

How to Do Agent-Based Simulations in the Future: From Modeling Social Mechanisms to Emergent Phenomena and Interactive Systems Design

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Abstract

Since the advent of computers, the natural and engineering sciences have enormously progressed. Computer simulations allow one to understand interactions of physical particles and make sense of astronomical observations, to describe many chemical properties *ab initio*, and to design energy-efficient aircrafts and safer cars. Today, the use of computational devices is pervasive. Offices, administrations, financial trading, economic exchange, the control of infrastructure networks, and a large share of our communication would not be conceivable without the use of computers anymore. Hence, it would be very surprising, if computers could not make a contribution to a better understanding of social and economic systems. While relevant also for the statistical analysis of data and data-driven efforts to reveal patterns of human interaction [1], we will focus here on the prospects of computer simulation of social and economic systems. More specifically, we will discuss the techniques of agent-based modeling (ABM) and multi-agent simulation (MAS), including the challenges, perspectives and limitations of the approach. In doing so, we will discuss a number of issues, which have not been covered by the excellent books and review papers available so far [2–10]. In particular, we will describe the different steps belonging to a thorough agent-based simulation study, and try to explain, how to do them right from a scientific perspective. To some extent, computer simulation can be seen as experimental technique for hypothesis testing and scenario analysis, which can be used complementary and in combination with experiments in real-life, the lab or the Web.

1 Why Develop and Use Agent-Based Models?

1.1 Potential of Computer Simulation in the Socio-Economic Sciences

It is well-known that the ways in which social scientists analyze human behavior, social interactions, and society vary largely. The methods range from qualitative to quantitative ones, and among the quantitative ones, some communities prefer detailed models with many variables and parameters, while others prefer simple or simplified models with a few variables and parameters only. Reference [11] discusses these different types of system description and their respective advantages and disadvantages. Overall, each method has its justification, and the choice of the proper method very much depends on the respective purpose. For example, the elaboration of applications such as new systems designs often requires a quite realistic and, hence, detailed description of all relevant aspects. In contrast, simple models may be used to get a better understanding of how social mechanisms work. They serve to reduce the complexity of a given system to an extent that allows to guide our thinking and provide an intuition how certain changes in the system would affect its dynamics and outcome.

The application of computational models is currently not common in the social and economic sciences. This is perhaps because many people consider them as intransparent and unreliable (as compared to analytical methods) and/or as unsuitable for prediction. These points will be addressed later on. In fact, if properly done, computer simulations *can* deliver reliable results beyond the range of analytical tractability (see Sec. 3.6). Moreover, we will show in Sec. 4.1 that prediction is not generally impossible for socio-economic systems and that it is even not necessary to improve a system (e.g. to reduce instabilities or vulnerabilities), see Sec. 4.2. Besides, the benefit of computational models is not restricted to prediction. Joshua Epstein, for example, discusses 16 other reasons to build models, including explanation, guiding data collection, revealing dynamical analogies, discovering new questions, illuminating core uncertainties, demonstrating tradeoffs, training practitioners, and last but not least decision support, particularly in crisis situations [12].

In fact, computer models can naturally complement classical research methods in the socio-economic sciences. For example, they allow one to test whether mechanisms and theories used to explain certain observed phenomena are sufficient to understand the respective empirical evidence, or whether there are gaps or inconsistencies in the explanation. Moreover, they allow one to study situations, for which analytical solutions cannot be found anymore, and to go beyond the idealizations and approximations of simple models. Without the exploration of model behaviors that can only be numerically determined, scientific analysis is often restricted to unrealistic models and to situations, which may be of little relevance for reality. For example, the financial crisis may have been the result of approximations and simplifications of economic models, which were not sufficiently justified (for a more detailed discussion of this point see Ref. [13]).

1.2 Equation-Based vs. Agent-Based Approach

Today, computer-simulation in the natural sciences and engineering mostly rely on equation-based modeling (e.g. of the dynamics of gases, fluids, or solid bodies). Such an approach would certainly be hard to transfer to the social sciences, as most system behaviors have not been formalized mathematically. A method that seems to be more suited for the computer simulation of socio-economic systems is agent-based modeling (ABM) [2–6]. The corresponding computational technique is called multi-agent simulation (MAS) or agent-based computational modeling (“ABC modeling”). Depending on the problem of interest, agents may for example represent individuals, groups, companies, or countries and their interactions. The behaviors and interactions of the agents may be formalized by equations, but more generally they may be specified through (decision) rules, such as if-then kind of rules or logical operations. This makes the modeling approach much more flexible. Besides, it is easily possible to consider individual variations in the behavioral rules (“heterogeneity”) and random influences or variations (“stochasticity”).

To give a clearer picture, let us provide below a list of properties, which may be given to an agent representing an individual:

- birth, death, and reproduction,
- individual needs of resources (e.g. to eat and drink),
- competition and fighting ability,
- toolmaking ability (e.g. the possibility to grow food, hunt etc.),
- perception,
- curiosity, exploration behavior, ability for innovation,
- emotions,
- memory and future expectations,
- mobility and carrying capacity,
- communication,
- learning and teaching ability,
- the possibility of trading and exchange,
- the tendency to have relationships with other agents (e.g. family or friendship ties etc.).

Agent-based computational models also appear ideal to study interdependencies between different human activities (both, symbiotic and competitive relationships) [14]. Therefore, they can shed new light on social and economic systems from an

“ecological” perspective [15]. In fact, evolutionary ecological models can also reflect the feature of steady innovation, which is typical for socio-economic systems. Moreover, such models appear particularly suited to study the sustainability and resilience of systems. Finally, they can be well combined with other simulation methods used in the natural and engineering sciences, including statistical physics, biology, and cybernetics.

1.3 Scientific Agent-Based Models vs. Computer Games

Agent-based computational modeling is also well suited for visualization. Here, it makes sense to distinguish agent-based models in science from computer games. While the latter may actually be *based* on agent-based models, they are often oriented at *believability*, i.e. at appearing as realistic as possible. However, computer games usually do not care too much about making realistic *assumptions*. That is, while the outcome may look realistic, the underlying mechanisms may not be well justified. As a consequence, they may not be suited to understand the outcome of the simulation, to draw conclusions, or to make predictions. Implications outside the exact settings that the game was prepared for are likely to be unreliable. Therefore, in many cases, such computer simulations will not produce useful knowledge beyond what has been put into the model.

Scientific agent-based models, in contrast, do often not invest into believability. That is, they intentionally make simplifications and may, for example, represent people by circles, or purposefully restrict themselves to very few properties from the list given in Sec. 1.2. Instead of focusing on a plausible appearance, they try to represent a few characteristic features (such as certain kinds of interaction mechanisms) more realistically. Such computer simulations should enable one to draw conclusions about previously unexperienced scenarios, i.e. they should be in reasonable agreement with later empirical observations or experimental results. In other words, scientific simulations are more focused on getting the processes rather than the visual representation right. They are interested in explanatory power.

Finally, agent-based simulations for engineering applications are often located somewhere between the two archetypical cases discussed above. However, rather than on the basis of the level of detail and believability, it also makes sense to classify models as follows:

- *Physical models* assume that individuals are mutually reactive to current (and/or past) interactions.
- *Economic models* assume that individuals respond to their future expectations and take decisions in a selfish way.
- *Sociological models* assume that individuals respond to their own and *other* people’s future expectations (and their past and current experiences as well).

1.4 Advantages of Agent-Based Simulations

Agent-based simulations are suited not only to reflect interactions between different individuals (and other entities). They allow one to start off with the descriptive power of verbal argumentation and to determine the implications of different hypotheses. From this perspective, computer simulation can provide “an orderly formal framework and explanatory apparatus” [16]. Other favorable features of agent-based simulations are [17]: modularity, great flexibility, large expressiveness, and the possibility to execute them in a parallelized way.

Agent-based models can be combined well with other kinds of models. For example, when simulating the interaction with the environment, the environment may be represented by a discrete or continuous field. Such an approach is pursued within the framework of active walker models [18, 19]. One can easily couple agent-based models with continuum models, such as gas-kinetic or fluid-dynamic models. Such an approach is, for example, used to simulate the evacuation of people in scenarios where poisonous gas spreads in the environment [20, 21]. A similar approach would be applied, when weather, environmental, or climate simulations would be combined with models of human response to the respective external conditions.

In certain contexts, for reasons of computational efficiency it may also be reasonable to replace an agent-based by an aggregate (“macroscopic”) simulation approach. For example, traffic flows can not only be well represented by a car-following (agent-based) model, but also by a fluid-dynamic one [22]. It is even possible to relate the car-following models with fluid-dynamic ones in an analytical way [23]. In other words, it is possible to construct a mathematical bridge between the micro- and macro-level of description [24]—something which would be very nice to have for economics and other fields as well.

In the economic sciences, multi-agent computer simulations make it possible to overcome limitations of the theoretical concept of *homo economicus* (the “perfect egoist”) [25], by relaxing idealized assumptions that are empirically not well enough supported. They also offer a possibility to go beyond the representative agent models of macroeconomics [26], and to establish a natural link between the micro- and macro-level description, considering heterogeneity, spatio-temporal variability, and fluctuations, which are known to change the dynamics and outcome of the system sometimes dramatically [13].

Finally, agent-based simulations are suited for detailed hypothesis-testing, i.e. for the study of the consequences of ex-ante hypotheses regarding the interactions of agents. Insofar, one could say that they can serve as a sort of magnifying glass or telescope (“socioscope”), which may be used to understand our reality better. It is usually no problem to apply methods from statistics and econometrics to simulation outcomes and to compare simulation results with actual data (after processing them in a way reflecting the measurement process). Moreover, by modeling the relationships on the level of individuals in a rule-based way, agent-based simulations allow one to produce characteristic features of the system as emergent phenomena without having to make a priori assumptions regarding the aggregate (“macroscopic”)

system properties.

1.5 Understanding Self-Organization and Emergence

Agent-based simulations are a suitable tool to study complex systems. Complex systems are systems with many interacting entities and non-linear interactions among them. Such systems may behave in a number of interesting and often unexpected (sometimes even paradoxical) ways, which justifies to call them complex:

- they may have several stationary states (a phenomenon known as “multi-stability”), and the resulting outcome may depend on the previous history (such as the size of occurring perturbations, the “initial state”, etc., and such history-dependencies are often called “hysteresis effect”) (see Figs. 1 and 3),
- they may be “out of equilibrium” and behave in non-stationary ways,
- they may “self-organize”, showing periodic or non-periodic oscillations, “chaotic” or “turbulent” behavior, or spatio-temporal pattern formation (such as stop-and-go waves in traffic flows),
- they are often robust to small perturbations, i.e. “relax” to their previous behavior (the “stable attractor”),
- consequently, they often resist external manipulation or control attempts,
- however, at so-called “tipping points”, small influences may cause a sudden and often unexpected “systemic shift” (“phase transition”), after which the system behaves substantially different (see Fig. 2),
- more generally, they may show new, “emergent” properties, which cannot be understood from the properties of their system elements (“the system is more than the sum of its parts”) (see Fig. 1),
- correlations may determine the system dynamics, and neglecting them may lead to completely wrong conclusions,
- during systemic shifts (so-called “phase transitions”) or due to a phenomenon called “self-organized criticality” (SOC), cascading effects on all scales (i.e. of any size) may occur, so that local factors may have a systemic (“global”) impact (“critical phenomena”),
- therefore, “extreme events” may happen with probabilities much higher than expected according to a normal distribution, and are distributed according to “(truncated) power laws” or other “fat tail distributions”,
- the system may have features such as reproduction, innovation, reinforcement learning, an expectation-related dynamics, etc., and
- there may be singularities after a finite time.

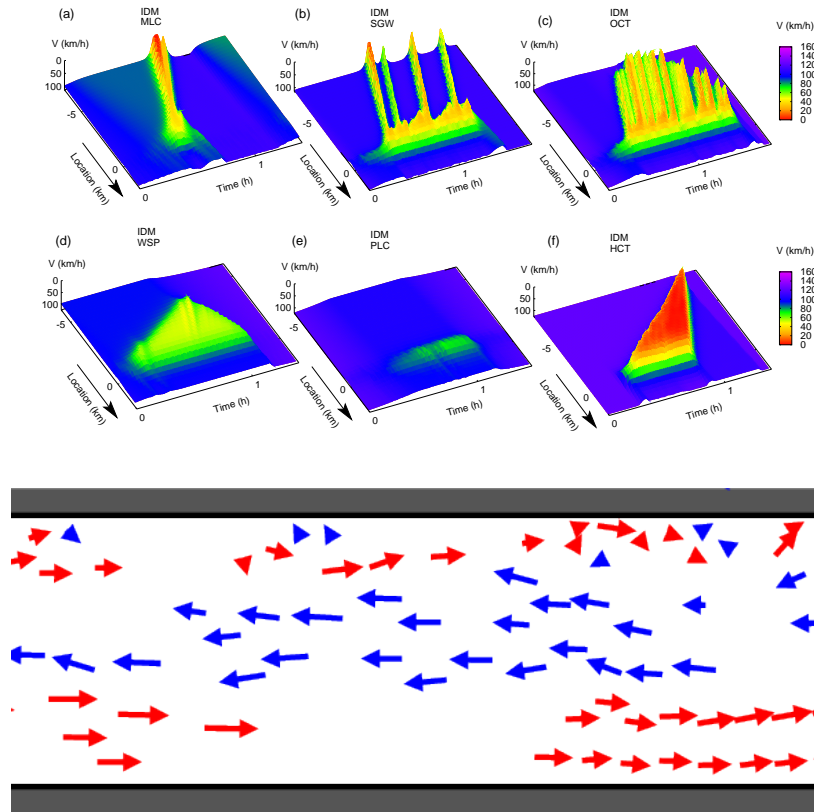


Figure 1: Top: Freeway traffic constitutes a dynamically complex system, as it involves the non-linear interaction of many independent driver-vehicle units with a largely autonomous behaviour. Their interactions can lead to the emergence of different kinds of traffic jams, depending on the traffic flow on the freeway, the bottleneck strength, and the initial condition (after Ref. [27]): a moving cluster (MC), a pinned localized cluster (PLC), stop-and-go waves (SGW), oscillating congested traffic (OCT), or homogeneous congested traffic (HCT). The different traffic patterns were produced by computer simulation of a freeway with an on-ramp at location $x = 0$ km using the intelligent driver model (IDM), which is a particular car-following model. The velocity as a function of the freeway location and time was determined from the vehicle trajectories (i.e. their spatio-temporal movement). During the first minutes of the simulation, the flows on the freeway and the on-ramp were increased from low values to their final values. The actual breakdown of free traffic flow was triggered by additional perturbations of the ramp flow. Bottom: In pedestrian counterflows one typically observes a separation of the opposite walking directions. This “lane formation” phenomenon has been reproduced here with the social force model [28].

Many of the above features are results of strong interactions in real or abstract space, or of network interactions within the system. Such interactions can often lead to counter-intuitive behaviors [29,30]. Here are a number of examples (most of them related to traffic systems, as they are well-known to everybody):

- Even when all drivers try to drive fluently, vehicles are sometimes stopped by “phantom traffic jams” (i.e. jams without an obvious reason such as an accident or bottleneck) [31,32].
- Stop-and-go traffic may occur despite the best attempts of drivers to move ahead smoothly [33,34].
- Even when the maximum road capacity is not reached, a temporary reduction in the traffic flow can cause a lasting traffic jam [35].
- Traffic jams do not occur in the bottleneck area, but upstream of it.
- Under certain conditions, speed limits can speed up traffic [31]. Similar “slower-is-faster effects” occur in urban traffic control, logistic systems, administrative processes, etc., i.e. delays at appropriate times and of suitable durations may reduce overall waiting times [36,37].
- While pedestrians moving in opposite directions normally organize into lanes, under certain conditions a “freezing-by-heating” effects or intermittent flows may occur [38].
- The maximization of system efficiency may lead to a breakdown of capacity and throughput [30].
- Sometimes, the combination of two dissatisfactory solutions can be the best solution [39,40].
- There is clearly a global diversity of opinions and behaviors, despite a rather strong tendency of local convergence [41].
- In repeated interactions, the “wisdom of crowds” mechanism may lead to collective error [42].
- One can often find cooperation among selfish individuals in social dilemma situations (where it seems more profitable to exploit the cooperativeness of others) [39].
- People often do not show the behavior they like and often show behaviors they do not like [43].

Generally, there is *still* a serious lack of understanding regarding the connections between the structure, dynamics, and function of complex networks such as technosocio-economic-environmental systems. Therefore, emergence often has an element of surprise. So far, for example, we do not understand emotions and consciousness, and we cannot even calculate the “fitness” of a behavioral strategy in

the computer. The most exciting open puzzles in science concern such emergent phenomena. It would be interesting to study, whether social and economic phenomena such as trust, solidarity, and economic value can be understood as emergent phenomena as well [13]. Agent-based simulations appear to be a promising approach to make scientific progress in these areas.

1.6 Examples of Agent-Based Models

The method of agent-based modeling is very versatile. It may be applied, for example, to the following problems:

- social influence and opinion formation [41,44],
- coalition formation [45,46],
- collective intelligence [47],
- social networks [48–50],
- group dynamics [51],
- social cooperation [52,53],
- social norms [14,54,55],
- social conflict [56,57],
- financial markets [58–60]
- competition and cooperation between firms [61,62],
- micro-economic models [63,64],
- macro-economic models [65,66],
- organization and managerial decisions [67],
- migration [68],
- agglomeration and segregation [69,70],
- urban and regional development [71–73],
- traffic dynamics [74,75],
- crowd dynamics [76,77],
- systemic risks in socio-economic systems [78,79].

and more [80–84].

1.7 Social Super-Computing

Multi-agent simulations are well suited for parallelization. Therefore, using super-computing power, it is possible in principle to run agent-based simulations with millions (or, in future, even billions) of agents. The following examples give an idea of the state-of-the art:

- A first successful application area was large-scale traffic simulation. The TRANSIMS project [85–87] of the Los Alamos National Institute (LANL), for example, has created agent-based simulations of whole cities such as Dallas [88] or Portland [89]. The approach has been recently extended to the simulation of the travel behavior of the 7.5 million inhabitants of Switzerland [90, 91]. These simulations are obviously based on parallel computing. They generate realistic individual activity patterns according to detailed statistical panel data (“travel diaries”) [92, 93], which are nowadays complemented by GPS data and public mobility data (e.g. from Greater London Area’s OYSTER Card). Other extensions look at interconnections between the traffic system and regional development [72, 73].
- Recent applications are studying contingency plans for large-scale evacuations of cities [94, 95]. The key aspect here is understanding the interdependency of infrastructure systems [96, 97] and their vulnerabilities through natural disasters, terrorist attacks, accidents, and other incidents. For example, the Los Alamos National Laboratories have already established a Critical Infrastructure Protection Decision Support System [98]. Its advanced simulation capabilities have already been extensively used during past emergencies.
- Large-scale simulations have also been applied to study and predict the spreading of diseases. While previous epidemic spreading models such as the SIR model [99–101] have neglected spatial interaction effects, recent models [102] take into account effects of spatial proximity [20], air traffic [103] and land transport [104], using TRANSIMS and other traffic simulation models. The current scientific development also tries to take into account behavioral changes, which may reduce the spreading rate of diseases.
- Furthermore, there are attempts to model financial markets with agent-based simulations. Two examples for this are the Santa-Fe Stock Market Simulator [105] and U-mart [106]. Recent attempts are even heading towards the simulation of the whole economic system (see for example the EU project EURACE [107]). Other simulation studies are trying to understand the evolution of cooperation [39], social norms [14, 54], language [108–110], and culture [3]. Such simulations explore the conditions under which trust, cooperation and other forms of “social capital” can thrive in societies (see also Ref. [13, 111]). They also show that the crust of civilization is disturbingly

vulnerable. Such simulations can reveal possible causes of breakdowns of social order. Examples for events where this actually happened are as diverse as the war in former Yugoslavia, lootings after earthquakes or other natural disasters, or the violent demonstrations we have recently seen in some European countries.

It appears logical that supercomputing will be ultimately moving on from applications in the natural and engineering sciences to simulations of social and economic systems, as more and more complex systems become understandable and the required data become available. And it is obvious that virtual three-dimensional worlds (such as Google Earth) are waiting to be filled with realistic life.

2 Principles of Agent-Based Modeling

After describing the potential of multi-agent simulations, let us now discuss the principles of how to craft agent-based models. A thorough scientific study involves a number of steps:

- First, one should clearly describe the evidence to be explained by the respective study. What are the empirical or experimental data or observations to be reproduced, or what are the “stylized facts”, i.e. the simplified, idealized properties of the system under consideration?
- Second, one should explain what is the purpose of the simulation? To understand a phenomenon? To get a more accurate description? To make predictions? To develop an application (e.g. a new traffic control)? In the social sciences, it is common to formulate a scientific puzzle, i.e. to describe a problem that is hard to understand and why. This could be an unexpected or even paradoxical individual or system behavior. Emergent system behaviors are particularly interesting candidates for the formulation of such a puzzle (“scientific mystery”).
- Next, one needs to decide how to choose the agents in the model. For example, when the competition of companies shall be studied, it may not be necessary to simulate all employees of all companies. It may be sufficient to choose the companies as the agents of the model. In fact, it can be shown mathematically (e.g. by eigenvalue analysis) that mutually coupled agents may jointly behave like one entity, i.e. one agent. An example for this is the quasi-species concept in the theory of evolution [112].
- After specifying the agents, one should formulate hypotheses regarding the underlying socio-economic processes or fundamental mechanisms leading to the particular system behavior that needs to be explained. Ideally, these mechanisms should be sociologically or economically justified, i.e. there should be some empirical evidence for the mechanisms on which the agent-based model is based. The transfer of models from other sciences (such as

spin, epidemic, or synchronization models) requires particular justification beyond saying that the resulting system behavior is reminiscent of features that have been observed elsewhere.

- When specifying the mechanisms underlying the multi-agent simulation, one should not put into the model assumptions what one wants to explain. The mechanisms on which the multi-agent simulations are based should be (at least) one level more elementary than the evidence to be understood. For example, the rich-gets-richer effect [113] may be used as an ingredient, if class formation shall be described. Moreover, “homophily” [114] may be assumed in models of coalition formation or solidarity. Moreover, social network characteristics may be used to explain the spreading of behaviors [115–117].

However, if the income distribution is to be explained, it is favorable not to start with the rich-gets-richer effect, but instead with a mechanism that is purely random and not biased in favor of anybody in the beginning. Moreover, even if this is not realistic, it would be interesting to start the computer simulation with identical wealth of everybody [118]. Furthermore, if social segregation is to be explained, one should not assume “homophily” already, but to let it evolve in a system that starts off with identical and non-segregated individuals [39, 119]. Finally, if group formation is to be explained, social network characteristics should not be assumed as an input [41]. They should, for example, result from certain rules regarding the formation and deletion of social ties [120, 121].

- Last but not least, one should compare the computer simulation results with the empirical evidence. Here, one should avoid to be selective, i.e. one should state what are the features that are correctly reproduced, and which ones are not. Pointing out the limitations of a model is equally important as underlining its explanatory power.

Note that, even though linear models may be sensitive to parameter variations or perturbations, they cannot explain self-organization or emergent phenomena in socio-economic systems. This underlines that the consideration of non-linear interaction mechanisms is crucial to understand many observations in social and economic systems [13].

When specifying the properties of agents and their interactions, it makes sense to select some from the list given in Sec. 1.2, but (by far) not all. As was pointed out before, the goal of scientific computer-simulation is usually not a realistically looking computer game, but the explanation of a set of observations from a minimum set of assumptions.

It is definitely not obvious how each of the points on the list in Sec. 1.2 is modeled best. Typically, there are a number of plausible alternatives. To gain an understanding and intuition of the studied system, simple assumptions (even if idealized) are often preferable over detailed or complicated ones. For example, rather

than assuming that individuals would strictly optimize over all options when decision are taken (as “homo economicus” would do), it seems justified to use simple decision heuristics, as evidence from social psychology suggests [122]. However, it would obviously be interesting to compare the implications of both modeling approaches (the classical economics and the heuristics-based one).

2.1 Number of Parameters and Choice of Model

When formulating the agent-based model, a number of aspects should be considered, as discussed in the following. The adequate number of model parameters and variables depends on the purpose of the model and its required degree of realism and accuracy. A model for the elaboration of practical applications tends to have more parameters than a model aimed at the fundamental understanding of social or economic mechanisms. However, one should keep in mind that parameter-rich models are hard to calibrate and may suffer from over-fitting. Hence, their predictive power may not be higher than that of simple models (see Sec. 3.2).

Generally, one can say that

- a model with meaningful parameters (which have a clear interpretation and are measurable) should be favored over a model with meaningless fit parameters,
- the same applies to a model with operational variables as compared to a model containing variables that cannot be measured,
- given the same number of parameters, an explanatory model should be preferred over a purely descriptive (“fit”) model,
- in case of a comparable goodness of fit in the model calibration step, one should choose the model with the better predictive power (i.e. which better matches data sets that have not been used for calibration),
- given a comparable predictive power of two models, one should select the simpler one (e.g. the one with analytical tractability or with fewer parameters) according to Einstein’s principle that *a model should be as simple as possible, but not simpler*.

The goodness of fit should be judged with established statistical methods, for example with the adjusted R -value or similar concepts considering the number of model parameters [123, 124]. This adjusted formula tries to compensate for the fact that it is easier to fit a data set with more parameters. For a further discussion of issues related to the calibration and validation of agent-based models see Sec. 3.2.

3 Implementation and Computer Simulation of Agent-Based Models

3.1 Coding Multi-Agent Simulations and Available Software Packages

Agent-based simulations may be programmed from scratch in most computer languages (e.g. C, C++, Java, etc.). This may be favorable from the perspective of computational efficiency. However, there are also more user-friendly ways of specifying agent-based models. This includes Swarm [129] and Repast [130], which provide low-level libraries, as well as graphical modelling environments such as Netlogo [131] or Sesam [132], which are more suited for beginners. Another recent software tool for the simulation of large crowds is MASSIVE [133]. For a comprehensive comparison of the features of available packages for agent-based simulations see Ref. [134] and Ref. [135].

3.1.1 Documentation

No matter how multi-agent simulations are implemented, it is crucial to document the code and the simulation scenarios well. Not only would the code be useless for any future collaboration partner, if it is not well documented. It is also quite easy to lose the overview over all the changes one may have experimented with. There are several publications, which report results that do not fit the model discussed in the paper. Such accidents happen often for one of the following reasons:

- the computer code has not been properly debugged – checked for mistakes, see Sec. 3.1.3 (e.g. variables are not properly initiated, or local variables are overwritten by global variables, or variables are mixed up, or there is a mismatch of different types of variables, such as integer values and real numbers),
- changes are made in the code, but not well documented and forgotten,
- readimade routines from software libraries are applied, but the preconditions for their use are not satisfied.

3.1.2 Plausibility Tests

The most famous mistake in a computer code was the explosion of a space shuttle caused by confusing a semicolon and a comma. However, not always do programming mistakes become evident in such a dramatic way. In fact, the danger is that they remain unnoticed. Computer codes do not automatically generate errors, when something is incorrectly implemented. On the contrary, most computer codes produce results independently of how reasonable the code or input is. Therefore, one should better have a healthy distrust towards any computational output. Computer

codes always require verification and proper testing, and it is crucial to check the plausibility of the output. Section 3.4.2 discusses some ways how this can be done.

3.1.3 Common Mistakes and Error Sources

To avoid errors in the computer code, the following precautions should be taken:

- Structuring the computer code into subroutines allows one to keep a better overview (and do a thorough, component-wise testing) as compared to one monolithic code.
- Different variants of the computer code should be distinguished with different version numbers¹, and the main features, particularly the changes with regard to the last version, should be clearly documented in the beginning of the code and, ideally, shown on the computer display and in the output file, when the code is run.
- One should not include (“hardcoding”) the parameters of the model in the computer code itself, but instead they should be read from a separate input file, which should also have an own version number. This parameter file should contain information, with what version of the computer code the parameter file needs to be run. It makes sense to write out the parameters and their values in the output file, to reduce unnoticed errors (such as adding a new parameter, which is erroneously initiated with the value of another parameter).
- Programming languages initiate variables and parameters in different ways (and some of them with random contents), so that it may easily remain unnoticed when a variable or parameter was not set. Therefore, all parameters and variables should be initialized immediately, when they are introduced in the code.
- Moreover, one should not make several significant changes at a time. For example, it is common to change only one parameter value and compare the new result with the previous one, before another parameter is changed. This helps to check the plausibility of the computational results.

3.2 Specification of Initial and Boundary Conditions, Interaction Network and Parameters

The simulation of agent-based models requires a number of specifications before the simulations can be run:

¹There exist a number of software packages aimed at supporting developers in versioning the code. They automatise several operations such as assigning sequential versions numbers, comparing different versions of files, undo or merging changes on the same files, etc. For examples, see [136–139].

- It is necessary to define the interaction network. For example, agents may interact in space, and their interaction frequency or strength may depend on their distance. However, in many cases, spatial proximity is just one factor or even irrelevant. In many cases, one needs to consider friendship networks or other interaction networks. It is well-known that the statistical properties of the interaction network may matter for the dynamics of the system dynamics. As a consequence, it may also be needed to run the simulation for different interaction networks. For example, interactions in square grids and in hexagonal grids may sometimes lead to qualitatively different outcomes. Moreover, the number of interaction partners may be relevant for the system dynamics, as may be a heterogeneity in the number of interaction partners or the existence of loops in the interaction network.
- One needs to specify the initial values of all variables (and if memory effects are considered as well, also their previous history). In many cases, it is common to make simplifying assumptions (e.g. all individuals are characterized by identical initial values, or more realistically, the initial variables are assumed to vary according to a particular distribution (e.g. a normal distribution)). It is advised to test the sensitivity of the model with respect to the initial condition.
- Furthermore, boundary conditions may have to be specified as well. For example, if a simulation of agents is performed in space, one needs to decide how the rules should look like at the boundary. One may decide to use a finite world such as a chess board. However, this may cause artificial behavior close to the boundaries of the simulation area. Therefore, in many cases one assumes “periodic” boundary conditions, which corresponds to a simulation on the surface of a torus. This is often simpler than the simulation on the surface of a sphere. Note, however, that an improper choice of the boundary conditions can sometimes produce artifacts, and that the simulated system size may affect the result. Therefore, one needs to have an eye on this and may have to test different specifications of the boundary conditions.
- Finally, it is necessary to specify the open model parameters, i.e. to calibrate the model. If the initial and boundary conditions, or the structure of the interaction network cannot be independently measured, they have to be treated as model parameters as well. In case there are enough empirical data, the parameters can be determined by minimizing an “error function”, which measures the difference between simulation results and empirical data. Note that the choice of a suitable error function may be relevant, but non-trivial. An improperly chosen function may not be able to differentiate well between good and bad models. For example, minimizing the error between actual and simulated trajectories of pedestrians does not manage to distinguish well between a simple extrapolation model and a circular repulsion model for pedestrians, although the latter avoids collisions and is much more real-

istic [127]. Or maximizing throughput may produce some unrealistic effects, such as the occurrence of pedestrians or drivers, who get trapped in a certain place [128], while a minimization of delay times ensures that everybody gets forward. Therefore, not only the model, but also the error function must be well chosen.

If there are not enough (or even no) empirical or experimental measurements to calibrate the model with, one can still try to find parameters, which “do the job”, i.e. which deliver plausibly looking simulation results. Such a qualitative simulation approach has, for example, been pursued in the early days of pedestrian modeling. Despite the simplicity of this method, it was surprisingly successful and managed to reproduce a number of striking self-organization phenomena observed in pedestrian crowds [127].

Finally note that, even when empirical or experimental data are available, the size of empirical data sets typically does not allow one to determine the model parameters accurately. Particularly, if the model contains many parameters, the reliability of the parameter estimates tends to be poor. In the worst case, this can imply dangerously misleading model predictions. To get an idea of the confidentiality intervals of parameters, one should therefore determine all parameter combinations which are compatible with the error bars of the empirical data to be reproduced.

3.2.1 Model Validation

A high goodness of fit during model calibration does not necessarily imply a high predictive power [125, 126], i.e. a good fit of new data sets. In many cases, one faces the problem of over-fitting (i.e. the risk of fitting noise or irrelevant details in the data). Therefore, it is necessary to determine the “predictive power” of a model by a suitable validation procedure.

In the ideal case, the model parameters can be measured independently (or can, at least be estimated by experts). If the parameters have a concrete meaning, it is often possible to restrict the parameters to a reasonable range. In case of non-meaningful fit parameters, however, this is not so simple (or even unfeasible).

One way of validating a model is to subdivide the empirical or experimental data into two non-overlapping parts: a calibration and a validation dataset. The calibration dataset is used to determine the model parameters, and the validation dataset is used to measure the goodness of fit reached with the model parameters determined in the calibration stage. In order to make this calibration and validation procedure independent of the way in which the original dataset is subdivided, the whole procedure should be performed either for all possible subdivisions into calibration and validation datasets or for a representative statistical sample of all possibilities. As each of these subdivisions delivers a separate set of parameters in the calibration step, this results in a distribution of model parameters, which are consistent with the data. From these distributions, one can finally determine aver-

age or most likely model parameters as well as confidence intervals. Furthermore, the distribution of goodness-of-fit values reached in the related validation steps reflects the predictive power of the model.

Another way of judging the power of a model is to determine the number of stylized facts that a model can reproduce. It sometimes makes sense to prefer a model that reproduces many different observations qualitatively well (for example, different observed kinds of traffic patterns) over a model whose goodness of fit is quantitatively better (for example, in terms of reproducing measured travel times). This applies in particular, if the model, which appears to be quantitatively superior, is not well consistent with the stylized facts [27]. To distinguish models of different quality, it can also be useful to measure the goodness of fit with several different error functions.

3.2.2 Sensitivity Analysis

As empirically or experimentally determined parameter values have a limited accuracy, one should also carry out a sensitivity analysis. For this, the simulation is performed with modified parameters to determine, how robust the simulation results are with respect to the choice of parameters. The sensitivity can be measured with Theil's inequality coefficient. Note that the parameters should be varied at least within the range of the confidence interval that determines the range, within which the actual parameters may vary according to the calibration dataset.

Besides determining the robustness of the model to the parameter specification, it is also recommended to test the robustness to the model assumptions themselves. By simulating variants of the model, one can figure out, which conclusions stay qualitatively the same, and which ones are changing (e.g., if the network characteristics or the system size or the learning rule are modified).

3.3 Performing Multi-Agent Simulations

3.3.1 Choice of the time discretization

The first problem relates to the choice of the time step Δt . In models that are discrete in time (such as deterministic cellular automata), the time step is assumed to be fixed, and the question may matter or not. When the model contains differential equations, however, the choice of Δt is always relevant. The rule is that large Δt speed up the simulation, but may lead to wrong results. For example, the discrete logistic equation $x_{n+1} = rx_n(1 - x_n)$ may behave very different from the continuous one $dx/dt = ax(1 - x)$ (the former one may perform a chaotic motion, while the latter one evolves smoothly in time).

To determine a suitable time discretization, Δt is reduced from moderate values (like $\Delta t = 1$ or 0.1) to smaller values, until the results do not change significantly anymore (e.g. less than 0.1 percent when Δt is chosen five times smaller).

3.3.2 Relevance of Considering Fluctuations

A widely recognized fact of socio-economic systems is that they do not behave deterministically. There are always sources of fluctuations (“noise”), such as errors, exploratory behavior, or the influence of unknown factors. For this reason, the probabilistic behavior should be reflected in the model. Neglecting “noise” may lead to misleading conclusions. For example, the zero-noise case may behave completely different from a system with just a little bit of noise, no matter how small it may be [119, 140].² In such cases, the result of the deterministic model without noise should be considered as artificial and of no practical relevance.

Note that the significance of fluctuations in techno-socio-economic-environmental systems is often wrongly judged. While large noise usually has a destructive influence on a system, as expected, it is quite surprising that small noise intensities can actually *trigger* structure formation or increase system performance [39, 141–143].

The implementation of noise in computer simulations is not fully trivial, i.e. mistakes can be easily made. First of all, no random number generator produces random numbers in a strict sense, but rather quasi-random numbers. In other words, there is a certain degree of statistical interdependency between computer-generated random numbers, and this may create artifacts. The quality of random number generators can be very different. Therefore, it makes sense to test the number generator in advance.

Moreover, when adding a Gaussian noise to differential equations (e.g. $dx/dt = f(x, t)$), it must be considered that the variance of the related diffusion process increases linearly with time. This implies that the prefactor of the noise term in the differential equation is not proportional to the time step Δt , but proportional to $\sqrt{\Delta t}$, which can be easily overlooked. This has to do with the particular properties of stochastic equations. (As a consequence, the discretized version of the above differential equation with noise would be $x(t + \Delta t) - x(t) = \Delta t f(x, t) + D\sqrt{\Delta t}$, where D allows to vary the noise strength.)

3.3.3 Simulating Statistical Ensembles

In case of a model containing noise, one needs to determine statistical distribution(s) (or, more simply, the averages and standard deviations), running the simulation many times with different sets of random numbers (i.e. with a differently initialized random number generator³). In other words, running a model with noise a single time is of relatively little meaning. Sometimes, it may serve to illustrate a “typical” system behavior (but then, the simulation run should not be selected by the authors; it should be randomly chosen to avoid a bias through selective presentation). In any case, a scientific paper should present the statistical distributions

²The reason for this is that deterministic systems may easily get trapped in local optima, which can be overcome by noise [119].

³Some random number generators do this automatically by coupling to the clock.

over sufficiently many (typically at least 100 simulation runs) or the mean value and variability (either one, two or three standard deviations, or quantiles, as it is done by box plots).

3.3.4 Choice of the Discretization Method

Finally, note that appropriate discretization schemes are also not trivial. Depending on how they are done, they may be quite inaccurate or inefficient. For example, the simplest possible discretization of the differential equation $dx/dt = f(x, t)$, namely $x(t + \Delta t) - x(t) = \Delta t f(x, t)$, may converge slowly. In case of so-called “stiff” systems of differential equations, the convergence may be so inefficient (due to different time scales on which different variables in the system change) that it may be completely useless. Certain discretizations may even not converge at all towards the correct solution, if Δt is not chosen extremely small. (For example, the solution procedure may face instabilities, which may be recognized by oscillatory values with growing amplitudes.) In such cases, it might be necessary to choose particular solution approaches. The situation for partial differential equations (which contain derivatives in several variables, such as space and time), is even more sophisticated. Normally, the trivial discretization does not work at all, and particular procedures such as the upwind scheme may be needed [144].

3.3.5 Performance and Scalability

Multi-agent simulations may require a considerable computational power for the following reasons:

- The dependence of the system dynamics on random fluctuations requires many simulation runs.
- Multi-agent simulations may involve a large number of agents.
- The simulation of rational behavior (i.e. systematic optimization over future developments resulting from behavioral alternatives) or of human cognitive and psychological processes (such as personality, memory, strategic decision-making, reflexivity, emotions, creativity etc.) is potentially quite resource demanding.
- The calibration of model parameters to empirical data also requires many simulation runs for various parameter combinations to determine the parameter set(s) with the minimum error.
- The parameter space needs to be scanned to determine possible system behaviors (see Sec. 3.4.5).
- Performing scenario analyses (with changed model assumptions) requires many additional simulation runs. A typical example is the method of “sensitivity analysis” to determine the robustness of a model (see Sec. 3.4.6).

- The visualization of simulation scenarios may require further substantial computer power (see Sec. 3.4.4).

For the above reasons, the performance of a multi-agent simulations can matter a lot. Unfortunately, this often makes it advantageous to write a specialized own computer program rather than using a general-purpose agent-based simulation platform. However, the performance can sometimes be increased by a factor of 100, 1000, or more by a number of measures such as

- using suitable compiler options to optimize the executable file of the computer program,
- avoiding output on a computer display (for example, by writing the numerical results into a file every 100 or 1000 time steps and visualizing the results afterwards),
- avoiding multiple entangled loops and performing loops in the right order (as is favorable for read/write operations),
- avoiding exponential functions, logarithms, and exponents, where possible,
- applying an efficient numerical integration method together with a proper time discretization (see Sec. 3.3.4),
- using appropriate parameter values (for example, dividing by small numbers often causes problems and, considering limitations of numerical accuracy, may create almost any output),
- starting with well-chosen initial conditions (e.g. an approximate analytical solution),
- considering that there are simple ways of determining certain quantities (e.g. the standard deviation can be easily determined from the sum and sum of squares of data values; a moving average can be determined by adding a new value and subtracting the last value; or an exponential average can be determined by multiplying the previous value of the average with a factor $q < 1$ and adding $(1 - q)$ times the new value).

The performance is often measured by the analysis of how the required computational time increases with the number N of agents in the system. In the ideal case, such a *scalability analysis* gives a linear (or constant) dependency on the “system size” N . In many cases, however, the computational time scales like a polynomial, or, what is even worse, like an exponential function. Hence, evaluating how a computer code scales with system size allows one to distinguish efficient implementations from inefficient ones. In computer science, the performance of algorithms is expressed by the “complexity” of an algorithm. For example, the term NP-hard refers to an algorithm which does not scale polynomially, which means that the computer time required for simulations explodes with the system size. In

such cases, only moderate system sizes are numerically tractable on a PC. Larger systems may still be treated by parallelization of the computer code and parallel processing on dozens or hundreds of processors, but NP-hard problems can be too demanding even for the biggest supercomputers. However, it is sometimes possible to reduce the complexity considerably by applying reasonable approximations. For example, the simulation of pedestrian crowds can be significantly accelerated by assuming that pedestrians do *not* interact if their distance is greater than a certain value. Moreover, many optimization problems can be approximately solved by using suitable “heuristics” [122, 145–147].

3.4 Presentation of Results

3.4.1 Reproducibility

As it is customary in other scientific areas, the result of multi-agent simulations must be presented in a way that allows other scientists to reproduce the results without having to ask the authors for details. In the ideal case, the source code underlying the computer simulations is published as supplementary information in a well documented form.

In order to be reproducible, a publication must contain all the information discussed in Sec. 3.2 (including the initial and boundary condition, kind of interaction network, and model parameters). Furthermore, it must be specified how the noise was implemented. The update rule (such as parallel or random sequential update) and the order of update steps must be provided as well as the full set of rules underlying the agent-based model. Any relevant approximations must be pointed out, and it may make sense to specify the numerical solution method and the way, in which random numbers and statistical distributions were produced.

For the sake of comfort, one should consider to provide parameter values in tables or figure captions. Moreover, besides specifying them, it is desirable to use meaningful names for each parameter and to explain the reasons, why the rules underlying the multi-agent simulation, certain initial and boundary conditions, particular network interactions, etc. were used.

3.4.2 Plausibility Considerations

As underlined before, there are quite a number of mistakes that can be made in multi-agent simulations (see Secs. 3.1.3 and 3.2). Therefore, the computer code and its single subroutines should be carefully checked. Moreover, it should be described what plausibility checks have been performed. For example, as has been pointed out in Sec. 3.1.2, the model may have exact or approximate solutions in certain limiting cases (e.g. when setting certain parameters to 0, 1 or very large values).

Furthermore, the computational results should have the right order of magnitude, and change in a plausible way over time or when the model parameters are modified. In addition, one should take into account that some variables are restricted to a certain range of values. For example, probabilities must always be

between 0 and 1. Besides, there may be so-called “constants of motion”. For example, probabilities must add up to 1 at any point in time, or the number of vehicles in a closed road network must stay the same. There may also be certain quantities, which should develop monotonously in time (e.g. certain systems have an entropy or Lyapunov function). All these features can be used to check the plausibility of simulation results. It may be enough to determine the values of such quantities every 1000 time steps.

Finally, any unexpected results must be tracked back to their origin, to make sure they are well understood. Seemingly paradoxical results must be carefully studied, and their origins and underlying mechanisms must be clearly identified and explained.

3.4.3 Error Bars, Statistical Analysis, and Significance of Results

Like empirical data, the data resulting from multi-agent simulations should be subject to statistical analysis. In particular, it is not sufficient to present single simulation runs or mean values of many simulation runs. No matter how far apart two mean values may be, this does not necessarily mean that the difference is statistically significant. The judgment of this requires a proper statistical analysis (such as a variance analysis) [148].

A minimum requirement to judge the significance of results is the presentation of error bars (and it should be stated whether they display one, two, or three standard deviations). Box plots (i.e. presenting the median, minimum and maximum value, and quantiles) are likely to give a better picture. Based on error bars or box plots, the significance of simulation results (and differences between different simulation settings) can be often visually assessed, but a thorough statistical analysis is clearly favorable.

When performing statistical analyses, it must be taken into account that the frequency distributions resulting in multi-agent simulations may not be of Gaussian type. One may find multi-modal distributions or strongly skewed, “fat-tail” distributions such as (truncated) power laws. We also point out that fitting power laws is tricky and that straight-forward fitting approaches may easily lead to wrong exponents and confidence intervals. Besides, one speaks of a power law only if there is a linear relationship in a log-log plot over at least two orders of magnitude (i.e. over a range of the horizontal axis that spans a factor 100). Deviations from power laws may be as meaningful as the power laws themselves and should be pointed out in the related discussion of results.

3.4.4 Visualization

Besides presenting tables with statistical analyses, visualization is a useful way of presenting scientific data, which is widely applied in the natural and engineering sciences. Some classical ways of representation are

- time-dependent plots,

- two- or three-dimensional plots indicating the interdependencies (or correlations) between two or three variables (which may require to project the full parameter space into two or three dimensions),
- pie charts or frequency distributions,
- representations of relative shares (percentages) and their changes over time,
- snapshots of spatial distributions or videos of their development in time, or
- illustrations of network dependencies.

In some sense, visualization is the art of transferring relevant information about complex interdependencies to the reader quickly and in an intuitive way. Today, large-scale data mining and massive computer simulations steadily create a need for new visualization techniques and approaches (see Ref. [111] for a more detailed discussion).

3.4.5 Scanning of Parameter Spaces and Phase Diagrams

An important way of studying the behavior of socio-economic systems with agent-based models is the scanning of the parameter space. As was pointed out in Sec. 1.5, systems composed of many non-linearly interacting agents are likely to produce a number of self-organized or emergent phenomena. It is, therefore, interesting to determine the conditions under which they occur. In particular, it is relevant to identify the separating lines (or hypersurfaces) in the parameter space that separate different system behaviors (“system states”) from each other, and to determine the character of the related “phase transitions”: One typically checks whether it is a continuous (“second-order”) transition, at which systemic properties start to change smoothly, or a discontinuous (“first-order”) transition, where a sudden regime shift occurs (see Fig. 2). The latter transitions occur, for example, in cases of hysteresis (“history dependence”), and their possible types have been studied by “catastrophe theory” [149] (see Fig. 2). Scientific paradigm shifts, for example, are typically first-order transitions [150], as are revolutions in societies [151, 152].

The parameter dependencies of the different kinds of system states (“phases”) and their separating lines or hypersurfaces are usually represented by “phase diagrams”. A particularly interesting case occurs, if the system displays multistability, i.e. were different stable states are possible, depending on the respective initial condition or history. For example, in traffic theory, various kinds of congested traffic states may result, depending on the size of perturbations in the traffic flow (see Fig. 3). That circumstance allows one to understand systems, which show apparently inconsistent behaviors. In fact, it is quite common for social systems that a certain characteristic system behavior is reported in one part of the world, while another one is observed elsewhere [153]. There may nevertheless be a common theory explaining both system behaviors, and history-dependence may be the reason for the different observations.

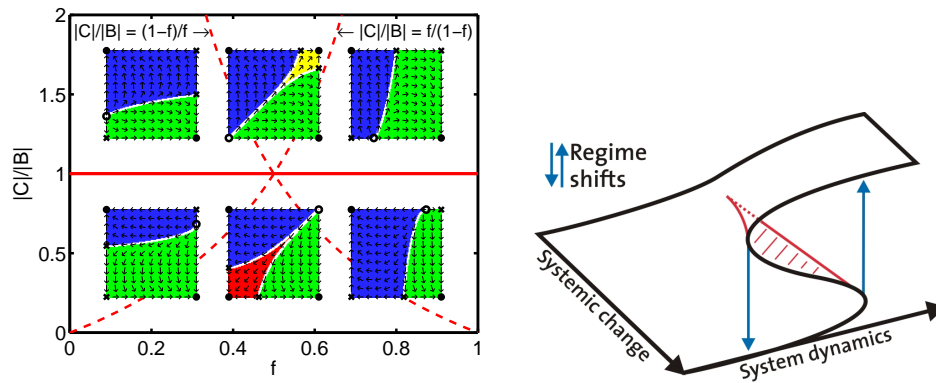


Figure 2: Left: Illustration of the parameter-dependent types of outcomes in the social norms game of two interacting populations with incompatible preferences (after Ref. [151]). f is the relative size of population 1 and $B = b - c < 0$ and $C = b + c > 0$ are model parameters, which depend on the benefit b of showing the individually preferred behavior, while c is the reward of conforming with the behavior of the respective interaction partner. Small arrows illustrate the vector field $(dp/dt, dq/dt)$ as a function of the fraction p of individuals in population 1 showing their preferred behavior (on the horizontal axis) and the corresponding fraction q in population 2 (on the vertical axis). Empty circles stand for unstable fix points (repelling neighboring trajectories), black circles represent stable fix points (attracting neighboring trajectories), and crosses represent saddle points (i.e. they are attractive in one direction and repulsive in the other). The basins of attraction of different stable fix points are represented in different shades of grey (colors) [green = population 1 sets the norm, blue = population 2 sets the norm, yellow = each population does what it prefers, red = nobody shows the preferred behavior]. The solid red lines indicates the threshold at which a continuous phase transition takes place, dashed lines indicate discontinuous phase transitions. Right: When a complex system is manipulated (e.g. by external control attempts), its system parameters, stability, and dynamics may be affected. This figure illustrates the occurrence of a so-called “cusp catastrophe”. It implies a discontinuous transition (“regime shift”) in system dynamics.

Scanning parameter spaces typically requires large computational resources. Even when using a computer pool, it may require weeks to determine a two-dimensional phase diagram at reasonable accuracy. Varying more parameters will consume even more time. However, one way of determining interesting areas of the parameter space is to use the “overlay method”, which simulates interactions in two-dimensional space, but additionally varies the parameter values in horizontal and vertical direction. In this way, one may get a quick impression of the spatio-temporal dynamics in different areas of the considered two-dimensional parameter

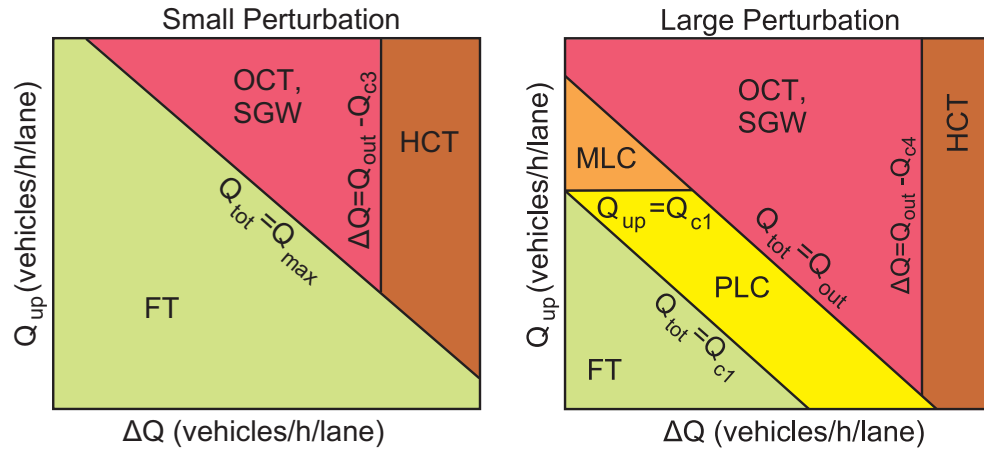


Figure 3: Schematic (idealized) phase diagrams of the traffic patterns expected for a particular traffic model as a function of the freeway traffic flow Q_{up} upstream of a ramp bottleneck and the on-ramp flow ΔQ (after Ref. [27]). The left figure is for negligible, the right figure for large perturbations in the traffic flow. The situation for medium-sized perturbations can lie anywhere between these two extremes. Different colors correspond to different possible traffic states (FT = free traffic flow, PLC = pinned localized cluster, MLC = moving localized cluster, OCT = oscillating congested traffic, SGW = stop-and-go waves, HCT = homogenous congested traffic, see Fig. 1). The equations next to the separating lines can be analytically calculated, but are not important here. For details see Ref. [27]).

space (see Fig. 4). After identifying the approximate location of separating lines between different phases (i.e. qualitatively different system behaviors), a fine-grained analysis (e.g. scaling analysis) can be made to reveal the detailed behavior next to the phase separation lines (or hypersurfaces).

3.4.6 Sensitivity Analysis

Preparing phase diagrams already provide good hints regarding the sensitivity of an agent-based model to parameter changes. Generally, within a given phase, there is not much variation of the system behavior, given the chosen parameter combinations are not too close to the lines (or hypersurfaces) separating it from other phases. However, when a transition line (or hypersurface) is crossed, significant changes in the system behavior are expected, particularly if the transition is of first order (i.e. discontinuous).

Beyond determining the phase diagram, the sensitivity can also be measured in terms of Theil's inequality coefficient [154]. It measures how different two time-dependent solutions are, when the model parameters (or initial conditions) are slightly changed. In a similar way, one may study how sensitive the model is

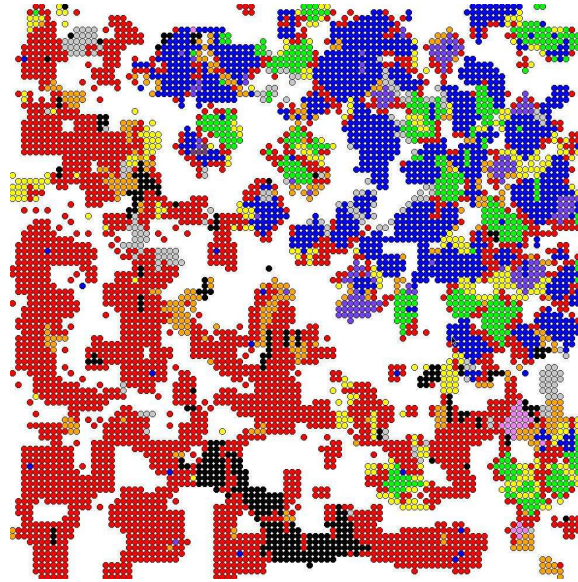


Figure 4: Overlay phase diagram for an evolutionary public goods game with success-driven migration as well as prosocial and antisocial punishment, representing different behaviors by different colors (recent work with Wenjian Yu). The prevalent behavior depends on the size of the model parameters (here: the punishment cost and fine), which is varied along the axes. One can see that people showing the same behavior tend to form clusters (a phenomenon called “homophily”). Moreover, cooperators (mainly green moralists and a few blue non-punishing cooperators) spread above a hyperbolic kind of line. Below it, defectors (red or black) flourish. The spreading of moralists above a certain punishment level gets rid of the conventional free-rider and the second-order free-rider problem. Mobility speeds up the convergence to the finally resulting strategy distribution. It also increases the green area of moralists, i.e. it pushes the hyperbolic separation line to lower punishment values. Defectors who punish non-punishers (grey) occur around the separating line. Defectors who punish defectors (yellow immoralists) occur in separation from each other (as loners). They require enough space, which they mainly find at low densities, or when mobility creates areas of low density. In the mixed phase of black and red, and in the mixed phase of blue and green, there is only a slow logarithmic coarsening, because the payoffs are the same. (This looks like a coexistence of two strategies, if the simulations are not run long enough.) The occurrence of black defectors who punish cooperators can explain the existence of antisocial punishers. Black antisocial punishers can exist basically at all punishment levels, if they can cluster together and are not in direct neighborhood to moralists.

towards the consideration of fluctuations (“noise”). A characterization can also be made by determining the Lyapunov exponents [155].

However, a multi-agent simulation may not only be sensitive to parameter changes. It may also be sensitive to minor modifications of the agent-based model itself. For example, slight changes of the interaction network (by adding or subtracting nodes or links) may impact the system behavior. Analyses of failure and attack tolerance demonstrate this very well [156]. To investigate so-called k -failures, one randomly removes k agents or links from the system and studies changes in the system performance. Similarly, one may investigate the impact of adding k links or agents. The method is capable to reveal certain kinds of “structural instabilities”. A further kind of structural instabilities may be caused by modifications in the rules determining the interactions of agents. Such modifications may reflect innovations, but also inaccuracies of the model as compared to reality. For example, “unknown unknowns” are factors overlooked in the model, but they may be discovered to a certain extent by varying the model assumptions. They may also be identified by comparing models of different researchers, focusing on their incompatible features.

3.5 Identification of the Minimum Model Ingredients

One important part of scientific analysis is the identification of the minimum set of rules required to explain certain empirical observations or “stylized facts” derived from them (i.e. simplified, idealized, characteristic features). Particularly in models with many parameters, it is sometimes difficult to understand the exact mechanism underlying a certain phenomenon. Therefore, one should attempt to successively reduce the model to a simpler one with less terms, parameters and/or variables, in order to find out under what conditions the phenomenon of interest disappears. It is clear that simplifications of the model will often reduce the level of quantitative agreement with empirical data. However, in many cases one is mainly interested in questions such as:

- Does the system have multiple stable states?
- Does the model behave in a history-dependent way?
- Does it produce an oscillatory behavior or a stable equilibrium?
- Is the statistical distribution Gaussian, bimodal, multi-modal or heavily skewed, e.g. a (truncated) power law?
- What kinds of observed patterns can the model reproduce?
- Is a linear or an equilibrium model sufficient to reproduce the observations, or does it miss out important facts?
- Are spatial (neighborhood) interactions important or not?
- Is a heterogeneity of agent properties relevant for the explanation or not?
- Does small or moderate noise have a significant influence on the system behavior or not?

- Are correlations important, or is a mean field approximation (“representative agent approach”), assuming well-mixed interactions good enough?
- How important are the degree distribution or other specifics of the the interaction network?

From statistical physics, it is known that all these factors *may* play a significant role for the system behavior [13], but this is not always the case. Therefore, the required ingredients of a model and appropriate level of sophistication very much depend on the phenomena to be described, on the purpose of the model, and the desired accuracy.

3.6 Gaining an Analytical Understanding

Besides providing a clearer understanding and intuition, simplifications also have another advantage: they can make models mathematically better tractable. In order to derive the essential phenomena in an analytical way (e.g. by means of stability or perturbation analyses), a radical simplification may be needed, and as a consequence, the resulting model will usually not reproduce empirical details, but just the stylized facts (such as the occurrence of certain kinds of instabilities, patterns, phase transitions/regime shifts, or phase diagrams with certain topological features) (see Ref. [27] for an example).

One advantage of analytical tractability is the circumstance that one can often derive parameter dependencies or scaling relationships. Frequently, such parameter dependencies are not obvious, and even a numerical analysis may not give a good picture, if several parameters are involved.

Analytical treatments often allow one to determine the location of stationary points and their stability properties. From this information, one can derive the fundamental features of the phase diagram, which gives a pretty good picture of the possible system behaviors. Therefore, the fundamental properties of a system may indeed be analytically understood. This is nicely illustrated by the example of multi-population evolutionary games [151]. Also the properties of freeway traffic flow and the possible congestion patterns have been analytically understood, despite their complexity [27] (see Fig 1).

3.7 Some Problems and Limitations of Computational Modeling

Despite all the virtues of mathematical modeling, one should not forget some possible problems. So far, it is not known what phenomena can be understood by agent-based models, and what are the fundamental limits of this approach. It is conceivable that there exist phenomena, which are irreducibly complex [11]. For example, the method of physics to reduce most observations to the behavior of individual particles and pair interactions may not be fully appropriate in socio-economic systems. Some phenomena require a more integrated treatment of the interactions between many agents. Public good games are just one example [157]. Recent models

of pedestrian interactions are also turning away from pair interaction approaches in favor of heuristics that respond to an integrated visual pattern [158]. The corresponding behaviors can still be treated by agent-based models, but one must be aware that they may have fundamental limitations as well.

Overestimating the power of models can be quite harmful for society, as the financial crisis has shown. For this reason, it is important to state known limitations of a model or, in other words, its range of validity. It should be made clear what the purpose of a particular model is, e.g. whether it serves to understand stylized facts or scientific puzzles better, or whether the model aims at predictions or real-world applications. When it comes to “predictions”, it should be said whether they are meant to be “forecasts” (in time) or “model implications” (in the sense of system states that are expected to occur when model parameters or initial conditions etc. are changed in a certain way).

In order to assess the reliability of a model, it is favorable to derive a number of predictions (or implications) of a model, which may later be verified or falsified. It is also good to point out advantages *and* disadvantages with regard to other existing models. This can give a better picture of the strengths and weaknesses of a certain approach, and guide further research aimed at developing models that can consistently explain a large set of empirical observations. It also helps to be aware of crucial modeling assumptions, on which the validity of the model implications depends. This is particularly important for models, which are practically applied.⁴

It must be underlined that specifying a model correctly is not simple. In many cases, different plausible mechanisms may exist that promise to explain the same observations [159]. For example, there are many possible explanation of power laws [160]. Moreover, if empirical data vary considerably (as it is common for socio-economic data), it may be difficult to decide empirically, which of the proposed models is the best [125, 126]. It may very well be that all known models are wrong, or that their parameter specification is wrong (as it happened in financial risk models that were calibrated with historical data). Actually, due to the implicit simplifications and approximations of most models, this is true most of the time [161], and this is why it is so important to state what a model is aimed to be useful for, and what are its limits. In other words, usually, models contain a grain of truth, but are not valid in every single respect. Consequently, a pluralistic modeling approach makes sense [11], and overlaying the implications of several models may give better results than the best available model itself (due to the “wisdom of crowds” effect or the law of large numbers).

⁴One should be aware that this may sooner or later happen to any model, if it promises to be useful to address real-world phenomena.

4 Practical Application of Agent-Based Models: Potentials and Limitations

Having discussed potential problems and limitations of computational modeling in several passages of Sec. 3.3, and in particular Sec. 3.7, we should not forget to point out, where agent-based models of socio-economic systems may actually be more powerful than one would think.

4.1 Stylized Facts and Prediction in Socio-Economic Systems

Some people think that the lack of collections of stylized facts in the socio-economic sciences may be a result of the non-existence of such facts due to the great degree of flexibility of social and economic systems. However, there *are* actually a number of stylized facts with a surprisingly large range of validity:

1. the Fisher equation of financial mathematics (which determines the relationship between nominal and real interest rates under inflation) [162],
2. the fat tail character of many financial and economic distributions [163, 164],
3. the Matthew effect (i.e. the rich-gets-richer effect) [113, 160]
4. Dunbar's number (limiting the number of people one can have stable social relations with) [109],
5. Pareto's principle (according to which roughly 80 percent of an effect comes from about 20 percent of the causes) [165],
6. Zipf's law (determining the distribution of city rank sizes and many other things) [166],
7. the gravity law (describing the distribution of trade flows and migration) [167–169],
8. Goodhart's law (according to which any observed statistical regularity, e.g. a risk model, breaks down once pressure is placed upon it for control purposes) [170, 171].

Complementary, a list of stylized facts regarding social norms can be found in Ref. [43].

Even when accepting that the above “laws” *tend* to apply,⁵ many people doubt the possibility to predict the behavior of socio-economic systems, while they believe that this is a precondition for crisis relief. However, both is not exactly true:

⁵probably, nobody would claim that they are always true

1. As pointed out before, there are many other purposes of modeling and simulation besides prediction [12]. For example, models may be used to get a picture of the robustness of a system to perturbations (i.e. applied to perform simulated “stress tests”, considering the effect of interactions among the entities constituting the system).
2. “Model predictions” should be better understood as “model implications” rather than “model forecasts”, e.g. a statements that say what systemic outcomes (e.g. cooperation, free-riding, or conflict) are expected to occur for certain (regulatory) boundary conditions [14, 54] (see Sec. 4.2.2).
3. It is important to recognize that forecasts *are* possible *sometimes*. A famous example is Moore’s law regarding the performance of computer chips (which is ultimately a matter of the innovation rate [172]). Moreover, while the detailed ups and downs of stock markets are hard to predict, the manipulation of interest rates by central banks leads, to a certain extent, to foreseeable effects. Also the sequence in which the real-estate market in the US affected the banking system, the US economy, and the world economy was quite logical. That is, even though the exact timing can often not be predicted, causality networks allow one to determine likely courses of events (see Fig. 5). In principle, this enables us to take counter-actions in order to avoid or mitigate the further spreading of the crisis [173].
4. As weather forecasts show, even unreliable short-term forecasts can be useful and of great economic value (e.g. for agriculture). Another example illustrating this is a novel self-control principle for urban traffic flows, which was recently invented [40, 174]. Although its anticipation of arriving platoons is of very short-term nature, it manages to reduce travel times, fuel consumption, and vehicle emissions.

In conclusion, prediction is limited in socio-economic systems, but more powerful than many people believe. Moreover, in certain contexts, it is not necessary to forecast the course of events. For example, in order to reduce problems resulting from bubbles in financial markets, it is not necessary to predict the time of their bursting or even to know about their concrete existence. The reason for this will be explained in Sec. 4.2.3. Furthermore, Sec. 4.2.2 shows how the often raised problem of self-destroying prophecies can be avoided.

4.2 Possibilities and Limitations in the Management of Socio-Economic Systems

4.2.1 Paradigm Shift from Controlling Systems to Managing Complexity

When trying to improve socio-economic systems, first of all, it must be stressed that the idea to *control* socio-economic systems is not only inadequate—it is also not working well. As socio-economic systems are complex systems, cause and effect

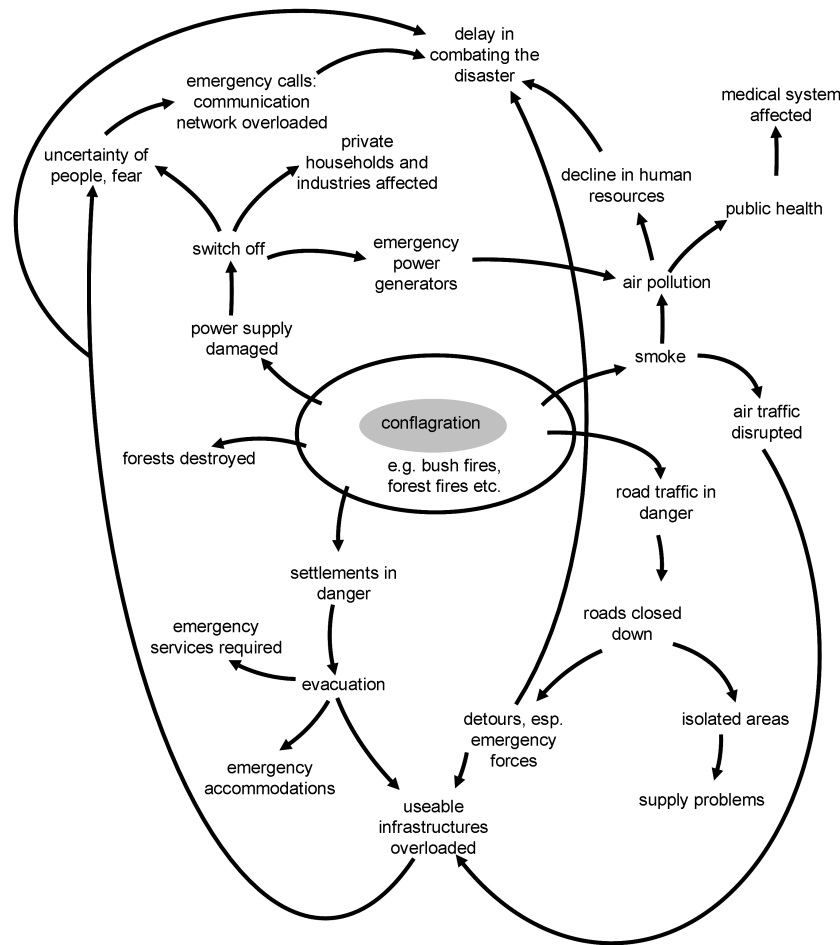


Figure 5: Illustration of cascading effects in techno-socio-economic systems triggered by forest fires (after Ref. [173]). Note that the largest damage of most disasters is caused by cascading effects, i.e. the systemic impact of an over-critical local perturbation.

are usually not proportional to each other. In many cases, complex systems tend to resist manipulation attempts (cf. “Goodhart’s law”), while close to so-called “tipping points” (or “critical points”), unexpected “regime shifts” (“phase transitions”, “catastrophes”) may happen. Consequently, complex systems cannot be controlled like a technical system (such as a car) [29].

The above property of systemic resistance is actually a result of the fact that complex systems often self-organize, and that their behavior is robust to not-too-large perturbations. While forcing complex systems tends to be expensive (if systemic resistance is strong) or dangerous (if an unexpected systemic shift is caused), the alternative to support the self-organization of the system appears to be promis-

ing. Such an approach “goes with the flow” (using the natural tendencies in the system) and is resource-efficient. Therefore, a reasonable way to manage complexity is to guide self-organization or facilitating coordination [29, 175].

In a certain sense, this self-organization or self-control approach moves away from classical regulation to mechanism design [176]. Regulation often corresponds to changing the boundary conditions, while mechanism design changes the interactions in the system in a way that reduces instabilities (e.g. due to delays) and avoids that the system is trapped in local optima (and continues its evolution to a system-optimal state). For example, slightly modifying the interaction of cars by special driver assistant systems can stabilize traffic flows and avoid bottleneck effects to a certain degree [177].

4.2.2 Self-Destroying Prophecies and Proper Design of Information Systems

It is often pointed out that socio-economic systems would not be predictable, because the reaction of people to information about the system would destroy the validity of the forecast. If done in the wrong way, this is actually true. Let us illustrate this by an example: Assume that all car drivers are given the same information about existing traffic jams. Then, drivers may most likely over-react, i.e. more drivers may use an alternative road than is required to reach a system-optimal distribution of traffic flows [178].

However, as has been shown by laboratory route choice experiments, an almost optimal route choice behavior may be reached by an information system that gives user-specific recommendations. In other words, some drivers would be asked to stay on the congested road, and others to leave it. When the recommendation system compensates for the fact that not everyone follows the recommendations, one can avoid over- or under-reactions of drivers in congestion scenarios [178].

One crucial issue of such individualized recommender systems, however, is their reliability. An unreliable system or one that is systematically biased, will be only poorly followed, and people will eventually compensate for biases [178]. Therefore, it is also essential to design the information system in a way that is fair to everyone. That is, nobody should have a *systematic* advantage. Nevertheless, the system should be flexible enough to allow a trading of temporary (dis)advantages. For example, somebody who was asked to take the slower road on a given day (and who would have a right to use the faster road on another day), may still use the faster road. However, he or she would have to pay a fee for this, which would be earned by somebody else, who would exchange his or her “ticket” for the faster road for a “ticket” for the slower road. In such a way, the system optimum state could still be maintained [178].

In summary, the above described information system would cheat nobody, and it would be flexible and fair. Only people who use the faster road more often than average would have to pay a road usage fee. A normal driver would either pay nothing on average: While he or she will pay on some days (when being under a pressure of time, while the recommendation asks to take the slower road), the same

amount of money can be earned on other days by taking the slower road. In other words, fair usage would be free of charge on average, and drivers would still have a freedom of route choice. The primary goal of the system would not be to suppress traffic flows through road pricing, but the pricing scheme would serve to reach a system optimal traffic state.

4.2.3 New Approaches and Designs to Manage Complexity

It is quite obvious that there is no single scheme, which allows one to manage *all* kinds of complex systems optimally, independently of their nature. The success of a management concept very much depends on the characteristics of the system, e.g. its degree of predictability. The systems design must account for this.

If *long-term forecasts* are possible, there must obviously be an almost deterministic relationship between input- and output-variables, which allows one to change the temporal development and final outcome of the system. If long-term predictability is *not* given, management attempts must be oriented at a sufficiently frequent re-adjustment, which requires a suitable monitoring of the system.

As the example of weather forecasts shows, even unreliable *short-term forecasts* can be very useful and economically relevant (e.g. for agriculture).⁶ The success principle in case of short-term forecasts is the flexible adjustment to the local conditions, while well predictable systems often perform well with a fixed (rather than variable) organization principle.

In systems where *no forecasts* over time are possible at all, it may still be feasible to improve the system behavior by modifying the statistical distribution. Taking the time-dependencies at stock markets for illustration, introducing a Tobin tax would reduce excessive levels of speculation (e.g. high-frequency trading). Moreover, introducing “noise” (further sources of unpredictability), could destroy undesirable correlations and impede insider trading [179].

These are just a few examples illustrating that there actually *are* possibilities to influence systems involving human behavior in a favorable way. A more detailed discussion of the issue of managing complexity is given in Refs. [29, 180–183].

4.2.4 Changing the Rules of the Game and Integrative Systems Design

Generally, there are two ways of influencing the dynamics and outcome of a system by changing the “rules of the game”. If the interaction in the system are weak, the system dynamics can be well influenced by modifying the boundary conditions of the system (i.e. by regulatory measures). However, if the interactions are strong, as in many social and economic processes, the self-organization of the system dominates the external influence. In this case, a modification of interactions in the system (“mechanism design”) seems to be more promising. (A good example for this is a traffic assistance system that reduces the likelihood of congestion

⁶Another example is the “self-control” of urban traffic flows, which is based on a special, traffic-responsive kind of decentralized traffic light control [40], see Sec. 4.2.1

by special driver assistance systems [177].) Of course, regulatory measures and mechanism design may also be combined with each other.

While mechanism design is relatively common in computer science and some other areas (e.g. in evolutionary game theory, mathematics, and partly in physics), it seems that these methods have not been extensively applied in the social sciences so far. For example, there are many different mechanisms to match supply and demand, and it would be interesting to know what systemic properties they imply (such as the level of stability, the efficiency of markets, the resulting wealth distribution, the creation of investment opportunities, etc.). It is also not clear how to reach the best combination of top-down and bottom-up elements in decision-making processes, and how to find the best balance between centralized and decentralized coordination approaches. All this poses interesting and practically relevant challenges that determine the prosperity and well-being of societies (see also Ref. [111]).

Moreover, in the past, mechanism design has been applied basically to subsystems, i.e. parts of the complex overall system we are living in. However, due to the interconnection of all sorts of (sub-)systems (e.g. of the traffic, supply, industrial, environmental, health and social systems), measures in one (sub)system may have undesired side effects on other (sub)systems. In fact, for fundamental reasons, it is quite frequent that taking the best action in one (sub)system affects another (sub)system in a negative way. Such partial improvements will usually not promote an optimal state of society (whatever the optimal state of society may be). A good example for this is the poor performance of traffic light controls that optimize locally (without a coordination with neighboring intersections) [177]. Unfortunately, such kinds of problems are not at all restricted to traffic systems. Undesirable feedback effects (like spill-over effects and mutual obstructions) are quite common for many networked systems, such as logistic or production systems, or even administrative processes [174].

4.2.5 Choice of the Goal Function

Improving a socio-economic system is far from trivial. Even if one would have a perfectly realistic and predictive model, the result may largely depend on the chosen goal function. An improper choice of the goal function can cause more harm than benefit. For example, maximizing the efficiency of a system may make it vulnerable to breakdowns [30]. Besides, there is a danger of misusing models to promote individual interests that are not compatible with human well-being. However, the following goals appear to be widely acceptable:

- increase the self-awareness of society,
- reduce vulnerability and risk,
- increase resilience (the ability to absorb societal, economic, or environmental shocks),

- avoid loss of control (sudden, large and unexpected systemic shifts),
- develop contingency plans,
- explore options for future challenges and opportunities,
- increase sustainability,
- facilitate flexible adaptation,
- promote fairness,
- increase social capital and the happiness of people,
- support social, economic and political inclusion and participation,
- balance between central and decentral (global and local) control,
- protect privacy and other human rights, pluralism and socio-bio-diversity,
- support collaborative forms of competition and vice versa (“coopetition”),
- promote human well-being.

If several goals shall be promoted at the same time, the question arises how to perform such multi-goal optimization. Most optimization methods used today eliminate heterogeneity in the system, i.e. there is one optimal solution which is applied to everyone. For socio-economic systems, this appears to be particularly problematic, as it tends to reduce socio-diversity and innovation. Besides, it is promoting average performance rather than individual strengths. A way to overcome this problem is suggested in the following.

The crucial question in this connection is, how to translate the performance values X_{ij} of the alternative systems i (measured on multiple scales j by goal functions g_j) into one scale. Traditionally, this is done by weighting each criterion j or goal function with a certain factor w_j . This results in the overall individual performance

$$x_i = \sum_j w_j x_{ij}, \quad (1)$$

where $x_{ij} = X_{ij}/\langle X_{ij} \rangle_i$ is the value X_{ij} , scaled by the average performance $\langle X_{ij} \rangle_i$ of all alternatives i . The overall individual performance values x_i can be ordered on a one-dimensional scale, i.e. ranked. Such an approach, however, promotes average performance rather than excellence, since excellence is typically characterized by extreme values on one or a few rating scales, but not on all of them.

In order to reward individual strengths of alternative system designs, one may proceed as follows: Political decision-makers could choose the weight they would like to attribute to each criterion or goal function, say $w_1 = 0.35$, $w_2 = 0.25$, $w_3 =$

0.25, and $w_4 = 0.15$ (assuming only 4 relevant goals in this example). An index that is favorable with respect to individual strengths, would for example be

$$y_i = \sum_j w_j x_{ij} + 0.1(y_{i1} + y_{i2} - y_{i3} - y_{i4}), \quad (2)$$

where the values y_{ij} correspond to the values x_{ij} , sorted according to their size in descending order. This formula overweights the particular strengths of each individual system i , and it is possible that different alternative systems perform equally well. Putting this into the context of the European Union for illustration, each country could choose a systemic design which fits the respective national strengths best. Hence, the “pluralistic” goal function (2) overcomes a number of problems of the optimization methods that are predominantly used today (namely, one-dimensional ranking scales, which measure the performance in an individually non-differentiated way, which typically creates one winner and many losers).

5 Summary, Discussion, and Outlook

In this contribution, we have presented an overview, how to do agent-based modeling (ABM) and multi-agent simulations (MAS) properly and how to avoid a number of traps associated with this research approach. In particular, we have discussed the potentials, limitations, and possible problems. Multi-agent simulations can be used for hypothesis testing and to get a better understanding of complex systems. They are flexible and allow one to reflect many characteristic features of technosocio-economic-environmental systems in a natural way (including heterogeneity and network interactions). As a result, one can expect insights into the different possible “states” or behaviors of a system and the preconditions for their occurrence. In particular, phase diagrams facilitate the representation of different characteristic phases and of transitions between them. Considering the possibility of multi-stability and history-dependence, phase diagram are also a promising approach to make sense of seemingly inconsistent empirical evidence.

We have underlined how essential it is to proceed carefully when modeling and simulating socio-economic systems. In particular, a publication should clearly describe

- the research question (challenge, “puzzle”, “mystery”) addressed, including the purpose of the model,
- the research methodology/approach used,
- the assumptions underlying the agent-based model,
- the current empirical or experimental evidence,
- implications or predictions that allow others to assess the explanatory power (e.g. through lab or Web experiments), as well as

- the expected range of validity and limitations of the approach.

Agent-based models can be published in a number of journals (searching for “agent-based model” or “multi-agent simulation” at <http://scholar.google.com> will give a good overview). However, studies presenting multi-agent simulations are currently still hard to publish in mainstream economic or social science journals, possibly because most authors do not back up their computational results with analytical ones. Proceeding more carefully when performing multi-agent simulations, as suggested in this contribution, will most likely increase the interest in this approach over time, particularly as it can produce significant results beyond the range of phenomena that are understandable through analytical methods. Finally, multi-agent simulations can also be useful to identify interesting experimental setups [142], considering in particular that experiments are costly and restricted to a small number of conditions and repetitions.

5.1 Future Prospects and Paradigm Shifts

Great prospects for agent-based modeling do not only result from the experience gained with multi-agent simulations, the availability of user-friendly simulation platforms, greater computer power, and improved visualization techniques. We also expect a number of paradigm shifts:

- The social sciences are currently experiencing a transition from a data-poor to a data-rich situation. This allows one to verify or falsify models, calibrate their parameters, and to move to data-driven modeling approaches [1, 158, 184]. Moreover, it will be possible to improve the level of detail, accuracy, and scale of agent-based models by orders of magnitude. At the same time, thanks to the availability of user-friendly simulation tools, the development times for multi-agent simulations will shrink dramatically.
- The application of methods from statistical physics and the theory of complex systems to socio-economic data generates a chance of moving beyond descriptive (fit) models towards explanatory models. The improved data situation supports the comprehension of the inner relationships and significant patterns of complex systems.
- New possibilities to mine real-time data (e.g. text mining of news, blogs, twitter feeds, etc.) create the opportunity to move from measurements with a delay (such as the classical ways of determining the gross national product or the number of people, who have got the flu) towards reliable real-time estimates (“nowcasting”) [185, 186]. Furthermore, using particular properties of spreading processes in networks, it seems even possible to achieve two-week forecasts (based on the method of “health sensors”) [187]. More generally, “reality mining” will facilitate multi-agent simulations of realistic scenarios, the determination of model parameters (and other relevant model inputs) on the fly, and the timely determination of advance warning signs. It

will also help to avoid destabilizing delays and to increase the efficiency of crisis response measures. (Delays are a major problem in the efficiency of disaster response management and mitigation [188].)

- Multi-agent simulations will integrate measurement-based data-mining and model-based simulation approaches. This approach goes beyond feeding in real-time inputs (such as initial and boundary conditions, parameters, and network characteristics) into multi-agent simulations: it performs a data-driven pattern recognition and modeling in parallel to related computer simulations and, thereby, combines the strengths of both methods to reach the optimum accuracy and predictability. For example, integrating two incomplete sets of traffic information (cross-sectional measurements and floating car data) and a fluid-dynamic real-time traffic simulation facilitates to reduce the delay times between the formation and detection of traffic jams by 50%, and to double the reliability of such traffic information.
- It will be possible to move from a batch processing of alternative simulation scenarios to interactive real-time specifications and scenario analyses. This will facilitate to explore policy options and “parallel worlds” (i.e. possible futures), as the situation evolves and pressing decisions must be taken. For example, evacuation scenarios in response to certain disasters have to be developed and evaluated quickly. More generally, interactive supercomputing would facilitate more flexible contingency plans that are tailored to the actual situation of crisis.
- Multi-agent simulations could be directly coupled with lab and web experiments. In fact, the decisions of agents in computer simulations could be taken by real people. Serious multi-player online-games provide the opportunity of involving a large number of people into the analysis of complex data and the exploration of realistic decision-making scenarios in virtual worlds, which realistically map possible future worlds. In this way, agent-based simulation approaches may be applied for crowd sourcing and eGovernance applications, to make use of the “wisdom of crowds”. For example, one could populate the three-dimensional virtual model of a new shopping center, railway station, or airport in order to find out, how well the architectures would fulfil its function, and to determine which design is favored by the future users.
- In the medium-term future, one can expect a confluence of real and virtual worlds. For example, Google Earth and similar virtual representations of the real world could be populated with simulated people or real ones. In fact, people agreeing to share their GPS coordinates could be represented in these worlds directly, to the level of detail they like. An augmented reality approach would allow people to share information about their interests, backgrounds, values, etc. The amount of information shared may be decided interactively or by the kinds of interaction partners (e.g. people are expected to

share private information more openly with people they consider to be their friends). Such augmented reality tools will be able to serve as “translator” or “adaptor” for people with different languages or cultural backgrounds, helping them to make themselves understandable to each other. The resulting techno-social systems would also offer many new opportunities for social and economic participation, both in the virtual and in the real world.

Given this development, we envision a new way of performing socio-economic research, which may be called “Social Supercomputing”. This approach would facilitate the integration of different kinds of data (e.g. demographic, socio-economic, and geographic data) and different kinds of simulation approaches (e.g. agent-based and equation-based ones), at an unprecedented scale and level of detail. It would enable simulations and interactivity on all scales. In fact, the computer simulation of techno-socio-economic systems on a global scale is a vision that appears to become feasible within the next 10 to 15 years, with unprecedented opportunities for societies and economies, if done in the right way. Some of the challenges for computer scientists and other researchers on the way are described in the Visioneer White papers (particularly Secs. 3.3 and 3.4.2 of Ref. [189]).

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