Exploiting Glue Clauses to Design Effective CDCL Branching Heuristics

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Outline

Introduction

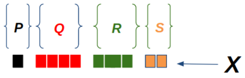
- In this work, I study Boolean Satisfiability (SAT)
  - Given a Boolean formula, the task is to **determine assignments** of the variables to satisfy that boolean formula, if one exists. Otherwise, report unsatisfiability ....
- SAT solving is **NP-Complete** $\rightarrow$ Intractable, in general.
- Modern SAT solvers $\rightarrow$ Conflict Directed Clause Learning (CDCL) Solvers.
  - **Applications in many domains**: Hardware design verification, Software testing, encryption, planning ..

Introduction

- Two basic SAT operations: **decision** and **propagation**.
- CDCL workflow:
  - decide $\rightarrow$ propagate $\rightarrow$ decide $\rightarrow$ propagate ....
  - decide $\rightarrow$ propagate $\rightarrow$ **conflict**
    - **conflict**: a clause cannot be satisfied wrt. the current partial assignment.
  - **conflict**$\rightarrow$ conflict analysis $\rightarrow$ clause learning and back-jumping.
- Conflict Generation at a **fast rate is crucial** for CDCL SAT solvers.
  - conflict$\rightarrow$ learned clause $\rightarrow$ space pruning.
- A CDCL SAT solver **learns clauses at a fast rate**.
  - May affect the overall speed of a solver.
  - Learnt clause DB management is necessary $\rightarrow$ **periodic reduction**.

Introduction

- One criterion for clause DB management is **Literal Block Distance (LBD)** score of the learned clauses.
  - Number of distinct decision levels in a learned clause.
    - The learned clause **X** has 4 decision levels: *P*, *Q*, *R* and *S*.



  - Lower the better.
- **Glue Clause:** Learned clauses with LBD score 2.
  - are known to **possess high pruning power**.
- In this work, we relate **Glue clauses to branching decisions**.
  - At any given state of the search:
    - **Glue Variable**: a variable that appears in at least one glue clauses.
    - **NonGlue Variable**: never appears in any of the glue clauses.

Contributions

- **Contribution I:**
  - We empirically show that
    - Decisions with glue variables are **more conflict efficient**.
    - CDCL branching heuristics show a **clear bias** toward Glue variables.
- **Contribution II:**
  - Developed a structure aware variable bumping scheme - Glue Bumping (GB)
    - **prioritizes** selection of Glue variables
  - Empirically evaluated the GB method on **four state-of-the-art CDCL SAT solvers**.
- **Contribution III:**
  - Have introduced the **Glue to Learned (G2L)** metric
    - G2L: fraction of the learned clauses that are glue.
    - consistently explain the performance of the GB method.

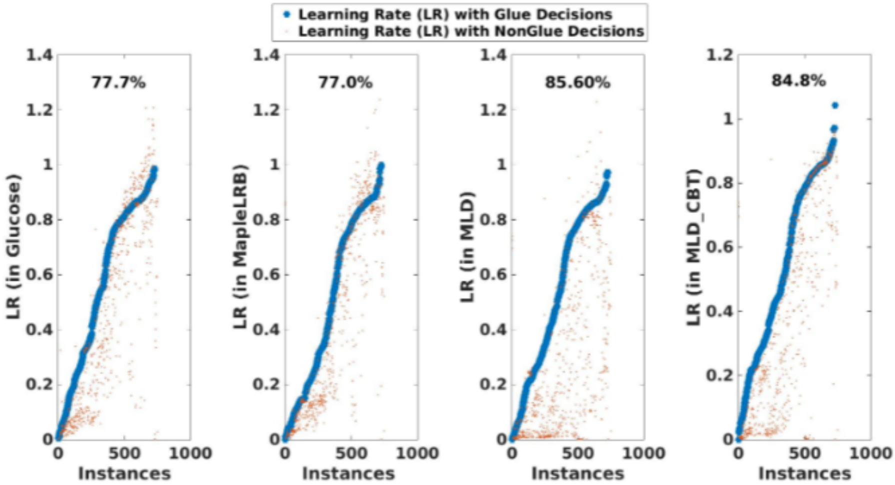
Notations

- For a run of a solver with a given SAT formula
  - **Learning Rate (LR)**
    - number of conflicts per decisions.
  - **Average LBD (aLBD)**
    - average LBD scores of the learned clauses derived from the generated conflicts.
  - **Glue** and **NonGlue decisions**
    The branching decision that selects
    - a Glue variable is called a **Glue decision**.
    - a NonGlue variable is called a **NonGlue decision**.

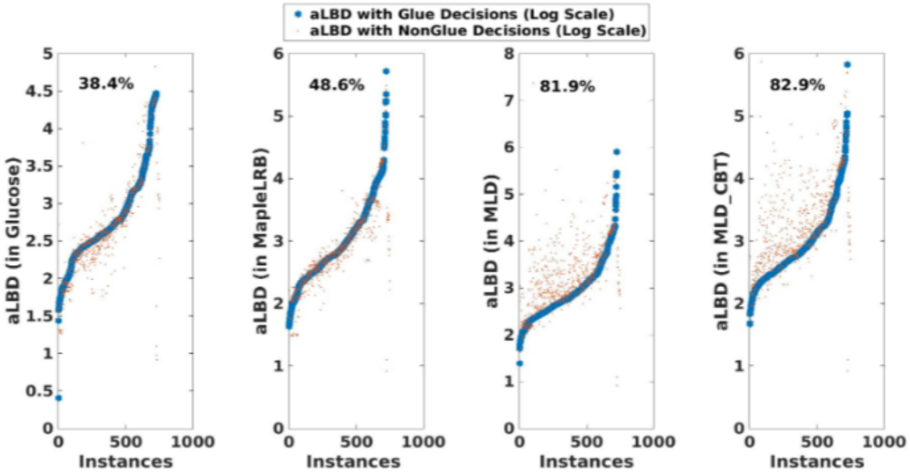
Contribution I: Conflict Efficiency of Glue Variables

- We study **LR** and **aLBD** over Glue and NonGlue decisions.
  - For all the maintrack instances from SAT-2017 and 2018 (750).
  - Using four state-of-the-art solvers:
    - **Glucose**,
    - **MaplePureLRB** (MapleLRB),
    - **MapleLCMDist** (MLD, winner of SAT-2017) and
    - **MapleLCMDistChronoBT** (MLD_CBT, winner of SAT-2018).
- For each run (time limit=5000s), we separately measure **LR** and **aLBD** over Glue and NonGlue decisions.

Conflict Efficiency of Glue Variables (LR)



Conflict Efficiency of Glue Variables (aLBD)



Biased Selection of Glue Variables

- For a run with a given solver for a given instance $\mathcal{F}$, we define
  - **Glue Percentage (GP):** $GP = \frac{\#Glue\_Variables\_in\_F}{\#Variable\_in\_F}$ *100

(A) Systems	(B) Average for Glue Variable	
	GP (B1)	Glue Decisions % (B2)
Glucose	25.32%	65.43%
MapleLRB	21.8%	63.14%
MLD	22.05%	47.60%
MLD_CBT	22.19%	48.76%

Contribution II: The Glue Bumping (GB) method

- How can we exploit this empirical characteristics of glue variables?
  - Glue Bumping: which bumps the **activity score of glue variables**
  - based on **appearance count** of a variable in glue clauses and its **current activity score**.
- **Glue Level (gl):**
  - Let G be the **set of learned glue clauses** so far.
  - *gl(v)* of a variable v is the appearance count of v in the glue clauses in G.

Alg. 1: Increase Glue Level

Input: *A newly learned glue clause* θ

1 **For** $i \leftarrow 1$ *to* $|\theta|$

2 $\quad v \leftarrow varAt(\theta, i)$

3 $\quad gl(v) \leftarrow gl(v) + 1$

4 **End**

Alg. 2: Bump Glue Variable

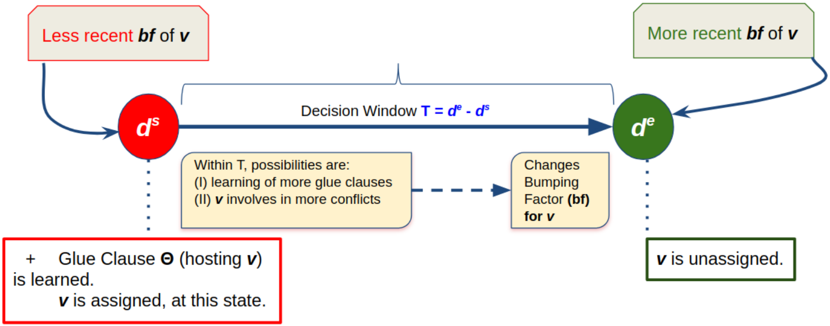
Input: *A glue variable* v

1 $bf_v \leftarrow activity(v) * \left(\frac{gl(v)}{|G|}\right)$

2 $activity(v) \leftarrow activity(v) + bf_v$

Delayed Bumping in GB

- GB delays the bumping of **v** until it is **unassigned by backtracking**.
  - θ is the latest learned clause and every variables are currently assigned.
  - $T = d^e - d^s > 0$ be the **decision window**.



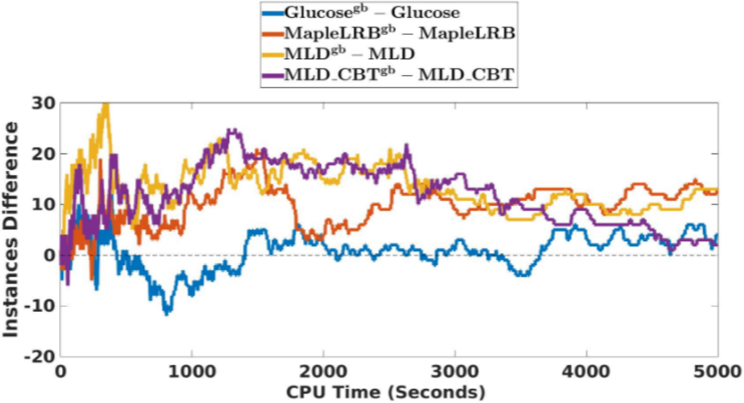
- Hence, GB method **delays the bumping until** d^e.

Empirical Evaluation

- Extended Glucose, MapleLRB, MLD and MLD_CBT with the GB.
- Performed experiments (timeout=5000s) $\rightarrow$ Apple-to-Apple comparison.
  - 13 additional instances solved by both MapleLRB^{gb} and MLD^{gb}.

Systems	SAT Comp-2017 and 2018			
	SAT	UNSAT	Total	PAR-2
Glucose	180	191	371	4167
Glucose^{gb}	**182 (+2)**	**193 (+2)**	**375 (+4)**	**4141**
MapleLRB	194	190	384	3966
MapleLRB^{gb}	**204 (+10)**	**193 (+3)**	**397 (+13)**	**3851**
MLD	235	207	442	3442
MLD^{gb}	**246 (+11)**	**209 (+2)**	**455 (+13)**	**3318**
MLD_CBT	238	215	453	3365
MLD_CBT^{gb}	**240 (+2)**	215 (+0)	**455 (+2)**	**3295**

Solve Time Comparison



Solve time comparisons. For any point above 0 in the vertical axis, our extensions solve more instances than their baselines at the time point in the horizontal axis.

Surprising observation for GLR and aLBD

- Better branching heuristics have **higher GLR and lower aLBD**, on average (Liang 2017 et. al.)
- We take two subsets (Extreme cases) into considerations:
  - $\textbf{GB}_{\textbf{exclusive}}$: Instances are solved by the GB extension, not by its baseline.
  - $\textbf{Baseline}_{\textbf{exclusive}}$: Instances are solved by the baseline, not by its GB extension.
- Expectations:
  - For $\textbf{GB}_{\textbf{exclusive}}$, our GB extensions achieve **higher GLR** and **lower aLBD**, on average.
  - For $\textbf{Baseline}_{\textbf{exclusive}}$, our GB extensions achieve **lower GLR** and **higher aLBD**, on average.
- We observe **almost opposite scenario**:
  - Average GLR <u>does not hold the expectations, at all</u>.
  - Average aLBD <u>is also inconsistent for these two subsets</u>.

Contribution III: G2L- A new measure for performance

(A) Systems	(B) Employed Heuristics	(C) $GB_{exclusive}$				(D) $Baseline_{exclusive}$			
		#inst	avg. GLR	avg. aLBD	avg. G2L	#inst	avg. GLR	avg. aLBD	avg. G2L
Glucose	{VSIDS}	33	**0.56**	28.60	0.0005	29	0.59	**18.52**	**0.0015**
$Glucose^{gb}$	$\{VSIDS\}^{gb}$		0.53	**24.69**	**0.0016**		**0.62**	20.14	0.00078
MapleLRB	{LRB}	27	**0.50**	26.06	0.00073	14	0.47	**30.75**	**0.00046**
$MapleLRB^{gb}$	$\{LRB\}^{gb}$		0.46	**20.38**	**0.00126**		**0.48**	32.02	0.00037
MLD	{Dist/VSIDS/LRB}	28	**0.55**	**23.60**	0.00029	15	0.53	26.70	**0.0011**
MLD^{gb}	$\{Dist/VSIDS/LRB\}^{gb}$		0.51	26.04	**0.00032**		**0.58**	**23.21**	0.0009
MLD_CBT	{Dist,VSIDS,LRB}	26	**0.49**	**26.08**	0.0006	24	0.51	29.64	**0.00065**
MLD_CBT^{gb}	$\{Dist/VSIDS/LRB\}^{gb}$		0.43	36.24	**0.0011**		**0.55**	**25.42**	0.00037

$$G2L = \frac{\#glue\_clauses}{\#learned\_clauses}$$

- **Better heuristic** for **an instance set** consistently achieves higher G2L.

Peculiarity of Glucose

- Lowest gains with **Glucose**. $\rightarrow$ why?
- **Glucose** already increases the score of some of the (glue) variables during conflict analysis.
- Hypothesis: GB in **Glucose**$^{\mathbf{gb}}$ creates imbalance.
- We lower the bumping factor with **high normalizing factor** $\rightarrow$ improved performance with **Glucose**$^{\mathbf{gb}}$.
  - Solves **11** additional instances.
    - In comparison, the version with **lower normalization factor** solves 4 additional instances.

Conclusions and Future Work

- Conclusions:
  - Decisions with Glue variables are conflict efficient.
  - GB method with delayed bumping of Glue variables.
  - Empirical evaluation shows performance gain.
  - G2L correlates well with performance.
- Future Work:
  - Relationships between normalized glue level and other centrality measures.
  - Design clause deletion heuristics based on the notion of glue level?
  - New branching heuristics based on G2L?