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Experiments and nonlocal continuum modeling of the size-dependent fracture in elastomers[1]

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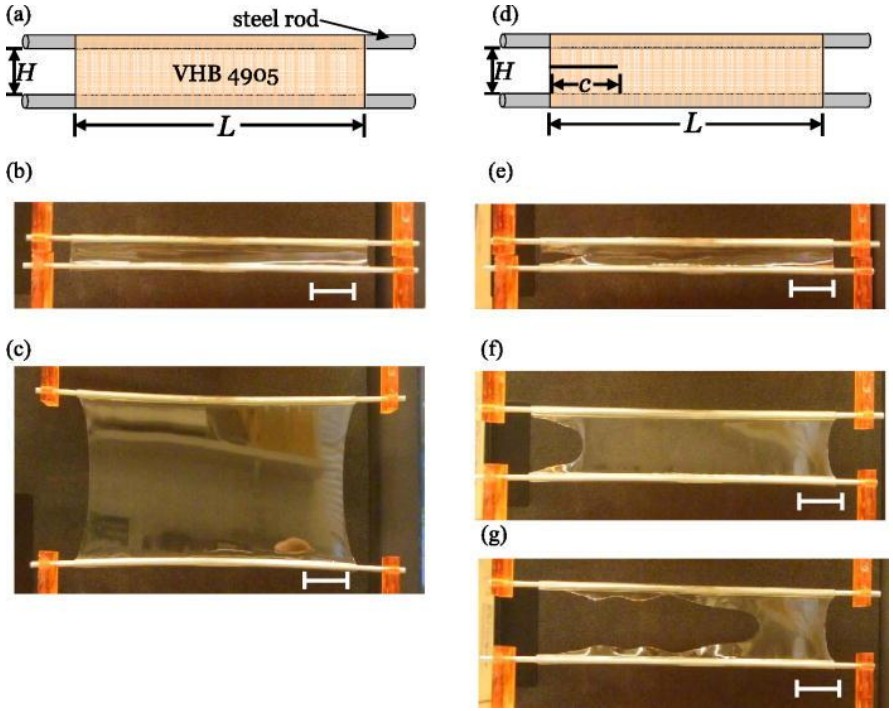
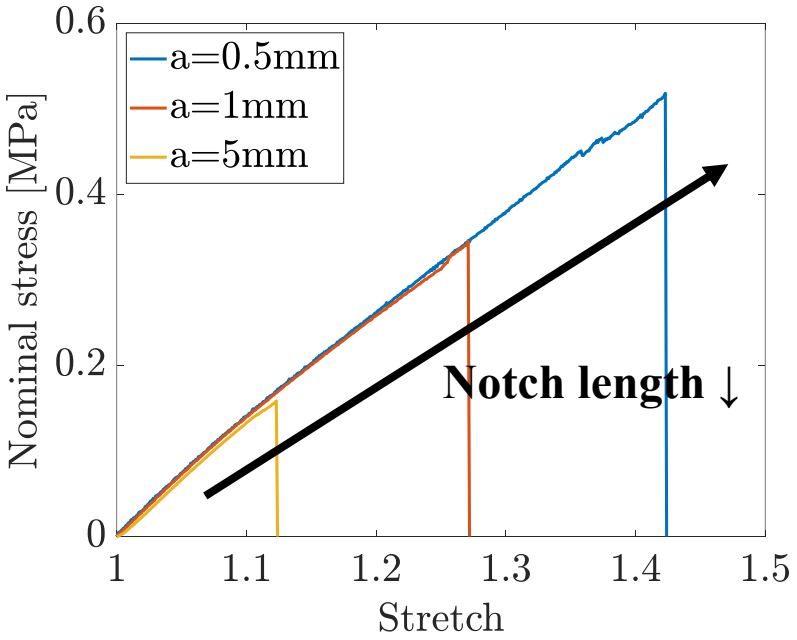
Fracture in elastomers

- Extreme, nonlinear deformation → nonlinear fracture
- Influenced by the size of flaws; **the size-dependent fracture**[1,2]
  - Rupture stretch increases as the specimen size decreases



a) The presence of flaws impacts the fracture behavior[3]

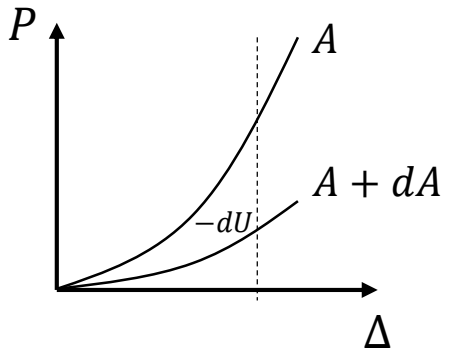


b) Size-dependent fracture in polydimethylsiloxane (PDMS) specimens

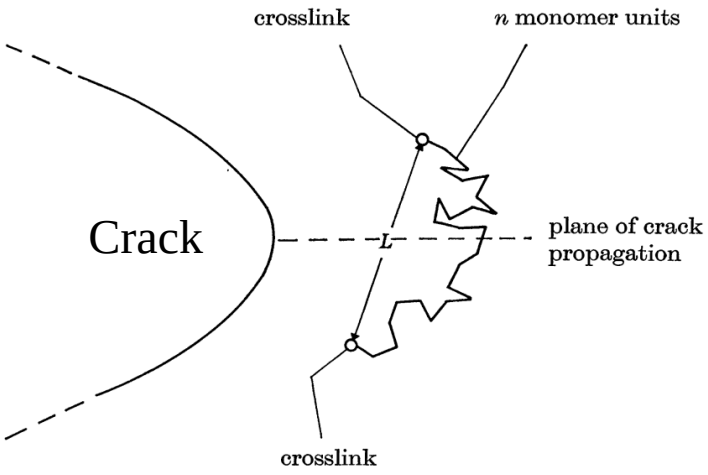
Fracture in elastomers

- Occurs when …
  - Macroscopically, **G reaches Γ**
    - Griffith theory[4,5]
    - G: Energy release rate
    - Γ: Fracture energy

$$G = -\frac{dU(\mathrm{P},\Delta)}{dA}$$



  - Microscopically, **ε_R reaches ε_R^f**
    - Lake-Thomas theory[6-8]
    - ε_R: Internal energy
    - ε_R^f: critical internal energy; bond dissociation energy



- These approaches are compatible (Lake and Thomas [6])

Objectives

- Predicting the **size-dependent fracture** in elastomers[1]
  - Experiments and numerical simulations were carried out
- **Internal energy-driven** fracture criterion; inspired by the Lake-Thomas model[6-8]

- Using the **phase-field model** rooted in the gradient-damage theory[8-12]
  - Mesh-insensitive crack propagation process
  - The internal energy-driven fracture criterion
  - Thermodynamics of the damage and fracture

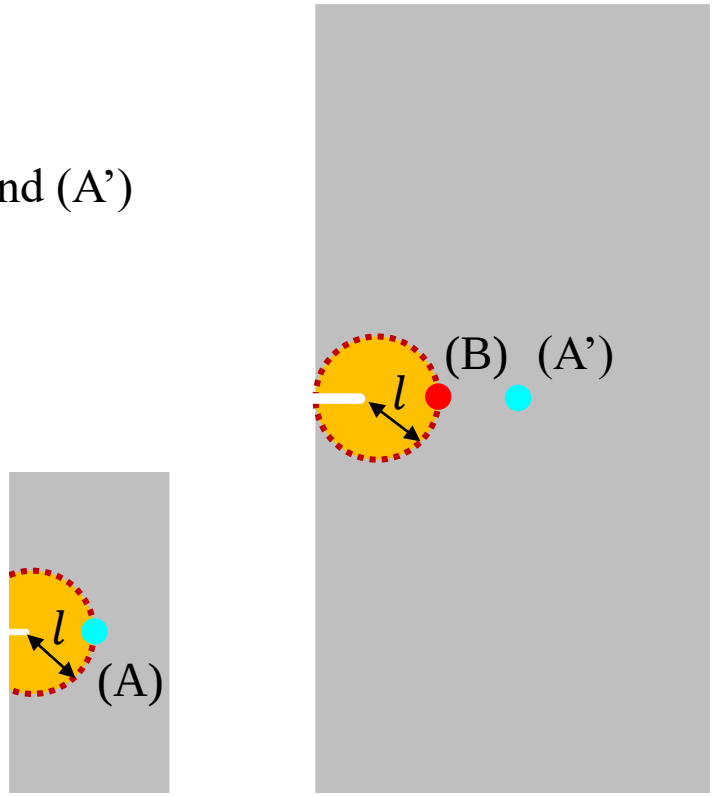
Size-dependent fracture & Fracture process zone

- Fracture process zone
  - Where the polymer chains rupture = Where the dissipation mainly occurs

- Stress at point (B) is larger than those at (A) and (A')
  - $\sigma_A = \sigma_{A'} < \sigma_B$
- → Free energy at point (B) is larger than those at (A) and (A')
  - $\psi_A = \psi_{A'} < \psi_B$
- → **ψ_B reaches the critical energy earlier than ψ_A**
- → **The larger specimen ruptures earlier**

The size of fracture process zone[1,2,13,14]:

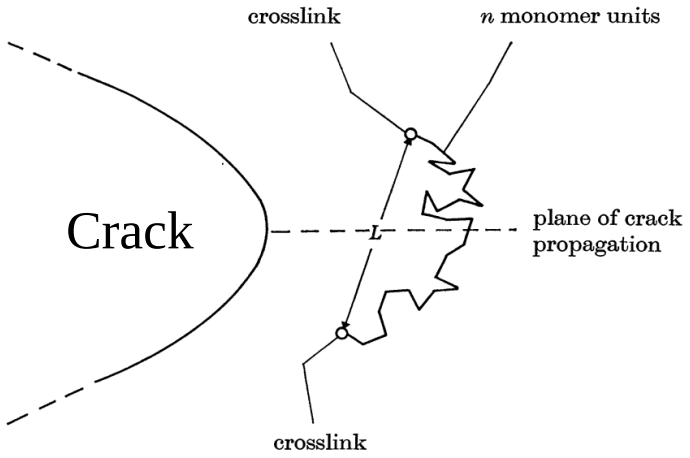
$$l = \frac{\Gamma}{W^*} = \frac{Fracture\ energy}{Critical\ deformation\ energy}$$



Nonlocal continuum modeling

- 1. The damage $d \in [0,1]$
  - d=0: intact
  - d=1: fully damaged
- **Internal energy-driven** fracture criterion
  - Inspired by the Lake-Thomas model[4]
  - Fracture = **Scission of polymer chains**
- Governing equations[8]
  - Macroforce balance Div $\mathbf{T}_R$=0
  - Microforce balance $\zeta \dot{d} = 2(1-d)\mathcal{H}_R - \hat{\varepsilon}_R^f (d - l'^2 \Delta d)$

a)[6]

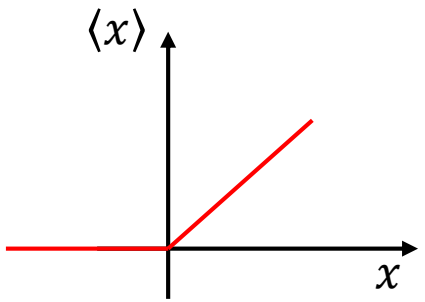


History function;
the fracture criterion

$$\mathcal{H}_R = \left\langle \varepsilon_R^0 - \varepsilon_R^f/2 \right\rangle, \quad \text{where } \langle x \rangle = \begin{cases} x & if\ x > 0 \\ 0 & if\ x \le 0 \end{cases}$$

Internal energy

b)



Nonlocal continuum modeling

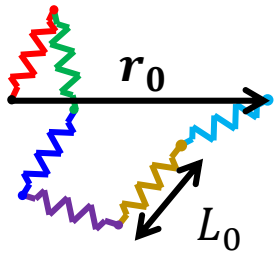
- **Internal energy** should be considered → **Bond stretch**[7,8]
  - Deformation = Chain configuration change + stretching of molecular bonds

- $\psi_R = (1-d)^2\left[\frac{1}{2}NnE_b(\lambda_b-1)^2+\frac{1}{2}K(J-1)^2\right] + Nk_b\theta n\left[\frac{\bar{\lambda}\lambda_b^{-1}}{\sqrt{n}}\beta+\ln\left(\frac{\beta}{\sinh\beta}\right)\right]+\frac{1}{2}\varepsilon_R^f l^2|\nabla d|^2$

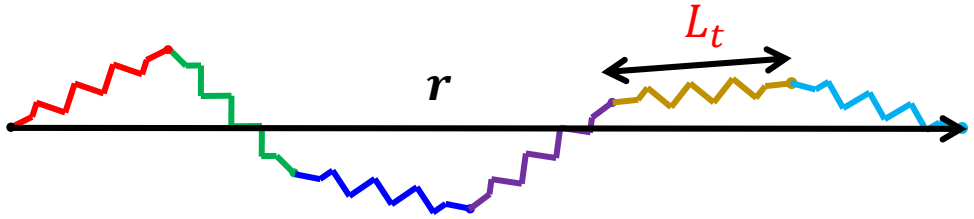
$(1-d)^2\varepsilon_R^0$; **Damage acts on the internal energy only** $-\theta\eta_R$; Entropic energy Nonlocal energy[8]

$$\bar{\lambda}=\frac{|\boldsymbol{r}|}{|\boldsymbol{r_0}|}$$
$$\lambda_b=\frac{L_t}{L_0}$$
$$\mathcal{L}(x)=\coth x-\frac{1}{x}$$
$$\beta=\mathcal{L}^{-1}\left(\frac{\bar{\lambda}\lambda_b^{-1}}{\sqrt{n}}\right)$$

a) Reference configuration



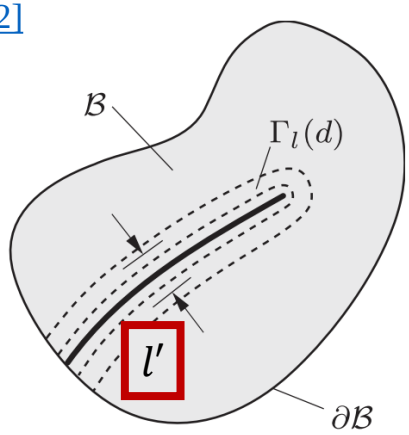
a) Deformed configuration



Nonlocal continuum modeling

- 2. Phase-field model rooted in the gradient-damage theory[8-12]
  - "Diffusive damage zone"

a)[12]



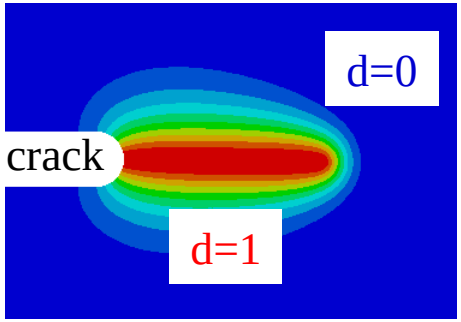
Intrinsic length scale l'

- Microforce balance $\zeta\dot{d} = 2(1-d)\mathcal{H}_R - \hat{\varepsilon}_{\mathrm{R}}^{f}(d - l'^2\Delta d)$

History function;
the fracture criterion

- The **intrinsic length scale l'** → the size of diffusive damage zone
  - A numerical parameter; ambiguous physical meaning

b)



Crack propagation;
at reference configuration

Nonlocal continuum modeling

- Assumption[1]: Diffusive damage zone = Fracture process zone
  - Regions of the damage evolution and the dissipation

- The size of fracture process zone

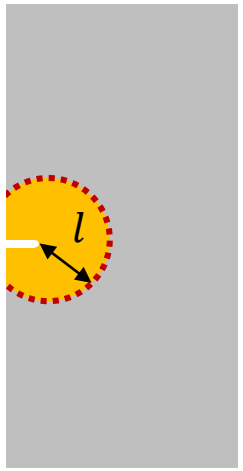
$$= \frac{\Gamma}{W^*} = \frac{Fracture\ energy}{Critical\ deformation\ energy}$$ → **Intrinsic length scale**

→ Identify the intrinsic length scale l **from experiments**

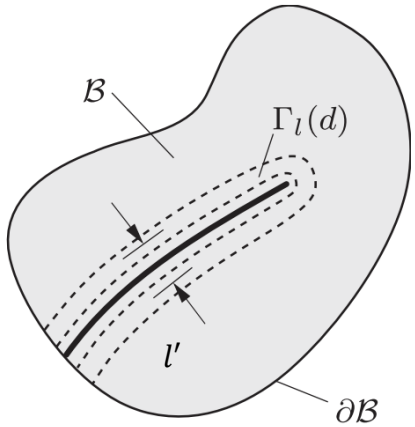
→ Apply to the phase field model

→ Predict the **size-dependent fracture** by numerical simulations[1]

a) Fracture process zone

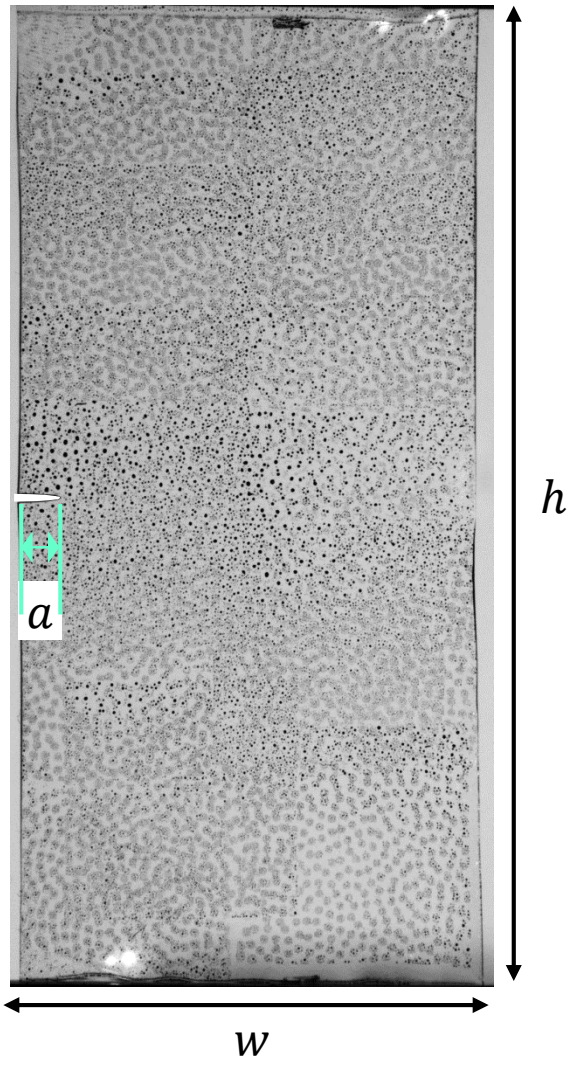


b)[12] Diffusive damage zone



Experimental procedures[1]

- Geometries
  - a = {0.5, 1, 5} mm
  - w = 10a, h = 20a, specimen thickness: 0.5mm
    - → w = {5, 10, 50} mm
    - → h = {10, 20, 100} mm
- Materials
  - PDMS
  - TangoPlus (3D-printed elastomer)
- Strain rate 0.01 s^{-1}, temperature ~21℃
- Digital image correlation (DIC) analysis
  - → Strain fields from experiments



The intrinsic length scale l

- $l = \frac{\Gamma}{W^*}$ → **Experimentally identified intrinsic length scale**[1]
- Γ: Fracture energy
  - from notched specimens
- W^*: Critical deformation energy
  - from unnotched specimens

PDMS

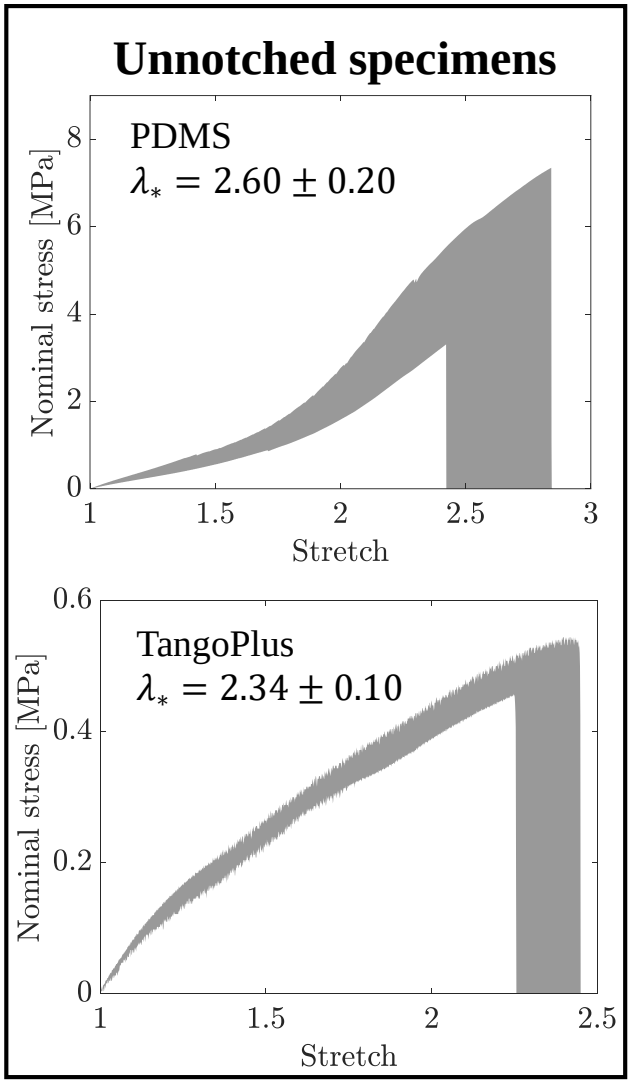
$\Gamma \approx 0.25\text{mJ/mm}^2$, $W^* \approx 2.7\text{mJ/mm}^3$

→ $\boldsymbol{l \approx 0.08mm}$

TangoPlus

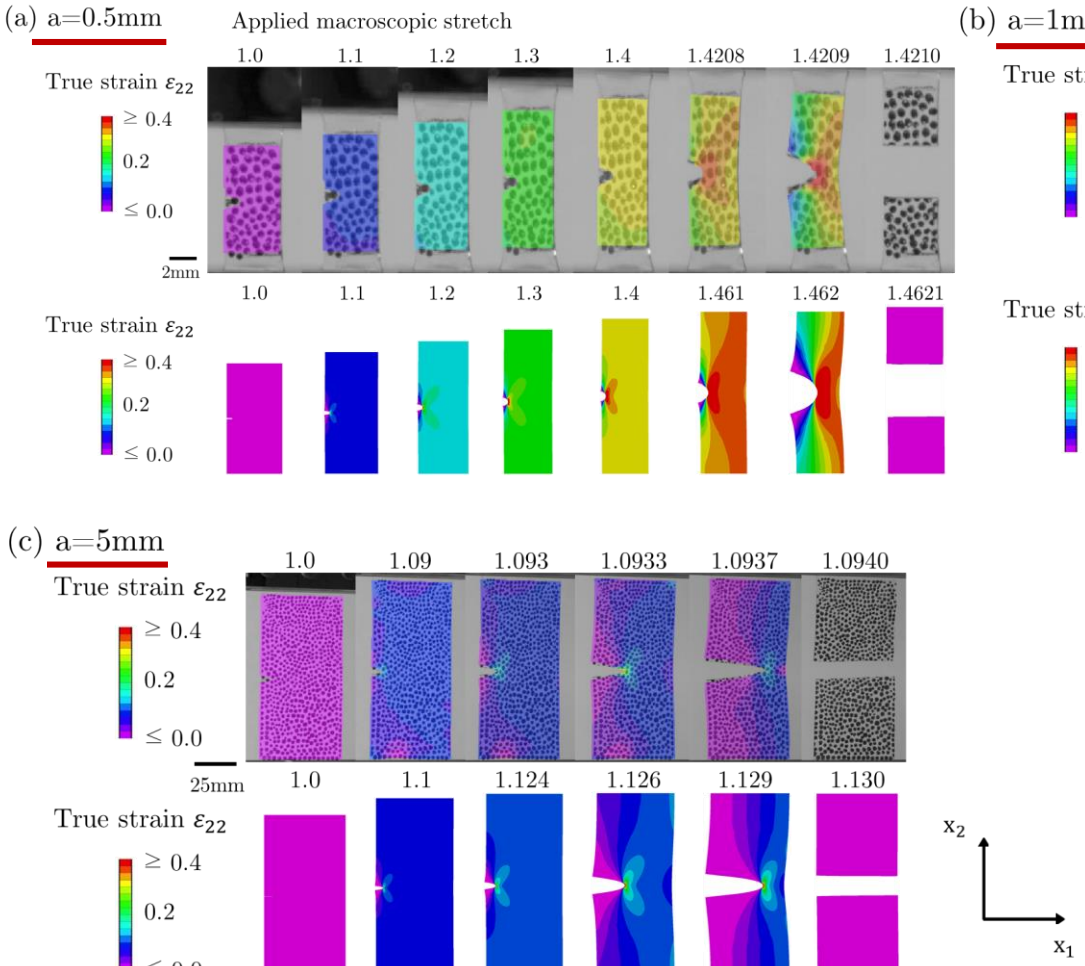
$\Gamma = 0.5\text{mJ/mm}^2$, $W^* \approx 0.45\text{mJ/mm}^3$

→ $\boldsymbol{l \approx 1mm}$



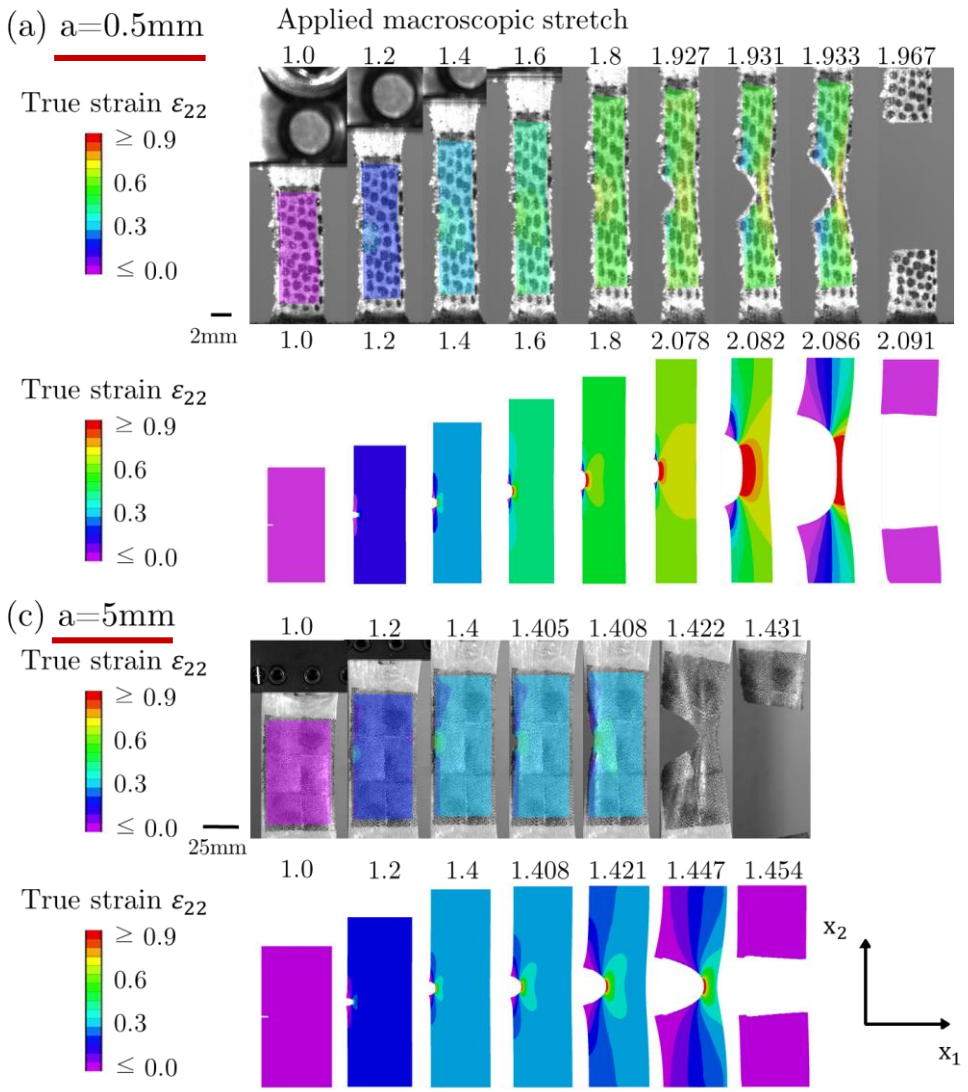
Results: Experiment vs. Numerical simulation[1]

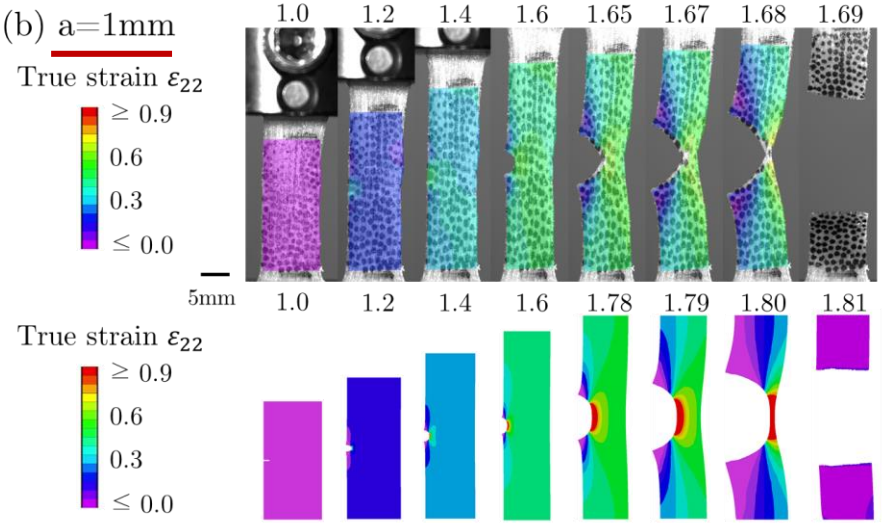
- Strain fields in **PDMS** specimens ($l = 0.08mm$)
  - Larger specimen ruptures earlier



Results: Experiment vs. Numerical simulation[1]

- Strain fields in **TangoPlus** specimens ($l = 1mm$)
  - Larger specimen ruptures earlier

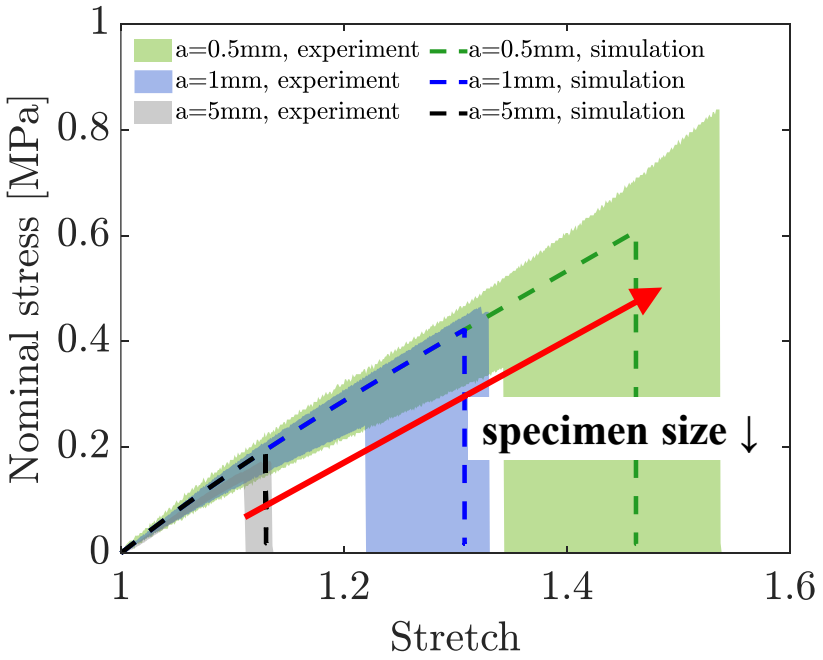




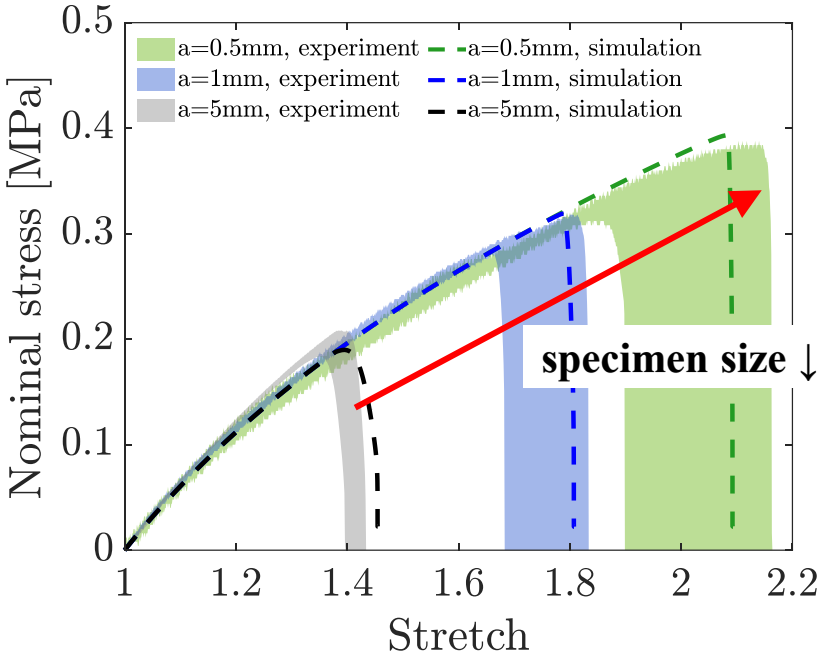
Results: Experiment vs. Numerical simulation[1]

- Notch lengths a = {0.5, 1, 5} mm
- Geometric similarity → **Identical initial stress-stretch response**
- Smaller notch length → Higher rupture stretch

a) PDMS



b) TangoPlus



Notch-length sensitivity[1]

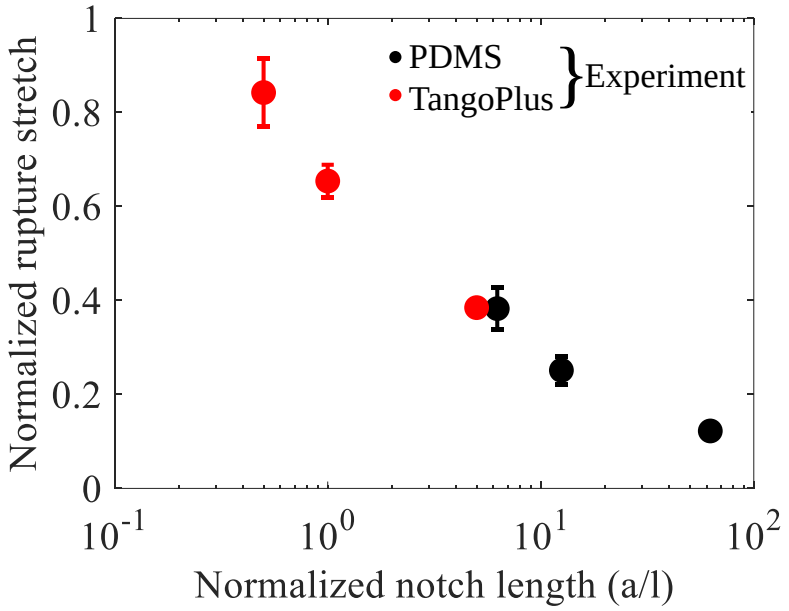
- PDMS vs. TangoPlus; same specimen sizes
  - PDMS: $l = 0.08mm$
  - TangoPlus: $l = 1mm$

  More than 10 times

- Normalized rupture stretch

  $$= \frac{\text{Rupture stretch of } \textbf{notched} \text{ specimens}}{\text{Rupture stretch of } \textbf{unnotched} \text{ specimens}}$$

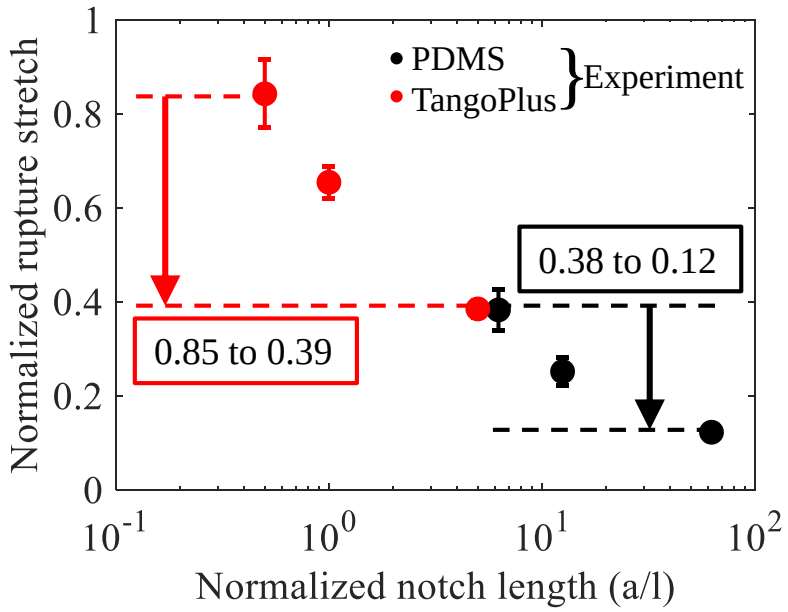
- Normalized notch length = $\frac{\text{Notch length (a)}}{\text{Intrinsic length scale } (l)}$

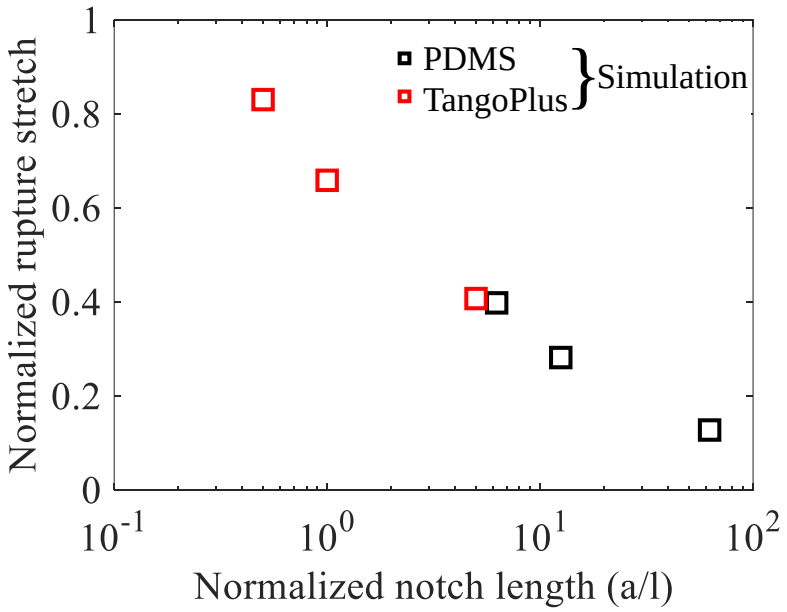


Notch-length sensitivity[1]

- PDMS vs. TangoPlus; same specimen sizes
  - PDMS: $l = 0.08mm$
  - TangoPlus: $l = 1mm$

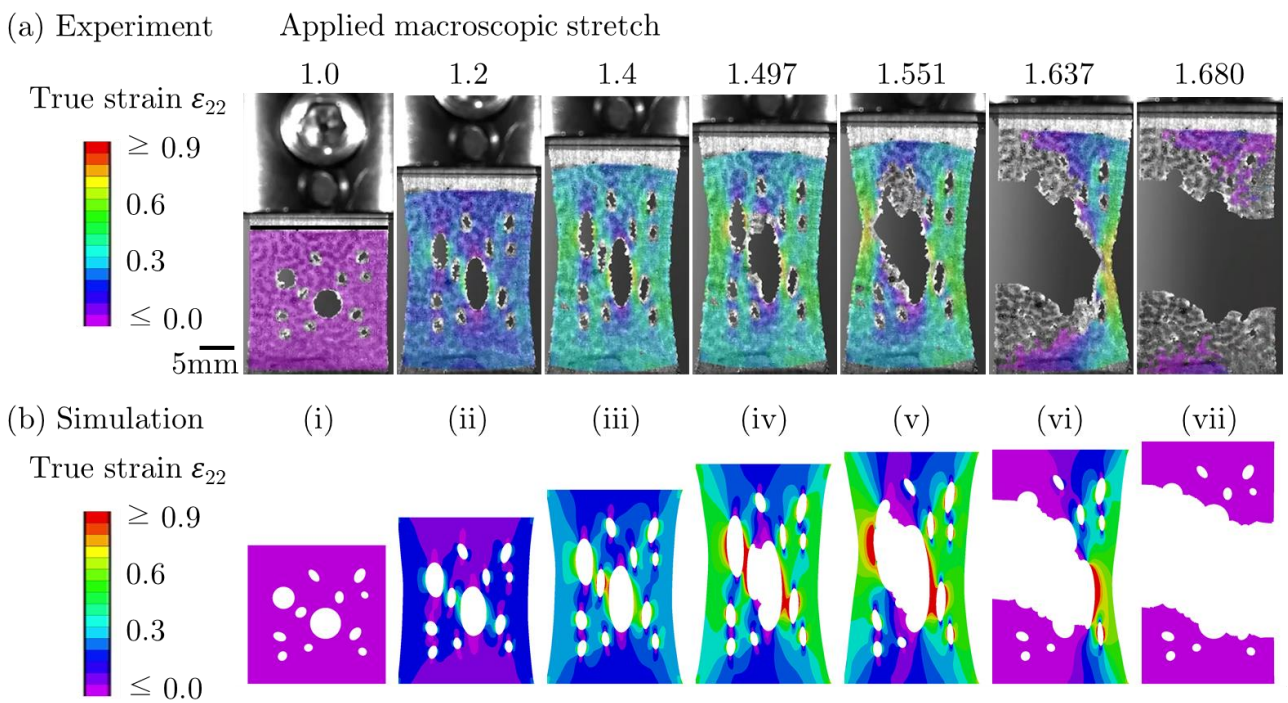
  More than 10 times
- a/l: 0.5~5 (TangoPlus; $l = 1mm$) → Highly notch length-sensitive
- a/l: 5~50 (PDMS; $l = 0.08mm$) → Less notch length-sensitive

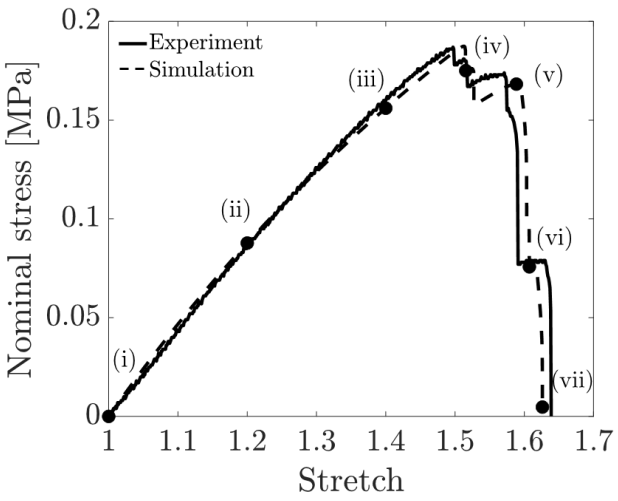




Randomly perforated specimen (TangoPlus)[1]

- Nicely predicted the response **without modification of parameters**
  - Progressive fracture of ligaments





Conclusion

- **Size-dependent fracture** is clearly observed in experiments
  - Rupture stretch increases as the notch length decreases
  - Size-dependence increases as the notch-root radius decreases
- **The intrinsic length scale** determines the size-dependent behavior
  - The intrinsic length scale l defines the size of diffusive damage zone / fracture process zone
  - The intrinsic length scales were identified from experiments
  - Normalized notch length (a/l) determines the size-dependence
- **Nonlocal continuum model** nicely predicted the fracture in elastomers
  - Nonlocal continuum model utilizes experimentally identified intrinsic length scales
  - The model captures the size-dependent fracture in elastomers
  - The model is capable of predicting the fracture of complex geometries

Reference

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