rotation and of producing both motoring and regeneration is called a four-quadrant variable-speed drive.

Power developed by a motor is given by the product of angular speed and torque. For multi-quadrant operation of drives the following conventions about the signs of torque and speed are useful: Motor speed is considered positive when rotating in forward direction. For drives which operate only in one direction, forward speed will be their normal speed. In loads involving up-and-down motions, the speed of motor which causes upward motion is considered forward motion. For reversible drives, forward speed is chosen arbitrarily. Then the rotation in the opposite direction gives reverse speed which is assigned the negative sign. Positive motor torque is defined as the torque which produces acceleration or the positive rate of change of speed in forward direction. Motor torque is taken negative if it produces retardation. Load torque is opposite in direction to the positive motor torque.

Fig. 7.33 shows the four-quadrant operation of drives. In quadrant I, developed power is positive, hence, the machine works as a motor supplying mechanical energy. Operation in quadrant I is, therefore, called forward motoring.

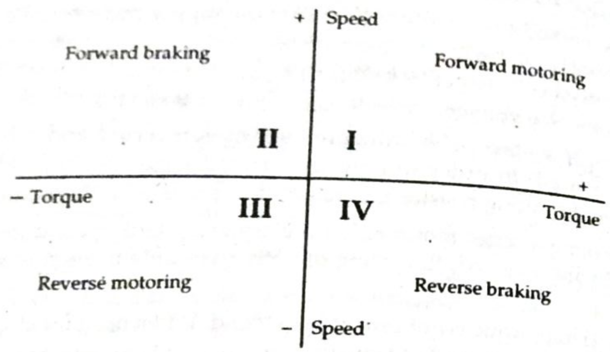


Fig. 7.33. *Four-quadrant operation*

Operation in the quadrant II represents braking, because in this part of the torque-speed plane the direction of rotation is positive and the torque is negative. The machine operates as a generator developing a negative torque which opposes motion. The kinetic energy of the rotating parts is available as electrical energy which may be supplied back to the mains, or in dynamic braking, dissipated in some resistance.

In the third quadrant which corresponds to the motor action in the reverse direction both speed and torque have negative values while the power is positive. Operation in quadrant III is similar to that in the first quadrant with direction of rotation reversed.

In the fourth quadrant, the torque is positive and the speed is negative. This quadrant corresponds to braking in reverse motoring.

The four-quadrant operation and its relationship to speed, torque, and power output are summarized in Table 7.1.

Table 7.1. Four-quadrant dc motor drive characteristics

Function	*Quadrant*	*Speed*	*Torque*	*Power output*
Forward motoring	I	+	+	+
Forward braking	II	+	–	–
Reverse motoring	III	–	–	+
Reverse braking	IV	–	+	–

Compressor, pump and fan type loads require operation in the first quadrant only, since their operation is unidirectional. They are one-quadrant drive systems.

Transportation drives require operation in both directions. The method of braking depends upon the conditions of availability of power supply. If regeneration is necessary, application in all four quadrants may be required. If not, operation is restricted to quadrants I and III dynamic braking or mechanical braking may be required. In hoist drives, a four-quadrant operation is needed.

7.32 PRESENT-DAY USES OF D.C. MACHINES

At present time bulk of electrical energy is generated in the form of alternating current. Hence the use of d.c. generators is very limited. They are mainly used in supplying excitation of small and medium range alternators. For industrial applications of d.c. like electrolytic processes, welding processes and variable speed motor drives, the present trend is to generate a.c. and then to convert a.c. into d.c. by rectifiers. Thus, dc generators have generally been superseded by rectified ac supplies for many applications.

Direct current motors are very commonly used as variable-speed drives and in applications where severe torque variations occur.

The main applications of the three types of d.c. motors are given below :

Series motors

These motors are used where high starting torque is required and speed can vary, for example, traction, cranes, etc.

Shunt motors

These motors are used where constant speed is required and starting conditions are not severe, for example, lathes, centrifugal pumps, fans, blowers, conveyors, lifts etc.

Compound motors

These motors are used where high starting torque and fairly constant speed is required, for example, presses, shears, conveyors, elevators, rolling mills, heavy planers etc.

Small d.c. machines (in fractional kilowatt ratings) are used primarily as control devices such as techogenerators for speed sensing and servomotors for positioning and tracking.

EXAMPLE 7.1. *A d.c. shunt machine, connected to 250 V supply, has an armature resistance (including brushes) of 0.12 Ω and the resistance of the field circuit is 100 Ω. Find the ratio of the speed as a generator to the speed as a motor, the line current in each case being 80 A.*

SOLUTION. $V = 250\ V,\quad I_L = 80\ A,\quad R_a = 0.12\ \Omega$

$$R_{sh} = 100\ \Omega,\quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{100} = 2.5\ A$$

Let suffixes 1 and 2 be used for generator and motor respectively.

Generator

$$I_{a_1} = I_L + I_{Sh} = 80 + 2.5 = 82.5 \text{ A}$$

$$E_1 = V + I_{a_1} R_a = 250 + 82.5 \times 0.12 = 259.9 \text{ V}$$

Motor

$$I_{a_2} = I_L - I_{Sh} = 80 - 2.5 = 77.5 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 77.5 \times 0.12 = 240.7$$

$$\frac{E_1}{E_2} = \frac{N_1 \Phi_1}{N_2 \Phi_2}$$

Since the field current is the same, $\Phi_2 = \Phi_1$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{259.9}{240.7} = 1.0797$$

EXAMPLE 7.2. *The armature resistance of a 200 V shunt motor is 0.4 Ω and no-load current is 2 A. When loaded and taking an armature current of 50 A, the speed is 1200 r.p.m. Find approximately the no-load speed.*

SOLUTION. $E_0 = V - I_{a_0} R_a = 200 - 2 \times 0.4 = 199.2 \text{ V}$

$$E_1 = V - I_{a_1} R_a = 200 - 50 \times 0.4 = 180 \text{ V}$$

$$\frac{N_1}{N_0} = \frac{E_1}{E_0} \times \frac{\Phi_0}{\Phi_1}$$

For a shunt motor, $\Phi_1 = \Phi_0$

$$\therefore \quad N_0 = \frac{E_0}{E_1} \times N_1 = \frac{199.2 \times 1200}{18}$$

$$= 1328 \text{ r.p.m.}$$

EXAMPLE 7.3. *A 250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 A. The total armature and shunt field resistance are respectively 0.2 Ω and 250 Ω. Calculate the speed when loaded and taking a current of 50 A, if the armature reaction weakens the field by 3%.*

SOLUTION. $I_L = 5 \text{ A}, \quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}, \quad I_{a_1} = I_{L_1} - I_{sh} = 5 - 1 = 4 \text{ A}$

$$E_1 = V - I_{a_1} R_a = 250 - 4 \times 0.2 = 249.2 \text{ V}$$

Armature current on load

$$I_{a_2} = I_{L_2} - I_{sh} = 50 - 1 = 49 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 49 \times 0.2 = 240.2 \text{ V}$$

$$\Phi_2 = 0.97\, \Phi_1$$

$$\frac{N_2}{N_1} = \frac{E_2 \Phi_1}{E_1 \Phi_2} = \frac{240.2\, \Phi_1}{249.2 \times 0.97\, \Phi_1}$$

$$N_2 = \frac{240.2 \times 1000}{249.2 \times 0.97} = 993.69 \text{ r.p.m.}$$

EXAMPLE 7.4. *A shunt generator delivers 50 kW at 250 V when running at 400 r.p.m. The armature and field resistance are 0.02 Ω and 50 Ω respectively. Calculate the speed of the machine when running as a shunt motor and taking 50 kW input at 250 V. Allow 1 V per brush for contact drop.*

SOLUTION. *As generator*

Load current $\quad I_L = \frac{50 \times 10^3}{250} = 200 \text{ A}$

Shunt field current $\quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \text{ A}$

Armature current $\quad I_{a_1} = I_{L_1} + I_{sh} = 200 + 5 = 205 \text{ A}$

Generated emf

$$E_1 = V + Ia_1 R_a + \text{voltage drop in the brushes}$$

$$= 250 + (205 \times 0.02) + 2 \times 1 = 256.1 \text{ V}$$

Speed of the generator $N_1 = 400$ r.p.m.

As motor

Input line current $\quad I_{L_2} = \frac{P_i}{V} = \frac{50 \times 10^3}{250} = 200 \text{ A}$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \text{ A}$$

Armature current $\quad I_{a_2} = I_{L_2} - I_{sh} = 200 - 5 = 195 \text{ A}$

$$E_2 = V - I_{a_2} R_a - \text{brush drop}$$

$$= 250 - 195 \times 0.02 - 2 \times 1 = 244.1 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2}$$

Since the field current is constant, $\Phi_2 = \Phi_1$

$$\therefore \quad N_2 = \frac{E_2}{E_1} N_1 = \frac{244.1}{256.1} \times 400 = 381.3 \text{ r.p.m.}$$

EXAMPLE 7.5. *A 4-pole, 250 V, wave-connected shunt motor gives 10 kW when running at 1000 r.p.m. and drawing armature and field currents of 60 A and 1 A respectively. It has 560 conductors. Its armature resistance is 0.2 Ω. Assuming a drop of 1 V per brush, determine : (a) total torque ; (b) useful torque ; (c) useful flux per pole ; (d) rotational losses ; (e) efficiency.*

SOLUTION. $E = V - I_a R_a - \text{brush drop}$

$$= 250 - 60 \times 0.2 - 2 \times 1 = 236 \text{ V}$$

(*a*) $\tau \times \frac{2\pi N}{60} = E I_a$

$$\tau = \frac{60 E I_a}{2\pi N} = \frac{60 \times 236 \times 60}{2\pi \times 1000} = 135.22 \text{ Nm}$$

(b) $\tau_{useful} = \tau_{shaft}$, $P_{out} = 10\text{ kW} = 10 \times 10^3\text{ W}$

$$\tau_{useful} \times \frac{2\pi N}{60} = P_{out}$$

$$\tau_{useful} = \frac{60 \times 10 \times 10^3}{2\pi \times 1000} = 95.49\text{ Nm}$$

(c) $E = \dfrac{N P \Phi Z}{60\,A}$

$$\Phi = \frac{60\,E\,A}{N P Z} = \frac{60 \times 236 \times 2}{1000 \times 4 \times 560}$$

$$= 0.0126\text{ Wb}$$

(d) Armature input $= V I_a = 250 \times 60 = 15000\text{ W}$

Armature copper loss $= I_a^2 R_a = (60)^2 \times 0.2 = 720\text{ W}$

Brush contact loss $= V_b I_a = 2 \times 60 = 120\text{ W}$

Power developed $= V I_a - I_a^2 R_a - V_b I_a$

$$= 15000 - 720 - 120 = 14160\text{ W}$$

Total power output + rotational losses = power developed

$\therefore$ rotational losses = power developed – total power output

$$= 14160 - 10000 = 4160\text{ W}$$

(e) Total input to motor

$$= VI = V(I_a + I_{sh})$$

$$= 250 \times (60 + 1) = 15250\text{ W}$$

Motor efficiency

$$= \frac{\text{motor output}}{\text{motor input}} \times 100\%$$

$$= \frac{10 \times 10^3}{15250} \times 100 = 65.57\%$$

EXAMPLE 7.6. *A 460 V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux and field current to be proportional.*

SOLUTION. $E_1 = V - I_{a_1} R_a = 460 - 40 \times 0.8 = 428\text{ A}$

$$E_2 = V - I_{a_2} R_a = 460 - 30 \times 0.8 = 436\text{ V}$$

Since $\Phi \propto I_a$, $\dfrac{\Phi_1}{\Phi_2} = \dfrac{I_{a_1}}{I_{a_2}}$

$$N_2 = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} N_1$$

$$= \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}} N_1 = \frac{436}{428} \times \frac{40}{30} \times 500 = 679\text{ r.p.m.}$$

$$\tau \propto \Phi I_a$$

$$\tau \propto I_a^2,\quad \tau_1 = k I_{a_1}^2,\ \tau_2 = k I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{k I_{a_2}^2}{k I_{a_1}^2} = \frac{I_{a_2}^2}{I_{a_1}^2} = \frac{(30)^2}{(40)^2} = \frac{9}{16}$$

Percentage change in torque

$$= \frac{\tau_1 - \tau_2}{\tau_1} \times 100$$

$$= \frac{\tau_1 - \frac{9}{16}\tau_1}{\tau_1} \times 100 = \frac{7}{16} \times 100 = 43.75\%$$

EXAMPLE 7.7. *A 220 V, d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 Ω. Assume that the magnetic circuit is unsaturated.*

SOLUTION. For a series motor $\Phi \propto I_a$

Torque $\tau \propto \Phi I_a \propto I_a^2$

$$\tau = k I_a^2$$

$$\tau_1 = k I_{a_1}^2,\quad \tau_2 = k I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$\frac{1}{2} = \frac{I_{a_2}^2}{(100)^2},\quad I_{a_2} = \frac{100}{\sqrt{2}} = 70.7\text{ A}$$

$$E_1 = V - I_{a_1} R_a = 220 - 100 \times 0.1 = 210\text{ V}$$

$$E_2 = V - I_{a_2} R_a = 220 - 70.7 \times 0.1 = 212.93\text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} = \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}}$$

$$\frac{N_2}{800} = \frac{212.93}{210} \times \frac{100}{70.7}$$

$$N_2 = 1147.3\text{ r.p.m.}$$

EXAMPLE 7.8. *A 110 kW belt-driven shunt generator running at 400 r.p.m. V busbars continues to run as a motor when the belt breaks. As a motor it takes Find the speed at which it will run as a motor if the resistance of the armature a are 0.025 Ω and 55 Ω respectively. Brush contact drop is 2 V.*

SOLUTION. *As generator*

Load current $I_{L_1} = \dfrac{110 \times 1000}{220} = 500$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ A}$$

Armature current

$$I_{a_1} = I_{L_1} + I_{sh} = 500 + 4 = 504 \text{ A}$$

Generated voltage

$$E_1 = V + I_{a_1} R_a + \text{brush contact drop}$$

$$= 220 + 504 \times 0.025 + 2 = 234.6 \text{ V}$$

As motor

When the belt breaks and the machine terminals of the generator remain connected across the brushes, the machine continues to run as a motor. The direction of the armature current is reversed compared to the direction of current in the armature while running as generator.

However, the direction of rotation remains the same.

The line input current to the motor

$$I_{L_2} = \frac{11 \times 1000}{220} = 50 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ A}$$

Armature current

$$I_{a_2} = I_{L_2} - I_{sh} = 50 - 4 = 46 \text{ A}$$

Back e.m.f. of the motor

$$E_2 = V - I_{a_2} R_a - \text{brush drop}$$

$$= 220 - 46 \times 0.025 - 2 = 216.85 \text{ V}$$

$$E = k N \Phi, \quad N = \frac{E}{k\Phi}$$

Speed of the motor

$$N_2 = \frac{E_2 \Phi_1}{E_1 \times \Phi_2} N_1$$

For the same machine, $\Phi_2 = \Phi_1$

$$\therefore \quad N_2 = \frac{E_2}{E_1} \times N_1 = \frac{216.85}{234.6} \times 400 = 369.7 \text{ r.p.m.}$$

EXAMPLE 7.9. *A 250 V d.c. shunt motor having an armature resistance of 0.25 Ω carries an armature current of 50 A and runs at 750 r.p.m. If the flux is reduced by 10%, find the speed. Assume that the load torque remains the same.*

SOLUTION. *Initial conditions*

$$V = 250 \text{ V}, \quad I_{a_1} = 50 \text{ A}, \quad R_a = 0.25\ \Omega, \quad N_1 = 750 \text{ r.p.m.}$$

$$E_1 = V - I_{a_1} R_a = 250 - 50 \times 0.25 = 237.5 \text{ V}$$

Conditions after reducing the flux

$$\Phi_2 = 0.9\,\Phi_1$$

Load torque $\tau \propto \Phi I_a$

Since the load torque remains the same

$$\tau_2 = \tau_1$$

$$\Phi_2 I_{a_2} = \Phi_1 I_{a_1}$$

$$I_{a_2} = \frac{\Phi_1}{\Phi_2} I_{a_1} = \frac{50}{0.9} = 55.56 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 55.6 \times 0.25 = 236.1 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2 \Phi_1}{E_1 \Phi_2}$$

$$N_2 = \frac{E_2 \Phi_1}{E_1 \Phi_2} N_1 = \frac{236.1 \times 750}{237.5 \times 0.9} = 828.5 \text{ r.p.m.}$$

EXAMPLE 7.10. *A 120 V d.c. shunt motor having an armature circuit resistance of 0.2 Ω and field circuit resistance of 60 Ω, draws a line current of 40 A at full load. The brush voltage drop is 3 V and rated full-load speed is 1800 r.p.m. Calculate : (a) the speed at half load ; (b) the speed at 125 per cent full load.*

SOLUTION. $V = 120$ V, $R_a = 0.2\ \Omega$, $R_{sh} = 60\ \Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2 \text{ A}, \quad I_L = 40 \text{ A}$$

$$I_{a_1} = I_L - I_{sh} = 40 - 2 = 38 \text{ A}$$

$$E_1 = V - I_a R_a - \text{brush drop}$$

$$= 120 - 38 \times 0.2 - 3 = 109.4 \text{ V}$$

At rated speed of 1800 r.p.m.,

$$E_1 = 109.4 \text{ V and } I_{a_1} = 38 \text{ A (full load)}$$

(a) At half load

Line current $I_{L_2} = \dfrac{40}{2} = 20$ A

$$I_{a_2} = I_{L_2} - I_{sh} = 20 - 2 = 18 \text{ A}$$

$$E_2 = V - I_a R_a - \text{brush drop}$$

$$= 120 - 18 \times 0.2 - 3 = 113.4 \text{ V}$$

If N_2 is the speed at half load,

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{113.4}{109.4} \times 1800 = 1865.8 \text{ r.p.m.}$$

(b) At 125 percent full load

Line current $I_{L_3} = 40 \times 1.25 = 50$ A

Armature current $I_{a_3} = I_{L_3} - I_{sh} = 50 - 2 = 48$ A

EXAMPLE 7.11. *A shunt wound motor has an armature resistance of 0.1 Ω. It is connected across 220 V supply. The armature current taken by the motor is 20 A and the motor runs at 800 r.p.m. Calculate the additional resistance to be inserted in series with the armature to reduce the speed to 520 r.p.m. Assume that there is no change in armature current.*

SOLUTION. $E_1 = V - I_{a_1} R_{a_1} = 220 - 20 \times 0.1 = 218$ V

$$E_2 = \frac{N_2 \Phi_2}{N_1 \Phi_1} E_1$$

Since $I_{sh} = \frac{V}{R_{sh}}$, the shunt field current I_{sh} remains constant, and, therefore

$$\Phi_2 = \Phi_1$$

$$E_2 = \frac{N_2}{N_1} E_1 = \frac{520}{800} \times 218 = 141.7 \text{ V}$$

If R_A is the additional resistance inserted in the armature circuit

$$E_2 = V - I_{a_2} (R_{a_1} + R_A)$$

$$141.7 = 220 - 20\,(0.1 + R_A)$$

$$R_A = 3.815\ \Omega$$

EXAMPLE 7.12. *A 240 V dc series motor takes 40 A when giving its rated output at 1500 r.p.m. Its resistance is 0.3 Ω. Calculate the value of resistance that must be added to obtain the rated torque (a) at starting, (b) at 1000 r.p.m.*

SOLUTION. Rated voltage $V = 240$ V

Rated current $I = I_a = 40$ A

$$N_1 = 1500 \text{ r.p.m.}, \quad R_a = 0.3\ \Omega$$

$$E = V - I_a R_a = 240 - 40 \times 0.3 = 228 \text{ V}$$

(a) *At starting*, back e.m.f. is zero. In order to obtain rated torque at rated current, an additional resistance R_1 is connected in series with the armature.

$$E_1 = V - I_a (R_a + R_1)$$

$$0 = 240 - 40\,(0.3 + R_1)$$

$$R_1 = \frac{240 - 12}{40} = 5.7\ \Omega$$

(b) Let R_2 be the resistance connected in series with the armature to obtain the rated torque at a speed of 1000 r.p.m.

$$E_2 = V - I_a (R_a + R_2)$$

$$E_2 = 240 - 40\,(0.3 + R_2) = 228 - 40\,R_2$$

$$\frac{N_2 \Phi_2}{N \Phi} = \frac{E_2}{E}$$

$$\frac{N_2 I_{a_2}}{N I_a} = \frac{E_2}{E}$$

Since $I_{a_2} = I_a$

$$\frac{N_2}{N} = \frac{E_2}{E}$$

$$\frac{1000}{1500} = \frac{228 - 40\,R_2}{228}, \quad R_2 = 1.9\ \Omega$$

EXAMPLE 7.13. *A 250 V shunt motor takes a current of 41 A and runs at 800 r.p.m. on full-load. Armature and field resistances are 0.2 Ω and 250 Ω respectively. If a resistance of 2 Ω is placed in series with the armature, determine :*

(a) *the speed at full-load torque ;*

(b) *the speed at double full-load torque ;*

(c) *the stalling torque in terms of the full-load torque.*

Assume that the flux remains constant throughout.

SOLUTION. (a) *Full-load conditions*

Armature current $I_{a_1} = I - I_{sh} = 41 - \frac{250}{250} = 40$ A

Back e.m.f. on full-load

$$E_1 = V - I_{a_1} R_a = 250 - 40 \times 0.2 = 24.2 \text{ V}$$

When a resistance of 2 Ω is placed in series with the armature, the back e.m.f. is

$$E_2 = V - I_{a_1} (0.2 + 2) = 250 - 40\,(2.2) = 162 \text{ V}$$

Since the flux remains constant

$$N_2 = \frac{E_2}{E_1} N_1 = \frac{162}{242} \times 800 = 535.54 \text{ r.p.m,}$$

(b) *Double full-load conditions*

At double full-load torque, the armature current

$$I_{a_2} = 40 \times 2 = 80 \text{ A}$$

With 2 Ω resistance in the armature circuit, the back e.m.f. at double full-load torque

$$E_3 = V - I_{a_2} (0.2 + 2) = 250 - 80\,(2.2) = 74 \text{ V}$$

$$N_3 = \frac{E_3}{E_1} N_1 = \frac{74}{242} \times 800 = 244.6 \text{ r.p.m.}$$

(c) *Stalling torque*

Under stalling conditions, the speed is zero and therefore the back e.m.f. is zero.

Let I_{a_0} be the armature current taken by the motor under stalling conditions.

$$\therefore \quad E_{b_0} = V - I_{a_0} (0.2 + 2)$$

$$0 = 250 - 2.2\,I_{a_0}$$

$$I_{a_u} = \frac{250}{2.2} = 113.64 \text{ A}$$

$$\tau \propto I_a \qquad (\because \Phi \text{ is constant})$$

Stalling torque $\propto$ stalling current

Full-load torque $\propto$ full-load armature current

$$\therefore \quad \frac{\text{stalling torque}}{\text{full-load torque}} = \frac{\text{stalling current}}{\text{full-load armature current}} = \frac{113.64}{40} = 2.84$$

$$\therefore \quad \text{stalling torque} = 2.84 \times \text{full-load torque.}$$

EXAMPLE 7.14. *A 200 V d.c. series motor runs at 1000 r.p.m. and takes 20 A. Combined resistance of armature and field is 0.4 Ω. Calculate the resistance to be inserted in series so as to reduce the speed to 800 r.p.m., assuming torque to vary as square of the speed and linear magnetization curve.*

SOLUTION. $I_L = I_{a_1} = 20$ A, $N_1 = 1000$ r.p.m., $N_2 = 800$ r.p.m.

$$E_1 = V - I_{a_1} R_{a_1} = 200 - 20 \times 0.4 = 192 \text{ V}$$

$$\frac{\tau_2}{\tau_1} = \frac{\Phi_2 I_{a_2}}{\Phi_1 I_{a_1}} = \left(\frac{I_{a_2}}{I_{a_1}}\right)^2 = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{800}{1000}\right)^2$$

$$\therefore \quad I_{a_2} = 0.8\, I_{a_1} = 0.8 \times 20 = 16 \text{ A}$$

$$E_2 = V - I_{a_2}(0.4 + R) = 200 - 16\,(0.4 + R)$$

$$E_2 = 193.6 - 16\,R$$

$$\frac{E_2}{E_1} = \frac{N_2 \Phi_2}{N_1 \Phi_1} = \frac{N_2 I_{a_2}}{N_1 I_{a_1}} = \frac{800 \times 16}{1000 \times 20} = 0.64$$

$$\frac{193.6 - 16\,R}{192} = 0.64$$

$$R = \frac{193.6 - 192 \times 0.64}{16} = 4.42\ \Omega$$

EXAMPLE 7.15. *A series motor, with an unsaturated magnetic circuit and 0.5 Ω total resistance, when running at a certain speed takes 60 A at 500 V. If the load torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.*

SOLUTION. $E_1 = V - I_a R_a = 500 - 60 \times 0.5 = 470$ V

$$N_2 = 0.75\, N_1$$

$$\tau \propto N^3$$

$$\frac{\tau_2}{\tau_1} = \left(\frac{N_2}{N_1}\right)^3 = (0.75)^3$$

$$\tau \propto \Phi\, I_a$$

$$\frac{\tau_2}{\tau_1} = \frac{\Phi_2 I_{a_2}}{\Phi_1 I_{a_1}} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$\therefore \quad \frac{I_{a_2}^2}{I_{a_1}^2} = \left(\frac{N_2}{N_1}\right)^3$$

$$\frac{I_{a_2}^2}{(60)^2} = (0.75)^3, \quad I_{a_2} = 60\sqrt{(0.75)^3} = 38.97 \text{ A}$$

Let R be the additional resistance to be connected in series with the armature.

$$E_2 = V - I_{a_2}(R_a + R) = 500 - 38.97\,(0.5 + R)\ ;\ E_2 = 480.5 - 38.97\,R$$

$$\frac{E_2}{E_1} = \frac{N_2 \Phi_2}{N_1 \Phi_1} = \frac{N_2 I_{a_2}}{N_1 I_{a_1}}$$

$$\frac{480.5 - 38.97\,R}{470} = \frac{0.75 \times 38.97}{60} \quad \Rightarrow \quad R = 6.455\ \Omega$$

EXAMPLE 7.16. *A 500 V shunt motor takes 4 A on no load. The armature resistance including that of brushes is 0.2 Ω and the field current is 1 A. Estimate the output and the efficiency when the input current is (a) 20 A, (b) 100 A.*

SOLUTION. Constant loss

= no-load power input – no load copper losses

$$p_c = VI_0 - (I_0 - I_{sh})^2 R_a = 500 \times 4 - (4 - 1)^2 \times 0.2 = 1998.2 \text{ W.}$$

(a) For 20 A input current

Power input $= VI = 500 \times 20 = 10000$ W

Armature copper loss $= (I - I_{sh})^2 R_a$

$= (20 - 1)^2 \times 0.2 = 72.2$ W.

Total losses $= p_c +$ armature copper loss

$= 1998.2 + 72.2 = 2070.4$ W.

Efficiency of motor

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{10000 - 2070.4}{10000} = 0.793 \text{ pu} = 79.3\%.$$

(b) For 100 A input current

Power input $= VI = 500 \times 100 = 50000$ W

Armature copper loss $= (I - I_{sh})^2 R_a = (100 - 1)^2 \times 0.2 = 1960$ W.

Total losses $= p_c +$ armature copper loss

$= 1998.2 + 1960 = 3958.2$ W.

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{50000 - 3958.2}{50000} = 0.921 \text{ pu} = 92.1\%$$

EXAMPLE 7.17. *The Hopkinson test on two shunt machines gave the following results for full load :*

Line voltage, 250 V ; line current excluding field currents, 50 A ; motor armature current, 380 A ; field currents, 5 A and 4.2 A. Calculate the efficiency of each machine. Armature resistance of each machine 0.02 Ω.

SOLUTION. In the problem it is mentioned that line current is 50 A excluding field currents. This indicates that the fields are separately excited. The losses supplied will only be the armature copper losses and stray losses, and *not* in the fields which are separately excited.

The generator armature current $I_{ga} = I_{ma} - I_L = 380 - 50 = 330$ A

The machine with smaller excitation acts as a motor.

Input from the supply = total losses in the set $= 250 \times 50 = 12500$ W

Copper loss in the motor armature $= I_{ma}^2 R_{am} = (380)^2 \times 0.02 = 2888$ W.

Copper loss in the generator armature $= I_{ga}^2 R_{ag} = (330)^2 \times 0.02 = 2178$ W

Total armature copper loss of the set $= 2888 + 2178 = 5066$ W.

$\therefore$ total stray loss of the set = input – total losses in the armatures

$= 12500 - 5066 = 7434$ W.

Stray loss per machine $= \dfrac{7434}{2} = 3717$ W

Motor efficiency

Input $= 250\,(380 + 4.2) = 96050$ W

Armature copper loss $= 2888$ W

Field copper loss $= 250 \times 4.2 = 1050$ W

Stray loss $= 3717$ W

Total losses $= 2888 + 1050 + 3717 = 7655$ W

The efficiency of motor

$$= \frac{\text{input} - \text{losses}}{\text{input}} \times 100 = \frac{96050 - 7655}{96050} \times 100 = 92.03\%$$

Generator efficiency

Output $= 250 \times 330 = 82500$ W

Armature copper loss $= 2178$ W

Field loss $= 250 \times 5 = 1250$ W

Stray loss $= 3717$ W

Total losses $= 2178 + 1250 + 3717 = 7145$ W.

The efficiency of generator

$$= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{82500}{82500 + 7145} \times 100 = 92.03\%.$$

EXERCISES

7.1 Explain what is meant by back e.m.f. Explain the principle of torque production in a d.c. motor.

7.2 Derive the torque equation of a d.c. motor.

7.3 What is the necessity of a starter for a d.c. motor. Explain, with a neat sketch, the working of a 3-point d.c. shunt motor starter, bringing out the protective features incorporated in it.

7.4 Discuss different methods of speed control of a d.c. motor.

7.5 Sketch the speed-load characteristics of a d.c. (*a*) shunt motor, (*b*) series motor, (*c*) cumulatively compounded motor. Account for the shape of the above characteristic curves.