Response to the reviewers of *Modeling Ionic Polymer-Metal Composites with Space-Time Adaptive Multimesh hp-FEM*

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Abstract. We are concerned with a model of ionic polymer-metal composite (IPMC) materials that consists of a coupled system of the Poisson and Nernst-Planck equations, discretized by means of the finite element method (FEM). We show that due to the transient character of the problem it is efficient to use adaptive algorithms that are capable of changing the mesh dynamically in time. We also show that due to large qualitative and quantitative differences between the two solution components, it is efficient to approximate them on different meshes using a novel adaptive multimesh *hp*-FEM. The study is accompanied with numerous computations and comparisons of the adaptive multimesh *hp*-FEM with several other adaptive FEM algorithms.

1 Reviewer #1

1. *The comparisons presented by the authors are interesting. One important aspect regarding the Poisson-Nernst-Planck equations might be missing. A characteristic quantity of the system is the dimensionless Debye length. It is well-known that the case of small Debye lengths provides severe numerical difficulties. Hence, it would be very interesting to see how hp-FEM are able to handle such cases. This might also lead to a different picture where possibly the electrostatic potential needs more adaptivity than the concentration of counter ions in contrast to examples considered so far.*

   Section 5.6 was added with a length scale study.

2. *last sentence and Figure 1 Could the authors give a schematic picture presenting the electrostatic forces causing the deformation/bending of the IPMC?*

   An explanation was added to the images in Figure 1.

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3. *3rd line "materials that includes"*

   Fixed.

4. *2nd paragraph, The computing power ... Are there references which elucidate this statement?*

   A reference was added to a previous study where solving the PNP system in 3D was discussed.

5. *1st sentence, ... principal difficulties ... As mentioned above, there are also other issues known for the Poisson-Nernst-Planck system in the case of small Debye lenghts. Are the authors sure that the motivation given in Section is numerical more demanding? If yes, are there references available?*

   In some practical cases such as in case of modeling the Ionic Polymer Metal Composites, often the values of diffusion constant D and dielectric permittivity ε are such that the gradients of φ do not become significantly high in a studied time scale, however, the gradients of c do. This is one reason why it is hard to find an optimal mesh, especially in case of a 3D domain.

6. *eq. (3.5) It might be more appropriate to present the dimensionless formulation of the problem instead of introducing the physically less motivated notations (3.5). Such a formulation then immediately provides the characteristic dimensionless parameter of the system, i.e. the Debye length.*

   This was very good point. The dimensionless form of the equations was derived. **All** the results were recalculated and each graph and image in the Results section was redone. Instead of using C and ϕ, the corresponding scaled variables c and φ are now used throughout the results section.

   Furthermore, this time, all the results were calculated using adaptivity where the relative error of the physical fields was considered separately. Previously, absolute errors were compared and the absolute error of the physical field ϕ was always very much smaller compared to C. Therefore no adaptivity of whatsoever was observed in case of ϕ. The new results (for instance, Figures 14 and 15) show that the mesh and the polynomial space of physical field φ is also adapted, however, not as much as in case of c. We feel that this approach makes physically more sense.

7. *3rd paragraph The reader might loose at this point the interest in reading further since there is no motivation which favors a hp-FEM discretization over traditional low-order FEM. One sentence regarding computational performance or a related comment might be useful ...*

   TODO Pavel

8. *2nd last sentence in 1st paragraph A short explanation, how Hermes further deals with this difference without going to much into details, might be interesting here ...*

TODO Pavel

9. *1st sentence in Sec. 5.2 Convergence plots of the error, which the authors mention, would be interesting...*

   Figure 17 shows an example error convergence in case of HP_ANISO, HP_ANISO_H, and HP_ISO.

10. *last paragraph "The results"*

    Fixed.

11. *Section 5.4, eq. (5.4) What does Hermes do in the case $e^n \geq \delta$?*

    Currently Hermes does not support the time step reduction. However, this is planned to be added in future. A comment was also added below (5.5): *At this point, the implementation does not support reducing the time step when $e^n \geq \delta$.*

2 Reviewer #2

1. *The authors have not given the initial values of C*

   The initial value was added below Eq. (1.1).

2. *The authors have not given the size of time-step and the relation of time step size and h; p or CFL stability condition.*

   TODO Pavel?