Guiding CDCL SAT Search via Random Exploration amid Conflict Depression

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February 4, 2020

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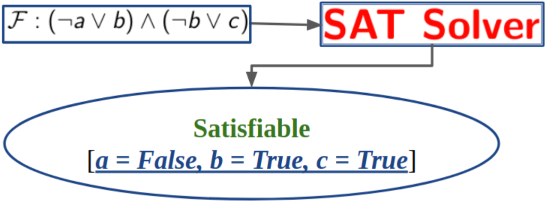
Introduction

- Boolean Satisfiability (SAT)
  - Given a SAT formula, determine assignments of the variables to satisfy a boolean formula, if one exists. Otherwise, unsatisfiable ....
    - A toy example:


$\mathcal{F} : (\neg a \vee b) \wedge (\neg b \vee c)$
SAT Solver
Satisfiable
[a = False, b = True, c = True]

- However, SAT solving is **NP-Complete** → Intractable, in general.
- Modern SAT solvers → Conflict Directed Clause Learning (CDCL).
  - **Applications in many practical domains**: Hardware design verification, Software testing, encryption, planning ..

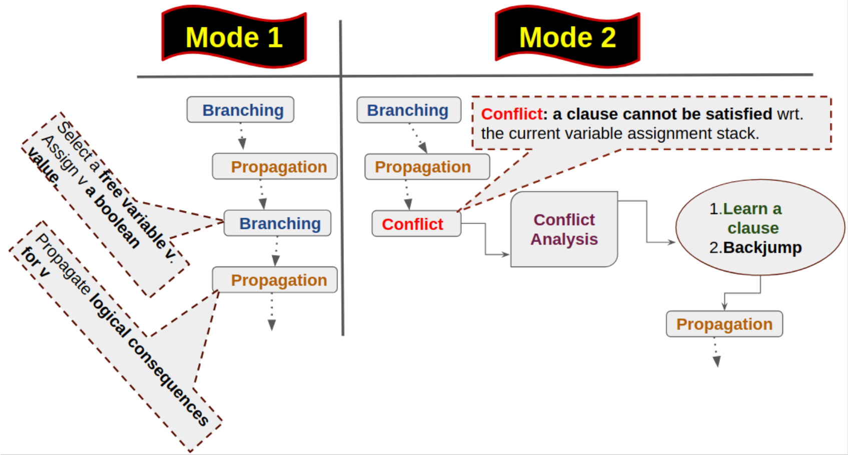
Contributions

In this work, we present

- **Contribution 1:** An empirical investigation of the CDCL SAT solving process to obtain insights on its conflict generation pattern.
- **Contribution 2:** A CDCL algorithmic extension, based on the obtained insights.
    - Employs random exploration, which is novel for CDCL SAT solving!
- **Contribution 3:** An extensive evaluation.

Background: How does a CDCL SAT solver works?

- Performs a backjumping tree-search to determine satisfiabilty.


Mode 1
Mode 2
Branching
Propagation
Branching
Propagation
Select a free variable v.
Assign v a boolean value.
Propagate logical consequences for v
Branching
Propagation
Conflict
Conflict: a clause cannot be satisfied wrt. the current variable assignment stack.
Conflict Analysis
1.Learn a clause
2.Backjump
Propagation

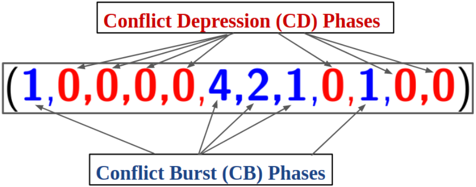
- **Restarts frequently**: abandons the current partial assignment and starts the search from the scratch.

Background: Importance of Fast Conflict Generation

- Conflict Generation at a **fast rate is crucial** for CDCL SAT solvers.
- **Conflict** ⟶ **Clause learning** ⟶ **Pruning** ⟶ **Faster Solving.**
- CDCL branching heuristics are **conflict-greedy**.
  - Example:
    Variable State Independent Decaying Sum (VSIDS)
    Learning Rate Based (LRB)
    - based on **look-back principle**
    - selection priority is based on **recent conflict involvements**.
    - **intuition**: such selection will generate more conflicts.
- CDCL branching heuristics **generate conflicts at a fast rate**.
  - On average, <u>1 conflict in 2 decisions</u>. (Liang 2017 et. al.)

Notions and Definitions

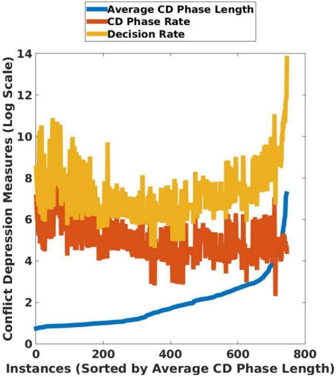
- We formulate **two novel notions:**
  - **Conflict Depression**: Sequence of one or more consecutive decisions with no conflict.
  - **Conflict Burst**: Sequence of one or more consecutive decisions with at least one conflict.

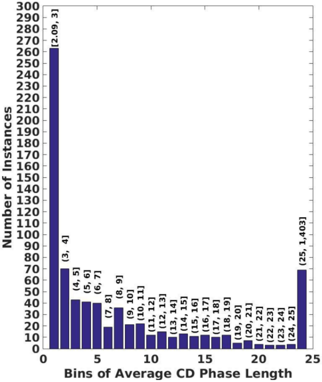


- Some Measures:
  - **Decision Rate**: number of decisions per restart.
  - **CD phase Rate**: number of CD phases per restarts.
  - **Average CD phase Length**

Contribution 1: CD phases with VSIDS

- We study CD phases with VSIDS.
  - CDCL solver: Glucose (uses VSIDS exclusively).
  - 750 maintrack instances from SAT-2017, 2018.
- Observations:
  - **1:** CD phases occur frequently with VSIDS. (left)
  - **2:** For many instances, avg. CD phase length are high! (right)





Propagation Depression during a CD phase

- During a CD phase, VSIDS decisions are ineffective to create conflicts and **only create truth value propagation**.
  - But, how much propagation ?

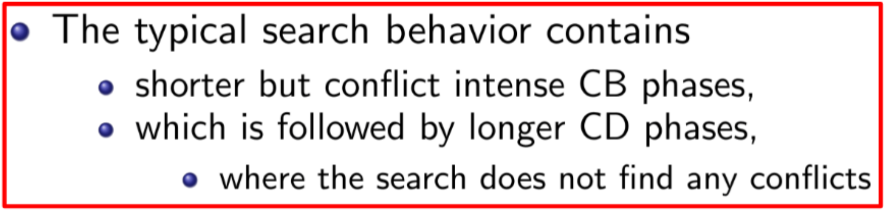
Observation 3:

- Propagtions in a CD phase is **10 times lower** than Propagations in a CB phase.
  ⟶ During a CD phase, VSIDS branching decisions **go through a propagation depression** as well !

Bursts of Conflict Generation

- **Observation 4:** On average,
  - Only 25% of the decisions produce at least one conflict.
    - Some of which produces **more than one conflicts.** How many of them?
  - 61% of the total conflict producing decisions, produces more than one conflict.
- **Conflict Burst (CB)** phases are **short**, but **conflict intense**.
  - Many conflicts within a short span of consecutive decisions.

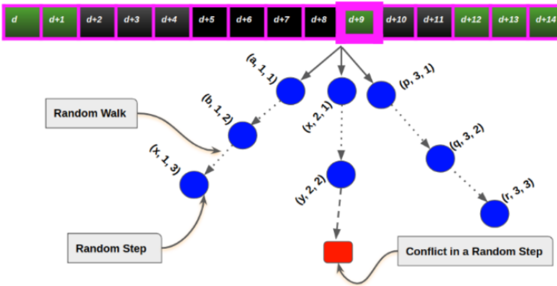
Summary of Empirical Observations



- The typical search behavior contains
  - shorter but conflict intense CB phases,
  - which is followed by longer CD phases,
    - where the search does not find any conflicts

Contribution 2: Random Exploration amid CD phases

- Can we do better amid a CD phase? VSIDS variable re-ranking?
- Formulated an exploration based CDCL framework named *expSAT*.
- **Main Idea:** Amid a substantial CD phase, **with a non-zero probability** ,
    - perform **exploration episodes** to identify conflict friendly variables.
    - Exploration Episode: a fixed number of **random walks**, with a fixed number of **steps per walk**.



Random Walk

Random Step

Conflict in a Random Step

Step Variables	Exploration Score
a	0
b	0
x	**bonus**
y	**bonus**
p	0
q	0
r	0

- **expVSIDS**: VSIDS+exploration score.
    - Selects the variable that maximizes the combined score.

Contribution 3: Empirical Evaluation

- Implemented expSAT on top of 4 CDCL solvers:
    - **glucose 4.2.1** (gLCM)
    - **MapleCOMSPS** (MplCOMSPS)
    - **Maple_CM** (MplCM)
    - **MapleLCMDist_ChronoBT** (MplCBT)
- Two test sets:
    - **Competition Benches**: 750 maintrack instances from SAT-2017 and 2018.
        - Time Out: 5,000 seconds
    - **SATCoin Benches**: 52 hard SATCoin benchmark instances generated by an instance generator submitted for SAT Competition 2018.
        - Time Out: 36,000 secs

Experimental Results

- Competition Benches Results: **Good-to-Strong** gains.

Solver Name	**Solved by Baseline**	**Solved by expSAT Extension**	**PAR2 Score Decrement**
gLCM	372	**379 (+7)**	1.59%
MplCOMSPS	412	**428 (+16)**	7.52%
MplCM	442	**443 (+1)**	0.3%
MplCBT	442	**451 (+9)**	2.88%

- Results with SATCoin Benches: **Very Strong** gains

Solver Name	**Solved by Baseline**	**Solved by expSAT Extension**
gLCM	7	**12 (+5)**
MplCOMSPS	4	**13 (+9)**
MplCM	1	**10 (+9)**
MplCBT	41	**43 (+2)**

paramAdapt: Exploration Parameter Adaptation

- **paramAdapt**
  - During the course of the expSAT search, it dynamically controls
    - **frequency of** exploration episodes,
    - **how much exploration** to perform in an exploration episode.
- **Experiments:**
  - Repeated the same experiments with both test sets.
  - **paramAdapt** implemented on top of our expSAT solvers.
- **Results:**
  - Compared **adaptive version** with **non-adaptive version** .
    - **mixed performance** over SATComp Benches.
    - **outstanding gains** over SATCoin Benches!

      - Biggest improvement: **baseline: 1, it's adaptive extension: 23**!

Analysis of the Solving Efficiency

- Analyze the experimental data for **glucose and eGLCM (non-adaptive)** to reveal further insights.
- Observation for **conflict efficiency**: In general,
    - Better solver for a subset of a problem **is more conflict efficient**.
        - Produces conflict at a fast rate, from which high quality clauses are learned.
- Observation for **average CD phase Length** : In general,
    - Better solver for a subset of a problem **reduces average CD phase length**.
    - exploration helps a solver **to escape from CD phases expeditiously**.

Conclusions and Future Work

- **Conclusions**:
    - Showed that VSIDS frequently undergoes the **pathological phase of CD**, in which branching decisions **are ineffective**.
    - To combat CD phases, **we proposed expSAT** that performs random exploration in the SAT search space.
    - **Empirically showed the effectiveness** of the expSAT approach.
- **Future Work**:
    - Integrate expSAT to **LRB based systems**.
    - Study exploration as in expSAT **to guide polarity selection**, e.g., by extending the phase-saving heuristic.
    - **Identify characteristics of SAT domains** which influence the effectiveness of exploration.