higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1040**(E)**(J29)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

29 July 2019 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Keep subsections of questions together.
5. Round off ALL calculations to THREE decimal places.
6. Use the correct symbols and units.
7. Start each NEW question on a new page.
8. Write neatly and legibly.

QUESTION 1

1.1 Given: $z = x^2 + 2xy + y^2$

Prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$ (3)

1.2 $Given: x = \sqrt{t}\ ; y = \frac{1}{\sqrt{t}}$

$Determine\ \frac{d^2y}{dx^2}\ in\ terms\ of\ t$

(3)
[6]

QUESTION 2

Determine $\int y dx$ if:

2.1 $y = x^2 e^{3x}$ (3)

2.2 $y = \cos^5 \frac{x}{5}$ (4)

2.3 $y = \tan^3 x \sec x$ (3)

2.4 $y = \frac{1}{9 - 4x - x^2}$ (4)

2.5 $y = \frac{1}{ab}\tan^{-1}\frac{bx}{a}$ (4)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{8x^2 - 2x + 3}{2x^3 - 2x^2 + x - 1}dx$

3.2 $\int \frac{(3x+2)(2x-3)}{(3x+2)^2 - (2x-3)^2}dx$

(2 × 6) **[12]**

QUESTION 4

Determine the general solution of each of the following:

4.1 $\frac{dy}{dx} = \tan x - y \cot x$

4.2 $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + 2y = 2e^{2x}$

(2 × 6) **[12]**

QUESTION 5

5.1 5.1.1 Draw the graphs of $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \pi$. Show the area bounded by the curves from $x = \frac{\pi}{2}$ to $x = \pi$. Show the representative strip that you will use to calculate the bounded area. (3)

5.1.2 Calculate the area described in QUESTION 5.1.1. (4)

5.1.3 Calculate the x-coordinate of the centroid of the area described in QUESTION 5.1.1. (6)

5.2 5.2.1 Draw the graph of $x^2 + y^2 = 25$. Show the area in the first quadrant of the graph between the x-axis and the y-axis. Show the representative strip that you will use to calculate the volume of the solid generated when this area rotates about the x-axis. (2)

5.2.2 Calculate the volume generated when the area described in QUESTION 5.2.1 rotates about the x-axis. (3)

5.2.3 Calculate the x-coordinate of the centre of gravity of the solid generated when the area in QUESTION 5.2.1 rotates about the x-axis. (4)

5.3 5.3.1 Neatly draw the graph of $y = -x^2 + 3x - 2$. Show the area bounded by the graph and the x-axis. Show the representative strip that you will use to calculate the area. (2)

5.3.2 Calculate the area described in QUESTION 5.3.1. (3)

5.3.3 Calculate the second moment of area about the y-axis of the area in QUESTION 5.3.1 and express the answer in terms of area. (4)

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5.4 5.4.1 A semicircular plate with a radius of 5 m is immersed in water with its wider end lying at the water level.

Draw a neat sketch of the plate and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the plate. (2)

5.4.2 Calculate the area moment of the plate about the water level. (6)

5.4.3 Calculate the depth of the centre of pressure on the plate if the second moment of area of the plate about the water level is given as 245,437 m^4 (1)

[40]

QUESTION 6

6.1 Calculate the length of the curve $2y = x^2$ between $(2;2)$ and $(4;8)$. (6)

6.2 Calculate the surface area generated, when the curve $x = \frac{1}{9}y^2$ from $y = 0$ to $y = 6$ rotates about the x-axis. (6)

[12]

TOTAL: 100

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MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

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$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)dx$
x^{n}	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^{n}	$a\, \frac{d}{dx}\, x^{n}$	$a \int x^{n}\, dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}\,(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}\,(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}\,ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\,\frac{d}{dx}\,f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}\,f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}\,f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a\sec^{2} ax$	$\frac{1}{a}\ln\,[\sec(ax)] + C$
$\cot ax$	$-a\,\mathrm{cosec}^{2}\, ax$	$\frac{1}{a}\ln\,[\sin(ax)] + C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln\,[\sec ax + \tan ax] + C$
$\mathrm{cosec}\, ax$	$-a\,\mathrm{cosec}\, ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right] + C$

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$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\ .\ f'(x)$	-
$\cos f(x)$	$-\sin f(x)\ .\ f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\ .\ f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x)\ .\ f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x)\ .\ f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x)\ .\ f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)\ dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\ g'(x)\ dx = f(x)\ g(x) - \int f'(x)\ g(x)\ dx$

$\int [f(x)]^n\ f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\ dx = \ln\ f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\ dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) + C$

$\int \sqrt{x^2 \pm b^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$

Applications of integration

AREAS

$A_x = \int_a^b y dx\ ;\ A_x = \int_a^b (y_1 - y_2)\ dx$

$A_y = \int_a^b x dy\ ;\ A_y = \int_a^b (x_1 - x_2)\ dy$

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VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xy dy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \qquad ; \qquad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

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GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\ (cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

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$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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