higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

29 MARCH 2019

This marking guideline consists of 1 pages.

The paper is marked out of 200 and divided by 2 to get a mark out of 100.

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DATE: 30 MARCH 2019

QUESTION 1

1.1 $z = \tan(xy)$

$$\frac{\partial z}{\partial x} = \sec^2(xy).y \quad ✓✓$$

$$\frac{\partial^2 z}{\partial x^2} = y.2\sec(xy).\sec(xy)\tan(xy).y \quad ✓✓✓✓$$

(6)

1.2

$$r = \sqrt{x^2 + y^2} \quad ✓$$

$$\Delta r = \frac{\partial r}{\partial x}\Delta x + \frac{\partial r}{\partial y}\Delta y \quad ✓$$

$$= \frac{x}{\sqrt{x^2 + y^2}}\Delta x + \frac{y}{\sqrt{x^2 + y^2}}\Delta y \quad ✓✓$$

$$= \frac{20}{\sqrt{20^2 + 10^2}}.2 + \frac{10}{\sqrt{20^2 + 10^2}}(-2) \quad ✓$$

$$= \frac{40 - 20}{\sqrt{20^2 + 10^2}} \quad ✓$$

$$= 0{,}894\,m$$

(6)

[12]

QUESTION 2

2.1 $\int ydx$

$=\int 1-\tan^4 3xdx$

$=\int(1-\tan^2 3x)\left(1+\tan^2 3x\right)dx$ ✓✓

$=\int(1-\tan^2 3x)\left(\sec^2 3x\right)dx$ ✓

$=\int(\sec^2 3x-\tan^2 3x\sec^2 3xdx$ ✓✓

$=\frac{1}{3}\tan 3x-\frac{1}{3}\frac{\tan^3 3x}{3}+c$ ✓✓✓

Or using $u=\tan 3x$

Alternative 1

$\int ydx$

$=\int 1-\tan^4 3xdx$

$\int(1-\tan^2 3x.\tan^2 3xdx$ ✓

$=\int(1-\tan^2 3x\left(\sec^2 3x-1\right)dx$ ✓

$= =\int(1-\tan^2 3x\sec^2 3x+\tan^2 3xdx$ ✓✓

$=x-\frac{1}{3}\frac{\tan^3 3x}{3}+\frac{1}{3}\tan 3x-x+c$ ✓✓✓

$=\frac{1}{3}\frac{\tan^3 3x}{3}+\frac{1}{3}\tan 3x+c$ ✓

Alternative 2

$\int 1-\frac{\sin^4 3x}{\cos^4 3x}dx$ ✓

$\int\frac{\cos^4 3x-\sin^4 3x}{\cos^4 3x}dx$ ✓

$=\int\frac{(\cos^2 3x+\sin^2 3x)(\cos^2 3x-\sin^2 3x)}{\cos^4 3x}dx$ ✓

$=\int\frac{\cos^2 3x-\sin^2 3x}{\cos^4 3x}dx$ ✓

$=\int\frac{\cos^2 3x}{\cos^4 3x}-\frac{\sin^2 3x}{\cos^4 3x}dx$ ✓

$=\int\sec^2 3x-\tan^2 3x\sec^2 3xdx$ ✓

$=\frac{1}{3}\tan 3x-\frac{1}{9}\tan^3 3x+c$ ✓✓

(8)

2.2 $\int x(\ln x)^2 dx$

$$= \frac{x^2}{2}(\ln x)^2 - \int 2\ln x . \frac{1}{x} . \frac{x^2}{2} dx \quad \checkmark\checkmark\checkmark$$

$$= \frac{x^2}{2}(\ln x)^2 - \int x \ln x dx \quad \checkmark$$

$$= \frac{x^2}{2}(\ln x)^2 - \left[\frac{x^2}{2}\ln x - \int \frac{1}{x} . \frac{x^2}{2} dx\right] \quad \checkmark\checkmark$$

$$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{2}\int x dx \quad \checkmark$$

$$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{2} . \frac{x^2}{2} + c \quad \checkmark$$

(8)

2.3 $\int (1-2\sin^2 2x)^2\,dx$

$\int (1-2\sin^2 2x)(1-2\sin^2 2x)dx$ ✓

$\int (1-4\sin^2 2x+4\sin^4 2x)dx$

$\int 1dx-4\int \sin^2 2xdx+4\int \sin^2 2x.\sin^2 2xdx$

$x-4\left(\frac{x}{2}-\frac{\sin 4x}{8}\right)+\int\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)dx$ ✓✓✓✓

$x-4\left(\frac{x}{2}-\frac{\sin 4x}{8}\right)+4\int \frac{1}{4}(1-2\cos 4x+\cos^2 4x)dx$ ✓

$x-2x+\frac{1}{2}\sin 4x+\int (1-2\cos 4x+\cos^2 4x)dx$

$x-2x+\frac{1}{2}\sin 4x+x-2\frac{\sin 4x}{4}+\frac{x}{2}+\frac{\sin 8x}{16}+C$ ✓✓

$\frac{x}{2}+\frac{\sin 8x}{16}+C$

OR

$\int (1-2\sin^2 2x)^2\,dx$

$\int\left[1-2\left(\frac{1}{2}-\frac{1}{2}\cos 4x\right)\right]^2 dx$ ✓✓

$\int (1-1+\cos 4x)^2\,dx$ ✓

$\int \cos^2 4xdx$ ✓

$\int\left(\frac{1}{2}+\frac{1}{2}\sin 8x\right)dx$ ✓

$\frac{1}{2}x+\frac{\cos 8x}{16}+c$ ✓✓✓

(8)

2.4

$$\boxed{f(x)=e^{-2x} \quad g'(x)=\sin 2x}$$

$$\int \frac{\sin 2x}{e^{2x}}dx$$

$$=\int e^{-2x}\sin 2x dx \quad \checkmark$$

$$=e^{-2x}.-\frac{\cos 2x}{2}-\int -2e^{-2x}.-\frac{\cos 2x}{2}dx \quad \checkmark\checkmark$$

$$=-\frac{1}{2}e^{-2x}\cos 2x-\int e^{-2x}\cos 2x dx$$

$$=-\frac{1}{2}e^{-2x}\cos 2x-\left[e^{-2x}.\frac{\sin 2x}{2}-\int -2e^{-2x}.\frac{\sin 2x}{2}dx\right] \quad \checkmark\checkmark$$

$$=-\frac{1}{2}e^{-2x}\cos 2x-\frac{1}{2}e^{-2x}.\sin 2x-\int e^{-2x}\sin 2x dx$$

$$2\int e^{-2x}\sin 2x dx=-\frac{1}{2}e^{-2x}\cos 2x-\frac{1}{2}e^{-2x}.\sin 2x \quad \checkmark$$

$$\int e^{-2x}\sin 2x dx=-\frac{1}{4}e^{-2x}\cos 2x-\frac{1}{4}e^{-2x}.\sin 2x+c \quad \checkmark\checkmark$$

Alternative

$$\boxed{f(x)=\sin 2x \qquad g'(x)=e^{-2x}}$$

$$\int e^{-2x}\sin 2x dx \quad \checkmark$$

$$=\sin 2x\frac{e^{-2x}}{-2}-\int 2\cos 2x\frac{e^{-2x}}{-2}dx \quad \checkmark\checkmark$$

$$=-\frac{1}{2}\sin 2x e^{-2x}+\int e^{-2x}\cos 2x\, dx$$

$$=-\frac{1}{2}\sin 2x e^{-2x}+\left[\cos 2x\frac{e^{-2x}}{-2}-\int -2\sin 2x\frac{e^{-2x}}{-2}dx\right] \quad \checkmark\checkmark$$

$$=-\frac{1}{2}\sin 2x e^{-2x}-\frac{1}{2}e^{-2x}\cos 2x-\int e^{-2x}\sin 2x\, dx$$

$$2\int e^{-2x}\sin 2x\, dx=-\frac{1}{2}\sin 2x e^{-2x}-\frac{1}{2}e^{-2x}\cos 2x+c \quad \checkmark$$

$$\int e^{-2x}\sin 2x\, dx=-\frac{1}{4}\sin 2x e^{-2x}-\frac{1}{4}e^{-2x}\cos 2x+c \quad \checkmark\checkmark$$

(8)

2.5

$$\int \frac{x^2-2}{x^4-4}dx$$

$$= \int \frac{x^2-2}{(x^2+2)(x^2-2)}dx \quad \checkmark$$

$$= \int \frac{1}{2+x^2}dx \quad \checkmark$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+c \quad \checkmark\checkmark$$

(4)

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QUESTION 3

3.1

$$\int \frac{x^2+x-7}{x^2+x-6}dx$$

$$\int \frac{(x^2+x-6)-1}{x^2+x-6}dx \quad \checkmark\checkmark$$

$$\int 1-\frac{1}{x^2+x-6}dx \quad \checkmark \qquad \checkmark$$

Or using long division

$$\frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)} = \frac{A}{(x+3)}+\frac{B}{(x-2)}$$

$$1 = A(x-2)+B(x+3) \quad \checkmark$$

$$x=2 \quad B=\frac{1}{5} \quad \checkmark$$

$$x=-3 \quad A=-\frac{1}{5} \quad \checkmark$$

$$1 = A(x-2)+B(x+3)$$
$$1 = Ax-2A+Bx+3B$$
$$A+B=0 \text{..........}(1) \Rightarrow B=-A$$
$$-2A+3B=1 \text{.........}(2)$$
$$-2A+3(-A)=1$$
$$-5A=1 \Rightarrow A=-\frac{1}{5} \quad and \quad B=\frac{1}{5}$$

$$\int 1-\frac{1}{x^2+x-6}dx = \int 1-\left[\frac{-\frac{1}{5}}{(x+3)}+\frac{\frac{1}{5}}{(x-2)}\right]dx \quad \checkmark\checkmark$$

$$= x+\frac{1}{5}\ln\ln(x+3)-\frac{1}{5}\ln(x-2)+c \quad \checkmark\checkmark\checkmark$$

$$= x+\frac{1}{5}\{(\ln(x+3)-\ln(x-2)\}+c$$

$$= x+\frac{1}{5}\ln\frac{x+3}{x-2}+c$$

(12)

3.2 $$\int \frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)}dx$$

$$\frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)}=\frac{A}{2x-1}+\frac{Bx+C}{x^2-2x+2} \quad \checkmark\checkmark$$

$$7x^2-12x+8=A(x^2-2x+2)+(Bx+C)(2x-1) \quad \checkmark$$

$$x=\frac{1}{2} \quad 7(\frac{1}{2})^2-12(\frac{1}{2})+8= A\{(\frac{1}{2})^2-2(\frac{1}{2})+2\} \Rightarrow \checkmark \quad \boxed{A=3} \quad \checkmark$$

$$7x^2-12x+8=Ax^2-2Ax+2A+2Bx^2+2Cx-Bx-C \quad \checkmark$$

$$A+2B=7 \qquad 3+2B=7 \qquad \boxed{B=2} \quad \checkmark$$

$$-2A+2C-B=-12$$

$$-2(3)+2C-2=-12 \qquad \boxed{C=-2} \quad \checkmark$$

$$\int \frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)}dx$$

$$=\int \frac{3}{2x-1}+\frac{2x-2}{x^2-2x+2}dx \quad \checkmark$$

$$=\frac{3}{2}\ln(2x-1)+\ln(x^2-2x+2)+c \quad \checkmark\checkmark\checkmark$$

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QUESTION 4

4.1 $\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y=3x-\sin x+\cos x$

$\frac{dy}{dx}-\frac{1}{x}y=x(3x-\sin x+\cos x)$ ✓

$e^{\int pdx}=e^{\int -\frac{1}{x}dx}$ ✓

$=e^{-\ln x}$ ✓

$=e^{\ln x^{-1}}$

$$\boxed{e^{-\ln x}=\frac{1}{e^{\ln x}}=\frac{1}{x}}$$

$=x^{-1} \quad =\frac{1}{x}$ ✓

$\int Qe^{\int pdx}dx=\int x(3x-\sin x+\cos x)\frac{1}{x}dx$ ✓

$=\int 3x-\sin x+\cos x)dx$ ✓

$=\frac{3}{2}x^2+\cos x+\sin x$ ✓✓

$\frac{y}{x}=\frac{3}{2}x^2+\cos x+\sin x+c$ ✓

$x=1;\ y=1 \quad \frac{1}{1}=\frac{3}{2}(1)^2+\cos(1)+\sin(1)+c$ ✓

$c=-1{,}882$ ✓

$\frac{3}{2}x^2+\cos x+\sin x-1{,}882$ ✓

$\frac{y}{x}=\frac{3}{2}x^2+cosx+sinx-1.882$ (12)

4.2

$$6\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=x^2$$

$$6r^2-r-1=0$$

$$(3r+1)(2r-1)=0 \quad ✓$$

$$r=-\frac{1}{3} \quad r=\frac{1}{2} \quad ✓$$

$$y_c=Ae^{-\frac{1}{3}x}+Be^{\frac{1}{2}x} \quad ✓$$

$$y=Cx^2+Dx+E \quad ✓$$

$$\frac{dy}{dx}=2Cx+D \qquad \frac{d^2y}{dx^2}=2C \quad ✓✓$$

$$6(2C)-(2Cx+D)-(Cx^2+Dx+E)=x^2 \quad ✓$$

$$12C-2Cx-2D-Cx^2-Dx-E=x^2$$

$$x^2\ :\ -C=1 \quad \therefore C=-1 \quad ✓$$

$$x:\ -2C-2D=0 \quad \therefore D=2 \quad ✓$$

$$12C-D-E=0 \quad \therefore E=-14 \quad ✓$$

$$y_p=-x^2+2x-14 \quad ✓$$

$$y=y_c+y_p$$

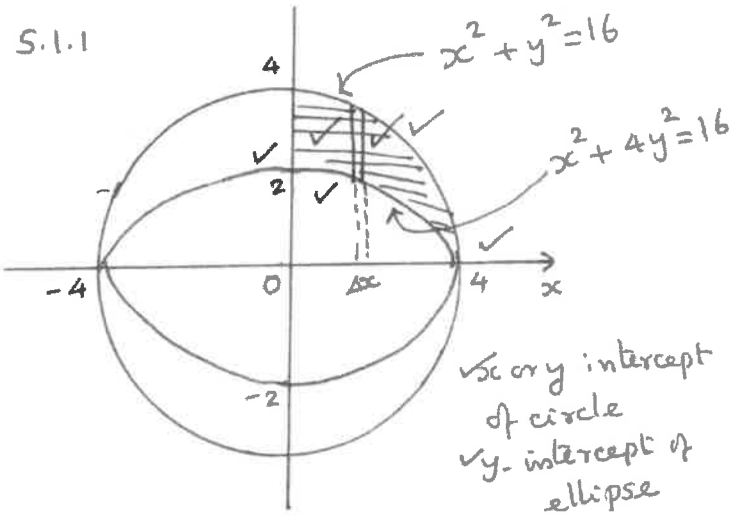
$$y=Ae^{-\frac{1}{3}x}+Be^{\frac{1}{2}x}-x^2+2x-14 \quad ✓$$

(12)

[24]

QUESTION 5

5.1 5.1.1



(6)

5.1.2 $Area =$

$$= \int_a^b y_1 - y_2 \, dx \quad \checkmark$$

$$= \int_0^4 \sqrt{16-x^2} - \frac{1}{2}\sqrt{16-x^2} \, dx \quad \checkmark\checkmark\checkmark$$

$$= \frac{1}{2}\int_0^4 \sqrt{16-x^2} \, dx \quad \checkmark$$

$$= \frac{1}{2}\left[\frac{16}{2}\sin^{-1}\frac{x}{4} + \frac{x}{2}\sqrt{16-x^2}\right]_0^4 \quad \checkmark\checkmark$$

$$= \frac{1}{2}\left[8\sin^{-1}\frac{4}{4} + \frac{4}{2}\sqrt{16-4^2} - \left\{8\sin^{-1}\frac{0}{4} + \frac{0}{2}\sqrt{16-0^2}\right\}\right] \quad \checkmark\checkmark$$

$$= \frac{1}{2}\left[8\sin^{-1}1\right] \quad = 6{,}283 \ or \quad 2\pi \ units^2 \quad \checkmark$$

,

(10)

5.1.3

$$\bar{x} = \frac{A_{m-y}}{A}$$

$$A_{m-y} = \int_a^b r dA$$

$$= \int_0^4 x\left[\sqrt{16-x^2} - \frac{1}{2}\sqrt{16-x^2}\right]dx \quad ✓✓✓$$

$$= \frac{1}{2}\int_0^4 x\sqrt{16-x^2}\,dx \quad ✓$$

$$= \frac{1}{2}(-\frac{1}{2})\int_0^4 -2x(16-x^2)^{\frac{1}{2}}dx \quad ✓$$

$$= -\frac{1}{4}\left[\frac{(16-x^2)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 \quad ✓$$

$$= -\frac{1}{6}\left[(16-x^2)\right]_0^4$$

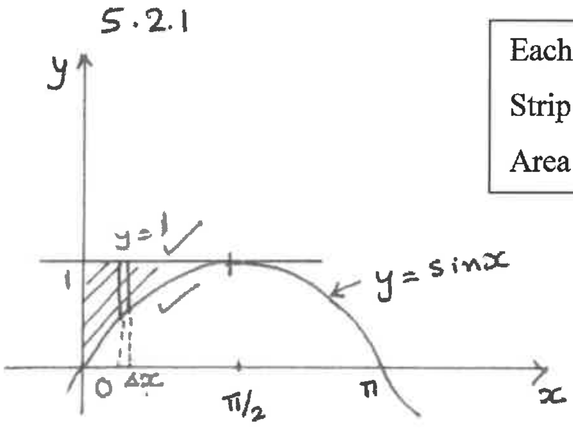
$$= -\frac{1}{6}\left[(16-4^2)^{\frac{3}{2}} - (16-0^2)^{\frac{3}{2}}\right] \quad ✓$$

$$= \frac{32}{3}units^2 \quad or \quad 10{,}667units^2 \quad ✓$$

$$\bar{x} = \frac{32/3}{6{,}283} = 1{,}698 \; units \quad ✓✓$$

(10)

5.2 5.2.1



Each grapl ✓✓
Strip ✓
Area ✓

(4)

5.2.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 \quad ✓$$

$$= \pi \int_0^{\frac{\pi}{2}} 1 - \sin^2 x dx \quad ✓✓ \qquad or \quad \pi \left[x - \left(\frac{x}{2} - \frac{\sin 2x}{4} \right) \right]_0^{\frac{\pi}{2}}$$

$$= \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad ✓ \qquad = \pi \left[x - \frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}}$$

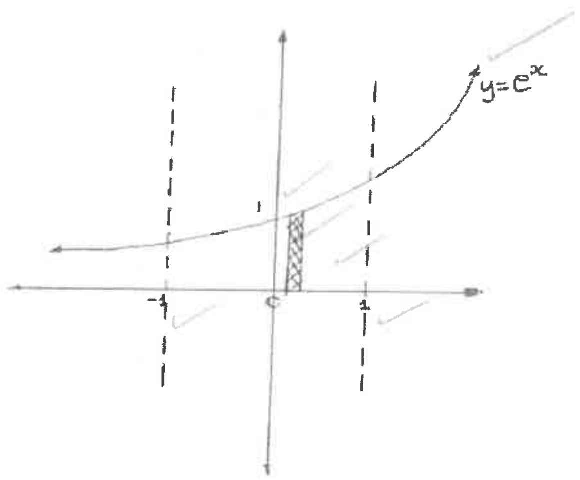
$$= \pi \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}} \quad ✓ \qquad = \pi \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \pi \left[\frac{\pi}{4} + \frac{\sin 0}{4} - \{0\} \right] \quad ✓✓$$

$$= \frac{\pi^2}{4} \text{units}^3 \quad (\text{or } 2,467) \quad ✓$$

(8)

5.3 5.3.1



(6)

5.3.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 \quad ✓$$

$$= \pi \int_{-1}^{1} (e^x)^2 dx \quad ✓✓$$

$$= \pi \left[\frac{e^{2x}}{2} \right]_{-1}^{1} \quad ✓$$

$=3,627\pi$ or $11,394\text{units}^3$

$$= \pi \left[\frac{e^2}{2} - \frac{e^{-2}}{2} \right] \quad ✓ \qquad or = \frac{\pi}{2} \left[e^2 - e^{-2} \right]$$

$=3,627\pi$ or $11,394\text{units}^3$ ✓

(6)

5.3.3

$$I_x = \frac{1}{2}\pi\rho\int_a^b y^4 dx \quad \checkmark\checkmark$$

$$= \frac{1}{2}\pi\rho\int_{-1}^{1} (e^x)^4 dx \quad \checkmark\checkmark$$

$$= \frac{1}{2}\pi\rho\left[\frac{e^{4x}}{4}\right]_{-1}^{1} \quad \checkmark$$

$$= \frac{1}{2}\pi\rho\left[\frac{e^4}{4} - \frac{e^{-4}}{4}\right] \quad or \frac{1}{8}\pi\rho\left(e^4 - e^{-4}\right) \quad \checkmark$$

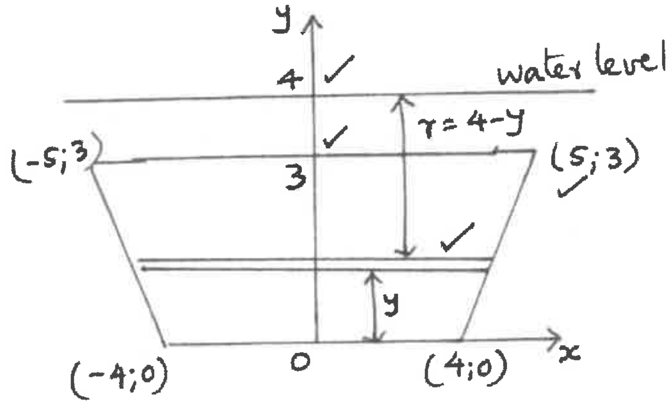
$$= 6{,}823\pi\rho \quad or 21{,}434\rho \quad \checkmark$$

$$= \frac{6{,}823\pi m}{3{,}627\pi} = \frac{21{,}434m}{11{,}394} \quad \checkmark\checkmark$$

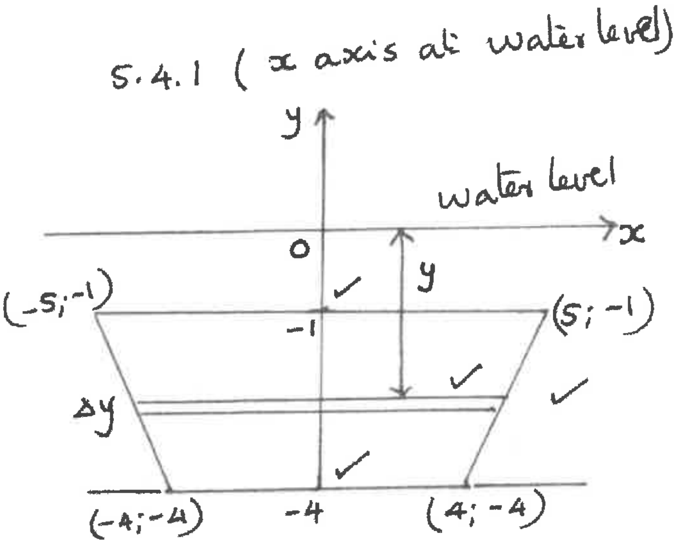
$$= 1{,}881m \quad \checkmark$$

(10)

5.4 5.4.1



Alternative



(4)

5.4.2 $m=\dfrac{3-0}{5-4} or \dfrac{0-3}{4-5}=3$ ✓

$y-0=3(x-4) \quad or \quad y-3=3(x-5)$

$y=3x-12$ ✓ $\quad or \quad y-3=3x-15$

$or \quad y=3x-12$

$x=\dfrac{y+12}{3}$

$=\int_a^b r dA$

$=\int_0^3 (4-y)2\dfrac{y+12}{3}dy$ ✓✓

$=\dfrac{2}{3}\int_0^3 (4-y)(y+12)dy$

$=\dfrac{2}{3}\int_0^3 (-8y-y^2+48)dy$ ✓

$=\dfrac{2}{3}\left[-4y^2-\dfrac{y^3}{3}+48y\right]_0^3$ ✓

$=\dfrac{2}{3}\left[-4(3)^2-\dfrac{(3)^3}{3}+48(3)-\{0\}\right]$ ✓

$=66m^3$ ✓

Alternative

With x-axis at the water level

$$m=\frac{-1+4}{5-4}\,or\,\frac{-4+1}{4-5}=3 \quad \checkmark$$

$$y+4=3(x-4) \quad or \quad y+1=3(x-5)$$

$$y=3x-16 \quad \checkmark \quad or \quad y=3x-15-1$$

$$or \quad y=3x-16$$

$$x=\frac{y+16}{3}$$

First moment of area $=\int_a^b rdA$

$$=\int_{-4}^{-1} y2\frac{y+16}{3}dy \quad \checkmark\checkmark$$

$$=\frac{2}{3}\int_{-4}^{-1} y^2+16ydy \quad \checkmark$$

$$=\frac{2}{3}\left[\frac{y^3}{3}+8y^2\right]_{-4}^{-1} \quad \checkmark$$

$$=\frac{2}{3}\left[\frac{-1}{3}+8-\{\frac{-64}{3}+8.16\}\right] \quad \checkmark$$

$$=-66m^3 \quad \checkmark$$

(8)

5.4.3 Second moment of area

$$=\int_a^b r^2 dA$$

$$=\int_0^3 (4-y)^2 2\frac{y+12}{3}dy \quad ✓✓$$

$$=\frac{2}{3}\int_0^3 (4-y)^2(y+12)dy$$

$$=\frac{2}{3}\int_0^3 (16-8y+y^2)(y+12)dy$$

$$=\frac{2}{3}\int_0^3 (-80y+4y^2+y^3+192)dy \quad ✓$$

$$=\frac{2}{3}\left[-40y^2+\frac{4}{3}y^3+\frac{y^4}{4}+192y\right]_0^3 \quad ✓$$

$$=\frac{2}{3}\left[-40(3)^2+\frac{4}{3}(3)^3+\frac{(3)^4}{4}+192(3)-\{0\}\right] \quad ✓$$

$$\boxed{\bar{y} = \frac{181,5}{66} ✓ = 2,75m ✓}$$

$$=181,5m^4 \quad ✓$$

Alternative

Second moment of area

$$=\int_a^b r^2 dA$$

$$=\int_{-4}^{-1} y^2 2\frac{y+16}{3}dy \quad ✓✓$$

$$=\frac{2}{3}\int_{-4}^{-1} y^3+16y^2 dy \quad ✓$$

$$=\frac{2}{3}\left[\frac{y^4}{4}+\frac{16y^3}{3}\right]_{-4}^{-1} \quad ✓$$

$$=\frac{2}{3}\left[\frac{1}{4}-\frac{16}{3}-\left\{\frac{(-4)^4}{4}+\frac{16(-4)^3}{3}\right\}\right]_{-4}^{-1} \quad ✓$$

$$=181,5m^4 \quad ✓$$

$$\bar{y}=\frac{181,5m^4}{-66m^3}=-2,75m \quad ✓✓$$

(8)

[80]

QUESTION 6

6.1 $x = e^{\theta} \sin\theta$

$$\frac{dx}{d\theta} = e^{\theta}\cos\theta + e^{\theta}\sin\theta \quad \checkmark$$

$$= e^{\theta}(\cos\theta + \sin\theta)$$

$$\left[\frac{dx}{d\theta}\right]^2 = e^{2\theta}(\cos\theta + \sin\theta)^2 \quad \checkmark$$

$$y = e^{\theta}\cos\theta \quad \checkmark$$

$$\frac{dy}{d\theta} = -e^{\theta}\sin\theta + e^{\theta}\cos\theta \quad \checkmark$$

$$= e^{\theta}(\cos\theta - \sin\theta)$$

$$\left[\frac{dy}{d\theta}\right]^2 = e^{2\theta}(\cos\theta - \sin\theta)^2 \quad \checkmark$$

$$\left[\frac{dx}{d\theta}\right]^2 + \left[\frac{dy}{d\theta}\right]^2 = e^{2\theta}\left[(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2\right] \quad \checkmark$$

$$= e^{2\theta}\left[\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right] \quad \checkmark$$

$$= e^{2\theta}[2] \quad \checkmark$$

Or using

$a^2 + b^2 = (a+b)^2 - 2ab$ $\qquad e^{2\theta}\left[(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2\right] = e^{2\theta}[2]$

$$S = \int_{\theta_1}^{\theta_2} \sqrt{\left[\frac{dx}{d\theta}\right]^2 + \left[\frac{dy}{d\theta}\right]^2}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{2e^{2\theta}}\, d\theta \quad \checkmark$$

$$= \sqrt{2}\int_0^{\frac{\pi}{3}} e^{\theta}\, d\theta$$

$$= \sqrt{2}\left[e^{\theta}\right]_0^{\frac{\pi}{3}} \quad \checkmark$$

$$= \sqrt{2}\left[e^{\frac{\pi}{3}} - e^0\right] \quad \checkmark$$

$$= 2{,}616 \text{ units} \quad \checkmark$$

(12)

6.2

$$y = \frac{3}{2}x$$

$$\frac{dy}{dx} = \frac{3}{2} \checkmark$$

$$\left[\frac{dy}{dx}\right]^2 = \frac{9}{4} \checkmark$$

$$1 + \left[\frac{dy}{dx}\right]^2 = 1 + \frac{9}{4} = \frac{13}{4} \checkmark\checkmark$$

$$A_x = 2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \checkmark$$

$$= 2\pi\int_2^4 \frac{3}{2}x\sqrt{\frac{13}{4}}dx \checkmark\checkmark\checkmark$$

$$= 2\pi\frac{3}{2}\frac{\sqrt{13}}{2}\int_2^4 xdx \checkmark$$

$$= \frac{3\sqrt{13}\pi}{2}\left[\frac{x^2}{2}\right]_2^4 \checkmark$$

$$= \frac{3\sqrt{13}\pi}{4}\left[4^2 - 2^2\right] \checkmark$$

$$= 9\sqrt{13}\pi \text{ or } 101{,}945 \text{ units}^2 \checkmark$$

$$x = \frac{2}{3}y$$

$$\frac{dx}{dy} = \frac{2}{3}$$

$$\left[\frac{dx}{dy}\right]^2 = \frac{4}{9}$$

$$1 + \left[\frac{dx}{dy}\right]^2 = 1 + \frac{4}{9} = \frac{13}{9}$$

$$A_x = 2\pi\int_a^b y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= 2\pi\int_3^6 y\sqrt{\frac{13}{9}}dy$$

$$= 2\pi\frac{\sqrt{13}}{3}\int_3^6 ydy$$

$$= \frac{2\sqrt{13}\pi}{3}\left[\frac{y^2}{2}\right]_3^6$$

$$= \frac{\sqrt{13}\pi}{3}\left[6^2 - 3^2\right]$$

$$= 9\sqrt{13}\pi \text{ or } 101{,}945 \text{ units}^2$$

(12)

[24]

TOTAL: 200