higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1040**(E)**(N17)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

17 November 2017 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. Round ALL calculations off to THREE decimal places.
7. ALL the formulae used must be written down.
8. Questions must be answered in BLUE or BLACK ink.
9. Write neatly and legibly.

QUESTION 1

1.1 Given: $z = \sin\,(\,(^{2}x + y^{2})$

Determine $y\dfrac{\partial z}{\partial x} - x\dfrac{\partial z}{\partial y}$ (2)

1.2 The equation of a curve is given by $x = \sin\theta$ and $y = \cos 2\theta$.

Determine the equation of the tangent to the curve where $\theta = \dfrac{\pi}{3}$. (4)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \sin x \cos 2x$ (2)

2.2 $y = \dfrac{x}{\sin^{2} x}$ (3)

2.3 $y = \sqrt{4 + 6x - 3x^{2}}$ (4)

2.4 $y = e^{3x}\sin 3x$ (4)

2.5 $y = \tan^{4} 7x$ (5)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{-2x^{2} + 2x - 5}{x^{4} + x}\,dx$ (7)

3.2 $\displaystyle\int \frac{x^{2} + x + 5}{(x+2)(x^{2} + 4x + 4)}\,dx$ (5)

[12]

QUESTION 4

4.1 Find the particular solution of $\frac{dy}{dx} - \frac{y}{x} = x + \ln x$ when $x = 2 \quad y = 1$ (6)

4.2 Determine the general solution of $\frac{d^2y}{dx^2} + 3y = e^{3x}$ (6)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = (x - 1)^2$ and $y = (x - 1)(5 - x)$.

Sketch the graphs and show the area enclosed by the graphs. Show the representative strip/element perpendicular to the *x*-axis. (4)

5.1.2 Calculate the volume when the area described in QUESTION 5.1.1 rotates about the *y*-axis. (Use a representative strip/element perpendicular to the *x*-axis.) (5)

5.2 5.2.1 Find the points of intersection of $y = x^2$ and $y = 2x + 3$.

Sketch the two graphs. Show the bounded area and the representative strip/element that you will use to calculate the area. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance from the x-axis of the centroid of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Make a neat sketch of the curve $4x^2 + 9y^2 = 36$. Show the representative strip/element (perpendicular to the *x*-axis) that you will use to calculate the volume when the enclosed area in the first quadrant bounded by the curve, the *x*-axis and the y-axis is rotated about the *x*-axis. (2)

5.3.2 Calculate the volume when the area described in QUESTION 5.3.1 rotates about the *x*-axis. (3)

5.3.3 Calculate the moment of inertia about the *x*-axis of the solid obtained when the area described in QUESTION 5.3.1 rotates about the *x*-axis.

Express the answer in terms of mass. (5)

5.4 5.4.1 A rectangular weir is vertically placed in a rectangular canal. The weir is 5 m wide and 2 m high. The top of the weir is 1 m below the water surface.

Make a neat sketch of the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the weir. (2)

5.4.2 Calculate the area moment of the weir about the water surface. (3)

5.4.3 Calculate the second moment of area of the weir about the water surface as well as the depth of the centre of pressure on the weir. (5)
[40]

QUESTION 6

6.1 Determine the x-intercepts of the parabola $y = -x^2 + 8x - 12$.

Calculate the length of the arch of this parabola which lies above the x-axis. (6)

6.2 Calculate the surface area generated when the curve $y = \frac{x^2}{16} - 2\ln x$ from $x = 1$ to $x = 3$ rotates about the y-axis. (6)
[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2\, x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec}\, x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$

$\int [f(x)]^n\,f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$

$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$

$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$

$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$

$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$

$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$

$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$

Applications of integration

AREAS

$A_x = \int_a^b y\,dx\,;\ A_x = \int_a^b (y_1 - y_2)\,dx$

$A_y = \int_a^b x\,dy\,;\ A_y = \int_a^b (x_1 - x_2)\,dy$

VOLUMES

$$V_x = p\int_a^b y^2 dx \,;\, V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \,;\, V_x = 2p\int_a^b xy dy$$

$$V_y = p\int_a^b x^2 dy \,;\, V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \,;\, V_y = 2p\int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = r\,V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = r\int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2py \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2px \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \square \; ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{dq}\left(\frac{dy}{dx}\right)\frac{dq}{dx}$$

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AMENDED MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

17 NOVEMBER 2017

This marking guideline consists of 16 pages.

MARKERS:
HLAKO MW(CM) 19/11/2017
TAUKOBONG AM(IM)
BUTHELEZI M
DINGALO TM
GUMBURA MC
LINDANE C
MOHLALA LT
NGOBESE NN
OKEOWO ST
THUNGANA TP AND OTHERS
MALEMA TAT(DMCM ACADEMIC) TMalema 19/01/2017

Each tick is half a mark √ **Total marks: 100**

QUESTION 1

1.1 $z = \sin(x^2 + y^2)$

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = y\cos(x^2+y^2).2x - x\cos(x^2+y^2).2y = 0 \quad √ \; √ \; √ \; √$$

~~(2)~~ 4

1.2 $y = \cos 2\theta$ √ $\qquad x = \sin\theta$

$$\frac{dy}{d\theta} = -2\sin 2\theta \qquad \frac{dx}{d\theta} = \cos\theta \quad √$$

$$\frac{dy}{dx} = \frac{-2\sin 2\theta}{\cos\theta} = \frac{-4\sin\theta\cos\theta}{\cos\theta} = -4\sin\theta \quad √$$

$$\theta = \frac{\pi}{3}\;; \quad \frac{dy}{dx} = -4\sin\frac{\pi}{3} = -4.\frac{\sqrt{3}}{2} = -2\sqrt{3} \quad √$$

$$\theta = \frac{\pi}{3}\;; \quad x = \sin\theta = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad √$$

$$y = \cos 2\theta = \cos 2.\frac{\pi}{3} = -\frac{1}{2} \quad √$$

Equation of tangent is:

$$y - y_1 = m(x - x_1)$$

$$\therefore y - (-\frac{1}{2}) = -2\sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right) \quad √$$

$$y + \frac{1}{2} = -2\sqrt{3}x + 3 \quad √$$

$$y = -2\sqrt{3}x + \frac{5}{2}$$

OR

$y = mx + c$

$\therefore -\frac{1}{2} = -2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + c$

$\therefore c = 5/2$

$\therefore y = -2\sqrt{3}x + 5/2$

~~(4)~~ 8
~~[6]~~ 12

QUESTION 2

NB: Students may want to use the other Double Angle Identity for $\cos 2x$.
i.e $\cos 2x = 1 - 2\sin^2 x$
and
$\cos 2x = \cos^2 x - \sin^2 x$.

2.1 $\int \sin x \cos 2x dx$ √

$= \int \sin x(2\cos^2 x - 1)dx$

$\int 2\sin x \cos^2 x - \sin x dx$ √

$= -\frac{2}{3}\cos^3 x + \cos x + C$ √ √

~~(2)~~ 4

2.2 $\int \frac{x}{\sin^2 x}$ √

$= \int x \operatorname{cosec}^2 x dx$

$= x(-\cot x) - \int 1(-\cot x)dx$ √ √ √

$= -x\cot x + \int \cot x dx$ √

$= -x\cot x + \ln(\sin x) + C$ √

~~(3)~~ 6

2.3 $\int \sqrt{4 + 6x - 3x^2}$

$4 + 6x - 3x^2 = -3x^2 + 6x + 4$

$= -3(x^2 - 2x - \frac{4}{3})$ √ √

$= -3\left[(x^2 - 2x + 1) - 1 - \frac{4}{3}\right]$ √

$= -3\left[(x-1)^2 - \frac{7}{3}\right]$ √

$= 3\left[\frac{7}{3} - (x-1)^2\right]$ √ *or* $7 - 3(x-1)^2$

$\int \sqrt{4 + 6x - 3x^2}\,dx = \int \sqrt{3[\frac{7}{3} - (x-1)^2]}\,dx$ √

$= \sqrt{3}\int \sqrt{\frac{7}{3} - (x-1)^2}\,dx$

$= \sqrt{3}\left[\frac{\frac{7}{3}}{2}\sin^{-1}\frac{x-1}{\frac{\sqrt{7}}{\sqrt{3}}} + \frac{x-1}{2}\sqrt{\frac{7}{3} - (x-1)^2}\right] + C$ √√

Or using $\frac{4ac - b^2}{4a} + a\left(x + \frac{b}{2a}\right)^2 = \frac{4(-3)4 - (6)^2}{4(-3)} - 3\left(x + \frac{6}{2(-3)}\right)^2 = 7 - 3(x-1)^2$

Or after completing the square and using $7 - 3(x-1)^2$

$\int \sqrt{7 - 3(x-1)^2}\,dx = \frac{7}{2\sqrt{3}}\sin^{-1}\frac{\sqrt{3}(x-1)}{\sqrt{7}} + \frac{x-1}{2}\sqrt{7 - 3(x-1)^2}$ +C

~~(4)~~ 8

2.4 $\int \tan^4 7x\,dx = \int \tan^2 7x \tan^2 7x\,dx$ √

$= \int \tan^2 7x(\sec^2 7x - 1)dx$ √

$= \int (\tan^2 7x \sec^2 7x - \tan^2 7x)dx$ √

$= \int (\tan^2 7x \sec^2 7x - (\sec^2 7x - 1)dx$ √

$= \int (\tan^2 7x \sec^2 7x - \sec^2 7x + 1)dx$ √

$= \frac{1}{7}\frac{\tan^3 7x}{3} - \frac{1}{7}\tan 7x + x + C$ √ √ √

~~(4)~~ 8

2.5 $\int e^{3x} \sin 3x\,dx$ $f(x) = e^{3x}$ $g'(x) = \sin 3x$

$= e^{3x}(-\frac{1}{3}\cos 3x) - \int 3e^{3x}(-\frac{1}{3}\cos 3x)dx$ √ √ √

$= -\frac{1}{3}e^{3x}\cos 3x + \int e^{3x}\cos 3x\,dx$ √

$= -\frac{1}{3}e^{3x}\cos 3x + [e^{3x}.\frac{1}{3}\sin 3x - \int 3e^{3x}.\frac{1}{3}\sin 3x\,dx]$ √ √ √

$= -\frac{1}{3}e^{3x}\cos 3x + e^{3x}.\frac{1}{3}\sin 3x - \int e^{3x}\sin 3x\,dx$ √ √

$\therefore \int e^{3x}\sin 3x\,dx + \int e^{3x}\sin 3x\,dx = -\frac{1}{3}e^{3x}\cos 3x + \frac{1}{3}e^{3x}\sin 3x$

$2\int e^{3x}\sin 3x\,dx = -\frac{1}{3}e^{3x}\cos 3x + \frac{1}{3}e^{3x}\sin 3x$

$\int e^{3x}\sin 3x\,dx = \frac{1}{2}[-\frac{1}{3}e^{3x}\cos 3x + \frac{1}{3}e^{3x}\sin 3x] + C$ √

Or $\frac{1}{6}e^{3x}[\sin 3x - \cos 3x] + C$

Alternatively, using $f(x) = \sin 3x$ and $g'(x) = e^{3x}$

~~(5)~~ 10
~~**[18]**~~ 36

QUESTION 3

3.1
$$\frac{-2x^2+2x-5}{x^4+x}$$
$$=\frac{-2x^2+2x-5}{x(x^3+1)} \quad \surd \quad \surd \quad \surd \quad \surd$$
$$=\frac{-2x^2+2x-5}{x(x+1)(x^2-x+1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{Cx+D}{x^2-x+1} \quad \surd$$
$$-2x^2+2x-5=A(x+1)(x^2-x+1)+Bx(x^2-x+1)+(Cx+D)x(x+1)$$

$x=0 \qquad -5=A$ $\surd$

$x=-1 \qquad -2-2-5=B(-1)(1+1+1) \qquad -9=-3B \qquad B=3$ $\surd$

$-2x^2+2x-5=A(x^3+1)+B(x^3-x^2+x)+Cx^3+Dx^2+Cx^2+Dx$ $\surd$

Equating $x^3 \qquad A+B+C=0 \qquad -5+3+C=0 \qquad C=2$ $\surd$
Equating $x \qquad B+D=2 \qquad 3+D=2 \qquad D=-1$ $\surd$

$$\therefore\int\frac{-5}{x}dx+\int\frac{3}{x+1}dx+\int\frac{2x-1}{x^2-x+1}dx \quad \surd$$
$$-5\ln x+3\ln(x+1)+\ln(x^2-x+1)+C \quad \surd\surd\surd$$

~~(7)~~ 14

3.2
$$\frac{x^2+x+5}{(x+2)(x^2+4x+4)}=\frac{x^2+x+5}{(x+2)(x+2)^2}=\frac{x^2+x+5}{(x+2)^3}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{(x+2)^3} \quad \surd$$

$x^2+x+5=A(x+2)^2+B(x+2)+C$ $\surd$

$x^2+x+5=A(x^2+4x+4)+Bx+2B+C$ $\surd$
$x^2+x+5=Ax^2+4Ax+4A+Bx+2B+C$ $\surd$ removed and given as indicated.
Reason: step is more important.
$A=1$ $\surd$
$4A+B=1$
$4+B=1 \qquad B=-3$ $\surd$
$4A+2B+C=5 \qquad 4-6+C=5 \qquad C=7$ $\surd$

$$\therefore\int\frac{1}{x+2}dx+\int\frac{-3}{(x+2)^2}dx+\int\frac{7}{(x+2)^3}dx \quad \checkmark$$
$$=\ln(x+2)+\frac{3}{x+2}-\frac{7}{2(x+2)^2}+C \quad \surd \quad \surd\surd$$

~~(5)~~ 10
~~[12]~~ 24

Please turn over

QUESTION 4

√ removed at all. Reason; they (ticks) are one more.

4.1 $$\frac{dy}{dx} - \frac{y}{x} = x + \ln x$$

$$e^{\int p dx} = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x} \quad \surd \; \surd \; \surd$$

$$\int Q e^{\int p dx} dx = \int (x + \ln x)\frac{1}{x} dx \quad \surd \; \surd$$

$$= \int (1 + \frac{1}{x}\ln x) dx \quad \surd$$

$$= x + \frac{(\ln x)^2}{2} \quad \surd \; \surd$$

$$y\frac{1}{x} = x + \frac{(\ln x)^2}{2} + C \quad \surd$$

$x = 2$, $y = 1$ $\qquad y\frac{1}{2} = 2 + \frac{(\ln 2)^2}{2} + C \quad \surd \qquad C = -1{,}740 \quad \surd$

$$y\frac{1}{x} = x + \frac{(\ln x)^2}{2} - 1{,}740 \quad \surd$$

Or

$$y = x^2 + x\frac{(\ln x)^2}{2} - 1{,}740x$$

~~(6)~~ 12

4.2 $$\frac{d^2y}{dx^2} + 3y = e^{3x}$$

$r^2 + 3 = 0 \quad \surd$

$r^2 = -3 \quad \surd$

$r = \pm\sqrt{-3} = \pm\sqrt{3}i \quad \surd$

$$y_c = e^{0.x}[A\cos\sqrt{3}x + B\sin\sqrt{3}x] \quad \surd \; \surd$$

$$= A\cos\sqrt{3}x + B\sin\sqrt{3}x$$

For y_p $y = Ce^{3x}$ √

$\frac{dy}{dx} = 3Ce^{3x}$ √ $\frac{d^2y}{dx^2} = 9Ce^{3x}$ √

√

$\therefore 9Ce^{3x} + 3Ce^{3x} = e^{3x}$ $12Ce^{3x} = e^{3x}$

$C = \frac{1}{12}$ √

$y_p = \frac{1}{12}e^{3x}$ √

$y = A\cos\sqrt{3}x + B\sin\sqrt{3}x + \frac{1}{12}e^{3x}$ √ ~~(6)~~ 12
~~[12]~~ 24

QUESTION 5

5.1 5.1.1 $y = (x-1)^2$

$y = (x-1)(5-x)$

$\therefore (x-1)^2 = (x-1)(5-x)$ √

$(x-1)^2 - (x-1)(5-x) = 0$

$(x-1)\{x-1-5+x)\} = 0$

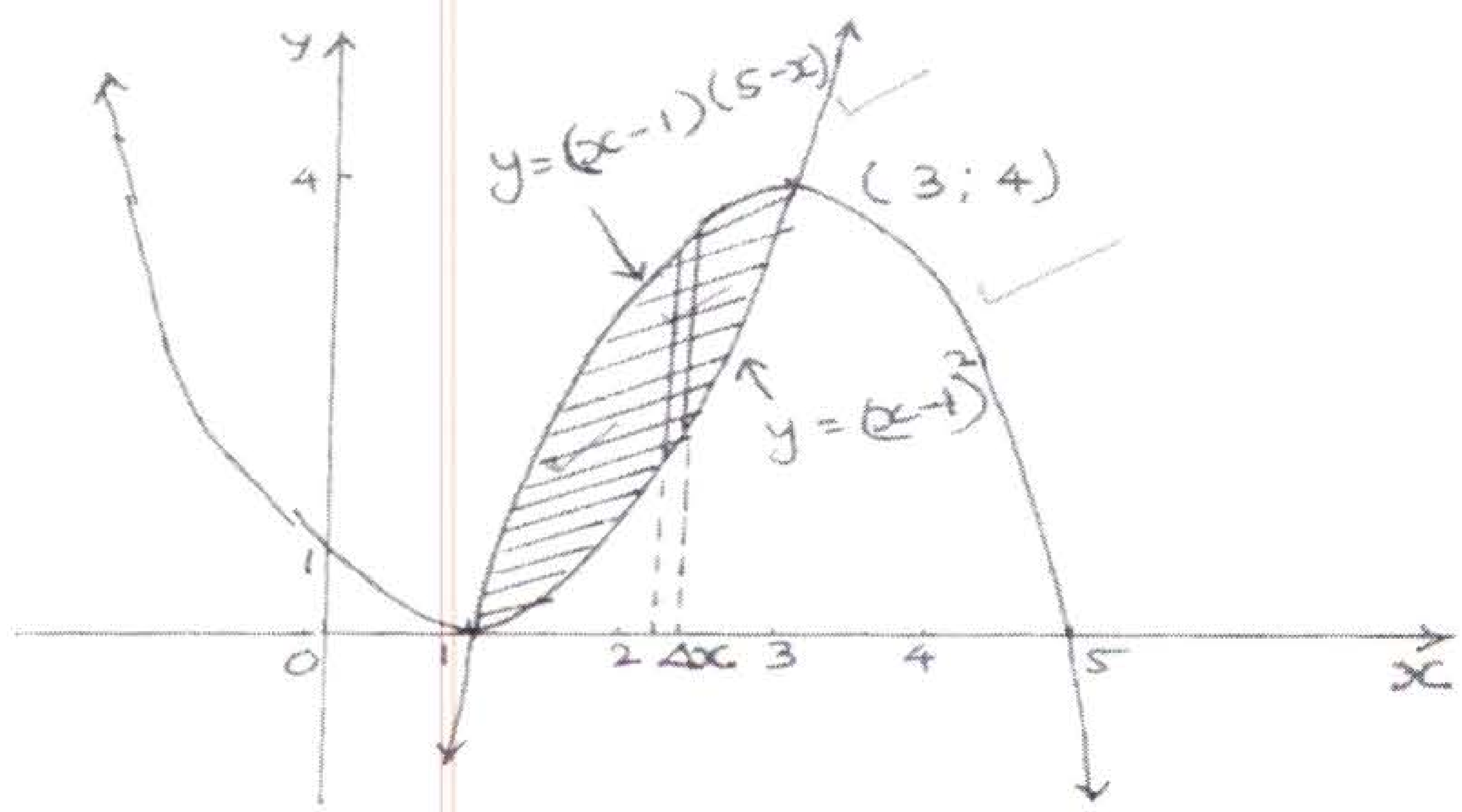
$(x-1)\{2x-6\} = 0$ √

$x = 1$ $x = 3$

$y = 0$ $y = 4$

$(1;0)$ $(3;4)$

√ √



√ each graph
√ strip
√ area ~~(4)~~ 8

5.1.2 $V_y = 2\pi\int_a^b x(y_1 - y_2)dx$ √

$= 2\pi\int_1^3 x\{(x-1)(5-x)-(x-1)^2\}dx$ √ √ √ √

$= 2\pi\int_1^3 x(x-1)\{(5-x)-(x-1)\}dx$

$= 2\pi\int_1^3 x(x-1)\{5-x-x+1\}dx$

$= 2\pi\int_1^3 x(x-1)\{6-2x\}dx$

$= 2\pi\int_1^3 x(6x-6-2x^2+2x)dx$

$= 2\pi\int_1^3 x(8x-6-2x^2)dx$ √

$= 2\pi\int_1^3 (8x^2-6x-2x^3)dx$ √

$= 2\pi\left[\frac{8x^3}{3}-\frac{6x^2}{2}-\frac{2x^4}{4}\right]_1^3$ √

$= 2\pi\left[\frac{8(3)^3}{3}-\frac{6(3)^2}{2}-\frac{2(3)^4}{4}-\{\frac{8}{3}-\frac{6}{2}-\frac{2}{4}\}\right]$ √

$= 10,667\pi$ √

NB: Students may want to multiply out the brackets and deal with.

$y=(x-1)^2$ as $y=x^2-2x+1$

$y=(x-1)(5-x)$ as $y=-x^2+6x-5$

1 for formula; 1 for limits; 3 for putting x and the equations of the curves; 2 for simplification; 1 for integration; 1 for substitution; 1 for the final answer ~~(5)~~ 10

5.2 5.2.1 $y = x^2 \qquad y = 2x+3$

$x^2 = 2x+3$

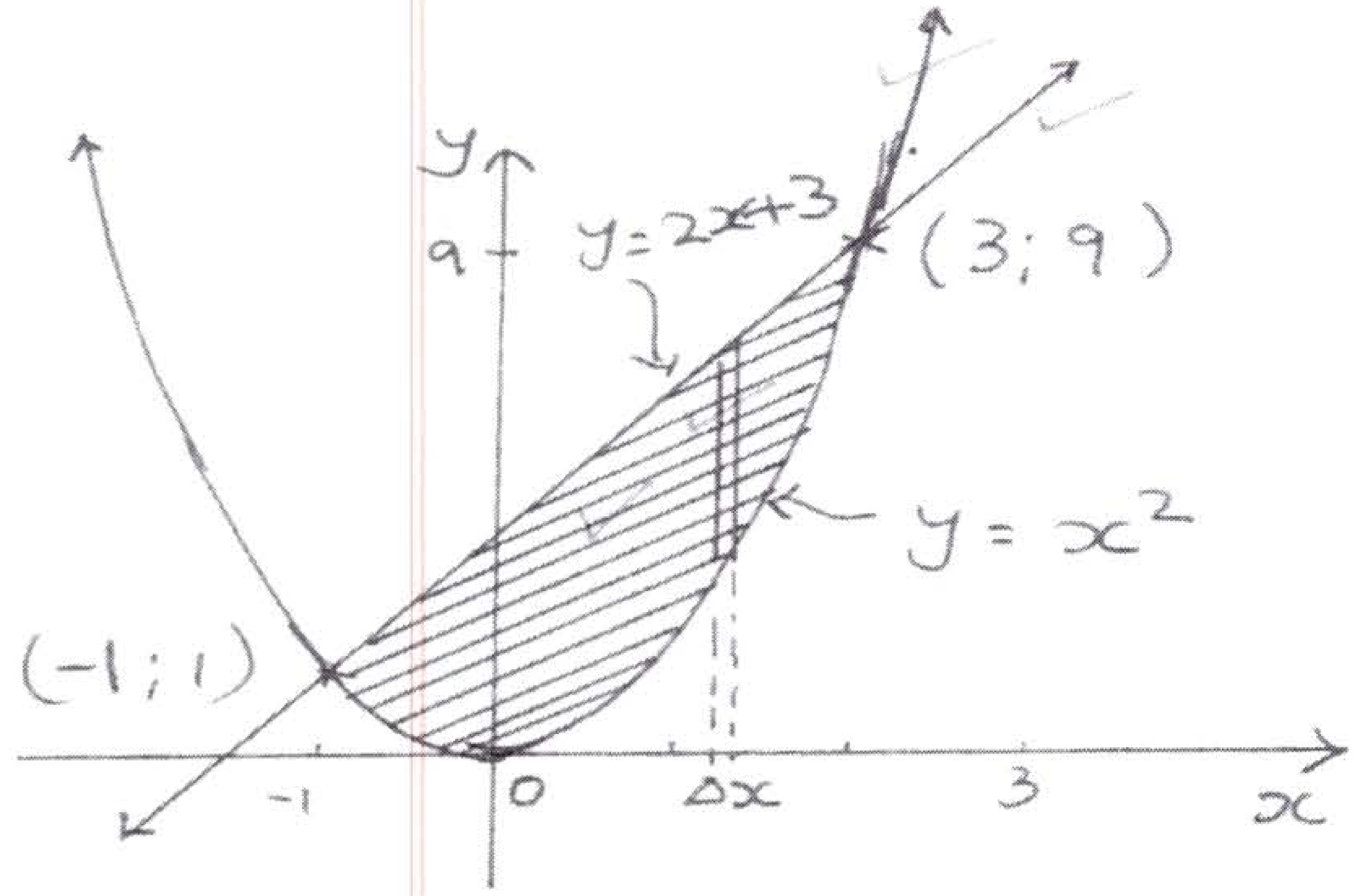
$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x=-1 \quad x=3$

$y=1 \quad y=9$

$(-1;1) \quad (3;9)$ √√



√ each graph
√ strip
√ area

~~(3)~~ 6

5.2.2

$A = \int_a^b y_1 - y_2 dx$ √

√ √

$= \int_{-1}^{3} 2x + 3 - x^2 dx$

√

$= \left[x^2 + 3x - \frac{x^3}{3} \right]_{-1}^{3}$

$= 9 + 9 - 9 - \{1 - 3 + \frac{1}{3}\}$ √

$= 10,667\, units^2$ √

1 for limits; 1 for substituting the equations of graphs

~~(3)~~ 6

5.2.3

$$A_{m-x} = \int_a^b \left(\frac{y_1 + y_2}{2}\right)(y_1 - y_2)dx \quad \surd$$

$$= \frac{1}{2}\int_a^b (y_1^2 - y_2^2)dx$$

$$\frac{1}{2}\int_{-1}^{3} (2x+3)^2 - (x^2)^2 dx \quad \surd\surd\surd$$

$$= \frac{1}{2}\int_{-1}^{3} (4x^2 + 12x + 9 - x^4)dx \quad \surd$$

$$= \frac{1}{2}\left[4\frac{x^3}{3} + 6x^2 + 9x - \frac{x^5}{5}\right]_{-1}^{3} \quad \surd$$

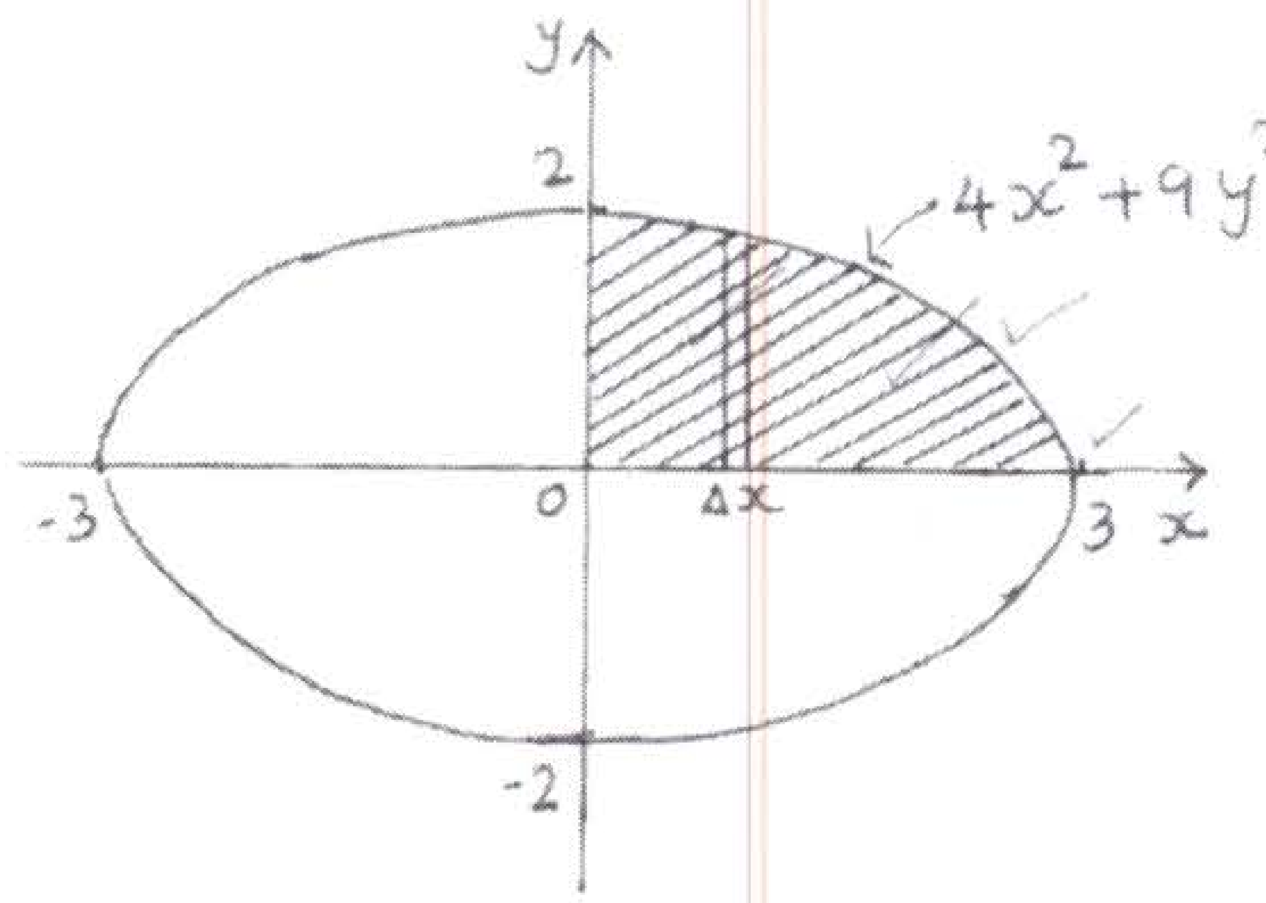
$$= \frac{1}{2}\left[4\frac{(3)^3}{3} + 6(3)^2 + 9(3) - \frac{(3)^5}{5} - \{4\frac{(-1)^3}{3} + 6(-1)^2 + 9(-1) - \frac{(-1)^5}{5}\}\right] \quad \surd$$

$$= 36\frac{4}{15} = 36,2667 units^3 \quad \surd$$

$$\bar{y} = \frac{A_{m-x}}{A} = \frac{36,2667}{10,6667} = 3,400 units \quad \surd \; \surd$$

~~(5)~~ 10

5.3 5.3.1



✓ graph
✓ area
✓ strip
✓ *x* and *y* intercepts

~~(2)~~ 4

5.3.2

$$V_x = \pi\int_a^b y^2 dx \quad \surd$$

$$= \pi\int_0^3 \frac{4}{9}(9 - x^2)dx \quad \surd\surd$$

$$= \frac{4}{9}\pi\left[9x - \frac{x^3}{3}\right]_0^3 \quad \surd$$

$$= \frac{4}{9}\pi\left[9(3) - \frac{(3)^3}{3} - 0\right] \quad \surd$$

$$= 8\pi units^3 = 25,1327 \quad \surd$$

~~(3)~~ 6

Please turn over

5.3.3 $I_x = \frac{1}{2}\pi\rho\int_a^b y^4 dx$ √ $4x^2 + 9y^2 = 36$ $9y^2 = 36 - 4x^2$

$y^2 = \frac{4}{9}(9 - x^2)$ √

$= \frac{1}{2}\pi\rho\int_0^3 \left[\frac{4}{9}(9 - x^2)\right]^2 dx$ √

$= \frac{16}{81}.\frac{1}{2}\pi\rho\int_0^3 (9 - x^2)^2 dx$ √

$= \frac{8}{81}\pi\rho\int_0^3 (81 - 18x^2 + x^4) dx$ √

$= \frac{8}{81}\pi\rho\left[81x - 6x^3 + \frac{1}{5}x^5\right]_0^3$ √

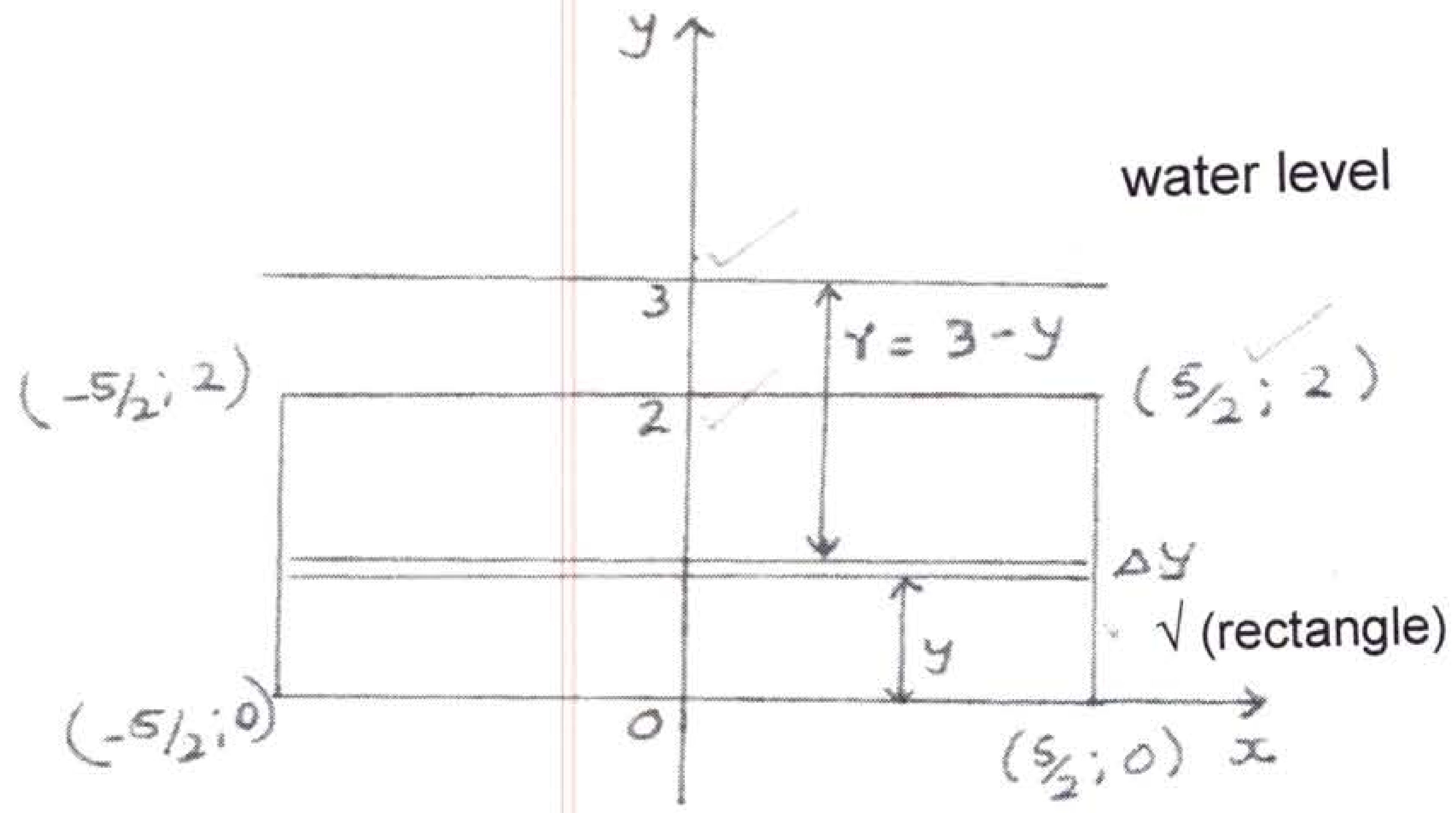
$= \frac{8}{81}\pi\rho\left[81(3) - 6(3)^3 + \frac{1}{5}(3)^5\right]$ √

$= 12,8\pi\rho = 40,212\rho$ √

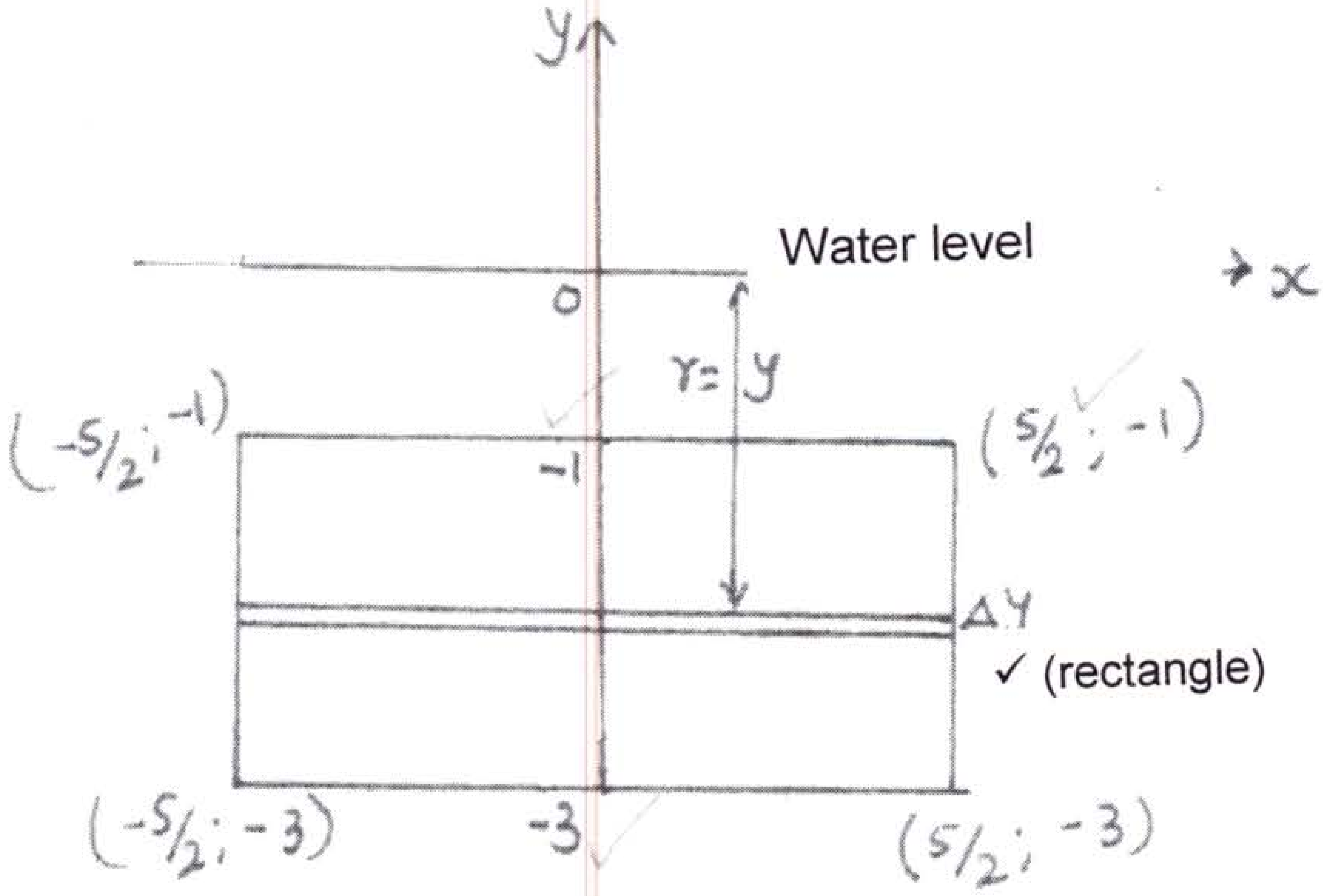
$= 40,2124\frac{m}{V}$ √

$= \frac{40,2124m}{25,1327} = 1,6m$ √ ~~(5)~~ 10

5.4 5.4.1



OR



~~(2)~~ 4

5.4.2 $x = \frac{5}{2}$

$$first\ moment = \int_a^b r\,dA$$

$$= \int_0^2 (3 - y)2.\frac{5}{2}dy \quad \surd\surd\surd$$

$$= 5\left[3y - \frac{y^2}{2}\right]_0^2 \quad \surd$$

$$= 5\left[3(2) - \frac{(2)^2}{2}\right] \quad \surd$$

$$= 20m^3 \quad \surd$$

OR

$$x = \frac{5}{2}$$

First moment

$$= \int_a^b r dA \quad \surd$$

$$= \int_{-3}^{-1} y(2x)dy \quad \surd$$

$$= \int_{-3}^{-1} y(5)dy \quad \surd$$

$$= 5\left[\frac{y^2}{2}\right]_{-3}^{-1} \quad \surd$$

$$= \frac{5}{2}\left[(-1)^2 - (-3)^2\right] \quad \surd$$

$$= -20m^3 \quad \surd$$

(3) 6

5.4.3 Second moment of area

$$= \int_a^b r^2 dA \quad \surd$$

$$= \int_0^2 (3-y)^2 2.\frac{5}{2}dy \quad \surd\surd\surd$$

$$= 5\int_0^2 (9-6y+y^2)dy \quad \surd$$

$$== 5\left[9y - 3y^2 + \frac{1}{3}y^3\right]_0^2 \quad \surd$$

$$= 5\left[9.(2) - 3(2)^2 + \frac{1}{3}(2)^3 - 0\right]_0^2 \quad \surd$$

$$= 43,3333m^4 \quad \surd$$

$$\bar{y} = \frac{43,3333m^4}{20m^3} = 2,167m \quad \surd\surd$$

OR

Second moment

$= \int_a^b r^2 dA$ √

$= \int_{-3}^{-1} y^2(2x)dy$ √ √ √ 1 for limits; 1 for y^2 1 for 2x

$= \int_{-3}^{-1} y^2(5)dy$ √

$= 5\left[\frac{y^3}{3}\right]_{-3}^{-1}$ √

$= \frac{5}{3}\left[(-1)^3 - (-3)^3\right]$ √

$= 43,333m^4$ √

$\overline{\overline{y}} = \frac{43,333m^4}{-20m^3}$ √ $= -2,167m$ √

~~(5)~~ 10
~~[40]~~ 80

QUESTION 6

6.1

$y = -x^2 + 8x - 12$

For x intercepts $y = 0$

$-x^2 + 8x - 12 = 0$ √

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$x = 2 \quad x = 6$ √

$y = -x^2 + 8x - 12$

$\frac{dy}{dx} = -2x + 8$ √

$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

√ √

NB: In determining the square of the derivative; other students may want to do the following:-

$\frac{dy}{dx} = -2x+8$

$\therefore \left(\frac{dy}{dx}\right)^2 = (-2x+8)^2 = (-2x+8)(-2x+8)$

$= 4x^2 - 32x + 64.$

$= \int_2^6 \sqrt{1 + (8-2x)^2}\, dx \qquad u = 8 - 2x \qquad \frac{du}{dx} = -2 \qquad dx = \frac{du}{-2}$

$x = 2 \quad u = 8 - 4 = 4$

$x = 6 \quad u = 8 - 12 = -4$

$= -\frac{1}{2}\int_4^{-4} \sqrt{1+u^2}\, du$ √

$= -\frac{1}{2}\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\{u + \sqrt{1+u^2}\}\right]_4^{-4}$ √√√

$= -\frac{1}{2}\left[\frac{-4}{2}\sqrt{17} + \frac{1}{2}\ln(-4+\sqrt{17}) - \{2\sqrt{17} + \frac{1}{2}\ln(4+\sqrt{17})\}\right]$ √

$= 9,294 units$ √√

~~(6)~~ 12

6.2

$$y=\frac{x^2}{16}-2\ln x \qquad \frac{dy}{dx}=\frac{x}{8}-\frac{2}{x}=\frac{x^2-16}{8x} \quad \surd\ \surd\ \surd$$

$$\left(\frac{dy}{dx}\right)^2=\left(\frac{x^2-16}{8x}\right)^2 \quad \surd$$

$$1+\left(\frac{dy}{dx}\right)^2=1+\frac{\left(x^2-16\right)^2}{(8x)^2}$$

$$=\frac{64x^2+x^4-32x^2+256}{64x^2} \quad \surd$$

$$=\frac{x^4+32x^2+256}{64x^2} \quad \surd$$

$$=\frac{\left(x^2+16\right)^2}{64x^2} \quad \surd$$

$$A_y=2\pi\int_a^b x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$=2\pi\int_1^3 x\sqrt{\frac{\left(x^2+16\right)^2}{64x^2}}\,dx$$

$$=2\pi\int_1^3 x\,\frac{\left(x^2+16\right)}{8x}\,dx \quad \surd$$

$$=\frac{\pi}{4}\int_1^3 x^2+16\,dx \quad \surd$$

$$=\frac{\pi}{4}\left[\frac{x^3}{3}+16x\right]_1^3 \quad \surd$$

$$\frac{\pi}{4}\left[\frac{(3)^3}{3}+16(3)-\left(\frac{1}{3}+16\right)\right] \quad \surd$$

$$31{,}940\,units^2 \quad \surd$$

~~(6)~~ 12
~~[12]~~ 24

TOTAL: ~~**100**~~ 200.

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1060**(E)**(M29)T

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

29 March 2017 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = z = 3\ln(x - 2y) - e^{\frac{x}{y}}$ determine:

1.1.1 $\dfrac{\partial z}{\partial x}$ (1)

1.1.2 $\dfrac{\partial z}{\partial y}$ (1)

1.2 If the parametric equations of a curve are given as $y = 6 - 2t^3$ and $x = -3t + t^2$ find the equation of the tangent to the curve at the point where $t = -1$. (4)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \cos^5 ax.\sin^3 ax$ (4)

2.2 $y = \dfrac{1}{\cot^4 \frac{x}{3}}$ (4)

2.3 $y = x^2.2^{3x}$ (4)

2.4 $y = \dfrac{1}{1 - x + 2x^2}$ (4)

2.5 $y = arc\cot x$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{-x^2 + 3x + 1}{(1 - 2x)^3}\, dx$ (5)

3.2 $\displaystyle\int \frac{x^2 - 29x + 5}{(x - 4)(x^2 + 3)x}\, dx$ (7)

[12]

QUESTION 4

4.1 Calculate the particular solution of :

$$x\frac{dy}{dx} - y = x\ln x \quad \text{if} \quad y = 3 \text{ when } x = e$$ (5)

4.2 Calculate the particular solution of:

$$\left(\frac{1}{2}\right)\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 2x^2 \quad \text{if} \quad y = 1 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 0$$ (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the two curves $y = x^3$ and $y = x$.

Make a neat sketch of the two curves and show the area bounded by the curves in the first quadrant. Show the representative strip/element that you will use to calculate the volume (use the shell method only) of the solid generated when the area bounded by the curves rotates about the *y*-axis. (3)

5.1.2 Calculate the volume described in QUESTION 5.1.1. (4)

5.2 5.2.1 Calculate the points of intersection of $y = x^2$ and $y^2 = 27x$.

Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element, perpendicular to the *x*-axis, that you will use to calculate the area bounded by the curves. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance from the centroid to the *x*-axis of the bounded area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Make a neat sketch of the graph $y = 3\sin x$ and show the representative strip/element that you will use to calculate the volume of the solid generated when the area bounded by the curve, the lines $x = 0$, $x = \frac{\pi}{4}$ and $y = 0$ rotates about the *x*-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid described in QUESTION 5.3.1. (5)

5.4 5.4.1 A vertical sluice gate in the form of a trapezium is 4 m high. The longest horizontal side is 6 m in length and 2 m below the water surface. The shortest side is 4 m in length and 6 m below the water surface.

Make a neat sketch of the sluice gate and show the representative strip/element that you will use to calculate the depth of the centre of pressure. Calculate the relation between the two variables *x* and *y*. (3)

5.4.2 Calculate, by using integration, the area moment of the sluice gate about the water surface. (4)

5.4.3 Calculate, by using integration, the second moment of area of the sluice gate about the water surface, as well as the depth of the centre of pressure on the sluice gate. (5)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve given by the parametric equations, $x = 2e^t \sin t$ and $y = 2e^t \cos t$, over the interval $0 \le t \le \frac{\pi}{2}$. (7)

6.2 Calculate the surface area generated when the arc of $x = 3y^3$ is rotated about the *y*-axis between $y = 1$ and $y = 2$. (5)

[12]

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}\ ;\ \sin x = \frac{1}{\operatorname{cosec} x}\ ;\ \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\dfrac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\dfrac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\dfrac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\dfrac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\dfrac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\dfrac{x}{2}-\dfrac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\dfrac{x}{2}+\dfrac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\dfrac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2 dx\ ;\ V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right) dx\ ;\ V_x = 2\pi\int_a^b xydy$$

$$V_y = \pi\int_a^b x^2 dy\ ;\ V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right) dy\ ;\ V_y = 2\pi\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A}\ ;\ \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m r^2$

GENERAL: $I = \int_a^b r^2 dm = r\int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2} r \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} r p \int_a^b y^4 dx \qquad I_y = \frac{1}{2} r p \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2py\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_x = \int_d^c 2py\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow \quad ye^{\int P\,dx} = \int Qe^{\int P\,dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

STANDARDISED GUIDELINE

T1060**(E)**(M29)T

NATIONAL CERTIFICATE

MATHEMATICS N6

29 MARCH 2017

This marking guideline consists of 17 pages.

CHIEF MARKER: HLAKO MW

INTERNAL MODERATOR: TAUKOBONG AM

DMCM(ACADEMIC): MALEMA TAT

$\checkmark$ = 1 MARK TOTAL: $\frac{200}{2} = 100$

NOTE: Do NOT subtract marks for incorrect units or units omitted.

QUESTION 1

1.1 1.1.1 $z = 3\ln(x - 2y) - e^{\frac{x}{y}}$

$$\frac{\partial z}{\partial x} = \overset{\checkmark}{\frac{3}{x-2y}} - \overset{\checkmark}{\frac{1}{y}e^{\frac{x}{y}}}$$

(2)

1.1.2 $$\frac{\partial z}{\partial y} = \overset{\checkmark}{\frac{3(-2)}{x-2y}} + \frac{x}{y^2}e^{\frac{x}{y}} \checkmark$$

or $$\frac{\partial z}{\partial y} = \frac{-6}{x-2y} + xy^{-2}e^{\frac{x}{y}}$$

(2)

1.2 $y = 6 - 2t^3$ $x = -3t + t^2$

$\frac{dy}{dt} = -6t^2$ $\checkmark$ $\frac{dx}{dt} = -3 + 2t$ $\checkmark$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-6t^2}{-3+2t} \checkmark$$

$$\therefore \quad m = \frac{dy}{dx} = \frac{-6t^2}{-3+2t}$$

$$= \frac{-6(-1)^2}{-3+2(-1)}$$

$$= \frac{6}{5} \text{ or } 1{,}2 \checkmark$$

Where $t = -1$: $y = 8$ $\checkmark$ and $x = 4$ $\checkmark$

Equation of the tangent to the curve:

$y = mx + c$ **.....** $(4:8)$

$$8 = (\frac{6}{5})(4) + c$$

$$\therefore \quad c = 3{,}2 \quad (\frac{16}{5} \; or \; 3\frac{1}{5}) \checkmark$$

$$\therefore \quad y = \frac{6}{5}x + \frac{16}{5} \checkmark \quad \text{or} \quad y = 1{,}2x + 3{,}2$$

(8)

[12]

QUESTION 2

2.1 $y = \int \cos^5 ax.\sin^3 ax dx$

$= \int \cos^5 ax.\sin^2 ax.\sin ax dx \checkmark$

$= \int \cos^5 ax.(1 - \overset{\checkmark}{\cos^2 ax})\sin ax dx$

$u = \cos ax$

$\therefore \frac{du}{dx} = -a\sin ax$

$\therefore du = -a\sin ax dx$

$= \overset{\checkmark}{-\frac{1}{a}}\int \overset{\checkmark}{u^5(1 - u^2)du}$

$= -\frac{1}{a}\int (u^5 - u^7)du \checkmark$

$= -\frac{1}{a}\left(\frac{u^6}{6} - \frac{u^8}{8}\right) + c \checkmark$

$= -\frac{1}{a}\left(\overset{\checkmark}{\frac{\cos^6 ax}{6}} - \overset{\checkmark}{\frac{\cos^8 ax}{8}}\right) + c$

$= -\frac{1}{6a}\cos^6 ax + \frac{1}{8a}\cos^8 ax + c$

or

$y = \int \cos^5 ax.\sin^3 ax dx$

$= \int \cos^5 ax.\sin^2 ax.\sin ax dx \checkmark$

$= \int \cos^5 ax.(1 - \overset{\checkmark}{\cos^2 ax})\sin ax dx$

$= \int \overset{\checkmark}{\cos^5 ax.\sin ax dx} - \int \overset{\checkmark}{\cos^7 ax.\sin ax dx}$

$= \overset{\checkmark}{-\frac{1}{a}}\int \cos^5 ax.-a\sin ax dx + \overset{\checkmark}{\frac{1}{a}}\int \cos^7 ax.-a\sin ax dx$

$= -\frac{1}{a}.\overset{\checkmark}{\frac{\cos^6 ax}{6}} + \frac{1}{a}.\overset{\checkmark}{\frac{\cos^8 ax}{8}} + c$

$= -\frac{1}{6a}\cos^6 ax + \frac{1}{8a}\cos^8 ax + c$ (8)

2.2 $y = \int \frac{1}{\cot^4 \frac{x}{3}} dx$

$= \int \tan^4 \frac{x}{3} dx$ ✓

$= \int (\tan^2 \frac{x}{3})^2 dx$ ✓

✓

$= \int (\sec^2 \frac{x}{3} - 1)(\tan^2 \frac{x}{3}) dx$

$= \int \sec^2 \frac{x}{3} . \tan^2 \frac{x}{3} dx - \int \tan^2 \frac{x}{3} dx$ ✓

$= 3\int \frac{1}{3} \sec^2 \frac{x}{3} . (\tan \frac{x}{3})^2 dx - \int \tan^2 \frac{x}{3} dx$

✓ ✓ ✓ ✓

$= 3. \frac{\tan^3 \frac{x}{3}}{3} - \left[\frac{1}{\frac{1}{3}} \tan \frac{x}{3} - x \right] + c$

Or $= \tan^3 \frac{x}{3} - 3\tan \frac{x}{3} + x + c$ (8)

2.3 $y = \int x^2 . 2^{3x} dx$

$f(x) = x^2$	$g'(x) = 2^{3x}$
$f'(x) = 2x$	$g(x) = \frac{2^{3x}}{3\ln 2}$

✓ ✓

$= x^2 . \frac{2^{3x}}{3\ln 2} - \int 2x . \frac{2^{3x}}{3\ln 2} dx$

$= x^2 . \frac{2^{3x}}{3\ln 2} - \frac{2}{3\ln 2} \int x . 2^{3x} dx$ ✓

$f(x) = x$	$g'(x) = 2^{3x}$
$f'(x) = 1$	$g(x) = \frac{2^{3x}}{3\ln 2}$

✓ ✓

$= x^2 . \frac{2^{3x}}{3\ln 2} - \frac{2}{3\ln 2} \left[x . \frac{2^{3x}}{3\ln 2} - \int \frac{2^{3x}}{3\ln 2} dx \right]$

✓ ✓

$= x^2 . \frac{2^{3x}}{3\ln 2} - \frac{2}{(3\ln 2)^2} . x . 2^{3x} + \frac{2}{(3\ln 2)^2} \int 2^{3x} dx$

✓

$= x^2 . \frac{2^{3x}}{3\ln 2} - \frac{2x . 2^{3x}}{(3\ln 2)^2} + \frac{2}{(3\ln 2)^2} . \frac{2^{3x}}{3\ln 2} + c$

$= x^2 . \frac{2^{3x}}{3\ln 2} - \frac{2x . 2^{3x}}{9\ln^2 2} + \frac{2 . 2^{3x}}{27\ln^3 2} + c$ (8)

2.4

$$y = \int \frac{1}{1 - x + 2x^2}\,dx$$

$$= 2x^2 - x + 1$$

$$= 2\left(x^2 - \frac{x}{2} + \frac{1}{2}\right) \quad ✓$$

$$= 2\left[\left(x - \frac{1}{4}\right)^2 + \frac{1}{2} - \frac{1}{16}\right] \quad ✓ \quad ✓$$

$$= 2\left[\left(x - \frac{1}{4}\right)^2 + \frac{7}{16}\right] \quad ✓$$

$$or = 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8} \qquad or = 2(x - 0{,}25)^2 + 0{,}875$$

$$= \int \frac{1}{1 - x + 2x^2}\,dx$$

$$= \int \frac{1}{2\left[\left(x - \frac{1}{4}\right)^2 + \frac{7}{16}\right]}\,dx \quad ✓$$

$$= \frac{1}{2}\int \frac{1}{\frac{7}{16} + \left(x - \frac{1}{4}\right)^2}\,dx \quad ✓$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{7}{16}}}\tan^{-1}\frac{\left(x - \frac{1}{4}\right)}{\sqrt{\frac{7}{16}}} + c \quad ✓ \quad ✓$$

$$or = \frac{2}{\sqrt{7}}\tan^{-1}\frac{\left(x - \frac{1}{4}\right)}{\frac{\sqrt{7}}{4}} + c$$

$$or = 0{,}756\tan^{-1}\frac{(x - 0{,}25)}{0{,}661} + c$$

$$or = \frac{2}{\sqrt{7}}\arctan\frac{4x - 1}{\sqrt{7}} + c$$

$$or \quad \int \frac{1}{\frac{7}{8} + 2\left(x - \frac{1}{4}\right)^2}\,dx \quad ✓$$

$$= \frac{1}{\sqrt{\frac{7}{8}}\cdot\sqrt{2}}\tan^{-1}\frac{\sqrt{2}\left(x - \frac{1}{4}\right)}{\sqrt{\frac{7}{8}}} + c \quad ✓✓ \quad ✓$$

Alternative

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

$$= 2\left(x + \frac{-1}{2(2)}\right) + \frac{4(2)(1) - (-1)^2}{4(2)}$$

$$= 2(x - 1/4)^2 + \frac{8 - 1}{8}$$

$$= 2(x - 1/4)^2 + \frac{7}{8}$$

2.5 $y = \int \cot^{-1} x\,dx$

$f(x) = \cot^{-1} x$	$g'(x) = 1$
$f'(x) = \frac{-1}{x^2+1}$	$g(x) = x$

✓ ✓

$= x.\cot^{-1} x - \int x.\frac{-1}{x^2+1}dx$

$= x.\cot^{-1} x + \frac{1}{2}\int \frac{2x}{x^2+1}dx$ ✓

$= x.\cot^{-1} x + \frac{1}{2}\ln(x^2+1) + c$ ✓

(4)

[36]

QUESTION 3

3.1 $\int \frac{-x^2+3x+1}{(1-2x)^3}\,dx$

$\frac{-x^2+3x+1}{(1-2x)^3} = \frac{A}{(1-2x)^3} + \frac{B}{(1-2x)^2} + \frac{C}{(1-2x)}$ ✓

$-x^2+3x+1 = A + B(1-2x) + C(1-2x)^2$ ✓

$-x^2+3x+1 = A + B - 2Bx + C - 4Cx + 4Cx^2$ ✓

let $x = \frac{1}{2}$ $\therefore A = \frac{9}{4}$ (2,25) ✓

Equate coeff of x^2: $\therefore C = -\frac{1}{4}$ (−0,25) ✓

Equate coeff of x: $\therefore B = -1$ ✓

$= \int \frac{\frac{9}{4}}{(1-2x)^3}dx + \int \frac{-1}{(1-2x)^2}dx + \int \frac{-\frac{1}{4}}{(1-2x)}dx$ ✓

$= -\frac{1}{2}.\frac{9}{4}\int (1-2x)^{-3}dx - \left(-\frac{1}{2}\right)\int (1-2x)^{-2}dx - \left(-\frac{1}{2}\right)\frac{1}{4}\int \frac{-2}{(1-2x)}dx$

✓ ✓ ✓

$= -\frac{9}{8}.\frac{(1-2x)^{-2}}{-2} + \frac{1}{2}.\frac{(1-2x)^{-1}}{-1} + \frac{1}{8}\ln(1-2x) + c$

$= \frac{9}{16(1-2x)^2} - \frac{1}{2(1-2x)} + \frac{1}{8}\ln(1-2x) + c$

(10)

3.2 $$\int \frac{x^2-29x+5}{(x-4)(x^2+3)x}\,dx$$

$$\frac{x^2-29x+5}{(x-4)(x^2+3)x}=\frac{A}{(x-4)}+\frac{Bx+C}{(x^2+3)}+\frac{D}{x} \quad ✓$$

$$x^2-29x+5=Ax(x^2+3)+x(Bx+C)(x-4)+D(x-4)(x^2+3) \quad ✓✓$$

$$x^2-29x+5=Ax^3+3Ax+Bx^3+Cx^2-4Bx^2-4Cx+Dx^3-4Dx^2+3Dx-12D \quad ✓✓$$

$$let \quad x=4 \quad \therefore\ A=-\frac{95}{76}=-\frac{5}{4} \quad (-1{,}25) \quad ✓$$

$$let \quad x=0 \quad \therefore\ D=-\frac{5}{12} \quad (-0{,}417) \quad ✓$$

$$\text{Equate coeff of } x^3: \quad B=\frac{5}{3} \quad (1{,}667) \quad ✓$$

$$\text{Equate } x^2: \quad C=6 \quad ✓$$

$$=\int \frac{-\frac{5}{4}}{(x-4)}dx+\int \frac{\frac{5}{3}x+6}{(x^2+3)}dx+\int \frac{-\frac{5}{12}}{x}dx \quad ✓$$

$$=-\frac{5}{4}\int \frac{1}{(x-4)}dx+\int \frac{\frac{5}{3}x}{(x^2+3)}dx+\int \frac{6}{(x^2+3)}dx-\frac{5}{12}\int \frac{1}{x}dx \quad ✓$$

✓ ✓ ✓✓ ✓ ✓

$$=-\frac{5}{4}\ln(x-4)+\frac{5}{3}.\frac{1}{2}\ln(x^2+3)+6.\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}-\frac{5}{12}\ln x+c$$

$$=-1{,}25\ln(x-4)+0{,}833\ln(x^2+3)+3{,}464\tan^{-1}\frac{x}{1{,}732}-0{,}416\ln x+c$$

(14)
[24]

QUESTION 4

4.1

$$x\frac{dy}{dx} - y = x\ln x$$

$$\frac{dy}{dx} - \frac{y}{x} = \ln x \quad ü$$

$$R = e^{\int -\frac{1}{x}dx} \quad ü$$
$$= e^{-\ln x} \quad ü$$
$$= x^{-1} \; or \; \frac{1}{x} \quad ü$$

ü ü

$$\frac{1}{x}.y = \int \frac{1}{x}.\ln x.dx$$

$$G/S: \frac{y}{x} = \frac{\ln^2 x}{2} + \quad ü$$

$$\frac{3}{e} = \frac{(\ln e)^2}{2} + c \quad ü$$

$$c = 0{,}604 \quad ü$$

$$P/S: \frac{y}{x} = \frac{\ln^2 x}{2} + 0{,}6 \quad ü$$

(10)

4.2 $\left(\frac{1}{2}\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=2x^2$

$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=4x^2$ ✓

$m^2+2m+2=0 \quad \therefore\ m=\frac{-2\pm\sqrt{4-4(2)}}{2} \quad \therefore\ m=-1\pm j$ ✓

$y_c=e^{-x}(A\cos x+B\sin x)$

$\therefore\ Find\ \ y_p: \quad y=Cx^2+Dx+E$ ✓

$\frac{dy}{dx}=2Cx+D$ ✓

$\frac{d^2y}{dx^2}=2C$ ✓

$\therefore\ 2C+2(2Cx+D)+2(Cx^2+Dx+E)=4x^2$ ✓

$2C+4Cx+2D+2Cx^2+2Dx+2E=4x^2$

Equate coefficients of x^2: $2C=4 \quad \therefore\ C=2$ ✓

Equate coefficients of x: $4C+2D=0 \quad \therefore\ D=-4$ ✓

Equate constants: $2C+2D+2E=0 \quad \therefore\ E=2$ ✓

$\therefore\ y_p=2x^2-4x+2$ ✓

$G/S: y=e^{-x}(A\cos x+B\sin x)+2x^2-4x+2$

$1=e^{-0}(A\cos 0+B\sin 0)+2 \quad \therefore\ A=-1$ ✓

$\frac{dy}{dx}=e^{-x}(-A\sin x+B\cos x)-e^{-x}(A\cos x+B\sin x)+4x-4$ ✓

$1=e^{-0}(-A\sin 0+B\cos 0)-e^{-0}(A\cos 0+B\sin 0)-4 \quad \therefore\ B=4$ ✓

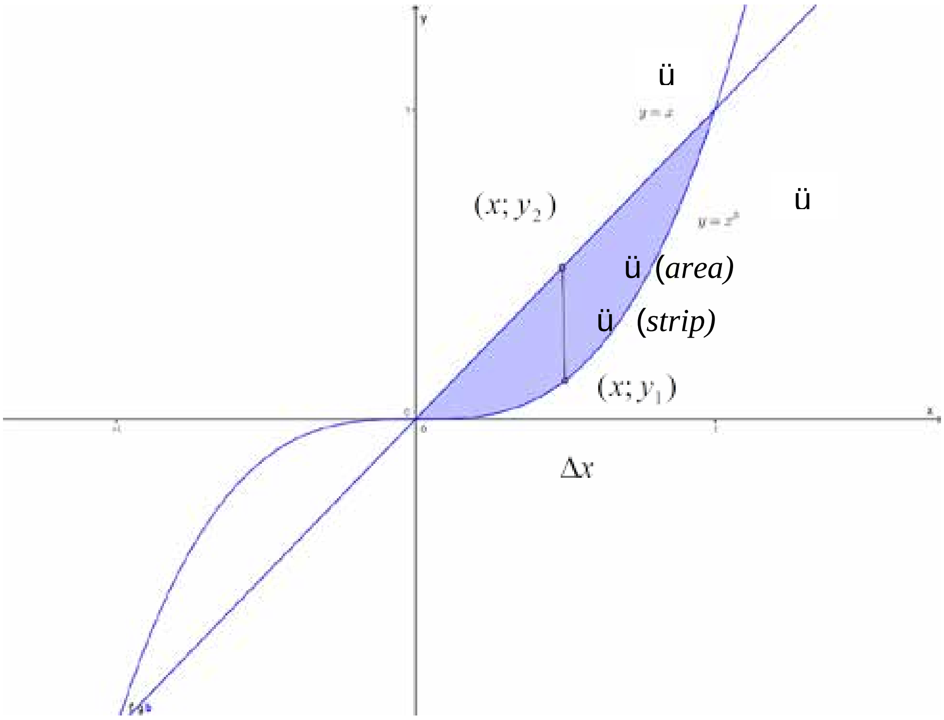
$\therefore\ P/S: y=e^{-x}(-\cos x+4\sin x)+2x^2-4$ ✓

(14)
[24]

QUESTION 5

5.1 5.1.1

$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x = 0; \quad x = 1; \ ✓ \quad x = -1$$
$$y = 0; \quad y = 1: \ ✓ \quad x = -1 \qquad \therefore \ (0;0), (1;1) \text{ and } (-1;-1)$$



(6)

5.1.2

$$\Delta V_y = 2\pi x \ ✓ \times (y_2 - y_1) \ ✓ \times \Delta x$$

$$V_y = 2\pi \int_0^1 x(y_2 - y_1)dx \ ✓$$

Incorrect limits: max 5 marks

$$= 2\pi \int_0^1 x(x - x^3)dx \quad ✓$$

$$= 2\pi \int_0^1 (x^2 - x^4)dx \quad ✓$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 \quad ✓$$

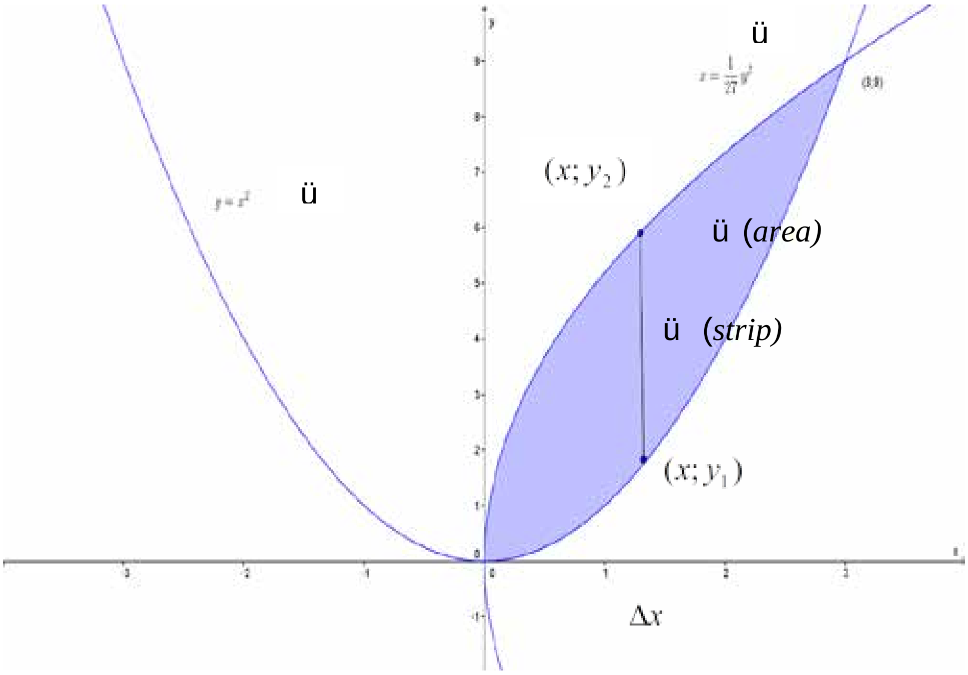
$$= 2\pi \left[\frac{(1)^3}{3} - \frac{(1)^5}{5}\right] \quad ✓$$

$$= \frac{4}{15}\pi \ \text{units}^3 \quad or \quad 0{,}267\pi \quad or \quad 0{,}838 \ \text{units}^3 \quad ✓$$

(8)

5.2 5.2.1

$$x^2 = \sqrt{27x}$$
$$x^4 = 27x$$
$$x^4 - 27x = 0$$
$$x(x^3 - 27) = 0$$
$$\therefore \ x = 0; \quad x^3 = 27$$
$$\therefore \ x = 3 \ ✓$$
$$\therefore \ y = 0; \quad y = 9 \quad ✓ \qquad \text{or} \qquad (0;0) \text{ and } (3;9)$$



(6)

5.2.2

$$\Delta A = (y_2 - y_1)\Delta x \quad ✓$$

✓

$$A = \int_0^3 (y_2 - y_1)dx$$

> *Incorrect limits: max 3 marks*

$$= \int_0^3 \left(\sqrt{27x} - x^2\right)dx \quad ✓$$

$$= \int_0^3 \left(\sqrt{27}x^{\frac{1}{2}} - x^2\right)dx$$

$$= \left[\frac{\sqrt{27}x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^3 \quad ✓$$

$$= \left[\frac{2}{3}.\sqrt{27}(3)^{\frac{3}{2}} - \frac{(3)^3}{3}\right] \quad ✓$$

$$= 9 \text{ units}^2 \quad ✓$$

(6)

5.2.3

$$A_{m-x} = (y_2 - y_1)\Delta x \times \frac{y_2 + y_1}{2} \quad ✓ ✓$$

$$= \frac{1}{2}\int_0^3 (y_2^{\,2} - y_1^{\,2})\,dx \quad ✓$$

Incorrect limits: max 6 marks

$$= \frac{1}{2}\int_0^3 [(27x) - (x^2)^2]\,dx \quad ✓$$

$$= \frac{1}{2}\int_0^3 (27x - x^4)\,dx \quad ✓$$

$$= \frac{1}{2}\left[\frac{27x^2}{2} - \frac{x^5}{5}\right]_0^3 \quad ✓$$

$$= \frac{1}{2}\left\{\left[\frac{27(3)^2}{2} - \frac{(3)^5}{5}\right] - \left[\frac{27(0)^2}{2} - \frac{(0)^5}{5}\right]\right\} \quad ✓$$

$$= 36{,}45 \quad \text{units}^3 \quad ✓$$

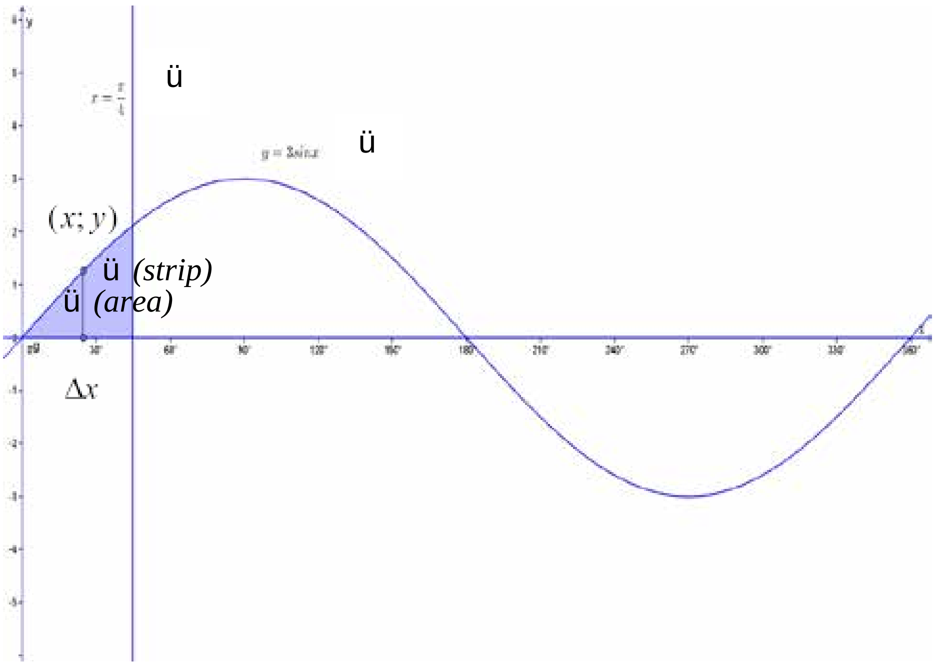
$$\bar{y} = \frac{A_{m-x}}{A}$$

$$= \frac{36{,}45}{9} \quad ✓$$

$$= 4{,}05 \text{ units} \quad ✓$$

(10)

5.3 5.3.1



(4)

5.3.2

$$\Delta V_x = \pi y^2 \Delta x \quad ✓$$

$$V_x = \pi \int_0^{\frac{\pi}{4}} y^2 dx \quad ✓$$

$$= \pi \int_0^{\frac{\pi}{4}} (3\sin^2 x)dx \quad ✓$$

$$= 9\pi \int_0^{\frac{\pi}{4}} (\sin^2 x)dx$$

$$= 9\pi \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{4}} \quad ✓$$

$$= 9\pi \left[\frac{\frac{\pi}{4}}{2} - \frac{\sin 2(\frac{\pi}{4})}{4}\right] \quad ✓$$

$$= 1{,}284\pi \text{ units}^3 \quad or \quad 4{,}035 \text{ units}^3 \quad ✓$$

(6)

5.3.3

$$\therefore \Delta I_x = \rho.\pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}}\right)^2 \quad ✓ \; ✓$$

$$I_x = \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} y^4 dx \quad ✓$$

$$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} (9\sin^2 x)^2 dx \quad ✓$$

$$= 81\frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} (\sin^2 x)^2 dx$$

$$= 81\frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 dx \quad ✓$$

$$= 81\frac{\rho\pi}{2}\int_0^{\frac{\pi}{4}} \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 2x^2\right)dx \quad ✓$$

$$= 81\frac{\rho\pi}{2}.\frac{1}{4}\int_0^{\frac{\pi}{4}} (1 - 2\cos 2x + \cos 2x^2)dx$$

$$\left(or \; 81\frac{\rho\pi}{2}.\frac{1}{4}\int_0^{\frac{\pi}{4}} \left(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right)dx\right)$$

✓ ✓

$$= \frac{81\rho\pi}{8}\left[x - \frac{2\sin 2x}{2} + \frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{81\rho\pi}{8}\left[\frac{\pi}{4} - \frac{2\sin 2(\frac{\pi}{4})}{2} + \frac{\frac{\pi}{4}}{2} + \frac{\sin 4(\frac{\pi}{4})}{8}\right] \quad ✓$$

$$= 6{,}866\pi\rho \quad \text{units}^4 \quad or \quad 21{,}569\rho \quad \text{units}^4 \quad ✓$$

(10)

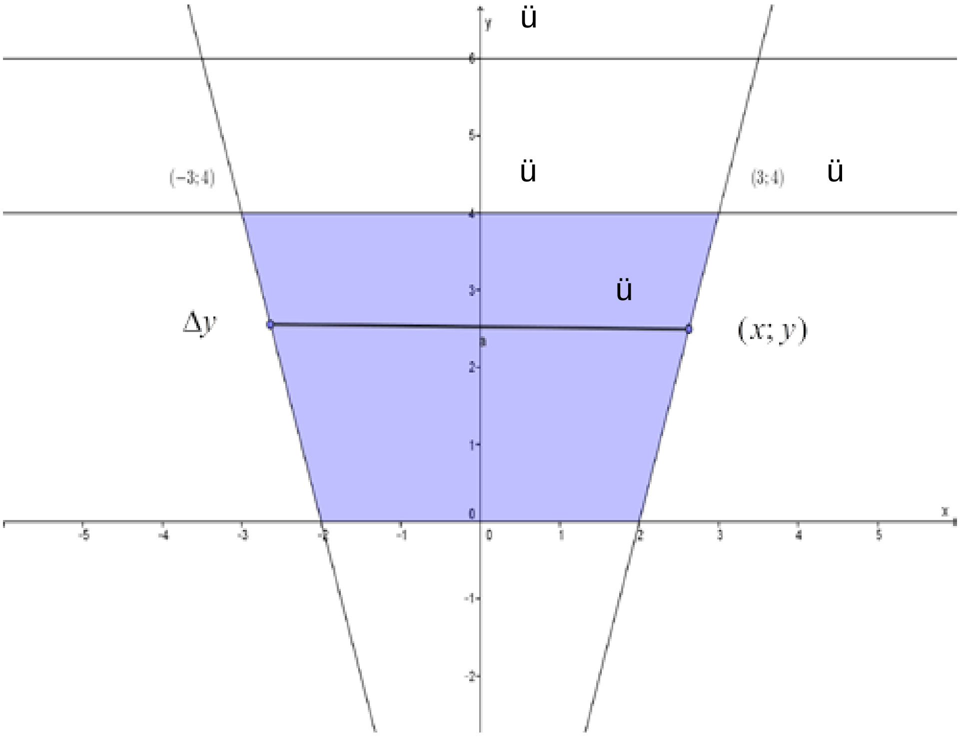
5.4 5.4.1

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x-2}=\frac{4-0}{3-2}\ ✓$$

$$y=4(x-2)$$

$$\therefore\ x=\frac{y}{4}+2\ ✓ \qquad \therefore\ dA=2(\frac{y}{4}+2)dy \quad \text{or} \quad \therefore\ dA=(\frac{y}{2}+4)dy$$



(6)

5.4.2

$$\int_0^2 r\,dA$$

✓ ✓ ✓

Incorrect limits: max 5 marks

$$=\int_0^4 (6-y)2(\frac{y}{4}+2)dy$$

$$=2\int_0^4 (\frac{6y}{4}-\frac{y^2}{4}-2y+12)dy\ ✓$$

$$=2\int_0^4 (\frac{3y}{2}-\frac{y^2}{4}-2y+12)dy \quad \text{or} \quad =2\int_0^4 (-\frac{y}{2}-\frac{y^2}{4}+12)dy$$

✓ ✓

$$=2\left[\frac{3y^2}{4}-\frac{y^3}{12}-\frac{2y^2}{2}+12y\right]_0^4$$

$$=2\left[\frac{3(4)^2}{4}-\frac{(4)^3}{12}-\frac{2(4)^2}{2}+12(4)\right]\ ✓$$

$= 77,333$ units^3 ✓ (8)

5.4.3 ✓ ✓ ✓

$$= \int_0^4 (6-y)^2\, 2\left(\frac{y}{4}+2\right)dy$$

Incorrect limits: max 6 marks

$$= 2\int_0^4 (6-y)^2\left(\frac{y}{4}+2\right)dy$$

$$= 2\int_0^4 \left(-15y+72-y^2+\frac{y^3}{4}\right)dy \ ✓$$

✓ ✓

$$= 2\left[-\frac{15y^2}{2}+72y-\frac{y^3}{3}+\frac{y^4}{16}\right]_0^4$$

$$= 2\left[-\frac{15(4)^2}{2}+72(4)-\frac{(4)^3}{3}+\frac{(4)^4}{16}\right] ✓$$

$= 325{,}333 \quad \text{units}^4$ ✓

$$\bar{y} = \frac{325{,}333}{77{,}333} \ ✓$$

$= 4{,}207$ units ✓ (10)

Alternative method

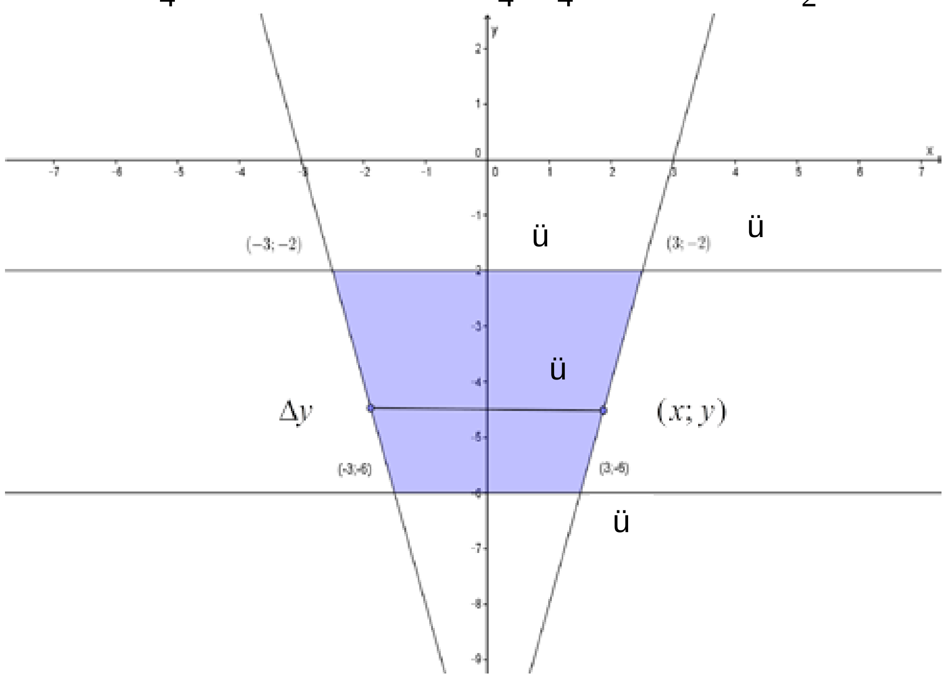
5.4 5.4.1

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-(-2)}{x-3} = \frac{-6-(-2)}{2-3}$$

$$y = 4x-14 \ ✓$$

$$\therefore \ x = \frac{1}{4}(y+14) \ ✓ \qquad \therefore \ dA = 2\left(\frac{y}{4}+\frac{14}{4}\right)dy \quad \text{or} \therefore \ dA = \left(\frac{y}{2}+7\right)dy$$



(6)

5.4.2 $\checkmark \ \checkmark \quad \checkmark$

$$\int_{-6}^{-2} y\left(\frac{y}{2}+7\right)dy$$

$$= \int_{-6}^{-2}\left(\frac{y^2}{2}+7y\right)dy \ \checkmark$$

Incorrect limits: max 5 marks

$\checkmark \quad \checkmark$

$$= \left[\frac{y^3}{6}+\frac{7y^2}{2}\right]_{-6}^{-2}$$

$$= \left[\frac{(-2)^3}{6}+\frac{7(-2)^2}{2}\right]-\left[\frac{(-6)^3}{6}+\frac{7(-6)^2}{2}\right] \checkmark$$

$= -77{,}333 \text{ units}^3 \checkmark$ (8)

5.4.3 $\checkmark \ \checkmark \quad \checkmark$

$$\int_{-6}^{-2} y^2\left(\frac{y}{2}+7\right)dy$$

$$= \int_{-6}^{-2}\left(\frac{y^3}{2}+7y^2\right)dy \ \checkmark$$

Incorrect limits: max 6 marks

$\checkmark \quad \checkmark$

$$= \left[\frac{y^4}{8}+\frac{7y^3}{3}\right]_{-6}^{-2}$$

$$= \left[\frac{(-2)^4}{8}+\frac{7(-2)^3}{3}\right]-\left[\frac{(-6)^4}{8}+\frac{7(-6)^3}{3}\right] \checkmark$$

$= 325{,}333 \text{ units}^3 \checkmark$

$$\bar{y} = \frac{324{,}333}{-77{,}333} \ \checkmark$$

$= -4{,}207 \text{ units} \checkmark$ (10)

[80]

QUESTION 6

6.1

$$x = 2e^t \sin t \qquad\qquad y = 2e^t \cos t$$

$$\frac{dx}{dt} = 2e^t \cos t + 2e^t \sin t \ \checkmark \qquad\qquad \frac{dy}{dt} = 2e^t(-\sin t) + 2e^t \cos t \ \checkmark$$

$$\left(\frac{dx}{dt}\right)^2 = [2e^t \cos t + 2e^t \sin t]^2 \ \checkmark \qquad\qquad \left(\frac{dy}{dt}\right)^2 = [2e^t(-\sin t) + 2e^t \cos t]^2 \ \checkmark$$

$$= 4e^{2t}[\cos t + \sin t]^2 \qquad\qquad = 4e^{2t}[(-\sin t) + \cos t]^2$$

$$= 4e^{2t}[\cos^2 t + 2\sin t \cos t + \sin^2 t] \ \checkmark \qquad\qquad = 4e^{2t}[\sin^2 t - 2\cos t \sin t + \cos^2 \ \checkmark$$

$$= 4e^{2t}[1 + 2\sin t \cos t] \qquad\qquad = 4e^{2t}[1 - 2\cos t \sin t]$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4e^{2t}[1 + 2\sin t \cos t] + 4e^{2t}[1 - 2\cos t \sin t] \ \checkmark$$

$$= 4e^{2t}[2]$$

$$= 8e^{2t} \ \checkmark$$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,.dt \ \checkmark\checkmark$$

$$s = \int_0^{\frac{\pi}{2}} \sqrt{8e^{2t}}\, dt$$

$$S = \int_0^{\frac{\pi}{2}} \sqrt{8}\,.e^t\, dt \ \checkmark$$

$$= \sqrt{8}\left[e^t\right]_0^{\frac{\pi}{2}} \ \checkmark$$

$$= \sqrt{8}\left[e^{\frac{\pi}{2}} - e^0\right] \ \checkmark$$

$$= 10{,}778 \ \text{ units} \checkmark$$

(14)

6.2

$$x = 3y^3$$

$$\frac{dx}{dy} = 9y^2 \checkmark$$

$$\left(\frac{dx}{dy}\right)^2 = \left(9y^2\right)^2 \checkmark$$

$$1+\left(\frac{dx}{dy}\right)^2 = 1+81y^4$$

$$Ay = 2\pi\int_a^b x.\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,.dy \checkmark \checkmark$$

$$A = 2\pi\int_1^2 x\sqrt{1+81y^4}\,dy \checkmark \checkmark$$

$$= 2\pi\int_1^2 3y^3(1+81y^4)^{\frac{1}{2}}\,dy$$

$$= \frac{6\pi}{324}\int_1^2 324y^3(1+81y^4)^{\frac{1}{2}}\,dy \checkmark$$

$$= \frac{6\pi}{324}\left[\frac{(1+81y^4)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2 \checkmark \quad \text{or} \quad = 0{,}0185\pi\left[\frac{(1+81y^4)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2$$

$$= \frac{6\pi}{324}.\frac{2}{3}\left[(1+81(2)^4)^{\frac{3}{2}} - (1+81(1)^4)^{\frac{3}{2}}\right] \checkmark$$

$$= 567{,}5\pi \text{ units}^2 \quad \text{or} \quad 1782{,}853 \text{ units}^2 \checkmark$$

(10)
[24]

TOTAL: 200

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Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T900**(E)**(N24)T
NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 November 2016 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Use only BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If z = $\tan(x^3y^2) + \mathrm{cosec}(xy^2)$ calculate the following:

1.1.1 $\frac{\partial z}{\partial x}$ (2)

1.1.2 $\frac{\partial z}{\partial y}$ (1)

1.2 The parametric equations of a function are given as $x = t^2$ and $y = 2t^5$.

Calculate the value of $\frac{d^2y}{dx^2}$ (3)

[6]

QUESTION 2

Determine $\int y\ dx$ if:

2.1 $y = \arcsin 3x$ (2)

2.2 $y = \frac{2}{\sec^4 2x}$ (4)

2.3 $y = \frac{1}{4x^2 + 12x + 24}$ (4)

2.4 $y = \mathrm{cosec}^5 4x.\cos^3 4x$ (4)

2.5 $y = x^3.\sin 2x$ (4)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{3x^2 - 3x + 1}{(2x+1)^2(x-1)}\ dx$ (6)

3.2 $\int \frac{x^4 + x^2 - 2}{x^3 + x}\ dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of

$$t\frac{dy}{dt} - 2y = t^2 - t + 1 \text{, if } t = 1 \text{ when } y = -\frac{1}{2}.$$ (5)

4.2 Calculate the particular solution of

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 6e^{\frac{x}{2}} \text{, if } y = 1 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 1 \text{ when } x = 0.$$ (7)

[12]

QUESTION 5

5.1 5.1.1 Make a neat sketch of the graph $y = 2\ln x$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, the line $y = 2$, the x-axis and the y-axis rotates about the x-axis. (2)

5.1.2 Calculate the volume generated if the area described in QUESTION 5.1.1 rotates about the x-axis. (5)

5.2 5.2.1 Calculate the points of intersection of the two curves $y = x + 6$ and $xy = -8$. Make a neat sketch of the two curves and show the area bounded by the curves in the second quadrant. Show the representative strip/element PARALLEL to the x-axis that you will use to calculate the area bounded by the curves. (3)

5.2.2 Calculate the area bounded by the two curves in the second quadrant described in QUESTION 5.2.1. (3)

5.2.3 Calculate the second moment of area when the area bounded by the two curves in the second quadrant described in QUESTION 5.2.1 is rotated about the x-axis. (4)

5.2.4 Express the answer in QUESTION 5.2.3 in terms of the area. (1)

5.3 5.3.1 Make a neat sketch of the graph $9x^2 + 4y^2 = 36$ and show the representative strip/element PERPENDICULAR to the x-axis that you will use to calculate the volume of the solid generated when the area bounded by the curve for $0 \le x \le 2$ rotates about the x-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid obtained when the area described in QUESTION 5.3.1 rotates about the x-axis. (5)

5.4 5.4.1 A water canal in the shape of a parabola is 5 m deep, 10 m wide at the top and full of water. A vertical retaining wall is placed in the canal with its top 1 m below the water surface.

Sketch the retaining wall and show the representative strip/element that you will use to calculate the area moment of the wall about the water level.

Calculate the relation between the two variables x and y. (4)

5.4.2 Calculate, by using integration, the area moment of the retaining wall about the water level in QUESTION 5.4.1. (3)

5.4.3 Calculate, by using integration, the second moment of area of the retaining wall described in QUESTION 5.4.1, about the water level as well as the depth of the centre of pressure on the retaining wall. (5)

[40]

QUESTION 6

6.1 Calculate the length of the curve $y = 2x^2 - 4$ between $x = 0$ and $x = 2$. (5)

6.2 Calculate the surface area of revolution generated by rotating the two curves, $x = a\cos^3\theta$ and $y = a\sin^3\theta$ about the x-axis, between $\theta = \frac{\pi}{2}$ and $\theta = -\frac{\pi}{2}$. (7)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \qquad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx \;;\; A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy \;;\; A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx \; ; V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy \; ; V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \text{r}\, V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = r\int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2} r \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} r\rho \int_a^b y^4 dx \qquad I_y = \frac{1}{2} r\rho \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \square \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N6

24 NOVEMBER 2016

This marking guideline consists of 20 pages.

✓ full mark

TOTAL: $\frac{200}{2} = 100$

NOTE: Do NOT subtract marks for incorrect units or units omitted

QUESTION 1

1.1 1.1.1 $z = \tan(x^3y^2) + \operatorname{cosec}(xy^2)$

$$\frac{\partial z}{\partial x} = 3x^2y^2\sec^2(x^3y^2) - y^2\operatorname{cosec}(xy^2)\cot(xy^2)$$ ✓✓ ✓ ✓✓ ✓✓ (4)

1.1.2 $$\frac{\partial z}{\partial y} = 2x^3y\sec^2(x^3y^2) - 2xy\operatorname{cosec}(xy^2)\cot(xy^2)$$ ✓✓ ✓✓ (2)

1.2 $x = t^2$ $y = 2t^5$

$\frac{dx}{dt} = 2t$ ✓✓ $\frac{dy}{dt} = 10t^4$ ✓✓

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{10t^4}{2t}$$ ✓✓

$$= 5t^3$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(5t^3\right) \times \frac{1}{2t}$$ ✓✓

$$= 15t^2 \times \frac{1}{2t}$$ ✓✓

$$= \frac{15t}{2} \quad or \quad 7{,}5t$$ ✓✓

(6)
[12]

QUESTION 2

2.1 $y = \int \sin^{-1} 3x\,dx$

$f(x) = \sin^{-1} 3x$ $\quad g'(x) = 1$

$f'(x) = \dfrac{3}{\sqrt{1-(3x)^2}}$ $\quad g(x) = x$

$\checkmark \quad \checkmark$

$$= x.\sin^{-1} 3x - \int x.\frac{3}{\sqrt{1-9x^2}}\,dx$$

$$= x.\sin^{-1} 3x - 3\int x.(1-9x^2)^{-\frac{1}{2}}\,dx$$

$\checkmark \quad \checkmark$

$$= x.\sin^{-1} 3x + \frac{3}{18}.\frac{(1-9x^2)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= x.\sin^{-1} 3x + \frac{1}{3}\sqrt{1-9x^2} + c$$

(4)

2.2 $y = \int \dfrac{2}{\sec^4 2x}\,dx$

$$= \int 2\cos^4 2x\,dx \quad \checkmark$$

$$= 2\int \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)^2 dx \quad \checkmark$$

$$= 2\int \left(\frac{1}{2} + \frac{1}{2}\cos 4x\right)\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= 2\int \left(\frac{1}{4} + \frac{1}{2}\cos 4x + \frac{1}{4}\cos^2 4x\right) dx \quad \checkmark$$

$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$

$$= 2\left[\frac{1}{4}x + \frac{1}{2}.\frac{\sin 4x}{4} + \frac{1}{4}\left(\frac{x}{2} + \frac{\sin 8x}{16}\right)\right] + c$$

$$= 2\left[\frac{1}{4}x + \frac{\sin 4x}{8} + \frac{x}{8} + \frac{\sin 8x}{64}\right] + c$$

$$= \frac{1}{2}x + \frac{\sin 4x}{4} + \frac{x}{4} + \frac{\sin 8x}{32} + c$$

$$= \frac{3}{4}x + \frac{\sin 4x}{4} + \frac{\sin 8x}{32} + c$$

(8)

2.3

$$y = \int \frac{1}{4x^2+12x+24}\,dx$$

$$= 4x^2+12x+24$$

$$= 4(x^2+3x+6) \quad \checkmark$$

$$= 4\left[\left(x+\frac{3}{2}\right)^2 \overset{\checkmark}{} + 6 - \left(\frac{3}{2}\right)^2 \overset{\checkmark}{}\right]$$

$$= 4\left[\left(x+\frac{3}{2}\right)^2 + \frac{15}{4}\right] \quad \checkmark$$

$$= 15 + 4\left(x+\frac{3}{2}\right)^2 \quad \checkmark$$

$$\therefore \int \frac{1}{4x^2+12x+24}\,dx$$

$$= \int \frac{1}{15+4\left(x+\frac{3}{2}\right)^2}\,dx \quad \checkmark$$

$$= \overset{\checkmark}{\frac{1}{2\sqrt{15}}} \tan^{-1} \overset{\checkmark}{\frac{2(x+\frac{3}{2})}{\sqrt{15}}} + c \quad \text{or} \quad = 0{,}129\tan^{-1}\frac{2x+3}{3{,}873} + c$$

Or

$$y = \int \frac{1}{4x^2+12x+24}\,dx$$

$$= \int \frac{1}{4(x^2+3x+6)}\,dx \quad \checkmark$$

$$= \overset{\checkmark}{\frac{1}{4}} \int \frac{1}{x^2+3x+\underset{\checkmark}{\left(\frac{3}{2}\right)^2} - \left(\frac{3}{2}\right)^2 + 6}\,dx \quad \checkmark$$

$$= \frac{1}{4}\int \frac{1}{\underset{\checkmark}{\left(x+\frac{3}{2}\right)^2} + \underset{\checkmark}{\frac{15}{4}}}\,dx$$

$$= \frac{1}{4}\left[\overset{\checkmark}{\frac{1}{\sqrt{\frac{15}{4}}}} \tan^{-1} \overset{\checkmark}{\frac{(x+\frac{3}{2})}{\sqrt{\frac{15}{4}}}}\right] + c \qquad \text{or} \quad = \frac{1}{4}\left[\frac{1}{\frac{\sqrt{15}}{2}} \tan^{-1} \frac{(x+\frac{3}{2})}{\frac{\sqrt{15}}{2}}\right] + c$$

(8)

2.4

$$y = \int \operatorname{cosec}^5 4x.\cos^3 4x\,dx$$

$\checkmark$

$$= \int \operatorname{cosec}^5 4x.\cos^2 4x.\cos 4x\,dx$$

$\checkmark$

$$= \int \operatorname{cosec}^5 4x.(1 - \sin^2 4x).\cos 4x\,dx$$

$\checkmark$

$$= \frac{1}{4}\int \frac{1}{u^5}.(1 - u^2)\,du \quad \checkmark$$

$u = \sin 4x$

$du = 4\cos 4x\,dx$

$\checkmark$

$$= \frac{1}{4}\int \left(\frac{1}{u^5} - \frac{1}{u}\right)du$$

$$= \frac{1}{4}\int \left(u^{-5} - \frac{1}{u}\right)du$$

$\checkmark$ $\checkmark$

$$= \frac{1}{4}\left[\frac{u^{-4}}{-4} - \ln u\right] + c$$

$$= \frac{1}{4}\left[\frac{\sin^{-4} 4x}{-4} - \ln(\sin 4x)\right] + c \quad \checkmark$$

Or $= -\dfrac{1}{16\sin^4 4x} - \dfrac{1}{4}\ln(\sin 4x) + c$ or $-\dfrac{1}{16}\operatorname{cosec}^4 4x - \dfrac{1}{4}\ln(\sin 4x) + c$ (8)

2.5

$$y = \int x^3.\sin 2x\,dx$$

$f(x) = x^3$ $\quad g'(x) = \sin 2x$

$f'(x) = 3x^2$ $\quad g(x) = -\dfrac{\cos 2x}{2}$

$\checkmark$ $\checkmark$

$$\int y\,dx = x^3\left(-\frac{\cos 2x}{2}\right) - \int 3x^2\left(-\frac{\cos 2x}{2}\right)dx$$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{2}\int x^2.\cos 2x\,dx$$

$f(x) = x^2$ $\quad g'(x) = \cos 2x$

$f'(x) = 2x$ $\quad g(x) = \dfrac{\sin 2x}{2}$

$\checkmark$ $\checkmark$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{2}\left[x^2.\frac{\sin 2x}{2} - \int 2x.\frac{\sin 2x}{2}\,dx\right]$$

$\checkmark$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x - \frac{3}{2}\int x.\sin 2x\,dx$$

$f(x) = x$ $\quad g'(x) = \sin 2x$

$f'(x) = 1$ $\quad g(x) = -\dfrac{\cos 2x}{2}$

$\checkmark$ $\checkmark$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x - \frac{3}{2}\left[x.-\frac{\cos 2x}{2} - \int -\frac{\cos 2x}{2}\,dx\right]$$

$\checkmark$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2.\sin 2x + \frac{3}{4}x.\cos 2x - \frac{3}{4}.\frac{\sin 2x}{2} + c$$

$$= -\frac{1}{2}x^3\cos 2x + \frac{3}{4}x^2\sin 2x + \frac{3}{4}x\cos 2x - \frac{3}{8}\sin 2x + c$$

(8)

[36]

QUESTION 3

3.1 $$\int \frac{3x^2-3x+1}{(2x+1)^2(x-1)}\,dx$$

$$\frac{3x^2-3x+1}{(2x+1)^2(x-1)}=\frac{A}{(2x+1)^2}+\frac{B}{(2x+1)}+\frac{C}{(x-1)} \quad ✓$$

$$3x^2-3x+1=A(x-1)+B(2x+1)(x-1)+C(2x \quad ✓$$

$$Let\ \ x=-\frac{1}{2}; \quad \therefore\ A=-\frac{13}{6} \quad (2{,}167) \quad ✓$$

$$Let\ \ x=1; \quad \therefore\ C=\frac{1}{9} \quad (0{,}111) \quad ✓$$

$$3x^2-3x+1=Ax-A+2Bx^2-Bx-B+4Cx^2+4Cx+C \quad ✓$$

$$\text{Equate coeff of } x^2: \quad 3=2B+4C \quad \therefore\ B=\frac{23}{18} \quad (1{,}278) \quad ✓$$

$$=\int\frac{-\frac{13}{6}}{(2x+1)^2}dx+\int\frac{\frac{23}{18}}{(2x+1)}dx+\int\frac{\frac{1}{9}}{(x-1)}dx \quad ✓$$

$$=-\frac{13}{6}\int(2x+1)^{-2}dx+\frac{23}{18}\int\frac{1}{(2x+1)}dx+\frac{1}{9}\int\frac{1}{(x-1)}dx$$

$$=-\frac{13}{6}\left(\frac{1}{2}\right)\left[\frac{(2x+1)^{-1}}{-1}\right]+\frac{23}{18}\left(\frac{1}{2}\right)\ln(2x+1)+\frac{1}{9}\ln(x-1)+c \quad ✓✓✓✓✓$$

$$=\frac{13}{12(2x+1)}+\frac{23}{36}\ln(2x+1)+\frac{1}{9}\ln(x-1)+c$$

$$=\frac{1{,}083}{(2x+1)}+0{,}639\ln(2x+1)+0{,}111\ln(x-1)+c$$

(12)

3.2 $\displaystyle\int \frac{x^4+x^2-2}{x^3+x}\,dx$

$$\rightarrow \quad x^3+x \overline{\big)\, x^4+x^2-2}\quad \text{quotient } x \ ✓$$
$$\underline{x^4+x^2}$$
$$-2$$

$= \displaystyle\int \left(x - \frac{2}{x^3+x}\right)dx$ ✓

$= \displaystyle\int x\,dx - \int \frac{2}{x^3+x}\,dx$

$\therefore \dfrac{2}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$ ✓ ✓

$2 = A(x^2+1)+(Bx+C)x$ ✓

$\therefore$ If $x=0$; A $=2$ ✓

$2 = Ax^2+A+Bx^2+Cx$ ✓

Equate coeff of x^2 : $\therefore B=-2$ ✓

Equate coeff of x : $\therefore C=0$ ✓

$= \displaystyle\int x\,dx + \int \frac{2}{x}\,dx + \int \frac{-2x}{x^2+1}\,dx$

✓ ✓ ✓

$= \dfrac{x^2}{2} - 2\ln x + \ln(x^2+1)+c$

Or

(1 mark for long division)

$= \displaystyle\int x\,dx + \int \frac{-2}{x^3+x}\,dx$ ✓

$\therefore \dfrac{-2}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$ ✓ ✓

$-2 = A(x^2+1)+(Bx+C)x$ ✓

$\therefore$ If $x=0$; A $=-2$ ✓

$2 = Ax^2+A+Bx^2+Cx$ ✓

Equate coeff of x^2 : $\therefore B=2$ ✓

Equate coeff of x : $\therefore C=0$ ✓

$= \displaystyle\int x\,dx + \int \frac{-2}{x}\,dx + \int \frac{2x}{x^2+1}\,dx$

✓ ✓ ✓

$= \dfrac{x^2}{2} - 2\ln x + \ln(x^2+1)+c$ (12)

[24]

QUESTION 4

4.1

$$t\frac{dy}{dt} - 2y = t^2 - t + 1$$

$$\frac{dy}{dt} - \frac{2y}{t} = t - 1 + \frac{1}{t} \quad \checkmark$$

$$R = e^{\int -\frac{2}{t}dt} \quad \checkmark$$

$$= e^{-2\ln t} \quad \checkmark$$

$$= e^{\ln t^{-2}}$$

$$= t^{-2} \quad \checkmark$$

$$= \frac{1}{t^2}$$

$$\frac{1}{t^2}.y = \int \frac{1}{t^2}.(t - 1 + \frac{1}{t})dt \quad \checkmark$$

$$\frac{y}{t^2} = \int (\frac{1}{t} - t^{-2} + t^{-3})dt \quad \checkmark$$

$$\frac{y}{t^2} = \ln t - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} + c \quad \checkmark$$

$$\boxed{or \quad \frac{y}{t^2} = \ln t + \frac{1}{t} - \frac{1}{2t^2} + c}$$

$$\frac{-\frac{1}{2}}{(1)^2} = \ln(1) - \frac{(1)^{-1}}{-1} + \frac{(1)^{-2}}{-2} + c \quad \checkmark$$

$$\therefore \quad c = -1 \quad \checkmark$$

$$\frac{y}{t^2} = \ln t - \frac{t^{-1}}{-1} + \frac{t^{-2}}{-2} - 1 \quad \checkmark$$

$$or \quad \frac{y}{t^2} = \ln t + \frac{1}{t} - \frac{1}{2t^2} - 1$$

(10)

4.2 $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = 6e^{\frac{x}{2}}$

$y_c: \quad m^2 + 2m + 2 = 0$ ü

$m = \dfrac{-2 \pm \sqrt{4 - 4(2)}}{2}$

$m = -1 \pm j$ ü

$y_c = e^{-x}(A\cos x + B\sin x)$ ü

To find y_p $\therefore\ y = Ce^{\frac{x}{2}}$ ü

$\dfrac{dy}{dx} = \dfrac{1}{2}Ce^{\frac{x}{2}}$ ü

$\dfrac{d^2y}{dx^2} = \dfrac{1}{4}Ce^{\frac{x}{2}}$ ü

$\dfrac{1}{4}Ce^{\frac{x}{2}} + 2(\dfrac{1}{2}Ce^{\frac{x}{2}}) + 2(Ce^{\frac{x}{2}}) = 6e^{\frac{x}{2}}$ ü

$\dfrac{13}{4}Ce^{\frac{x}{2}} = 6e^{\frac{x}{2}}$

$\therefore\ C = \dfrac{24}{13}$ (1,846) ü

$\therefore\ y_p = \dfrac{24}{13}e^{\frac{x}{2}}$ ü

$y = e^{-x}(A\cos x + B\sin x) + \dfrac{24}{13}e^{\frac{x}{2}}$ ü

$1 = e^{-0}(A\cos 0 + B\sin 0) + \dfrac{24}{13}e^{\frac{0}{2}}$

$\therefore\ A = -\dfrac{11}{13}$ $(-0,846)$ ü

$\dfrac{dy}{dx} = e^{-x}(-A\sin x + B\cos x) - e^{-x}(A\cos x + B\sin x) + \dfrac{1}{2}.\dfrac{24}{13}e^{\frac{x}{2}}$ ü

$1 = e^{-0}(-A\sin 0 + B\cos 0) - e^{-0}(A\cos 0 + B\sin 0) + \dfrac{12}{13}e^{\frac{0}{2}}$

$\therefore\ B = -\dfrac{10}{13}$ $(-0,769)$ ü

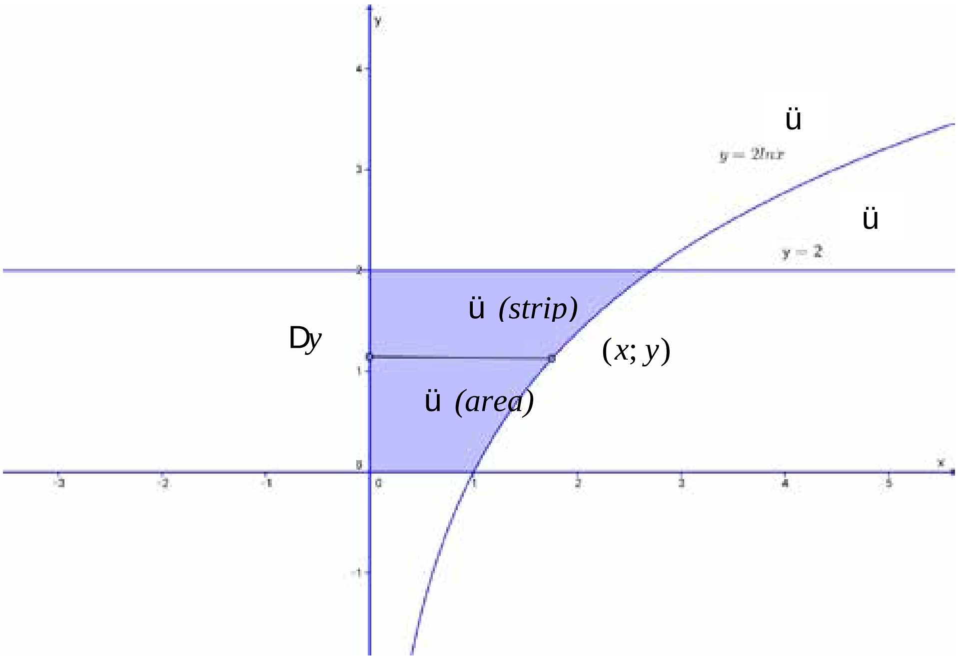
$\therefore\ y = e^{-x}(-\dfrac{11}{13}\cos x - \dfrac{10}{13}\sin x) + \dfrac{24}{13}e^{\frac{x}{2}}$ ü

$or \quad y = e^{-x}(-0,864\cos x - 0,769\sin x) + \dfrac{24}{13}e^{\frac{x}{2}}$ (14)

[24]

QUESTION 5

5.1 5.1.1



(4)

5.1.2

$$\Delta V_x = 2\pi y x \times \Delta y \quad ✓ ✓$$

$$V_x = 2\pi \int_0^2 xy\,dy \quad ✓$$

$$= 2\pi \int_0^2 e^{\frac{y}{2}}.y\,dy \quad ✓$$

Incorrect limits: max 6 marks

$$= 2\pi \left[y.2e^{\frac{y}{2}} - \int 2e^{\frac{y}{2}}dy \right]_0^2 \quad ✓ ✓$$

$$= 2\pi \left[2ye^{\frac{y}{2}} - 2\int e^{\frac{y}{2}}dy \right]_0^2$$

$$= 2\pi \left[2ye^{\frac{y}{2}} - 2.\frac{e^{\frac{y}{2}}}{\frac{1}{2}} \right]_0^2 \quad ✓$$

$f(y) = y$ $f'(y) = 1$	$g'(y) = e^{\frac{y}{2}}$ $g(y) = \frac{e^{\frac{y}{2}}}{\frac{1}{2}} = 2e^{\frac{y}{2}}$

$$= 4\pi \left[ye^{\frac{y}{2}} - 2e^{\frac{y}{2}} \right]_0^2$$

$$= 4\pi \left\{ \left[2e^{\frac{2}{2}} - 2e^{\frac{2}{2}} \right] - \left[0 - 2e^{\frac{0}{2}} \right] \right\} \quad ✓ ✓$$

$$= 8\pi \quad units^3 \quad or \quad 25{,}133 \quad units^3 \quad ✓$$

(10)

5.2 5.2.1

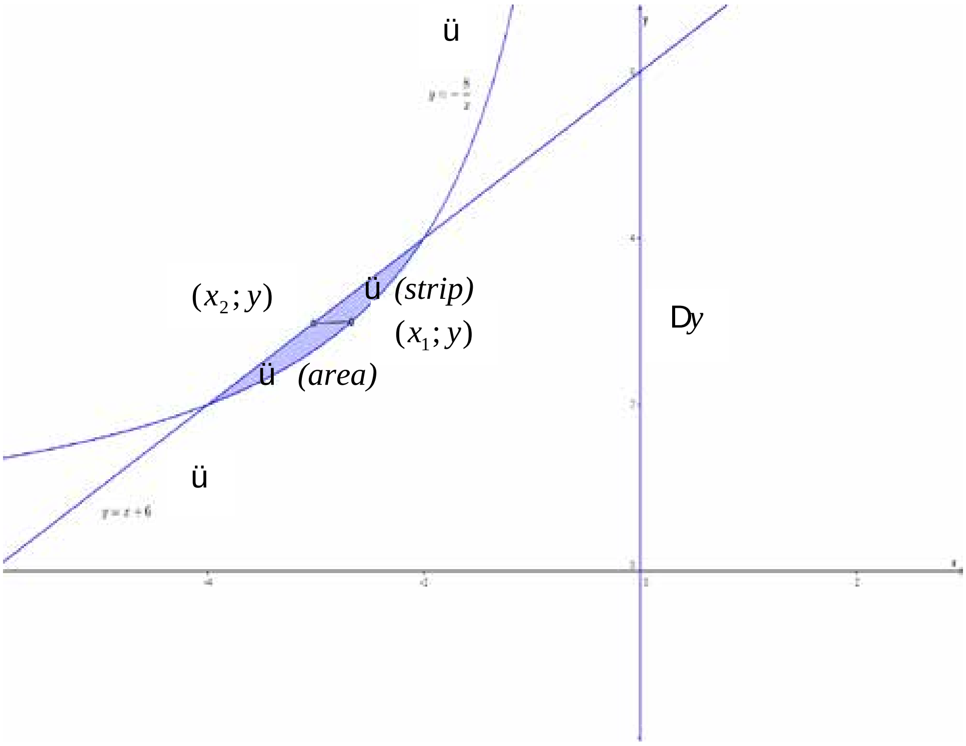
$$x+6=-\frac{8}{x}$$

$$x^2+6x+8=0$$

$$(x+4)(x+2)=0$$

$$x=-4;\quad x=-2 \quad ✓$$

$$y=2;\quad y=4 \quad ✓ \quad \therefore\ (-4;2) \text{ and } (-2;4)$$



(6)

5.2.2

$$\Delta A=(x_2-x_1)\Delta y \quad ✓$$

✓

$$A=\int_2^4 (x_2-x_1)\,dy$$

Incorrect limits: max 3 marks

$$=\int_2^4\left(-\frac{8}{y}-(y-6)\right)dy \quad ✓$$

$$=\left[-8\ln y-\frac{y^2}{2}+6y\right]_2^4 \quad ✓$$

$$=\left[-8\ln 4-\frac{4^2}{2}+6(4)\right]-\left[-8\ln 2-\frac{2^2}{2}+6(2)\right] \quad ✓$$

$$=0{,}455\ units^2 \quad ✓$$

(6)

5.2.3 $\therefore \Delta I = (x_2 - x_1)\Delta y \times y^2$ ✓ ✓ ✓

$I = \int_2^4 \left(-\frac{8}{y} - y + 6\right) y^2\,dy$ ✓

Incorrect limits: max 5 marks

$= \int_2^4 (-8y - y^3 + 6y^2)\,dy$ ✓

$= \left[-\frac{8y^2}{2} - \frac{y^4}{4} + \frac{6y^3}{3}\right]_2^4$ ✓

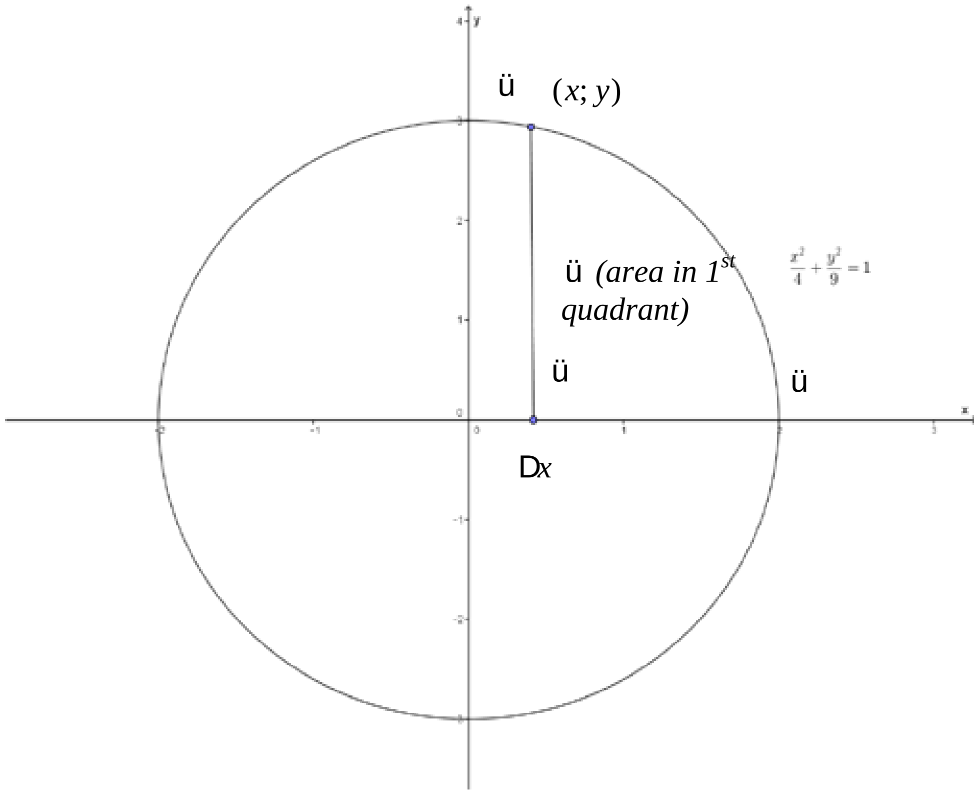
$= \left[-4(4)^2 - \frac{(4)^4}{4} + 2(4)^3\right] - \left[-4(2)^2 - \frac{(2)^4}{4} + 2(2)^3\right]$ ✓

$= 4\ units^4$ ✓ (8)

5.2.4 $I = \frac{4}{0{,}455} \times A$ ✓

$= 8{,}781\,A(units^4)$ ✓ (2)

5.3 5.3.1



(4)

5.3.2

$\Delta V_x = \pi y^2 \Delta x$ ✓

✓

$V_x = \pi \int_0^2 y^2\,dx$

Incorrect limits: max 3 marks

$= \pi \int_0^2 9\left(1 - \frac{x^2}{4}\right)dx$ ✓

$= 9\pi \int_0^2 \left(1 - \frac{x^2}{4}\right)dx \quad or \quad \frac{9}{4}\pi \int_0^2 (4 - x^2)\,dx$

$= 9\pi \left[x - \frac{x^3}{12}\right]_0^2$ ✓

$= 9\pi \left[2 - \frac{2^3}{12}\right]$ ✓

$= \frac{36}{3}\pi \quad or\ 12\pi \quad units^3 \quad or \quad 37{,}699 \quad units^3$ ✓

(6)

5.3.3

$\Delta M = \rho . \Delta V_x$

$= \rho . \pi y^2 \Delta x$

✓ ✓

$\therefore \ \Delta I_x = \rho . \pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}}\right)^2$

✓

$I_x = \frac{\rho \pi}{2} \int_0^2 y^4\,dx$ ✓

Incorrect limits: max 7 marks

$= \frac{\rho \pi}{2} \int_0^2 \left[9\left(1 - \frac{x^2}{4}\right)\right]^2 dx$ ✓

$= \frac{\rho \pi}{2} \int_0^2 \left[9\left(\frac{4 - x^2}{4}\right)\right]^2 dx$

$= \frac{81\rho \pi}{16.2} \int_0^2 (4 - x^2)^2\,dx$ ✓

$= \frac{81\rho \pi}{32} \int_0^2 (16 - 8x^2 + x^4)\,dx$ ✓

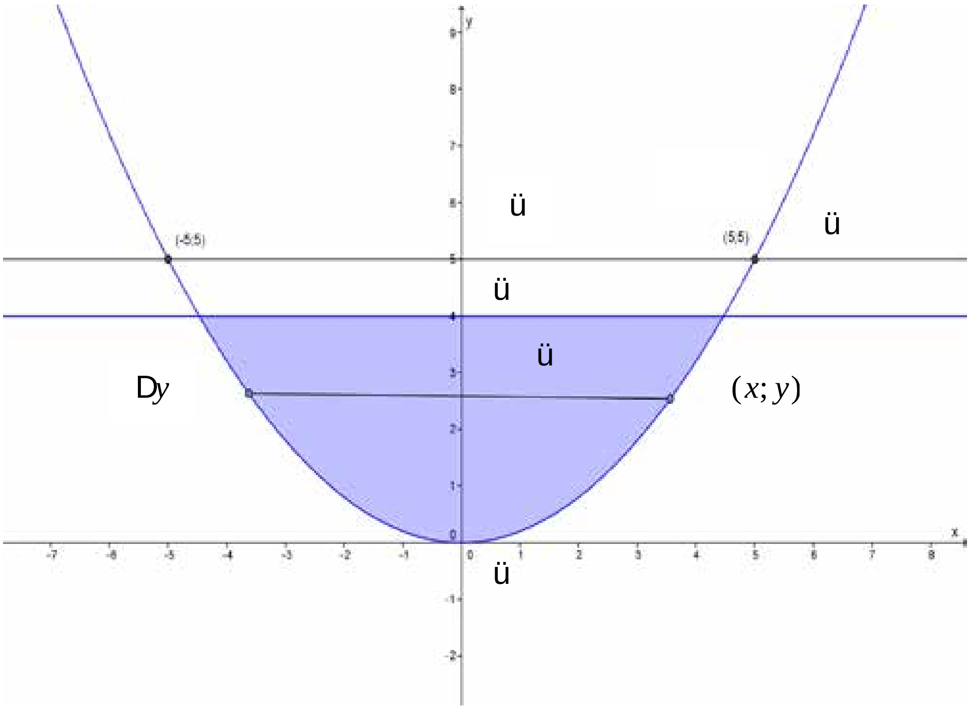
$= \frac{81\rho \pi}{32} \left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]_0^2$ ✓

$= \frac{81\rho \pi}{32} \left[16(2) - \frac{8(2)^3}{3} + \frac{(2)^5}{5}\right]$ ✓

$= 43{,}2\pi\rho \quad units^4 \quad or \quad 135{,}717\rho$ ✓

(10)

5.4 5.4.1



$y = ax^2$

$5 = a(5)^2 \quad a = \frac{1}{5}$ ✓

$y = \frac{1}{5}x^2$ ✓ ∴ $x = \sqrt{5y}$ or $x = \sqrt{5}y^{\frac{1}{2}}$ ✓ ∴ $dA = 2(\sqrt{5}y^{\frac{1}{2}})dy$ (8)

5.4.2 $\int_0^4 r\,dA$

✓ ✓ ✓

$= \int_0^4 (5 - y)2\sqrt{5}y^{\frac{1}{2}}dy$

Incorrect limits: max 4 marks

$= 2\sqrt{5}\int_0^4 (5y^{\frac{1}{2}} - y^{\frac{3}{2}})dy$ ✓

$= 2\sqrt{5}\left[\frac{5y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4$ ✓

$= 2\sqrt{5}\left[\frac{2}{3}.5(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}}\right]$

$= 62{,}014$ units^3 ✓ (6)

5.4.3 $\int_0^4 r^2 dA$

✓ ✓ ✓

$= \int_0^4 (5-y)^2 \, 2\sqrt{5}\, y^{\frac{1}{2}} dy$

Incorrect limits: max 6 marks

$= 2\sqrt{5}\int_0^4 (25-10y+y^2)\, y^{\frac{1}{2}} dy$ ✓

$= 2\sqrt{5}\int_0^4 (25y^{\frac{1}{2}} - 10y^{\frac{3}{2}} + y^{\frac{5}{2}})\, dy$ ✓

$= 2\sqrt{5}\left[\frac{25y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{10y^{\frac{5}{2}}}{\frac{5}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4$ ✓

$= 2\sqrt{5}\left[\frac{2}{3}.25(4)^{\frac{3}{2}} - \frac{2}{5}.10(4)^{\frac{5}{2}} + \frac{2}{7}(4)^{\frac{7}{2}}\right]$ ✓

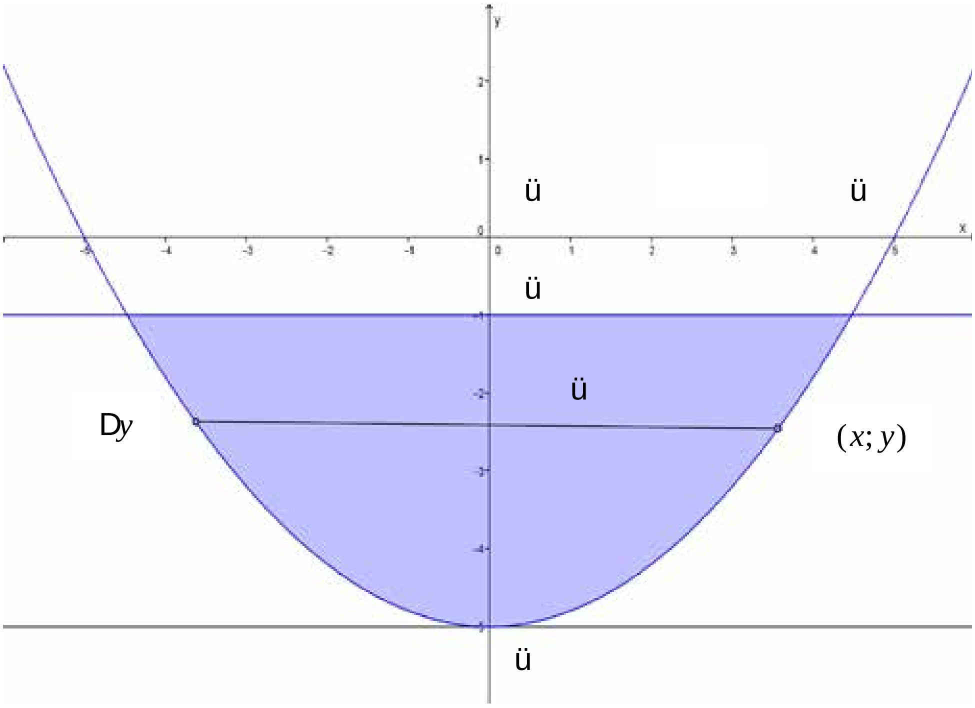
$= 187{,}404$ units^4 ✓

$\bar{y} = \frac{187{,}404}{62{,}014}$ ✓

$= 3{,}022$ *units* ✓ (10)

Alternative method

5.4 5.4.1



$y = ax^2 + bx - 5$

$0 = a(5)^2 + b(5) - 5$......(5;0)

\ $1 = 5a + 6$....(1)

$0 = a(-5)^2 + b(-5) - 5$......(-5;0)

\ $1 = 5a - 6$....(2)

ü

\ $a = \frac{1}{5}$ *and* $b = 0$

$y = \frac{1}{5}x^2 - 5$ ü

\ $x = \sqrt{5y + 25}$ or $x = \sqrt{5}(y+5)^{\frac{1}{2}}$ ü \ $dA = 2[\sqrt{5}(y+5)^{\frac{1}{2}}]dy$ (8)

5.4.2 $\int_{-5}^{-1} r\,dA$

✓ ✓ ✓

$$= \int_{-5}^{-1} y.2\sqrt{5}(y+5)^{\frac{1}{2}}\,dy$$

Incorrect limits: max 4 marks

$$= 2\sqrt{5}\int_{-5}^{-1} y(y+5)^{\frac{1}{2}}\,dy$$

Let $u = y+5$
$du = dy$
$y = u - 5$

$$= 2\sqrt{5}\int_{-5}^{-1} (u-5)(u)^{\frac{1}{2}}\,du \ ✓$$

$$= 2\sqrt{5}\int_{-5}^{-1} (u^{\frac{3}{2}} - 5u^{\frac{1}{2}})\,du$$

$$= 2\sqrt{5}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 5.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{y=-5}^{y=-1} \ ✓$$

$$= 2\sqrt{5}\left[\frac{2}{5}(y+5)^{\frac{5}{2}} - 5.\frac{2}{3}(y+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

$$= 2\sqrt{5}\left[\frac{2}{5}(-1+5)^{\frac{5}{2}} - 5.\frac{2}{3}(-1+5)^{\frac{3}{2}}\right]_{-5}^{-1}$$

$= -62,014$ units^3 ✓ (6)

5.4.3 $\int_{0}^{4} r^2 \, dA$

✓ ✓ ✓

$= \int_{-5}^{-1} y^2 . 2\sqrt{5}(y+5)^{\frac{1}{2}} \, dy$

Incorrect limits: max 6 marks

$= 2\sqrt{5}\int_{-5}^{-1} y^2 (y+5)^{\frac{1}{2}} \, dy$ ✓

Let $u = y+5$
$du = dy$
$y = (u-5)^2$

$= 2\sqrt{5}\int_{-5}^{-1} (u-5)^2 (u)^{\frac{1}{2}} \, du$ ✓

$= 2\sqrt{5}\int_{-5}^{-1} (u^2 - 10u + 25)u^{\frac{1}{2}} \, du$

$= 2\sqrt{5}\int_{-5}^{-1} (u^{\frac{5}{2}} - 10u^{\frac{3}{2}} + 25u^{\frac{1}{2}}) \, du$

$= 2\sqrt{5}\left[\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{10u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{25u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{y=-5}^{y=-1}$ ✓

$= 2\sqrt{5}\left[\frac{(y+5)^{\frac{7}{2}}}{\frac{7}{2}} - \frac{10(y+5)^{\frac{5}{2}}}{\frac{5}{2}} + \frac{25(y+5)^{\frac{3}{2}}}{\frac{3}{2}}\right]_{5}^{-1}$ ✓

$= 2\sqrt{5}\left[\frac{2}{7}(-1+5)^{\frac{7}{2}} - \frac{2}{5}.10(-1+5)^{\frac{5}{2}} + \frac{2}{3}.25(-1+5)^{\frac{3}{2}}\right]$

$= 187{,}404$ units^4 ✓

$\bar{y} = \frac{187{,}404}{-62{,}014}$ ✓

$= -3{,}022$ *units* ✓ (10)

[80]

QUESTION 6

Incorrect limits: max 7 marks

6.1

$$y = 2x^2 - 4$$

$$\frac{dy}{dx} = 4x \quad ✓$$

$$\left(\frac{dy}{dx}\right)^2 = (4x)^2 \quad ✓$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+(4x)^2 \quad ✓$$

$$= 1+16x^2$$

✓

$$S = \int_0^2 \sqrt{1+16x^2}\,dx \quad ✓$$

$$= \int_0^2 \sqrt{16\left(\frac{1}{16}+x^2\right)}\,dx \quad ✓$$

✓

$$= 4\int_0^2 \sqrt{\left(x^2+\frac{1}{16}\right)}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{x^2+\frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(x+\sqrt{x^2+\frac{1}{16}}\right)\right]_0^2 \quad ✓$$

$$= 4\left\{\left[\frac{2}{2}\sqrt{2^2+\frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(2+\sqrt{2^2+\frac{1}{16}}\right)\right] - \left[\frac{0}{2}\sqrt{0^2+\frac{1}{16}} + \left(\frac{\frac{1}{16}}{2}\right)\ln\left(0+\sqrt{0^2+\frac{1}{16}}\right)\right]\right\} \quad ✓$$

$= 8{,}409$ units ✓ (10)

6.2 $x = a\cos^3\theta$ and $y = a\sin^3\theta$

$\dfrac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$ ✓ $\qquad \dfrac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$ ✓

$\left(\dfrac{dx}{d\theta}\right)^2 = (-3a\cos^2\theta\sin\theta)^2$ ✓ $\qquad \left(\dfrac{dy}{d\theta}\right)^2 = (3a\sin^2\theta\cos\theta)^2$ ✓

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = (-3a\cos^2\theta\sin\theta)^2 + (3a\sin^2\theta\cos\theta)^2 \ ✓$$

$$= 9a^2\cos^4\theta\sin^2\theta + 9a^2\sin^4\theta\cos^2\theta$$

$$= 9a^2\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)$$

$$= 9a^2\cos^2\theta\sin^2\theta \ ✓$$

$$A = 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta \ ✓✓$$

Incorrect limits: max 11 marks

$$= 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\sin^3\theta\sqrt{9a^2\cos^2\theta\sin^2\theta}\,d\theta \ ✓$$

$$= 2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a\sin^3\theta . 3a\cos\theta\sin\theta\ d\theta \ ✓$$

$$= 6a^2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4\theta.\cos\theta\ d\theta \ ✓$$

$$= 6a^2\pi\left[\frac{(\sin\theta)^5}{5}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \ ✓$$

$$= 6a^2\pi\left[\frac{(\sin\frac{\pi}{2})^5}{5} - \frac{(\sin-\frac{\pi}{2})^5}{5}\right] \ ✓$$

$$= \frac{12}{5}a\pi \text{ units}^2 \text{ or } 2{,}4a\pi \text{ or } 7{,}54a \text{ units}^2 \ ✓$$

(14)
[24]

200 ÷ 2

TOTAL: 100

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T900(**E**)(A5)T
AUGUST EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

5 August 2016 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order but subsections of questions must be kept together.
5. Show ALL intermediate steps.
6. ALL formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink
8. Write neatly and legibly.

QUESTION 1

1.1 If z = $\cos^4(3x-4y)$ determine $\dfrac{\partial z}{\partial y}$. (2)

1.2 Calculate $\dfrac{d^2y}{dx^2}$ of the parametric equations $x = \ln u^3$ and $y = e^{3u}$ at the point where $u = 1$. (4)
[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \sqrt{67+4x-2x^2}$ (4)

2.2 $y = x^2.\arctan x$ (3)

2.3 $y = \tan^3 7x.\tan^2 7x$ (5)

2.4 $y = \cos^4 \dfrac{x}{2}.\sin^3 \dfrac{x}{2}$ (4)

2.5 $y = ax\ln x$ (2)
[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{x^2+x-1}{x^2(x^2-5x+6)}\,dx$ (5)

3.2 $\displaystyle\int \frac{x^2-5x+3}{(x+3)(x^2+6)}\,dx$ (7)
[12]

QUESTION 4

4.1 Calculate the particular solution of :

$$\left(\frac{\sin x}{\cos x}\right)\frac{dy}{dx} = y + \frac{1}{\cos x} \quad \text{if} \quad y = 2 \text{ when } x = \frac{\pi}{5}$$ (6)

4.2 Calculate the general solution of:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3e^{-2x}$$ (6)

[12]

QUESTION 5

5.1 5.1.1 Sketch the graph of $y = 2e^{2x}$ and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the graph $x = 0$, $y = 0$ and $x = 1$ rotates about the y-axis. (2)

5.1.2 Calculate the volume described in QUESTION 5.1.1. (5)

5.2 5.2.1 Make a neat sketch of the graph $y = 3\cos\frac{x}{2}$ and show the representative strip/element that you will use to calculate the area bounded by the graph, the x-axis, the y-axis and the line $x = \pi$. (2)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance of the centroid from the x-axis of the bounded area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Calculate the points of intersection of the TWO curves $y - 5x = 0$ and $x^2 = \frac{y}{5}$. Make a neat sketch of the TWO curves and show the area bounded by the curves. Show the representative strip/element that you will use to calculate the volume of the solid generated when the bounded area rotates about the x-axis. (3)

5.3.2 Calculate the magnitude of the volume described in QUESTION 5.3.1 by means of integration. (4)

5.3.3 Calculate the moment of inertia if the bounded area described in QUESTION 5.3.1 is rotated about the x-axis. (5)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the mass. (1)

5.4 5.4.1 A vertical weir in a rectangular canal is 3 m high and 4 m wide. The top of the weir is in the water surface.

Make a neat sketch of the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the weir.

Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate, by using integration, the area moment of the weir about the water surface. (3)

5.4.3 Calculate, by using integration, the second moment of area of the weir about the water surface, as well as the depth of the centre of pressure on the weir. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the parabola $10y = x^2$ from the origin to the point where $x = 4$. (6)

6.2 Calculate the surface area generated when the curve given by the parametric equations $x = e^t$ and $y = 2e^{2t}$ rotates about the y-axis between $t = 0$ and $t = 1$. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left\lvert\tan\left(\frac{ax}{2}\right)\right\rvert+C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\,.\,f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\,f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\,f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x)\,.\,f'(x)$	-
$\sec f(x)$	$\sec f(x)\tan f(x)\,.\,f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x)\cot f(x)\,.\,f'(x)$	-
$\sin^{-1} f(x)$	$\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\dfrac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\dfrac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\dfrac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\dfrac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\dfrac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\dfrac{x}{2}-\dfrac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\dfrac{x}{2}+\dfrac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\dfrac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int \left[f(x)\right]^n\, f'(x)\, dx = \frac{\left[f(x)\right]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln\, f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2} \sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2\pi \int_a^b xydy$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2\pi \int_a^b xydx$$

AREA MOMENTS

$A_{m-x} = rdA$ $\qquad$ $A_{m-y} = rdA$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \therefore ye^{\int P dx} = \int Qe^{\int P dx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

AUGUST EXAMINATION

MATHEMATICS N6

5 AUGUST 2016

This marking guideline consists of 17 pages.

✓= ½ MARK TOTAL: $\frac{200}{2}=100$

NOTE: Do not subtract marks for incorrect units or units omitted

QUESTION 1

1.1 $z = \cos^4(3x-4y)$

✓ ✓ ✓ ✓

$$\frac{\partial z}{\partial y} = 4\cos^3(3x-4y).-(-4)\sin(3x-4y)$$

$$= 16\cos^3(3x-4y)\sin(3x-4y)$$ (4)

1.2 $x = \ln u^3$ $y = e^{3u}$ at the point where $u = 1$

$\frac{dx}{du} = \frac{3}{u}$ ✓ $\frac{dy}{du} = 3e^{3u}$ ✓

$$\frac{dy}{dx} = \frac{\frac{dy}{du}}{\frac{dx}{du}} = \frac{3e^{3u}}{\frac{3}{u}}$$ ✓

$$\therefore \frac{dy}{dx} = e^{3u}.u$$ ✓

$$\frac{d^2y}{dx^2} = \frac{d}{du}\left(\frac{dy}{dx}\right)\times\frac{du}{dx}$$

$$= \frac{d}{du}\left(e^{3u}.u\right)\times\frac{u}{3}$$ ✓

✓

$$= (e^{3u}+3e^{3u}.u)\times\frac{u}{3}$$

$$= \frac{e^{3u}.u+3e^{3u}.u^2}{3}$$

$$= \frac{e^3+3e^3}{3}$$ ✓

$$= 26{,}781 \text{ or } = \frac{4e^3}{3}$$ ✓

(8)
[12]

QUESTION 2

2.1

$$y = \int \sqrt{67 + 4x - 2x^2}\,dx$$

$$= -2x^2 + 4x + 67$$

$$= -2\left(x^2 - 2x - \frac{67}{2}\right) \quad ✓$$

✓ ✓

$$= -2\left[(x-1)^2 - \frac{67}{2} - 1\right]$$

$$= -2\left[(x-1)^2 - \frac{69}{2}\right] \quad ✓$$

$$= 2\left[\frac{69}{2} - (x-1)^2\right] \quad or \quad = 2[34{,}5 - (x-1)^2]$$

$$= \int \sqrt{2\left[\frac{69}{2} - (x-1)^2\right]}\,dx \quad ✓$$

✓ ✓ ✓

$$= \sqrt{2}\left[\frac{69}{2}\sin^{-1}\frac{(x-1)}{\sqrt{\frac{69}{2}}} + \frac{(x-1)}{2}\sqrt{\frac{69}{2} - (x-1)^2}\right] + c$$

$$= \frac{69\sqrt{2}}{2}\sin^{-1}\frac{(x-1)}{\sqrt{\frac{69}{2}}} + \frac{\sqrt{2}(x-1)}{2}\sqrt{\frac{69}{2} - (x-1)^2} + c$$

OR

$$= 48{,}79\sin^{-1}\frac{(x-1)}{5{,}874} + 0{,}707(x-1)\sqrt{34{,}5 - (x-1)^2} + c$$

(8)

2.2

$$y=\int x^2\arctan x.dx$$

$$f(x)=\tan^{-1}x \qquad g'(x)=x^2$$

$$f'(x)=\frac{1}{x^2+1}\ \checkmark \qquad g(x)=\frac{x^3}{3}\ \checkmark$$

$$=\frac{x^3}{3}.\tan^{-1}x-\int\left(\frac{1}{x^2+1}\right)\left(\frac{x^3}{3}\right)dx$$

$$=\frac{x^3}{3}.\tan^{-1}x-\frac{1}{3}\int\frac{x^3}{x^2+1}dx$$

$$=\frac{x^3}{3}.\tan^{-1}x-\frac{1}{3}\int\left(x\ \checkmark-\frac{x}{x^2+1}\ \checkmark\right)dx$$

$$=\frac{x^3}{3}.\tan^{-1}x-\frac{1}{3}.\frac{x^2}{2}\ \checkmark+\frac{1}{6}\ln(x^2+1)+c\ \checkmark$$

$$\rightarrow\quad \begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & \underline{x^3+x} \\ & -x \end{array}$$

(6)

2.3

$$y=\int\tan^3 7x.\tan^2 7xdx$$

$$=\int\tan^3 7x.(\sec^2 7x-1)dx\ \checkmark$$

$$=\int\tan^3 7x.\sec^2 7x)dx\ \checkmark-\int\tan^3 7xdx\ \checkmark$$

$$=\frac{1}{7}\int(\tan 7x)^3.7\sec^2 7xdx\ \checkmark-\int\tan^2 7x.\tan 7xdx\ \checkmark$$

$$=\frac{1}{7}.\frac{\tan^4 7x}{4}\ \checkmark-\int(\sec^2 7x-1).\tan 7xdx\ \checkmark$$

$$=\frac{\tan^4 7x}{28}-\int\sec^2 7x.\tan 7xdx+\int\tan 7xdx\ \checkmark$$

$$=\frac{\tan^4 7x}{28}-\frac{1}{7}.\frac{\tan^2 7x}{2}\ \checkmark+\frac{1}{7}\ln(\sec 7x)+c\ \checkmark$$

$$=\frac{\tan^4 7x}{28}-\frac{\tan^2 7x}{14}+\frac{1}{7}\ln(\sec 7x)+c$$

(10)

2.4

$$y = \int \cos^4 \frac{x}{2}.\sin^3 \frac{x}{2}dx$$

$$= \int \cos^4 \frac{x}{2}.\sin^2 \frac{x}{2}.\sin\frac{x}{2}dx \quad \checkmark$$

$$\checkmark$$

$$= \int \cos^4 \frac{x}{2}.(1 - \cos^2 \frac{x}{2})\sin\frac{x}{2}dx$$

$$u = \cos\frac{x}{2}$$

$$\therefore \quad \frac{du}{dx} = -\frac{1}{2}\sin\frac{x}{2}$$

$$\therefore \quad du = -\frac{1}{2}\sin\frac{x}{2}dx$$

$$\checkmark \quad \checkmark$$

$$= -2\int u^4(1 - u^2)du$$

$$= -2\int (u^4 - u^6)du \quad \checkmark$$

$$= -2\left(\frac{u^5}{5} - \frac{u^7}{7}\right) + c \quad \checkmark$$

$$\checkmark \qquad \checkmark$$

$$= -2\left(\frac{\cos^5 \frac{x}{2}}{5} - \frac{\cos^7 \frac{x}{2}}{7}\right) + c$$

$$= -\frac{2\cos^5 \frac{x}{2}}{5} + \frac{2\cos^7 \frac{x}{2}}{7} + c$$

OR

$$y = \int \cos^4 \frac{x}{2}.\sin^3 \frac{x}{2}dx$$

$$\checkmark$$

$$= \int \cos^4 \frac{x}{2}.\sin^2 \frac{x}{2}.\sin\frac{x}{2}dx$$

$$\checkmark$$

$$= \int \cos^4 \frac{x}{2}.(1 - \cos^2 \frac{x}{2})\sin\frac{x}{2}dx$$

$$\checkmark \qquad \checkmark$$

$$= \int \cos^4 \frac{x}{2}\sin\frac{x}{2}dx - \int \cos^6 \frac{x}{2}\sin\frac{x}{2}dx$$

$$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= -2.\frac{\cos^5 \frac{x}{2}}{5} - (-2)\frac{\cos^7 \frac{x}{2}}{7} + c$$

$$= -\frac{2\cos^5 \frac{x}{2}}{5} + \frac{2\cos^7 \frac{x}{2}}{7} + c$$

(8)

2.5 $y = \int ax \ln x dx$

$f(x) = \ln x \qquad g'(x) = ax$

$f'(x) = \frac{1}{x} \qquad g(x) = \frac{ax^2}{2}$

$\checkmark \qquad \checkmark$

$$= \ln x \left(\frac{ax^2}{2}\right) - \int \frac{1}{x}\left(\frac{ax^2}{2}\right) dx$$

$$= \frac{ax^2}{2}.\ln x - \frac{a}{2}\int x dx \ \checkmark$$

$$= \frac{ax^2}{2}.\ln x - \frac{a}{2}.\frac{x^2}{2} + c \ \checkmark$$

$$= \frac{ax^2}{2}.\ln x - \frac{ax^2}{4} + c$$

(4)

[36]

QUESTION 3

3.1 $\int \frac{x^2 + x - 1}{x\ (x^2 - 5x + 6)}\ dx$

$$\frac{x^2 + x - 1}{x(x^2 - 5x + 6)} = \frac{x^2 + x - 1}{x\ (x - 3)(x - 2)} = \frac{A}{x} + \frac{B}{(x - 3)} + \frac{C}{(x - 2)} \quad \checkmark$$

$$x^2 + x - 1 = A(x - 3)(x - 2) + Bx(x - 2) + Cx(x - 3) \quad \checkmark$$

$$x^2 + x - 1 = Ax^2 - 5Ax + 6A + Bx^2 - 2B + Cx^2 - 3Cx \quad \checkmark$$

$let \quad x = 0 \quad \therefore \ A = -\frac{1}{6} \ (-0{,}167) \ \checkmark$

$x = 3 \quad \therefore \ B = \frac{11}{3} \ (3{,}667) \ \checkmark$

$x = 2 \quad \therefore \ C = -\frac{5}{2} \ (2{,}5) \ \checkmark$

$$= \int \frac{-\frac{1}{6}}{x} dx + \int \frac{\frac{11}{3}}{(x - 3)} dx + \int \frac{-\frac{5}{2}}{(x - 2)} dx \quad \checkmark$$

$\checkmark \qquad \checkmark \qquad \checkmark$

$$= -\frac{1}{6}\ln x + \frac{11}{3}\ln(x - 3) - \frac{5}{2}\ln(x - 2) + c$$

$$= -0{,}167\ln x + 3{,}667\ln(x - 3) - 2{,}5\ln(x - 2) + c$$

(10)

3.2 $\int \frac{x^2-5x+3}{(x-3)(x^2+6)}\,dx \quad \therefore \quad \frac{x^2-5x+3}{(x-3)(x^2+6)} = \overset{\checkmark}{\frac{A}{(x-3)}} + \frac{Bx+C}{(x^2+6)} \checkmark$

$$x^2-5x+3 = A(x^2+6)+(Bx+C)(x-3) \checkmark$$

$$x^2-5x+3 = Ax^2+6A+Bx^2+Cx-3Bx-3C \checkmark$$

$$let \quad x=3 \quad \therefore \; A=-\frac{1}{5} \quad \left(-\frac{3}{15} \quad or \quad -0{,}2\right) \checkmark$$

$$Equate \text{ cooeff of } x^2: \quad B=\frac{6}{5} \quad (1{,}2) \checkmark \quad Equate \quad x: \quad C=-\frac{7}{5} \quad (1{,}4) \checkmark$$

$$=\int \frac{-\frac{1}{5}}{(x-3)}dx+\int \frac{\frac{6}{5}x-\frac{7}{5}}{(x^2+6)}dx \checkmark$$

$$=-\frac{1}{5}\int \frac{1}{(x-3)}dx+\int \frac{\frac{6}{5}x}{(x^2+6)} \checkmark \int \frac{-\frac{7}{5}}{(x^2+6)}dx$$

$$=\overset{\checkmark}{-\frac{1}{5}\ln(x-3)}+\overset{\checkmark}{\frac{6}{5}}.\overset{\checkmark}{\frac{1}{2}\ln(x^2+6)}-\overset{\checkmark}{\frac{7}{5}.\frac{1}{\sqrt{6}}}\overset{\checkmark}{\tan^{-1}\frac{x}{\sqrt{6}}}+c$$

$$=-0{,}2\ln(x-3)+0{,}6\ln(x^2+6)-0{,}572\tan^{-1}\frac{x}{2{,}45}+c$$

(14)
[24]

QUESTION 4

4.1

$$\left(\frac{\sin x}{\cos x}\right)\frac{dy}{dx} = y + \frac{1}{\cos x}$$

$$\frac{dy}{dx} - \frac{y}{\frac{\sin x}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} \quad ✓$$

$$\frac{dy}{dx} - \frac{y}{\tan x} = \frac{1}{\sin x} \quad ✓$$

$$\frac{dy}{dx} - \frac{y}{\tan x} = \text{cosec}x \quad ✓$$

$$\overset{✓}{\frac{1}{\sin x}}.y = \overset{✓}{\int} \text{cosec}x.\text{cosec}x\,dx$$

$$y.\text{cosec}x = \int \text{cosec}^2 x.dx$$

$$y.\text{cosec}x = -\cot x + c \quad ✓$$

$$2.\text{cosec}\frac{\pi}{5} = -\cot\frac{\pi}{5} + c \quad ✓$$

$$c = 4{,}779 \quad ✓$$

$$y.\text{cosec}x = -\cot x + 4{,}779 \quad ✓$$

$$e^{\int \frac{1}{\tan x}dx} \quad ✓$$

$$e^{\int \cot x dx}$$

$$= e^{-\ln(\sin x)} \quad ✓$$

$$= (\sin x)^{-1} \ or \ \frac{1}{\sin x}$$

$$or \quad \text{cosec}x \quad ✓$$

(12)

4.2

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 3e^{-2x}$$

$$y_c: \quad m^2 + 5m + 6 = 0 \quad ✓$$

$$(m+3)(m+2) = 0 \quad ✓$$

$$\therefore \ \overset{✓}{m = -3}; \quad \overset{✓}{m = -2}$$

$$y_c = Ae^{-3x} + Be^{-2x} \quad ✓$$

$$\therefore \ y_p = Cxe^{-2x} \quad ✓$$

$$\frac{dy}{dx} = Cx(-2e^{-2x}) + Ce^{-2x} \quad ✓$$

$$\frac{d^2y}{dx^2} = Cx(4e^{-2x}) + C(-2e^{-2x}) - 2Ce^{-2x} \quad ✓$$

$$= 4Cxe^{-2x} - 4Ce^{-2x}$$

$$\therefore \ 4Cxe^{-2x} - 4Ce^{-2x} + 5(-2Cxe^{-2x}) + Ce^{-2x} + 6(Cxe^{-2x}) = 3e^{-2x} \quad ✓$$

$$Ce^{-2x} = 3e^{-2x}$$

$$\therefore \ C = 3 \quad ✓$$

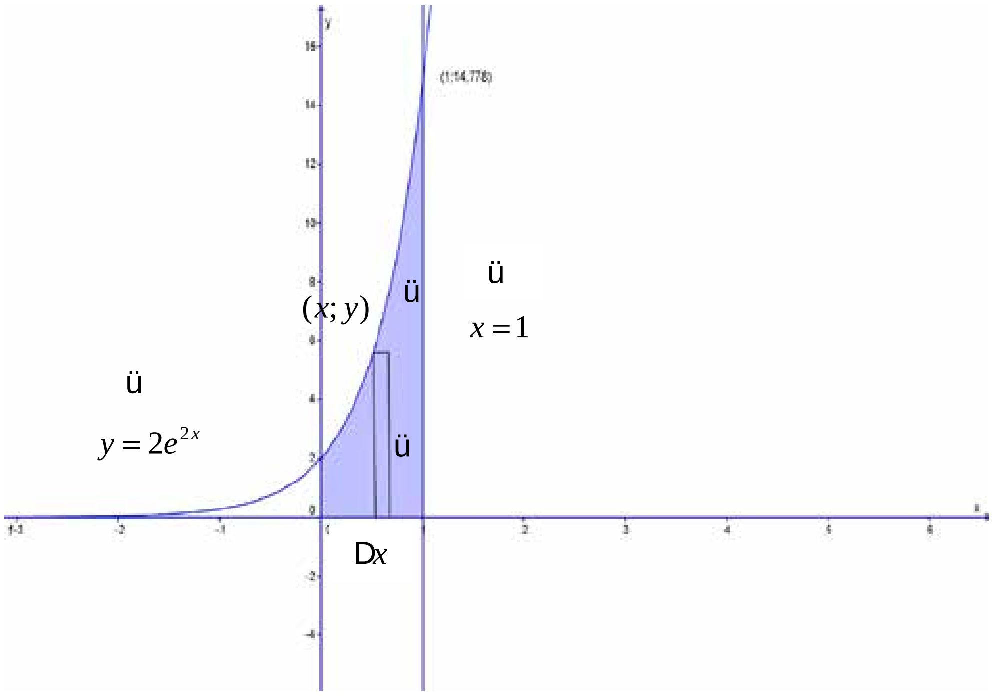
$$\therefore \ y_p = 3xe^{-2x} \quad ✓$$

$$y = Ae^{-3x} + Be^{-2x} + 3xe^{-2x} \quad ✓$$

(12)
[24]

QUESTION 5

5.1 5.1.1



(4)

5.1.2 $\Delta V_y = 2\pi x \times y \times \Delta x$ ✓ ✓ ✓

Incorrect limits: max 7 marks

$f(x) = x$	$g'(x) = e^{2x}$
$f'(x) = 1$	$g(x) = \dfrac{e^{2x}}{2}$

$$V_y = 2\pi\int_0^1 xy\,dx \quad ✓$$

$$= 2\pi\int_0^1 x(2e^{2x})dx \quad ✓$$

$$= 4\pi\int_0^1 x(e^{2x})dx$$

$$= 4\pi\left\{\left[x\left(\frac{e^{2x}}{2}\right)\right]_0^1 - \int_0^1 \left(\frac{e^{2x}}{2}\right)dx\right\} \quad ✓ \quad ✓$$

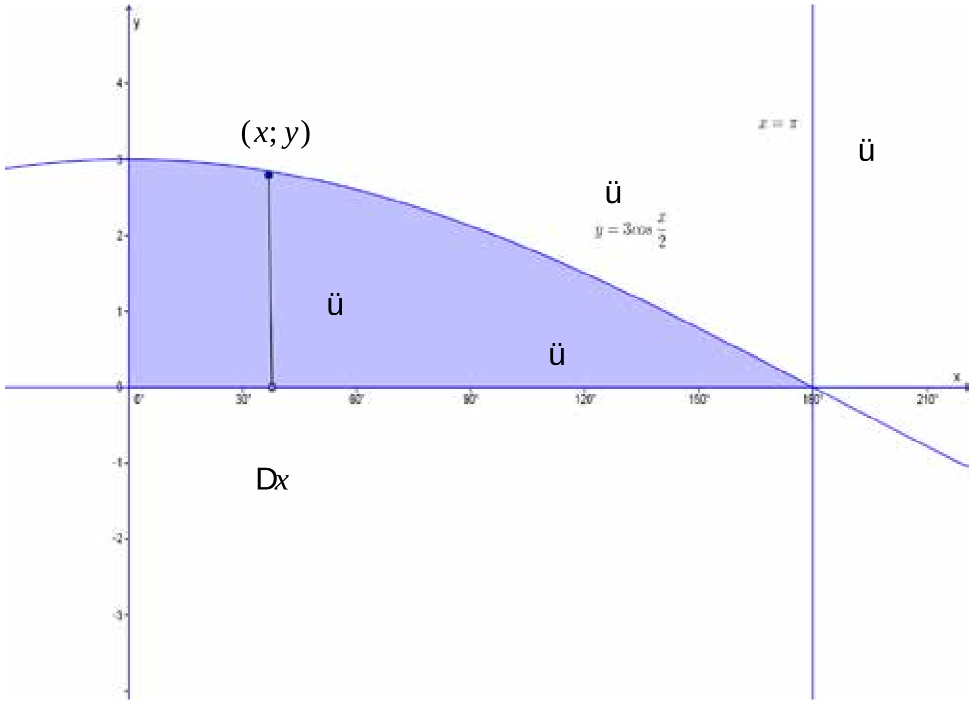
$$= 4\pi\left[x\left(\frac{e^{2x}}{2}\right) - \frac{e^{2x}}{4}\right]_0^1 \quad ✓$$

$$= 4\pi\left\{\left[(1)\left(\frac{e^{2(1)}}{2}\right) - \frac{e^{2(1)}}{4}\right] - \left[0 - \frac{e^{2(0)}}{4}\right]\right\} \quad ✓$$

$$= 8{,}389\pi \quad units^3 \quad or \quad 26{,}355 \quad units^3 \quad ✓$$

(10)

5.2 5.2.1



(4)

5.2.2 $\Delta A = y\Delta x$ ✓

$$A = \int_0^{\pi} y\,dx \quad ✓$$

$$= \int_0^{\pi} 3\cos\frac{x}{2}\,dx \quad ✓$$

$$= \left[\frac{3\sin\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi} \quad ✓$$

$$= \left[\frac{3\sin\frac{\pi}{2}}{\frac{1}{2}} - \frac{3\sin\frac{0}{2}}{\frac{1}{2}}\right] \quad ✓$$

$$= 6 \quad units^2 \quad ✓$$

(6)

Incorrect limits: max 3 marks

5.2.3

$$\Delta M_x = y\Delta x \times \frac{y}{2} \quad ✓✓$$

$$M_x = \frac{1}{2}\int_0^{\pi} y^2\,dx \quad ✓$$

$$= \frac{1}{2}\int_0^{\pi} \left(3\cos\frac{x}{2}\right)^2 dx \quad ✓$$

$$= \frac{9}{2}\int_0^{\pi} \cos^2\frac{x}{2}\,dx$$

$$= 4{,}5\left[\frac{x}{2} + \frac{\sin 2(\frac{1}{2})x}{4(\frac{1}{2})}\right]_0^{\pi} \quad ✓ \quad ✓$$

$$= 4{,}5\left[\frac{\pi}{2} + \frac{\sin\pi}{2}\right] \quad ✓$$

$$= 2{,}25\pi \ \text{units}^3 \quad \text{or} \quad 7{,}069 \ \text{units}^3 \quad ✓$$

$$I_x = \frac{7{,}069}{6} \quad ✓$$

$$= 1{,}178 \ \text{units} \quad ✓$$

(10)

Incorrect limits: max 6 marks

5.3 5.3.1

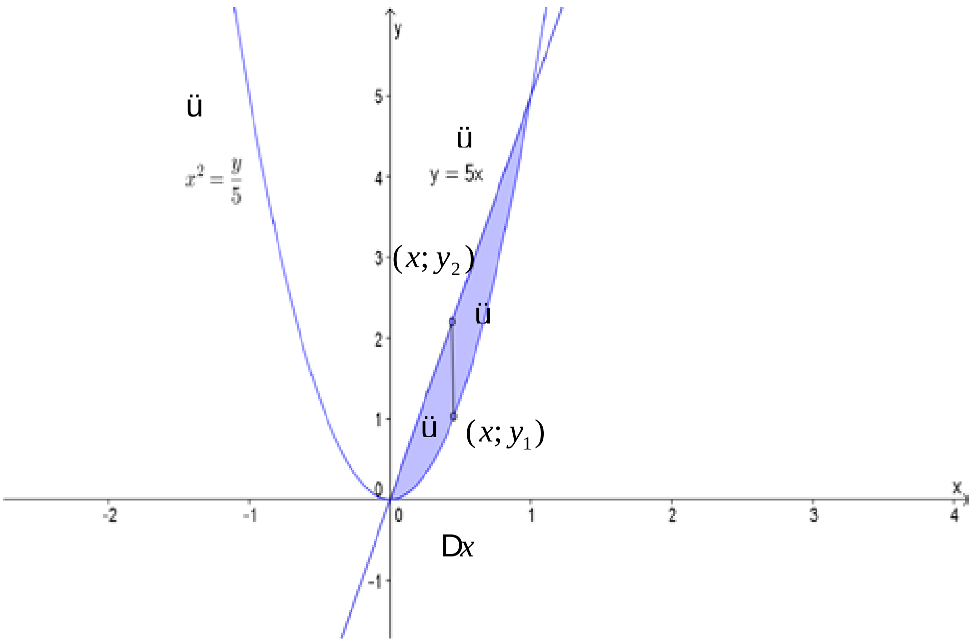
$$5x^2 = 5x$$

$$5x^2 - 5x = 0$$

$$5x(x-1) = 0$$

$$x = 0; \quad x = 1 \quad ✓$$

$$y = 0; \quad y = 5 \quad ✓ \quad \therefore (0;0) \ \text{and} \ (1;5)$$



(6)

5.3.2 $\Delta V_x = 2\pi y(x_2 - x_1)\Delta y$ ✓ ✓ ✓

$V_x = 2\pi \int_0^5 y(x_2 - x_1)dy$ ✓

Incorrect limits: max 4 marks

$= 2\pi \int_0^5 y\left(\sqrt{\frac{y}{5}} - \frac{y}{5}\right)dy$ ✓

$= 2\pi \int_0^5 \left(\frac{1}{\sqrt{5}} y^{\frac{3}{2}} - \frac{1}{5} y^2\right)dy$ ✓

$= 2\pi \left[\frac{1}{\sqrt{5}} \cdot \frac{y^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1}{5} \cdot \frac{y^3}{3}\right]_0^5$ ✓

$= 2\pi \left[\frac{1}{\sqrt{5}} \cdot \frac{(5)^{\frac{5}{2}}}{\frac{5}{2}} - \frac{1}{5} \cdot \frac{(5)^3}{3}\right]$

$= 3{,}333\pi \quad units^3 \quad or \quad 10{,}472 \quad units^3$ ✓

Or strip perpendicular to the *x-axis*

$\Delta V_x = \pi(y_2^2 - y_1^2)\Delta x$ ✓ ✓ ✓

$V_x = \pi \int_0^1 (y_2^2 - y_1^2)dx$ ✓

$= \pi \int_0^1 (5x)^2 - (5x^2)^2 \, dx$ ✓

$= \pi \int_0^1 (25x^2 - 25x^4)dx$ ✓

$= \pi \left[\frac{25x^3}{3} - \frac{25x^5}{5}\right]_0^1$ ✓

$= \pi \left[\frac{25(1)^3}{3} - \frac{25(1)^5}{5}\right]$

$= 3{,}333\pi \quad units^3 \quad or \quad 10{,}472 \quad units^3$ ✓ (8)

5.3.3 $\Delta I_x = 2\pi\rho(x_2 - x_1)y\Delta y \times y^2$ ✓ ✓ ✓

Incorrect limits: max 7 marks

$$I_x = 2\pi\rho \int_0^5 y^3(x_2 - x_1)dy$$ ✓ ✓

$$= 2\pi\rho \int_0^5 y^3\left(\sqrt{\frac{y}{5}} - \frac{y}{5}\right)dy$$ ✓

$$= 2\pi\rho \int_0^5 \left(\frac{1}{\sqrt{5}}y^{\frac{7}{2}} - \frac{1}{5}y^4\right)dy$$ ✓

$$== 2\pi\rho \left[\frac{1}{\sqrt{5}}\cdot\frac{y^{\frac{9}{2}}}{\frac{9}{2}} - \frac{1}{5}\cdot\frac{y^5}{5}\right]_0^5$$ ✓

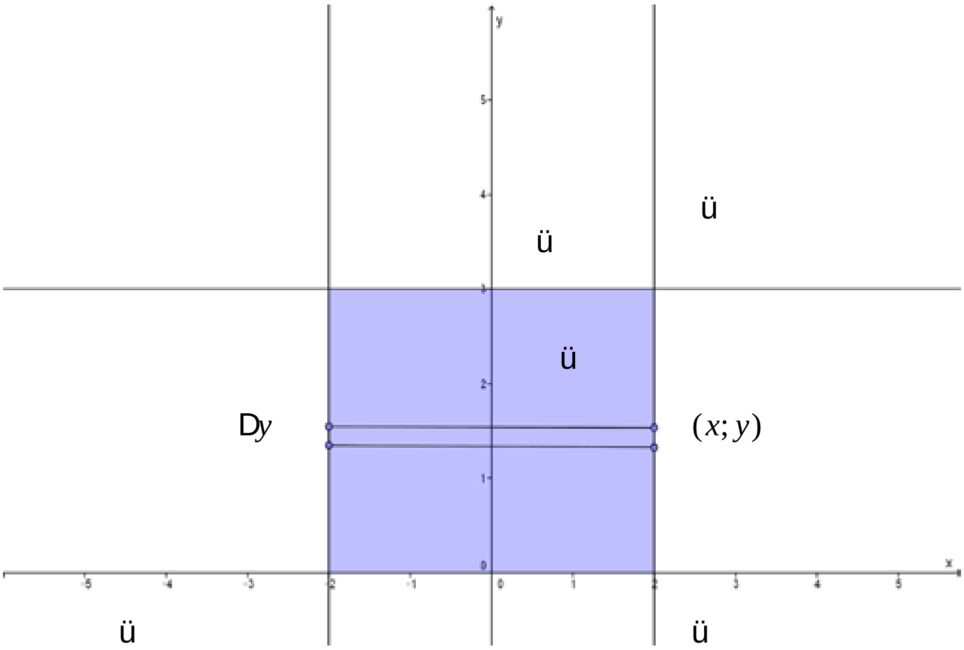
$$= 2\pi\rho \left[\frac{1}{\sqrt{5}}\cdot\frac{(5)^{\frac{9}{2}}}{\frac{9}{2}} - \frac{1}{5}\cdot\frac{(5)^5}{5}\right]$$ ✓

$$= 27{,}778\pi\rho \quad units^4 \quad or \quad 87{,}267\rho \quad units^4$$ ✓ (10)

5.3.4 $$I_x = \frac{87{,}267m}{26{,}155}$$ ✓

$$= 1{,}062 \quad m$$ ✓ (2)

5.4 5.4.1



$x = 2$

$\therefore \; dA = 2(2)dy$ ✓

or $\therefore \; dA = 4dy$ (6)

5.4.2 ✓ ✓ ✓

$$\int_0^3 (3-y)4dy$$

$$= 4\int_0^3 (3-y)dy$$

$$= 4\left[3y - \frac{y^2}{2}\right]_0^3 \quad ✓$$

$$= 4\left[3(3) - \frac{(3)^2}{2}\right] \quad ✓$$

$$= 18 \ \text{units}^3 \quad ✓$$

Incorrect limits: max 3 marks

(6)

5.4.3 ✓ ✓

$$\int_0^3 (3-y)^2 4dy$$

Incorrect limits: max 5 marks

$$= 4\int_0^3 (9 - 6y + y^2)dy \quad ✓$$

$$= 4\left[9y - \frac{6y^2}{2} + \frac{y^3}{3}\right]_0^3 \quad ✓$$

$$= 4\left[9(3) - \frac{6(3)^2}{2} + \frac{(3)^3}{3}\right]_0^3 \quad ✓$$

$$= 36 \quad \text{units}^4 \quad ✓$$

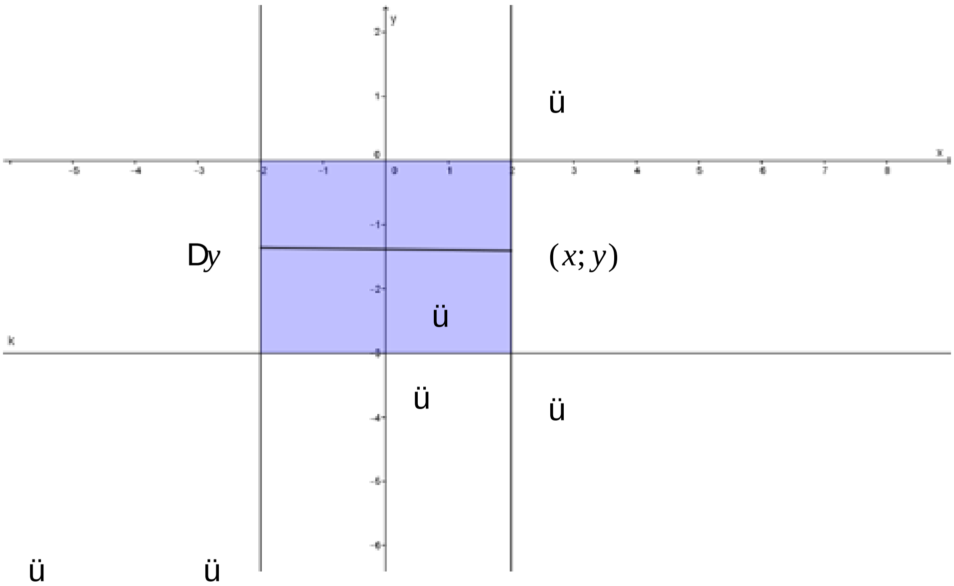
$$\bar{y} = \frac{36}{18} \quad ✓$$

$$= 2 \ units \quad ✓$$

(8)

OR ALTERNATIVE METHOD

5.4 5.4.1



$x = 2 \quad \therefore \quad dA = 2(2)dy \quad$ or $\quad \therefore \quad dA = 4dy$ (6)

5.4.2 ✓ ✓ ✓

$= \int_{3}^{0} y.4dy$

$= 4\int_{3}^{0} ydy$ ✓

$=4\left[\frac{y^2}{2}\right]_{3}^{0}$ ✓

$=4\left[\frac{(0)^2}{2} - \frac{(-3)^2}{2}\right]$

$= -18$ units^3 ✓

Incorrect limits: max 4 marks

(6)

5.4.3 $\int_{3}^{0} r^2 dA$

✓ ✓ ✓

$= \int_{3}^{0} y^2.4dy$

$= 4\int_{3}^{0} y^2dy$

$= 4\left[\frac{y^3}{3}\right]_{3}^{0}$ ✓

$= 4\left[\frac{0^3}{3} - \frac{(-3)^3}{3}\right]$ ✓

$= 36$ units^4 ✓

$\bar{\bar{y}} = \frac{36}{-18}$ ✓

$= -2\ units$ ✓

Incorrect limits: max 5 marks

(8)
[80]

QUESTION 6

6.1

$$10y = x^2$$

$$y = \frac{1}{10}x^2 \quad ✓$$

$$\frac{dx}{dy} = \frac{2}{10}x \quad ✓$$

$$\frac{dx}{dy} = \frac{1}{5}x$$

$$\left(\frac{dx}{dy}\right)^2 = \left(\frac{1}{5}x\right)^2 \quad ✓$$

$$1+\left(\frac{dx}{dy}\right)^2 = 1+\left(\frac{1}{5}x\right)^2 \quad ✓$$

$$= 1+\frac{x^2}{25}$$

$$= \frac{25+x^2}{25} \quad ✓$$

✓

$$S = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$= \int_0^4 \sqrt{\frac{25+x^2}{25}}\,dx \quad ✓$$

$$= \frac{1}{5}\int_0^4 \sqrt{25+x^2}\,dx \quad ✓$$

✓ ✓

$$= \frac{1}{5}\left[\frac{x}{2}\sqrt{25+x^2} + \frac{25}{2}\ln\left(x+\sqrt{25+x^2}\right)\right]_0^4$$

$$= \frac{1}{5}\left[\frac{4}{2}\sqrt{25+4^2} + \frac{25}{2}\ln\left(4+\sqrt{25+4^2}\right) - \frac{25}{2}\ln\left(\sqrt{25}\right)\right] \quad ✓$$

$= 4{,}393$ units ✓ (12)

6.2 $x = e^{t}$ and $y = 2e^{2t}$

$\dfrac{dx}{dt} = e^{t}$ ✓ $\dfrac{dy}{dt} = 4e^{2t}$ ✓

$\left(\dfrac{dx}{dt}\right)^2 = (e^{t})^2$ ✓ $\left(\dfrac{dy}{dt}\right)^2 = (4e^{2t})^2$ ✓

$\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 = e^{2t} + 16e^{4t}$ ✓

$= e^{2t}(1+16e^{2t})$

✓

$A_y = \int_0^1 2\pi x\sqrt{e^{2t}(1+16e^{2t})}\,dt$ ✓

$= 2\pi\int_0^1 e^{t}\sqrt{e^{2t}(1+16e^{2t})}\,dt$ ✓

$= \dfrac{2\pi}{32}\int_0^1 32e^{2t}(1+16e^{2t})^{\frac{1}{2}}\,dt$ ✓

$= \dfrac{\pi}{16}\left[\dfrac{(1+16e^{2t})^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$ ✓

$= \dfrac{\pi}{24}\left[(1+16e^{2(1)})^{\frac{3}{2}} - (1+16e^{2(0)})^{\frac{3}{2}}\right]$ ✓

$=51{,}322\pi$ units^2 or $161{,}233$ units^2 ✓

(12)
[24]

TOTAL: 200 ÷ 2
= 100

higher education & training

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Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T980**(E)**(A6)T
APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

6 April 2016 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = -5x^3y^2 - y^4 + 3x^2y$, determine $\dfrac{\partial^2 z}{\partial x \partial y}$ (2)

1.2 Given: $I = \dfrac{V}{R}$

Calculate the change in I if V decreases with 5 volts and R with 8 ohms. The original value of V is 30 volts and of R is 10 ohms. (4)
[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \sin^4 5x \cos^3 5x$ (5)

2.2 $y = \dfrac{1}{\sqrt{16x - x^2}}$ (3)

2.3 $y = \sin^4 mx$ (4)

2.4 $y = e^{\frac{x}{2}}.\cos 3x$ (6)
[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{-x^2 + 3x + 4}{x(1-2x)^2}\, dx$ (6)

3.2 $\displaystyle\int \frac{10x^2 + 7x + 1}{(2x^2+1)(4x-1)}\, dx$ (6)
[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$2\sin x\frac{dy}{dx} - y(\sin 2x) = \frac{2\sin x}{\sec x} \quad \text{at } (0;1)$$

(5)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2x + 3, \text{ if } y = 1 \text{ when } x = 0 \text{ and } \frac{dy}{dx} = 2 \text{ when } x = 0.$$

(7)
[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the two curves $y = \frac{3}{x}$ and $y + x - 4 = 0$. Make a neat sketch of the curves and show the area, in the first quadrant, bounded by the curves. Show the representative strip/element that you will use to calculate the volume (use the SHELL method only) generated if the area bounded by the curves rotates about the y-axis. (3)

5.1.2 Use the SHELL method to calculate the volume generated if the area, described in QUESTION 5.1.1, bounded by the two curves $y = \frac{3}{x}$ and $y + x - 4 = 0$, rotates about the y-axis. (5)

5.2 5.2.1 Make a neat sketch of the graph $y = \tan x$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, the ordinates $y = 0$ and $x = \frac{\pi}{3}$ rotates about the x-axis. (2)

5.2.2 Calculate the volume generated if the area, described in QUESTION 5.1.1, rotates about the x-axis. (3)

5.2.3 Calculate the volume moment about the y-axis as well as the distance of the centre of gravity from the y-axis. (6)

5.3 5.3.1 Calculate the points of intersection of the two curves $y = 2x^2$ and $x = \frac{y}{3}$. Make a neat sketch of the curves and show the area bounded by the curves. Show the representative strip/element, PERPENDICULAR to the x-axis, that you will use to calculate the area bounded by the curves. (3)

5.3.2 Calculate the area described in QUESTION 5.3.1, bounded by the two curves $y = 2x^2$ and $x = \frac{y}{3}$. (3)

5.3.3 Calculate the second moment of area of the area described in QUESTION 5.3.1 about the *y*-axis. (4)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the area. (1)

5.4 5.4.1 A weir in the form of a trapezium is 2 m high, 10 m wide at the top and 4 m wide at the bottom. The top of the weir is in the water surface.

Sketch the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the retaining wall.

Calculate the relation between the two variables *x* and *y*. (3)

5.4.2 Calculate, by using integration, the area moment of the weir about the water level. (3)

5.4.3 Calculate, by using integration, the second moment of area of the weir about the water level, as well as the depth of the centre of pressure on the weir. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve described by the parametric equations, $x = 5(\cos t + t\sin t)$ and $y = 5(\sin t - t\cos t)$, between the points $t = 0$ and $t = \pi$. (6)

6.2 Calculate the surface area generated when the curve of $y = \sqrt{16x}$, over the interval $1 \le x \le 4$, is rotated about the *x*-axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any other applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} \text{ ; } \sin x = \frac{1}{\operatorname{cosec} x} \text{ ; } \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx}\,f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\,\frac{d}{dx}\,x^n$	$a \int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}\,(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}\,(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}\,ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\,\frac{d}{dx}\,f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}\,f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}\,f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln\left[\sec(ax)\right]+C$
$\cot ax$	$-a\,\text{cosec}^2\,ax$	$\frac{1}{a}\ln\left[\sin(ax)\right]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln\left[\sec ax+\tan ax\right]+C$
$\text{cosec}\,ax$	$-a\,\text{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\dfrac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\dfrac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\dfrac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\dfrac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\dfrac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\dfrac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\dfrac{x}{2}-\dfrac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\dfrac{x}{2}+\dfrac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\dfrac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\,f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx \; ; V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy \; ; V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \mathrm{r}\, V$

DEFINITION: $I = m\, r^2$

GENERAL

$$I = \int_a^b r^2 dm = r\int_a^b r^2 dV$$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}r\int_a^b r^2 dV$$

$$I_x = \frac{1}{2}rp\int_a^b y^4 dx \qquad I_y = \frac{1}{2}rp\int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2py\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_x = \int_d^c 2py\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_y = \int_a^b 2px\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_y = \int_d^c 2px\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \square \quad ye^{\int P dx} = \int Qe^{\int P dx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N6

6 APRIL 2016

This marking guideline consists of 17 pages.

TOTAL: $\frac{200}{2} = 100$

NOTE: Do NOT subtract marks for incorrect units or units omitted.

QUESTION 1

1.1 $z = -5x^3y^2 - y^4 + 3x^2y$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-10x^3y - 4y^3 + 3x^2) \quad ✓✓✓$$

$$= -30x^2y + 6x \quad ✓$$

(4)

1.2 $I = \frac{V}{R}$

$I = VR^{-1}$

$$\Delta I = \frac{\partial I}{\partial V}\Delta V + \frac{\partial I}{\partial R}\Delta R \quad ✓✓$$

$$= R^{-1}\Delta V - VR^{-2}\Delta R \quad ✓✓$$

$$= \frac{1}{R}\Delta V - \frac{V}{R^2}\Delta R \quad ✓$$

$$= \frac{1}{(10)}(-5) - \frac{30}{(10)^2}(-8) \quad ✓✓$$

$= 1,9$ A ✓

(8)

[12]

QUESTION 2

2.1

$$y = \int \sin^4 5x \cos^3 5x\,dx$$

✓

$$= \int \sin^4 5x \cos^2 5x.\cos 5x\,dx$$

✓

$$= \int \sin^4 5x(1 - \sin^2 5x)\cos 5x\,dx$$

$$= \frac{1}{5}\int \sin^4 5x(1 - \sin^2 5x)5\cos 5x\,dx$$

$u = \sin 5x$

$du = 5\cos 5x\,dx$

✓ ✓ ✓

$$= \frac{1}{5}\int u^4(1 - u^2)\,du$$

✓

$$= \frac{1}{5}\int u^4 - u^6)\,du$$

✓ ✓

$$= \frac{1}{5}\left[\frac{u^5}{5} - \frac{u^7}{7}\right] + c$$

✓ ✓

$$= \frac{1}{5}\left[\frac{\sin^5 5x}{5} - \frac{\sin^7 5x}{7}\right] + c$$

OR

$$= \frac{1}{25}\sin^5 5x - \frac{1}{35}\sin^7 5x + c$$

OR

$$y = \int \sin^4 5x \cos^3 5x\,dx$$

✓

$$= \int \sin^4 5x \cos^2 5x.\cos 5x\,dx$$

✓

$$= \int \sin^4 5x(1 - \sin^2 5x)\cos 5x\,dx$$

✓ ✓ ✓ ✓

$$= \int \sin^4 5x.\cos 5x\,dx - \int \sin^6 5x.\cos 5x\,dx$$

$u = \sin 5x$

$du = 5\cos 5x\,dx$

✓ ✓ ✓ ✓

$$= \frac{1}{5}.\frac{\sin^5 5x}{5} - \frac{1}{5}.\frac{\sin^7 5x}{7} + c$$

(10)

2.2 $y = \int \frac{1}{\sqrt{16x - x^2}}\,dx$

$= -x^2 + 16x$

$= -(x^2 - 16x)$ ✓

✓ ✓

$= -[(x-8)^2 - 64]$

$= 64 - (x-8)^2$ ✓

$= \int \frac{1}{\sqrt{64 - (x-8)^2}}\,dx$ ✓

$= \sin^{-1}\frac{(x-8)}{8} + c$ ✓

(6)

2.3 $y = \int \sin^4 mx\,dx$

$= \int (\sin^2 mx)^2\,dx$ ✓

$= \int (\frac{1}{2} - \frac{1}{2}\cos 2mx)^2\,dx$ ✓

$= \int (\frac{1}{2} - \frac{1}{2}\cos 2mx)(\frac{1}{2} - \frac{1}{2}\cos 2mx)\,dx$

$= \int (\frac{1}{4} - \frac{1}{2}\cos 2mx + \frac{1}{4}\cos^2 2mx)\,dx$ ✓

✓ ✓ ✓ ✓ ✓

$= \frac{1}{4}x - \frac{1}{2}.\frac{\sin 2mx}{2m} + \frac{1}{4}\left[\frac{x}{2} + \frac{\sin 4mx}{8m}\right] + c$

$= \frac{1}{4}x - \frac{\sin 2mx}{4m} + \frac{x}{8} + \frac{\sin 4mx}{32m} + c$

(8)

2.4

$$y = \int e^{\frac{x}{2}}.\cos 3x\,dx$$

$$f(x) = e^{\frac{x}{2}} \qquad g'(x) = \cos 3x$$

$$f'(x) = \frac{1}{2}e^{\frac{x}{2}} \qquad g(x) = \frac{\sin 3x}{3}$$

$$\int y\,dx = \overset{\checkmark}{e^{\frac{x}{2}}(\frac{\sin 3x}{3})} - \overset{\checkmark}{\int \frac{1}{2}e^{\frac{x}{2}}(\frac{\sin 3x}{3})dx}$$

$$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x - \frac{1}{6}\overset{\checkmark}{\int} e^{\frac{x}{2}}.\sin 3x\,dx$$

$$f(x) = e^{\frac{x}{2}} \qquad g'(x) = \sin 3x$$

$$f'(x) = \frac{1}{2}e^{\frac{x}{2}} \qquad g(x) = -\frac{\cos 3x}{3}$$

$$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x - \frac{1}{6}\left[\overset{\checkmark}{e^{\frac{x}{2}}.-\frac{\cos 3x}{3}} - \overset{\checkmark}{\int \frac{1}{2}e^{\frac{x}{2}}.-\frac{\cos 3x}{3}dx}\right]$$

$$= \frac{1}{3}e^{\frac{x}{2}}\sin 3x + \overset{\checkmark}{\frac{1}{18}e^{\frac{x}{2}}.\cos 3x} - \overset{\checkmark}{\frac{1}{36}\int e^{\frac{x}{2}}.\cos 3x\,dx}$$

$$\overset{\checkmark}{I} = \frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x - \frac{1}{36}I \checkmark$$

$$\therefore \frac{37}{36}I = \overset{\checkmark}{\frac{1}{3}e^{\frac{x}{2}}\sin 3x} + \overset{\checkmark}{\frac{1}{18}e^{\frac{x}{2}}.\cos 3x} \checkmark$$

$$I = \frac{36}{37}(\frac{1}{3}e^{\frac{x}{2}}\sin 3x + \frac{1}{18}e^{\frac{x}{2}}.\cos 3x) + c$$

$$= 0{,}973(0{,}333e^{\frac{x}{2}}\sin 3x + 0{,}054e^{\frac{x}{2}}.\cos 3x) + c$$

$$= 0{,}324e^{\frac{x}{2}}\sin 3x + 0{,}054e^{\frac{x}{2}}.\cos 3x + c$$

OR

$$y = \int e^{\frac{x}{2}}.\cos 3x\,dx$$

$$f(x) = \cos 3x \qquad g'(x) = e^{\frac{x}{2}}$$

$$f'(x) = -3\sin 3x \qquad g(x) = \frac{e^{\frac{x}{2}}}{\frac{1}{2}} = 2e^{\frac{x}{2}}$$

$$\int y\,dx = \overset{\checkmark}{2e^{\frac{x}{2}}\cos 3x} - \overset{\checkmark}{\int(-3\sin 3x)2e^{\frac{x}{2}}dx}$$

$$= 2e^{\frac{x}{2}}\cos 3x + 6\overset{\checkmark}{\int}\sin 3x.e^{\frac{x}{2}}dx$$

$$f(x) = \sin 3x \qquad g'(x) = e^{\frac{x}{2}}$$

$$f'(x) = 3\cos 3x \qquad g(x) = 2e^{\frac{x}{2}}$$

$$= 2e^{\frac{x}{2}}\cos 3x + 6\left[\overset{\checkmark}{\sin 3x.2e^{\frac{x}{2}}} - \overset{\checkmark}{\int 3\cos 3x.2e^{\frac{x}{2}}dx}\right]$$

$$= 2e^{\frac{x}{2}}\cos 3x + \overset{\checkmark}{12\sin 3x.e^{\frac{x}{2}}} - \overset{\checkmark}{36\int\cos 3x.e^{\frac{x}{2}}dx}$$

$$I = 2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x - 36I$$ ü ü

$$\therefore\ 37.I = 2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x$$ ü ü ü

$$I = \frac{1}{37}(2e^{\frac{x}{2}}\cos 3x + 12e^{\frac{x}{2}}.\sin 3x) + c$$

$$= 0{,}054e^{\frac{x}{2}}\cos 3x + 0{,}324e^{\frac{x}{2}}\sin 3x + c$$ (12)

[36]

QUESTION 3

3.1 $$\int \frac{-x^2+3x+4}{x(1-2x)^2}\,dx$$

$$\frac{-x^2+3x+4}{x(1-2x)^2} = \frac{A}{x} + \frac{B}{(1-2x)^2} + \frac{C}{(1-2x)}$$ ü

$$-x^2+3x+4 = A(1-2x)^2 + Bx + Cx(1-2x)$$ ü

$$Let\ \ x = 0; \quad \therefore\ A = 4$$ ü

$$Let\ \ x = \frac{1}{2}; \quad \therefore\ B = \frac{21}{2} \quad (10{,}5)$$ ü

$$-x^2+3x+4 = A - 4Ax + 4Ax^2 + Bx + Cx - 2Cx^2$$ ü

$$\text{Equate coeff of } x^2: \quad C = \frac{17}{2} \quad (8{,}5)$$ ü

$$= \int \frac{4}{x}dx + \int \frac{\frac{21}{2}}{(1-2x)^2}dx + \int \frac{\frac{17}{2}}{(1-2x)}dx$$ ü

ü ü ü ü ü

$$= 4\ln x - \frac{21}{4}.\frac{(1-2x)^{-1}}{-1} - \frac{17}{4}\ln(1-2x) + c$$

$$= 4\ln x + \frac{21}{4(1-2x)} - \frac{17}{4}\ln(1-2x) + c$$

$$= 0{,}444\ln x + \frac{5{,}25}{(1-2x)} - 1{,}444\ln(1-2x) + c$$ (12)

3.2 $$\int \frac{10x^2+7x+1}{(2x^2+1)(4x-1)}\,dx$$

$$\frac{10x^2+7x+1}{(2x^2+1)(4x-1)} = \frac{Ax+B}{2x^2+1} + \frac{C}{4x-1} \quad ✓$$

$$10x^2+7x+1 = (Ax+B)(4x-1) + C(2x^2+1) \quad ✓$$

$$10x^2+7x+1 = 4Ax^2+4Bx-Ax-B+2Cx^2+C \quad ✓$$

$$let \quad x=\frac{1}{4} \quad \therefore \; C=3 \quad ✓$$

Equate coeff of x^2: $A=1$ ✓ $Equate \quad x$: $B=2$ ✓

$$= \int \frac{x+2}{2x^2+1}dx + \int \frac{3}{4x-1}dx \quad ✓$$

$$\quad ✓$$

$$= \int \frac{x}{2x^2+1}dx + \int \frac{2}{2x^2+1}dx + \int \frac{3}{4x-1}dx$$

$$✓ \qquad ✓ \qquad ✓ \qquad ✓$$

$$= \frac{1}{4}\ln(2x^2+1) + 2\left(\frac{1}{\sqrt{2}}\right)\arctan\sqrt{2}x + \frac{3}{4}\ln(4x-1) + c$$

(12)

[24]

QUESTION 4

4.1 $$2\sin x\frac{dy}{dx} - y(\sin 2x) = \frac{2\sin x}{\sec x}$$

$$\frac{dy}{dx} - \frac{y(2\sin 2x)}{2\sin x} = \frac{1}{\sec x} \quad ✓$$

$$\quad ✓$$

$$\frac{dy}{dx} - \frac{y(2\sin x\cos x)}{2\sin x} = \frac{1}{\sec x}$$

$$\frac{dy}{dx} - y.\cos x = \cos x \quad ✓$$

$$✓$$

$$e^{\int \cos x.dx} = e^{-\sin x} \quad ✓$$

$$e^{-\sin x}.y = \int e^{-\sin x}.\cos x dx \quad ✓$$

$$= -e^{-\sin x} + c \quad ✓$$

$$e^{-\sin 0}.(1) = -e^{-\sin 0} + c \quad ✓$$

$$\therefore \; c=2 \quad ✓$$

$$e^{-\sin x}.y = -e^{-\sin x} + 2 \quad ✓$$

(10)

4.2

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2x + 3$$

$y_c: \quad m^2 - 7m + 6 = 0$ ✓

$(m - 6)(m - 1) = 0$

$m = 6; \quad m = 1$

$y_c = Ae^{6x}$ ✓

To find $y_p \quad \therefore \quad y = Cx + D$ ✓

$\frac{dy}{dx} = C$ ✓

$\frac{d^2y}{dx^2} = 0$ ✓

$0 - 7C + 6Cx + 6D = 2x + 3$ ✓

$C = \frac{1}{3} \quad (0{,}333)$ ✓

$-7C + 6D = 3 \quad \therefore \quad D = \frac{8}{9} \quad (0{,}889)$ ✓

$\therefore \quad y_p = \frac{1}{3}x + \frac{8}{9}$

$y = Ae^{6x} + Be^{x} + \frac{1}{3}x + \frac{8}{9}$ ✓

$1 = A + B + \frac{8}{9} \quad \therefore \quad A + B = \frac{1}{9}$ ✓

$\frac{dy}{dx} = 6Ae^{6x} + Be^{x} + \frac{1}{3}$ ✓

$2 = 6A + B + \frac{1}{3}$

$\therefore \quad A = 0{,}311 \quad \left(\frac{14}{45}\right)$ ✓

$and \quad B = -0{,}2 \quad \left(-\frac{9}{45} \; or \; -\frac{1}{5}\right)$ ✓

$\therefore \quad y = \frac{14}{45}e^{6x} - \frac{1}{5}e^{x} + \frac{1}{3}x + \frac{8}{9}$ ✓

$y = 0{,}311e^{6x} - 0{,}2e^{x} + 0{,}333x + 0{,}889$

(14)
[24]

QUESTION 5

5.1 5.1.1

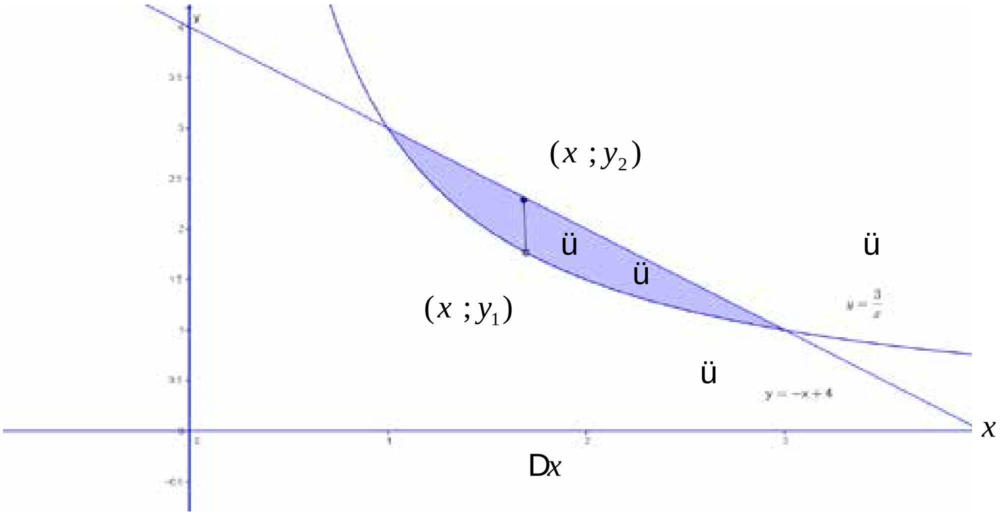
$$\frac{3}{x} = -x + 4$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3; \quad x = 1 \quad ✓$$

$$y = 1; \quad y = 3 \quad ✓ \quad \therefore (3;1) \text{ and } (1;3)$$



(6)

5.1.2

$$\overset{✓}{\Delta V_y} = \overset{✓}{2\pi x} \times \overset{✓}{(y_2 - y_1)} \times \Delta x$$

$$V_y = 2\pi \overset{✓}{\int_1^3} x(y_2 - y_1)dx$$

Incorrect limits: max 7 marks

$$= 2\pi \int_1^3 x(\overset{✓}{-x+4} - \overset{✓}{\frac{3}{x}})dx$$

$$= 2\pi \int_1^3 (-x^2 + 4x - 3)dx \quad ✓$$

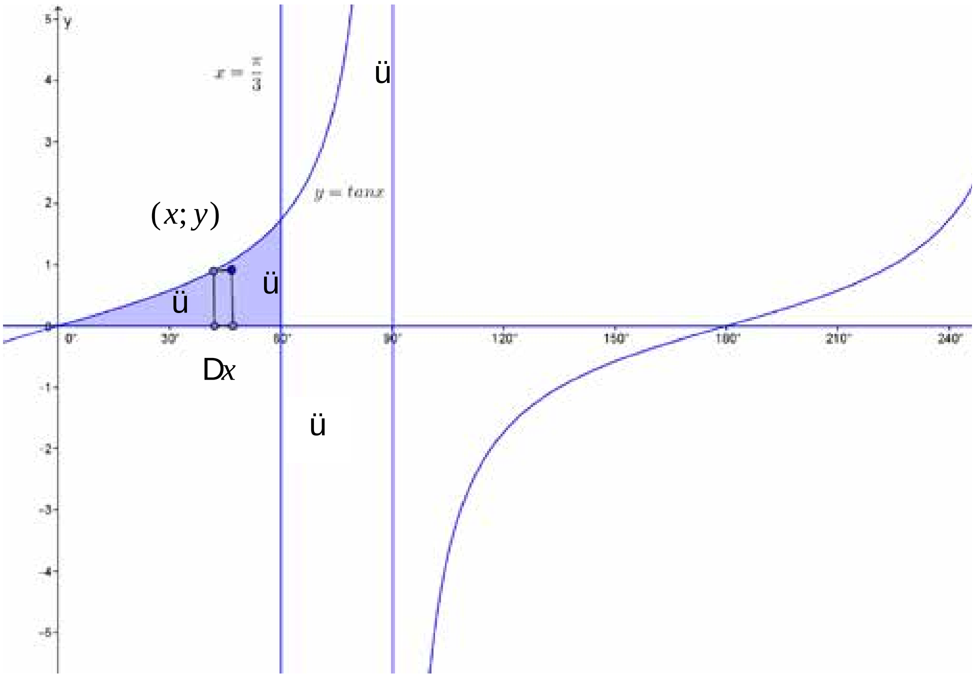
$$= 2\pi \left[-\frac{x^3}{3} + \frac{4x^2}{2} - 3x \right]_1^3 \quad ✓$$

$$= 2\pi \left\{ \left[-\frac{(3)^3}{3} + 2(3)^2 - 3(3) \right] - \left[-\frac{(1)^3}{3} + 2(1)^2 - 3(1) \right] \right\} ✓ \quad ✓$$

$$= 2{,}667\pi \quad units^3 \quad or \quad 8{,}278 \quad or \quad \frac{8}{3}\pi \quad units^3 \quad ✓✓$$

(10)

5.2 5.2.1



(4)

5.2.2 $\Delta V_x = \pi y^2 \Delta x$ ✓

✓

$$V_x = \pi \int_0^{\frac{\pi}{3}} y^2 \, dx$$

Incorrect limits: max 3 marks

$$= \pi \int_0^{\frac{\pi}{3}} (\tan^2 x) \, dx \quad ✓$$

$$= \pi \left[(\tan x - x) \right]_0^{\frac{\pi}{3}} \quad ✓$$

$$= \pi \left[(\tan \frac{\pi}{3} - \frac{\pi}{3}) - (\tan 0 - 0) \right] \quad ✓$$

$$= 0{,}685\pi \quad units^3 \quad or \quad 2{,}152\ units^3 \quad ✓$$ (6)

5.2.3 $\Delta M_y = \pi y^2 \Delta x \times x$ ✓✓

> Incorrect limits: max 8 marks

$$\therefore M_y = \pi \int_0^{\frac{\pi}{3}} x y^2 \, dx$$ ✓

$f(x) = x$	$g'(x) = \tan^2 x$
$f'(x) = 1$	$g(x) = \tan x - x$

$$= \pi \int_0^{\frac{\pi}{3}} x(\tan^2 x) \, dx$$ ✓

$$= \pi \left\{ [x(\tan x - x)]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} (\tan x - x) \, dx \right\}$$ ✓✓✓

$$= \pi \left\{ [x(\tan x - x)]_0^{\frac{\pi}{3}} - \left[\ln \sec x - \frac{x^2}{2} \right]_0^{\frac{\pi}{3}} \right\}$$ ✓✓✓

$$= \pi \left\{ \left[\frac{\pi}{3} \left(\tan \frac{\pi}{3} - \frac{\pi}{3}\right) \right] - \left[\ln \sec \frac{\pi}{3} - \frac{(\frac{\pi}{3})^2}{2} \right] \right\}$$ ✓✓✓

$$= 0{,}572\pi \quad units^3 \quad or \quad 1{,}798 \quad u^3$$ ✓

$$\bar{x} = \frac{1{,}798}{2{,}152}$$ ✓

$$= 0{,}836 \; units$$ ✓ (12)

5.3 5.3.1

$$2x^2 = 3x$$
$$2x^2 - 3x = 0$$
$$x(2x - 3) = 0$$
$$x = 0; \quad x = \frac{3}{2}$$ ✓
$$y = 0; \quad y = 4\frac{1}{2}$$ ✓✓ $\therefore$ (0;0) and $(\frac{3}{2};\frac{9}{2})$ (6)

5.3.2

$$\Delta A = (y_2 - y_1)\Delta x \quad \checkmark$$

$$A = \int_{\frac{1}{2}}^{1} (y_2 - y_1)\,dx \quad \checkmark$$

Incorrect limits: max 3 marks

$$= \int_{\frac{1}{2}}^{1} (3x - 2x^2)\,dx \quad \checkmark$$

$$= \left[\frac{3x^2}{2} - \frac{2x^3}{3}\right]_{\frac{1}{2}}^{1} \quad \checkmark$$

$$= \left[\frac{3(1)^2}{2} - \frac{2(1)^3}{3}\right] - \left[\frac{3(\frac{1}{2})^2}{2} - \frac{2(\frac{1}{2})^3}{3}\right] \quad \checkmark$$

$$= 0{,}542 \ \ units^2 \quad \checkmark$$

(6)

5.3.3

$$\therefore \Delta I_y = (y_2 - y_1)\Delta x \times x^2 \quad \checkmark \checkmark$$

$$= \int_{\frac{1}{2}}^{1} (3x - 2x^2)x^2\,dx \quad \checkmark \checkmark$$

Incorrect limits: max 5 marks

$$= \int_{\frac{1}{2}}^{1} (3x^3 - 2x^4)\,dx \quad \checkmark$$

$$= \left[\frac{3x^4}{4} - \frac{2x^5}{5}\right]_{\frac{1}{2}}^{1} \quad \checkmark$$

$$= \left[\frac{3(1)^4}{4} - \frac{2(1)^5}{5}\right] - \left[\frac{3(\frac{1}{2})^4}{4} - \frac{2(\frac{1}{2})^5}{5}\right] \quad \checkmark$$

$$= 0{,}316 \quad units^4 \quad \checkmark$$

(8)

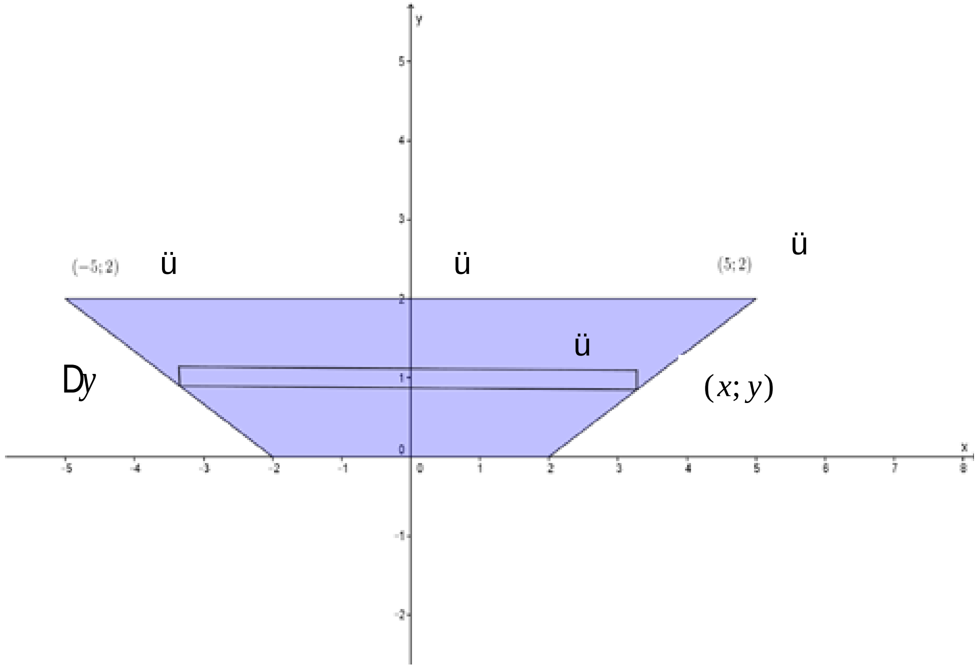
5.3.4

$$I = \frac{0{,}316}{0{,}542} \quad \checkmark$$

$$= 0{,}553 \quad A \quad \checkmark$$

(2)

5.4 5.4.1



$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x-2}=\frac{2-0}{5-2} \quad ✓$$

$$y=\frac{2}{3}(x-2)$$

$$\therefore\ x=\frac{3}{2}y+2 \quad ✓ \qquad \therefore\ dA=2\left(\frac{3}{2}y+2\right)dy \quad \text{or} \quad \therefore\ dA=(3y+4)dy$$ (6)

5.4.2

$$\int_0^2 r\,dA$$

✓ ✓ ✓

Incorrect limits: max 4 marks

$$=\int_0^2 (2-y)(3y+4)dy$$

$$=\int_0^2 (6y+8-3y^2-4y)dy \quad ✓$$

$$=\left[\frac{6y^2}{2}+8y-\frac{3y^3}{3}-\frac{4y^2}{2}\right]_0^2 \quad ✓$$

$$=\left[\frac{6(2)^2}{2}+8(2)-\frac{3(2)^3}{3}-\frac{4(2)^2}{2}\right]$$

$= 12\ \text{units}^3$ ✓ (6)

5.4.3 $\int_0^2 r^2 dA$

✓ ✓

Incorrect limits: max 5 marks

$= \int_0^2 (2 - y)^2(3y + 4)dy$

$= \int_0^2 (12y - 12y^2 + 3y^3 + 16 - 16y + 4y^2)dy$ ✓

$= \left[\frac{12y^2}{2} - \frac{12y^3}{3} + \frac{3y^4}{4} + 16y - \frac{16y^2}{2} + \frac{4y^3}{3}\right]_0^2$ ✓

$= \left[6(2)^2 - 4(2)^3 + \frac{3(2)^4}{4} + 16(2) - 8(2)^2 + \frac{4}{3}(2)^3\right]$ ✓

$= 14,667$ units^4 ✓

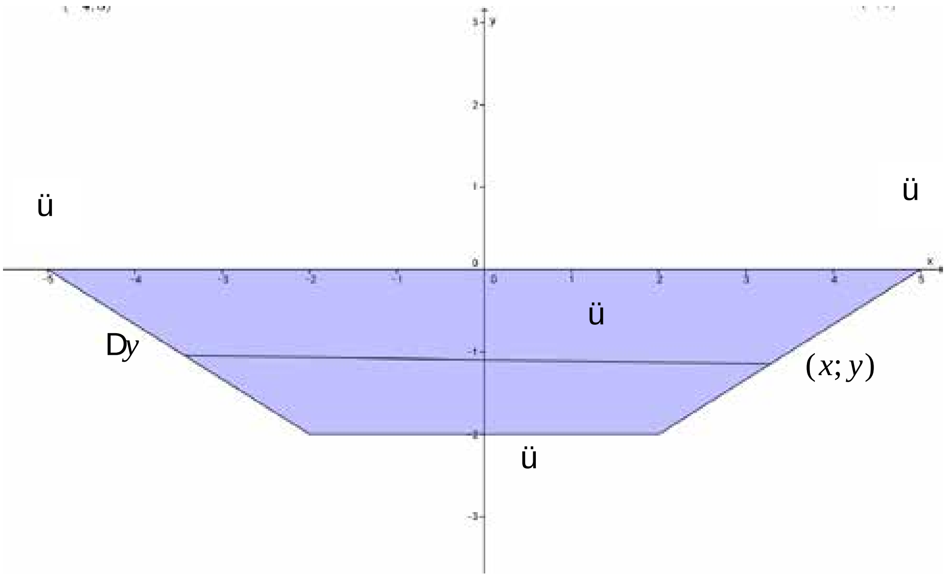
$\bar{y} = \frac{14,667}{12}$ ✓

$= 1,222\ units$ ✓

(8)

Or alternative method

5.4 5.4.1



$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y - 0}{x - 5} = \frac{-2 - 0}{2 - 5}$

$y = \frac{2}{3}(x - 5)$ ✓

$\therefore\ x = \frac{3}{2}y + 5$ ✓ $\therefore\ dA = 2(\frac{3}{2}y + 5)dy$ or $\therefore\ dA = (3y + 10)dy$

(6)

5.4.2 ✓ ✓ ✓

$$= \int_{-2}^{0} y(3y+10)\,dy$$

Incorrect limits: max 4 marks

$$= \int_{-2}^{0} (3y^2+10y)\,dy \quad ✓$$

$$= \left[\frac{3y^3}{3}+\frac{10y^2}{2}\right]_{-2}^{0} \quad ✓$$

$$= \left[\frac{3(0)^3}{3}+\frac{10(0)^2}{2}\right]-\left[\frac{3(-2)^3}{3}+\frac{10(-2)^2}{2}\right] \quad ✓$$

$$= -12 \text{ units}^3 \quad ✓$$

(6)

5.4.3 ✓ ✓

$$= \int_{-2}^{0} y^2(3y+10)\,dy$$

Incorrect limits: max 5 marks

$$= \int_{-2}^{0} (3y^3+10y^2)\,dy \quad ✓$$

$$= \left[\frac{3y^4}{4}+\frac{10y^3}{3}\right]_{-2}^{0} \quad ✓$$

$$= \left[\frac{3(0)^4}{4}+\frac{10(0)^3}{3}\right]-\left[\frac{3(-2)^4}{4}+\frac{10(-2)^3}{3}\right] \quad ✓$$

$$= 14,667 \text{ units}^4 \quad ✓$$

$$\bar{y} = \frac{14,667}{-12} \quad ✓$$

$$= -1,222 \text{ units} \quad ✓$$

(8)

[80]

QUESTION 6

6.1

$x = 5(\cos t + t\sin t)$ and $y = 5(\sin t - t\cos t)$

$\frac{dx}{dt} = 5(-\sin t + t\cos t + \sin t)$ ✓ $\qquad \frac{dy}{dt} = 5(\cos t + t\sin t - \cos t)$ ✓

Or $\frac{dx}{dt} = -5\sin t + 5t\cos t + 5\sin t$ $\qquad$ OR $\frac{dy}{dt} = 5\cos t + 5t\sin t - 5\cos t$

$= 5t\cos t$ ✓ $\qquad = 5t\sin t$ ✓

$\left(\frac{dx}{dt}\right)^2 = (5t\cos t)^2 \qquad \left(\frac{dy}{dt}\right)^2 = (5t\sin t)^2$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 25t^2\cos^2 t + 25t^2\sin^2 t$ ✓

$= 25t^2(\cos^2 t + \sin^2 t)$

$= 25t^2$ ✓

✓

S = $\int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Incorrect limits: max 9 marks

$= \int_0^{\pi} \sqrt{25t^2}\,dt$ ✓

$= 5\int_0^{\pi} t.dt$ ✓

$= 5\left[\frac{t^2}{2}\right]_0^{\pi}$ ✓

$= \frac{5}{2}\left[\pi^2\right]$ ✓

$= 2{,}5\pi^2$ units or 24,674 units ✓ (12)

6.2

$y = \sqrt{16x}$

$y = 4x^{\frac{1}{2}}$ ✓

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ ✓

$\left(\frac{dy}{dx}\right)^2 = 4x^{-1}$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4}{x}$ ✓

$= \frac{x+4}{x}$ ✓

✓

$A_x = 2\pi\int_1^4 y\sqrt{\frac{x+4}{x}}\,dx$ ✓

Incorrect limits: max 9 marks

$\checkmark$

$$= 2\pi \int_1^4 4x^{\frac{1}{2}} \sqrt{\frac{x+4}{x}}\,dx$$

$$= 8\pi \int_1^4 \sqrt{x+4}\,dx \ \checkmark$$

$$= 8\pi \int_1^4 (x+4)^{\frac{1}{2}}\,dx$$

$$= 8\pi \left[\frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \ \checkmark$$

$$= \frac{16\pi}{3} \left[(x+4)^{\frac{3}{2}} \right]_1^4$$

$$= \frac{16\pi}{3} \left[(8)^{\frac{3}{2}} - (5)^{\frac{3}{2}} \right]_1 \ \checkmark$$

$= 61{,}051\ \pi$ units^2 or $191{,}798$ units^2 $\checkmark$ (12)

[24]

TOTAL: 100

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T970**(E)**(N24)T
NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 November 2015 (X-Paper)
9:00–12:00

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = e^{\frac{x}{y}}.\sin y$ calculate the following:

1.1.1 $\dfrac{\partial z}{\partial x}$ (1)

1.1.2 $\dfrac{\partial z}{\partial y}$ (2)

1.2 The length of the base of a right-angled triangle is 3 m and the perpendicular height is 5 m. In constructing the triangle an error of 0,1 m is made with the base and an error of $-$ 0,13 m is made with the perpendicular height. Use partial differentiation to determine the error made with the area calculation. (3)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \arctan 3x$ (2)

2.2 $y = \sin^3 2x . \cos^7 2x$ (4)

2.3 $y = \dfrac{1}{\sqrt{5x - 2x^2}}$ (4)

2.4 $y = \left(\dfrac{x^2}{2} + 1\right)\cos 2x$ (4)

2.5 $y = \tan^5 ax$ (4)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^2 - x - 4}{(x+4)^3}\,dx$ (5)

3.2 $\int \frac{-x^2 + 7x}{(1-3x)(x^2+1)}\,dx$ (7)

[12]

QUESTION 4

4.1 Solve the following differential equation:

$(1-x^2)dy - xydx = 2(1-x^2)dx$ (5)

4.2 Calculate the particular solution of:

$\frac{d^2y}{dx^2} + 9y = x^2 + 4x$ if $y = 1$ when $x = 0$ and $\frac{dy}{dx} = 2$ when $x = 0$. (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the graphs $x^2 = 2y$ and $y = \frac{x}{2} + 1$.

Sketch the graphs and show the representative strip/element that you will use to calculate the area bounded by the graphs and the line $x = 0$ in the first quadrant. (3)

5.1.2 Calculate the magnitude of the area described in QUESTION 5.1.1. (3)

5.1.3 Calculate the area moment described in QUESTION 5.1.1 about the x-axis as well as the distance of the centroid of the first quadrant area from the x-axis. (5)

5.2 5.2.1 Make a neat sketch of the graph $y = 5\sin x$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, $y = 0$ and $x = \frac{\pi}{2}$ is rotated about the x-axis. (2)

	5.2.2	Calculate the volume described in QUESTION 5.2.1.	(3)
	5.2.3	Calculate the moment of inertia of the solid generated when the area described in QUESTION 5.2.1 rotates about the *x*-axis and express the answer in terms of the mass.	(5)
5.3	5.3.1	Sketch the graph of $y=\frac{2}{x^2+1}$ in the first quadrant and show the representative strip/element that you will use to calculate the area bounded by the graph $y=\frac{2}{x^2+1}$, $x=0$, $x=3$ and $y=0$.	(2)
	5.3.2	Calculate the area described in QUESTION 5.3.1.	(3)
	5.3.3	Calculate the second moment of area described in QUESTION 5.3.1 with respect to the y-axis.	(4)
5.4	5.4.1	A trapezoidal weir is 3 m high, 8 m wide at the top and 6 m wide at the bottom. The top of the weir is 2 m below the surface of the water. Sketch the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the retaining wall.	
		Calculate the relation between the two variables x and y.	(3)
	5.4.2	Calculate, by using integration, the area moment of the weir about the water level.	(3)
	5.4.3	Calculate, by using integration, the second moment of area of the weir about the water level, as well as the depth of the centre of pressure on the weir.	(4) **[40]**

QUESTION 6

6.1	Calculate the length of the curve described by the parametric equations, $x=3\sin^3\theta$ and $y=3\cos^3\theta$, between $x=\frac{\pi}{4}$ and $x=0$.	(6)
6.2	Calculate the surface area generated when the curve represented by $x=\sqrt{25-y^2}$, between $y=-5$ and $y=5$, rotates about the y-axis by means of integration.	(6) **[12]**

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} \text{ ; } \sin x = \frac{1}{\operatorname{cosec} x} \text{ ; } \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec(ax)] + C$
$\cot ax$	$-a \,\text{cosec}^2 ax$	$\frac{1}{a} \ln [\sin(ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\text{cosec}\, ax$	$-a \,\text{cosec}\, ax \cot ax$	$\frac{1}{a} \ln \left[\tan\left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\,.\,f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\,f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\,f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x)\,.\,f'(x)$	-
$\sec f(x)$	$\sec f(x)\tan f(x)\,.\,f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x)\cot f(x)\,.\,f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx \;;\; A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy \;;\; A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \rho\int_a^b y^2dx \; ; V_x = \rho\int_a^b \left(y_1^2 - y_2^2\right)dx \; ; V_x = 2\rho\int_a^b xydy$$

$$V_y = \rho\int_a^b x^2dy \; ; V_y = \rho\int_a^b \left(x_1^2 - x_2^2\right)dy \; ; V_y = 2\rho\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2dA \qquad ; \qquad I_y = \int_a^b r^2dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \mathrm{r}\,V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u_1}^{u_2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u_1}^{u_2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N6

24 NOVEMBER 2015

This marking guideline consists of 18 pages.

✓ $= \frac{1}{2}$ MARK:	TOTAL: $\frac{200}{2} = 100$

Note: Do not subtract marks for incorrect units or units omitted

QUESTION 1

1.1 1.1.1

$$z = e^{\frac{x}{y}}.\sin y$$

✓ ✓

$$\frac{\partial z}{\partial x} = \frac{1}{y}e^{\frac{x}{y}}.\sin y$$

(2)

1.1.2

$$\frac{\partial z}{\partial y} = e^{\frac{x}{y}} ✓ \cos y ✓ + (-xy^{-2} ✓ .e^{\frac{x}{y}} \sin y ✓)$$

(4)

1.2

$$\Delta A = \left(\frac{\partial A}{\partial b}\right)\Delta b + \left(\frac{\partial A}{\partial h}\right)\Delta h \quad ✓$$

✓ ✓

$$= \frac{1}{2}h(\Delta b) + \frac{1}{2}b(\Delta h)$$

✓ ✓

$$= \frac{1}{2}(5)(0{,}1) + \frac{1}{2}(3)(-0{,}13)$$

$$= 0{,}055\ m^2 \quad ✓$$

(6)

[12]

QUESTION 2

2.1

$$y = \int \arctan 3x.dx$$

$$f(x) = \tan^{-1} 3x \qquad g'(x) = 1$$

$$f'(x) = \frac{3}{(3x)^2 + 1} \qquad g(x) = x$$

✓ ✓

$$= x.\tan^{-1} 3x + \int \frac{3x}{9x^2 + 1}.dx$$

$$= x.\tan^{-1} 3x + \frac{3}{18}\int \frac{18x}{9x^2 + 1}dx \quad ✓$$

$$= x.\tan^{-1} 3x + \frac{1}{6}\ln(9x^2 + 1) + c \quad ✓$$

(4)

2.2 $y = \int \sin^3 2x \,.\cos^7 2x dx$

✓

$= \int \sin^2 2x.\sin 2x.\cos^7 2x dx$

✓

$= \int \left(1 - \cos^2 2x\right)\sin 2x.\cos^7 2x dx$

$u = \cos 2x$

$\therefore \dfrac{du}{dx} = -2\sin 2x$

$\therefore du = -2\sin 2x dx$

✓ ✓

$= -\dfrac{1}{2}\int \left(1 - u^2\right)u^7 du$

$= -\dfrac{1}{2}\int \left(u^7 - u^9\right) du$ ✓

$= -\dfrac{1}{2}\left(\dfrac{u^8}{8} - \dfrac{u^{10}}{10}\right)$ ✓

✓ ✓

$= -\dfrac{1}{2}\left(\dfrac{\cos^8 2x}{8} - \dfrac{\cos^{10} 2x}{10}\right) + c$

$= -\dfrac{\cos^8 2x}{16} + \dfrac{\cos^{10} 2x}{20} + c$

Or

$y = \int \sin^3 2x \,.\cos^7 2x dx$

✓

$= \int \sin^2 2x.\sin 2x.\cos^7 2x dx$

✓

$= \int \left(1 - \cos^2 2x\right)\sin 2x.\cos^7 2x dx$

✓ ✓

$= \int (\sin 2x \cos^7 2x - \sin 2x \cos^9 2x)dx$

$= \int (\sin 2x \cos^7 2x)dx - \int (\sin 2x \cos^9 2x)dx$

$u = \cos 2x$

$\therefore \dfrac{du}{dx} = -2\sin 2x$

$\therefore dx = \dfrac{du}{-2\sin 2x}$

✓ ✓ ✓ ✓

$= -\dfrac{1}{2}\left(\dfrac{\cos^8 2x}{8}\right) + \dfrac{1}{2}\left(\dfrac{\cos^{10} 2x}{10}\right) + c$ (8)

2.3

$$y = \int \frac{1}{\sqrt{5x - 2x^2}}\,dx$$

$$= -2x^2 + 5x$$

$$= -2(x^2 - \frac{5}{2}x) \quad \checkmark$$

$$\checkmark \qquad \checkmark$$

$$= -2[(x - \frac{5}{4})^2 - \frac{25}{16}]$$

$$= 2[\frac{25}{16} - (x - \frac{5}{4})^2] \quad \checkmark$$

$$= \int \frac{1}{\sqrt{2[\frac{25}{16} - (x - \frac{5}{4})^2]}}dx \quad \checkmark$$

$$\checkmark$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{[\frac{25}{16} - (x - \frac{5}{4})^2]}}dx$$

$$\checkmark \qquad \checkmark$$

$$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{(x - \frac{5}{4})}{\frac{5}{4}} + c$$

or

$$= \frac{1}{\sqrt{2}} \sin^{-1} \frac{4}{5}\left(x - \frac{5}{4}\right) + c$$

or

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4}{5}x - 1\right) + c$$

(8)

2.4 $y = \int \left(\frac{x^2}{2}+1\right)\cos 2x\,dx$

✓ ✓

$\int y\,dx = \left(\frac{x^2}{2}+1\right)\frac{\sin 2x}{2} - \int x.\frac{\sin 2x}{2}\,dx$

$f(x) = \frac{x^2}{2}+1 \quad g'(x) = \cos 2x$

$f'(x) = x \quad g(x) = \frac{\sin 2x}{2}$

✓

$= \left(\frac{x^2}{2}+1\right)\frac{\sin 2x}{2} - \frac{1}{2}\int x.\sin 2x\,dx$

✓ ✓ ✓

$= \left(\frac{x^2}{2}+1\right)\frac{\sin 2x}{2} - \frac{1}{2}\left[x.\frac{-\cos 2x}{2} - \int \frac{-\cos 2x}{2}\,dx\right]$

$f(x) = x \quad g'(x) = \sin 2x$

$f'(x) = 1 \quad g(x) = \frac{-\cos 2x}{2}$

✓ ✓

$= \left(\frac{x^2}{2}+1\right)\frac{\sin 2x}{2} + \frac{1}{4}x.\cos 2x - \frac{1}{4}\left(\frac{\sin 2x}{2}\right) + c$

$= \left(\frac{x^2}{2}+1\right)\frac{\sin 2x}{2} + \frac{1}{4}x.\cos 2x - \frac{1}{8}\sin 2x + c$

Or

$y = \int \frac{x^2}{2}\cos 2x\,dx + \int \cos 2x\,dx$ ✓

$f(x) = \frac{x^2}{2} \quad g'(x) = \cos 2x$

$f'(x) = x \quad g(x) = \frac{\sin 2x}{2}$

✓ ✓

$= \left(\frac{x^2}{2}\right)\frac{\sin 2x}{2} - \int x.\frac{\sin 2x}{2}\,dx + \int \cos 2x\,dx$

$f(x) = x \quad g'(x) = \sin 2x$

$f'(x) = 1 \quad g(x) = \frac{-\cos 2x}{2}$

✓ ✓

$= \frac{x^2}{4}\sin 2x - \frac{1}{2}\left[x.\frac{-\cos 2x}{2} - \int \frac{-\cos 2x}{2}\,dx\right] + \frac{\sin 2x}{2} + c$

✓

$= \frac{x^2}{4}\sin 2x + \frac{1}{4}x.\cos 2x + \frac{1}{4}\int \frac{-\cos 2x}{2}\,dx + \frac{\sin 2x}{2} + c$

✓ ✓

$= \frac{x^2}{4}\sin 2x + \frac{1}{4}x.\cos 2x - \frac{1}{4}.\frac{\sin 2x}{2} + \frac{\sin 2x}{2} + c$ (8)

2.5 $y = \int \tan^5 ax\, dx$

$= \int \tan^2 ax.\tan^3 ax\, dx$ ✓

✓

$= \int (\sec^2 ax - 1).\tan^3 ax\, dx$

✓ ✓

$= \int (\sec^2 ax.\tan^3 ax)dx - \int \tan^3 ax\, dx$

$= \int (\sec^2 ax.\tan^3 ax)dx - \int \tan^2 ax.\tan axdx$

$= \int (\sec^2 ax)(\tan ax)^3 dx - \int (\sec^2 ax - 1).\tan axdx$

$= \frac{1}{a}\int (a\sec^2 ax)(\tan ax)^3 dx - \frac{1}{a}\int a\sec^2 ax.\tan axdx + \int \tan ax\, dx$ ✓

✓ ✓ ✓

$= \frac{1}{a}.\frac{\tan^4 ax}{4} - \frac{1}{a}.\frac{\tan^2 ax}{2} + \frac{1}{a}\ln(\sec ax) + c$

$= \frac{1}{4a}\tan^4 ax - \frac{1}{2a}\tan^2 ax + \frac{1}{a}\ln(\sec ax) + c$

(8)

[36]

QUESTION 3

3.1 $\int \frac{x^2 - x - 4}{(x+4)^3}\, dx$

$\frac{x^2 - x - 4}{(x+4)^3} = \frac{A}{(x+4)^3} + \frac{B}{(x+4)^2} + \frac{C}{x+4}$ ✓

$x^2 - x - 4 = A + B(x+4) + C(x+4)^2$ ✓

$= A + Bx + 4B + Cx^2 + 8Cx + 16C$ ✓

$let \quad x = -4 \quad \therefore \quad A = 16$ ✓

$Equate \quad x^2 : \quad C = 1$ ✓ $\quad Equate \quad x : \quad B = -9$ ✓

$= \int \frac{16}{(x+4)^3}dx + \int \frac{-9}{(x+4)^2}dx + \int \frac{1}{x+4}dx$ ✓

✓ ✓ ✓

$= 16.\frac{(x+4)^{-2}}{-2} - 9.\frac{(x+4)^{-1}}{-1} + \ln(x+4) + c$

$= \frac{-8}{(x+4)^2} + \frac{9}{x+4} + \ln(x+4) + c$

(10)

3.2 $\int \frac{-x^2+7x}{(1-3x)(x^2+1)}\,dx$

✓ ✓

$$\frac{-x^2+7x}{(1-3x)(x^2+1)} = \frac{A}{1-3x} + \frac{Bx+C}{x^2+1}$$

$$-x^2+7x = A(x^2+1) + (Bx+C)(1-3x) \quad ✓$$

$$-x^2+7x = Ax^2 + A + Bx + C - 3Bx^2 - 3Cx \quad ✓$$

$$let \quad x = \frac{1}{3} \qquad \therefore \; A = 2 \quad ✓$$

$$Equate \quad x^2: \quad B = 1 \quad ✓$$

$$Equate \quad x: \quad C = -2 \quad ✓$$

$$\int \frac{2}{1-3x}dx + \int \frac{x-2}{x^2+1}dx \quad ✓$$

✓

$$= \int \frac{2}{1-3x}.dx + \int \frac{x}{x^2+1}dx + \int \frac{-2}{x^2+1}dx$$

✓ ✓ ✓ ✓ ✓

$$= \frac{2}{-3}\ln(1-3x) + \frac{1}{2}\ln(x^2+1) - 2\tan^{-1}x + c$$

(14)

[24]

QUESTION 4

4.1 $(1-x^2)dy - xydx = 2(1-x^2)dx$

$$\frac{dy}{dx} - \frac{xy}{1-x^2} = 2 \quad ✓$$

✓ ✓

$$\sqrt{1-x^2}.y = \int \sqrt{1-x^2}.2dx$$

$$\sqrt{1-x^2}.y = 2\int \sqrt{1-x^2}dx \quad ✓$$

✓ ✓ ✓

$$\sqrt{1-x^2}.y = 2\left(\frac{1}{2}\sin^{-1}x + \frac{x}{2}\sqrt{1-x^2}\right) + c$$

(10)

$$e^{\int \frac{x}{1-x^2}.dx} \quad ✓$$

$$= e^{\frac{1}{2}\ln(1-x^2)} \quad ✓$$

$$= (1-x^2)^{\frac{1}{2}} \quad ✓$$

or

$$= \sqrt{1-x^2}$$

4.2

$$\frac{d^2y}{dx^2} + 9y = x^2 + 4x$$

$y_c: \quad m^2 + 9 = 0$ ✓

$m = \pm\sqrt{-9}$

$\therefore \; m = 0 \pm 3j$

$y_c = A\cos 3x + B\sin 3x$ ✓

$\therefore \; y = Cx^2 + Dx + E$ ✓

$\frac{dy}{dx} = 2Cx + D$ ✓

$\frac{d^2y}{dx^2} = 2C$ ✓

$\therefore \; 2C + 9Cx^2 + 9Dx + 9E = x^2 + 4x$ ✓

Equate coefficients of x^2: $9C = 1 \quad \therefore \; C = \frac{1}{9}$ (0,111) ✓

Equate coefficients of x: $9D = 4 \quad \therefore \; D = \frac{4}{9}$ (0,444) ✓

Equate constants: $2C + 9E = 0 \quad \therefore \; E = -\frac{2}{81}$ (0,025) ✓

$\therefore \; y_p = \frac{1}{9}x^2 + \frac{4}{9}x - \frac{2}{81}$

$y = A\cos 3x + B\sin 3x + \frac{1}{9}x^2 + \frac{4}{9}x - \frac{2}{81}$ ✓

$1 = A + \frac{2}{81} \quad \therefore$ A $= \frac{83}{81}$ (1,025) ✓

$\frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x + \frac{2}{9}x + \frac{4}{9}$ ✓

$2 = 3B + \frac{4}{9} \quad \therefore \; B = \frac{14}{27}$ (0,519) ✓

$y = \frac{83}{81}\cos 3x + \frac{14}{27}\sin 3x + \frac{1}{9}x^2 + \frac{4}{9}x - \frac{2}{81}$ ✓

(14)

[24]

QUESTION 5

5.1 5.1.1

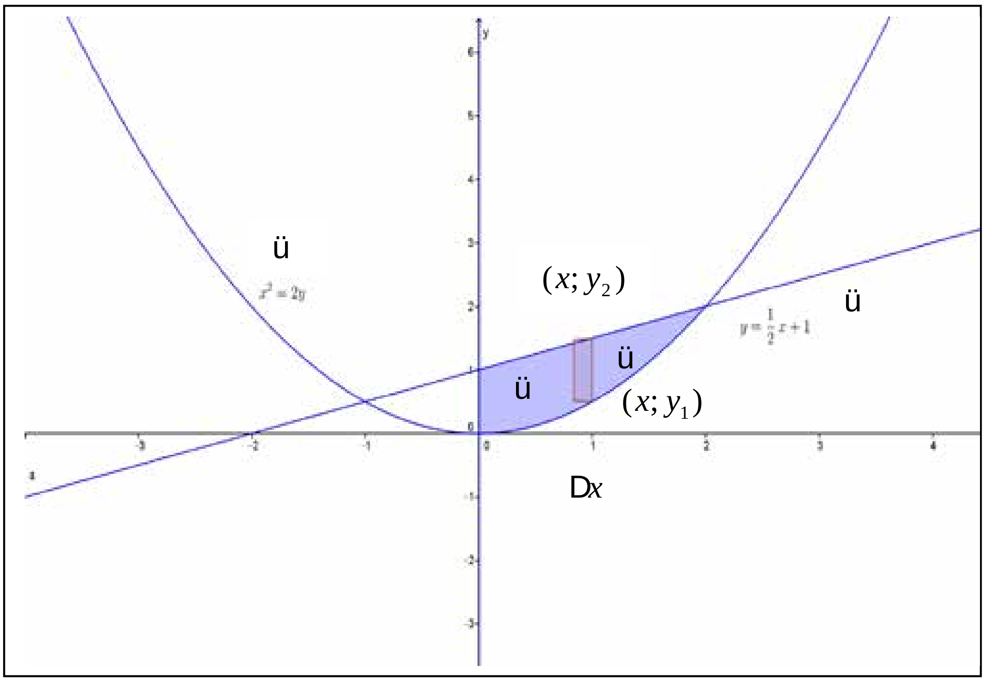
$$\frac{x^2}{2} = \frac{x}{2} + 1$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$\therefore \quad x = 2; \quad x = -1$ ✓ ✓

$y = 2; \ y = \frac{1}{2}$ or $(2;2) \ ; \ (-1;\frac{1}{2})$



(6)

5.1.2

$\Delta A = (y_2 - y_1)\Delta x$ ✓

✓

$$A = \int_0^2 (y_2 - y_1)dx$$

$$= \int_0^2 \left(\frac{x}{2} + 1 - \frac{1}{2}x^2\right)dx \ ✓$$

$$= \left[\frac{1}{2}\cdot\frac{x^2}{2} + x - \frac{1}{2}\cdot\frac{x^3}{3}\right]_0^2 \ ✓$$

$$= \left[\frac{(2)^2}{4} + (2) - \frac{(2)^3}{6}\right] ✓$$

$$= 1{,}667 \quad units^2 \ ✓$$

Incorrect limits: max 3 marks

(6)

5.1.3 ✓ ✓

$$A_{m-x} = (y_2 - y_1)\Delta x \times \frac{y_2 + y_1}{2}$$

✓

$$= \frac{1}{2}\int_0^2 (y_2{}^2 - y_1{}^2)dx$$

Incorrect limits: max 6 marks

$$= \int_0^2 [(\frac{x}{2}+1)^2 - (\frac{1}{2}x^2)^2]dx \text{ ✓}$$

$$= \int_0^2 (\frac{x^2}{4} + x + 1 - \frac{1}{4}x^4)dx \text{ ✓}$$

$$= \frac{1}{2}\left[\frac{x^3}{12} + \frac{x^2}{2} + x - \frac{1}{20}x^5\right]_0^2 \text{ ✓}$$

$$= \frac{1}{2}\left[\frac{(2)^3}{12} + \frac{(2)^2}{2} + (2) - \frac{1}{20}(2)^5\right] \text{ ✓}$$

$$= 1{,}533 \quad units^3 \text{ ✓}$$

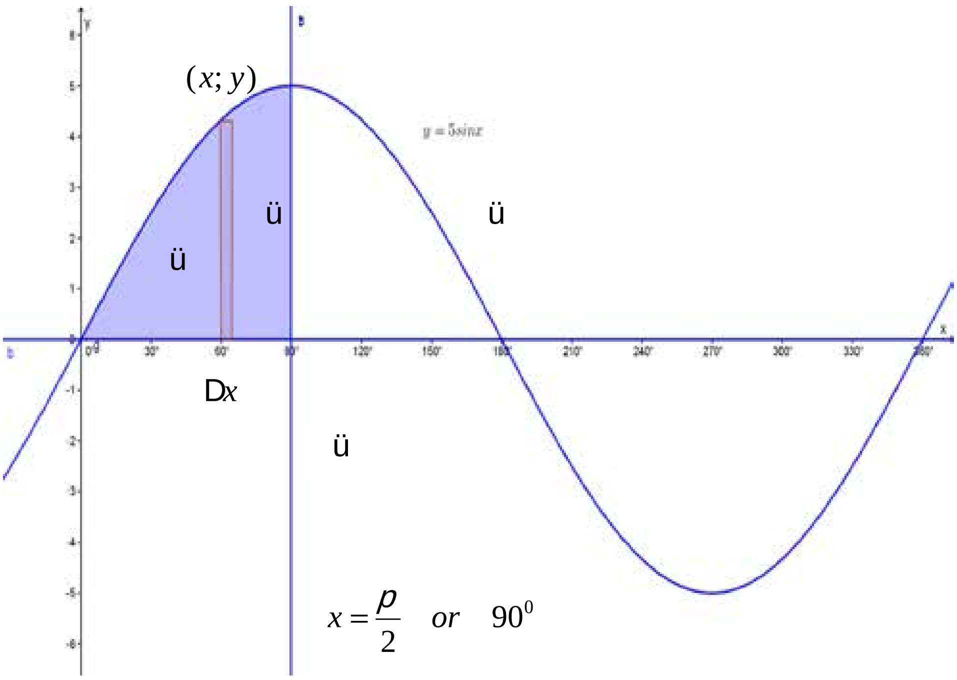
$$\bar{y} = \frac{A_{m-x}}{A}$$

$$= \frac{1{,}533}{1{,}667} \text{ ✓}$$

$$= 0{,}92\ units \text{ ✓}$$

(10)

5.2 5.2.1



(4)

5.2.2 $\Delta V_x = \pi y^2 \Delta x$ ✓

$$V_y = \pi \int_0^{\frac{\pi}{2}} y^2\,dx \quad ✓$$

Incorrect limits: max 3 marks

$$= \pi \int_0^{\frac{\pi}{2}} (5\sin x)^2\,dx \quad ✓$$

$$= \pi \int_0^{\frac{\pi}{2}} (25\sin^2 x)\,dy$$

$$= 25\pi \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} \quad ✓$$

$$= 25\pi \left[\frac{\left(\frac{\pi}{2}\right)}{2} - \frac{\sin 2\left(\frac{\pi}{2}\right)}{4}\right] \quad ✓$$

$$= 19{,}635\pi \quad units^3 \quad or \quad 61{,}685 \quad units^3 \quad ✓$$

(6)

5.2.3 $\Delta M = \rho.\Delta V_x$

$$= \rho.\pi y^2 \Delta x$$

$$\therefore \Delta I_x = \rho.\pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}}\right)^2 \quad ✓ \; ✓$$

$$I_x = \frac{\rho\pi}{2} \int_0^{\frac{\pi}{2}} y^4\,dx \quad ✓$$

Incorrect limits: max 6 marks

$$= \frac{\rho\pi}{2} \int_0^{\frac{\pi}{2}} (5\sin x)^4\,dx \quad ✓$$

$$= \frac{625\rho\pi}{2} \int_0^{\frac{\pi}{2}} (\sin^2 x)^2\,dx$$

$$= 312{,}5\rho\pi \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 dx \quad ✓$$

$$= \frac{312{,}5\pi\rho}{4} \int_0^{\frac{\pi}{2}} \left(1 - 2\cos 2x + \cos^2 2x\right)dx$$

$$= 78{,}125\pi\rho \left[\left(x - \frac{2\sin 2x}{2} + \frac{x}{2} + \frac{\sin 4x}{8}\right)\right]_0^{\frac{\pi}{2}} \quad ✓$$

$$= 78{,}125\pi\rho \left[\left(\frac{\pi}{2} - \sin 2\left(\frac{\pi}{2}\right) + \frac{\frac{\pi}{2}}{2} + \frac{\sin 4\left(\frac{\pi}{2}\right)}{8}\right)\right] \quad ✓$$

$$= 184{,}078\pi\rho \quad units^4 \quad or \quad 578{,}298\rho \quad ✓$$

$$I_x = \frac{578{,}298m}{61{,}685} \quad ✓$$

$$= 9{,}375\,m$$

(10)

OR

ALTERNATIVE METHOD

$= \rho.\pi y^2 \Delta x$ ✓

$\therefore \Delta I_x = \rho.\pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}}\right)^2$ ✓

✓

$I_x = \frac{\rho\pi}{2}\int_0^{\frac{\pi}{2}} y^4 dx$ ✓

$= \frac{\rho\pi}{2}\int_0^{\frac{\pi}{2}} (5\sin x)^4 dx$

$= \frac{625\rho\pi}{2}\int_0^{\frac{\pi}{2}} (\sin^2 x)^2 dx$ ✓

$= \frac{625\rho\pi}{2}\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)^2 dx$

$= 312{,}5\pi\rho \int_0^{\frac{\pi}{2}} \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right) dx$ ✓

$= 312{,}5\pi\rho \left[\left(\frac{1}{4}x - \frac{1}{2}.\frac{\sin 2x}{2} + \frac{1}{4}\left(\frac{x}{2} + \frac{\sin 4x}{8}\right)\right)\right]_0^{\frac{\pi}{2}}$ ✓

$= 312{,}5\pi\rho \left[\frac{1}{4}\left(\frac{\pi}{2}\right) - \frac{1}{2}.\frac{\sin 2\left(\frac{\pi}{2}\right)}{2} + \frac{1}{4}\left(\frac{\frac{\pi}{2}}{2} + \frac{\sin 4\left(\frac{\pi}{2}\right)}{8}\right)\right]$ ✓ ✓

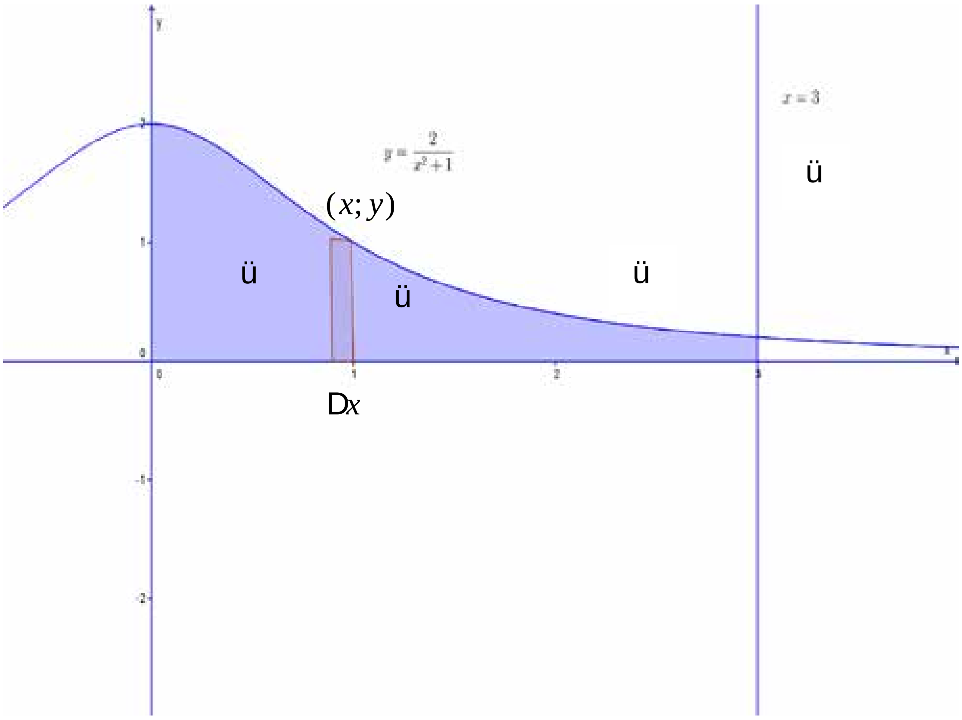
$= 184{,}078\pi\rho \quad units^4 \quad or \quad 578{,}298\rho$

$I_x = \frac{578{,}298m}{61{,}685}$ ✓

$= 9{,}375\ m$ ✓

(10)

5.3 5.3.1



(4)

5.3.2

$\Delta A = y\Delta x$ ✓

✓

$A = \int_0^3 y\,dx$

Incorrect limits: max 3 marks

$= \int_0^3 \left(\frac{2}{1+x^2}\right)dx$ ✓

$= \left[2\tan^{-1}x\right]_0^3$ ✓

$= 2\left[\tan^{-1}3 - \tan^{-1}0\right]$ ✓

$= 2{,}498 \quad units^2$ ✓

(6)

5.3.3

$\Delta I_y = y\Delta x \times x^2$ ✓

✓ ✓

$I_y = \int_0^3 x^2 y\,dx$

$= \int_0^3 x^2\left(\frac{2}{x^2+1}\right)dx$ ✓

$= 2\int_0^3 \frac{x^2}{x^2+1}dx$

$$\begin{array}{r|l} & 1 \\ \hline x^2+1 & x^2 \\ & \underline{x^2+1} \\ & -1 \end{array}$$

$= 2\int_0^3 \left(1 - \frac{1}{x^2+1}\right)dx$ ✓

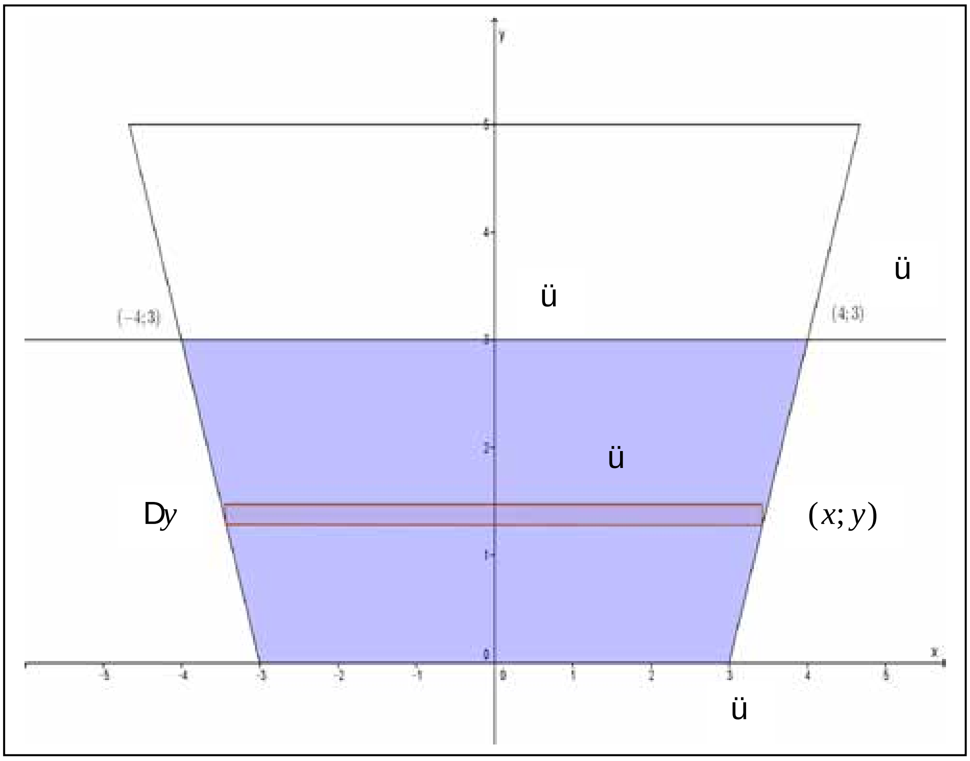
$= 2\left[x - \tan^{-1}x\right]_0^3$ ✓

$= 2\left[3 - \tan^{-1}3\right]$ ✓

$= 3{,}502 \quad units^4$ ✓

(8)

5.4 5.4.1



$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-3}{x-4}=\frac{0-3}{3-4}\text{........}(4;3)\ and\ (3;0)$$

$$y=3x-9\ ✓$$

$$\therefore\ x=\frac{1}{3}y+3\ ✓\qquad \therefore\ dA=2(\frac{1}{3}y+3)dy\quad \text{or}\quad \therefore\ dA=\frac{2}{3}(y+9)dy$$

(6)

5.4.2

$$\int_0^3 r\,dA$$

✓ ✓ ✓

$$=\int_0^3(5-y).2(\frac{1}{3}y+3)dy$$

$$=2\int_0^3(5-y)(\frac{1}{3}y+3)dy$$

$$=2\int_0^3(\frac{5}{3}y+15-\frac{1}{3}y^2-3y)dy\ ✓$$

$$=2\left[\frac{5}{3}.\frac{y^2}{2}+15y-\frac{1}{3}.\frac{y^3}{3}-\frac{3y^2}{2}\right]_0^3\ ✓$$

$$=2\left[\frac{5}{3}.\frac{3^2}{2}+15(3)-\frac{1}{3}.\frac{3^3}{3}-\frac{3(3^2)}{2}\right]$$

$= 72$ units^3 ✓

(6)

5.4.3 $\int_0^3 r^2 dA$

$\checkmark \quad \checkmark$

$= \int_0^3 (5-y)^2 . 2(\frac{1}{3}y+3)dy$

$= 2\int_0^3 (5-y)^2 (\frac{1}{3}y+3)dy$

$= 2\int_0^3 (\frac{25}{3}y - \frac{10}{3}y^2 + \frac{1}{3}y^3 + 75 - 30y + 3y^2)dy$ $\checkmark$

$= 2\left[\frac{25}{3}(\frac{y^2}{2}) - \frac{10}{3}(\frac{y^3}{3}) + \frac{1}{3}(\frac{y^4}{4}) + 75y - \frac{30y^2}{2} + \frac{3y^3}{3}\right]_0^3$ $\checkmark$

$= 2\left[\frac{25}{3}(\frac{3^2}{2}) - \frac{10}{3}(\frac{3^3}{3}) + \frac{1}{3}(\frac{3^4}{4}) + 75(3) - \frac{30(3)^2}{2} + \frac{3(3)^3}{3}\right]_0^3$ $\checkmark$

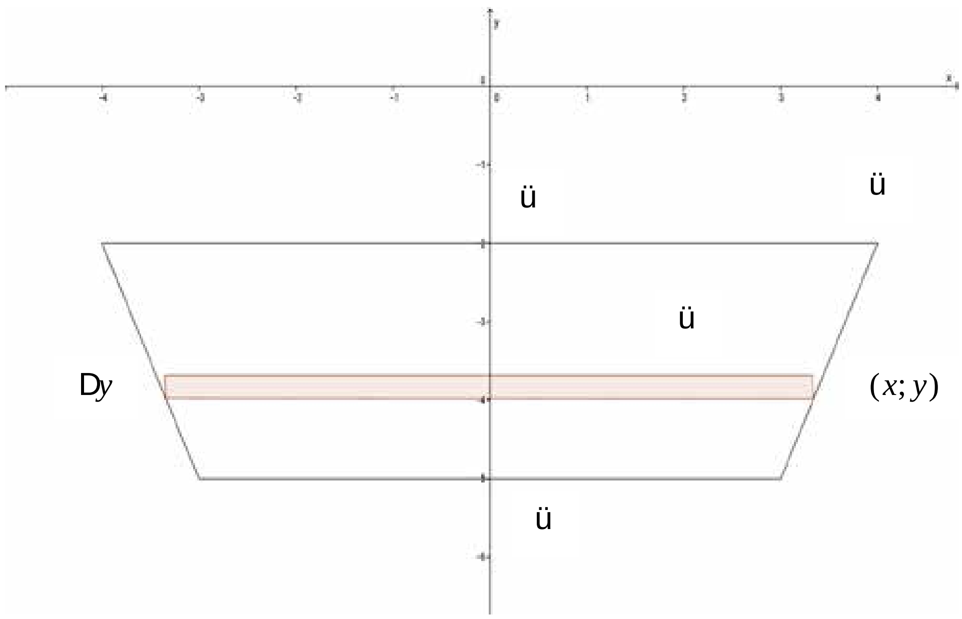
$= 262,5 \quad \text{units}^4$ $\checkmark$

$\bar{y} = \frac{262,5}{72}$ $\checkmark$

$= 3,646\ units$ $\checkmark$ (8)

Or alternative method

5.4 5.4.1



$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{y-(-2)}{x-4} = \frac{-5-(-2)}{3-4}$........$(4;-2)\ and\,(3;-5)$

$y = 3x - 14$ $\checkmark$

$\therefore\ x = \frac{1}{3}y + \frac{14}{3}$ $\checkmark$ $\quad \therefore\ dA = 2(\frac{1}{3}y + \frac{14}{3})dy \quad$ 0r $\therefore\ dA = \frac{2}{3}(y+14)dy$ (6)

5.4.2 $\int_{-5}^{-2} r\,dA$

✓ ✓ ✓

$= \int_{-5}^{-2} y.2(\frac{1}{3}y + \frac{14}{3})dy$

$=2\int_{-5}^{-2} (\frac{1}{3}y^2 + \frac{14}{3}y)dy$ ✓

$=2\left[\frac{1}{3}.\frac{y^3}{3} + \frac{14}{3}.\frac{y^2}{2}\right]_{-5}^{-2}$ ✓

$=2\left\{\left[\frac{1}{9}.(-2)^3 + \frac{14}{6}.(-2)^2\right] - \left[\frac{1}{9}.(-5)^3 + \frac{14}{6}.(-5)^2\right]\right\}$ ✓

$= -72$ units^3 ✓ (6)

5.4.3 $\int_{-5}^{-2} r^2\,dA$

✓ ✓ ✓

$= \int_{-5}^{-2} y^2.2(\frac{1}{3}y + \frac{14}{3})dy$

$= \frac{2}{3}\int_{-5}^{-2} (y^3 + 14y^2)dy$

$= \frac{2}{3}\left[\frac{y^4}{4} + 14.\frac{y^3}{3}\right]_{-5}^{-2}$ ✓

$= \frac{2}{3}\left\{\left[\frac{(-2)^4}{4} + 14.\frac{(-2)^3}{3}\right] - \left[\frac{(-5)^4}{4} + 14.\frac{(-5)^3}{3}\right]\right\}$ ✓

$= 262,5$ units^4 ✓

$\bar{y} = \frac{262,5}{-72}$ ✓

$= -3,646\ units$ ✓

(8)

[80]

QUESTION 6

6.1

$x = 3\sin^3\theta$ $\qquad\qquad$ $y = 3\cos^3\theta$

$\dfrac{dx}{d\theta} = 9(\sin\theta)^2\cos\theta$ ✓ $\qquad$ $\dfrac{dy}{d\theta} = 9(\cos\theta)^2(-\sin\theta)$ ✓

$\left(\dfrac{dx}{d\theta}\right)^2 = [9(\sin\theta)^2\cos\theta]^2$ ✓ $\qquad$ $\left(\dfrac{dy}{d\theta}\right)^2 = [9(\cos\theta)^2(-\sin\theta)]^2$ ✓

$\left(\dfrac{dx}{d\theta}\right)^2 + \left(\dfrac{dy}{d\theta}\right)^2 = 81(\sin\theta)^4\cos^2\theta + 81(\cos\theta)^4\sin^2\theta$ ✓

$= 81\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= 81\sin^2\theta\cos^2\theta$ ✓

✓

$S = \int_0^{\frac{\pi}{4}} \sqrt{81\sin^2\theta\cos^2\theta}\, d\theta$ ✓

$S = 9\int_0^{\frac{\pi}{4}} \sin\theta\cos\theta\, d\theta$ ✓

$= 9\left[\dfrac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{4}}$ ✓ $\qquad$ or $\qquad$ $= -9\left[\dfrac{\cos^2\theta}{2}\right]_0^{\frac{\pi}{4}}$

$= 9\left[\dfrac{\sin^2\frac{\pi}{4}}{2} - \dfrac{\sin^2 0}{2}\right]$ ✓ $\qquad$ $= -9\left[\dfrac{\cos^2\frac{\pi}{4}}{2} - \dfrac{\cos^2 0}{2}\right]$

$=$ 4,5 units ✓

(12)

6.2 $x=\sqrt{25-y^2}$

$$\frac{dx}{dy}=\frac{1}{2}(25-y^2)^{-\frac{1}{2}}.-2y \quad ✓$$

$$\frac{dx}{dy}=\frac{-y}{\sqrt{25-y^2}} \quad ✓$$

$$\left(\frac{dx}{dy}\right)^2=\left(\frac{-y}{\sqrt{25-y^2}}\right)^2 \quad ✓$$

$$1+\left(\frac{dx}{dy}\right)^2=1+\left(\frac{-y}{\sqrt{25-y^2}}\right)^2 \quad ✓$$

$$=1+\frac{y^2}{25-y^2}$$

$$=\frac{25}{25-y^2} \quad ✓$$

$$A=2\pi\overset{✓}{\int_{-5}^{5}}x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \quad ✓$$

$$=2\pi\overset{✓}{\int_{-5}^{5}}\sqrt{25-y^2}.\sqrt{\frac{25}{25-y^2}}dy$$

$$=2\pi\int_{-5}^{5}5\,dy \quad ✓$$

$$=10\pi(y)_{-5}^{5} \quad ✓$$

$$=10\pi[5-(-5)] \quad ✓$$

$$=100\pi\text{ units}^2 \quad \text{or} \quad 314{,}159\text{ units}^2 \quad ✓$$

(12)
[24]

TOTAL: 200

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T980**(E)**(A6)T
AUGUST EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

6 August 2015 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $z = e^{-x}.\cos y$ prove that $\dfrac{\partial^2 z}{\partial y \partial x} = \dfrac{\partial^2 z}{\partial x \partial y}$ (3)

1.2 If $x = 4\sin\theta$ and $y = \cos^3\theta$, calculate $\dfrac{dy}{dx}$ at the point where $\theta = \dfrac{\pi}{6}$ (3)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \cot^4 \dfrac{x}{2}$ (4)

2.2 $y = \sqrt{12 - 6x - 3x^2}$ (4)

2.3 $y = e^{3x} \sin x$ (5)

2.4 $y = \dfrac{\cos^3 4x}{\sin^2 4x}$ (5)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{x^2 - 5x + 4}{(x^2 + 5)(x - 3)}\, dx$ (6)

3.2 $\displaystyle\int \frac{4x^3 + x^2 - 2x + 1}{x^2 - 3x + 2}\, dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$2\dfrac{dy}{dx} - \dfrac{6y}{x - 1} = 2(x - 1)^2$ at (3;1) (5)

4.2 Calculate the particular solution of:

$\dfrac{d^2 y}{dx^2} - 6\dfrac{dy}{dx} + 5y = 5e^{5x}$, if $x = 0$ when $y = 3$ and $\dfrac{dy}{dx} = 0$ when $x = 0$. (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of the two curves $y = -x^2 + 4$ and $y = -4x^2 + 4$. Make a neat sketch of the curves and show the area, in the first quadrant, bounded by the curves and the x-axis. Show the representative strip/element that you will use to calculate the volume generated if the area, in the first quadrant, bounded by the curves rotates about the y-axis. (3)

5.1.2 Calculate the volume generated if the area, as described in QUESTION 5.1.1, bounded by the two curves $y = -x^2 + 4$ and $y = -4x^2 + 4$ rotates about the y-axis. (4)

5.2 5.2.1 Sketch the graph of $y = \sqrt{9 - x^2}$ and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the graph and the x-axis rotates about the y-axis. (2)

5.2.2 Calculate the volume described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance of the centre of gravity from the x-axis of the solid generated when the area described in QUESTION 5.2.1 rotates about the y-axis. (5)

5.3 5.3.1 Make a neat sketch of the graph $\frac{x^2}{4} - y^2 = 1$ and show the representative strip/element that you will use to calculate the volume of the solid generated when the area, in the first quadrant, bounded by the graph, the y-axis, the x-axis and the line $y = 2$ is rotated about the y-axis. (2)

5.3.2 Calculate the volume of the solid generated as described in QUESTION 5.3.1. (3)

5.3.3 Calculate the moment of inertia of the solid generated when the area described in QUESTION 5.3.1 rotates about the y-axis. (5)

5.3.4 Express the answer in QUESTION 5.3.3 in terms of the mass. (1)

5.4 5.4.1 A vertical sluice gate, in the form of a parabola, is installed in a dam wall. The sluice gate is 4 m high and 8 m wide at the top. The top of the sluice gate lies in the water level. Sketch the vertical sluice gate and show the representative strip/element that you will use to calculate the area moment of the sluice gate about the water level. (2)

5.4.2 Calculate the relation between the two variables x and y that you will need to calculate the area moment of the sluice gate about the water level by means of integration. (2)

5.4.3 Calculate the area moment of the sluice gate about the water level by means of integration. (4)

5.4.4 Calculate the second moment of area of the sluice gate about the water level, as well as the depth of the centre of pressure on the sluice gate by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $y + ln\, x = \frac{x^2}{8}$ between the points $x = 1$ and $x = e$. (6)

6.2 The curve with parametric equations $x = 6t + t^2$ and $y = 2t + 6$ revolves about the x-axis between 0 £ t £ 2.

Calculate the surface area thus generated. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \mathrm{cosec}^2\, x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}\ ;\ \sin x = \frac{1}{\mathrm{cosec}\ x}\ ;\ \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln[\sec(ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln[\sin(ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln[\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln\left[\tan\left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\dfrac{d}{dx}f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\,.\,f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\,f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\,f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x)\,.\,f'(x)$	-
$\sec f(x)$	$\sec f(x)\tan f(x)\,.\,f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x)\cot f(x)\,.\,f'(x)$	-
$\sin^{-1} f(x)$	$\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\dfrac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\dfrac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\dfrac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\dfrac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\dfrac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\dfrac{x}{2}-\dfrac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\dfrac{x}{2}+\dfrac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\dfrac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2dx \,;\, V_x = p\int_a^b \left(y_1^2 - y_2^2\right)dx \,;\, V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2dy \,;\, V_y = p\int_a^b \left(x_1^2 - x_2^2\right)dy \,;\, V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2dA \qquad ; \qquad I_y = \int_a^b r^2dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \,; \qquad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \text{r}\, V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2dm = r\int_a^b r^2dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\; dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\; dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\; du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

AUGUST EXAMINATION

MATHEMATICS N6

6 AUGUST 2015

This marking guideline consists of 18 pages.

QUESTION 1

1.1 $z = e^{-x}.\cos y$

$$\frac{\partial z}{\partial x} = -e^{-x}.\cos y \quad ✓ \qquad\qquad \frac{\partial z}{\partial y} = -e^{-x}.\sin y \quad ✓$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \qquad\qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial y}\left(-e^{-x}.\cos y\right) \quad ✓ \qquad\qquad = \frac{\partial}{\partial x}\left(-e^{-x}.\sin y\right) \quad ✓$$

$$= -e^{-x}(-\sin y) \quad ✓ \qquad\qquad = -(-e^{-x})(\sin y) \quad ✓$$

$$= e^{-x}.\sin y \qquad\qquad = e^{-x}.\sin y$$

$$\therefore \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

(6 x ½) (3)

1.2 $x = 4\sin\theta$ and $y = \cos^3\theta$

$$\frac{dy}{d\theta} = 4\cos\theta \quad ✓ \qquad\qquad \frac{dx}{d\theta} = 3\cos^2\theta(-\sin\theta) \quad ✓$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{-3\cos^2\theta\sin\theta}{4\cos\theta} \quad ✓$$

$$\therefore \frac{dy}{dx} = -\frac{3}{4}\cos\theta\sin\theta \quad ✓$$

$$= -\frac{3}{4}\cos\frac{\pi}{6}\sin\frac{\pi}{6} \quad ✓$$

$$= -0,325 \quad ✓$$

(6 x ½) (3)
[6]

QUESTION 2

2.1

$$y = \int \cot^4 \frac{x}{2}\,dx$$

$$= \int \cot^2 \frac{x}{2} . \cot^2 \frac{x}{2}\,dx \quad ✓$$

✓

$$= \int (\operatorname{cosec}^2 \frac{x}{2} - 1) . \cot^2 \frac{x}{2}\,dx$$

✓ ✓

$$= \int (\operatorname{cosec}^2 \frac{x}{2} . \cot^2 \frac{x}{2})dx - \int \cot^2 \frac{x}{2}\,dx$$

✓ ✓ ✓ ✓

$$= -2 . \frac{\cot^3 \frac{x}{2}}{3} - \left[\frac{1}{\frac{1}{2}} \cot \frac{x}{2} - x \right] + c$$

$$= -\frac{2}{3}\cot^3 \frac{x}{2} + 2\cot \frac{x}{2} + x + c$$

(8 x ½) (4)

2.2

$$y = \int \sqrt{12 - 6x - 3x^2}\,dx$$

$$= -3x^2 - 6x + 12$$

$$= -3(x^2 + 2x - 4) \quad ✓$$

✓ ✓

$$= -3[(x+1)^2 - 4 - 1]$$

$$= -3[(x+1)^2 - 5]$$

$$= 3[5 - (x+1)^2] \quad ✓$$

$$= \int \sqrt{3\left[5 - (x+1)^2\right]}\,dx \quad ✓$$

✓ ✓ ✓

$$= \sqrt{3}\left[\frac{5}{2} \sin^{-1} \frac{(x+1)}{\sqrt{5}} + \frac{(x+1)}{2}\sqrt{5 - (x+1)^2} \right] + c$$

$$= \frac{5\sqrt{3}}{2} \sin^{-1} \frac{(x+1)}{\sqrt{5}} + \frac{\sqrt{3}(x+1)}{2}\sqrt{5 - (x+1)^2} + c$$

Or

$$= 4{,}33 \sin^{-1} \frac{(x+1)}{2{,}236} + 0{,}866(x+1)\sqrt{5 - (x+1)^2} + c$$

(8 x ½) (4)

2.3 $y = \int e^{3x} \sin x \, dx$

$f(x) = e^{3x}$ $\quad g'(x) = \sin x$
$f'(x) = 3e^{3x}$ $\quad g(x) = -\cos x$

$$\int y\,dx = e^{3x}(-\cos x) - \int 3e^{3x}(-\cos x)dx \quad \checkmark \checkmark$$

$f(x) = e^{3x}$ $\quad g'(x) = \cos x$
$f'(x) = 3e^{3x}$ $\quad g(x) = \sin x$

$$= e^{3x}(-\cos x) + 3\int e^{3x}.\cos x dx \quad \checkmark$$

$$= e^{3x}(-\cos x) + 3[e^{3x}.\sin x - \int 3e^{3x}.\sin x dx] \quad \checkmark \checkmark$$

$$= e^{3x}(-\cos x) + 3e^{3x}.\sin x - 9\int e^{3x}.\sin x dx \quad \checkmark \checkmark$$

$$I = e^{3x}(-\cos x) + 3e^{3x}.\sin x - 9I \quad \checkmark$$

$$\therefore \quad 10I = e^{3x}(-\cos x) + 3e^{3x}.\sin x \quad \checkmark$$

$$I = \frac{1}{10}(-e^{3x}.\cos x + 3e^{3x}.\sin x) + c \quad \checkmark$$

Or

$$\int y\,dx = \frac{e^{3x}}{3}.\sin x - \int \cos x.\frac{e^{3x}}{3}dx \quad \checkmark \checkmark$$

$f(x) = \sin x$ $\quad g'(x) = e^{3x}$
$f'(x) = \cos x$ $\quad g(x) = \frac{e^{3x}}{3}$

$$= \frac{1}{3}e^{3x}.\sin x - \frac{1}{3}\int e^{3x}.\cos x.dx \quad \checkmark$$

$f(x) = \cos x$ $\quad g'(x) = e^{3x}$
$f'(x) = -\sin x$ $\quad g(x) = \frac{e^{3x}}{3}$

$$= \frac{1}{3}e^{3x}.\sin x - \frac{1}{3}\left[\frac{1}{3}e^{3x}.\cos x - \int -\sin x.\frac{e^{3x}}{3}.dx\right] \quad \checkmark \checkmark$$

$$= \frac{1}{3}e^{3x}.\sin x - \frac{1}{9}e^{3x}.\cos x - \frac{1}{9}\int e^{3x}.\sin x dx \quad \checkmark \checkmark$$

$$= \frac{1}{3}e^{3x}.\sin x - \frac{1}{9}e^{3x}.\cos x - \frac{1}{9}I$$

$$\therefore \quad \frac{10}{9}I = \frac{1}{3}e^{3x}.\sin x - \frac{1}{9}e^{3x}.\cos x \quad \checkmark$$

$$I = \frac{9}{10}\left(\frac{1}{3}e^{3x}.\sin x - \frac{1}{9}e^{3x}.\cos x\right) + c \quad \checkmark$$

(10 x ½) (5)

2.4 $y = \int \frac{\cos^3 4x}{\sin^2 4x}dx$

✓

$= \int \frac{\cos^2 4x.\cos 4x}{\sin^2 4x}dx$

✓

$= \int \frac{(1-\sin^2 4x).\cos 4x}{\sin^2 4x}dx$

✓ ✓ ✓

$= \frac{1}{4}\int \frac{(1-u^2)}{u^2}du$

$u = \sin 4x$

$du = 4\cos 4x dx$

✓

$= \frac{1}{4}\int (u^{-2} - 1)du$

✓ ✓

$= \frac{1}{4}\left[\frac{u^{-1}}{-1} - u\right] + c$

✓ ✓

$= -\frac{1}{4\sin 4x} - \frac{1}{4}\sin 4x + c$

Or

$= \frac{1}{4}\left(-\frac{1}{\sin 4x} - \sin 4x\right) + c$

(10 x ½) (5)

[18]

QUESTION 3

3.1

$$\int \frac{x^2 - 5x + 4}{(x^2+5)(x-3)}\,dx$$

$$\frac{x^2 - 5x + 4}{(x^2+5)(x-3)} = \frac{Ax+B}{x^2+5} + \frac{C}{x-3} \quad ✓$$

$$x^2 - 5x + 4 = (Ax+B)(x-3) + C(x^2+5) \quad ✓$$

$$x^2 - 5x + 4 = Ax^2 + Bx - 3Ax - 3B + Cx^2 + 5C \quad ✓$$

$$let \quad x = 3 \quad \therefore \; C = -\frac{1}{7} \quad (-0{,}143) \quad ✓$$

$$Equate \quad x^2: \quad A = \frac{8}{7} \quad (1{,}143) \quad ✓$$

$$Equate \quad x: \quad B = -\frac{11}{7} \quad (-1{,}572) \quad ✓$$

$$\int \frac{1{,}143x - 1{,}572}{x^2+5}dx + \int \frac{-0{,}143}{x-3}dx \quad ✓$$

$$= \int \frac{1{,}143x}{x^2+5}.dx ✓ + \int \frac{-1{,}572}{x^2+5}dx + \int \frac{-0{,}143}{x-3}dx$$

$$= \frac{1{,}143}{2}\ln(x^2+5) ✓ - 1{,}572 ✓ \left(\frac{1}{\sqrt{5}}\right) ✓ \arctan\frac{x}{\sqrt{5}} - 0{,}143\ln(x-3) ✓ + c$$

$$= 0{,}572\ln(x^2+5) - 0{,}703\arctan\frac{x}{\sqrt{5}} - 0{,}143\ln(x-3) + c$$

Or

$$= \frac{8}{7}.\frac{1}{2}\ln(x^2+5) - \frac{11}{7}\left(\frac{1}{\sqrt{5}}\arctan\frac{x}{\sqrt{5}}\right) - \frac{1}{7}\ln(x-3) + c$$

(12 x ½) (6)

3.2 $\displaystyle\int \frac{4x^3 + x^2 - 2x + 1}{x^2 - 3x + 2}\,dx$

$$\rightarrow \quad x^2 - 3x + 2 \,\overline{\big)\, 4x^3 + x^2 - 2x + 1} \quad \text{quotient: } 4x + 13 \ ✓$$

$$\underline{4x^3 - 12x^2 + 8x}$$

$$13x^2 - 10x + 1$$

$$\underline{13x^2 - 39x + 26}$$

$$29x - 25 \ ✓$$

$$= \int \left(4x + 13 + \frac{29x - 25}{x^2 - 3x + 2}\right) dx \qquad ✓ \qquad ✓$$

$$= \int 4x\,dx + \int 13\,dx + \int \frac{29x - 25}{x^2 - 3x + 2}\,dx \qquad \rightarrow \qquad \frac{29x - 25}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$29x - 25 = A(x-1) + B(x-2) \ ✓$$

$$\therefore \text{ If } x = 2 \ ; \quad A = 33 \ ✓$$

$$\therefore \text{ If } x = 1 \ ; \quad B = -4 \ ✓$$

$$= \int 4x\,dx + \int 13\,dx + \int \frac{33}{x-2}\,dx + \int \frac{-4}{x-1}\,dx \ ✓$$

$$\quad ✓ \qquad ✓ \qquad ✓ \qquad ✓$$

$$= \frac{4x^2}{2} + 13x + 33\ln(x-2) - 4\ln(x-1) + c$$

(12 x ½) (6)

[12]

QUESTION 4

4.1

$$2\frac{dy}{dx} - \frac{6y}{x-1} = 2(x-1)^2$$

$$\frac{dy}{dx} - \frac{6y}{2(x-1)} = (x-1)^2 \checkmark$$

$$\frac{dy}{dx} - \frac{3y}{x-1} = (x-1)^2 \checkmark$$

$$\checkmark$$

$$e^{\int P.dx} = e^{\ln(x-1)^{-3}} = \frac{1}{(x-1)^3} \checkmark \text{ or } (x-1)^{-3}$$

$$Ry = \int \frac{1}{(x-1)^3}(x-1)^2 dx \quad \checkmark$$

$$\frac{1}{(x-1)^3}.y = \int (x-1)^{-1} dx \quad \checkmark$$

$$\frac{y}{(x-1)^3} = \ln(x-1) + c \quad \checkmark$$

$$\frac{1}{(3-1)^3} = \ln(3-1) + c \quad \checkmark$$

$$\frac{1}{8} = \ln(2) + c$$

$$\therefore \ c = -0{,}568 \quad \checkmark$$

$$\frac{y}{(x-1)^3} = \ln(x-1) - 0{,}568 \checkmark$$

(10 x ½) (5)

4.2 $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 5y = 5e^{5x}$

$y_c: \quad m^2 - 6m + 5 = 0$ ✓

$(m - 5)(m - 1) = 0$

$m = 5; \quad m = 1$ ✓

$y_c = Ae^{5x} + Be^{x}$ ✓

To find y_p :

$\therefore \ y = Cxe^{5x}$ ✓

$\dfrac{dy}{dx} = 5Cxe^{5x} + Ce^{5x}$ ✓

$\dfrac{d^2y}{dx^2} = 25Cxe^{5x} + 5Ce^{5x} + 5Ce^{5x}$ ✓

$= 25Cxe^{5x} + 10Ce^{5x}$

$25Cxe^{5x} + 10Ce^{5x} - 6(5Cxe^{5x} + Ce^{5x}) + 5Cxe^{5x} = 5e^{5x}$ ✓

$4Ce^{5x} = 5e^{5x}$

$C = \dfrac{5}{4} \quad or \quad 1{,}25$ ✓

$\therefore \ y_p = \dfrac{5}{4}xe^{5x}$ ✓

$y = Ae^{5x} + Be^{x} + \dfrac{5}{4}xe^{5x}$ ✓

(0; 3)………. $3 = Ae^0 + Be^0$

A = 3 – B…..(2)

$\dfrac{dy}{dx} = 5Ae^{5x} + Be^{5x} + \dfrac{25}{4}xe^{5x} + \dfrac{5}{4}e^{5x}$ ✓

$0 = 5A + B + \dfrac{5}{4}$

$\therefore \ B = 4{,}063 \quad \left(\dfrac{65}{16}\right)$ ✓

$and \quad A = -1{,}063 \quad \left(-\dfrac{17}{16}\right)$ ✓

$\therefore \ y = -1{,}063e^{5x} + 4{,}063xe^{5x} + \dfrac{5}{4}xe^{5x}$ ✓

(14 x ½) (7)

[12]

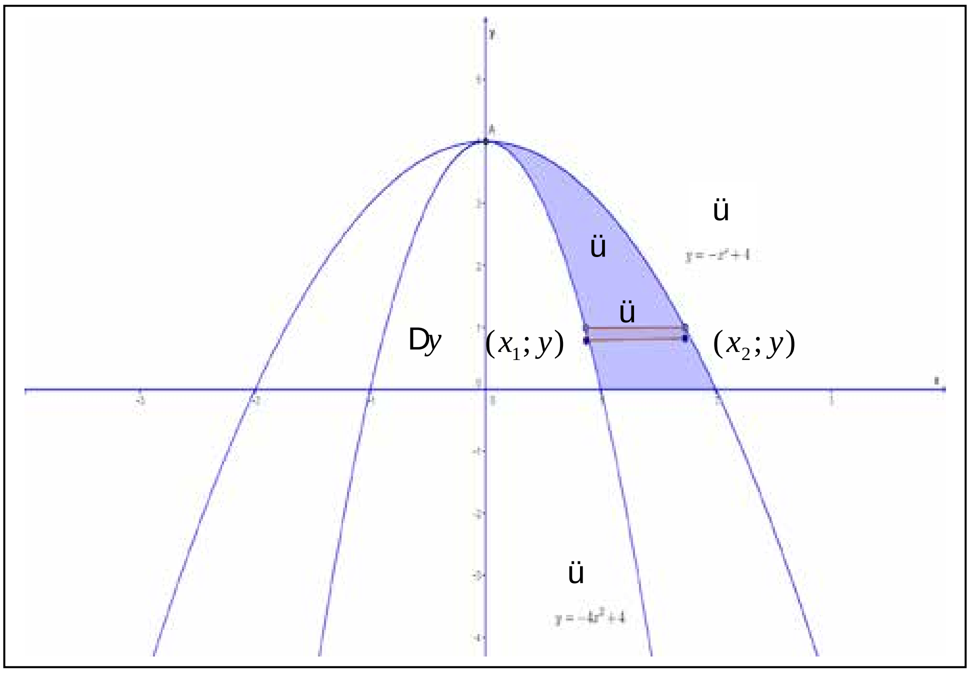
QUESTION 5

5.1 5.1.1

$$-x^2+4=-4x^2+4$$

$$3x^2=0 \qquad ✓✓$$

$$x=0 \qquad \therefore\ (0;4)$$



(6 x ½) (3)

5.1.2

$$\Delta V_y=\pi(x_2^2-x_1^2)\Delta y \quad ✓$$

$$V_y=\pi\int_0^4 (x_2^2-x_1^2)\,dy \quad ✓$$

Incorrect limits: max 2½ marks

$$=\pi\int_0^4\left[4-y-\left(\frac{4-y}{4}\right)\right]dy \quad ✓\ ✓$$

$$=\pi\int_0^4\left[3-\frac{3}{4}y\right]dy \quad ✓$$

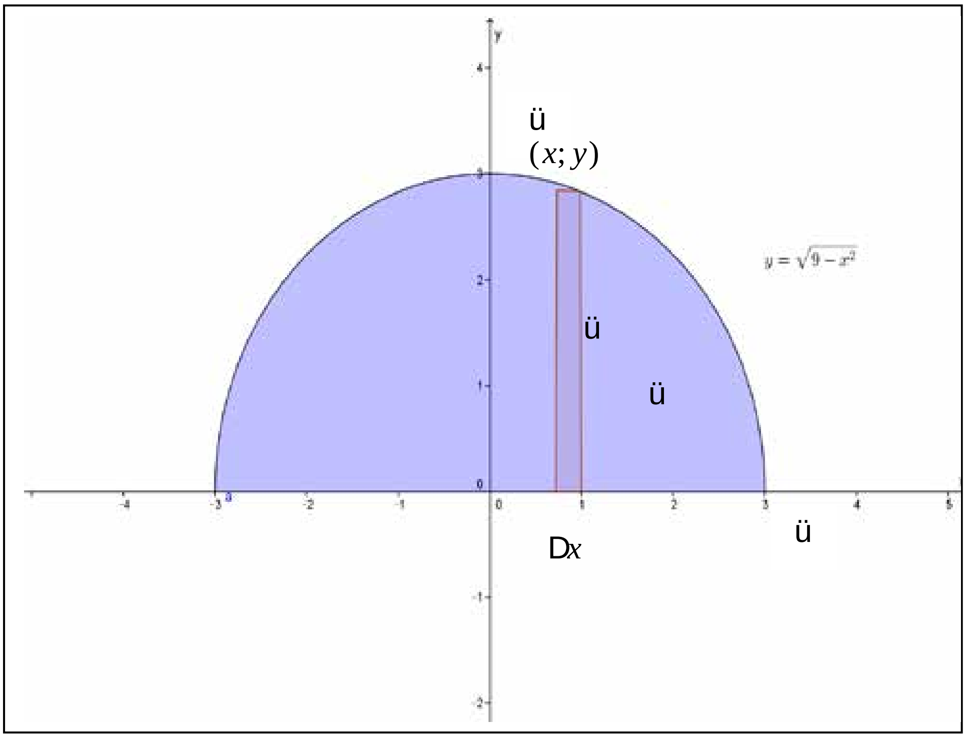
$$=\pi\left[3y-\frac{3}{8}y^2\right]_0^4 \quad ✓$$

$$=\pi\left[3(4)-\frac{3}{8}(4)^2\right] \quad ✓$$

$$=6\pi \text{ units}^3 \text{ or } 18{,}85 \text{ units}^3 \quad ✓$$

(8 x ½) (4)

5.2 5.2.1



(4 x ½) (2)

5.2.2

$$\Delta V_y = 2\pi x \times y \times \Delta x \quad ✓$$

✓

$$V_y = 2\pi x \int_0^3 xy\, dx$$

$\boxed{\text{Incorrect limits: max 1½ marks}}$

$$= 2\pi \int_0^3 x.\sqrt{9 - x^2}\, dx \quad ✓$$

$$= \frac{2\pi}{-2} \int_0^3 -2x.(9 - x^2)^{\frac{1}{2}} dx$$

$$= -\pi \left[\frac{(9 - x^2)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3 \quad ✓$$

$$= -\frac{2\pi}{3} \left[(9 - x^2)^{\frac{3}{2}} \right]_0^3$$

$$= -\frac{2\pi}{3} \left[(9 - 3^2)^{\frac{3}{2}} - (9 - 0^2)^{\frac{3}{2}} \right] \quad ✓$$

$$= 18\pi \quad units^3 \quad or \quad 56{,}549 \quad units^3 \quad ✓$$

Alternative – Strip: perpendicular to the y-axis

$\Delta V_y = \pi x^2 \Delta y$ ✓✓

$V_y = \pi \int_0^3 x^2 dy$ ✓

$= \pi \int_0^3 (9 - y^2) dy$ ✓

$= \pi \left[9y - \frac{y^3}{3} \right]_0^3$ ✓

$= \pi \left[9(3) - \frac{3^3}{3} \right]$ ✓

$= 18\pi \quad units^3 \quad or \quad 56{,}549 \quad units^3$ ✓ (6 x ½) (3)

5.2.3 $\Delta M_x = \Delta V_y \times y$

$= \pi x^2 \Delta y \times y$ ✓ ✓

$\therefore M_x = \pi \int_0^3 x^2 y dy$ ✓✓

Incorrect limits: max 3 marks

$= \pi \int_0^3 (9 - y^2) y dy$ ✓

$= \pi \int_0^3 (9y - y^3) dy$ ✓

$= \pi \left[\frac{9y^2}{2} - \frac{y^4}{4} \right]_0^3$ ✓

$= \pi \left[\frac{9(3)^2}{2} - \frac{3^4}{4} \right]$ ✓

$= 20{,}25\pi \quad units^3 \quad or \quad \frac{81}{4}\pi \quad or \quad 63{,}617\, u^3$ ✓

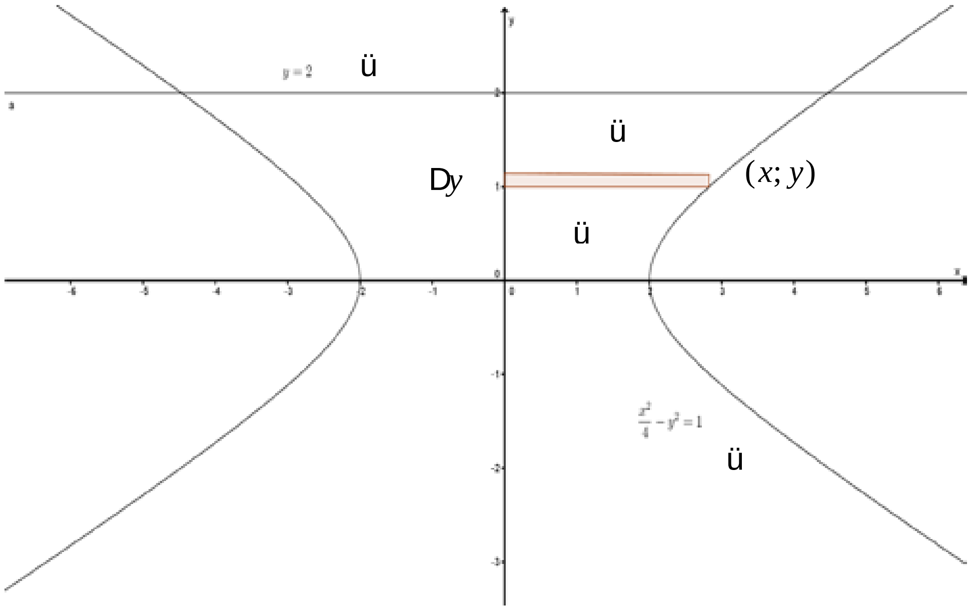
$\bar{y} = \frac{M_x}{V_y}$

$= \frac{63{,}617}{56{,}549}$ ✓

$= 1{,}125\, units$ ✓

(10 x ½) (5)

5.3 5.3.1



(4 x ½) (2)

5.3.2

$$\Delta V_y = \pi x^2 \Delta y \quad ü$$

ü

$$V_y = \pi \int_0^2 x^2 dy$$

Incorrect limits: max 1½ marks

$$= \pi \int_0^2 (4y^2 + 4) dy \quad ü \quad = 4\pi \int_0^2 (y^2 + 1) dy$$

$$= \pi \left[\frac{4y^3}{3} + 4y \right]_0^2 \quad ü$$

$$= \pi \left[\frac{4(2)^3}{3} + 4(2) \right] \quad ü$$

$$= 18{,}667\pi \quad units^3 \quad or \quad 58{,}643 \quad units^3 \quad or \quad \frac{56}{3}\pi \quad ü$$

(6 x ½) (3)

5.3.3

$$\Delta M = \rho.\Delta V_y$$
$$= \rho.\pi x^2 \Delta y$$

✓ ✓ ✓
$$\therefore \Delta I_y = \rho.\pi x^2 \Delta y \times \left(\frac{x}{\sqrt{2}}\right)^2$$

✓
$$= \frac{\rho\pi}{2}\int_0^2 x^4 dy \quad ✓$$

Incorrect limits: max 3½ marks

✓
$$= \frac{\rho\pi}{2}\int_0^2 \left[16(y^2+1)^2\right] dy$$
$$= \frac{16\rho\pi}{2}\int_0^2 \left[y^4+2y^2+1\right] dy \quad ✓$$
$$= 8\rho\pi\left[\frac{y^5}{5}+\frac{2y^3}{3}+y\right]_0^2 \quad ✓ ✓$$
$$= 8\rho\pi\left[\frac{2^5}{5}+\frac{2(2)^3}{3}+2\right] \quad ✓ ✓$$
$$= 109{,}867\pi\rho \quad or \quad 345{,}156\rho \quad units^4 \quad ✓$$

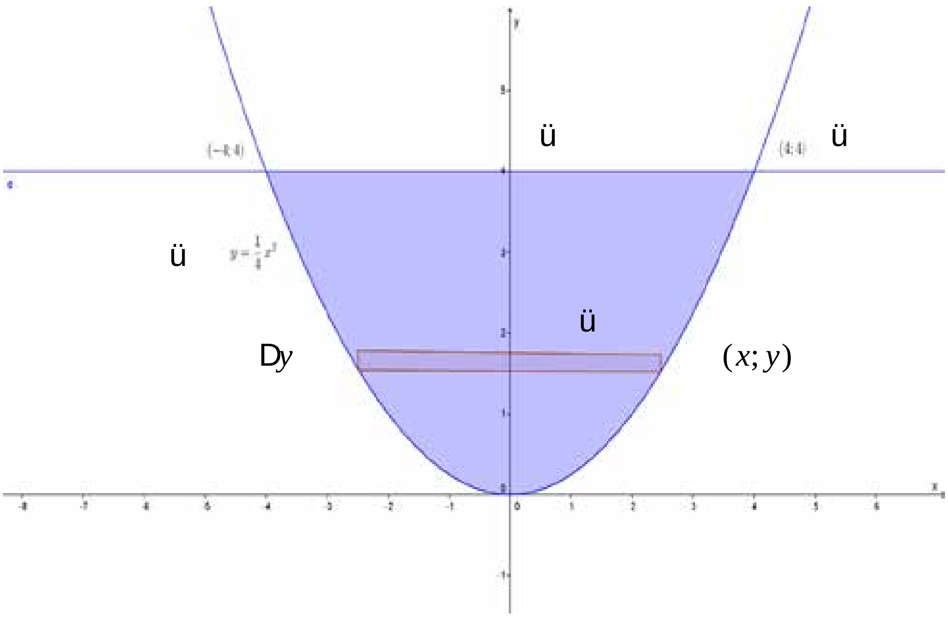
(10 x ½) (5)

5.3.4

$$I = \frac{345{,}156\ m}{58{,}643} \quad ✓$$
$$= 5{,}886\ \ m \quad ✓$$

(2 x ½) (1)

5.4 5.4.1



5.4.2

$y = ax^2 ....(4;4)$

$4 = a(4)^2$ ✓

$\therefore\ a = \frac{1}{4}$ ✓

$\therefore\ y = \frac{1}{4}x^2$ ✓

$\therefore\ x = \sqrt{4y} \quad or \quad x = 2\sqrt{y}$ ✓

$dA = 2x\,dy \quad \therefore\ dA = 4\sqrt{y}\,dy$

5.4.3

$\int_0^4 r\,dA$

✓ ✓ ✓

$= \int_0^4 (4 - y).4\sqrt{y}\,dy$

Incorrect limits: max 2½ marks

$=4\int_0^4 (4y^{\frac{1}{2}} - y^{\frac{3}{2}})dy$ ✓

✓ ✓

$=4\left[\frac{4y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4$

$=4\left[\frac{8}{3}(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}}\right]$ ✓

$= 34{,}133$ units^3 ✓

5.4.4

$\int_0^4 r^2\,dA$

✓ ✓ ✓

$= \int_0^4 (4 - y)^2.4\sqrt{y}\,dy$

Incorrect limits: max 2½ marks

$=4\int_0^4 (16y^{\frac{1}{2}} - 8y^{\frac{3}{2}} + y^{\frac{5}{2}})dy$ ✓

$=4\left[\frac{16y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{8y^{\frac{5}{2}}}{\frac{5}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^4$ ✓

$=4\left[\frac{32}{3}(4)^{\frac{3}{2}} - \frac{16}{5}(4)^{\frac{5}{2}} + \frac{2}{7}y^{\frac{7}{2}}\right]$

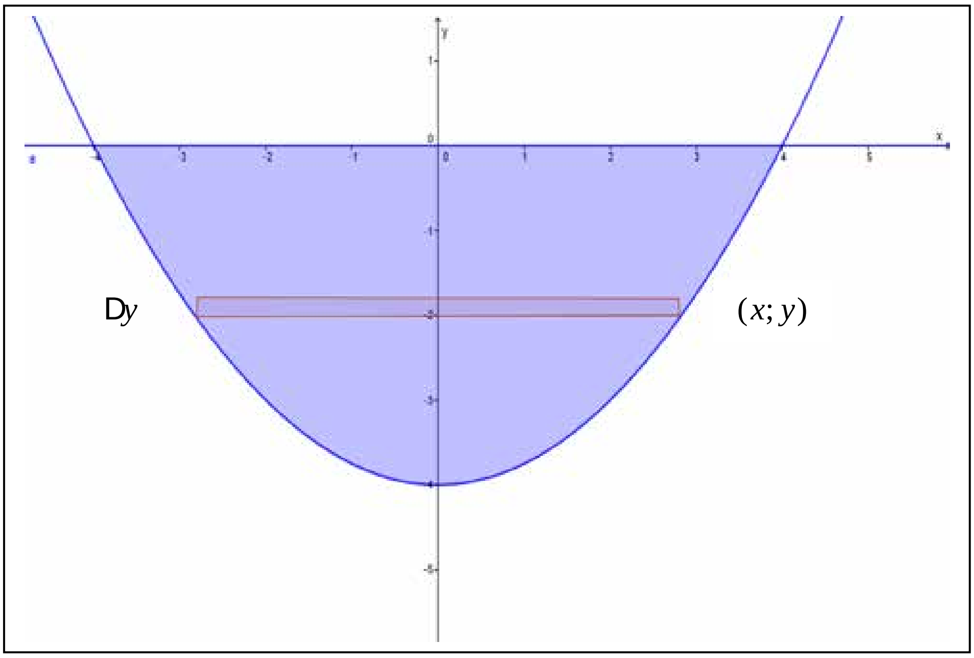
$=78{,}019$ units^4 ✓

$\bar{y} = \frac{78{,}019}{34{,}133}$ ✓

$= 2{,}286 \quad units$ ✓

Or Alternative method

5.4 5.4.1



(4 x ½) (2)

5.4.2

$y = ax^2 - 4$........(4;0)

$0 = a(4)^2 - 4$ ✓

$\therefore\ a = \dfrac{1}{4}$ ✓

$\therefore\ y = \dfrac{1}{4}x^2 - 4$ ✓

$\therefore\ x = \sqrt{4(y+4)}\ or \quad x = 2\sqrt{y+4}$ ✓

$dA = 2x\,dy \quad \therefore\ dA = 4\sqrt{y+4}\,dy$

(4 x ½) (2)

5.4.3

$\int_0^4 r\,dA$

✓ ✓ ✓

$= \int_{-4}^{0} y.4\sqrt{y+4}\,dy$

Incorrect limits: max 2½ marks

$=4\int_{-4}^{0} y(y+4)^{\frac{1}{2}}\,dy$

$=4\int_0^4 (u-4)(u)^{\frac{1}{2}}\,du$ ✓

$=4\int_0^4 (u^{\frac{3}{2}} - 4u^{\frac{1}{2}})\,du$ ✓

$=4\left[\dfrac{2}{5}u^{\frac{5}{2}} - 4(\dfrac{2}{3})u^{\frac{3}{2}}\right]_0^4$ ✓

$=4\left[\dfrac{2}{5}(4)^{\frac{5}{2}} - 4(\dfrac{2}{3})(4)^{\frac{3}{2}}\right]_0^4$ ✓

$= -34{,}133\ \text{units}^3$ ✓ (8 x ½) (4)

5.4.4 $\int_0^4 r^2 dA$

✓ ✓ ✓

$= \int_4^0 y^2 .4\sqrt{y+4}\,dy$

Incorrect limits: max 2½ marks

$=4\int_4^0 y^2 (y+4)^{\frac{1}{2}} dy$

$=4\int_0^4 (u-4)^2 (u)^{\frac{1}{2}} du$ ✓

$=4\int_0^4 (u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}}) du$

$=4\left[\frac{2}{7}u^{\frac{7}{2}} - 8(\frac{2}{5})u^{\frac{5}{2}} + 16(\frac{2}{3})u^{\frac{3}{2}}\right]_0^4$ ✓

$=4\left[\frac{2}{7}(4)^{\frac{7}{2}} - 8(\frac{2}{5})(4)^{\frac{5}{2}} + 16(\frac{2}{3})(4)^{\frac{3}{2}}\right]_0^4$

$=78{,}019$ units^4 ✓

$\bar{\bar{y}} = \frac{78{,}019}{-34{,}133}$ ✓

$= -2{,}286$ *units* ✓ (8 x ½) (4)

[40]

QUESTION 6

6.1 $y + ln\,x = \frac{x^2}{8}$

$y = \frac{1}{8}x^2 - \ln x$ ✓

$\frac{dy}{dx} = \frac{2}{8}x - \frac{1}{x}$ ✓

$\frac{dy}{dx} = \frac{1}{4}x - \frac{1}{x}$

or $y + ln\,x = \frac{x^2}{8}$

$\frac{dy}{dx} + \frac{1}{x} = \frac{1}{4}x$ ✓

$\frac{dy}{dx} = \frac{1}{4}x - \frac{1}{x}$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + (\frac{1}{4}x - \frac{1}{x})^2$ ✓

$= 1 + \frac{x^2}{16} - \frac{1}{2} + \frac{1}{x^2}$

$= \frac{(x^2+4)^2}{16x^2}$ ✓

Note: If $S = \int_e^1 \sqrt{\frac{(x^2+4)^2}{16x^2}}\,dx$:

Incorrect limits: max 4½ marks

✓

$$S = \int_1^e \sqrt{\frac{(x^2+4)^2}{16x^2}}\,dx \ ✓$$

$$S = \int_1^e \frac{x^2+4}{4x}\,dx \ ✓$$

$$= \int_1^e \left(\frac{1}{4}x + 4x^{-1}\right)dx \ ✓ \qquad \textbf{\textit{Or}} \qquad = \frac{1}{4}\int_1^e \left(x + \frac{4}{x}\right)dx \ ✓$$

✓ ✓ ✓ ✓

$$= \left[\frac{x^2}{8} + 4\ln x\right]_1^e \qquad\qquad = \frac{1}{4}\left[\frac{x^2}{2} + 4\ln x\right]_1^e$$

$$= \left(\frac{e^2}{8} + 4\ln e\right) - \left(\frac{1}{8} + 4\ln 1\right) ✓ \qquad\qquad = \frac{1}{4}\left(\frac{e^2}{2} + 4 - \frac{1}{2}\right) ✓$$

$= 1{,}799$ units ✓ $\qquad$ $= 1{,}799$ units ✓ $\qquad$ (12 x ½) $\qquad$ (6)

6.2 $x = 6t + t^2$ and $y = 2t + 6$

$$\frac{dx}{dt} = 6 + 2t \ ✓ \qquad \frac{dy}{dt} = 2 \ ✓$$

$$\left(\frac{dx}{dt}\right)^2 = (6+2t)^2 \ ✓ \qquad \left(\frac{dy}{dt}\right)^2 = (2)^2 \ ✓$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (6+2t)^2 + 4 \ ✓$$

$$= 4t^2 + 24t + 40$$

✓

$$A = 2\pi\int_0^2 y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \ ✓$$

Incorrect limits: max 4½ marks

$$= 2\pi\int_0^2 (2t+6)\sqrt{4(t^2+6t+10)}\,dt \ ✓$$

$$= 4\pi\int_0^2 (2t+6)(t^2+6t+10)^{\frac{1}{2}}\,dt \ ✓$$

$$= 4\pi\left[\frac{(t^2+6t+10)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 \ ✓$$

$$= \frac{8\pi}{3}\left[\sqrt{[2^2+6(2)+10]^3} - \sqrt{10^3}\right] ✓$$

$= 269{,}205\pi$ units^2 or $845{,}731$ units^2 ✓ $\qquad$ (12 x ½) $\qquad$ (6)

[12]

TOTAL: 100

higher education & training

Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T980**(E)**(M31)T

APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

31 March 2015 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If $z = arc\cot\left(\frac{y^2}{x}\right)$ calculate the following:

1.1.1 $\frac{\partial z}{\partial x}$ (1)

1.1.2 $\frac{\partial z}{\partial y}$ (1)

1.2 A rectangular container with a length of 3,5 m, breadth of 1,6 m and a depth of 0,8 m is expanding along its length at 0,04 m/s and along its breadth at 0,035 m/s. The container is contracting along its depth at 0,05 m/s.

Calculate the approximate change in the volume of the container.

HINT: $\Delta V = \frac{\partial V}{\partial l}\Delta l + \frac{\partial V}{\partial b}\Delta b + \frac{\partial V}{dh}\Delta h$ (4)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \frac{1}{\operatorname{cosec}^3 ax}$ (4)

2.2 $y = \sqrt{37 - 18x - 9x^2}$ (5)

2.3 $y = \cot^5 2x$ (5)

2.4 $y = e^{2x}\sin 2x$ (4)

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QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{33x^2 - 7x + 6}{(3x-1)^2(2x+3)}\,dx$ (6)

3.2 $\int \frac{x^2 + 3x - 3}{(2-x)(x^2+1)}\,dx$ (6)

[12]

QUESTION 4

4.1 Solve the following differential equation:

$$x^2.\sin x\,dx - y\,dx = x\,dy \qquad (5)$$

4.2 Calculate the particular solution of:

$$2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 20e^t, \text{ if } y = 0 \text{ when } t = 0 \text{ and } \frac{dy}{dt} = 0 \text{ when } t = 0. \qquad (7)$$

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QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = 4 - x^2$ and $y = 4 - 4x^2$. Make a neat sketch of the two curves and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the two curves is rotated about the y-axis. (2)

5.1.2 Calculate the magnitude of the volume described in QUESTION 5.1.1 by means of integration. (3)

5.2 5.2.1 Sketch the graph of $y = e^{3x}$. Show the representative strip/element that you will use to calculate the area bounded by the graph, $x = 0$, $y = 0$ and $x = 2$. (2)

5.2.2 Calculate the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the second moment of area about the y-axis of the area described in QUESTION 5.2.1. (6)

5.2.4 Express the second moment of area, calculated in QUESTION 5.2.3, in terms of the area. (1)

5.3 5.3.1 Calculate the points of intersection of $x + y - 7 = 0$ and $xy = 6$. Make a neat sketch of the two graphs and show the representative strip/element that you will use to calculate the volume generated when the area bounded by the two graphs is rotated about the x-axis. (3)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (4)

5.3.3 Calculate the moment of inertia of the solid generated when the area described in QUESTION 5.3.1 is rotated about the x-axis. Express the answer in terms of the mass. (6)

5.4 5.4.1 A vertical retaining wall in a canal is 8 m wide at the top, 4 m wide at the bottom and 4 m high. The top of the retaining wall lies in the water level. Make a neat sketch of the retaining wall and show the representative strip/element that you will use to calculate the area moment of the wall about the water level.

Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate the area moment of the wall about the water level. (3)

5.4.3 Calculate the second moment of area of the wall about the water level as well as the depth of the centre of pressure on the wall by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $y = ln\ \sec x$ between $x = \frac{p}{6}$ and $x = \frac{p}{3}$. (5)

6.2 Calculate the surface area generated when the curve represented by $y = \sqrt{16 - x^2}$ is rotated about the x-axis. (7)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}\ ;\ \sin x = \frac{1}{\operatorname{cosec} x}\ ;\ \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} \right) \right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx \; ; V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy \; ; V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \qquad ; \qquad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \; ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \mathrm{r}\, V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N6

31 MARCH 2015

This marking guideline consists of 13 pages.

QUESTION 1

1.1 $z = \text{arc}\cot\left(\frac{y^2}{x}\right)$

$\therefore z = \cot^{-1}\left(\frac{y^2}{x}\right)$

1.1.1 $\frac{\partial z}{\partial x} = \frac{-y^2x^{-2}}{\left[\frac{y^2}{x}\right]^2+1}$ ✓ ✓

$= \frac{-\frac{y^2}{x^2}}{\left[\frac{y^2}{x}\right]^2+1}$ (2)

1.1.2 $\frac{\partial z}{\partial y} = \frac{-2yx^{-1}}{\left[\frac{y^2}{x}\right]^2+1}$ ✓ ✓

$= \frac{-\frac{2y}{x}}{\left[\frac{y^2}{x}\right]^2+1}$ (2)

1.2 $\Delta V = \frac{\partial V}{\partial l}\Delta l + \frac{\partial V}{\partial b}\Delta b + \frac{\partial V}{\partial h}\Delta h$

$= bh\,\Delta l$ ✓ $+ lh\,\Delta b$ ✓ $+ lb\,\Delta h$ ✓

$= (1,6)(0,8)(0,04)$ ✓ $+ (3,5)(0,8)(0,035)$ ✓ $+ (3,5)(1,6)(-0,05)$ ✓ ✓

$= -0,131\ m^3/s$ ✓

(8)

[12]

QUESTION 2

2.1.
$$\int \frac{1}{\operatorname{cosec}^3 ax}\,dx$$
$$= \int \sin^3 ax\,dx$$
$$= \int \sin^2 ax \, \sin ax \, dx$$
$$= \int (1-\cos^2 ax)\sin ax\,dx$$
$$= \int (1-u^2)\,du$$
$$= -\frac{1}{a}\left[u - \frac{u^3}{3}\right] + c$$
$$= -\frac{1}{a}\left[\cos ax - \frac{1}{3}\cos^3 ax\right] + c$$

$u = \cos ax$

$du = -a \sin ax$

(8)

2.2.
$$\int \sqrt{37 - 18x - 9x^2}\,dx$$
$$= \int \sqrt{9\left[\frac{46}{9} - (x+1)^2\right]}\,dx$$
$$= 3\int \sqrt{\frac{46}{9} - (x+1)^2}\,dx$$
$$= 3\left[\frac{\frac{46}{9}}{2}\sin^{-1}\frac{(x+1)}{\sqrt{\frac{46}{9}}} + \frac{(x+1)}{2}\sqrt{\frac{46}{9} - (x+1)^2}\right] + c$$
or
$$= 7{,}667 \sin^{-1}\frac{(x+1)}{2{,}261} + \frac{(x+1)}{2}\sqrt{\frac{46}{9} - (x+1)^2} + c$$

$$-9x^2 - 18x + 37$$
$$= -9\left[x^2 + 2x - \frac{37}{9}\right]$$
$$= -9\left[(x+1)^2 - \frac{37}{9} - 1\right]$$
$$= -9\left[(x+1)^2 - \frac{46}{9}\right]$$
$$= 9\left[\frac{46}{9} - (x+1)^2\right]$$
$$= 9\left[5{,}111 - (x+1)^2\right]$$

OR
$$\int \sqrt{-9\left[x^2 + 2x + (1)^2 - (1)^2 - \frac{37}{9}\right]}\,dx$$
$$= \int \sqrt{-9\left[(x+1)^2 - 1 - \frac{37}{9}\right]}\,dx$$
$$= \int \sqrt{-9\left[(x+1)^2 - \frac{46}{9}\right]}\,dx$$
$$= \int \sqrt{46 - 9(x+1)^2}\,dx$$
$$= \frac{46}{2(\sqrt{9})}\sin^{-1}\frac{3(x+1)}{\sqrt{46}} + \frac{(x+1)}{2}\sqrt{46 - 9(x+1)^2} + c$$
$$= 7{,}667 \sin^{-1}\frac{3(x+1)}{6{,}782} + \frac{(x+1)}{2}\sqrt{46 - 9(x+1)^2} + c$$

(10)

2.3 $\int \cot^5 2x \, dx$

$= \int \cot^2 2x . \cot^3 2x \, dx$

$= \int (\operatorname{cosec}^2 2x - 1) \cot^3 2x \, dx$

$= \int (\operatorname{cosec}^2 2x . \cot^3 2x) \, dx - \int \cot^3 2x \, dx$

$= -\frac{1}{2} \left(\frac{\cot^4 2x}{4} \right) - \int \cot^2 2x . \cot 2x \, dx$

$= -\frac{1}{8} \cot^4 2x - \int (\operatorname{cosec}^2 2x - 1) \cot 2x \, dx$

$= -\frac{1}{8} \cot^4 2x - \int (\operatorname{cosec}^2 2x . \cot 2x) \, dx + \int \cot 2x \, dx$

$= -\frac{1}{8} \cot^4 2x - (-\frac{1}{2}) \left(\frac{\cot^2 2x}{2} \right) + \frac{1}{2} \ln(\sin 2x) + c$

$= -\frac{1}{8} \cot^4 2x + \frac{1}{4} \cot^2 2x + \frac{1}{2} \ln(\sin 2x) + c$

(10)

2.4 $\int e^{2x} \sin 2x \, dx$

$f(x) = e^{2x}$ $\quad g'(x) = \sin 2x$

$f'(x) = 2e^{2x}$ $\quad g(x) = \frac{-\cos 2x}{2}$

$= e^{2x} . \frac{-\cos 2x}{2} - \int 2e^{2x} . \frac{-\cos 2x}{2} \, dx$

$= -\frac{1}{2} e^{2x} . \cos 2x + \int e^{2x} . \cos 2x \, dx$

$f(x) = e^{2x}$ $\quad g'(x) = \cos 2x$

$f'(x) = 2e^{2x}$ $\quad g(x) = \frac{\sin 2x}{2}$

$= -\frac{1}{2} e^{2x} \cos 2x + \left[e^{2x} . \frac{\sin 2x}{2} - \int 2e^{2x} . \frac{\sin 2x}{2} \, dx \right]$

$= -\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} . \sin 2x - \int e^{2x} . \sin 2x \, dx$

$\therefore I = -\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x - I$

$2I = -\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x$

$\therefore I = \frac{1}{2} \left(-\frac{1}{2} e^{2x} \cos 2x + \frac{1}{2} e^{2x} \sin 2x \right) + c$

(8)

OR $\int e^{2x} \sin 2x \, dx$

$f(x) = \sin 2x$ $\quad g'(x) = e^{2x}$

$f'(x) = 2\cos 2x$ $\quad g(x) = \frac{e^{2x}}{2}$

$= \sin 2x . \frac{e^{2x}}{2} - \int 2\cos 2x . \frac{e^{2x}}{2} \, dx$

$= \frac{1}{2} e^{2x} \sin 2x - \int \cos 2x . e^{2x} \, dx$

$f(x) = \cos 2x$ $\quad g'(x) = e^{2x}$

$f'(x) = -2\sin 2x$ $\quad g(x) = \frac{e^{2x}}{2}$

$= \frac{1}{2} e^{2x} \sin 2x - \left[\cos 2x . \frac{e^{2x}}{2} - \int -2\sin 2x . \frac{e^{2x}}{2} \, dx \right]$

$= \frac{1}{2} e^{2x} \sin 2x - \frac{1}{2} e^{2x} \cos 2x - \int \sin 2x . e^{2x} \, dx$

$I = \frac{1}{2} e^{2x} \sin 2x - \frac{1}{2} e^{2x} \cos 2x - I$

$2I = \frac{1}{2} e^{2x} \sin 2x - \frac{1}{2} e^{2x} \cos 2x$

$\therefore I = \frac{1}{2} (\frac{1}{2} e^{2x} \ldots$

QUESTION 3

3.1 $\int \frac{33x^2-7x+6}{(3x-1)^2(2x+3)}\,dx$

$\frac{33x^2-7x+6}{(3x-1)^2(2x+3)} = \frac{A}{(3x-1)^2} + \frac{B}{(3x-1)} + \frac{C}{2x+3}$ ✓

$33x^2-7x+6 = A(2x+3)+B(3x-1)(2x+3)+C(3x-1)^2$ ✓

Let $x=\frac{1}{3}$; $\therefore A=2$ ✓

$x=-\frac{3}{2}$; $\therefore C=3$ ✓

$\therefore 33x^2-7x+6 = 2Ax+3A+6Bx^2+7Bx-3B+9Cx^2-6Cx+C$ ✓

Equate coeff of x^2; $33=6B+9C$ $\therefore B=1$ ✓

$\therefore \int \frac{2}{(3x-1)^2}\,dx + \int \frac{1}{(3x-1)}\,dx + \int \frac{3}{(2x+3)}\,dx$ ✓

$= 2\int (3x-1)^{-2}\,dx + \int \frac{1}{3x-1}\,dx + 3\int \frac{1}{2x+3}\,dx$

$= \frac{2}{3}$ ✓ $\cdot \frac{(3x-1)^{-1}}{-1}$ ✓ $+ \frac{1}{3}\ln(3x-1)$ ✓ $+ \frac{3}{2}$ ✓ $\ln(2x+3)$ ✓ $+ C$ (12)

$= -\frac{2}{3(3x-1)} + \frac{1}{3}\ln(3x-1) + \frac{3}{2}\ln(2x+3) + C$

3.2 $\int \frac{x^2+3x-3}{(2-x)(x^2+1)}\,dx$

$\frac{x^2+3x-3}{(2-x)(x^2+1)} = \frac{A}{(2-x)} + \frac{Bx+C}{(x^2+1)}$ ✓

$x^2+3x-3 = A(x^2+1)+(Bx+C)(2-x)$ ✓

Let $x=2$; $\therefore A=\frac{7}{5}$ ✓ (1,4)

$\therefore x^2+3x-3 = Ax^2+A+2Bx+2C-Bx^2-Cx$ ✓

Equate coeff of x^2: $1=A-B$ $\therefore B=\frac{2}{5}$ ✓ (0,4)

" " " x: $3=2B-C$ $\therefore C=-\frac{11}{5}$ ✓ (2,2)

$\therefore \int \frac{\frac{7}{5}}{(2-x)}\,dx + \int \frac{\frac{2}{5}x-\frac{11}{5}}{(x^2+1)}\,dx$ ✓

$= \frac{7}{5}\int \frac{1}{(2-x)}\,dx + \int \frac{\frac{2}{5}x}{x^2+1}\,dx$ ✓ $- \int \frac{\frac{11}{5}}{x^2+1}\,dx$

$= \frac{7}{5}\int \frac{1}{(2-x)}\,dx + \frac{1}{5}\int \frac{2x}{x^2+1}\,dx - \frac{11}{5}\int \frac{1}{x^2+1}\,dx$

$= -\frac{7}{5}$ ✓ $\ln(2-x)$ ✓ $+ \frac{1}{5}$ ✓ $\ln(x^2+1)$ ✓ $- \frac{11}{5}\cdot\tan^{-1}x$ ✓ $+ C$ (12)

[24]

QUESTION 4

4.1
$$x^2 \sin x\, dx - y\, dx = x\, dy$$
$$x^2 . \sin x - y = x\frac{dy}{dx}$$
$$x\frac{dy}{dx} + y = x^2 . \sin x$$
$$\frac{dy}{dx} + \frac{y}{x} = x . \sin x \checkmark$$
$$\therefore R = e^{\int \frac{1}{x} dx} \checkmark$$
$$= e^{\ln x} \checkmark$$
$$= x \checkmark$$
$$\therefore Ry = \int RQ\, dx + c$$
$$x.y \checkmark = \int x . x \sin x \checkmark\, dx + c$$
$$xy = \int x^2 \sin x\, dx + c$$

$F(x) = x^2$ $\quad g'(x) = \sin x$
$F'(x) = 2x$ $\quad g(x) = -\cos x$

$$xy = x^2(-\cos x) \checkmark - \int 2x(-\cos x) \checkmark\, dx + c$$
$$xy = -x^2\cos x + 2\int x \cos x\, dx + c$$

$F(x) = x$ $\quad g'(x) = \cos x$
$F'(x) = 1$ $\quad g(x) = \sin x$

$$xy = -x^2\cos x + 2\left[x \sin x - \int \sin x\, dx \checkmark + c\right]$$
$$xy = -x^2\cos x + 2x\sin x + 2\cos x + c \checkmark$$

(10)

4.2
$$2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 20e^t$$
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 10e^t \checkmark$$
$$m^2 + 2m + 2 = 0 \checkmark$$
$$m = \frac{-2 \pm \sqrt{4-4(1)(2)}}{2}$$
$$m = -1 \pm j \checkmark$$
$$\therefore y_c = e^{-t}(A\cos t + B\sin t) \checkmark$$
y_p:
$$y = Ce^t \checkmark$$
$$\frac{dy}{dt} = Ce^t \checkmark$$
$$\frac{d^2y}{dt^2} = Ce^t \checkmark$$
$$\therefore Ce^t + 2Ce^t + 2Ce^t = 10e^t \checkmark$$
$$5C = 10$$
$$\therefore C = 2 \checkmark$$
$$\therefore y_p = 2e^t$$

$\therefore y = e^{-t}(A\cos t + B\sin t) + 2e^{t}$ ✓

$0 = e^{0}(A\cos 0 + B\sin 0) + 2e^{0}$

$\therefore A = -2$ ✓

$\frac{dy}{dt} = e^{-t}(-A\sin t + B\cos t) - e^{-t}(A\cos t + B\sin t) + 2e^{t}$ ✓

$0 = e^{0}(-A\sin 0 + B\cos 0) - e^{0}(A \quad 0 + B \quad 0) + 2e^{0}$

$0 = B - A + 2$

$0 = B - (-2) + 2$

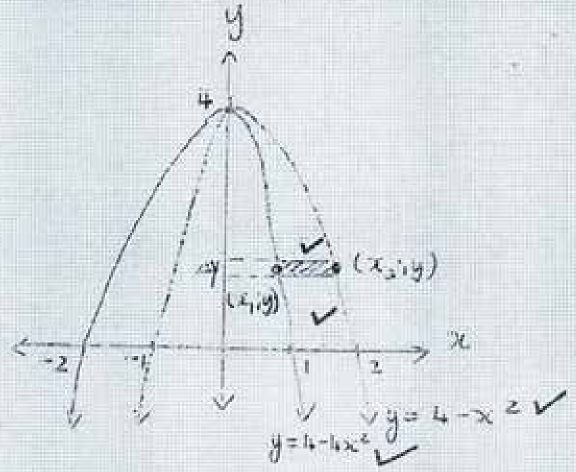
$\therefore B = -4$ ✓

$\therefore \underline{y = e^{-t}(-2\cos t - 4\sin t) + 2e^{t}}$ ✓ (14)

[24]

QUESTION 5

5.1 5.1.1.



(4)

5.1.2 $\Delta V = \pi(x_2^2 - x_1^2)\Delta y$ ✓

$$V = \pi\int_0^4 (x_2^2 - x_1^2)\,dy$$ ✓

$$= \pi\int_0^4 \left(4 - y - 1 + \frac{y}{4}\right)dy$$ ✓

$$= \pi\int_0^4 \left(3 - \frac{3}{4}y\right)dy$$

$$= \pi\left[3y - \frac{3y^2}{8}\right]_0^4$$ ✓

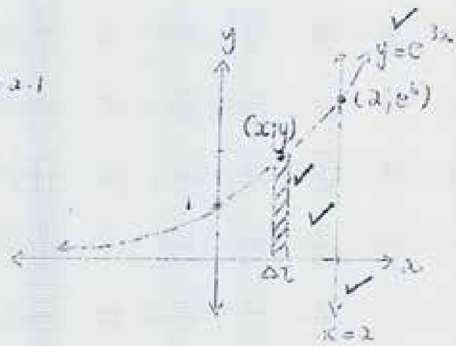
$$= \pi\left[3(4) - \frac{3}{8}(4)^2\right]$$ ✓

$$= 6\pi \text{ units}^3 \quad / \quad 18,85 \text{ units}^3$$ ✓

(6)

8

5.2 5.2.1



(4)

5.2.2 $\Delta A = y\,\Delta x$

$$\therefore A = \int_0^2 y\,dx$$
$$= \int_0^2 e^{3x}\,dx$$
$$= \left[\frac{e^{3x}}{3}\right]_0^2$$
$$= \frac{1}{3}\left[e^{3(2)} - e^{3(0)}\right]$$
$$= \underline{134,143 \text{ units}^2}$$

(6)

5.2.3 $\Delta I_y = y\,\Delta x \cdot x^2$

$$I_y = \int_0^2 y x^2\,dx$$
$$= \int_0^2 e^{3x} x^2\,dx$$

$f(x) = x^2 \quad g'(x) = e^{3x}$
$f'(x) = 2x \quad g(x) = \frac{e^{3x}}{3}$

$$= \left[x^2 \cdot \frac{e^{3x}}{3} - \int 2x\,\frac{e^{3x}}{3}\,dx\right]_0^2$$
$$= \left[\frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x\,e^{3x}\right]_0^2$$

$f(x) = x \quad g'(x) = e^{3x}$
$f'(x) = 1 \quad g(x) = \frac{e^{3x}}{3}$

$$= \left[\frac{1}{3}x^2 e^{3x} - \frac{2}{3}\left(x \cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx\right)\right]_0^2$$
$$= \left[\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{9}\int e^{3x}\,dx\right]_0^2$$
$$= \left[\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{9}\cdot\frac{e^{3x}}{3}\right]_0^2$$
$$= \left[\frac{1}{3}(2)^2 e^{3(2)} - \frac{2}{9}(2)e^{3(2)} + \frac{2}{27}e^{3(2)}\right] - \left[\frac{2}{27}e^0\right]$$
$$= \underline{388,413 \text{ units}^4}$$

(12)

5.2.4
$$\frac{I_y}{A} = \frac{388,413}{134,143}$$
$$= \underline{2,896 \text{ A units}}$$

(2)

5.3 5.3.1 $x + y - 7 = 0 \quad ; \quad xy = 6$

$\therefore y = -x + 7 \quad ; \quad \therefore y = \frac{6}{x}$

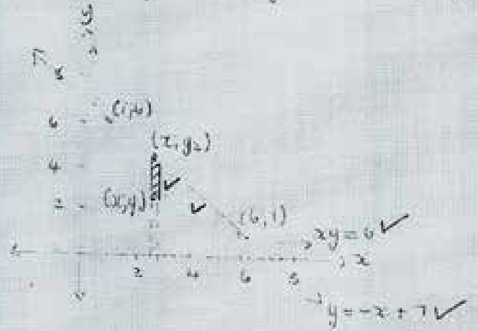
$\therefore -x + 7 = \frac{6}{x}$

$x^2 - 7x + 6 = 0$

$(x - 6)(x - 1) = 0$

$\therefore x = 6 \; ; \; x = 1 \quad \Big\} \; (6;1)$ ✓

$y = 1 \; ; \; y = 6 \quad \Big\} \; (1;6)$ ✓



(6)

5.3.2 $\Delta V_x = \pi\left(y_2^2 - y_1^2\right)\Delta x$ ✓

$V_x = \pi \int_1^6 \left(y_2^2 - y_1^2\right) dx$ ✓

$= \pi \int_1^6 \left[(7 - x)^2 - \left(\frac{6}{x}\right)^2\right] dx$ ✓ ✓

$= \pi \int_1^6 \left[49 - 14x + x^2 - \frac{36}{x^2}\right] dx$ ✓

$= \pi \left[49x - 7x^2 + \frac{x^3}{3} + \frac{36}{x}\right]_1^6$ ✓

$= \pi\left\{\left[49(6) - 7(6)^2 + \frac{1}{3}(6)^3 + 6\right] - \left[49 - 7 + \frac{1}{3} + 36\right]\right\}$ ✓

$= 41{,}667\,\pi$ units^3

or $= 130{,}9$ units^3 ✓

(8)

OR 5.3.2

$\rightarrow$ 5.3.1 Strip parallel to x-axis

$\Delta V = \pi\left(x_2^2 - x_1^2\right)\Delta y$ ✓

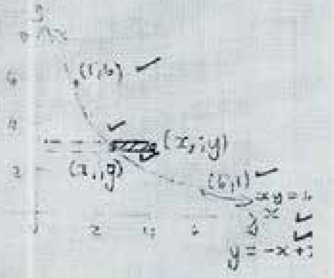
$V = \pi \int_1^6 (7 - y)^2 - \left(\frac{6}{y}\right)^2 dy$ ✓ ✓ ✓

$= \pi \int_1^6 \left(49 - 14y + y^2 - \frac{36}{y^2}\right) dy$ ✓

$= \pi \left[49y - \frac{14y^2}{2} + \frac{y^3}{3} + \frac{36}{y}\right]_1^6$ ✓

$= \pi\left[\left[49(6) - 7(6)^2 + \frac{1}{3}(6)^3 + 6\right] - \left[49 - 7 + \frac{1}{3} + 36\right]\right]$ ✓

$= 41{,}667\,\pi$ units^3 ✓



5.3.3 Strip parallel to x-axis

$$\Delta I_x = \rho 2\pi y \Delta y (x_2 - x_1) \times y^2$$

$$I_x = 2\pi\rho \int_1^6 y^3 (x_2 - x_1) dy$$

$$= 2\pi\rho \int_1^6 y^3 \left(7 - y - \frac{6}{y}\right) dy$$

$$= 2\pi\rho \int_1^6 \left(7y^3 - y^4 - \frac{6}{y^2}\right) dy$$

$$= 2\pi\rho \left[\frac{7y^4}{4} - \frac{y^5}{5} - 2y^3\right]_1^6$$

$$= 2\pi\rho \left\{\left[\frac{7}{4}(6)^4 - \frac{1}{5}(6)^5 - 2(6)^3\right] - \left[\frac{7}{4} - \frac{1}{5} - 2\right]\right\}$$

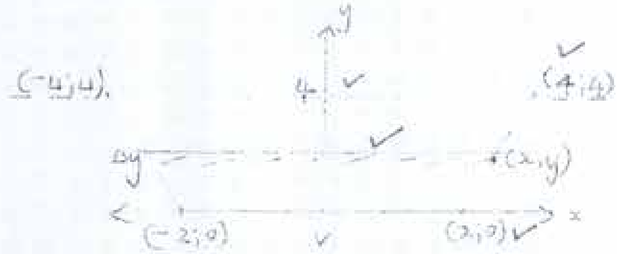
$$= 1767{,}146\rho \quad \text{or} \quad 562{,}5\pi\rho$$

$$\therefore I_y = \frac{1767{,}146m}{130{,}9}$$

$$= 13{,}5 \text{ m}$$

(12)

5.4 5.4.1



$$\frac{y-0}{x-2}=\frac{4-0}{4-2} \qquad \begin{matrix}(2;0) & (4;4)\\ x_1 y_1 & x_2 y_2\end{matrix}$$

$$\therefore y = 2x-4$$

$$x = \tfrac{1}{2}y + 2$$

$$\Delta A = 2x\,\Delta y$$

$$\therefore dA = 2\left(\tfrac{1}{2}y+2\right)dy$$

$$= (y+4)\,dy \qquad (6)$$

5.4.2
$$\int_0^4 (4-y)(y+4)\,dy$$
$$= \int_0^4 (4y - y^2 + 16 - 4y)\,dy$$
$$= \int_0^4 (-y^2+16)\,dy$$
$$= \left[\frac{-y^3}{3} + 16y\right]_0^4$$
$$= -\tfrac{1}{3}(4)^3 + 16(4)$$
$$= 42,667 \text{ units}^3 \qquad (6)$$

5.4.3
$$\int_0^4 (4-y)^2(y+4)\,dy$$
$$= \int_0^4 (16-8y+y^2)(y+4)\,dy$$
$$= \int_0^4 (16y - 8y^2 + y^3 + 64 - 32y + 4y^2)\,dy$$
$$= \int_0^4 (y^3 - 16y - 4y^2 + 64)\,dy$$
$$= \left[\frac{y^4}{4} - \frac{16y^2}{2} - \frac{4y^3}{3} + 64y\right]_0^4$$
$$= \left[\tfrac{1}{4}(4)^4 - 8(4)^2 - \tfrac{4}{3}(4)^3 + 64(4)\right]$$
$$= 106,667 \text{ units}^4$$

$$\therefore \bar{y} = \frac{106,667}{42,667}$$
$$= 2,499$$
$$\approx 2,5 \text{ units}$$

5.4.1

$$\frac{y-0}{x-4}=\frac{-4-0}{2-4}$$

$$y = 2x-8$$

$$x = \tfrac{1}{2}y + 4$$

$$\Delta A = 2x\,\Delta y$$

$$dA = 2\left(\tfrac{1}{2}y+4\right)dy$$

$$= (y+8)\,dy$$

5.4.2
$$\int_{-4}^{0} (y)(y+8)\,dy$$
$$= \int_{-4}^{0} (y^2+8y)\,dy$$
$$= \left[\frac{y^3}{3} + 8\cdot\frac{y^2}{2}\right]_{-4}^{0}$$
$$= [0] - \left[\tfrac{1}{3}(-4)^3 + 4(-4)^2\right]$$
$$= -42,667 \text{ units}^3$$

5.4.3
$$\int_{-4}^{0} (y^2)(y+8)\,dy$$
$$= \int_{-4}^{0} (y^3 + 8y^2)\,dy$$
$$= \left[\frac{y^4}{4} + \frac{8y^3}{3}\right]_{-4}^{0}$$
$$= [0] - \left[\tfrac{1}{4}(-4)^4 + \tfrac{8}{3}(-4)^3\right]$$
$$= 106,667 \text{ units}^4$$

$$\bar{y} = \frac{106,667}{-42,667}$$
$$= -2,499$$
$$\approx -2,5$$

QUESTION 6

6.1 $y = \ln \sec x$

$\frac{dy}{dx} = \frac{\sec x \tan x}{\sec x}$ ✓

$= \tan x$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x$ ✓

$= \sec^2 x$ ✓

$\therefore S = \int_{\pi/6}^{\pi/3} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ ✓ ✓

$= \int_{\pi/6}^{\pi/3} \sqrt{\sec^2 x}\, dx$

$= \int_{\pi/6}^{\pi/3} \sec x\, dx$ ✓

$= \left[\ln(\sec x + \tan x)\right]_{\pi/6}^{\pi/3}$ ✓

$= \ln\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) - \ln\left(\sec\frac{\pi}{6} + \tan\frac{\pi}{6}\right)$ ✓

$= 0{,}768$ unit ✓

(10)

6.2 $y = \sqrt{16 - x^2}$

$y = (16 - x^2)^{\frac{1}{2}}$ ✓

$\frac{dy}{dx} = \frac{1}{2}(16 - x^2)^{-\frac{1}{2}} \cdot -2x$ ✓ ✓

$= \frac{-x}{\sqrt{16 - x^2}}$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{-x}{\sqrt{16 - x^2}}\right)^2$ ✓

$= 1 + \frac{x^2}{16 - x^2}$

$= \frac{16}{16 - x^2}$ ✓

$A_x = \int_{-4}^{4} 2\pi y \sqrt{\frac{16}{16 - x^2}}\, dx$ ✓ ✓ or $2\int_0^4 2\pi \sqrt{\frac{16}{16 - x^2}}\, dx$

$= 2\pi \int_{-4}^{4} \sqrt{16 - x^2} \cdot \sqrt{\frac{16}{16 - x^2}}\, dx$ ✓

$= 8\pi \int_{-4}^{4} dx$ ✓ ✓

$= 8\pi \left[x\right]_{-4}^{4}$ ✓

$= 8\pi \left[4 - (-4)\right]$ ✓

$= 64\pi$ unit^2

or $= 201{,}062$ unit^2 ✓

(14)

[24]

TOTAL: 100

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T970**(E)**(N14)T
NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

14 November 2014 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If $z = e^x \ln (x^2 y)$ determine the values of:

1.1.1 $\dfrac{\partial z}{\partial x}$

1.1.2 $\dfrac{\partial z}{\partial y}$

1.1.3 $\dfrac{\partial^2 z}{\partial x \partial y}$

(3 x 1) (3)

1.2 In measuring the length of the perpendicular sides of a right-angled triangle, errors of 0,4% are made with each length.

Calculate the maximum possible error made when calculating the area.

HINT:

The approximate error can be determined by $\Delta A = \dfrac{\partial A}{\partial a}\Delta a + \dfrac{\partial A}{\partial b}\Delta b$ (3)

[6]

QUESTION 2

Determine $\int y \, dx$ if:

2.1 $y = \dfrac{x^2}{2}.\tan^2 x$ (4)

2.2 $y = \sin^5 2x.\cos^3 2x$ (4)

2.3 $y = x^2 e^{4x}$ (4)

2.4 $y = \dfrac{1}{\sqrt{10 - 2x - x^2}}$ (3)

2.5 $y = \cot^4 3x$ (3)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^3+1}{x(x-1)^3}\,dx$ (6)

3.2 $\int \frac{3x^3-6x^2-2x+12}{x^2(x^2-4x+6)}\,dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$\tan x\frac{dy}{dx}-y=\frac{e^x\sin x}{\cot x} \text{ at } (2;0).$$ (6)

4.2 Calculate the general solution of:

$$2\frac{d^2y}{dx^2}+8\frac{dy}{dx}+8y=16x^2$$ (6)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y=2x+6$ and $y=9-x^2$. Make a neat sketch of the two graphs and show the representative strip/element that you will use to calculate the volume of the solid generated when the area bounded by the graphs is rotated about the x-axis. (3)

5.1.2 Calculate the volume described in QUESTION 5.1.1 by means of integration. (3)

5.1.3 Calculate the volume moment of the solid about the y-axis as well as the x-ordinate of the centre of gravity of the solid described in QUESTION 5.1.1. (4)

5.2 5.2.1 Make a neat sketch of the graphs $y=-x+1$ and $y-2x-1=0$. Show the representative strip/element that you will use to calculate the area bounded by the graphs and the line $x=1$. (3)

5.2.2 Calculate, by means of integration, the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the area moment about the x-axis and also the distance of the centroid from the x-axis of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Make a neat sketch of the graph $y = 3\sin 2x$. Show the representative strip/element that you will use to calculate the volume generated when the area bounded by the graph $y = 0$ and $x = \frac{p}{4}$ is rotated about the x-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1 by means of integration. (4)

5.3.3 Calculate the moment of inertia of the solid generated when the area described in QUESTION 5.3.1 is rotated about the x-axis. (5)

5.4 5.4.1 A vertical sluice gate, in the form of a parabola, is installed in a dam wall. The top of the sluice gate is 9 m wide and lies in the water level. The sluice gate is 3 m high. Make a neat sketch of the sluice gate and show the representative strip/element that you will use to calculate the area moment of the sluice gate about the water level. (2)

5.4.2 Calculate the relation between x and y that you will need to calculate the area moment of the sluice gate about the water level. (1)

5.4.3 Calculate the depth of the centre of pressure on the sluice gate by means of integration if the second moment of area of the sluice gate about the water level is given as 37,056 units^4. (5)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $x^2 = y^3$ between $y = 1$ and $y = 3$. (5)

6.2 Calculate the surface area generated when the curve with parametric equations $x = e^{3t}\sin t$ and $y = e^{3t}\cos t$ rotates about the x-axis for $0 £ t £ \frac{p}{2}$. (7)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2\, x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}\ ; \sin x = \frac{1}{\text{cosec}\ x}\ ; \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\text{cosec} f(x)$	$-\text{cosec} f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1 - [f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2 + 1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2 + 1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2 - 1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2 - 1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int f'(x)\,g(x)\,dx$$

$$\int [f(x)]^n\, f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\,dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx \,;\, V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \,;\, V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy \,;\, V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \,;\, V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \qquad ; \qquad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \mathrm{r}\, V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = r\int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \square \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N6

14 NOVEMBER 2014

This marking guideline consists of 14 pages.

QUESTION 1

1.1 $z = e^x \ln(x^2 y)$

1.1.1
$$\frac{\partial z}{\partial x} = e^x \cdot \frac{2xy}{x^2 y} \checkmark + e^x \ln(x^2 y) \checkmark$$
$$= e^x \cdot \frac{2}{x} + e^x \ln(x^2 y)$$

(2)

1.1.2
$$\frac{\partial z}{\partial y} = e^x \checkmark \cdot \frac{x^2}{x^2 y} \checkmark$$
$$= \frac{e^x}{y}$$

(2)

1.1.3
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}\left(\frac{e^x}{y}\right) \checkmark$$
$$= \frac{e^x}{y} \checkmark$$

(2)

1.2
$$A = \frac{1}{2} ab$$
$$\frac{\partial A}{\partial a} = \frac{1}{2} b \checkmark$$
$$\frac{\partial A}{\partial b} = \frac{1}{2} a \checkmark$$

$$\Delta A = \frac{\partial A}{\partial a} \Delta a + \frac{\partial A}{\partial b} \Delta b$$
$$= \frac{1}{2} b \left(\frac{4}{100} a\right) \checkmark + \frac{1}{2} a \left(\frac{4}{100} b\right) \checkmark$$
$$= \frac{1}{2} ab \left(\frac{8}{100}\right) \checkmark$$
$$= 8\% \checkmark \text{ of } A$$

(6)
[12]

QUESTION 2

2.1

$$\int \frac{x}{2} \cdot \tan^2 x \, dx$$
$$= \int \frac{x}{2} (\sec^2 x - 1) \, dx$$
$$= \int \frac{x}{2} \cdot \sec^2 x \, dx - \int \frac{x}{2} \, dx$$

$f(x) = \frac{x}{2}$ $\quad g'(x) = \sec^2 x$
$f'(x) = \frac{1}{2}$ $\quad g(x) = \tan x$

$$= \frac{x}{2} \cdot \tan x - \int \frac{1}{2} \tan x \, dx - \int \frac{x}{2} \, dx$$
$$= \frac{x}{2} \cdot \tan x - \frac{1}{2} \ln(\sec x) - \frac{1}{2} \cdot \frac{x^2}{2} + c$$
$$= \frac{1}{2} x \tan x - \frac{1}{2} \ln(\sec x) - \frac{1}{4} x^2 + c$$

OR $\int \frac{x}{2} \tan^2 x \, dx$

$f(x) = \frac{x}{2}$ $\quad g'(x) = \tan^2 x$
$f'(x) = \frac{1}{2}$ $\quad g(x) = \tan x - x$

$$= \frac{x}{2} (\tan x - x) - \int \frac{1}{2} (\tan x - x) \, dx$$
$$= \frac{x}{2} \tan x - \frac{1}{2} x^2 - \frac{1}{2} \int \tan x \, dx + \frac{1}{2} \int x \, dx$$
$$= \frac{x}{2} \tan x - \frac{1}{2} x^2 - \frac{1}{2} \ln(\sec x) + \frac{1}{2} \cdot \frac{x^2}{2} + c$$
$$= \frac{x}{2} \tan x - \frac{1}{4} x^2 - \frac{1}{2} \ln(\sec x) + c$$

(8)

2.2

$$\int \sin^5 2x \cdot \cos^3 2x \, dx$$
$$= \int \sin^5 2x \cdot \cos^2 2x \cdot \cos 2x \, dx$$
$$= \int \sin^5 2x \, (1 - \sin^2 2x) \cos 2x \, dx$$

$u = \sin 2x$
$du = 2\cos 2x \, dx$

$$= \frac{1}{2} \int u^5 (1 - u^2) \, du$$
$$= \frac{1}{2} \int (u^5 - u^7) \, du$$
$$= \frac{1}{2} \left[\frac{u^6}{6} - \frac{u^8}{8} \right] + c$$
$$= \frac{1}{2} \left[\frac{\sin^6 2x}{6} - \frac{\sin^8 2x}{8} \right] + c$$
$$= \frac{1}{12} \sin^6 2x - \frac{1}{16} \sin^8 2x + c$$

OR

$$\int (\sin^2 2x)^2 \sin 2x \cdot \cos^3 2x \, dx$$
$$= \int (1 - \cos^2 2x)^2 \sin 2x \cdot \cos^3 2x \, dx$$

$u = \cos 2x$
$du = -2 \sin 2x \, dx$

$$= -\frac{1}{2} \int (1 - u^2)^2 \, u^3 \, du$$
$$= -\frac{1}{2} \int (u^3 - 2u^5 + u^7) \, du$$
$$= -\frac{1}{2} \left[\frac{u^4}{4} - \frac{2u^6}{6} + \frac{u^8}{8} \right] + c = -\frac{1}{2} \left[\frac{\cos^4 2x}{4} - \frac{\cos^6 2x}{3} + \frac{\cos^8 2x}{8} \right] + c$$

(8)

2.3

$$\int x^2 e^{4x}\,dx \qquad \begin{array}{ll} f(x) = x^2 & g'(x) = e^{4x} \\ f'(x) = 2x & g(x) = \frac{e^{4x}}{4} \end{array}$$

$$= x^2 . \frac{e^{4x}}{4} - \int 2x . \frac{e^{4x}}{4}\,dx$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\int x . e^{4x}\,dx \qquad \begin{array}{ll} f(x) = x & g'(x) = e^{4x} \\ f'(x) = 1 & g(x) = \frac{e^{4x}}{4} \end{array}$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{2}\left[x . \frac{e^{4x}}{4} - \int 1 . \frac{e^{4x}}{4}\,dx\right]$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}x e^{4x} + \frac{1}{8} . \frac{e^{4x}}{4} + C$$

$$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}x . e^{4x} + \frac{1}{32}e^{4x} + C$$

(8)

2.4

$$\int \frac{1}{\sqrt{10 - 2x - x^2}}\,dx$$

$$= \int \frac{1}{\sqrt{11 - (x+1)^2}}\,dx$$

$$= \sin^{-1}\frac{(x+1)}{\sqrt{11}} + c$$

$$= \sin^{-1}\frac{(x+1)}{3,317} + c$$

$$-x^2 - 2x + 10$$
$$= -\left[x^2 + 2x - 10\right]$$
$$= -\left[(x+1)^2 - 10 - 1\right]$$
$$= -\left[(x+1)^2 - 11\right].$$
$$= 11 - (x+1)^2$$

(6)

2.5

$$\int \cot^4 3x\,dx.$$

$$= \int \cot^2 3x . \cot^2 3x\,dx$$

$$= \int (\operatorname{cosec}^2 3x - 1)\cot^2 3x\,dx$$

$$= \int (\operatorname{cosec}^2 3x . \cot^2 3x)\,dx - \int \cot^2 3x\,dx$$

$$= -\frac{1}{3} . \frac{\cot^3 3x}{3} - \left[-\frac{1}{3}\cot 3x - x\right] + c$$

$$= -\frac{1}{9}\cot^3 3x + \frac{1}{3}\cot 3x + x + c$$

(6)
[36]

QUESTION 3

3.1

$$\int \frac{x^3+1}{x(x-1)^3}\,dx$$

$\therefore \dfrac{x^3+1}{x(x-1)^3} = \dfrac{A}{x} + \dfrac{B}{(x-1)^3} + \dfrac{C}{(x-1)^2} + \dfrac{D}{(x-1)}$ ✓

$\therefore x^3+1 = A(x-1)^3 + Bx + Cx(x-1) + Dx(x-1)^2$ ✓

Let $x = 0$; $\therefore A = -1$ ✓

$x = 1$; $\therefore B = 2$ ✓

$\therefore x^3+1 = Ax^3 - Ax^2 - 2Ax^2 + 2Ax + Ax - A + Bx + Cx^2 - Cx + Dx^3 - 2Dx^2 + Dx$ ✓

Equate coeff. of x^3: $1 = A + D$ $\therefore D = 2$ ✓

" " " x^2: $0 = -A - 2A + C - 2D$ $\therefore C = 1$ ✓

$\therefore \displaystyle\int \frac{-1}{x}\,dx + \int \frac{2}{(x-1)^3}\,dx + \int \frac{1}{(x-1)^2}\,dx + \int \frac{2}{(x-1)}\,dx$ ✓

$= -\displaystyle\int \frac{1}{x}\,dx + 2\int (x-1)^{-3}\,dx + \int (x-1)^{-2}\,dx + 2\int \frac{1}{(x-1)}\,dx$

$= -\ln x$ ✓ $+ 2 \cdot \dfrac{(x-1)^{-2}}{-2}$ ✓ $+ \dfrac{(x-1)^{-1}}{-1}$ ✓ $+ 2\ln(x-1)$ ✓ $+ C$

$= -\ln x - \dfrac{1}{(x-1)^2} - \dfrac{1}{(x-1)} + 2\ln(x-1) + C$

(12)

3.2

$$\int \frac{3x^3 - 6x^2 - 2x + 12}{x^2(x^2-4x+6)}\,dx$$ ✓

$\therefore \dfrac{3x^3-6x^2-2x+12}{x^2(x^2-4x+6)} = \dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{Cx+D}{x^2-4x+6}$ ✓

$\therefore 3x^3 - 6x^2 - 2x + 12 = A(x^2-4x+6) + Bx(x^2-4x+6) + (Cx+D)x^2$ ✓

Let $x = 0$; $\therefore A = 2$ ✓

$\therefore 3x^3 - 6x^2 - 2x + 12 = Ax^2 - 4Ax + 6A + Bx^3 - 4Bx^2 + 6Bx + Cx^3 + Dx^2$ ✓

Equate coeff of x^3: $3 = B + C$

" " x^2: $-6 = A - 4B + D$ $\therefore -8 = -4B + D$

" " x: $-2 = -4A + 6B$ $\therefore B = 1$ ✓

$C = 2$ ✓

$D = -4$ ✓

$\therefore \displaystyle\int \frac{2}{x^2}\,dx + \int \frac{1}{x}\,dx + \int \frac{2x-4}{x^2-4x+6}\,dx$ ✓

$= \displaystyle\int 2x^{-2}\,dx + \int \frac{1}{x}\,dx + \int \frac{2x-4}{x^2-4x+6}\,dx$

$= 2 \cdot \dfrac{x^{-1}}{-1}$ ✓ $+ \ln x$ ✓ $+ \ln(x^2-4x+6)$ ✓ $+ C$

$= -\dfrac{2}{x} + \ln x + \ln(x^2-4x+6) + C$

(12)
[24]

QUESTION 4

4.1

$$\tan x \frac{dy}{dx} - y = \frac{e^x \sin^2 x}{\cot x}$$

$$\frac{dy}{dx} - \frac{y}{\tan x} = \frac{e^x . \sin x}{\cot x . \tan x} \checkmark$$

$$\frac{dy}{dx} - y . \cot x = e^x . \sin x \checkmark$$

$$\therefore R = e^{\int -\cot x \, dx} \checkmark$$

$$= e^{-\ln(\sin x)} \checkmark$$

$$= e^{\ln(\sin x)^{-1}}$$

$$= \frac{1}{\sin x} \checkmark \;|\; \operatorname{cosec} x$$

$$\therefore Ry = \int RQ \, dx + c$$

$$\frac{1}{\sin x} . (y) \checkmark = \int \frac{1}{\sin x} . e^x \sin^2 x \, dx + c \checkmark$$

$$y . \frac{1}{\sin x} = \int e^x \, dx + c \checkmark$$

$$\frac{y}{\sin x} = e^x + c \checkmark$$

$$\frac{0}{\sin 2} = e^2 + c \checkmark \ldots (2;0)$$

$$c = -e^2 \checkmark (-7,389)$$

$$\therefore \frac{y}{\sin x} = e^x - e^2 \checkmark \quad \text{or} \quad \frac{y}{\sin x} = e^x - 7,389$$

(12)

4.2

$$2\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 8y = 16x^2$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 8x^2 \checkmark$$

$$\therefore m^2 + 4m + 4 = 0 \checkmark$$

$$(m+2)(m+2) = 0$$

$$\therefore m = -2 \checkmark$$

$$y_c = Ae^{-2x} + Bxe^{-2x} \checkmark$$

$$y_p : \quad y = Cx^2 + Dx + E \checkmark$$

$$\frac{dy}{dx} = 2Cx + D \checkmark$$

$$\frac{d^2y}{dx^2} = 2C \checkmark$$

$$2C + 4(2Cx + D) + 4(Cx^2 + Dx + E) = 8x^2 \checkmark$$

$$2C + 8Cx + 4D + 4Cx^2 + 4Dx + 4E = 8x^2$$

Equate coeff of x^2 : $4C = 8 \quad \therefore C = 2$ ✓

" " " x : $8C + 4D = 0 \quad \therefore D = -4$ ✓

" constants : $2C + 4D + 4E = 0 \quad \therefore E = -3$ ✓

$$y = Ae^{-2x} + Bxe^{-2x} + 2x^2 - 4x - 3 \checkmark$$

(12)
[24]

QUESTION 5

5.1 5.1.1

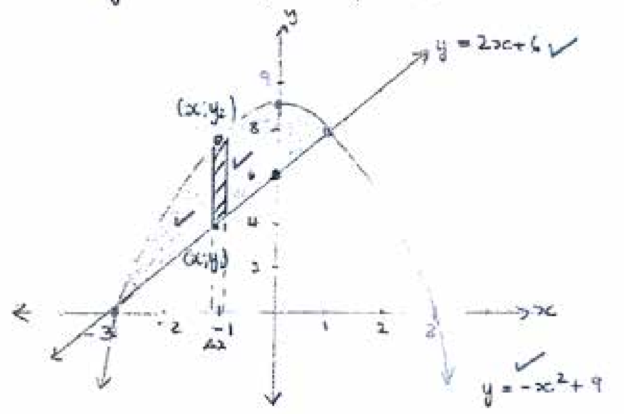
$$2x+6 = 9-x^2$$

$$\therefore x^2+2x-3 = 0$$

$$(x+3)(x-1) = 0$$

$$\therefore x = -3 \; ; \; x = 1 \quad (-3;0) \checkmark$$

$$y = 0 \; ; \; y = 8 \quad (1;8) \checkmark$$



(6)

5.1.2

$$\Delta V_x = \pi\,(y_2^2 - y_1^2)\,\Delta x \checkmark$$

$$V_x = \pi \int_{-3}^{1} (y_2^2 - y_1^2)\,dx \checkmark$$

$$= \pi \int_{-3}^{1} \left[(-x^2+9)^2 - (2x+6)^2\right] dx \checkmark$$

$$= \pi \int_{-3}^{1} \left[x^4 - 18x^2 + 81 - 4x^2 - 24x - 36\right] dx$$

$$= \pi \int_{-3}^{1} \left[x^4 - 22x^2 - 24x + 45\right] dx$$

$$= \pi \left[\frac{x^5}{5} - \frac{22x^3}{3} - \frac{24x^2}{2} + 45x\right]_{-3}^{1} \checkmark$$

$$= \pi\left\{\left[\tfrac{1}{5}(1)^5 - \tfrac{22}{3}(1)^3 - 12(1)^2 + 45(1)\right] - \left[\tfrac{1}{5}(-3)^5 - \tfrac{22}{3}(-3)^3 - 12(-3)^2 + 45(-3)\right]\right\} \checkmark$$

$$= 119{,}467\,\pi \text{ units}^3$$

$$\text{or} = 375{,}316 \text{ units}^3 \checkmark$$

(6)

5.1.3

$$\Delta M_y = \pi (y_2^2 - y_1^2)\Delta x \times x$$

$$\therefore M_y = \pi \int_{-3}^{1} (y_2^2 - y_1^2)\, x \, dx$$

$$= \pi \int_{-3}^{1} \left[(-x^2+9)^2 - (2x+6)^2\right] x \, dx$$

$$= \pi \int_{-3}^{1} \left[x^5 - 22x^3 - 24x^2 + 45x\right] dx$$

$$= \pi \left[\frac{x^6}{6} - \frac{22x^4}{4} - \frac{24x^3}{3} + \frac{45x^2}{2}\right]_{-3}^{1}$$

$$= \pi \left\{\left[\frac{1}{6} - \frac{22}{4} - 8 + \frac{45}{2}\right] - \left[\frac{1}{6}(-3)^6 - \frac{22}{4}(-3)^4 - 8(-3)^3 + \frac{45}{2}(-3)^2\right]\right\}$$

$$= \pi \left\{\left[\frac{1}{6} - \frac{11}{2} - 8 + \frac{45}{2}\right] - \left[\frac{1}{6}(729) - \frac{11}{2}(81) - 8(-27) + \frac{45}{2}(9)\right]\right\}$$

$$\left.\begin{aligned} &= -85{,}333\,\pi \\ &= -268{,}083 \end{aligned}\right\}$$

$$\therefore \bar{x} = \frac{-268{,}083}{375{,}316}$$

$$= -0{,}714 \text{ units}$$

(8)

5.2 5.2.1

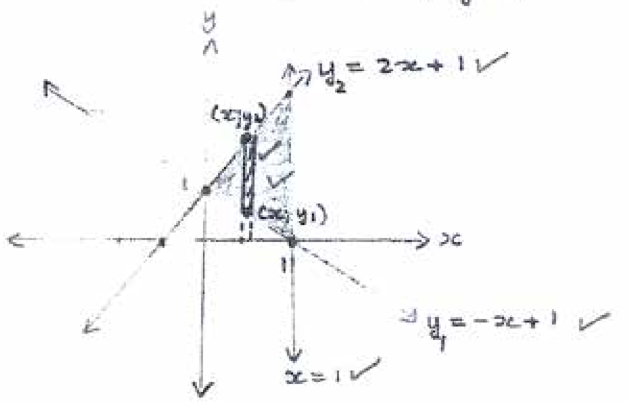
$y = -x + 1$; $y - 2x - 1 = 0$

$\therefore y = 2x + 1$

$\therefore -x + 1 = 2x + 1$

$3x = 0$

$x = 0$; $y = 1$ $(0;1)$



(6)

5.2.2

$$\Delta A = (y_2 - y_1)\,\Delta x \checkmark$$
$$A = \int_0^1 (y_2 - y_1)\,dx \checkmark$$
$$= \int_0^1 [(2x+1) - (-x+1)]\,dx \checkmark$$
$$= \int_0^1 (2x+1+x-1)\,dx$$
$$= \int_0^1 (3x)\,dx \checkmark$$
$$= \left[\frac{3x^2}{2}\right]_0^1 \checkmark$$
$$= \frac{3}{2}(1)^2$$
$$= \frac{3}{2} \text{ units}^2 \ / \ 1{,}5 \text{ units}^2 \checkmark$$

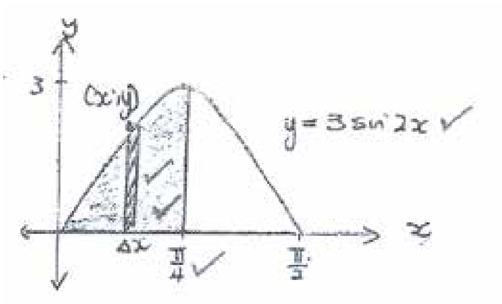
(6)

5.2.3

$$\Delta M_x = (y_2 - y_1)\,\Delta x \times \frac{y_2 + y_1}{2} \checkmark \checkmark$$
$$= \frac{1}{2}\int_0^1 (y_2^2 - y_1^2)\,dx \checkmark$$
$$= \frac{1}{2}\int_0^1 [(2x+1)^2 - (-x+1)^2]\,dx \checkmark$$
$$= \frac{1}{2}\int_0^1 [3x^2 + 6x]\,dx \checkmark$$
$$= \frac{1}{2}\left[\frac{3x^3}{3} + \frac{6x^2}{2}\right]_0^1 \checkmark$$
$$= \frac{1}{2}[(1)^3 + 3(1)^2] \checkmark$$
$$= 2 \text{ units}^3 \checkmark$$
$$\therefore \bar{y} = \frac{2}{1{,}5} \checkmark$$
$$= 1{,}333 \text{ units} \checkmark$$

(10)

5.3 5.3.1



(4)

5.3.2

$$\Delta V = \pi y^2 \Delta x \checkmark$$
$$V = \pi \int_0^{\frac{\pi}{4}} y^2 \, dx \checkmark$$
$$= \pi \int_0^{\frac{\pi}{4}} (3\sin 2x)^2 \, dx \checkmark$$
$$= 9\pi \int_0^{\pi/4} \sin^2 2x \, dx \checkmark$$
$$= 9\pi \left[\frac{x}{2} - \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{4}} \checkmark \checkmark$$
$$= 9\pi \left[\frac{\pi/4}{2} - \frac{\sin 4(\frac{\pi}{4})}{8}\right] \checkmark$$
$$= \frac{9}{8}\pi^2 \text{ units}^3$$
$$\text{or } = \underline{11{,}103 \text{ units}^3} \checkmark$$

(8)

5.3.3

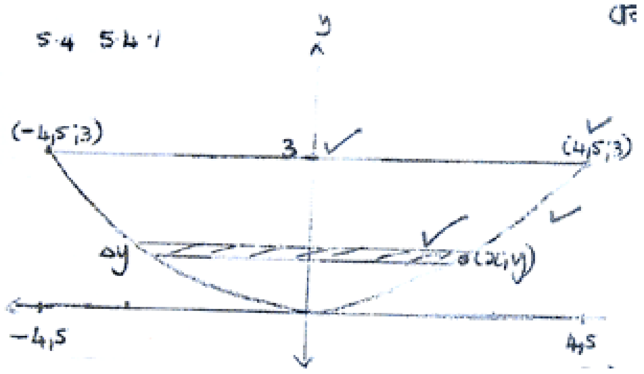
$$\Delta I_x = \rho \pi y^2 \Delta x \times \left(\frac{y}{\sqrt{2}}\right)^2 \checkmark \checkmark$$
$$= \frac{\rho\pi}{2} \int_0^{\pi/4} y^4 \, dx \checkmark$$
$$= \frac{\rho\pi}{2} \int_0^{\pi/4} (3\sin 2x)^4 \, dx \checkmark$$
$$= \frac{81\rho\pi}{2} \int_0^{\pi/4} \sin^4 2x \, dx \checkmark$$
$$= \frac{81\rho\pi}{2} \int_0^{\pi/4} (\sin^2 2x)^2 \, dx$$
$$= \frac{81\rho\pi}{2} \int_0^{\pi/4} \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right) dx \checkmark$$
$$= \frac{81\rho\pi}{2} \int_0^{\pi/4} \left(\frac{1}{4} - \frac{1}{2}\cos 4x + \frac{1}{4}\cos^2 4x\right) dx \checkmark$$
$$= \frac{81\rho\pi}{2} \left[\frac{1}{4}x - \frac{1}{2} \cdot \frac{\sin 4x}{4} + \frac{1}{4}\left(\frac{x}{2} + \frac{\sin 8x}{16}\right)\right]_0^{\pi/4} \checkmark$$
$$= \frac{81\rho\pi}{2} \left[\frac{1}{4}\left(\frac{\pi}{4}\right) - \frac{1}{8}\sin 4\left(\frac{\pi}{4}\right) + \frac{\pi/4}{8} + \frac{1}{64}\sin 8\left(\frac{\pi}{4}\right)\right] \checkmark$$
$$= \underline{19{,}621\rho} \checkmark$$

OR

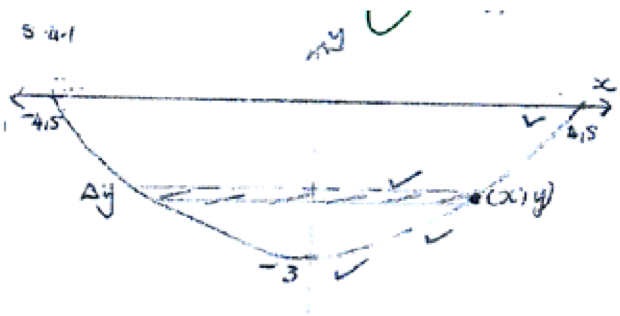
$$\Delta I_x = \text{mass} \times (\text{distance})^2$$
$$\Delta I_x = \frac{9\pi^2}{8}\rho \times \left(\frac{y}{\sqrt{2}}\right)^2 \Delta x \checkmark \checkmark$$
$$\therefore I_x = \frac{9\pi^2}{16}\rho \int_0^{\pi/4} y^2 \, dx \checkmark \checkmark$$
$$= \frac{9\pi^2}{16}\rho \int_0^{\pi/4} (3\sin 2x)^2 \, dx \checkmark$$
$$= \frac{81\pi^2}{16}\rho \int_0^{\pi/4} \sin^2 2x \, dx \checkmark$$
$$= \frac{81\pi^2}{16}\rho \left[\frac{x}{2} - \frac{\sin 4x}{8}\right]_0^{\pi/4} \checkmark$$
$$= \frac{81\pi^2}{16}\rho \left[\frac{\pi/4}{2} - \frac{\sin 4(\pi/4)}{8}\right] \checkmark$$
$$= \underline{19{,}621\rho} \checkmark$$

(10)

5.4 5.4.1



OR



(4)

5.4.2

5.4.2 $y = ax^2$ .... $(4,5;3)$

$3 = a(4,5)^2$

$\therefore y = 0,148\, x^2$ ✓ or $y = \frac{12}{81} x^2$

$\therefore x = \sqrt{\frac{1}{0,148}}\; y^{\frac{1}{2}}$ ✓

or $x = 2,6\, y^{1/2}$

$\Delta A = 2x\, \Delta y \Rightarrow 2(2,6y^{\frac{1}{2}})\, dy$ (2)

OR

4.2 $y = a(x^2 - 20,25)$;

$\therefore y = \frac{3}{20,25}(x^2 - 20,25)$

$\therefore y = 0,148x^2 - 3$ ✓

$x = \sqrt{\frac{y+3}{0,148}}$ or ✓ $x = \sqrt{6,757y + 20,27}$

$A = 2(6,757y + 20,27)^{\frac{1}{2}}\, dy$

(2)

5.4.3

$$\int_0^3 (3-y)\,5{,}2\,y^{\frac{1}{2}}\,dy$$
$$= 5{,}2\int_0^3 \left(3y^{\frac{1}{2}} - y^{3/2}\right)dy$$
$$= 5{,}2\left[\frac{3y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2}\right]_0^3$$
$$= 5{,}2\left[\frac{2}{3}\cdot 3(3)^{3/2} - \frac{2}{5}(3)^{5/2}\right]$$
$$= 21{,}616 \text{ units}^3$$

$$\therefore \bar{y} = \frac{37{,}056}{21{,}616}$$
$$= 1{,}714 \text{ units}$$

OR

5.4.3 $\int_{-3}^{0} y \cdot 2\,(6{,}757y+20{,}27)^{\frac{1}{2}}dy$

$u = 6{,}757y+20{,}27$

$\frac{du}{dy} = 6{,}757$

$dy = \frac{du}{6{,}757}$

$y = \frac{u-20{,}27}{6{,}757}$

$= 0{,}148u - 3$

$$= 2\int_0^{20{,}27} (0{,}148u-3)\,u^{\frac{1}{2}} \cdot \frac{du}{6{,}757}$$
$$= \frac{2}{6{,}757}\int_0^{20{,}27} \left(0{,}148u^{3/2} - 3u^{1/2}\right)du$$
$$= 0{,}296\left[\frac{0{,}148u^{5/2}}{5/2} - \frac{3u^{3/2}}{3/2}\right]_0^{20{,}27}$$
$$= 0{,}296\left[\frac{2}{5}(0{,}148)(20{,}27)^{5/2} - \frac{2}{3}(3)(20{,}27)^{3/2}\right]$$
$$= -21{,}616 \text{ units}^3$$

$$\therefore \bar{y} = \frac{37{,}057}{-21{,}616}$$
$$= -1{,}714 \text{ units}$$

(10)
[80]

QUESTION 6

6.1

$$x^2 = y^3$$
$$x = y^{3/2} \checkmark$$
$$\frac{dx}{dy} = \frac{3}{2}y^{\frac{1}{2}} \checkmark$$
$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \left(\frac{3}{2}y^{\frac{1}{2}}\right)^2 \checkmark$$
$$= 1 + \frac{9}{4}y$$
$$= \frac{4+9y}{4} \checkmark$$
$$\therefore S_y = \int_1^3 \checkmark \sqrt{\frac{4+9y}{4}}\,dy \checkmark$$
$$= \frac{1}{2}\int_1^3 (4+9y)^{\frac{1}{2}}\,dy$$
$$= \frac{1}{2}\cdot\frac{1}{9}\int_1^3 9(4+9y)^{\frac{1}{2}}\,dy$$
$$= \frac{1}{18}\cdot\left[\frac{(4+9y)^{3/2}}{\frac{3}{2}}\right]_1^3 \checkmark$$
$$= \frac{1}{18}\cdot\frac{2}{3}\left[(4+9(3))^{3/2} - (4+9(1))^{3/2}\right] \checkmark$$
$$= 4,657 \text{ unit} \checkmark$$

(10)

6.2

2. $x = e^{3t}\sin t \qquad y = e^{3t}\cos t$

$$\frac{dx}{dt} = e^{3t}(\cos t) + 3e^{3t}\sin t \checkmark \qquad \frac{dy}{dt} = e^{3t}(-\sin t) + 3e^{3t}\cos t \checkmark$$
$$\frac{dx}{dt} = e^{3t}(\cos t + 3\sin t) \qquad \frac{dy}{dt} = e^{3t}(-\sin t + 3\cos t)$$
$$\left(\frac{dx}{dt}\right)^2 = e^{6t}(\cos t + 3\sin t)^2 \checkmark \qquad \frac{dy}{dt} = e^{6t}(-\sin t + 3\cos t)^2 \checkmark$$
$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{6t}(\cos t + 3\sin t)^2 + e^{6t}(-\sin t + 3\cos t)^2$$
$$= 10e^{6t} \checkmark$$

$$\therefore A = 2\pi\int_0^{\frac{\pi}{2}} \checkmark\, y\sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2}\,dt$$
$$= 2\pi\int_0^{\pi/2} e^{3t}\cos t \checkmark \sqrt{10e^{6t}}\,dt \checkmark$$
$$= 2\pi\sqrt{10}\int_0^{\pi/2} e^{6t}\cos t\,dt \checkmark$$
$$= 2\pi\sqrt{10}\left[e^{6t}\sin t - \int 6e^{6t}\sin t\,dt\right]_0^{\frac{\pi}{2}} \checkmark \qquad f(t) = e^{6t} \qquad g'(t) = \cos t$$
$$= 2\pi\sqrt{10}\left[e^{6t}\sin t - 6\int e^{6t}\sin t\,dt\right]_0^{\pi/2} \checkmark \qquad f'(t) = 6e^{6t} \qquad g(t) = \sin t$$
$$= 2\pi\sqrt{10}\left[e^{6t}\sin t - 6\left(e^{6t}\cdot -\cos t - \int 6e^{6t}\cdot -\cos t\,dt\right)\right]_0^{\frac{\pi}{2}} \qquad f(t) = e^{6t} \qquad g'(t) = \sin t$$
$$= 2\pi\sqrt{10}\left[e^{6t}\sin t + 6e^{6t}\cos t - 36\int e^{6t}\cos t\,dt\right]_0^{\pi/2} \qquad f'(t) = 6e^{6t} \qquad g(t) = -\cos t$$

$\Rightarrow = e^{6t}\sin t + 6e^{6t}\cos t - 36\int e^{6t}\cos t\, dt$ ✓✓ 15

$I = e^{6t}\sin t + 6e^{6t}\cos t - 36 I$ .... Let $I = \int e^{6t}\cos t\, dt$

$37I = e^{6t}\sin t + 6e^{6t}\cos t$

$\therefore I = \frac{1}{37}\left(e^{6t}\sin t + 6e^{6t}\cos t\right)$

$\therefore \quad A = 2\pi\sqrt{10}\left[\frac{1}{37}\left(e^{6t}\sin t + 6e^{6t}\cos t\right)\right]_0^{\pi/2}$ ✓

$= \frac{2\pi\sqrt{10}}{37}\left[e^{6t}(\sin t + 6\cos t)\right]_0^{\pi/2}$

$= \frac{2\pi\sqrt{10}}{37}\left[e^{6(\frac{\pi}{2})}(\sin\frac{\pi}{2} + 6\cos\frac{\pi}{2}) - e^{6(0)}(\sin 0 + 6\cos 0)\right]$ ✓

$= \frac{2\pi\sqrt{10}}{37}\left[e^{3\pi} - 6\right]$

$= 2117{,}127\,\pi$ units^2 ✓

OR $= 6651{,}152$ units^2

(14)

[24]

200 ÷ 2

TOTAL: 100

higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T960**(E)(**J24)T
AUGUST EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 July 2014 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and a formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If $w = \tan 3x \,.\, \cot \frac{y}{3}$, determine $\frac{\partial^2 w}{\partial x^2}$ (2)

1.2 The parametric equations of a function are given as:

$y = \frac{3a}{1+a^2}$ and $x = \frac{4}{1+a^2}$, calculate the values of the following:

1.2.1 $\frac{dy}{dx}$

1.2.2 $\frac{d^2y}{dx^2}$

(2 × 2) (4)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \tan^4 2x$ (4)

2.2 $y = \sqrt{12 - 8x - 2x^2}$ (4)

2.3 $y = \dfrac{\sin^3 \frac{x}{3} \,.\, \sec^2 \frac{x}{3}}{1 + \tan^2 \frac{x}{3}}$ (4)

2.4 $y = 3x^2 \,.\, \cot^{-1} x$ (4)

2.5 $y = x \ln ax$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{4+3x-x^2}{x(3-x)^2}\,dx$ (6)

3.2 $\int \frac{x^2+6x+12}{x(2-x^2)}\,dx$ (6)
[12]

QUESTION 4

4.1 Find the equation of the curve with gradient $\frac{dy}{dx}=x^2-\frac{2y}{x}$ and which passes through (1;1). (5)

4.2 Calculate the particular solution of:

$\frac{d^2s}{dt^2}-9s=e^t$, if $t=0$ when $s=1$ and $\frac{ds}{dt}=3$ when $t=0$. (7)
[12]

QUESTION 5

5.1 5.1.1 Sketch of the curve of $y=-x^2+3x$ and show the representative strip/element that you will use to calculate the volume (by using the SHELL-method only) generated when the area bounded by the curve, $x=0$ and the line $y=3$ is rotated about the y-axis. (2)

5.1.2 Use the shell-method to calculate the volume described in QUESTION 5.1.1. (4)

5.2 5.2.1 Make a neat sketch of the graph $x^2-y^2=1$ and show the area bounded by the graph and $x=2$. Show the representative strip/element that you will use to calculate the bounded area. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1. (4)

5.2.3 Calculate the x-ordinate of the centroid of the area described in QUESTION 5.2.1. (5)

5.3 5.3.1 Calculate the points of intersection of the graphs $y = 3x$ and $y = 3x^2$. Make a neat sketch of the two graphs and show the representative strip/element (Parallel to the y -axis) that you will use to calculate the volume if the area bounded by the two graphs is rotated about the y -axis. (3)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (4)

5.3.3 Calculate the moment of inertia of the solid obtained when the area in QUESTION 5.3.1 is rotated about the y -axis and express the answer in terms of the mass. (5)

5.4 5.4.1 A vertical weir in a rectangular canal is 6 m wide and 3 m high. The top of the weir is 3 m below the water surface. Make a neat sketch of the weir and show the representative strip/element that you will use to calculate the depth of the centre of pressure on the weir. (2)

5.4.2 Calculate the area moment of the weir about the water surface by means of integration. (4)

5.4.3 Calculate the second moment of area of the weir about the water surface, as well as the depth of the centre of pressure on the weir by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the arc length of the curve given by the parametric equations

$x = 3(\cos\theta + \theta\sin\theta)$ and $y = 3(\sin\theta - \theta\cos\theta)$ between $\theta = 0$ and $\theta = \frac{\pi}{2}$. (6)

6.2 Determine, by integration, the surface area generated when revolving $\frac{x^2}{36} + \frac{y^2}{4} = 1$, about the x -axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} \; ; \; \sin x = \frac{1}{\operatorname{cosec} x} \; ; \; \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x \ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln[\sec(ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln[\sin(ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln[\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln\left[\tan\left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \,.\, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \,.\, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \,.\, f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x) \,.\, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \,.\, f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x) \,.\, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\,;\; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\,;\; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx\,;\, V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx\,;\, V_x = 2p\int_a^b xy dy$$

$$V_y = p\int_a^b x^2 dy\,;\, V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy\,;\, V_y = 2p\int_a^b xy dx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A}\,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = r V$

DEFINITION: $I = m r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2} \int_a^b r^2 dm = \frac{1}{2} \rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2} \rho \pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2} \rho \pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\; du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\; du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\; dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\; du$$

$$\frac{dy}{dx} + Py = Q \quad \square \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

AUGUST EXAMINATION

MATHEMATICS N6

August 2014

This marking guideline consists of 13 pages.

QUESTION 1

1.1. $w = \tan 3x \, . \cot \frac{y}{3}$

$$\frac{dw}{dx} = \cot \frac{y}{3} \, . \, 3\sec^2 3x$$

$$= 3\cot \frac{y}{3} \sec^2 3x$$

$$\frac{d^2w}{dx^2} = 2 . 3\cot \frac{y}{3} \, . \sec 3x \, . \, 3\sec 3x \tan 3x$$

$$= 18 \cot \frac{y}{3} . \sec^2 3x \, . \tan 3x$$

(4)

1.2 $y = \frac{3a}{1+a^2}$ $\quad\quad$ $x = \frac{4}{1+a^2}$

1.2.1. $\frac{dy}{da} = \frac{(1+a^2)(3) - 3a(2a)}{(1+a^2)^2}$ $\quad\quad$ $\frac{dx}{da} = -4(1+a^2)^{-2}, 2a$

$= \frac{3+3a^2-6a^2}{(1+a^2)^2}$ $\quad\quad$ $= -\frac{8a}{(1+a^2)^2}$

$= \frac{3-3a^2}{(1+a^2)^2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{da}}{\frac{dx}{da}}$$

$$= \frac{\frac{3-3a^2}{(1+a^2)^2}}{\frac{-8a}{(1+a^2)^2}}$$

$$= \frac{3-3a^2}{-8a}$$

$$= -\frac{3}{8a} + \frac{3a}{8}$$

(4)

1.2.2 $\frac{d^2y}{dx^2} = \frac{d}{da}\left(\frac{dy}{dx}\right) \times \frac{da}{dx}$

$$= \frac{d}{da}\left(-\frac{3}{8a} + \frac{3a}{8}\right) \times \frac{(1+a^2)^2}{-8a}$$

$$= \left(\frac{3}{8a^2} + \frac{3}{8}\right)\left[\frac{(1+a^2)^2}{-8a}\right]$$

$$= \left(\frac{3+3a^2}{8a^2}\right)\left[\frac{(1+a^2)^2}{-8a}\right]$$

$$= \frac{3(1+a^2)}{8a^2} \times \frac{(1+a^2)^2}{-8a}$$

$$= -\frac{3}{64a^3}(1+a^2)^3$$

(4)
[12]

QUESTION 2

2.1. $\int \tan^4 2x \, dx$

$= \int \tan^2 2x \cdot \tan^2 2x \, dx$

$= \int (\sec^2 2x - 1) \tan^2 2x \, dx$

$= \int \sec^2 2x \cdot \tan^2 2x \, dx - \int \tan^2 2x \, dx$

$= \frac{1}{2} \cdot \frac{\tan^3 2x}{3} - \left[\frac{1}{2} \tan 2x - x\right] + c$

$= \frac{1}{6} \tan^3 2x - \frac{1}{2} \tan 2x + x + c$

(8)

2.2. $\int \sqrt{12 - 8x - 2x^2} \, dx$

$= \int \sqrt{2\left[10 - (x+2)^2\right]} \, dx$

$= \sqrt{2} \int \sqrt{10 - (x+2)^2} \, dx$

$-2x^2 - 8x + 12$

$= -2\left[x^2 + 4x - 6\right]$

$= -2\left[(x+2)^2 - 6 - 4\right]$

$= -2\left[(x+2)^2 - 10\right]$

$= 2\left[10 - (x+2)^2\right]$

$= \sqrt{2}\left[\frac{10}{2} \sin^{-1} \frac{(x+2)}{\sqrt{10}} + \frac{(x+2)}{2} \sqrt{10 - (x+2)^2}\right] + c$

$= \sqrt{2}\left[5 \sin^{-1} \frac{(x+2)}{3{,}162} + \frac{x+2}{2} \sqrt{10 - (x+2)^2}\right] + c$

(8)

OR $\int \sqrt{-2\left[x^2 + 4x - 6\right]} \, dx$

$= \int \sqrt{-2\left[x^2 + 4x + (2)^2 - (2)^2 - 6\right]} \, dx$

$= \int \sqrt{-2\left[(x+2)^2 - 4 - 6\right]} \, dx$

$= \int \sqrt{-2\left[(x+2)^2 - 10\right]} \, dx$

$= \int \sqrt{20 - 2(x+2)^2} \, dx$

$= \frac{20}{2\sqrt{2}} \sin^{-1} \frac{\sqrt{2}(x+2)}{\sqrt{20}} + \frac{(x+2)}{2} \sqrt{20 - 2(x+2)^2} + c$

$= 7{,}071 \sin^{-1} \frac{1{,}414(x+2)}{4{,}472} + \frac{(x+2)}{2} \sqrt{20 - 2(x+2)^2} + c$

2.3.
$$\int \frac{\sin^3\frac{x}{3}\cdot\sec^2\frac{x}{3}}{1+\tan^2\frac{x}{3}}\,dx$$
$$= \int \frac{\sin^3\frac{x}{3}\cdot\sec^2\frac{x}{3}}{\sec^2\frac{x}{3}}\,dx$$
$$= \int \sin^3\frac{x}{3}\,dx$$
$$= \int \sin^2\frac{x}{3}\cdot\sin\frac{x}{3}\,dx$$
$$= \int \left(1-\cos^2\frac{x}{3}\right)\sin\frac{x}{3}\,dx$$
$$= -3\int(1-u^2)\,du$$
$$= -3\left[u-\frac{u^3}{3}\right]+c$$
$$= -3\left[\cos\frac{x}{3}-\frac{1}{3}\cos^3\frac{x}{3}\right]+c$$

$u = \cos\frac{x}{3}$
$du = -\frac{1}{3}\sin\frac{x}{3}$

(8)

2.4.
$$\int 3x^2\cdot\cot^{-1}x\,dx$$

$f(x) = \cot^{-1}x \quad g'(x) = 3x^2$
$f'(x) = \frac{-1}{x^2+1} \quad g(x) = \frac{3x^3}{3} = x^3$

$$= \cot^{-1}x\cdot x^3 - \int \frac{-1}{x^2+1}\cdot x^3\,dx$$
$$= x^3\cdot\cot^{-1}x + \int \frac{x^3}{x^2+1}\,dx$$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & x^3+x \\ \hline & -x \end{array}$$

$$= x^3\cot^{-1}x + \int\left(x - \frac{x}{x^2+1}\right)dx$$
$$= x^3\cot^{-1}x + \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + c$$

(8)

2.5.
$$\int x\ln ax\,dx$$
$$= \ln ax\left(\frac{x^2}{2}\right) - \int\frac{1}{x}\cdot\frac{x^2}{2}\,dx$$
$$= \frac{1}{2}x^2\ln ax - \frac{1}{2}\int x\,dx$$
$$= \frac{1}{2}x^2\ln ax - \frac{1}{2}\cdot\frac{x^2}{2}+c$$
$$= \frac{1}{2}x^2\ln ax - \frac{1}{4}x^2+c$$

$f(x) = \ln ax \quad g'(x) = x$
$f'(x) = \frac{1}{x} \quad g(x) = \frac{x^2}{2}$

(4)

[36]

QUESTION 3

3.1 $\int \frac{4+3x-x^2}{x(3-x)^2}\,dx$

$\therefore \frac{4+3x-x^2}{x(3-x)^2} = \frac{A}{x} + \frac{B}{(3-x)^2} + \frac{C}{(3-x)}$ ✓

$\therefore 4+3x-x^2 = A(3-x)^2 + Bx + Cx(3-x)$ ✓

Let $x = 0$; $\therefore A = \frac{4}{9}$ ✓ $(0{,}444)$

$x = 3$; $\therefore B = \frac{4}{3}$ ✓ $(1{,}333)$

$\therefore 4+3x-x^2 = 9A - 6Ax + Ax^2 + Bx + 3Cx - Cx^2$ ✓

Equate coeff of x^2: $-1 = A - C$ $\therefore C = \frac{13}{9}$ ✓ $(1{,}444)$

$\therefore \int \frac{\frac{4}{9}}{x}\,dx + \int \frac{\frac{4}{3}}{(3-x)^2}\,dx + \int \frac{\frac{13}{9}}{(3-x)}\,dx$ ✓

$= \frac{4}{9}\int \frac{1}{x}\,dx + \frac{4}{3}(3-x)^{-2}\,dx + \frac{13}{9}\int \frac{1}{3-x}\,dx$

$= \frac{4}{9}\ln x$ ✓ $-$ ✓ $\frac{4}{3}\cdot\frac{(3-x)^{-1}}{-1}$ ✓ $+ \frac{13}{9}(-1)$ ✓ $\ln(3-x)$ ✓ $+ C$

$= \frac{4}{9}\ln x + \frac{4}{3(3-x)} - \frac{13}{9}\ln(3-x) + C$

or $= 0{,}444\ln x + \frac{1{,}333}{(3-x)} - 1{,}444\ln(3-x) + C$ (12)

3.2 $\int \frac{x^2+6x+12}{x(2-x^2)}\,dx$

$\therefore \frac{x^2+6x+12}{x(\sqrt{2}-x)(\sqrt{2}+x)}$ ✓ $= \frac{A}{x} + \frac{B}{(\sqrt{2}-x)} + \frac{C}{(\sqrt{2}+x)}$ ✓

$\therefore x^2+6x+12 = A(\sqrt{2}-x)(\sqrt{2}+x) + Bx(\sqrt{2}+x) + Cx(\sqrt{2}-x)$ ✓

Let $x=0$; $\therefore A = 6$ ✓

$x = \sqrt{2}$; $\therefore B = 5{,}621$ ✓

$x = -\sqrt{2}$; $\therefore C = -1{,}379$ ✓

$\therefore \int \frac{6}{x}\,dx + \int \frac{5{,}621}{(\sqrt{2}-x)}\,dx + \int \frac{-1{,}379}{(\sqrt{2}+x)}\,dx$ ✓

$= 6\int \frac{1}{x}\,dx + 5{,}621\int \frac{1}{\sqrt{2}-x}\,dx - 1{,}379\int \frac{1}{\sqrt{2}+x}\,dx$

$= 6\ln x$ ✓ $-$ ✓ $5{,}621\ln(\sqrt{2}-x)$ ✓ $-$ ✓ $1{,}379\ln(\sqrt{2}+x)$ ✓ $+ C$ (12)

[24]

QUESTION 4

4.1. $\frac{dy}{dx} = x^2 - \frac{2y}{x}$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = x^2$ ✓

$R = e^{\int \frac{2}{x} dx}$ ✓

$= e^{2 \ln x}$ ✓

$= e^{\ln x^2}$

$= x^2$ ✓

$\therefore Ry = \int RQ\, dx + c$

$x^2 y = \int x^2 . x^2\, dx + c$ ✓ ✓

$x^2 y = \int x^4\, dx + c$

$x^2 y = \frac{x^5}{5} + c$ ✓

$(1)^2(1) = \frac{(1)^5}{5} + c$ ✓ .... $(1;1)$

$c = \frac{4}{5}$ ✓ $(0,8)$

$\therefore x^2 y = \frac{1}{5} x^5 + \frac{4}{5}$ ✓ (10)

4.2. $\frac{d^2s}{dt^2} - 9s = e^t$

$\therefore m^2 - 9 = 0$ ✓

$(m-3)(m+3) = 0$

$\therefore m = -3 \; ; m = 3$ ✓

$S_c = Ae^{3t} + Be^{-3t}$ ✓

S_p : $s = Ce^t$ ✓

$\frac{ds}{dt} = Ce^t$ ✓

$\frac{d^2s}{dt^2} = Ce^t$ ✓

$\therefore Ce^t - 9Ce^t = e^t$ ✓

$-8Ce^t = e^t$

$\therefore C = -\frac{1}{8}$ ✓

$\therefore S_p = -\frac{1}{8}e^t$ ✓

$s = Ae^{3t} + Be^{-3t} - \frac{1}{8}e^t$ ✓

$1 = A + B - \frac{1}{8} \quad \therefore A + B = \frac{9}{8} \quad (1{,}125)$

$\frac{ds}{dt} = 3Ae^{3t} - 3Be^{-3t} - \frac{1}{8}e^t$ ✓

$3 = 3A - 3B - \frac{1}{8} \quad \therefore 3A - 3B = \frac{25}{8} \quad (3{,}125)$

$\therefore \quad A = \frac{9}{8} - B$

and $\quad 3A - 3B = \frac{25}{8}$

$\therefore 3(\frac{9}{8} - B) - 3B = \frac{25}{8}$

$\therefore \quad B = \frac{1}{24} \ (0{,}417)$ ✓ and $A = \frac{26}{24} \ (1{,}083)$ ✓

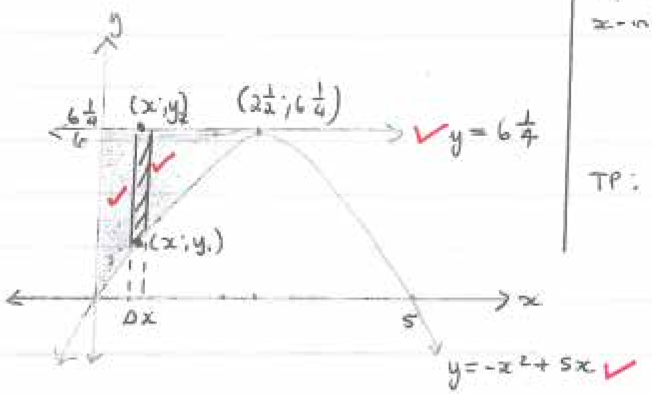
$\therefore \quad s = 1{,}083\,e^{3t} + 0{,}417\,e^{-3t} - \frac{1}{8}e^{t}$ ✓ (14)

[24]

or $\quad s = \frac{26}{24}e^{3t} + \frac{1}{24}e^{-3t} - \frac{1}{8}e^{t}$

QUESTION 5

5.1. 5.1.1.



$y = -x^2 + \ x$

x-intercepts : $0 = -x^2 + \ x$

$0 = x(-x + \ )$

$\therefore x = 0; \ x =$

TP : $x = \frac{-5}{2(-1)} = \frac{5}{2} = 2\frac{1}{2}$

$y = 6\frac{1}{4} \qquad \therefore (2\frac{1}{2}; 6\frac{1}{4})$

or $(2{,}5; 6{,}25)$

(4)

5.1.2. $\Delta V = 2\pi x$ ✓ $\times (y_2 - y_1)$ ✓ $\times \Delta x$ ✓

$\therefore \quad V = 2\pi \int_0^{5/2} x(y_2 - y_1)\,dx$ ✓

$= 2\pi \int_0^{5/2} x(6\frac{1}{4} + x^2 - 5x)\,dx$ ✓

$= 2\pi \int_0^{5/2} (6\frac{1}{4}x + x^3 - 5x^2)\,dx$

$= 2\pi \left[\frac{25}{4}\cdot\frac{x^2}{2} + \frac{x^4}{4} - \frac{5x^3}{3}\right]_0^{5/2}$ ✓

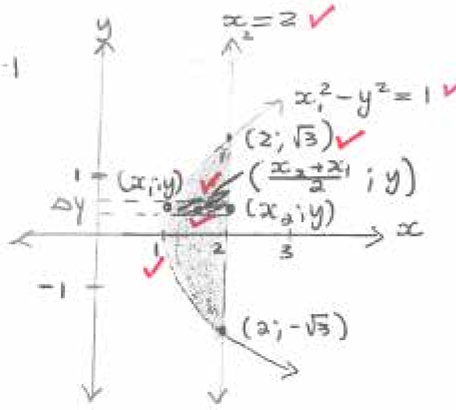
$= 2\pi \left[\frac{25}{8}(\frac{5}{2})^2 + \frac{1}{4}(\frac{5}{2})^4 - \frac{5}{3}(\frac{5}{2})^3\right]$ ✓

$= 6{,}51\,\pi$ units^3 ✓

or $= 20{,}453$ units^3

(8)

5.2 5.2.1



$$x^2 - y^2 = 1 \quad \ldots \quad x = 2$$
$$2^2 - y^2 = 1$$
$$y^2 = 3$$
$$\therefore y = \pm\sqrt{3}$$

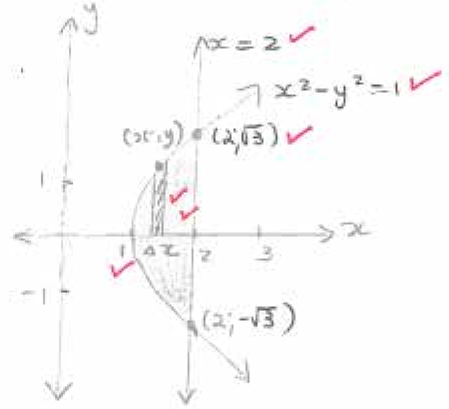
(6)

5.2.2 $\Delta A = (x_2 - x_1)\Delta y$

$$\therefore A = 2\int_0^{\sqrt{3}} (x_2 - x_1)\,dy \quad \text{or} \quad \int_{-\sqrt{3}}^{\sqrt{3}} (x_2 - x_1)\,dy$$
$$= 2\int_0^{\sqrt{3}} \left(2 - \sqrt{1+y^2}\right)dy$$
$$= 2\left[2y - \left(\tfrac{y}{2}\sqrt{1+y^2} + \tfrac{1}{2}\ln\left(y + \sqrt{1+y^2}\right)\right)\right]_0^{\sqrt{3}}$$
$$= 2\left[2\sqrt{3} - \tfrac{\sqrt{3}}{2}\sqrt{1+3} - \tfrac{1}{2}\ln\left(\sqrt{3} + \sqrt{1+3}\right)\right]$$
$$= \underline{2{,}147 \text{ units}^2}$$

(8)

5.2.1 OR



(6)

5.2.2 $\Delta A = 2y\,\Delta x \qquad \therefore A = 2\int_1^2 y\,dx$

$$\therefore A = 2\int_1^2 \sqrt{x^2 - 1}\,dx$$
$$= 2\left[\tfrac{x}{2}\sqrt{x^2-1} - \tfrac{1}{2}\ln\left(x + \sqrt{x^2-1}\right)\right]_1^2$$
$$= 2\left[\tfrac{2}{2}\sqrt{(2)^2-1} - \tfrac{1}{2}\ln\left(2 + \sqrt{2^2-1}\right)\right]$$
$$= \underline{2{,}147 \text{ units}^2}$$

(8)

5.2.3 $\Delta M_y = (x_2 - x_1)\Delta y \times \frac{x_2 + x_1}{2}$

$\therefore M_y = \frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}} (x_2^2 - x_1^2)\,dy$

$= \frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}} [4 - (1+y^2)]\,dy$

$= \frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}} [4 - 1 - y^2]\,dy$

$= \frac{1}{2}\int_{-\sqrt{3}}^{\sqrt{3}} [3 - y^2]\,dy$

$= \frac{1}{2}\left[3y - \frac{y^3}{3}\right]_{-\sqrt{3}}^{\sqrt{3}}$

$= \frac{1}{2}\left\{\left[3(\sqrt{3}) - \frac{1}{3}(\sqrt{3})^3\right] - \left[3(-\sqrt{3}) - \frac{1}{3}(-\sqrt{3})^3\right]\right\}$

$= \frac{1}{2}\left\{\left[3\sqrt{3} - \frac{1}{3}(\sqrt{3})^3 + 3\sqrt{3} - \frac{1}{3}(\sqrt{3})^3\right]\right\}$

$= 3{,}464$

$\therefore \bar{x} = \frac{M_y}{A}$

$= \frac{3{,}464}{2{,}147}$

$= \underline{1{,}613 \text{ units}}$

(10)

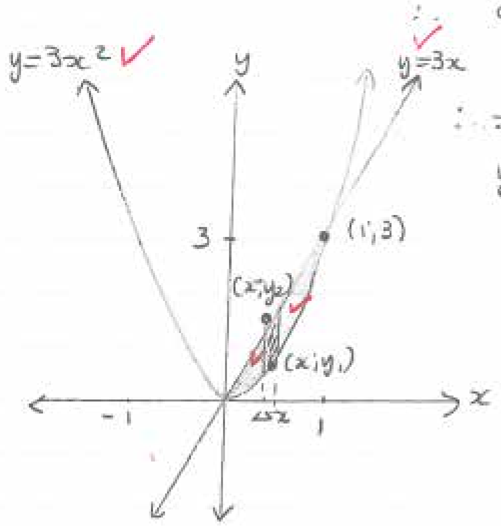
5.3 5.3.1 $y = 3x$ $y = 3x^2$

Points of intersection: $3x = 3x^2$

$\therefore 0 = 3x^2 - 3x$

$0 = 3x(x-1)$

$\therefore x = 0\ ;\ x = 1$
$y = 0\ ;\ y = 3$ $\Big\}$ $(0;0)$ $(1;3)$



(6)

5.3.2. $\Delta V_y = 2\pi x (y_2 - y_1)\Delta x$ ✓ ✓

$$V_y = 2\pi\int_0^1 x(y_2 - y_1)\,dx \quad ✓$$
$$= 2\pi\int_0^1 x(3x - 3x^2)\,dx \quad ✓$$
$$= 2\pi\int_0^1 (3x^2 - 3x^3)\,dx \quad ✓$$
$$= 2\pi\left[\frac{3x^3}{3} - \frac{3x^4}{4}\right]_0^1 \quad ✓$$
$$= 2\pi\left[(1)^3 - \tfrac{3}{4}(1)^4\right] \quad ✓$$
$$= 0{,}5\pi \text{ or } 1{,}571 \text{ units}^3 \quad ✓$$

(8)

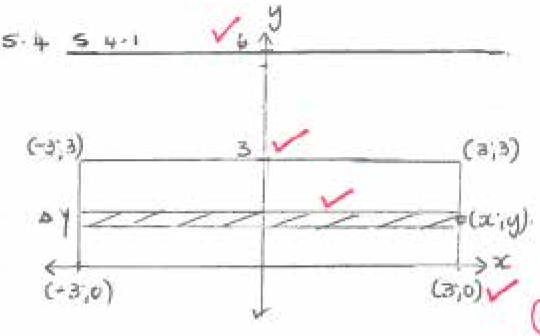
5.3.3. $\Delta I_y = \rho\, 2\pi x (y_2 - y_1)\Delta x \times x^2$ ✓ ✓

$$I_y = 2\pi\rho\int_0^1 x^3(y_2 - y_1)\,dx \quad ✓$$
$$= 2\pi\rho\int_0^1 x^3(3x - 3x^2)\,dx \quad ✓$$
$$= 2\pi\rho\int_0^1 (3x^4 - 3x^5)\,dx \quad ✓$$
$$= 2\pi\rho\left[\frac{3x^5}{5} - \frac{3x^6}{6}\right]_0^1 \quad ✓$$
$$= 2\pi\rho\left[\tfrac{3}{5}(1)^5 - \tfrac{1}{2}(1)^6\right] \quad ✓$$
$$= 0{,}2\pi\rho \text{ or } 0{,}628\rho \quad ✓$$

$$\therefore I_y = \frac{1{,}571\,m}{0{,}628} \quad ✓ \quad \left(\frac{1{,}571\,M}{\Delta V_y}\right)$$
$$= 2{,}502\,m \quad ✓$$

(10)

5.4 5.4.1



(4)

5.4.2 $\Delta A = 2x \cdot \Delta y$

$A = 2(3)\,dy \quad \ldots\ldots x = 3$

$\therefore A = 6\,dy$

$$\int_0^3 (6-y)\,6\,dy$$

$$= 6\int_0^3 (6-y)\,dy$$

$$= 6\left[6y - \frac{y^2}{2}\right]_0^3$$

$$= 6\left[6(3) - \tfrac{1}{2}(3)^2\right]$$

$$= \underline{81 \text{ units}^3}$$

(8)

5.4.3 $$\int_0^3 (6-y)^2\,6\,dy$$

$$= 6\int_0^3 (36 - 12y + y^2)\,dy$$

$$= 6\left[36y - \frac{12y^2}{2} + \frac{y^3}{3}\right]_0^3$$

$$= 6\left[36y - 6y^2 + \tfrac{1}{3}y^3\right]_0^3$$

$$= 6\left[36(3) - 6(3)^2 + \tfrac{1}{3}(3)^3\right]$$

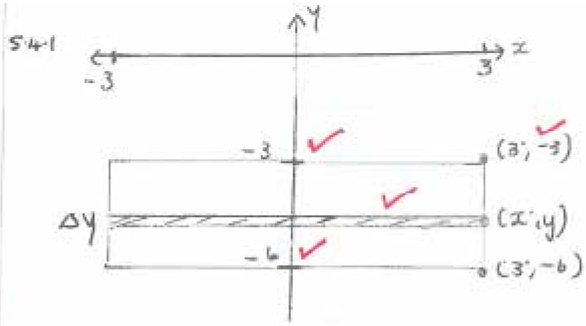
$$= \underline{378 \text{ units}^4}$$

$$\therefore \bar{\bar{y}} = \frac{378}{81}$$

$$= \underline{4{,}667 \text{ units}}$$

(8)

5.4.1



5.4.2. $\Delta A = 2x\,\Delta y$

$\therefore A = 2(3)\,dy \quad \ldots\ldots x = 3$

$A = 6\,dy$

$$\int_{-6}^{-3} y \cdot 6\,dy$$

$$= 6\int_{-6}^{-3} y\,dy$$

$$= 6\left[\frac{y^2}{2}\right]_{-6}^{-3}$$

$$= 3\left[(-3)^2 - (-6)^2\right]$$

$$= \underline{-81 \text{ units}^3}$$

5.4.3. $$\int_{-6}^{-3} y^2 \cdot 6\,dy$$

$$= 6\int_{-6}^{-3} y^2\,dy$$

$$= 6\left[\frac{y^3}{3}\right]_{-6}^{-3}$$

$$= 2\left[(-3)^3 - (-6)^3\right]$$

$$= \underline{378 \text{ units}^4}$$

$$\therefore \bar{\bar{y}} = \frac{378}{-81}$$

$$= \underline{-4{,}667 \text{ units}}$$

[80]

QUESTION 6

6.1.

$x = 3(\cos\theta + \theta\sin\theta)$

$\frac{dx}{d\theta} = 3(-\sin\theta + \theta\cos\theta + \sin\theta)$

$= 3\theta\cos\theta$

$y = 3(\sin\theta - \theta\cos\theta)$

$\frac{dy}{d\theta} = 3(\cos\theta - \theta.-\sin\theta - \cos\theta)$

$= 3(\cos\theta + \theta\sin\theta - \cos\theta)$

$= 3\theta\sin\theta$

$\left(\frac{dx}{d\theta}\right)^2 = (3\theta\cos\theta)^2$ ✓

$= 9\theta^2\cos^2\theta$

$\left(\frac{dy}{d\theta}\right)^2 = (3\theta\sin\theta)^2$ ✓

$= 9\theta^2\sin^2\theta$

$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 9\theta^2\cos^2\theta + 9\theta^2\sin^2\theta$ ✓

$= 9\theta^2$ ✓

$\therefore S = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$ ✓ ✓

$= \int_0^{\pi/2} \sqrt{9\theta^2}\, d\theta$ ✓

$= 3\int_0^{\pi/2} \theta\, d\theta$ ✓

$= 3\left[\frac{\theta^2}{2}\right]_0^{\pi/2}$ ✓

$= \frac{3}{2}\left[\left(\frac{\pi}{2}\right)^2 - 0\right]$

$= \frac{3\pi^2}{8}$ units

or $= 3,701$ units ✓

(12)

6.2.

$\frac{x^2}{36} + \frac{y^2}{4} = 1$

$\frac{2x}{36} + \frac{2y}{4}\cdot\frac{dy}{dx} = 0$ ✓

$\therefore \frac{dy}{dx} = -\frac{2x}{36} \times \frac{4}{2y}$

$= -\frac{x}{9y}$ ✓

$\longrightarrow y^2 = 4\left(1 - \frac{x^2}{36}\right)$

$y^2 = 4\left(\frac{36 - x^2}{36}\right)$

$\therefore y^2 = \frac{36 - x^2}{9}$

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{-x}{9y}\right)^2$ ✓

$= 1 + \frac{x^2}{81y^2}$

$= 1 + \frac{x^2}{81\left(\frac{36 - x^2}{9}\right)}$ ✓

$= 1 + \frac{x^2}{9(36 - x^2)}$

$= \frac{9(36 - x^2) + x^2}{9(36 - x^2)}$

$= \frac{324 - 8x^2}{9(36 - x^2)}$ ✓

12

$$\therefore \quad A = 2\int_0^6 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad \int_{-6}^{6} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\int_0^6 2\pi \sqrt{\frac{36-x^2}{9}} \cdot \sqrt{\frac{324-8x^2}{9(36-x^2)}}\, dx$$

$$= 4\pi \int_0^6 \frac{1}{9} \sqrt{324-8x^2}\, dx$$

$$= \frac{4\pi}{9}\left[\frac{324}{2\sqrt{8}} \sin^{-1} \frac{\sqrt{8}x}{18} + \frac{x}{2}\sqrt{324-8x^2}\right]_0^6$$

$$= \frac{4\pi}{9}\left[\frac{162}{\sqrt{8}} \sin^{-1} \frac{\sqrt{8}(6)}{18} + \frac{6}{2}\sqrt{324-8(6)^2}\right]$$

$$\left.\begin{aligned} &= 39{,}335\,\pi \text{ units}^2 \\ \text{or } &= \underline{123{,}575 \text{ units}^2} \end{aligned}\right\}$$

(12)

[24]

TOTAL: 100

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higher education & training

Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T960**(E)**(M27)T
APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

27 March 2014 (Y-Paper)
13:00–16:00

Calculators may be used.

This question paper consists of 5 pages and 1 formula sheet of 7 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N6

TIME: 3 HOURS

MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If $z = \cos e^{xy}$, calculate the following:

1.1.1 $\dfrac{\partial z}{\partial x}$ (1)

1.1.2 $\dfrac{\partial z}{\partial y}$ (1)

1.2 If $x = \sin^3\theta$ and $y = \cos^3\theta$, calculate $\dfrac{d^2y}{dx^2}$. (4)

[6]

QUESTION 2

Determine $\int y\,dx$ if:

2.1 $y = \dfrac{1}{3x^2 - x + 1}$ (4)

2.2 $y = \sin^4 3x$ (4)

2.3 $y = e^{-2x}.\cos x$ (5)

2.4 $y = \operatorname{cosec}^3 2x \,.\, \cos^3 2x$ (5)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{5x - 2}{x^2(x^2 + 4)}\,dx$ (6)

3.2 $\displaystyle\int \frac{-18x^3 + 9x^2 + 15x + 12}{(2x+1)^2(1 - x)^2}\,dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$\frac{dy}{dx} - \frac{3y}{x} = x^4 . \sin x \,, \text{ if } x = 1 \text{ when } y = 2.$$ (6)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 5e^{3x}, \text{ if } x = 0 \text{ when } y = 3 \text{ and } x = 0 \text{ when } \frac{dy}{dx} = 2.$$ (6)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection in the first quadrant of the curve $y = -x^3 + 4x$ and the straight line $4y - 7x = 0$. Make a neat sketch of the graphs and clearly indicate the area bounded by the graphs. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graphs rotates about the *x*-axis. (3)

5.1.2 Calculate the magnitude of the volume described in QUESTION 5.1.1 by means of integration. (4)

5.2 5.2.1 Make a neat sketch of the graph $y = 2\cos x$ and show the representative strip/element (PARALLEL to the *y*-axis) that you will use to calculate the volume generated when the area bounded by the graph, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$ is rotated about the *x*-axis. (2)

5.2.2 Calculate the volume described in QUESTION 5.2.1. (3)

5.2.3 Calculate the distance of the centre of gravity from the *y*-axis when the area described in QUESTION 5.2.1 is rotated about the *x*-axis. (5)

5.3 5.3.1 Calculate the points of intersection of $y = \frac{1}{2}\sin x$ and $y = \frac{1}{2}\cos x$. Make a neat sketch of the graphs and show the representative strip/element that you will use to calculate the area bounded by the graphs and the *y*-axis. (3)

5.3.2 Calculate the area described in QUESTION 5.3.1. (4)

5.3.3 Calculate the *y*-ordinate of the centroid of the area described in QUESTION 5.3.1. (5)

5.4 5.4.1 A water canal in the shape of an isosceles triangle with its vertex pointing downwards contains water 4 m deep. The top of a sluice gate is in the water level and 4 m wide.

Sketch the canal and show the representative strip/element that you will use to calculate the area moment of the sluice gate about the water level.

Calculate the relation between the two variables x and y. (3)

5.4.2 Calculate the area moment of the sluice gate about the water level by means of integration. (4)

5.4.3 Calculate the second moment of area of the sluice gate about the water level, as well as the depth of the centre of pressure on the sluice gate by means of integration. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $2y^2 = x^3$ between $x = 0$ and $x = 2$. (6)

6.2 Calculate the surface area generated by rotating the curve represented by the parametric equations, $x = \frac{t^2}{2} + t$ and $y = t + 1$, about the x-axis for $0 \le t \le 3$. (6)

[12]

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x}\ ;\ \sin x = \frac{1}{\operatorname{cosec} x}\ ;\ \cos x = \frac{1}{\sec x}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
ax^n	$a \frac{d}{dx} x^n$	$a \int x^n \, dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e} . \ln a . \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx} ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} \frac{d}{dx} f(x)$	-
$a^{f(x)}$	$a^{f(x)} . \ln a . \frac{d}{dx} f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	-
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a} \ln [\sec (ax)] + C$
$\cot ax$	$-a \operatorname{cosec}^2 ax$	$\frac{1}{a} \ln [\sin (ax)] + C$
$\sec ax$	$a \sec ax \tan ax$	$\frac{1}{a} \ln [\sec ax + \tan ax] + C$
$\operatorname{cosec} ax$	$-a \operatorname{cosec} ax \cot ax$	$\frac{1}{a} \ln \left[\tan \left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\text{cosec} f(x)$	$-\text{cosec} f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1 - [f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2 + 1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2 + 1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2 - 1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2 - 1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a} \cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\ f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\ dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx\ ;\ V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx\ ;\ V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy\ ;\ V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy\ ;\ V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad\qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A}\ ;\ \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \qquad ; \qquad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \qquad ; \qquad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \mathrm{r}\,V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho\int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2}mr^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho\int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi\int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi\int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_x = \int_d^c 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_y = \int_a^b 2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_y = \int_d^c 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\ du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\ dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

APRIL EXAMINATION

MATHEMATICS N6

27 MARCH 2014

This marking guideline consists of 14 pages.

QUESTION 1

1.1 $z = \cos e^{xy}$

1.1.1 $\frac{dz}{dx} = ye^{xy}\left(-\sin e^{xy}\right)$ ✓ ✓

$= -ye^{xy}\sin e^{xy}$ (2)

1.1.2 $\frac{dz}{dy} = -xe^{xy}\sin e^{xy}$ ✓ ✓ (2)

1.2 $x = \sin^3\theta$ $\qquad y = \cos^3\theta$

$\frac{dx}{d\theta} = 3\sin^2\theta\cos\theta$ ✓ ✓ $\qquad \frac{dy}{d\theta} = 3\cos^2\theta(-\sin\theta)$ ✓ ✓

$= -3\cos^2\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$= \frac{-3\cos^2\theta\sin\theta}{3\sin^2\theta\cos\theta}$ ✓

$= -\frac{\cos\theta}{\sin\theta}$

$= -\cot\theta$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \times \frac{d\theta}{dx}$

$= \frac{d}{d\theta}(-\cot\theta)$ ✓ $\times \frac{1}{3\sin^2\theta\cos\theta}$ ✓

$= \frac{\operatorname{cosec}^2\theta}{3\sin^2\theta\cos\theta}$ ✓

$= \frac{1}{3\sin^2\theta\cos\theta\sin^2\theta}$

OR

$\frac{d}{d\theta}\left(\frac{-\cos\theta}{\sin\theta}\right)$ ✓ $\times \frac{1}{3\sin^2\theta\cos\theta}$ ✓

$= \frac{\sin\theta(\sin\theta)-(-\cos\theta)(\cos\theta)}{\sin^2\theta}$ ✓ $\times \frac{1}{3\sin^2\theta\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta} \times \frac{1}{3\sin^2\theta\cos\theta}$

$= \frac{1}{3\sin^4\theta\cos\theta}$

(8)

[12]

QUESTION 2

2.1 $\int \frac{1}{3x^2 - x + 1}\,dx$

$3x^2 - x + 1$

$= 3\left[x^2 - \frac{x}{3} + \frac{1}{3}\right]$

$= 3\left[(x - \frac{1}{6})^2 + \frac{1}{3} - \frac{1}{36}\right]$

$= 3\left[(x - \frac{1}{6})^2 + \frac{11}{36}\right]$

$= 3\left[(x - \frac{1}{6})^2 + 0{,}306\right]$

$= 3\left[0{,}306 + (x - \frac{1}{6})^2\right]$

$= \int \frac{1}{3\left[(x - \frac{1}{6})^2 + 0{,}306\right]}\,dx$

$= \frac{1}{3} \cdot \frac{1}{\sqrt{0{,}306}} \tan^{-1} \frac{(x - \frac{1}{6})}{\sqrt{0{,}306}}$

$= 0{,}603 \tan^{-1} \frac{(x - \frac{1}{6})}{0{,}553}$

(8)

or $\int \frac{1}{3\left[x^2 - \frac{x}{3} + (\frac{1}{6})^2 - (\frac{1}{6})^2 + \frac{1}{3}\right]}\,dx$

$= \int \frac{1}{3\left[(x - \frac{1}{6})^2 + \frac{11}{36}\right]}\,dx$

$= \int \frac{1}{\frac{11}{12} + 3(x - \frac{1}{6})^2}\,dx$

$= \frac{1}{\sqrt{\frac{11}{12}} \cdot \sqrt{3}} \tan^{-1} \frac{\sqrt{3}(x - \frac{1}{6})}{\sqrt{\frac{11}{12}}} + c$

$= 0{,}603 \tan^{-1} 1{,}809(x - \frac{1}{6}) + c$

2.2 $\int \sin^4 3x\,dx$

$= \int (\sin^2 3x)^2\,dx$

$= \int (\frac{1}{2} - \frac{1}{2}\cos 6x)^2\,dx$

$= \int (\frac{1}{4} - \frac{1}{2}\cos 6x + \frac{1}{4}\cos^2 6x)\,dx$

$= \frac{1}{4}x - \frac{1}{2} \cdot \frac{\sin 6x}{6} + \frac{1}{4}\left[\frac{x}{2} + \frac{\sin 12x}{24}\right] + c$

$= \frac{1}{4}x - \frac{1}{12}\sin 6x + \frac{x}{8} + \frac{1}{96}\sin 12x + c$

Or $\frac{3}{8}x - \frac{1}{12}\sin 6x + \frac{1}{96}\sin 12x + c$

(8)

2.3. $\int e^{-2x} \cos x \, dx$

$f(x) = e^{-2x}$ $\quad g'(x) = \cos x$
$f'(x) = -2e^{-2x}$ $\quad g(x) = \sin x$

$= e^{-2x} \cdot \sin x - \int -2e^{-2x} \cdot \sin x \, dx$ ✓✓

$= e^{-2x} \cdot \sin x + 2\int e^{-2x} \sin x \, dx$ ✓

$f(x) = e^{-2x}$ $\quad g'(x) = \sin x$
$f'(x) = -2e^{-2x}$ $\quad g(x) = -\cos x$

$= e^{-2x} \cdot \sin x + 2\left[e^{-2x}(-\cos x) - \int e^{-2x}(-\cos x)\, dx\right]$ ✓✓

$= e^{-2x} \sin x - 2e^{-2x} \cos x + 2\int e^{-2x} \cos x \, dx$ ✓✓

$\therefore I = e^{-2x} \sin x - 2e^{-2x} \cos x + 2I$ ✓

$3I = e^{-2x} \sin x - 2e^{-2x} \cos x$ ✓ (10)

$\therefore I = \frac{1}{3}\left(e^{-2x} \sin x - 2e^{-2x} \cos x\right)$ ✓

OR $\int e^{-2x} \cos x \, dx$

$f(x) = \cos x$ $\quad g'(x) = e^{-2x}$
$f'(x) = -\sin x$ $\quad g(x) = \frac{e^{-2x}}{-2}$

$= \cos x \cdot \frac{e^{-2x}}{-2} - \int -\sin x \cdot \frac{e^{-2x}}{-2} \, dx$ ✓✓

$= -\frac{1}{2} \cos x \cdot e^{-2x} - \frac{1}{2}\int \sin x \cdot e^{-2x} \, dx$ ✓

$f(x) = \sin x$ $\quad g'(x) = e^{-2x}$
$f'(x) = \cos x$ $\quad g(x) = \frac{e^{-2x}}{2}$

$= -\frac{1}{2} \cos x \cdot e^{-2x} - \frac{1}{2}\left[\sin x \cdot \frac{e^{-2x}}{2} - \int \cos x \cdot \frac{e^{-2x}}{2} \, dx\right]$ ✓✓

$= -\frac{1}{2} \cos x \cdot e^{-2x} - \frac{1}{4} \sin x \cdot e^{-2x} + \frac{1}{4}\int \cos x \cdot e^{-2x} \, dx$ ✓✓

$\therefore I = -\frac{1}{2} \cos x \, e^{-2x} - \frac{1}{4} \sin x \cdot e^{-2x} + \frac{1}{4} I$ ✓

$\frac{3}{4} I = -\frac{1}{2} \cos x \, e^{-2x} - \frac{1}{4} \sin x \, e^{-2x}$ ✓

$I = \frac{4}{3}\left(-\frac{1}{2} \cos x \, e^{-2x} - \frac{1}{4} \sin x \cdot e^{-2x}\right)$ ✓

2.4 $\int \text{cosec}^3 2x \cdot \cos^3 2x \, dx$ 4

$= \int \text{cosec}^3 2x \cdot \cos^2 2x \cdot \cos 2x \, dx$ ✓

$= \int \text{cosec}^3 2x \, (1 - \sin^2 2x) \cos 2x \, dx$ ✓

$= \frac{1}{2} \int \frac{1}{u^3} (1 - u^2) \, du$ ✓✓✓

$u = \sin 2x$

$du = 2\cos 2x \, dx$

$= \frac{1}{2} \int (u^{-3} - u^{-1}) \, du$ ✓

$= \frac{1}{2} \left[\frac{u^{-2}}{-2} - \ln u \right] + c$ ✓✓

$= \frac{1}{2} \left[\frac{1}{-2\sin^2 2x} - \ln(\sin 2x) \right] + c$ ✓✓

$= -\frac{1}{4\sin^2 2x} - \frac{1}{2} \ln(\sin 2x) + c$

(10)

[36]

QUESTION 3

3.1 $\displaystyle\int \frac{5x-2}{x^2(x^2+4)}\,dx$

$\therefore \dfrac{5x-2}{x^2(x^2+4)} = \dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{Cx+D}{(x^2+4)}$ ✓

$\therefore 5x-2 = A(x^2+4) + Bx(x^2+4) + (Cx+D)x^2$ ✓

Let $x=0$; $\therefore \underline{A = -\frac{1}{2}}$ ✓ $(-0{,}5)$

$5x-2 = Ax^2 + 4A + Bx^3 + 4Bx + Cx^3 + Dx^2$

Equate coeff of x^3: $0 = B + C$ $\therefore \underline{C = -\frac{5}{4}}$ ✓ $(-1{,}25)$

" " " x^2: $0 = A + D$ $\therefore \underline{D = \frac{1}{2}}$ ✓ $(0{,}5)$

" " " x: $5 = 4B$ $\therefore \underline{B = \frac{5}{4}}$ ✓ $(1{,}25)$

$\therefore \displaystyle\int \frac{-\frac{1}{2}}{x^2}\,dx + \int \frac{\frac{5}{4}}{x}\,dx + \int \frac{-\frac{5}{4}x + \frac{1}{2}}{(x^2+4)}\,dx$ ✓

$= -\frac{1}{2}\displaystyle\int x^{-2}\,dx + \frac{5}{4}\int \frac{1}{x}\,dx + \int \frac{-\frac{5}{4}x}{x^2+4}\,dx + \frac{1}{2}\int \frac{1}{x^2+4}\,dx$

$= -\frac{1}{2}\cdot\frac{x^{-1}}{-1}$ ✓ $+ \frac{5}{4}\ln x$ ✓ $- \frac{5}{4}(\frac{1}{2})\ln(x^2+4)$ ✓ $+ \frac{1}{2}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2}$ ✓ $+ C$

$= \underline{\dfrac{1}{2x} + \dfrac{5}{4}\ln x - \dfrac{5}{8}\ln(x^2+4) + \dfrac{1}{4}\tan^{-1}\dfrac{x}{2} + c}$ (12)

3.2 $\displaystyle\int \frac{-18x^3+9x^2+15x+12}{(2x+1)^2(1-x)^2}\,dx$

$\therefore \dfrac{-18x^3+9x^2+15x+12}{(2x+1)^2(1-x)^2} = \dfrac{A}{(2x+1)^2} + \dfrac{B}{(2x+1)} + \dfrac{C}{(1-x)^2} + \dfrac{D}{(1-x)}$ ✓

$\therefore -18x^3+9x^2+15x+12 = A(1-x)^2 + B(2x+1)(1-x)^2 + C(2x+1)^2 + D(1-x)(2x+1)^2$ ✓

Let $x = -\frac{1}{2}$; $\underline{A = 4}$ ✓

$x = 1$; $\underline{C = 2}$ ✓

$\therefore -18x^3+9x^2+15x+12 = A - 2Ax + Ax^2 + 2Bx - 4Bx^2 + 2Bx^3 + B - 2Bx + Bx^2 + 4Cx^2 + 4Cx + C + 4Dx^2 + 4Dx + D - 4Dx^3 - 4Dx^2 - Dx$ ✓

Equate coeff of x^3: $-18 = 2B - 4D$

" " " x^2: $9 = A - 4B + B + 4C + 4D - 4D$ $\therefore \underline{B = 1}$ ✓ and $\underline{D = 5}$ ✓

$\therefore \displaystyle\int \frac{4}{(2x+1)^2}\,dx + \int \frac{1}{(2x+1)}\,dx + \int \frac{2}{(1-x)^2}\,dx + \int \frac{5}{(1-x)}\,dx$ ✓

$= 4\displaystyle\int (2x+1)^{-2}\,dx + \int \frac{1}{(2x+1)}\,dx + 2\int (1-x)^{-2}\,dx + 5\int \frac{1}{(1-x)}\,dx$

$= \frac{4}{2}\cdot\frac{(2x+1)^{-1}}{-1}$ ✓ $+ \frac{1}{2}\ln(2x+1)$ ✓ $+ 2(-1)\cdot\frac{(1-x)^{-1}}{-1}$ ✓ $+ 5(-1)\ln(1-x)$ ✓ $+ c$

$= \underline{-\dfrac{2}{2x+1} + \dfrac{1}{2}\ln(2x+1) + \dfrac{2}{1-x} - 5\ln(1-x) + c}$

(12)

[24]

QUESTION 4

4.1. $\frac{dy}{dx} - \frac{3y}{x} = x^4 . \sin x$

$R = e^{\int -\frac{3}{x} dx}$ ✓

$= e^{-3\ln x}$ ✓

$= e^{\ln x^{-3}}$

$= x^{-3}$ ✓

$= \frac{1}{x^3}$

$\therefore Ry = \int RQ \, dx + c$

$\frac{1}{x^3} y$ ✓ $= \int \frac{1}{x^3} . x^4 \sin x \, dx + c$ ✓

$\frac{1}{x^3} y = \int x . \sin x \, dx + c$ ✓

$f(x) = x \quad g'(x) = \sin x$

$f'(x) = 1 \quad g(x) = -\cos x$

$\frac{1}{x^3} . y = x(-\cos x) - \int -\cos x \, dx + c$ ✓

$\frac{1}{x^3} y = -x \cos x + \int \cos x \, dx + c$

$\frac{1}{x^3} y = -x \cos x + \sin x + c$ ✓

$\frac{1}{(1)^3} . 2 = -(1)\cos(1) + \sin(1) + c \quad \ldots (1;2)$

$2 = -\cos 1 + \sin 1 + c$

$\therefore c = 1,699$ ✓

$\therefore \frac{1}{x^3} . y = -x\cos x + \sin x + 1,699$ ✓ (10)

4.2. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 5e^{3x}$

$\therefore m^2 - 5m + 6 = 0$ ✓

$(m-3)(m-2) = 0$

$\therefore m = 3 \; ; \; m = 2$ ✓

$y_c = Ae^{3x} + Be^{2x}$ ✓

$y_p : y = Cxe^{3x}$ ✓

$\frac{dy}{dx} = Cx.3e^{3x} + Ce^{3x}$ ✓

$\frac{d^2y}{dx^2} = Cx.9e^{3x} + C.3e^{3x} + 3Ce^{3x}$ ✓

$= 9Cxe^{3x} + 6Ce^{3x}$

$\therefore 9Cxe^{3x} + 6Ce^{3x} - 5(3Cxe^{3x} + Ce^{3x}) + 6Cxe^{3x} = 5e^{3x}$ ✓

$9Cxe^{3x} + 6Ce^{3x} - 15Cxe^{3x} - 5Ce^{3x} + 6Cxe^{3x} = 5e^{3x}$

$C = 5$ ✓

$\therefore y_p = 5xe^{3x}$

$\therefore y = Ae^{3x} + Be^{2x} + 5xe^{3x}$

$3 = A + B \qquad \therefore A = 3 - B$

$\therefore \frac{dy}{dx} = 3Ae^{3x} + 2Be^{2x} + 5x \cdot 3e^{3x} + 5e^{3x}$

$2 = 3A + 2B + 5$

$-3 = 3A + 2B \qquad \therefore -3 = 3(3 - B) + 2B$

$\underline{y} = 12$

$\therefore \quad 9$

$\therefore \underline{y = -9e^{3x} + 12e^{2x}} \qquad 3x$

$\rightarrow$

(14)

[24]

QUESTION 5

5.1 5.1.1 $y = -x^3 + 4x$ $\qquad 4y - 7x = 0$

$\therefore -x^3 + 4x = \frac{7x}{4}$

$-4x^3 + 16x - 7x = 0$

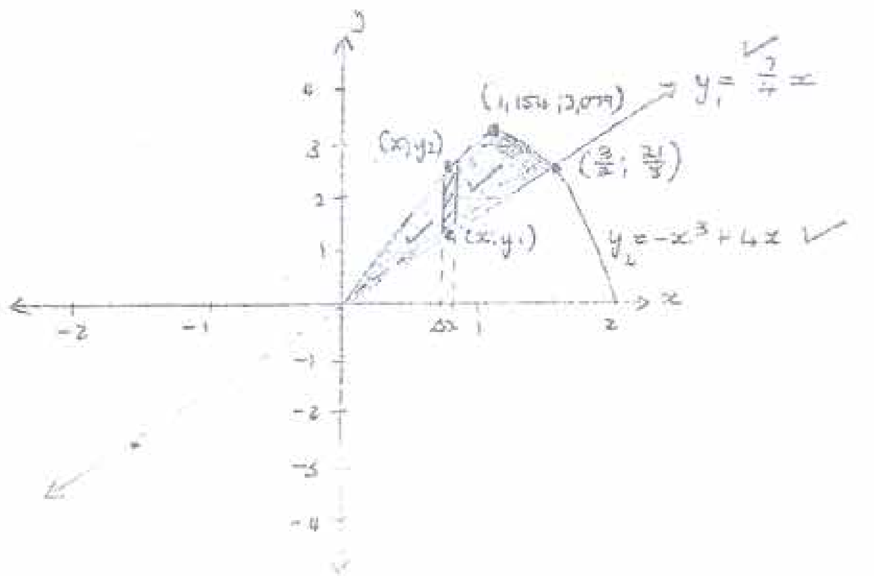
$-4x^3 + 9x = 0$

$\therefore 0 = x(4x^2 - 9)$

$x = 0 \quad ; \quad x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$

$y = 0 \quad ; \quad y = \pm\frac{21}{8}$

$\therefore (0;0)\ (\frac{3}{2}; \frac{21}{8}) ; (-\frac{3}{2}; -\frac{21}{8})$



(6)

5.1.2 $\Delta V_x = \pi(y_2^2 - y_1^2)\Delta x$

$V_x = \pi\int_0^{3/2} (-x^3+4x)^2 - (\frac{7}{4}x)^2\,dx$

$= \pi\int_0^{3/2} (16x^2 - 8x^4 + x^6 - \frac{49}{16}x^2)\,dx$

$= \pi\left[\frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7} - \frac{49x^3}{48}\right]_0^{3/2}$

$= \pi\left[\frac{16}{3}(\frac{3}{2})^3 - \frac{8}{5}(\frac{3}{2})^5 + \frac{1}{7}(\frac{3}{2})^7 - \frac{49}{48}(\frac{3}{2})^3\right]$

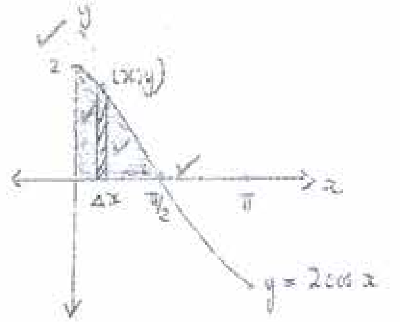
$= 4,846\,\pi$ units^3

or $= 15,223$ units^3

(8)

9

5.2. 5.2.1.



(4)

5.2.2. $\Delta V_x = \pi y^2 \Delta x$

$\therefore V_x = \pi \int_0^{\pi/2} (2\cos x)^2 \, dx$

$= 4\pi \int_0^{\pi/2} \cos^2 x \, dx$

$= 4\pi \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\pi/2}$

$= 4\pi \left[\frac{\pi/2}{2} + \frac{1}{4} \sin 2\left(\frac{\pi}{2}\right) \right]$

$= \underline{\pi^2 \text{ or } 9{,}87 \text{ unit}^3}$

(6)

5.2.3. $\Delta M_y = \pi y^2 \Delta x \times x$

$M_y = \pi \int_0^{\pi/2} y^2 x \, dx$

$= \pi \int_0^{\pi/2} (2\cos x)^2 x \, dx$

$= 4\pi \int_0^{\pi/2} x (\cos^2 x) \, dx$

$f(x) = x$	$g'(x) = \cos^2$
$f'(x) = 1$	$g(x) = \frac{x}{2} + \frac{\sin 2x}{4}$

$= 4\pi \left[x\left(\frac{x}{2} + \frac{\sin 2x}{4}\right) - \int \left(\frac{x}{2} + \frac{\sin 2x}{4}\right) dx \right]_0^{\pi/2}$

$= 4\pi \left[\frac{x^2}{2} + \frac{x \sin 2x}{4} - \frac{x^2}{4} + \frac{\cos 2x}{8} \right]_0^{\pi/2}$

$= 4\pi \left[\frac{\left(\frac{\pi}{2}\right)^2}{2} + \frac{\frac{\pi}{2} \sin 2\left(\frac{\pi}{2}\right)}{4} - \frac{1}{4}\left(\frac{\pi}{2}\right)^2 + \frac{1}{8} \cos 2\left(\frac{\pi}{2}\right) - \frac{\cos 2(0)}{8} \right]$

$= 4\pi \left[\frac{\pi^2}{8} + \frac{\pi}{8}(0) - \frac{\pi^2}{16} - \frac{1}{8} - \frac{1}{8} \right]$

$= \underline{1{,}467\pi \text{ or } 4{,}61 \text{ unit}^4}$

$\therefore \bar{x} = \frac{4{,}61}{9{,}87} \quad \left(\frac{M_y}{V_x}\right)$

$= \underline{0{,}467 \text{ unit}}$

(10)

5.3 5.3.1

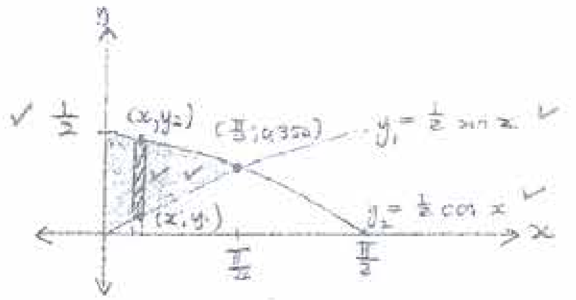
$\frac{1}{2}\sin x = \frac{1}{2}\cos x$

$\therefore \frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \tan^{-1} 1$

$= \frac{\pi}{4}$ or $0{,}785 \Rightarrow (\frac{\pi}{4}; 0{,}354)$ ✓



(6)

5.3.2

$\Delta A = (y_2 - y_1)\, dx$ ✓

$\therefore A = \int_0^{\pi/4} (y_2 - y_1)\, dx$ ✓

$= \int_0^{\pi/4} (\frac{1}{2}\cos x - \frac{1}{2}\sin x)\, dx$ ✓

$= \frac{1}{2}\int_0^{\pi/4} (\cos x - \sin x)\, dx$

$= \frac{1}{2}\left[\sin x + \cos x\right]_0^{\pi/4}$ ✓ ✓

$= \frac{1}{2}\left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - \sin 0 - \cos 0\right]$ ✓

$= 0{,}207$ units^2 ✓

(8)

5.3.3

$\Delta M_x = (y_2 - y_1)\, dx \times \left(\frac{y_2 + y_1}{2}\right)$ ✓ ✓

$\therefore M_x = \frac{1}{2}\int_0^{\pi/4} (y_2^2 - y_1^2)\, dx$ ✓

$= \frac{1}{2}\int_0^{\pi/4} \left[(\frac{1}{2}\cos x)^2 - (\frac{1}{2}\sin x)^2\right] dx$ ✓

$= \frac{1}{2}\int_0^{\pi/4} \left[\frac{1}{4}\cos^2 x - \frac{1}{4}\sin^2 x\right] dx$

$= \frac{1}{8}\int_0^{\pi/4} \left[\cos^2 x - \sin^2 x\right] dx$

$= \frac{1}{8}\int_0^{\pi/4} \cos 2x\, dx$ ✓

$= \frac{1}{8}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$ ✓

$= \frac{1}{16}\left[\sin 2(\frac{\pi}{4})\right]$ ✓

$= \frac{1}{16}$ units^3 or $0{,}0625$ units^3 ✓

$\therefore \bar{y} = \frac{0{,}0625}{0{,}207}$ ✓

$= 0{,}302$ units ✓

(10)

$-x$

$c \quad \ldots (2;4) \quad ; c=0$

$\Rightarrow x = \frac{1}{2}y$

$2x \, \Delta y$

$(\frac{1}{2}y)\, dy$

$y\, dy$

(6)

$(4-y)\, y\, dy$

$(4y - y^2)\, dy$

$\frac{4y^2}{2} - \frac{y^3}{3}\Big]_0^4$

$(4)^2 - \frac{1}{3}(4)^3\Big]$

667 units^3

(8)

$4\,(4-y)^2\, y\, dy$

$\int_0^4 (16y - 8y^2 + y^3)\, dy$

$\frac{16y^2}{2} - \frac{8y^3}{3} + \frac{y^4}{4}\Big]_0^4$

$\Big[8(4)^2 - \frac{8}{3}(4)^3 + \frac{1}{4}(4)^4\Big]$

$21{,}333$ units^4

$y = 2x$

$\Delta A = 2x$

$\therefore dA = 2(\frac{1}{2}$

$= (y +$

5.4.2. $\int_{-4}^{0} y\,($

$= \int_{-4}^{0} (y^2$

$= \Big[\frac{y^3}{3}$

$= \Big[-\frac{1}{3}(-4$

$= -10{,}66$

5.4.3. $\int_{-4}^{0} y$

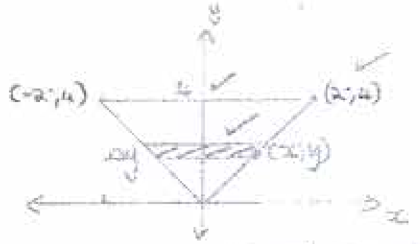
$= \int_{-4}^{0} (y$

$= \Big[\frac{y^4}{4}$

$= \Big[-\frac{1}{4}(-4)$

$= 21{,}333$

5.4 5.4.1



$y = mx + c \quad \ldots (2;4) \; ; c = 0$

$4 = m(2)$

$\therefore y = 2x \Rightarrow x = \frac{1}{2}y$

$\Delta A = 2x\,\Delta y$

$\therefore dA = 2\left(\frac{1}{2}y\right) dy$

$= y\,dy$ (6)

5.4.2 $\int_0^4 (4-y)\,y\,dy$

$= \int_0^4 (4y - y^2)\,dy$

$= \left[\frac{4y^2}{2} - \frac{y^3}{3}\right]_0^4$

$= \left[2(4)^2 - \frac{1}{3}(4)^3\right]$

$= 10{,}667 \text{ units}^3$ (8)

5.4.3 $\int_0^4 (4-y)^2\,y\,dy$

$= \int_0^4 (16y - 8y^2 + y^3)\,dy$

$= \left[\frac{16y^2}{2} - \frac{8y^3}{3} + \frac{y^4}{4}\right]_0^4$

$= \left[8(4)^2 - \frac{8}{3}(4)^3 + \frac{1}{4}(4)^4\right]$

$= 21{,}333 \text{ units}^4$

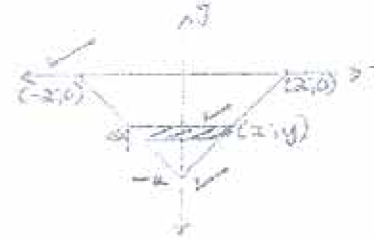
$\therefore \bar{\bar{y}} = \frac{21{,}333\,u^4}{10{,}667\,u^3}$

$= 1{,}999$ } or

≈ 2 units (8)

OR

5.4.1



$y = 2x - 4 \Rightarrow x = \frac{1}{2}y + 2$

$\Delta A = 2x\,\Delta y$

$\therefore dA = 2\left(\frac{1}{2}y + 2\right) dy$

$= (y + 4)\,dy$

5.4.2 $\int_{-4}^0 y(y+4)\,dy$

$= \int_{-4}^0 (y^2 + 4y)\,dy$

$= \left[\frac{y^3}{3} + \frac{4y^2}{2}\right]_{-4}^0$

$= \left[-\frac{1}{3}(-4)^3 - 2(-4)^2\right]$

$= -10{,}667 \text{ units}^3$

5.4.3 $\int_{-4}^0 y^2(y+4)\,dy$

$= \int_{-4}^0 (y^3 + 4y^2)\,dy$

$= \left[\frac{y^4}{4} + \frac{4y^3}{3}\right]_{-4}^0$

$= \left[-\frac{1}{4}(-4)^4 - \frac{4}{3}(-4)^3\right]$

$= 21{,}333 \text{ units}^4$

$\therefore \bar{\bar{y}} = \frac{21{,}333\,u^4}{-10{,}667\,u^3}$

$= -1{,}999$ units }

≈ 2 units

[80]

QUESTION 6

6.1 $2y^2 = x^3$

$y^2 = \frac{1}{2}x^3$ ✓

$y = \sqrt{\frac{1}{2}x^3}$

$\therefore y = \frac{1}{\sqrt{2}} x^{3/2}$ ✓

$\frac{dy}{dx} = \frac{3}{2} \cdot \frac{1}{\sqrt{2}} x^{\frac{1}{2}}$ ✓

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{3}{2} \cdot \frac{1}{\sqrt{2}} x^{\frac{1}{2}}\right)^2$ ✓

$= 1 + \frac{9}{8} x$ ✓

$= \frac{8 + 9x}{8}$

$\therefore S = \int_0^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ ✓ ✓

$= \int_0^2 \sqrt{\frac{8+9x}{8}}\, dx$ ✓

$= \int_0^2 \frac{1}{\sqrt{8}} (8 + 9x)^{\frac{1}{2}} dx$

$= \frac{1}{\sqrt{8}} \cdot \frac{1}{9} \int_0^2 (8 + 9x)^{\frac{1}{2}} 9\, dx$ ✓

$= \frac{1}{9\sqrt{8}} \left[\frac{(8+9x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2$ ✓

$= \frac{1}{9\sqrt{8}} \times \frac{2}{3} \left[(8 + 9(2))^{3/2} - (8)^{\frac{3}{2}}\right]$ ✓

$= 2,879$ units ✓

(12)

6.2. $x = \frac{t^2}{2} + t$ $\qquad y = t + 1$

$\frac{dx}{dt} = t + 1$ ✓ $\qquad \frac{dy}{dt} = 1$ ✓

$\left(\frac{dx}{dt}\right)^2 = (t+1)^2$ ✓ $\qquad \left(\frac{dy}{dt}\right)^2 = 1$ ✓

$= t^2 + 2t + 1$

$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^2 + 2t + 1 + 1$ ✓

$= t^2 + 2t + 2$

$\therefore A = 2\pi \int_0^3 y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ ✓ ✓

$= 2\pi \int_0^3 (t+1) \sqrt{t^2 + 2t + 2}\, dt$ ✓

$= \frac{2\pi}{2} \int_0^3 2(t+1)(t^2 + 2t + 2)^{1/2} dt$

$= \pi \left[\frac{(t^2+2t+2)^{3/2}}{3/2}\right]_0^3$ ✓ ✓

$= \frac{2\pi}{3} \left\{\left[(3)^2 + 2(3) + 2\right]^{3/2} - \left[2\right]^{3/2}\right\}$ ✓

$\left.\begin{array}{l} = 44,843\pi \text{ units}^2 \\ \text{or} = 140,878 \text{ units}^2 \end{array}\right\}$ ✓

(12)

TOTAL: 100

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1060**(E)**(N13)T

NOVEMBER EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

13 November 2013 (X-Paper)
09:00 – 12:00

Calculators may be used.

This question paper consists of 5 pages and 7 formula sheets.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 If z = $\cos 2xy.\sin 3x$, calculate the value of:

1.1.1 $\frac{\partial z}{\partial x}$ (1)

1.1.2 $\frac{\partial z}{\partial y}$ (1)

1.1.3 $\frac{\partial^2 z}{\partial y^2}$ (1)

1.2 The volume of a cylinder is given as $V = \pi r^2 h$. Calculate the change in the volume if the radius increases by 3% while the height decreases by 2%.

HINT: $\Delta V = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$ (3)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = -x^2 . \cot^{-1} x$ (4)

2.2 $y = \frac{\cos^3 x}{\operatorname{cosec}^7 x}$ (4)

2.3 $y = \frac{1}{18 + 9x + 3x^2}$ (4)

2.4 $y = \sin^5 4x$ (4)

2.5 $y = x \ln 3x$ (2)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{6x^3 + 8x^2 + 2x - 3}{(x^2 + 1)x^2}\, dx$ (6)

3.2 $\int \frac{3x^2 - 5x + 4}{(x-2)^2 (x-1)^2}\, dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$\left(\frac{1}{\sin x}\right)\frac{dy}{dx} + y = 1 \text{ if } y = 3 \text{ when } x = 0$$ (5)

4.2 Calculate the particular solution of:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = e^{2t}, \text{ if } y = -1 \text{ when } t = 0 \text{ and } \frac{dy}{dt} = 1 \text{ when } t = 0$$ (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = \frac{3}{x}$ and $y + x - 4 = 0$. Make a neat sketch of the two curves and show the area bounded by the curves in the first quadrant. Show the representative strip/element that you will use to calculate the volume (use the SHELL METHOD only) generated if the area bounded by the curves rotates about the *x*-axis. (3)

5.1.2 Use the SHELL METHOD to calculate the volume described in QUESTION 5.1.1 by means of integration. (4)

5.2 5.2.1 Calculate the points of intersection of $x^2 = \frac{1}{2}y$ and $y = x + 3$. Sketch the graphs and show the representative strip/element that you will use to calculate the area bounded by the graphs. (3)

5.2.2 Calculate the magnitude of the area described in QUESTION 5.2.1. (3)

5.2.3 Calculate the area moment of the bounded area, described in QUESTION 5.2.1, about the *x*-axis as well as the distance of the centroid from the *x*-axis. (6)

5.3 5.3.1 Make a neat sketch of the curve $y = \sqrt{25 - x^2}$ and show the representative strip/element (PERPENDICULAR to the *y*-axis) that you will use to calculate the volume generated when the area bounded by the curve and the x-axis rotates about the *y*-axis. (2)

5.3.2 Calculate the volume described in QUESTION 5.3.1. (3)

5.3.3 Calculate the distance of the centre of gravity from the *x*-axis of the solid generated when the area, described in QUESTION 5.3.1, is rotated about the *y*-axis. (5)

5.4 A water canal in the form of a parabola is 4 m deep, 10 m wide at the top and full of water. The top of a vertical retaining wall is 1 m below the surface of the water.

5.4.1 Sketch the water canal and show the representative strip that you will use to calculate the depth of the centre of pressure on the retaining wall.

Calculate the relation between the two variables x and y. (4)

5.4.2 Calculate the area moment of the retaining wall about the water level by means of integration. (3)

5.4.3 Calculate the second moment of area of the retaining wall about the water level, as well as the depth of the centre of pressure on the retaining wall by means of integration. (4)

[40]

QUESTION 6

6.1 Given: $f(x) = x^2$.

Calculate the length of the parabolic arc if $x \in [0 ; 1]$ (6)

6.2 Calculate the surface area generated when the arc of the curve given by the parametric equations $y = 4t^2$ and $x = 4t$, between t = 1 and t = 2, rotates about the y-axis. (6)

[12]

TOTAL: 100

FORMULA SHEETS

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \mathrm{cosec}^2\, x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$sin\ (\mathrm{A} \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\mathrm{cosec}\ x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x) \, . \, f'(x)$	-
$\cos f(x)$	$-\sin f(x) \, . \, f'(x)$	-
$\tan f(x)$	$\sec^2 f(x) \, . \, f'(x)$	-
$\cot f(x)$	$-\text{cosec}^2 f(x) \, . \, f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x) \, . \, f'(x)$	-
$\text{cosec}\, f(x)$	$-\text{cosec}\, f(x) \cot f(x) \, . \, f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\text{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2} - \frac{\sin(2ax)}{4a} + C$
$\cos^2(ax)$	-	$\frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax) - x + C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2dx\,;\, V_x = p\int_a^b \left(y_1^2 - y_2^2\right)dx\,;\, V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2dy\,;\, V_y = p\int_a^b \left(x_1^2 - x_2^2\right)dy\,;\, V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A}\,;\, \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENTS OF AREA

$$I_x = \int_a^b r^2dA \quad ; \quad I_y = \int_a^b r^2dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = density × volume

$M = r\,V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\bar{\bar{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\,du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\,du$$

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$S = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2}\,du$$

$$\frac{dy}{dx}+Py=Q \quad \Rightarrow \quad ye^{\int Pdx}=\int Qe^{\int Pdx}\,dx$$

$$y = Ae^{r_1x}+Be^{r_2x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A+Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

NOVEMBER EXAMINATION

MATHEMATICS N6

13 NOVEMBER 2013

This marking guideline consists of 14 pages.

QUESTION 1

1.1. $z = \cos 2xy \cdot \sin 3x$

1.1.1. $\frac{\partial z}{\partial x} = \cos 2xy\,(3\cos 3x) + \sin 3x\,(-2y \sin 2xy)$ ✓ ✓ (1)

1.1.2 $\frac{\partial z}{\partial y} = -2x \sin 2xy \cdot \sin 3x$ ✓ ✓ (1)

1.1.3 $\frac{\partial^2 z}{\partial y^2} = -4x^2 \cos 2xy \cdot \sin 3x$ ✓ ✓ (1)

1.2 $\Delta V = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$ .... Given

$= 2\pi rh(\Delta r) + \pi r^2(\Delta h)$ ✓ ✓

$= 2\pi rh\left(\frac{3}{100}r\right) + \pi r^2\left(-\frac{2}{100}h\right)$ ✓ ✓

$= \pi r^2 h\left(\frac{6}{100} - \frac{2}{100}\right)$ ✓

$= \pi r^2 h\left(\frac{4}{100}\right)$ ✓

$= 4\%$ of the Volume / $\pi r^2 h$ (3)

Or $= 2\pi rh(\Delta r) + \pi r^2(\Delta h)$ ✓ ✓

$= 2\pi rh(0{,}03\,r) + \pi r^2(-0{,}02\,h)$ ✓ ✓

$= \pi r^2 h(0{,}06 - 0{,}02)$ ✓

$= \pi r^2 h\,(0{,}04)$

$= 0{,}04\ V$ / $\frac{4}{100}V$ ✓

$= 4\%$ of Volume / $\pi r^2 h$

[6]

QUESTION 2

2.1 $\int -x^2 . \cot^{-1}x \, dx$

$f(x) = \cot^{-1}x \quad g'(x) = -x^2$

$f'(x) = \frac{-1}{x^2+1} \quad g(x) = -\frac{x^3}{3}$

$= \cot^{-1}x\left(-\frac{x^3}{3}\right) - \int \frac{-1}{x^2+1} . -\frac{x^3}{3} \, dx$

$= -\frac{1}{3}x^3 . \cot^{-1}x - \frac{1}{3}\int \frac{x^3}{x^2+1} dx$

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 \\ & x^3+x \\ \hline & -x \end{array}$$

$= -\frac{1}{3}x^3 . \cot^{-1}x - \frac{1}{3}\int \left(x - \frac{x}{x^2+1}\right) dx$

$= -\frac{1}{3}x^3 . \cot^{-1}x - \frac{1}{3} . \frac{x^2}{2} + \frac{1}{3}\int \frac{x}{x^2+1} dx$

$= -\frac{1}{3}x^3 . \cot^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\ln(x^2+1) + C$ (4)

2.2. $\int \frac{\cos^3 x}{\operatorname{cosec}^7 x} dx$

$= \int \cos^3 x . \sin^7 x \, dx$

$= \int \cos^2 x . \cos x . \sin^7 x \, dx$

$= \int (1-\sin^2 x)\cos x . \sin^7 x \, dx$ • $u = \sin x$

$= \int (1-u^2)\, u^7 \, du$ $du = \cos x \, dx$

$= \int (u^7 - u^9)\, du$

$= \frac{u^8}{8} - \frac{u^{10}}{10} + C$

$= \frac{1}{8}\sin^8 x - \frac{1}{10}\sin^{10} x + C$ (4)

OR $\int \cos^3 x . \sin^7 x \, dx$

$= \int \cos^2 x . \cos x . \sin^7 x \, dx$

$= \int (1-\sin^2 x)\cos x \; \sin^7 x \, dx$

$= \int \cos x . \sin^7 x \, dx - \int \cos x . \sin^9 x \, dx$

$= \frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C$ (both constants correct)

OR $\int \cos^3 x . \sin^7 x \, dx$

$= \int \cos^3 x . (\sin^2 x)^3 \sin x \, dx$

$= \int \cos^3 x \, (1-\cos^2 x)^3 \sin x \, dx$ • $u = \cos x$

$= -\int u^3(1-u^2)^3 \, du$ $du = -\sin x \, dx$

$= -\int (u^3 - 3u^5 + 3u^7 - u^9)\, du$

$$= -\left(\frac{u^4}{4} - \frac{3u^6}{6} + \frac{3u^8}{8} - \frac{u^{10}}{10}\right) + c \checkmark$$

$$= -\left(\frac{1}{4}\cos^4 x - \frac{1}{2}\cos^6 x + \frac{3}{8}\cos^8 x - \frac{1}{10}\cos^{10} x\right) + c \checkmark\checkmark$$

$$\text{or} = -\frac{1}{4}\cos^4 x + \frac{1}{2}\cos^6 x - \frac{3}{8}\cos^8 x + \frac{1}{10}\cos^{10} x + c$$

2.3. $\int \frac{1}{18 + 9x + 3x^2}\,dx$

$$= \int \frac{1}{3\left[(x + \frac{3}{2})^2 + \frac{15}{4}\right]}\,dx \checkmark$$

$$= \frac{1}{3}\checkmark \int \frac{1}{(x + \frac{3}{2})^2 + \frac{15}{4}}\,dx$$

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{\frac{15}{4}}}\checkmark \tan^{-1} \frac{(x + 3/2)}{\sqrt{\frac{15}{4}}} \checkmark + c$$

$3x^2 + 9x + 18$

$= 3(x^2 + 3x + 6) \checkmark$

$= 3\left[(x + \frac{3}{2})^2 + 6 - \frac{9}{4}\right] \checkmark$

$= 3\left[(x + \frac{3}{2})^2 + \frac{15}{4}\right] \checkmark$

or $3\left[(x + 1,5)^2 + 3,75\right]$

or $11,25 + 3(x + \frac{3}{2})^2$ / $\frac{45}{4} + 3(x + \frac{3}{2})^2$

(4)

or $0,172 \tan^{-1} \frac{2(x + \frac{3}{2})}{3,873} + c$

or $\int \frac{1}{3(x^2 + 3x + 6)}\,dx$

$$= \int \frac{1}{3\left[x^2 + 3x + (\frac{3}{2})^2 - (\frac{3}{2})^2 + 6\right]}\checkmark dx$$

$$= \int \frac{1}{3\left[(x + \frac{3}{2})^2 + \frac{15}{4}\right]}\checkmark\,dx$$

$$= \int \frac{1}{\frac{45}{4} + 3(x + \frac{3}{2})^2}\checkmark dx$$

$$= \frac{1}{\sqrt{\frac{45}{4}} \cdot \sqrt{3}}\checkmark \tan^{-1} \frac{\sqrt{3}(x + \frac{3}{2})}{\sqrt{\frac{45}{4}}}\checkmark + c$$

$$= 0,172 \tan^{-1} \frac{1,732(x + \frac{3}{2})}{3,354} + c$$

2.4. $\int \sin^5 4x\,dx$

$$= \int (\sin^2 4x)^2 \checkmark \sin 4x\,dx$$

$$= \int (1 - \cos^2 4x)^2 \checkmark \sin 4x\,dx$$

$$= -\frac{1}{4}\checkmark \int (1 - u^2)^2\,du \checkmark$$

$$= -\frac{1}{4}\int (1 - 2u^2 + u^4)\,du$$

$$= -\frac{1}{4}\left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right] + c \checkmark$$

$$= -\frac{1}{4}\left[\cos 4x \checkmark - \frac{2}{3}\cos^3 4x \checkmark + \frac{1}{5}\cos^5 4x\checkmark\right] + c$$

$$= -\frac{1}{4}\cos 4x + \frac{1}{6}\cos^3 4x - \frac{1}{20}\cos^5 4x$$

- $u = \cos 4x$

$du = -4\sin 4x\,dx$

(4)

$$\text{Or } \int (\sin^2 4x)^2 \sin 4x \, dx$$

$$= \int (1-\cos^2 4x)^2 \sin 4x \, dx$$

$$= \int (1 - 2\cos^2 4x + \cos^4 4x) \sin 4x \, dx$$

$$= \int \sin 4x \, dx - 2\int (\cos 4x)^2 \sin 4x \, dx + \int (\cos 4x)^4 \sin 4x \, dx$$

$$= \frac{-\cos 4x}{4} + \frac{2}{4} \cdot \frac{\cos^3 4x}{3} - \frac{1}{4} \cdot \frac{\cos^5 4x}{5} + c$$

$$= -\frac{1}{4}\cos 4x + \frac{1}{6}\cos^3 4x - \frac{1}{20}\cos^5 4x + c$$

2.5 $\int x \ln 3x \, dx$

$$f(x) = \ln 3x \qquad g'(x) = x$$

$$f'(x) = \frac{1}{x} \qquad g(x) = \frac{x^2}{2}$$

$$= \ln 3x\left(\frac{x^2}{2}\right) - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx$$

$$= \frac{1}{2}x^2 \ln 3x - \frac{1}{2}\int x \, dx$$

$$= \frac{1}{2}x^2 \ln 3x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$= \frac{1}{2}x^2 \ln 3x - \frac{1}{4}x^2 + c$$

(2)
[18]

QUESTION 3

3.1 $\int \frac{6x^3+8x^2+2x-3}{(x^2+1)x^2}\,dx$

$\frac{6x^3+8x^2+2x-3}{(x^2+1)x^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x^2} + \frac{D}{x}$ ✓

$\therefore 6x^3+8x^2+2x-3 = x^2(Ax+B)+C(x^2+1)+Dx(x^2+1)$ ✓

Let $x=0$; $\therefore C=-3$ ✓

(Method)

$6x^3+8x^2+2x-3 = Ax^3+Bx^2+Cx^2+C+Dx^3+Dx$ ✓

Equate coeff. of x^3: $6=A+D$ $\therefore A=4$ ✓

" " " x^2: $8=B+C$ $\therefore B=11$ ✓

" " " x: $2=$ $\therefore D=2$ ✓

$\therefore \int\frac{4x+11}{x^2+1}\,dx + \int\frac{-3}{x^2}\,dx + \int\frac{2}{x}\,dx$ ✓

$= 4\int\frac{x}{x^2+1}\,dx + \int\frac{11}{x^2+1}\,dx - 3\int\frac{1}{x^2}\,dx + 2\int\frac{1}{x}\,dx$

$= \frac{4}{2}\ln(x^2+1) + 11\tan^{-1}x - 3\cdot\frac{x^{-1}}{-1} + 2\ln x + c$

$= 2\ln(x^2+1) + 11\tan^{-1}x + \frac{3}{x} + 2\ln x + c$ ✓ ✓ ✓ ✓ (6)

OR 3.1 $\frac{6x^3+8x^2+2x-3}{(x^2+1)x^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2}$ ✓

$\therefore 6x^3+8x^2+2x-3 = x^2(Ax+B)+(Cx+D)(x^2+1)$ ✓

$6x^3+8x^2+2x-3 = Ax^3+Bx^2+Cx^3+Cx+Dx^2+D$ ✓(M)

Equate coeff. of x^3: $6=A+C$ $\therefore A=4$ ✓

" " " x^2: $8=B+D$ $\therefore B=11$ ✓

" " " x: $2=C$ $\therefore C=2$ ✓

" constants: $-3=D$ $\therefore D=-3$ ✓

$\therefore \int\frac{4x+11}{x^2+1}\,dx + \int\frac{2x-3}{x^2}\,dx$ ✓

$= \int\frac{4x}{x^2+1}\,dx + \int\frac{11}{x^2+1}\,dx + \int\frac{2}{x}\,dx - \int\frac{3}{x^2}\,dx$

$= 2\ln(x^2+1) + 11\tan^{-1}x + 2\ln x - 3.\frac{x^{-1}}{-1} + c$

$= 2\ln(x^2+1) + 11\tan^{-1}x + 2\ln x + \frac{3}{x} + c$ ✓ ✓ ✓ ✓

3.2. $\int \frac{3x^2-5x+4}{(x-2)^2(x-1)^2}\,dx$

$$\Rightarrow \frac{3x^2-5x+4}{(x-2)^2(x-1)^2} = \frac{A}{(x-2)^2} + \frac{B}{(x-2)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)} \checkmark$$

$$3x^2-5x+4 = A(x-1)^2 + B(x-2)(x-1)^2 + C(x-2)^2 + D(x-1)(x-2)^2 \checkmark$$

$$\therefore 3x^2-5x+4 = Ax^2-2Ax+A+Bx^3-2Bx^2+Bx-2Bx^2+4Bx-2B+Cx^2-4Cx+4C+$$

$$Dx^3-4Dx^2+4Dx-Dx^2+4Dx-4D \checkmark\text{(method)}$$

Let $x=2$; $\therefore \underline{A=6}$ $\checkmark$

$x=1$; $\therefore \underline{C=2}$ $\checkmark$

Equate coeff. of x^3: $0=B+D$

" " " x^2: $3 = A-2B-2B+C-4D-D$ $\therefore -5=-4B-5D$

$\therefore \underline{D=5}$; $\checkmark$ $\underline{B=-5}$ $\checkmark$

$$\Rightarrow \int\frac{6}{(x-2)^2}\,dx + \int\frac{-5}{(x-2)}\,dx + \int\frac{2}{(x-1)^2}\,dx + \int\frac{5}{(x-1)}\,dx \checkmark\checkmark$$

$$= 6\cdot\frac{(x-2)^{-1}}{-1} \checkmark - 5\ln(x-2) \checkmark + 2\cdot\frac{(x-1)^{-1}}{-1} \checkmark + 5\ln(x-1) + c$$

$$= -\frac{6}{x-2} - 5\ln(x-2) - \frac{2}{x-1} + 5\ln(x-1) + c$$

(6)
[12]

QUESTION 4

4.1 $\left(\frac{1}{\sin x}\right)\frac{dy}{dx} + y = 1$

$\frac{dy}{dx} + y.\sin x = \sin x$ ✓ ✓

$\therefore R = e^{\int \sin x \, dx}$ ✓

$= e^{-\cos x}$ ✓

$\therefore y.e^{-\cos x} = \int e^{-\cos x}.\sin x \, dx$ ✓ ✓

$y.e^{-\cos x} = e^{-\cos x} + c$ ✓

$3.e^{-\cos 0} = e^{-\cos 0} + c$ ✓ ... $(0;3)$

$\therefore c = \frac{2}{e} \;\Big|\; 2e^{-1} \;\Big|\; 0{,}736$ ✓

$\therefore y.e^{-\cos x} = e^{-\cos x} + \frac{2}{e}$ ✓ (5)

4.2 $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = e^{2t}$

$m^2 - m - 2 = 0$ ✓

$(m-2)(m+1) = 0$

$\therefore m = 2;\ m = -1$ ✓

$\therefore y_c = Ae^{2t} + Be^{-t}$ ✓

$y_p: \quad y = Cte^{2t}$ ✓

$\frac{dy}{dt} = Ct(2e^{2t}) + C.e^{2t}$ ✓

$\frac{d^2y}{dt^2} = Ct(4e^{2t}) + C.2e^{2t} + 2Ce^{2t}$ ✓

$= 4Cte^{2t} + 4Ce^{2t}$

$\therefore 4Cte^{2t} + 4Ce^{2t} - 2Cte^{2t} - Ce^{2t} - 2Cte^{2t} = e^{2t}$ ✓

$4Ct + 4C - 2Ct - C - 2Ct = 1$

$3C = 1$

$\therefore C = \frac{1}{3}$ ✓

$\therefore y_p = \frac{1}{3}te^{2t}$ ✓

$\therefore y = Ae^{2t} + Be^{-t} + \frac{1}{3}te^{2t}$ ✓

$-1 = A + B$

$\frac{dy}{dt} = 2Ae^{2t} - Be^{-t} + \frac{1}{3}t.2e^{2t} + \frac{1}{3}e^{2t}$ ✓

$1 = 2A - B + \frac{1}{3} \quad \therefore \frac{2}{3} = 2A - B \quad \therefore B = -\frac{8}{9}\ (-0{,}889)$ ✓ $;\ A = -\frac{1}{9}\ (-0{,}111)$ ✓

$\therefore y = -\frac{1}{9}e^{2t} - \frac{8}{9}e^{-t} + \frac{1}{3}te^{2t}$ ✓ (7)

[12]

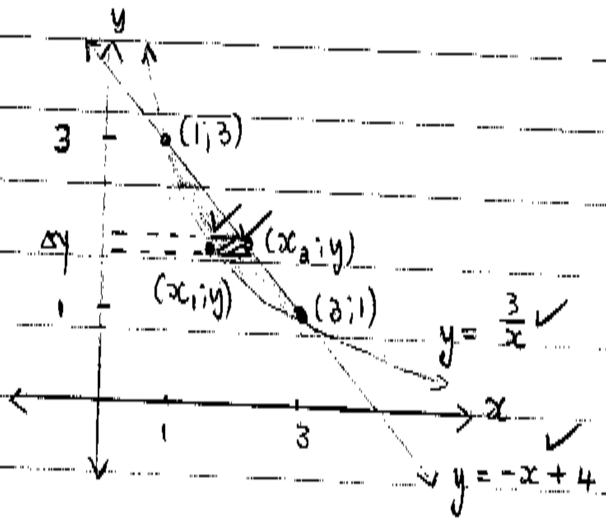
QUESTION 5

5.1 5.1.1 $\frac{3}{x} = -x + 4$

$0 = x^2 - 4x + 3$

$0 = (x - 3)(x - 1)$

$\left.\begin{array}{l} \therefore x = 3 \; ; \; x = 1 \\ y = 1 \; ; \; y = 3 \end{array}\right\}$ (3;1) and (1;3)



(3)

5.1.2 $\Delta V = 2\pi y \,(x_2 - x_1)\, \Delta y$

$V_x = 2\pi \int_1^3 y(x_2 - x_1)\, dy$

$= 2\pi \int_1^3 y\left(4 - y - \frac{3}{y}\right) dy$

$= 2\pi \int_1^3 (4y - y^2 - 3)\, dy$

$= 2\pi \left[\frac{4y^2}{2} - \frac{y^3}{3} - 3y\right]_1^3$

$= 2\pi \left[2y^2 - \frac{1}{3}y^3 - 3y\right]_1^3$

$= 2\pi \left\{\left[2(3)^2 - \frac{1}{3}(3)^3 - 3(3)\right] - \left[2(1)^2 - \frac{1}{3}(1)^3 - 3(1)\right]\right\}$

$= 2{,}667\pi$ / $8{,}378$ units^3 / $\frac{8\pi}{3}$ units^3

(4)

5.2 5.2.1 $x^2 = \frac{1}{2}y$ and $y = x + 3$

$\therefore y = 2x^2$

$\therefore 2x^2 = x + 3$

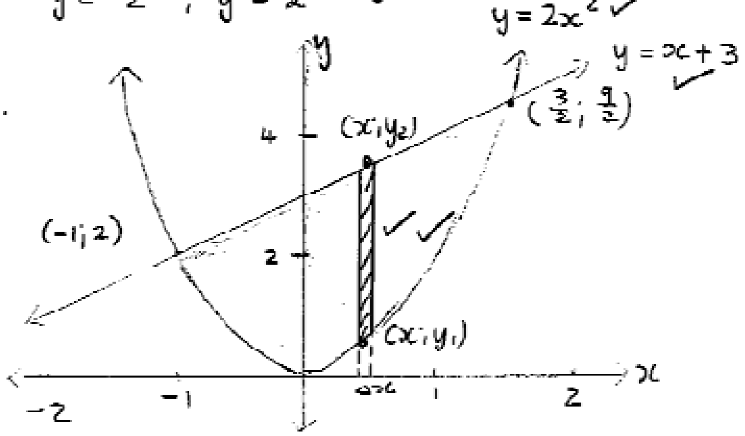
$2x^2 - x - 3 = 0$

$(2x - 3)(x + 1) = 0$

$\therefore x = \frac{3}{2}$; ✓ $x = -1$

$y = \frac{9}{2}$; ✓ $y = 2$

$\left.\right\}$ $(\frac{3}{2}; \frac{9}{2})$ and $(-1; 2)$



(3)

5.2.2 $\Delta A = (y_2 - y_1)\Delta x$ ✓

$A = \int_{-1}^{\frac{3}{2}} (y_2 - y_1)\, dx$ ✓ $\qquad \int_{-1}^{1,5}$

$= \int_{-1}^{3/2} (x + 3 - 2x^2)\, dx$ ✓

$= \left[\frac{x^2}{2} + 3x - \frac{2x^3}{3}\right]_{-1}^{\frac{3}{2}}$ ✓

$= \left[\frac{1}{2}(\frac{3}{2})^2 + 3(\frac{3}{2}) - \frac{2}{3}(\frac{3}{2})^3\right] - \left[\frac{1}{2}(-1)^2 + 3(-1) - \frac{2}{3}(-1)^3\right]$ ✓

$= 5,208$ units^2 ✓

(3)

5.2.3 $\Delta M_x = (y_2 - y_1)\Delta x \times \frac{y_2 + y_1}{2}$ ✓ ✓

$M_x = \frac{1}{2}\int_{-1}^{3/2} (y_2^2 - y_1^2)\, dx$ ✓ ✓

$= \frac{1}{2}\int_{-1}^{1,5} \left[(x + 3)^2 - (2x^2)^2\right] dx$ ✓ ✓

$= \frac{1}{2}\int_{-1}^{1,5} (x^2 + 6x + 9 - 4x^4)\, dx$ ✓

$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{6x^2}{2} + 9x - \frac{4x^5}{5}\right]_{-1}^{1,5}$ ✓

$= \frac{1}{2}\left\{\left[\frac{1}{3}(1,5)^3 + 3(1,5)^2 + 9(1,5) - \frac{4}{5}(1,5)^5\right] - \left[\frac{1}{3}(-1)^3 + 3(-1)^2 + 9(-1) - \frac{4}{5}(-1)^5\right]\right\}$ ✓

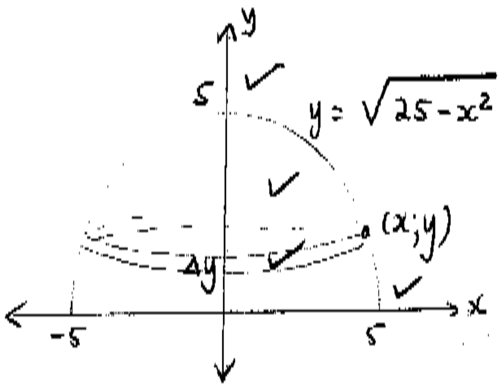
$= 10,417$ units^2 ✓

$\therefore \bar{y} = \frac{10,417}{5,208}$ ✓

$= 2$ units ✓

(6)

5.3 5.3.1



(2)

5.3.2

$$\Delta V_y = \pi x^2 \Delta y$$

$$V_y = \pi \int_0^5 x^2 \, dy$$

$$= \pi \int_0^5 (25 - y^2) \, dy \qquad \bullet\ y = \sqrt{25 - x^2} \quad \therefore x^2 = 25 - y^2$$

$$= \pi \left[25y - \frac{y^3}{3} \right]_0^5$$

$$= \pi \left[25(5) - \frac{1}{3}(5)^3 \right]$$

$$= 83{,}333\pi \ / \ 261{,}799 \text{ units}^3$$

(3)

5.3.3

$$\Delta M_x = \pi x^2 \Delta y \times y$$

$$M_x = \pi \int_0^5 x^2 y \, dy$$

$$= \pi \int_0^5 (25 - y^2) y \, dy$$

$$= \pi \int_0^5 (25y - y^3) \, dy$$

$$= \pi \left[\frac{25y^2}{2} - \frac{y^4}{4} \right]_0^5$$

$$= \pi \left[\frac{25}{2}(5)^2 - \frac{1}{4}(5)^4 \right]$$

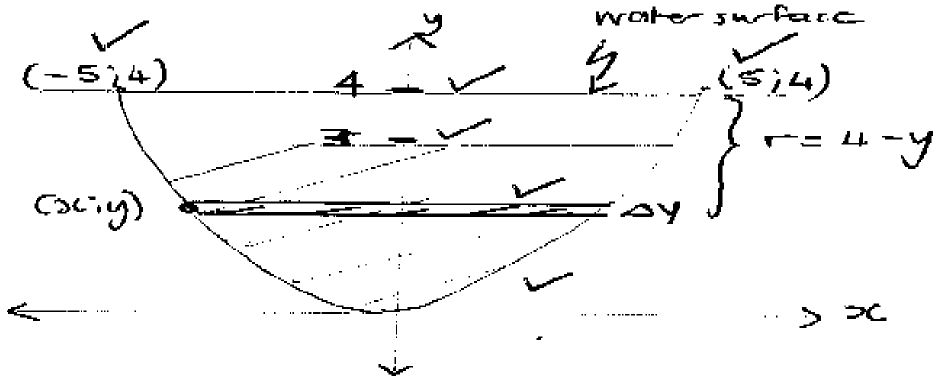
$$= 156{,}25\pi \ / \ 490{,}874 \text{ units}^4$$

$$\therefore \bar{y} = \frac{490{,}874}{261{,}799} \ / \ \frac{156{,}25\pi}{83{,}333\pi}$$

$$= 1{,}875 \text{ units}$$

(5)

5.4 5.4.1



$$y = ax^2 \quad \ldots\ldots (5;4)$$
$$4 = a(5)^2$$
$$\therefore y = \frac{2}{5}x^2$$
$$\therefore x = \sqrt{\frac{5}{2}y} \quad \text{or} \quad x = 2{,}5\sqrt{y}$$
$$\therefore \Delta A = 2x\Delta y$$
$$dA = 2 \cdot 2{,}5\sqrt{y}\,dy$$
$$= 5\sqrt{y}\,dy$$

(4)

5.4.2.
$$\int_0^3 r\,dy$$
$$= \int_0^3 (4-y) \cdot 5\sqrt{y}\,dy$$
$$= 5\int_0^3 (4y^{\frac{1}{2}} - y^{\frac{3}{2}})\,dy$$
$$= 5\left[\frac{4y^{3/2}}{3/2} - \frac{y^{5/2}}{\frac{5}{2}}\right]_0^3$$
$$= 5\left[\frac{2}{3}(4)(3)^{3/2} - \frac{2}{5}(3)^{\frac{5}{2}}\right]$$
$$= 38{,}105\ m^3$$

(3)

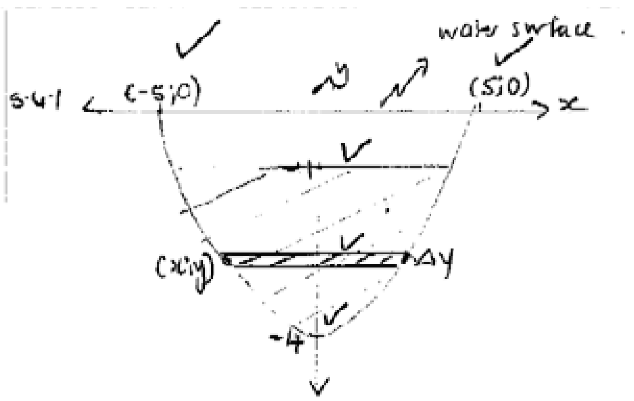
5.4.3.
$$\int_0^3 r^2\,dy$$
$$= \int_0^3 (4-y)^2\, 5\sqrt{y}\,dy$$
$$= 5\int_0^3 (16y^{\frac{1}{2}} - 8y^{\frac{3}{2}} + y^{\frac{5}{2}})\,dy$$
$$= 5\left[16\frac{y^{\frac{3}{2}}}{3/2} - 8\frac{y^{5/2}}{5/2} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^3$$
$$= 5\left[16 \cdot \frac{2}{3}(3)^{\frac{3}{2}} - \frac{2}{5} \cdot 8(3)^{\frac{5}{2}} + \frac{2}{7}(3)^{\frac{7}{2}}\right]$$
$$= 94{,}52\ m^4$$
$$\therefore \bar{\bar{y}} = \frac{94{,}52}{38{,}105} = 2{,}481$$

(4)

OR

5.4

5.4.1



$y = ax^2 + bx - 4$

$0 = a(5)^2 + b(5) - 4 \quad \ldots (5;0)$

$4 = 25a + 5b \quad ——— ①$

$0 = a(-5)^2 + b(-5) - 4 \quad \ldots (-5;0)$

$4 = 25a - b \quad ——— ②$

$\therefore a = \frac{4}{25} \;;\; b = 0$

$\therefore y = \frac{4}{25}x^2 - 4 \;;\; x = \frac{5}{2}\sqrt{y+4}\, dy$

$\therefore dA = 2 \cdot \frac{5}{2}\sqrt{y+4}\, dy \;;\; dA = 5\sqrt{y+4}\, dy$ (6)

5.4.2 $\int_{-4}^{-1} r\, dy$

$= \int_{-4}^{-1} y \cdot 5\sqrt{y+4}\, dy$

$= 5\int_{-4}^{-1} y(y+4)^{\frac{1}{2}} dy \quad \bullet\ u = y+4 \;;\; du = dy$

$= 5\int_{-4}^{-1} (u-4)u^{\frac{1}{2}} du$

$= 5\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{4u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-4}^{-1}$

$= 5\left[\frac{2}{5}(y+4)^{5/2} - 4 \cdot \frac{2}{3}(y+4)^{\frac{3}{2}}\right]$

$= 5\left[\frac{2}{5}(-1+4)^{5/2} - \frac{8}{3}(-1+4)^{3/2}\right]$

$= -38{,}105 \text{ m}^3$ (6)

5.4.3 $\int_{-4}^{-1} r^2\, dy$

$= \int_{-4}^{-1} y^2\, 5\sqrt{y+4}\, dy$

$= 5\int_{-4}^{-1} y^2(y+4)^{\frac{1}{2}} dy \quad \bullet\ u = y+4 \;;\; du = dy$

$= 5\int_{-4}^{-1} (u-4)^2 u^{\frac{1}{2}} du$

$= 5\int_{-4}^{-1} (u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}})\, du$

$= 5\left[\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - 8 \cdot \frac{u^{5/2}}{5/2} + \frac{16u^{3/2}}{3/2}\right]_{-4}^{-1}$

$= 5\left[\frac{2}{7}(y+4)^{7/2} - \frac{16}{5}(y+4)^{5/2} + \frac{32}{3}(y+4)^{\frac{3}{2}}\right]_{-4}^{-1}$

$= 5\left[\frac{2}{7}(-1+4)^{\frac{7}{2}} - \frac{16}{5}(-1+4)^{\frac{5}{2}} + \frac{32}{3}(-1+4)^{\frac{3}{2}}\right]$

$= 94{,}52 \text{ m}^4$

$\therefore \bar{\bar{y}} = \frac{94{,}52}{-38{,}105}$

$= -2{,}481 \text{ m}$ (4)

[40]

QUESTION 6

6.1.
$$y = x^2$$
$$\frac{dy}{dx} = 2x$$
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^2$$
$$\therefore \quad S = \int_0^1 \sqrt{1+4x^2}\, dx \qquad \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \int_0^1 \sqrt{4\left(\tfrac{1}{4}+x^2\right)}\, dx$$
$$= 2\int_0^1 \sqrt{\tfrac{1}{4}+x^2}\, dx$$
$$= 2\left[\frac{x}{2}\sqrt{\tfrac{1}{4}+x^2} + \frac{\frac{1}{4}}{2}\ln\left(x+\sqrt{\tfrac{1}{4}+x^2}\right)\right]_0^1$$
$$= \left[x\sqrt{\tfrac{1}{4}+x^2} + \tfrac{1}{4}\ln\left(x+\sqrt{\tfrac{1}{4}+x^2}\right)\right]_0^1$$
$$= \left[1\sqrt{\tfrac{1}{4}+1} + \tfrac{1}{4}\ln\left(1+\sqrt{\tfrac{1}{4}+1}\right)\right] - \left[0 + \tfrac{1}{4}\ln\sqrt{\tfrac{1}{4}}\right]$$
$$= 1{,}479 \text{ units}$$

(6)

6.2.
$$y = 4t^2 \qquad x = 4t$$
$$\frac{dy}{dt} = 8t \qquad \frac{dx}{dt} = 4$$
$$\left(\frac{dy}{dt}\right)^2 = 64t^2 \qquad \left(\frac{dx}{dt}\right)^2 = 16$$
$$\therefore \left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 = 64t^2+16 \quad \text{or} \quad 16(4t^2+1) \; *$$
$$\therefore A_y = \int_1^2 2\pi x \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$
$$= 2\pi\int_1^2 4t\sqrt{64t^2+16}\, dt$$
$$= 8\pi\int_1^2 (64t^2+16)^{\frac{1}{2}}\, dt$$
$$= \frac{8\pi}{128} \cdot \left[\frac{(64t^2+16)^{3/2}}{\frac{3}{2}}\right]_1^2$$
$$= \frac{\pi}{24}\left\{\left[64(2)^2+16\right]^{3/2} - \left[64(1)^2+16\right]^{3/2}\right\}$$
$$= 157{,}1\,\pi \;/\; 493{,}544 \text{ units}^2$$

OR
$$A_y = 2\pi\int_1^2 4t\sqrt{16(4t^2+1)}\, dt \qquad \bullet (5) + (7) = (12)$$
$$= 32\pi\int_1^2 t\,(4t^2+1)^{\frac{1}{2}}\, dt$$
$$= \frac{32\pi}{8} \cdot \left[\frac{(4t^2+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^2$$
$$= \frac{8}{3}\pi\left\{\left[4(2)^2+1\right]^{3/2} - \left[4(1)^2+1\right]\right\}$$
$$= 157{,}1\,\pi \;/\; 493{,}544 \text{ units}^2$$

(6)
[12]

TOTAL: 100

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1100**(E)**(J24)T
AUGUST EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

24 July 2013 (X-Paper)
09:00–12:00

Calculators may be used.

This question paper consists of 5 pages and 7 formula sheets.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in BLUE or BLACK ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 If $z = -4x^4y - x^3y^3$**,** determine the value of:

1.1.1 $\dfrac{\partial^2 z}{\partial x \partial y}$ (1)

1.1.2 $\dfrac{\partial^2 z}{\partial y^2}$ (1)

1.1.3 $\left(\dfrac{\partial^2 z}{\partial y^2}\right)^2$ (1)

1.2 If the parametric equations of a function are given as $x = \ln(3 - t)$ and $y = 2t^3 - t$, calculate $\dfrac{d^2y}{dx^2}$ when $t = -1$. (3)

[6]

QUESTION 2

Determine $\int y\, dx$ if:

2.1 $y = \sin^3 \pi x \,.\, \cos^5 \pi x$ (4)

2.2 $y = e^{5x}.\sin 2x$ (5)

2.3 $y = \sqrt{79 + 4x - 2x^2}$ (5)

2.4 $y = \cos^4 x$ (4)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\displaystyle\int \frac{-28x^3 + 40x^2 - 18x + 4}{x(1 - 2x)^3}\, dx$ (6)

3.2 $\displaystyle\int \frac{2x^2 - 2x + 2}{2x^2 - 5x + 2}\, dx$ (6)

[12]

QUESTION 4

4.1 Calculate the particular solution of:

$$y + \sec x = \frac{dy}{dx}.\tan x \text{ if } y = 2 \text{ when } x = \frac{p}{4}.$$ (6)

4.2 Calculate the general solution of:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 6x - 2$$

(6)
[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = \sqrt{x}$ and $y = x^3$. Make a neat sketch of the two curves and show the area bounded by the curves. Show the representative strip/element that you will use to calculate the volume (use the SHELL METHOD only) generated if the area bounded by the curves rotates about the y-axis. (3)

5.1.2 Use the SHELL METHOD to calculate the volume described in QUESTION 5.1.1 by means of integration. (4)

5.2 5.2.1 Calculate the points of intersection of $xy = 32$ and $y = -18 + x$. Sketch the graphs and show the representative strip/element (PERPENDICULAR to the x-axis) that you will use to calculate the volume when the area bounded by the graphs rotates about the x-axis. (3)

5.2.2 Calculate the volume described in QUESTION 5.2.1. (4)

5.2.3 Calculate the distance of the centre of gravity from the y-axis of the solid generated when the area described in QUESTION 5.2.1 rotates about the x-axis. (6)

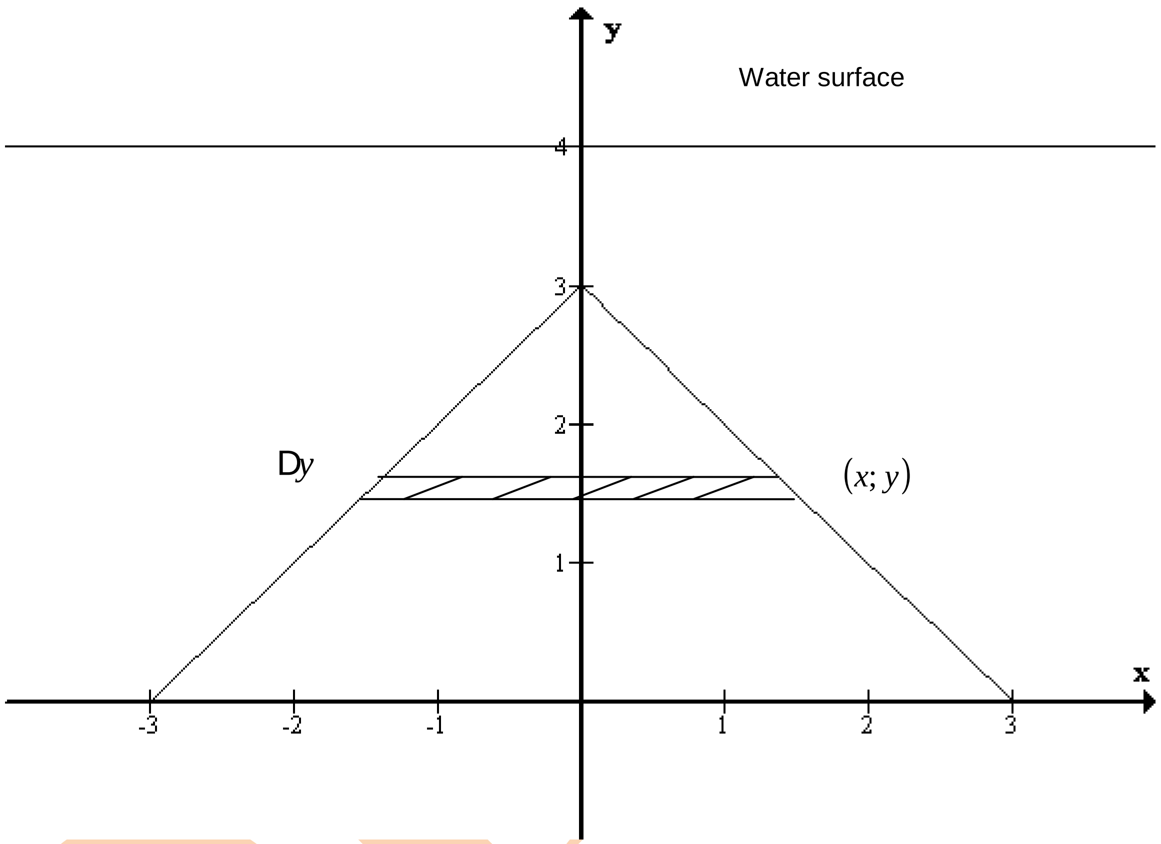
5.3 5.3.1 Make a neat sketch of the graph $y = e^{\frac{x}{2}}$. Show the representative strip/element that you will use to calculate the volume generated if the area bounded by the graph, the line $x = 4$, the y-axis and the x-axis rotate about the x-axis. (3)

5.3.2 Calculate the moment of inertia of the solid generated when the area described in QUESTION 5.3.1 rotates about the x-axis. (6)

5.3.3 Express the answer in QUESTION 5.3.2 in terms of the mass. (2)

5.4 An isosceles triangular plate with a base of 6 m is below the water surface as indicated.

Calculate the depth of the centre of pressure on the triangular plate by means of integration.



(9)
[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $y = \frac{1}{8}x^2 - \ln x$ between $x = 1$ and $x = 4$. (6)

6.2 Calculate the surface area generated by revolving the curve given by $y = \sin x$ for $\frac{\pi}{2} \le x \le \pi$ about the x-axis. (6)
[12]

TOTAL: 100

FORMULA SHEETS

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\,\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}.\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx}\,f(x)$	$\int f(x)\,dx$
$\sin f(x)$	$\cos f(x)\,.\ f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\ f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\ f'(x)$	-
$\cot f(x)$	$-\operatorname{cosec}^2 f(x)\,.\ f'(x)$	-
$\sec f(x)$	$\sec f(x)\tan f(x)\,.\ f'(x)$	-
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x)\cot f(x)\,.\ f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)\,dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b}\sin^{-1}\frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a + bx}{a - bx}\right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left[x + \sqrt{x^2 \pm b^2}\right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b}\ln\left[bx + \sqrt{b^2x^2 \pm a^2}\right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = p\int_a^b y^2 dx \; ; V_x = p\int_a^b \left(y_1^2 - y_2^2\right) dx \; ; V_x = 2p\int_a^b xydy$$

$$V_y = p\int_a^b x^2 dy \; ; V_y = p\int_a^b \left(x_1^2 - x_2^2\right) dy \; ; V_y = 2p\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A} \; ; \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENTS OF AREA

$$I_x = \int_a^b r^2 dA \quad ; \quad I_y = \int_a^b r^2 dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{V_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{V_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = density × volume

$M = r\, V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\overline{\overline{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \ldots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \ldots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_a^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \Box \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

AUGUST EXAMINATION

MATHEMATICS N6

24 JULY 2013

This marking guideline consists of 12 pages.

QUESTION 1

1.1 1.1.1

$$z = -4x^4y - x^3y^3$$

$$\frac{\partial z}{\partial y} = -4x^4 - 3x^3y^2 \checkmark$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(-4x^4 - 3x^3y^2\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \underline{-16x^3 - 9x^2y^2} \checkmark$$

OR

$$\frac{\partial z}{\partial x} = -16x^3y - 3x^2y^3 \checkmark$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(-16x^3y - 3x^2y^3\right)$$

$$\therefore \frac{\partial^2 z}{\partial x \partial y} = -16x^3 - 9x^2y^2 \checkmark$$

1.1.2

$$\frac{\partial^2 z}{\partial y^2} = \underline{-18x^2y} \checkmark \checkmark$$

$$\left(\frac{\partial^2 z}{\partial y^2}\right)^2 = \underline{324x^4y} \checkmark \checkmark$$

1.1.3

$$x = \ln(3-t) \qquad y = 2t^3 - t$$

$$\frac{dx}{dt} = \frac{-1}{3-t} \checkmark \qquad \frac{dy}{dt} = 6t^2 - 1 \checkmark$$

(3 x 1) (3)

1.2

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{6t^2-1}{\frac{-1}{3-t}} \checkmark$$

$$= \underline{-(6t^2-1)(3-t)}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left[-(6t^2-1)(3-t)\right] \times \frac{3-t}{-1} \checkmark$$

$$= -(6t^2-1)(-1) + (3-t)(-12t) \times -(3-t) \checkmark$$

$$= (6t^2 - 1 - 36t + 12t^2)(-3+t)$$

$$= (18t^2 - 36t - 1)(-3+t)$$

$$= \underline{-212} \checkmark$$

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QUESTION 2

2.1

$$\int \sin^3 \pi x \cdot \cos^5 \pi x \, dx$$
$$= \int \sin^2 \pi x \cdot \sin \pi x \cdot \cos^5 \pi x \, dx \checkmark$$
$$= \int (1-\cos^2 \pi x) \cdot \sin \pi x \cdot \cos^5 \pi x \, dx \checkmark$$
$$= -\frac{1}{\pi}\int (1-u^2)\, u^5 \, du \checkmark \checkmark$$
$$= -\frac{1}{\pi}\int (u^5-u^7)\, du \checkmark$$
$$= -\frac{1}{\pi}\left[\frac{u^6}{6}-\frac{u^8}{8}\right]+c \checkmark$$
$$= -\frac{1}{\pi}\left[\frac{\cos^6 \pi x}{6}\checkmark - \frac{\cos^8 \pi x}{8}\right]\checkmark + c$$
$$= -\frac{1}{6\pi}\cos^6 \pi x + \frac{1}{8\pi}\cos^8 \pi x + c$$

- $u = \cos \pi x$

$du = -\pi \sin \pi x$

OR
$$= \int \sin^2 \pi x \cdot \sin \pi x \cdot \cos^5 \pi x \, dx \checkmark$$
$$= \int (1-\cos^2 \pi x) \sin \pi x \cdot \cos^5 \pi x \, dx \checkmark$$
$$= \int (\sin \pi x \cdot \cos^5 \pi x)\checkmark dx - \int (\sin \pi x \cdot \cos^7 \pi x)\checkmark dx$$
$$= -\frac{1}{\pi}\checkmark \cdot \frac{\cos^6 \pi x}{6}\checkmark + \frac{1}{\pi}\checkmark \cdot \frac{\cos^8 \pi x}{8}\checkmark + c$$
$$= -\frac{1}{6\pi}\cos^6 \pi x + \frac{1}{8\pi}\cos^8 \pi x + c$$

OR
$$= \int \sin^3 \pi x \cdot (\cos^2 \pi x)^2 \checkmark \cos \pi x \, dx$$
$$= \int \sin^3 \pi x \, (1-\sin^2 \pi x)^2 \checkmark \cos \pi x \, dx$$
$$= \frac{1}{\pi}\int u^3(1-u^2)^2 \, du \checkmark \checkmark$$
$$= \frac{1}{\pi}\int u^3(1-2u^2+u^4)\, du$$
$$= \frac{1}{\pi}\int (u^3-2u^5+u^7)\, du \checkmark$$
$$= \frac{1}{\pi}\left[\frac{u^4}{4}-\frac{2u^6}{6}+\frac{u^8}{8}\right]+c$$
$$= \frac{1}{\pi}\left[\frac{\sin^4 \pi x}{4}\checkmark - \frac{2\sin^6 \pi x}{6}\checkmark + \frac{\sin^8 \pi x}{8}\checkmark\right]+c$$
$$\text{or } = \frac{1}{4\pi}\sin^4 \pi x - \frac{1}{3\pi}\sin^6 \pi x + \frac{1}{8\pi}\sin^8 \pi x + c$$

- $u = \sin \pi x$

$du = \pi \cos \pi x$

or
$$\int \sin^3 \pi x \,(\cos^2 \pi x)^2 \checkmark \cos \pi x \, dx$$
$$= \int \sin^3 \pi x \,(1-\sin^2 \pi x)^2 \checkmark \cos \pi x$$
$$= \int \sin^3 \pi x (1-2\sin^2 \pi x + \sin^4 \pi x)\cos \pi x \, dx$$
$$= \int \sin^3 \pi x \cdot \cos \pi x \, dx \checkmark - \int 2\sin^5 \pi x \cos \pi x \, dx \checkmark + \int \sin^7 \pi x \cos \pi x \, dx \checkmark$$
$$= \frac{1}{\pi}\cdot \frac{\sin^4 \pi x}{4}\checkmark - \frac{2}{\pi}\cdot \frac{\sin^6 \pi x}{6}\checkmark + \frac{1}{\pi}\cdot \frac{\sin^8 \pi x}{8}\checkmark + c$$

(4)

2.2

$$\int e^{5x} \cdot \sin 2x \, dx \qquad \left| \begin{array}{ll} f(x) = e^{5x} & g'(x) = \sin 2x \\ f'(x) = 5e^{5x} & g(x) = -\frac{\cos 2x}{2} \end{array} \right.$$

$$= e^{5x} \cdot \frac{-\cos 2x}{2} - \int 5e^{5x} \cdot \frac{-\cos 2x}{2} \, dx$$

$$= -\frac{1}{2} e^{5x} \cdot \cos 2x + \frac{5}{2} \int e^{5x} \cdot \cos 2x \, dx$$

$$\left| \begin{array}{ll} f(x) = e^{5x} & g'(x) = \cos 2x \\ f'(x) = 5e^{5x} & g(x) = \frac{\sin 2x}{2} \end{array} \right.$$

$$= -\frac{1}{2} e^{5x} \cdot \cos 2x + \frac{5}{2} \left[e^{5x} \cdot \frac{\sin 2x}{2} - \int 5e^{5x} \cdot \frac{\sin 2x}{2} \, dx \right]$$

$$= -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \cdot \sin 2x - \frac{25}{4} \int e^{5x} \cdot \sin 2x \, dx$$

$$\therefore I = -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x - \frac{25}{4} I$$

$$\therefore I + \frac{25}{4} I = -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x$$

$$\frac{29}{4} I = -\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x$$

$$\therefore I = \frac{4}{29} \left[-\frac{1}{2} e^{5x} \cos 2x + \frac{5}{4} e^{5x} \sin 2x \right] + c$$

$$\text{or} \quad \underline{I = -\frac{2}{29} e^{5x} \cos 2x + \frac{5}{29} e^{5x} \sin 2x + c}$$

OR

$$\int e^{5x} \cdot \sin 2x \, dx \qquad \left| \begin{array}{ll} f(x) = \sin 2x & g'(x) = e^{5x} \\ f'(x) = 2\cos 2x & g(x) = \frac{e^{5x}}{5} \end{array} \right.$$

$$= \sin 2x \cdot \frac{e^{5x}}{5} - \int 2\cos 2x \cdot \frac{e^{5x}}{5} \, dx$$

$$= \frac{1}{5} e^{5x} \cdot \sin 2x - \frac{2}{5} \int \cos 2x \cdot e^{5x} \, dx$$

$$\left| \begin{array}{ll} f(x) = \cos 2x & g'(x) = e^{5x} \\ f'(x) = -2\sin 2x & g(x) = \frac{e^{5x}}{5} \end{array} \right.$$

$$= \frac{1}{5} e^{5x} \sin 2x - \frac{2}{5} \left[\cos 2x \cdot \frac{e^{5x}}{5} - \int -2\sin 2x \cdot \frac{e^{5x}}{5} \, dx \right]$$

$$= \frac{1}{5} e^{5x} \cdot \sin 2x - \frac{2}{25} e^{5x} \cdot \cos 2x - \frac{4}{25} \int \sin 2x \cdot e^{5x} \, dx$$

$$\therefore I = \frac{1}{5} e^{5x} \sin 2x - \frac{2}{25} e^{5x} \cos 2x - \frac{4}{25} I$$

$$\therefore I + \frac{4}{25} I = \frac{1}{5} e^{5x} \sin 2x - \frac{2}{25} e^{5x} \cos 2x$$

$$\frac{29}{25} I = \frac{1}{5} e^{5x} \sin 2x - \frac{2}{25} e^{5x} \cos 2x$$

$$I = \frac{25}{29} \left(\frac{1}{5} e^{5x} \sin 2x - \frac{2}{25} e^{5x} \cos 2x \right) + c$$

$$\text{Or} \quad \underline{I = \frac{5}{29} e^{5x} \sin 2x - \frac{2}{29} e^{5x} \cos 2x + c}$$

(5)

2.3

$$\int \sqrt{79 + 4x - 2x^2}\, dx$$

$$-2x^2 + 4x + 79$$
$$= -2\left[x^2 - 2x - \tfrac{79}{2}\right]$$
$$= -2\left[(x-1)^2 - \tfrac{79}{2} - 1\right]$$
$$= -2\left[(x-1)^2 - \tfrac{81}{2}\right]$$
$$= 2\left[\tfrac{81}{2} - (x-1)^2\right]$$
$$\text{or } 2\left[40{,}5 - (x-1)^2\right]$$

$$= \int \sqrt{2\left[\tfrac{81}{2} - (x-1)^2\right]}\, dx$$
$$= \sqrt{2}\int \sqrt{\tfrac{81}{2} - (x-1)^2}\, dx$$
$$=$$
$$= \sqrt{2}\left[\frac{\frac{81}{2}}{2} \sin^{-1}\frac{(x-1)}{\sqrt{\frac{81}{2}}} + \frac{(x-1)}{2}\sqrt{\tfrac{81}{2} - (x-1)^2}\right] + c$$
$$\text{or } \sqrt{2}\left[\frac{40{,}5}{2} \sin^{-1}\frac{(x-1)}{\sqrt{40{,}5}} + \frac{(x-1)}{2}\sqrt{40{,}5 - (x-1)^2}\right] + c$$

OR
$$\int \sqrt{-2\left[x^2 - 2x - \tfrac{79}{2}\right]}\, dx$$
$$= \int \sqrt{-2\left[(x^2 - 2x + (1)^2 - (1)^2 - \tfrac{79}{2}\right]}\, dx$$
$$= \int \sqrt{-2\left[(x-1)^2 - \tfrac{81}{2}\right]}\, dx$$
$$= \int \sqrt{2\left[\tfrac{81}{2} - (x-1)^2\right]}\, dx$$

$$= \int \sqrt{81 - 2(x-1)^2}\, dx$$
$$= \frac{81}{2\sqrt{2}} \sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{81}} + \frac{(x-1)}{2}\sqrt{81 - 2(x-1)^2} \quad c$$
$$= 28{,}638 \sin^{-1} 0{,}157(x-1) + \frac{(x-1)}{2}\sqrt{81 - 2(x-1)^2} + c$$

(5)

2.4

$$\int \cos^4 x\, dx$$
$$= \int (\cos^2 x)^2\, dx$$
$$= \int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)^2 dx$$
$$= \int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right) dx$$
$$= \int \left(\tfrac{1}{4} + \tfrac{1}{2}\cos 2x + \tfrac{1}{4}\cos^2 2x\right) dx$$
$$= \tfrac{1}{4}x + \tfrac{1}{2}\cdot\frac{\sin 2x}{2} + \tfrac{1}{4}\left[\frac{x}{2} + \frac{\sin 4x}{8}\right] + c$$
$$= \tfrac{1}{4}x + \tfrac{1}{4}\sin 2x + \frac{x}{8} + \tfrac{1}{32}\sin 4x + c$$
$$\text{or } = \tfrac{3}{8}x + \tfrac{1}{4}\sin 2x + \tfrac{1}{32}\sin 4x$$

or
$$\int (\cos^2 x)^2\, dx$$
$$= \int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2x\right)^2 dx$$
$$= \tfrac{1}{4}\int (1 + \cos 2x)^2\, dx$$

$$= \tfrac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$
$$= \tfrac{1}{4}\left[x + \frac{2\sin 2x}{2} + \left(\frac{x}{2} + \frac{\sin 4x}{8}\right)\right] + c$$
$$= \tfrac{1}{4}x + \tfrac{1}{4}\sin 2x + \frac{x}{8} + \tfrac{1}{32}\sin 4x + c$$

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QUESTION 3

3.1

$$\int \frac{-28x^3+40x^2-18x+4}{x(1-2x)^3}\,dx$$

$$\Rightarrow \frac{-28x^3+40x^2-18x+4}{x(1-2x)^3} = \frac{A}{x} + \frac{B}{(1-2x)^3} + \frac{C}{(1-2x)^2} + \frac{D}{(1-2x)} \checkmark$$

$$\therefore -28x^3+40x^2-18x+4 = A(1-2x)^3 + Bx + Cx(1-2x) + Dx(1-2x)^2 \checkmark$$

Let $x = 0$; $\therefore A = 4$ ✓

$x = \frac{1}{2}$; $\therefore B = 3$ ✓

✓ (method)

$$-28x^3+40x^2-18x+4 = -8Ax^3+12Ax^2-6Ax+A+Bx+Cx-2Cx^2+Dx-4Dx^2+4Dx^3$$

Equate coeff of x^3: $-28 = -8A + 4D$ $\therefore D = 1$ ✓

" " " x^2: $40 = 12A - 2C - 4D$ $\therefore C = 2$ ✓

$$\therefore \int \frac{4}{x}\,dx + \int \frac{3}{(1-2x)^3}\,dx + \int \frac{2}{(1-2x)^2}\,dx + \int \frac{1}{(1-2x)}\,dx \checkmark$$

$$= 4\ln x + 3\int (1-2x)^{-3}\,dx + 2\int (1-2x)^{-2}\,dx + \int \frac{1}{(1-2x)}\,dx$$

$$= 4\ln x + \frac{3}{(-2)} \cdot \frac{(1-2x)^{-2}}{-2} - \frac{(1-2x)^{-1}}{-1} - \frac{1}{2}\ln(1-2x) + c$$

$$= \underline{4\ln x \checkmark + \frac{3}{(1-2x)^2} \checkmark + \frac{1}{(1-2x)} \checkmark - \frac{1}{2}\ln(1-2x) \checkmark + c}$$

(6)

3.2

$$\int \frac{2x^2-2x+2}{2x^2-5x+2}\,dx$$

$$\Rightarrow \begin{array}{r|l} & 1 \checkmark \\ 2x^2-5x+2 & 2x^2-2x+2 \\ & \underline{2x^2-5x+2} \\ & 3x \checkmark \end{array}$$

$$\therefore \frac{2x^2-2x+2}{2x^2-5x+2} = 1 + \frac{3x}{2x^2-5x+2} \checkmark$$

$$\Rightarrow \frac{3x}{2x^2-5x+2} \rightarrow \frac{3x}{(2x-1)(x-2)} = \frac{A}{(2x-1)} + \frac{B}{(x-2)} \checkmark$$

$$\therefore 3x = A(x-2) + B(2x-1) \checkmark$$

$$3x = Ax + 2A + 2Bx - B \checkmark$$

Let $x = \frac{1}{2}$; $\therefore A = -1$ ✓

$x = 2$; $B = 2$ ✓

$$\therefore \rightarrow \int \frac{-1}{(2x-1)}\,dx + \int \frac{2}{(x-2)}\,dx \checkmark$$

$$= -\frac{1}{2}\ln(2x-1) + 2\ln(x-2) + c$$

$$\therefore \int \frac{2x^2-2x+2}{2x^2-5x+2}\,dx = \underline{x \checkmark - \frac{1}{2}\ln(2x-1) \checkmark + 2\ln(x-2) \checkmark + c}$$

(6)
[12]

QUESTION 4

4.1

$$y + \sec x = \frac{dy}{dx} \cdot \tan x$$

$$\sec x = \frac{dy}{dx} \cdot \tan x - y \checkmark$$

$$\therefore \frac{dy}{dx} - \frac{y}{\tan x} = \frac{\sec x}{\tan x} \checkmark$$

$$\frac{dy}{dx} - (\cot x)y = \operatorname{cosec} x$$

$$\rightarrow \quad IR = e^{\int -\cot x} \checkmark$$

$$= e^{-\ln(\sin x)} \checkmark$$

$$= (\sin x)^{-1} \Big/ \frac{1}{\sin x} \checkmark$$

$$= \operatorname{cosec} x$$

$$\rightarrow \quad y \cdot \operatorname{cosec} x = \int \operatorname{cosec} x \cdot \operatorname{cosec} x \, dx$$

$$= \int \operatorname{cosec}^2 x \, dx$$

$$y \cdot \operatorname{cosec} x = -\cot x + c \checkmark$$

$$2 \cdot \operatorname{cosec} \frac{\pi}{4} = -\cot \frac{\pi}{4} + c \checkmark \quad \ldots\left(\frac{\pi}{4}; 2\right)$$

$$\therefore c = 3{,}828$$

$$\rightarrow \quad \underline{y \cdot \operatorname{cosec} x = -\cot x + 3{,}829} \checkmark$$

(6)

4.2

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 6x - 2$$

$$m^2 - 4m + 3 = 0 \checkmark$$

$$(m-3)(m-1) = 0 \checkmark$$

$$\therefore m = 3 \; ; \; m = 1 \checkmark$$

$$y_c = Ae^{3x} + Be^{x} \checkmark$$

$$y_p: \; y = Cx + D \checkmark$$

$$\frac{dy}{dx} = C \checkmark$$

$$\frac{d^2y}{dx^2} = 0 \checkmark$$

$$\therefore \; -4C + 3(Cx + D) = 6x - 2 \checkmark$$

$$-4C + 3Cx + 3D = 6x - 2$$

Equate coeff of x: $3C = 6 \quad \therefore C = 2$ ✓

" constants : $-4C + 3D = -2 \quad \therefore D = 2$ ✓

$$\therefore y_p = 2x + 2 \checkmark$$

$$\underline{y = Ae^{3x} + Be^{x} + 2x + 2} \checkmark$$

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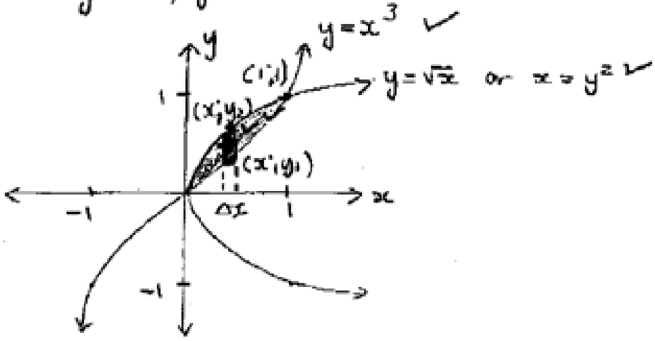
[12]

QUESTION 5

5.1 5.1.1

$$\sqrt{x} = x^3$$
$$x = x^6$$
$$\therefore\ 0 = x(x^5 - 1)$$
$$\left.\begin{array}{l}\therefore\ x = 0\ ;\ x = 1 \\ \quad y = 0\ ;\ y = 1\end{array}\right\}(0;0) \text{ and } (1;1)$$



(3)

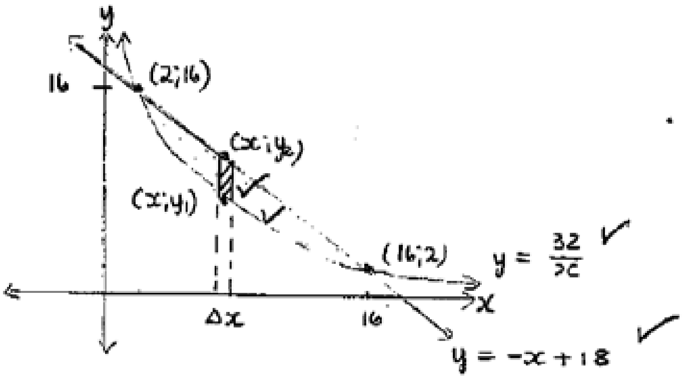
5.1.2

$$\Delta V = 2\pi x\,(y_2 - y_1)\Delta x$$
$$\therefore\ V_y = 2\pi\int_0^1 x(y_2 - y_1)\,dx$$
$$= 2\pi\int_0^1 x(\sqrt{x} - x^3)\,dx$$
$$= 2\pi\int_0^1 (x^{\frac{3}{2}} - x^4)\,dx$$
$$= 2\pi\left[\frac{x^{5/2}}{5/2} - \frac{x^5}{5}\right]_0^1$$
$$= 2\pi\left[\frac{2}{5}(1)^{5/2} - \frac{1}{5}(1)^5\right]$$
$$= 0{,}4\pi \ /\ 1{,}257 \text{ unit}^3$$

(4)

5.2 5.2.1

$$xy = 32 \qquad y = -x + 18$$
$$\therefore \frac{32}{x} = -x + 18$$
$$x^2 - 18x + 32 = 0$$
$$(x - 16)(x - 2) = 0$$
$$\therefore x = 16 \; ; \; x = 2$$
$$y = 2 \; ; \; y = 16$$
$(16;2)$ and $(2;16)$



(3)

5.2.2

$$\Delta V = \pi (y_2^2 - y_1^2)\Delta x$$
$$V_x = \pi \int_2^{16} (y_2^2 - y_1^2)\,dx$$
$$= \pi \int_2^{16} \left[(-x+18)^2 - \left(\frac{32}{x}\right)^2\right] dx$$
$$= \pi \int_2^{16} \left[x^2 - 36x + 324 - \frac{1024}{x^2}\right] dx$$
$$= \pi \left[\frac{x^3}{3} - \frac{36x^2}{2} + 324x - \frac{1024x^{-1}}{-1}\right]_2^{16}$$
$$= \pi\left\{\left[\tfrac{1}{3}(16)^3 - 18(16)^2 + 324(16) + \frac{1024}{16}\right] - \left[\tfrac{1}{3}(2)^3 - 18(2)^2 + 324(2) + \frac{1024}{2}\right]\right\}$$
$$= 2873{,}51 \text{ units}^3 \; / \; 914{,}667\pi \text{ units}^3$$

(4)

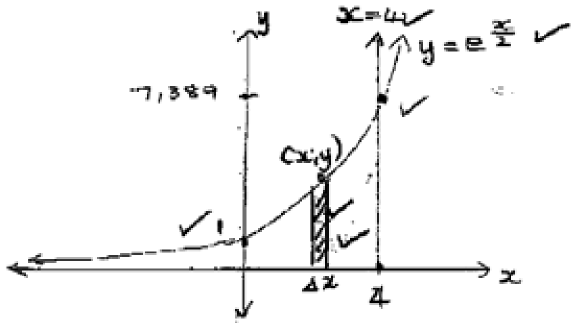
5.2.3

$$\Delta M_y = \pi (y_2^2 - y_1^2)\Delta x \times x$$
$$M_y = \pi \int_2^{16} (y_2^2 - y_1^2)x\,dx$$
$$= \pi \int_2^{16} \left(x^2 - 36x + 324 - \frac{1024}{x^2}\right)x\,dx$$
$$= \pi \int_2^{16} \left(x^3 - 36x^2 + 324x - \frac{1024}{x}\right)dx$$
$$= \pi \left[\frac{x^4}{4} - \frac{36x^3}{3} + \frac{324x^2}{2} - 1024\ln x\right]_2^{16}$$
$$= \pi\left\{\left[\tfrac{1}{4}(16)^4 - 12(16)^3 + 162(16)^2 - 1024\ln 16\right] - \left[\tfrac{1}{4}(2)^4 - 12(2)^3 + 162(2)^2 - 1024\ln 2\right]\right\}$$
$$= 6018{,}652\pi \quad \text{or } 18908{,}152 \text{ units}^4$$
$$\therefore \bar{x} = \frac{18908{,}152}{2873{,}51} = 6{,}581 \text{ units}$$

(6)

5.3 5.3.1

$$y = e^{\frac{x}{2}} \quad ; \quad x = 4$$



(3)

5.3.2

$$\Delta V_x = \pi y^2 \Delta x$$
$$V_x = \pi \int_0^4 y^2 \, dx$$
$$= \pi \int_0^4 e^x \, dx \qquad \bullet \left(e^{\frac{x}{2}}\right)^2$$
$$= \pi \cdot \left[e^x\right]_0^4$$
$$= \pi \left[e^4 - e^0\right]$$
$$= \underline{e^4 \pi \;/\; 54,598\pi \;/\; 171,525 \text{ units}^3}$$

$$\Delta I_x = \rho \cdot \pi y^2 \Delta x \cdot \left(\frac{y}{\sqrt{2}}\right)^2$$
$$I_x = \frac{\rho\pi}{2} \int_0^4 y^4 \, dx$$
$$= \frac{\rho\pi}{2} \int_0^4 e^{2x} \, dx \qquad \bullet \left(e^{\frac{x}{2}}\right)^4$$
$$= \frac{\rho\pi}{2} \cdot \left[\frac{e^{2x}}{2}\right]_0^4$$
$$= \frac{\rho\pi}{4} \left[e^{2(4)} - e^{2(0)}\right]$$
$$= \underline{2340,454\rho \;/\; 744,989\pi\rho}$$

(6)

5.3.3

$$I_x = \frac{2340,454 \text{ m}}{171,525}$$
$$= \underline{13,645 \text{ m}}$$

(2)

5.4

$y = -x + 3$ ✓ or $y = x + 3$

$\therefore x = 3 - y$ ✓ $\therefore x = y - 3$

$\therefore \Delta A = 2x \Delta y$ or $\Delta A = 2x\Delta y$

$dy = 2(3-y)\,dy$ $dA = 2(y-3)dy$

or $dy = 6 - 2y\,dy$ $dA = 2y - 6\,dy$

$\int_0^3 r\,dA$ ✓

$= \int_0^3 (4-y) \cdot 2(3-y)\,dy$ ✓ ✓

$= 2\int_0^3 (12 - 7y + y^2)\,dy$ ✓

$= 2\left[12y - \frac{7}{2}y^2 + \frac{y^3}{3}\right]_0^3$ ✓

$= 2\left[12(3) - \frac{7}{2}(3)^2 + \frac{1}{3}(3)^3\right]$ ✓

$= 27\text{ m}^3$ ✓

$\int_0^3 r^2\,dA$ ✓

$= \int_0^3 (4-y)^2\, 2(3-y)\,dy$ ✓ ✓

$= 2\int_0^3 (48 - 40y + 11y^2 - y^3)\,dy$ ✓

$= 2\left[48y - \frac{40}{2}y^2 + \frac{11y^3}{3} - \frac{y^4}{4}\right]_0^3$ ✓

$= 2\left[48(3) - 20(3)^2 + \frac{11}{3}(3)^3 - \frac{1}{4}(3)^4\right]$ ✓

$= 85,5\text{ m}^4$ ✓

$\therefore \bar{y} = \frac{85,5}{27}$ ✓

$= 3,167\text{ m}$ ✓

(9)
[40]

QUESTION 6

6.1

$$F(x) = \frac{1}{8}x^2 - \ln x$$
$$\therefore F'(x) = \frac{2}{8}x - \frac{1}{x} ✓$$
$$\therefore \frac{dy}{dx} = \frac{1}{4}x - \frac{1}{x} ✓$$
$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{4}x - \frac{1}{x}\right)^2 ✓$$
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{1}{4}x - \frac{1}{x}\right)\left(\frac{1}{4}x - \frac{1}{x}\right) ✓$$
$$= \frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}$$
$$= \frac{x^4 + 8x^2 + 16}{16x^2}$$
$$= \frac{(x^2+4)(x^2+4)}{16x^2} ✓$$
$$\therefore S = \int_1^4 \sqrt{\frac{(x^2+4)^2}{16x^2}}\,dx ✓ \qquad \bullet\ S = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= \int_1^4 \frac{x^2+4}{4x}\,dx ✓$$
$$= \int_1^4 \left(\frac{1}{4}x + \frac{1}{x}\right)dx ✓$$
$$= \left[\frac{1}{4}\cdot\frac{x^2}{2} + \ln x\right]_1^4 ✓$$
$$= \left[\frac{1}{8}(4)^2 + \ln 4\right] - \left[\frac{1}{8}(1)^2 + \ln 1\right] ✓$$
$$= \underline{3{,}261 \text{ units}} ✓$$

(6)

6.2

$$y = \sin x$$
$$\frac{dy}{dx} = \cos x ✓$$
$$\left(\frac{dy}{dx}\right)^2 = \cos^2 x ✓$$
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \cos^2 x ✓$$
$$\therefore A_x = \int_{\pi/2}^{\pi} 2\pi y \sqrt{1+\cos^2 x}\,dx ✓ \qquad \bullet \int_{x_1}^{x_2} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= 2\pi \int_{\pi/2}^{\pi} \sin x \sqrt{1+\cos^2 x}\,dx \qquad \bullet\ u = \cos x$$
$$= 2\pi \int_{\pi/2}^{\pi} \sin x \sqrt{1+u^2}\cdot -\frac{du}{\sin x} ✓ \qquad \frac{du}{dx} = -\sin x$$
$$= -2\pi \int_{\pi/2}^{\pi} \sqrt{1+u^2}\,du ✓ \qquad dx = -\frac{du}{\sin x}$$
$$= -2\pi \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u + \sqrt{1+u^2}\right)\right]_{\pi/2}^{\pi} ✓$$
$$= -2\pi \left[\frac{\cos x}{2}\sqrt{1+\cos^2 x} + \frac{1}{2}\ln\left(\cos x + \sqrt{1+\cos^2 x}\right)\right]_{\pi/2}^{\pi} ✓$$
$$= -2\pi\left\{\left[\frac{1}{2}\cos\pi\sqrt{1+\cos^2\pi} + \frac{1}{2}\ln\left(\cos\pi + \sqrt{1+\cos^2\pi}\right)\right] - \left[\frac{1}{2}\cos\frac{\pi}{2}\sqrt{1+\cos^2\frac{\pi}{2}} + \frac{1}{2}\ln\left(\cos\frac{\pi}{2} + \sqrt{1+\cos^2\frac{\pi}{2}}\right)\right]\right\}$$
$$= \underline{2{,}296\,\pi\ /\ 7{,}213 \text{ units}^2} ✓$$

(6)
[12]

TOTAL: 100

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Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1100**(E)**(N13)T
NOVEMBER 2012

NATIONAL CERTIFICATE

MATHEMATICS N6

(16030186)

13 November (X-Paper)
09:00 – 12:00

This question paper consists of 5 pages and a 7-page formula sheet.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Questions may be answered in any order, but subsections of questions must be kept together.
5. Show ALL the intermediate steps.
6. ALL the formulae used must be written down.
7. Questions must be answered in blue or black ink.
8. Marks indicated are percentages.
9. Write neatly and legibly.

QUESTION 1

1.1 Given: $z = xe^{y} - ye^{x}$

Calculate the values of the following:

1.1.1 $\dfrac{\partial^2 z}{\partial y \partial x}$ (1)

1.1.2 $\dfrac{\partial^2 z}{\partial x^2}$ (1)

1.1.3 $\dfrac{\partial^2 z}{\partial y^2}$ (1)

1.2 The volume of a cylinder is given as V = $\pi r^2 h$ when r = 3,5 cm and h = 12 cm. Calculate the approximate increase in the volume when the radius increases by 1,2 cm and the height decreases by 6 cm. (3)

[6]

QUESTION 2

Determine $\int y \, dx$ if:

2.1 $y = \dfrac{1}{x^2 + x + 7}$ (3)

2.2 $y = \dfrac{x}{2} . \tan^2 x$ (3)

2.3 $y = \dfrac{\sin^5 x}{\cos^2 x}$ (5)

2.4 $y = arc\cos x$ (2)

2.5 $y = \dfrac{3}{\sec^4 3x}$ (5)

[18]

QUESTION 3

Use partial fractions to calculate the following integrals:

3.1 $\int \frac{x^2 - x}{x^2 - 4}\, dx$ (5)

3.2 $\int \frac{10x^2 + 7x + 1}{(1 - 4x)(2x^2 + 1)}\, dx$ (7)

[12]

QUESTION 4

Determine the particular solutions of the following:

4.1 $\frac{dy}{dx} - \frac{2y}{x+2} = (x + 2)^3$ at (3;1) (5)

4.2 $\frac{d^2y}{dx^2} - 49y = e^{7x}$, given that $x = 0$ when $y = 0$ and

$\frac{dy}{dx} = \frac{1}{2}$ when $x = 0$. (7)

[12]

QUESTION 5

5.1 5.1.1 Calculate the points of intersection of $y = -x^2 + x + 12$ and $y = -2x + 12$. Sketch the graphs and show the representative strip/element that you would use to calculate the area bounded by the TWO graphs. (3)

5.1.2 Calculate, by means of integration, the area described in QUESTION 5.1.1. (3)

5.1.3 Calculate the area moment of the enclosed area about the x-axis as well as the y-ordinate of the centroid of the area. (5)

5.2 5.2.1 Calculate the co-ordinates of the points of intersection of $y = -x$ and $2y = -x + 6$. Sketch the graphs and show the area bounded by the graphs and $x = 0$. Show the representative strip/element that you would use to calculate the area bounded by the graphs. (3)

5.2.2 Calculate the area described in QUESTION 5.2.1 (3)

5.2.3 Calculate the second moment of area described in QUESTION 5.2.1 about the x-axis and express the answer in terms of the area. (4)

5.3 5.3.1 Make a neat sketch of $y = 2\sin 2x$. Show the representative strip/element that you would use to calculate the volume generated when the area bounded by the curve $y = 2\sin 2x$ and the x- axis, for $0 \leq x \leq \frac{\pi}{2}$, rotates about the y- axis. (2)

5.3.2 Calculate, by means of integration, the volume described in QUESTION 5.3.1. (4)

5.3.3 Calculate the moment of inertia of a solid obtained when the area described in QUESTION 5.3.1 rotates about the y-axis. (6)

5.4 5.4.1 A vertical sluice gate, in the form of a rectangle, is installed in a dam wall. The horizontal side is 6 m and 1 m below the surface of the water. The vertical side is 5 m. Make a neat sketch of the sluice gate and show the representative strip/element that you would use to calculate the depth of the centre of pressure. Calculate the relation between the TWO variables x and y. (3)

5.4.2 Calculate the second area moment of the sluice gate about the water level and the depth of the centre of pressure on the sluice gate, by means of integration, if the first area moment of the sluice gate about the water level is given as $105\,\text{units}^3$. (4)

[40]

QUESTION 6

6.1 Calculate the length of the curve represented by $6xy = x^4 + 3$ between the points $x = 1$ and $x = 3$. (6)

6.2 Calculate the surface area of revolution generated by rotating the curve described by the parametric equations $x = 6t + t^2$ and $y = 2t + 6$ from t = 0 to t = 1 about the x- axis. (6)

[12]

TOTAL: 100

MATHEMATICS N6

FORMULA SHEET

Any applicable formula may also be used.

Trigonometry

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\operatorname{cosec} x}$; $\cos x = \dfrac{1}{\sec x}$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}+C \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n\,dx$
e^{ax+b}	$e^{ax+b}.\frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)}+C$
a^{dx+e}	$a^{dx+e}.\ln a.\frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)}+C$
$\ln(ax)$	$\frac{1}{ax}\cdot\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}\frac{d}{dx}f(x)$	-
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx}f(x)$	-
$\ln f(x)$	$\frac{1}{f(x)}\cdot\frac{d}{dx}f(x)$	-
$\sin ax$	$a\cos ax$	$-\frac{\cos ax}{a}+C$
$\cos ax$	$-a\sin ax$	$\frac{\sin ax}{a}+C$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$-a\,\mathrm{cosec}^2\,ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a\sec ax\tan ax$	$\frac{1}{a}\ln[\sec ax+\tan ax]+C$
$\mathrm{cosec}\,ax$	$-a\,\mathrm{cosec}\,ax\cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\sin f(x)$	$\cos f(x)\,.\; f'(x)$	-
$\cos f(x)$	$-\sin f(x)\,.\; f'(x)$	-
$\tan f(x)$	$\sec^2 f(x)\,.\; f'(x)$	-
$\cot f(x)$	$-\mathrm{cosec}^2 f(x)\,.\; f'(x)$	-
$\sec f(x)$	$\sec f(x) \tan f(x)\,.\; f'(x)$	-
$\mathrm{cosec}\, f(x)$	$-\mathrm{cosec}\, f(x) \cot f(x)\,.\; f'(x)$	-
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	-
$\tan^{-1} f(x)$	$\frac{f'(x)}{[f(x)]^2+1}$	-
$\cot^{-1} f(x)$	$\frac{-f'(x)}{[f(x)]^2+1}$	-
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	-
$\sin^2(ax)$	-	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	-	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	-	$\frac{1}{a}\tan(ax)-x+C$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)\, dx$
$\cot^2(ax)$	-	$-\frac{1}{a}\cot(ax) - x + C$

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln f(x) + C$$

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + C$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + C$$

$$\int \sqrt{a^2 - b^2x^2}\, dx = \frac{a^2}{2b} \sin^{-1} \frac{bx}{a} + \frac{x}{2}\sqrt{a^2 - b^2x^2} + C$$

$$\int \frac{dx}{a^2 - b^2x^2} = \frac{1}{2ab} \ln \left(\frac{a + bx}{a - bx} \right) + C$$

$$\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2} \ln \left[x + \sqrt{x^2 \pm b^2} \right] + C$$

$$\int \frac{dx}{\sqrt{b^2x^2 \pm a^2}} = \frac{1}{b} \ln \left[bx + \sqrt{b^2x^2 \pm a^2} \right] + C$$

Applications of integration

AREAS

$$A_x = \int_a^b y\,dx \; ; \; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x\,dy \; ; \; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2dx\ ;\ V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right)dx\ ;\ V_x = 2\pi\int_a^b xydy$$

$$V_y = \pi\int_a^b x^2dy\ ;\ V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right)dy\ ;\ V_y = 2\pi\int_a^b xydx$$

AREA MOMENTS

$$A_{m-x} = rdA \qquad A_{m-y} = rdA$$

CENTROID

$$\bar{x} = \frac{A_{m-y}}{A} = \frac{\int_a^b rdA}{A}\ ;\ \bar{y} = \frac{A_{m-x}}{A} = \frac{\int_a^b rdA}{A}$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2dA \quad ; \quad I_y = \int_a^b r^2dA$$

VOLUME MOMENTS

$$V_{m-x} = \int_a^b rdV \quad ; \quad V_{m-y} = \int_a^b rdV$$

CENTRE OF GRAVITY

$$\bar{x} = \frac{v_{m-y}}{V} = \frac{\int_a^b rdV}{V} \quad ; \quad \bar{y} = \frac{v_{m-x}}{V} = \frac{\int_a^b rdV}{V}$$

MOMENTS OF INERTIA

Mass = Density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

CIRCULAR LAMINA

$$I_z = \frac{1}{2} m r^2$$

$$I = \frac{1}{2}\int_a^b r^2 dm = \frac{1}{2}\rho \int_a^b r^2 dV$$

$$I_x = \frac{1}{2}\rho\pi \int_a^b y^4 dx \qquad I_y = \frac{1}{2}\rho\pi \int_a^b x^4 dy$$

CENTRE OF FLUID PRESSURE

$$\overline{\overline{y}} = \frac{\int_a^b r^2 dA}{\int_a^b r dA}$$

$$\frac{f(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots \frac{Z}{(ax+b)^n}$$

$$\frac{f(x)}{(ax+b)^3\,(cx+d)^3} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \frac{D}{(cx+d)} + \frac{E}{(cx+d)^2} + \frac{F}{(cx+d)^3}$$

$$\frac{f(x)}{(ax^2+bx+c)(dx+e)^n} = \frac{Ax+F}{ax^2+bx+c} + \frac{B}{dx+e} + \frac{C}{(dx+e)^2} + \dots + \frac{Z}{(dx+e)^n}$$

$$A_x = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_x = \int_d^c 2\pi y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A_y = \int_d^c 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$A_x = \int_{u1}^{u2} 2\pi y \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$A_y = \int_{u1}^{u2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}\, du$$

$$\frac{dy}{dx} + Py = Q \quad \Rightarrow \quad ye^{\int Pdx} = \int Qe^{\int Pdx}\, dx$$

$$y = Ae^{r_1 x} + Be^{r_2 x} \quad r_1 \neq r_2$$

$$y = e^{rx}(A + Bx) \quad r_1 = r_2$$

$$y = e^{ax}[A\cos bx + B\sin bx] \quad r = a \pm ib$$

$$\frac{d^2 y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx}$$

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higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

13 NOVEMBER 2012

This marking guideline consists of 15 pages.

QUESTION 1

1.1

$$z = xe^{y} - ye^{x}$$
$$\frac{\partial z}{\partial x} = e^{y} - ye^{x}$$

1.1.1

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(e^{y} - ye^{x}\right)$$
$$\frac{\partial^2 z}{\partial y \partial x} = e^{y} - e^{x}$$

(2)

1.1.2

$$\frac{\partial^2 z}{\partial x^2} = -ye^{x}$$

(2)

1.1.3

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial y}\left(xe^{y} - e^{x}\right)$$
$$= xe^{y}$$

(2)

1.2

$$V = \pi r^2 h$$
$$\frac{\partial V}{\partial r} = 2\pi r h$$
$$\frac{\partial V}{\partial h} = \pi r^2$$

$$\Delta V = \frac{\partial V}{\partial r}(\Delta r) + \frac{\partial V}{\partial h}(\Delta h)$$
$$= 2\pi r h(\Delta r) + \pi r^2(\Delta h)$$
$$= 2\pi(3{,}5)(12)(1{,}2) + \pi(3{,}5)^2(-6)$$
$$= 85{,}765 \text{ cm}^3$$

(6)
[12]

QUESTION 2

2.1

$$\int \frac{1}{x^2+x+7}\,dx$$

$$= \int \frac{1}{(x+\frac{1}{2})^2+\frac{35}{4}}\,dx$$

$$= \frac{1}{\frac{\sqrt{35}}{2}}\tan^{-1}\frac{(x+\frac{1}{2})}{\frac{\sqrt{35}}{2}}+c$$

$$= \frac{2}{\sqrt{35}}\tan^{-1}\frac{2(x+\frac{1}{2})}{\sqrt{35}}+c$$

$$= 0{,}338\tan^{-1}\frac{(2x+1)}{5{,}916}+c$$

x^2+x+9

$= \left[(x+\frac{1}{2})^2+9-\frac{1}{4}\right]$

$= (x+\frac{1}{2})^2+\frac{35}{4}$

(6)

2.2

$$\int \frac{x}{2}\cdot\tan^2 x\,dx$$

$f(x)=\frac{x}{2}$ $g'(x)=\tan^2 x$

$f'(x)=\frac{1}{2}$ $g(x)=\tan x - x$

$$= \frac{x}{2}(\tan x - x) - \int \frac{1}{2}(\tan x - x)\,dx$$

$$= \frac{x}{2}(\tan x - x) - \int \frac{1}{2}\tan x\,dx + \int \frac{1}{2}x\,dx$$

$$= \frac{x}{2}\tan x - \frac{x^2}{2} - \frac{1}{2}\ln(\sec x) + \frac{1}{2}\cdot\frac{x^2}{2} + C$$

$$= \frac{x}{2}\tan x - \frac{x^2}{2} - \frac{1}{2}\ln(\sec x) + \frac{1}{4}x^2 + c$$

(6)

OR

$$\int \frac{x}{2}\cdot\tan^2 x\,dx$$

$$= \int \frac{x}{2}(\sec^2 x - 1)\,dx$$

$$= \int\left(\frac{x}{2}\cdot\sec^2 x\right)dx - \int \frac{x}{2}\,dx$$

$$= \frac{1}{2}\int x\cdot\sec^2 x\,dx - \frac{1}{2}\int x\,dx$$

$f(x)=x$ $g'(x)=\sec^2 x$

$f'(x)=1$ $g(x)=\tan x$

$$= \frac{1}{2}\left[x\cdot\tan x - \int \tan x\,dx\right] - \frac{1}{2}\cdot\frac{x^2}{2} + c$$

$$= \frac{1}{2}x\tan x - \frac{1}{2}\ln(\sec x) - \frac{1}{4}x^2 + c$$

2.3

$$\int \frac{\sin^5 x}{\cos^2 x}\,dx$$
$$= \int \frac{(\sin^2 x)^2 . \sin x}{\cos^2 x}\,dx$$
$$= \int \frac{(1-\cos^2 x)^2 . \sin x}{\cos^2 x}\,dx$$

$u = \cos x$
$du = -\sin x\,dx$

$$= -\int \frac{(1-u^2)^2}{u^2}\,du$$
$$= -\int \frac{(1-2u^2+u^4)}{u^2}\,du$$
$$= -\int (u^{-2} - 2 + u^2)\,du$$
$$= -\left[\frac{u^{-1}}{-1} - 2u + \frac{u^3}{3}\right] + c$$
$$= -\left[\frac{1}{-\cos x} - 2\cos x + \frac{1}{3}\cos^3 x\right] + c$$

(10)

or
$$= \frac{1}{\cos x} + 2\cos x - \frac{1}{3}\cos^3 x + c$$

OR
$$\int \frac{(\sin^2 x)^2 . \sin x}{\cos^2 x}\,dx$$
$$= \int \frac{(1-\cos^2 x)(1-\cos^2 x)\sin x}{\cos^2 x}\,dx$$
$$= \int \frac{(1-2\cos^2 x + \cos^4 x)\sin x}{\cos^2 x}\,dx$$
$$= \int \frac{(\sin x - 2\cos^2 x . \sin x + \cos^4 x . \sin x)\,dx}{\cos^2 x}$$
$$= \int \sin x . (\cos x)^{-2}\,dx - \int 2\sin x\,dx + \int \cos^2 x \sin x\,dx$$
$$= -\frac{(\cos x)^{-1}}{-1} + 2\cos x - \frac{\cos^3 x}{3} + c$$

or
$$= \frac{1}{\cos x} + 2\cos x - \frac{1}{3}\cos^3 x + c$$

2.4

$$\int \cos^{-1} x\,dx$$

$f(x) = \cos^{-1} x \quad g'(x$
$f'(x) = -\frac{1}{\sqrt{1-x^2}} \quad g(x$

$$= x . \cos^{-1} x - \int -\frac{1}{\sqrt{1-x^2}} . x\,dx$$
$$= x . \cos^{-1} x + \int x(1-x^2)^{-1/2}\,dx$$
$$= x . \cos^{-1} x - \frac{1}{2}\int -2x(1-x^2)^{-\frac{1}{2}}\,dx$$
$$= x . \cos^{-1} x - \frac{1}{2} . \frac{(1-x^2)^{1/2}}{1/2} + c$$
$$= x . \cos^{-1} x - (1-x^2)^{\frac{1}{2}} + c$$

(2)

2.5

$$\int \frac{3}{\sec^4 3x}\,dx$$

$$= \int 3\cos^4 3x\,dx$$

$$= 3\int (\cos^2 3x)^2\,dx$$

$$= 3\int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right)^2 dx$$

$$= 3\int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right)\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right) dx$$

$$= 3\int \left(\tfrac{1}{4} + \tfrac{1}{2}\cos 6x + \tfrac{1}{4}\cos^2 6x\right) dx$$

$$= 3\left[\tfrac{1}{4}x + \tfrac{1}{2}\cdot\frac{\sin 6x}{6} + \tfrac{1}{4}\left(\frac{x}{2} + \frac{\sin 12x}{4.6}\right)\right] + c$$

$$= \frac{3}{4}x + \frac{1}{4}\sin 6x + \frac{3}{8}x + \frac{1}{32}\sin 12x + c$$

(10)

$$\int 3\cos^4 3x\,dx$$

$$= 3\int (\cos^2 3x)^2\,dx$$

$$= 3\int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right)^2 dx$$

$$= 3\int \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right)\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 6x\right) dx$$

$$= \frac{3}{4}\int (1 + \cos 6x)(1 + \cos 6x)\,dx$$

$$= \frac{3}{4}\int (1 - 2\cos 6x + \cos^2 6x)\,dx$$

$$= \frac{3}{4}\left[x - 2\cdot\frac{\sin 6x}{6} + \left(\frac{x}{2} + \frac{\sin 12x}{24}\right)\right] + c$$

$$= \frac{3}{4}x - \frac{1}{4}\sin 6x + \frac{3}{8}x + \frac{1}{32}\sin 12x + c$$

[18]

QUESTION 3

3.1

$$\int \frac{x^2 - x}{x^2 - 4}\, dx$$

$$x^2 - 4 \overline{\big)\, x^2 - x \,}\quad 1$$

$$\underline{x^2 - 4}$$

$$-x + 4 \checkmark$$

$$\therefore \frac{x^2 - x}{x^2 - 4} = 1 - \frac{-x+4}{x^2-4}$$

$$\Rightarrow \int 1\, dx - \int \frac{-x+4}{x^2-4} \checkmark\, dx$$

$$\Rightarrow \frac{-x+4}{(x-2)(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x+2)} \checkmark$$

$$-x + 4 = A(x+2) + B(x-2) \checkmark$$

Let $x = 2$: $A = \frac{1}{2}$ ✓

$x = -2$: $B = -\frac{3}{2}$ ✓

$$\therefore \int 1\, dx - \int \frac{\frac{1}{2}}{(x-2)}\, dx + \int \frac{-\frac{3}{2}}{(x+2)}\, dx \checkmark$$

$$= \int 1\, dx - \frac{1}{2}\int \frac{1}{x-2}\, dx - \frac{3}{2}\int \frac{1}{x+2}\, dx$$

$$= x \checkmark - \frac{1}{2}\ln(x-2) \checkmark - \frac{3}{2}\ln(x+2) \checkmark + C$$

(10)

3.2

$$\int \frac{10x^2+7x+1}{(1-4x)(2x^2+1)}\,dx$$

$$\Rightarrow \frac{10x^2+7x+1}{(1-4x)(2x^2+1)} = \frac{A}{(1-4x)} + \frac{Bx+C}{(2x^2+1)}$$

$$10x^2+7x+1 = A(2x^2+1) + (Bx+C)(1-4x)$$

Let $x = \frac{1}{4}$; $\therefore A = 3$

$$\therefore 10x^2+7x+1 = 2Ax^2 + A + Bx + C - 4Bx^2 - 4Cx$$

Equate coeff. of x^2: $10 = 2A - 4B \quad \therefore B = -1$

x : $7 = B - 4C \quad \therefore C = -2$

$$\therefore \int \frac{3}{1-4x}\,dx + \int \frac{-x-2}{2x^2+1}\,dx$$

$$= 3\int \frac{1}{1-4x}\,dx - \int \frac{x}{2x^2+1}\,dx - \int \frac{2}{2x^2+1}\,dx$$

$$= \frac{3}{-4}\int \frac{-4}{1-4x}\,dx - \frac{1}{4}\int \frac{4x}{2x^2+1} - 2\int \frac{1}{2x^2+1}\,dx$$

$$= -\frac{3}{4}\ln(1-4x) - \frac{1}{4}\ln(2x^2+1) - 2\left(\frac{1}{\sqrt{2}}\right)\tan^{-1}\sqrt{2}x + C$$

(14)
[24]

QUESTION 4

4.1

$$\frac{dy}{dx} - \frac{2y}{x+2} = (x+2)^3$$

$$\therefore R = e^{-\int \frac{2}{x+2}\,dx}$$

$$= e^{-2\ln(x+2)}$$

$$= e^{\ln(x+2)^{-2}}$$

$$= (x+2)^{-2} \Big/ \frac{1}{(x+2)^2}$$

$$\frac{1}{(x+2)^2}\cdot y = \int \frac{1}{(x+2)^2}\cdot (x+2)^3\,dx$$

$$= \int (x+2)\,dx$$

$$\frac{y}{(x+2)^2} = \frac{x^2}{2} + 2x + c \quad \ldots\ldots (3;1)$$

$$\frac{1}{(3+2)^2} = \frac{3^2}{2} + 2(3) + c$$

$$c = -10{,}46$$

$$\therefore \frac{y}{(x+2)^2} = \frac{x^2}{2} + 2x - 10{,}46$$

(10)

4.2

$$\frac{d^2y}{dx^2} - 49y = e^{7x}$$

$$m^2 - 49 = 0 \checkmark$$

$$(m-7)(m+7) = 0$$

$$\therefore m = 7 \text{ or } m = -7 \checkmark$$

$$\therefore y_c = Ae^{7x} + Be^{-7x} \checkmark$$

$$y_p: \; y = Cxe^{7x} \checkmark$$

$$\frac{dy}{dx} = Ce^{7x} + 7Cxe^{7x} \checkmark$$

$$\frac{d^2y}{dx^2} = 7Ce^{7x} + 7Ce^{7x} + 49Cxe^{7x} \checkmark$$

$$= 14Ce^{7x} + 49Cxe^{7x}$$

$$\therefore 14Ce^{7x} + 49Cxe^{7x} - 49Cxe^{7x} = e^{7x} \checkmark$$

$$\therefore 14C = 1$$

$$C = \frac{1}{14} \; (0{,}071) \checkmark$$

$$\therefore y_p = \frac{1}{14}xe^{7x} \checkmark$$

$$\therefore y = Ae^{7x} + Be^{-7x} + \frac{1}{14}xe^{7x} \checkmark$$

$$0 = A + B \qquad \therefore A = -B$$

$$\frac{dy}{dx} = 7Ae^{7x} - 7Be^{-7x} + \frac{1}{14}e^{7x} + \frac{1}{2}xe^{7x} \checkmark$$

$$\frac{1}{2} = 7A - 7B - \frac{1}{14}$$

$$\frac{4}{7} = 7A - 7B \qquad \therefore \frac{4}{7} = 7(-B) - 7B$$

$$\frac{4}{7} = -14B$$

$$B = -\frac{2}{49} \; (-0{,}041) \checkmark \qquad \therefore A = \frac{2}{49} \; (0{,}041) \checkmark$$

$$\therefore y = \frac{2}{49}e^{7x} - \frac{2}{49}e^{-7x} + \frac{1}{14}xe^{7x} \checkmark$$

$$\text{or } y = 0{,}041e^{7x} - 0{,}041e^{-7x} + 0{,}071e^{7x}$$

(14)
[24]

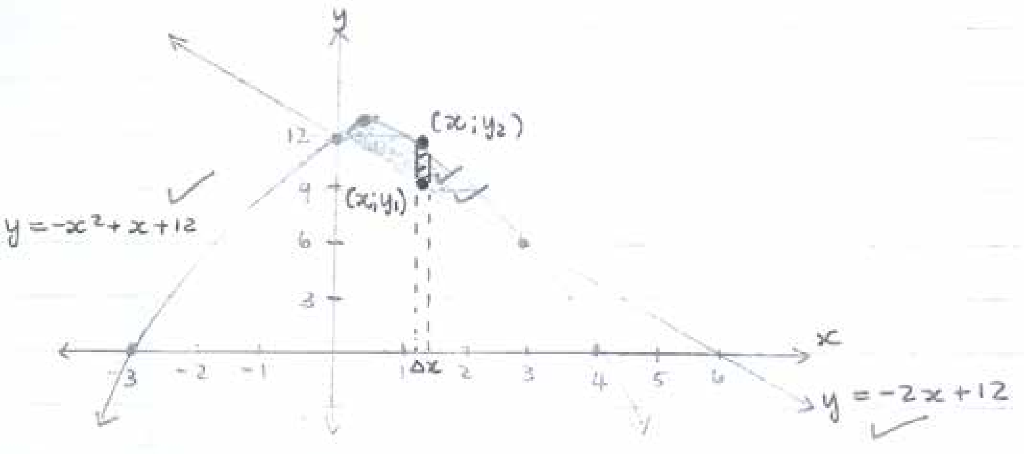
QUESTION 5

5.1 5.1.1

$$-x^2 + x + 12 = -2x + 12$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$
$$\therefore x = 0 \quad \text{or} \quad x = 3 \checkmark$$
$$y = 12 \qquad y = 6 \checkmark$$

or

$(0;12)$ ($\checkmark$)
$(3;6)$ ($\checkmark$)



(4 + 2) (6)

5.1.2

$$\Delta A = (y_2 - y_1)\Delta x \checkmark$$
$$A = \int_0^3 \checkmark (y_2 - y_1)\,dx$$
$$= \int_0^3 (-x^2 + x + 12 + 2x - 12)\,dx \checkmark$$
$$= \int_0^3 (-x^2 + 3x)\,dx$$
$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_0^3 \checkmark$$
$$= \left[-\frac{1}{3}(3)^3 + \frac{3}{2}(3)^2\right] \checkmark$$
$$= \underline{4,5 \text{ units}^2} \checkmark$$

(6)

5.1.3

$$\Delta M_x = (y_2 - y_1)\,\Delta x \times \frac{y_2 + y_1}{2}$$

$$M_x = \frac{1}{2}\int_0^3 (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^3 \left[(-x^2+x+12)^2 - (-2x+12)^2\right] dx$$

$$= \frac{1}{2}\int_0^3 (x^4 - 2x^3 - 27x^2 + 72x)\,dx$$

$$= \frac{1}{2}\left[\frac{x^5}{5} - \frac{2x^4}{4} - \frac{27x^3}{3} + \frac{72x^2}{2}\right]_0^3$$

$$= \frac{1}{2}\left[\frac{1}{5}(3)^5 - \frac{1}{2}(3)^4 - 9(3)^3 + 36(3)^2\right]$$

$$= 44,55 \text{ units}^3$$

$$\therefore \bar{y} = \frac{44,55}{4,5}$$

$$= 9,9 \text{ units}$$

(10)

5.2 5.2.1

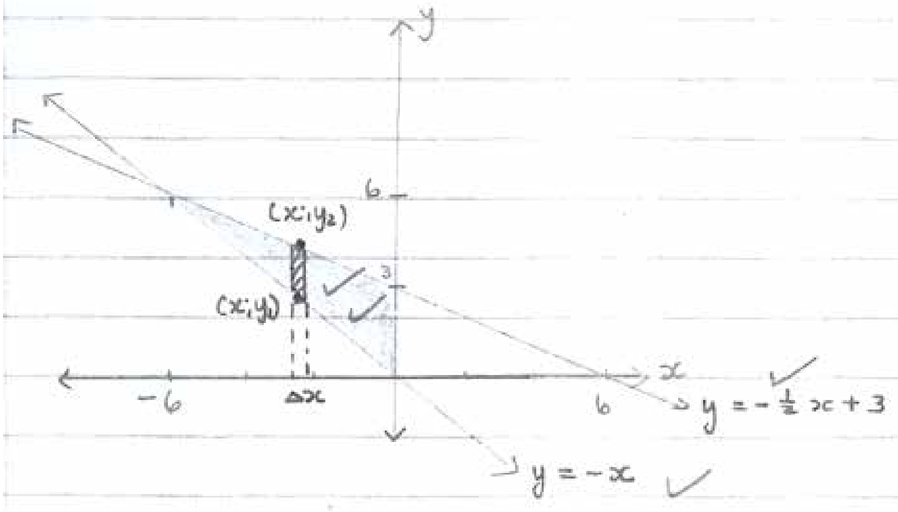
$$-\frac{1}{2}x + 3 = -x$$

$$x - 6 = 2x$$

$$-6 = x$$

$$\therefore y = 6$$

OR $(-6;6)$



(6)

5.2.2

$$\Delta A = (y_2 - y_1)\Delta x$$
$$A = \int_{-6}^{0} (y_2 - y_1)\,dx$$
$$= \int_{-6}^{0} \left(-\tfrac{1}{2}x + 3 + x\right) dx$$
$$= \left[-\tfrac{1}{2}\cdot\tfrac{x^2}{2} + 3x + \tfrac{x^2}{2}\right]_{-6}^{0} \quad / \left[\tfrac{1}{4}x^2 + 3x\right]_{-6}^{0}$$
$$= \left[-\tfrac{1}{4}x^2 + 3x + \tfrac{1}{2}x^2\right]$$
$$= [0] - \left[-\tfrac{1}{4}(-6)^2 + 3(-6) + \tfrac{1}{2}(-6)^2\right]$$
$$= 9 \text{ units}^2$$

(6)

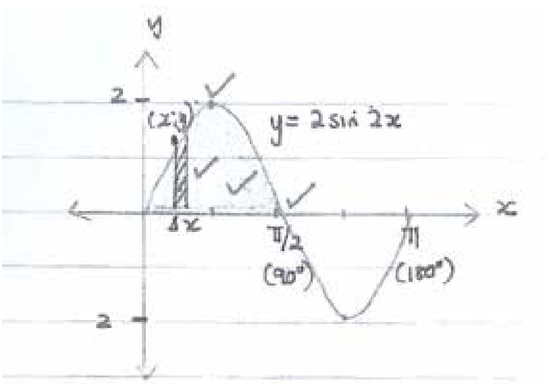
5.2.3

$$\Delta I = (y_2 - y_1)\Delta x \times x^2$$
$$I = \int_{-6}^{0} (y_2 - y_1)x^2\,dx$$
$$= \int_{-6}^{0} \left(-\tfrac{1}{2}x + 3 + x\right)x^2\,dx$$
$$= \int_{-6}^{0} \left(-\tfrac{1}{2}x^3 + 3x^2 + x^3\right) dx \quad / \int_{-6}^{0} \left(\tfrac{1}{2}x^3 + 3x^2\right) dx$$
$$= \left[-\tfrac{1}{2}\cdot\tfrac{x^4}{4} + \tfrac{3x^3}{3} + \tfrac{x^4}{4}\right]_{-6}^{0}$$
$$= 0 - \left[-\tfrac{1}{4}(-6)^4 + (-6)^3 + \tfrac{1}{4}(-6)^4\right]$$
$$= 54 \text{ units}^4$$

$$\therefore I_y = \frac{54A}{9}$$
$$= 6A$$

(8)

5.3 5.3.1



(4)

5.3.2

$$\Delta V = 2\pi x \times y \times \Delta x$$

$$V_y = 2\pi\int_0^{\pi/2} x\,y\,dx$$

$$= 2\pi\int_0^{\pi/2} x\,(2\sin 2x)\,dx$$

$f(x) = x \quad g'(x) = 2\sin 2x$

$f'(x) = 1 \quad g(x) = -\frac{2\cos 2x}{2}$

$$= 2\pi\left[-x\cos 2x - \int -\cos 2x\,dx\right]_0^{\pi/2}$$

$$= 2\pi\left[-x\cos 2x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= 2\pi\left[-\frac{\pi}{2}\cos 2(\tfrac{\pi}{2}) + \tfrac{1}{2}\sin 2(\tfrac{\pi}{2})\right]$$

$$= \pi^2 \,/\, 9{,}8 \text{ units}^3$$

(8)

5.3.3

$$\Delta I_y = \rho(2\pi x y \Delta x) \times x^2$$

$$I_y = 2\pi\rho\int_0^{\pi/2} x^3 y\,dx$$

$$= 2\pi\rho\int_0^{\pi/2} x^3(2\sin 2x)\,dx$$

$f(x) = x^3 \quad g'(x) = 2\sin 2x$

$f'(x) = 3x^2 \quad g(x) = -\frac{2\cos 2x}{2}$

$$= 2\pi\rho\left[-x^3.\cos 2x - \int 3x^2(-\cos 2x)\,dx\right]_0^{\pi/2}$$

$$= 2\pi\rho\left[-x^3\cos 2x + 3\int x^2\cos 2x\,dx\right]_0^{\pi/2}$$

$f(x) = x^2 \quad g'(x) = \cos 2x$

$f'(x) = 2x \quad g(x) = \frac{\sin 2x}{2}$

$$= 2\pi\rho\left[-x^3\cos 2x + 3\left(\tfrac{1}{2}x^2\sin 2x - \int 2x.\frac{\sin 2x}{2}\,dx\right)\right]_0^{\pi/2}$$

$$= 2\pi\rho\left[-x^3\cos 2x + \frac{3}{2}x^2\sin 2x - 3\int x\sin 2x\,dx\right]_0^{\pi/2}$$

$f(x) = x \quad g'(x) = \sin 2x$

$f'(x) = 1 \quad g(x) = \frac{-\cos 2x}{2}$

$$= 2\pi\rho\left[-x^3\cos 2x + \frac{3}{2}x^2\sin 2x - 3\left(-\tfrac{1}{2}x\cos 2x + \int\tfrac{1}{2}\cos 2x\,dx\right)\right]_0^{\pi/2}$$

$$= 2\pi\rho\left[-x^3\cos 2x + \frac{3}{2}x^2\sin 2x + \frac{3}{2}x\cos 2x - \frac{3}{2}\cdot\frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= 2\pi\rho\left\{\left[-(\tfrac{\pi}{2})^3\cos 2(\tfrac{\pi}{2}) + \tfrac{3}{2}(\tfrac{\pi}{2})^2\sin 2(\tfrac{\pi}{2}) + \tfrac{3}{2}(\tfrac{\pi}{2})\cos 2(\tfrac{\pi}{2}) - \tfrac{3}{4}\sin 2(\tfrac{\pi}{2})\right] - [0]\right\}$$

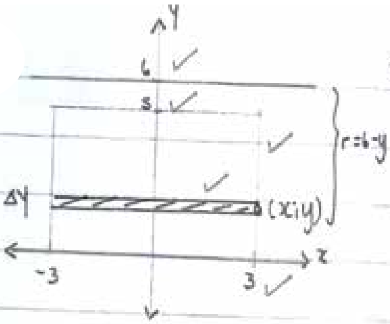
$$= 3{,}039\pi\rho$$

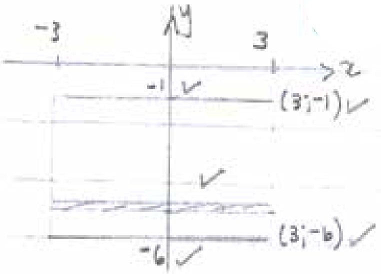
Or $= 9{,}548\,\rho$

(12)

OR

5.4 5.4.1





$x = 3$

$\therefore \Delta A = 2x\,\Delta y$

$dA = 2(3)\,dy$

$dA = 6\,dy$

$x = 3$

$\therefore dA = 6\,dy$

(6)

5.4.2

$$\int_0^5 r^2\,dA$$
$$= \int_0^5 (6-y)^2\,6\,dy$$
$$= 6\int_0^5 (36 - 12y + y^2)\,dy$$
$$= 6\left[36y - \frac{12y^2}{2} + \frac{y^3}{3}\right]_0^5$$
$$= 6\left[36(5) - 6(5)^2 + \tfrac{1}{3}(5)^3\right]$$
$$= 430 \text{ units}^4$$

$$\bar{\bar{y}} = \frac{430}{105}$$
$$= 4{,}095 \text{ units}$$

$$\therefore \int_{-6}^{-1} y^2\,6\,dy$$
$$= 6\int_{-6}^{-1} y^2\,dy$$
$$= 6\left[\frac{y^3}{3}\right]_{-6}^{-1}$$
$$= 2\left[(-1)^3 - (-6)^3\right]$$
$$= 430 \text{ units}^4$$

$$\therefore \bar{\bar{y}} = \frac{430}{105}$$
$$= 4{,}095 \text{ units}$$

(8)
[80]

QUESTION 6

6.1

$$6xy = x^4 + 3$$
$$y = \frac{1}{6}x^3 + \frac{1}{2}x^{-1} \checkmark$$
$$\frac{dy}{dx} = \frac{3}{6}x^2 - \frac{1}{2}x^{-2} \checkmark$$
$$= \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$$
$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)\left(\frac{x^2}{2} - \frac{1}{2x^2}\right) \checkmark$$
$$= \frac{x^4}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4x^4}$$
$$= \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4} \checkmark$$
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^8 - 2x^4 + 1}{4x^4} \checkmark$$
$$= \frac{x^8 + 2x^4 + 1}{4x^4}$$
$$\therefore S = \int_1^3 \checkmark \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \checkmark$$
$$= \int_1^3 \sqrt{\frac{x^8 + 2x^4 + 1}{4x^4}}\, dx$$
$$= \int_1^3 \sqrt{\frac{(x^4+1)(x^4+1)}{4x^4}}\, dx \checkmark$$
$$= \int_1^3 \frac{x^4+1}{2x^2} \checkmark$$
$$= \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right) dx \checkmark$$
$$= \left[\frac{1}{2}\cdot\frac{x^3}{3} + \frac{1}{2}\cdot\frac{x^{-1}}{-1}\right]_1^3 \checkmark$$
$$= \frac{1}{2}\left\{\left[\frac{1}{3}(3)^3 - \frac{1}{3}\right] - \left[\frac{1}{3}(1)^3 - \frac{1}{1}\right]\right\}$$
$$= \frac{28}{6} \;/\; \frac{14}{3} \;/\; 4{,}667 \text{ units} \checkmark$$

(12)

6.2

$$x = 6t + t^2 \qquad y = 2t + 6$$

$$\frac{dx}{dt} = 6 + 2t \checkmark \qquad \frac{dy}{dt} = 2 \checkmark$$

$$\left(\frac{dx}{dt}\right)^2 = (6+2t)^2 \checkmark \qquad \left(\frac{dy}{dt}\right)^2 = 4 \checkmark$$

$$\therefore \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 36 + 24t + 4t^2 + 4 \checkmark$$

$$= 4t^2 + 24t + 40$$

$$\therefore A = \int_0^1 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \checkmark \checkmark$$

$$= 2\pi \int_0^1 (2t+6) \sqrt{4t^2 + 24t + 40}\, dt \checkmark$$

$$= 2\pi \int_0^1 (2t+6) \sqrt{4(t^2 + 6t + 10)}\, dt \checkmark$$

$$= 4\pi \int_0^1 (2t+6)(t^2 + 6t + 10)^{\frac{1}{2}}\, dt$$

$$= 4\pi \left[\frac{(t^2 + 6t + 10)^{3/2}}{3/2}\right]_0^1 \checkmark$$

$$= \frac{8\pi}{3}\left\{\left[(1)^2 + 6(1) + 10\right]^{3/2} - \left[10^{3/2}\right]\right\} \checkmark$$

$$= \underline{322{,}286 \text{ units}^2} \;/\; 102{,}587\pi \text{ units}^2 \checkmark$$

(12)
[24]

TOTAL: 100