

Applications of String Functions

Searching for a Sub-string in a String (KMP)

- Concatenate the substring to the beginning of the string, separated by a character that does not occur in either string.
- Run the prefix function on the concatenated string and count how many times the length of the substring appears.

Counting the Number of Prefixes in a String

- Run the prefix function on the string.
- For each prefix of length i : $\text{ans}[\text{pi}[i-1]] += \text{ans}[i]$
- Add one for each original (non-counted) prefix.

```
vll ans(n+1, 1); // Set to one for the actual prefix
for (int i = 0; i < n; i++) ans[pi[i]]++;
for (int i = n-1; i > 0; i--) ans[pi[i-1]] += ans[i];
```

The Number of Different Substrings in a String

- Start with an empty string.
- Add the next character of the original string and reverse.
- Run the prefix function to find the maximum length of substrings already found.
- New substrings added = $|S| + 1 - \text{maxPi}$, accumulated over all steps.

Compressing a String

- Given a string s of length n , we want the shortest compressed representation.
- After running the prefix function:
 - If $n \% (n - \text{pi}[n-1]) == 0$, the string is compressible, with minimum size $n - \text{pi}[n-1]$.
 - Otherwise, it is not compressible.

Creating Automation for String Matching

- After preprocessing a string with the prefix function, we can build an automaton to allow matching in $O(m)$ per query (after $O(n)$ preprocessing).
- This is useful for many repeated searches with the same pattern (KMP alone would take $O(n+m)$ per query).
- Build a table $\text{aut}[\text{state}][\text{charNumber}]$ where:
 - state = index in the pattern
 - charNumber = current character
- Depending on the character, move forwards or backwards to a new state.
 - Example: reading 'a' might move us from state 1 \rightarrow 2, while reading 'b' moves state 2 \rightarrow 0.