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TTK4135 – Lecture 20
Summing up / Nonlinear MPC / Control applications

Lecturer: Lars Imsland

Outline

- Summing up optimization

- Today: Control and (nonlinear) optimization
  - Nonlinear MPC, some “practical”/industrial issues on MPC
  - Reference: F&H 4.5, 4.6

- Some examples from literature using concepts from this course

Numerical optimization in a nutshell (in this course)

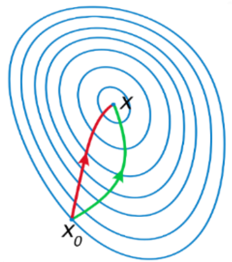
- Unconstrained optimization
  - Search directions: Steepest descent, Newton, Quasi-Newton
  - Globalization: Line-search and (possibly) Hessian modification (**Alg. 6.1**)
- Constrained optimization
  - Optimality conditions, KKT
  - Linear programming: Standard form, KKT conditions, BFPs, SIMPLEX method (**Proc. 13.1**)
  - Quadratic programming: EQP/QP, KKT system, Active set method (**Alg. 16.3**)
  - Nonlinear programming: Nonlinear equations (**Alg. 11.1**), SQP-method (**Alg. 18.3**)

Learning goal Ch. 2, 3 and 6: Understand this slide

Line-search unconstrained optimization

$$\min_x f(x)$$

1. Initial guess x_0
2. While termination criteria not fulfilled
   a) Find descent direction p_k from x_k
   b) Find appropriate step length α_k ; set $x_{k+1} = x_k + \alpha_k p_k$
   c) $k = k+1$
3. $x_M = x^*$? (possibly check sufficient conditions for optimality)



A comparison of steepest descent and Newton's method. Newton's method uses curvature information to take a more direct route. (wikipedia.org)

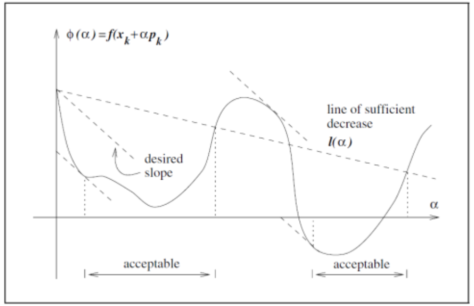
Termination criteria:
Stop when first of these become true:

- $\|\nabla f(x_k)\| \leq \epsilon$ (necessary condition)
- $\|x_k - x_{k-1}\| \leq \epsilon$ (no progress)
- $\|f(x_k) - f(x_{k-1})\| \leq \epsilon$ (no progress)
- $k \leq k_{\max}$ (kept on too long)

Descent directions:

- Steepest descent
  $p_k = -\nabla f(x_k)$
- Newton
  $p_k = -(\nabla^2 f(x_k))^{-1} \nabla f(x_k)$
- Quasi-Newton
  $p_k = -B_k^{-1} \nabla f(x_k)$
  $B_k \approx \nabla^2 f(x_k)$

Step length line search (Wolfe):



How to calculate derivatives – Ch. 8

How many iterations? (Convergence rates)

Quasi-Newton: BFGS method

$$p_k = -B_k^{-1} \nabla f(x_k)$$
$$B_k \approx \nabla^2 f(x_k)$$
$$H_k = B_k^{-1}$$

Algorithm 6.1 (BFGS Method).

Given starting point x_0, convergence tolerance $\epsilon > 0$,
inverse Hessian approximation H_0;
$k \leftarrow 0$;
while $\|\nabla f_k\| > \epsilon$;
Compute search direction

$$p_k = -H_k \nabla f_k;$$

Set $x_{k+1} = x_k + \alpha_k p_k$ where α_k is computed from a line search procedure to satisfy the Wolfe conditions (3.6);
Define $s_k = x_{k+1} - x_k$ and $y_k = \nabla f_{k+1} - \nabla f_k$;
Compute H_{k+1} by means of (6.17);
$k \leftarrow k + 1$;
end (while)

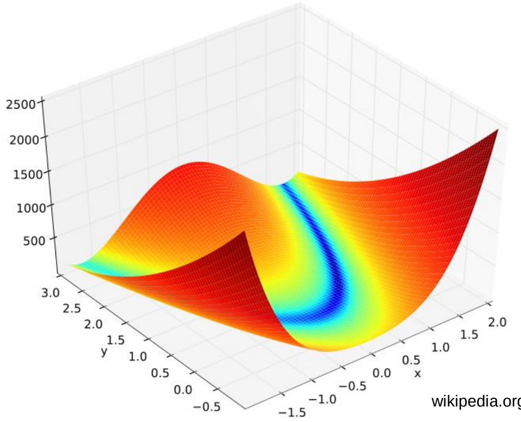
We use only gradient!

$$H_{k+1} = (I - \rho_k s_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T$$

Example (from book)

- Using steepest descent, BFGS and inexact Newton on Rosenbrock function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$



wikipedia.org

- Iterations from starting point (-1.2,1):
  - Steepest descent: 5264
  - BFGS: 34
  - Newton: 21

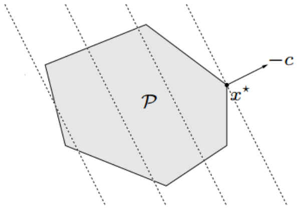
- Last iterations; value of $\|x_k - x^*\|$

steepest descent	BFGS	Newton
1.827e-04	1.70e-03	3.48e-02
1.826e-04	1.17e-03	1.44e-02
1.824e-04	1.34e-04	1.82e-04
1.823e-04	1.01e-06	1.17e-08

Types of Constrained Optimization Problems

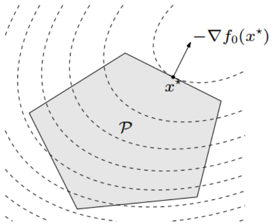
- Linear programming
  - Convex problem
  - Feasible set polyhedron

$$\begin{aligned} \min \quad & c^\top x \\ \text{subject to} \quad & Ax \leq b \\ & Cx = d \end{aligned}$$



- Quadratic programming
  - Convex problem if $P \geq 0$
  - Feasible set polyhedron

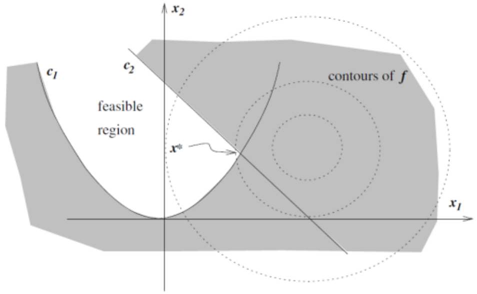
$$\begin{aligned} \min \quad & \frac{1}{2} x^\top P x + q^\top x \\ \text{subject to} \quad & Ax \leq b \\ & Cx = d \end{aligned}$$



- Nonlinear programming
  - In general non-convex!

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to} \quad & g(x) = 0 \\ & h(x) \geq 0 \end{aligned}$$

$$\min_{x \in \mathbb{R}^n} f(x) \qquad \text{subject to} \quad \begin{aligned} c_i(x) &= 0, \quad i \in \mathcal{E}, \\ c_i(x) &\geq 0, \quad i \in \mathcal{I}. \end{aligned}$$



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KKT conditions (Theorem 12.1)

$$\min_{x \in \mathbb{R}^n} f(x) \qquad \text{subject to} \quad \begin{array}{ll} c_i(x) = 0, & i \in \mathcal{E}, \\ c_i(x) \geq 0, & i \in \mathcal{I}. \end{array}$$

KKT-conditions (First-order necessary conditions): If x^* is a local solution and LICQ holds, then there exist λ^* such that

$$\begin{aligned} \nabla_x \mathcal{L}(x^*, \lambda^*) &= 0, && && \text{(stationarity)} \\ c_i(x^*) &= 0, && \forall i \in \mathcal{E}, && \\ c_i(x^*) &\geq 0, && \forall i \in \mathcal{I}, && \text{(primal feasibility)} \\ \lambda_i^* &\geq 0, && \forall i \in \mathcal{I}, && \text{(dual feasibility)} \\ \lambda_i^* c_i(x^*) &= 0, && \forall i \in \mathcal{E} \cup \mathcal{I}. && \text{(complementarity condition/ complementary slackness)} \end{aligned}$$

Starting point for all algorithms for constrained optimization in this course!

Linear programming, standard form and KKT

LP: $\min_{x \in \mathbb{R}^n} c^T x \quad \text{subject to} \quad \begin{cases} a_i x = b_i, & i \in \mathcal{E} \\ a_i x \geq b_i, & i \in \mathcal{I} \end{cases}$

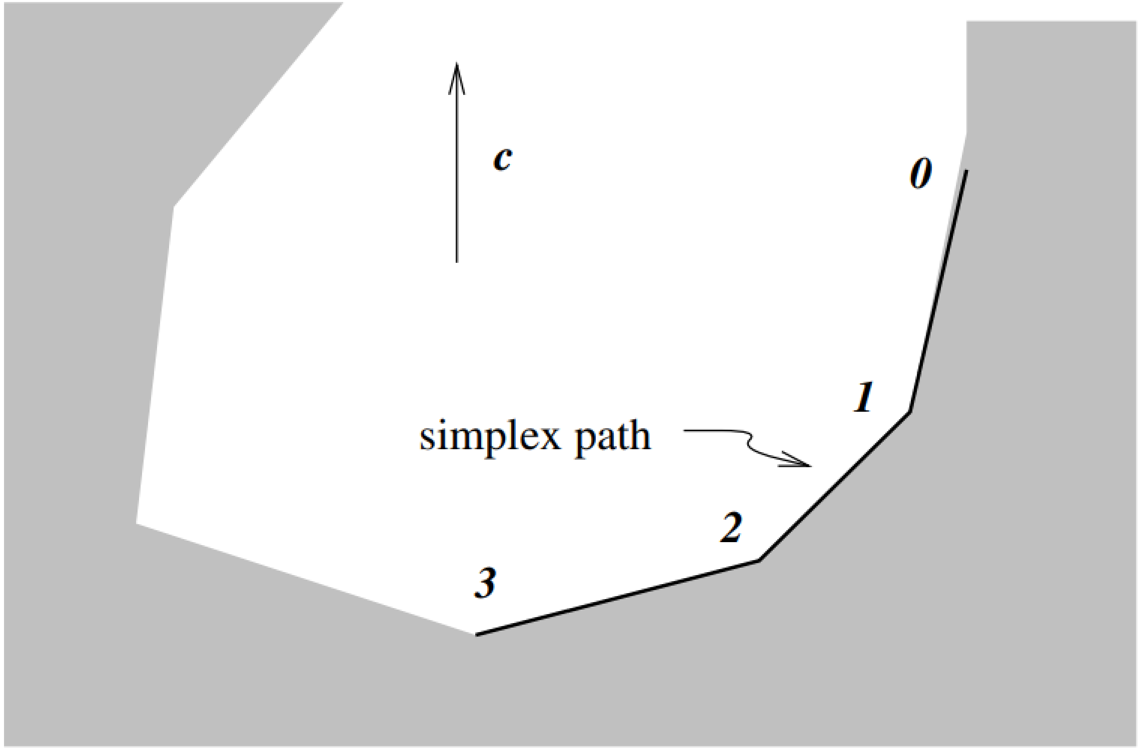
LP, standard form: $\min_{x \in \mathbb{R}^n} c^T x \quad \text{subject to} \quad \begin{cases} Ax = b \\ x \geq 0 \end{cases}$

Lagrangian: $\mathcal{L}(x, \lambda, s) = c^T x - \lambda^T (Ax - b) - s^T x$

KKT-conditions (LPs: necessary *and* sufficient for optimality):

$$
\begin{aligned}
A^T \lambda^* + s^* &= c, \\
Ax^* &= b, \\
x^* &\geq 0, \\
s^* &\geq 0, \\
x_i^* s_i^* &= 0, \quad i = 1, 2, \ldots, n
\end{aligned}
$$

Simplex method: BFP and KKT



General QP problem

$$\begin{aligned}\min_x \; & \frac{1}{2}x^\top G x + x^\top c \\ \text{s.t. } & a_i^\top x = b_i, \qquad i \in \mathcal{E} \\ & a_i^\top x \geq b_i, \qquad i \in \mathcal{I}\end{aligned}$$

- Lagrangian

$$\mathcal{L}(x^*, \lambda^*) = \frac{1}{2}x^\top G x + x^\top c - \sum_{i \in \mathcal{E} \cup \mathcal{I}} \lambda_i (a_i^\top x - b_i)$$

- KKT conditions

General:

$$\begin{aligned} & Gx^* + c - \sum_{i \in \mathcal{E} \cup \mathcal{I}} \lambda_i^* a_i = 0 \\ & a_i^\top x^* = b_i, \qquad i \in \mathcal{E} \\ & a_i^\top x^* \geq b_i, \qquad i \in \mathcal{I} \\ & \lambda_i^* \geq 0, \qquad i \in \mathcal{I} \\ & \lambda_i^*(a_i^\top x^* - b_i) = 0, \qquad i \in \mathcal{E} \cup \mathcal{I} \end{aligned}$$

Defined via active set:

$$\begin{aligned} & \mathcal{A}(x^*) = \mathcal{E} \cup \{ i \in \mathcal{I} | a_i^\top x^* = b_i \} \\ & Gx^* + c - \sum_{i \in \mathcal{A}(x^*)} \lambda_i^* a_i = 0 \\ & a_i^\top x^* = b_i, \qquad i \in \mathcal{A}(x^*) \\ & a_i^\top x^* \geq b_i, \qquad i \in \mathcal{I} \backslash \mathcal{A}(x^*) \\ & \lambda_i^* \geq 0, \qquad i \in \mathcal{A}(x^*) \cap \mathcal{I} \end{aligned}$$

Active set method for convex QP

Algorithm 16.3 (Active-Set Method for Convex QP).

Compute a feasible starting point x_0;
Set $\mathcal{W}_0$ to be a subset of the active constraints at x_0;
for $k = 0, 1, 2, \ldots$
    Solve (16.39) to find p_k;
    if $p_k = 0$
        Compute Lagrange multipliers $\hat{\lambda}_i$ that satisfy (16.42),
                with $\hat{\mathcal{W}} = \mathcal{W}_k$;
        if $\hat{\lambda}_i \geq 0$ for all $i \in \mathcal{W}_k \cap \mathcal{I}$
            stop with solution $x^* = x_k$;
        else
            $j \leftarrow \arg\min_{j \in \mathcal{W}_k \cap \mathcal{I}} \hat{\lambda}_j$;
            $x_{k+1} \leftarrow x_k$; $\mathcal{W}_{k+1} \leftarrow \mathcal{W}_k \backslash \{j\}$;
    else (* $p_k \neq 0$ *)
        Compute α_k from (16.41);
        $x_{k+1} \leftarrow x_k + \alpha_k p_k$;
        if there are blocking constraints
            Obtain $\mathcal{W}_{k+1}$ by adding one of the blocking
                constraints to $\mathcal{W}_k$;
        else
            $\mathcal{W}_{k+1} \leftarrow \mathcal{W}_k$;
end (for)

$$\min_p \quad \tfrac{1}{2} p^T G p + g_k^T p \tag{16.39a}$$

$$\text{subject to} \quad a_i^T p = 0, \quad i \in \mathcal{W}_k. \tag{16.39b}$$

$$\sum_{i \in \hat{\mathcal{W}}} a_i \hat{\lambda}_i = g = G\hat{x} + c, \tag{16.42}$$

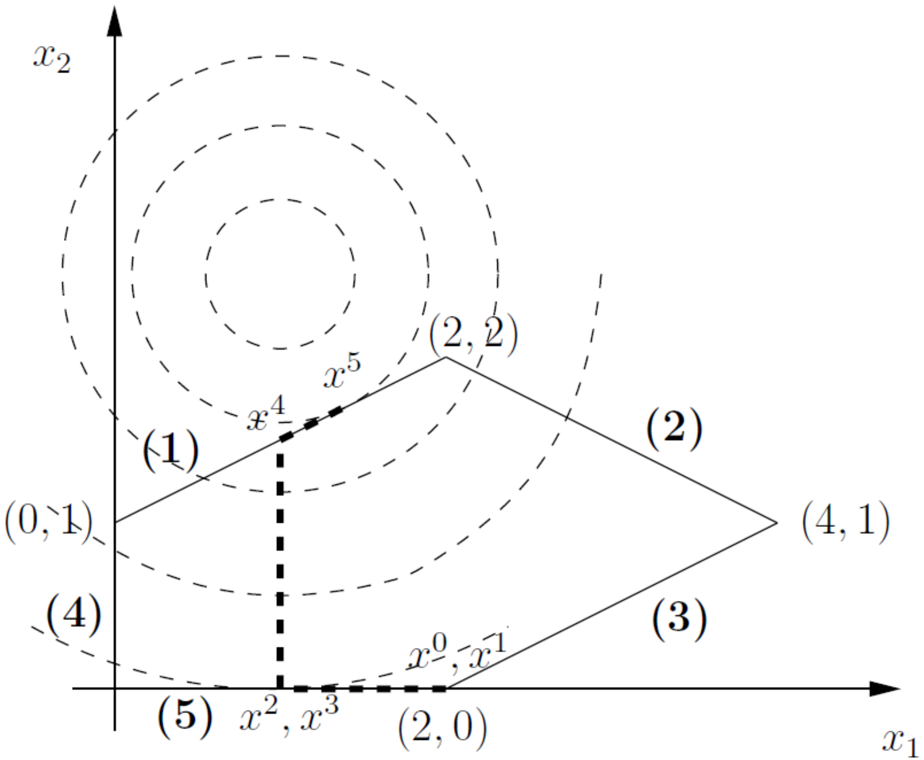
$$\alpha_k \stackrel{\text{def}}{=} \min\left(1, \min_{i \notin \mathcal{W}_k,\, a_i^T p_k < 0} \frac{b_i - a_i^T x_k}{a_i^T p_k}\right). \tag{16.41}$$

No degeneracy and *G>0*: Active set method converges in finite number of iterations.

Example 16.4

$$\min_x q(x) = (x_1 - 1)^2 + (x_2 - 2.5)^2$$

$$\begin{aligned} \text{subject to} \quad x_1 - 2x_2 + 2 &\geq 0 & (1)\\ -x_1 - 2x_2 + 6 &\geq 0 & (2)\\ -x_1 + 2x_2 + 2 &\geq 0 & (3)\\ x_1 &\geq 0 & (4)\\ x_2 &\geq 0 & (5) \end{aligned}$$



$$G = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad c = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$$

$$a_1 = \begin{bmatrix} 1 & -2 \end{bmatrix}^\mathsf{T}, \quad b_1 = -2$$

$$a_2 = \begin{bmatrix} -1 & -2 \end{bmatrix}^\mathsf{T}, \quad b_2 = -6$$

$$a_3 = \begin{bmatrix} -1 & 2 \end{bmatrix}^\mathsf{T}, \quad b_3 = -2$$

$$a_4 = \begin{bmatrix} 1 & 0 \end{bmatrix}^\mathsf{T}, \quad b_4 = 0$$

$$a_5 = \begin{bmatrix} 0 & 1 \end{bmatrix}^\mathsf{T}, \quad b_5 = 0$$

Local SQP-algorithm for solving NLPs

Only equality constraints:

$$\begin{aligned} &\min f(x) \\ &\text{subject to } c(x) = 0 \end{aligned}$$

Algorithm 18.1 (Local SQP Algorithm for solving (18.1)).

Choose an initial pair (x_0, λ_0); set $k \leftarrow 0$;
repeat until a convergence test is satisfied
  Evaluate f_k, ∇f_k, $\nabla^2_{xx}\mathcal{L}_k$, c_k, and A_k;
  Solve (18.7) to obtain p_k and l_k;
  Set $x_{k+1} \leftarrow x_k + p_k$ and $\lambda_{k+1} \leftarrow l_k$;
end (repeat)

$$\begin{aligned} \min_p \quad & f_k + \nabla f_k^T p + \tfrac{1}{2} p^T \nabla^2_{xx}\mathcal{L}_k p \\ \text{subject to} \quad & A_k p + c_k = 0. \end{aligned}$$

Extension to inequality constraints – IQP method:

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{subject to} \quad \begin{cases} c_i(x) = 0, & i \in \mathcal{E} \\ c_i(x) \geq 0, & i \in \mathcal{I} \end{cases}$$

$$\begin{aligned} \min_p \quad & f_k + \nabla f_k^T p + \tfrac{1}{2} p^T \nabla^2_{xx}\mathcal{L}_k p \\ \text{subject to} \quad & \nabla c_i(x_k)^T p + c_i(x_k) = 0, \quad i \in \mathcal{E}, \\ & \nabla c_i(x_k)^T p + c_i(x_k) \geq 0, \quad i \in \mathcal{I}. \end{aligned}$$

Convergence:

- Close to the solution, a solution to the approximation has same active set as a solution to the original problem (Thm 18.1).
- Therefore, once the optimal active set is found, Algorithm 18.1 with inequalities behaves just like a problem with only equalities. That is, very good (local) convergence.
- In addition: Far from the solution, the SQP approach is usually able to improve the estimate of the active set and guide the iterates toward a solution.

A practical line search SQP method

$$\min_p \quad f_k + \nabla f_k^T p + \tfrac{1}{2} p^T \nabla_{xx}^2 \mathcal{L}_k p \qquad (18.11a)$$

$$\text{subject to} \quad \nabla c_i(x_k)^T p + c_i(x_k) = 0, \quad i \in \mathcal{E}, \qquad (18.11b)$$

$$\nabla c_i(x_k)^T p + c_i(x_k) \geq 0, \quad i \in \mathcal{I}. \qquad (18.11c)$$

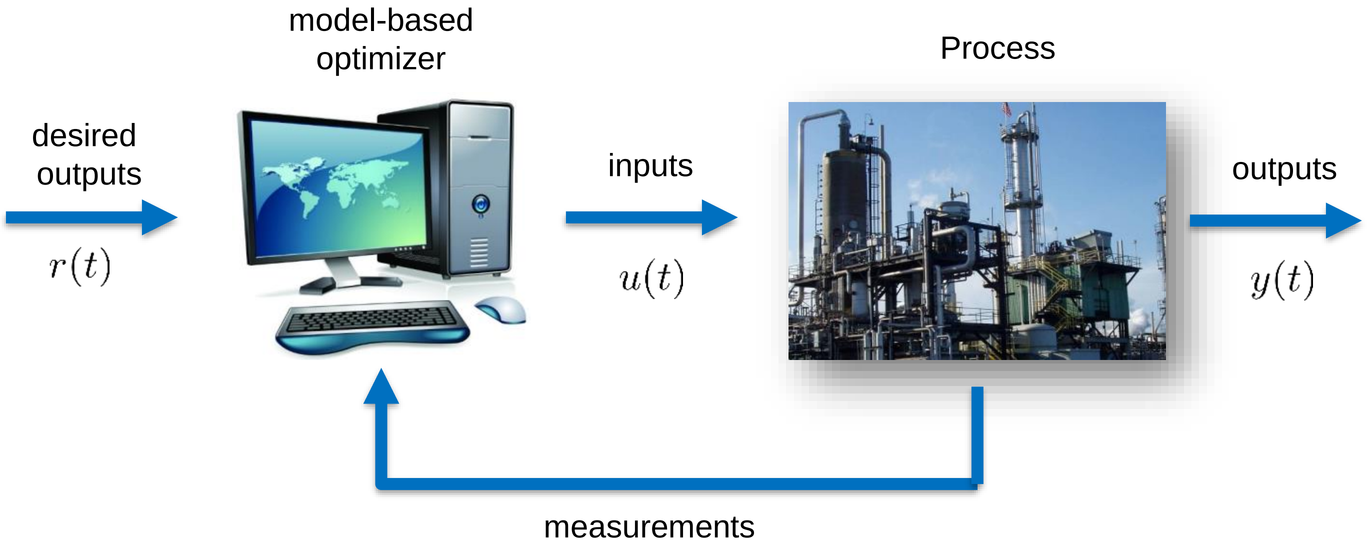
Algorithm 18.3 (Line Search SQP Algorithm).

Choose parameters $\eta \in (0, 0.5)$, $\tau \in (0, 1)$, and an initial pair (x_0, λ_0);
Evaluate $f_0, \nabla f_0, c_0, A_0$;
If a quasi-Newton approximation is used, choose an initial $n \times n$ symmetric positive definite Hessian approximation B_0, otherwise compute $\nabla_{xx}^2 \mathcal{L}_0$;
repeat until a convergence test is satisfied
  Compute p_k by solving (18.11); let $\hat{\lambda}$ be the corresponding multiplier;
  Set $p_\lambda \leftarrow \hat{\lambda} - \lambda_k$;
  Choose μ_k to satisfy (18.36) with $\sigma = 1$;
  Set $\alpha_k \leftarrow 1$;
  while $\phi_1(x_k + \alpha_k p_k; \mu_k) > \phi_1(x_k; \mu_k) + \eta \alpha_k D_1(\phi(x_k; \mu_k) p_k)$
    Reset $\alpha_k \leftarrow \tau_\alpha \alpha_k$ for some $\tau_\alpha \in (0, \tau]$;
  end (while)
  Set $x_{k+1} \leftarrow x_k + \alpha_k p_k$ and $\lambda_{k+1} \leftarrow \lambda_k + \alpha_k p_\lambda$;
  Evaluate $f_{k+1}, \nabla f_{k+1}, c_{k+1}, A_{k+1}$, (and possibly $\nabla_{xx}^2 \mathcal{L}_{k+1}$);
  If a quasi-Newton approximation is used, set
    $s_k \leftarrow \alpha_k p_k$ and $y_k \leftarrow \nabla_x \mathcal{L}(x_{k+1}, \lambda_{k+1}) - \nabla_x \mathcal{L}(x_k, \lambda_{k+1})$,
  and obtain B_{k+1} by updating B_k using a quasi-Newton formula;
end (repeat)

Compare Alg. 6.1:

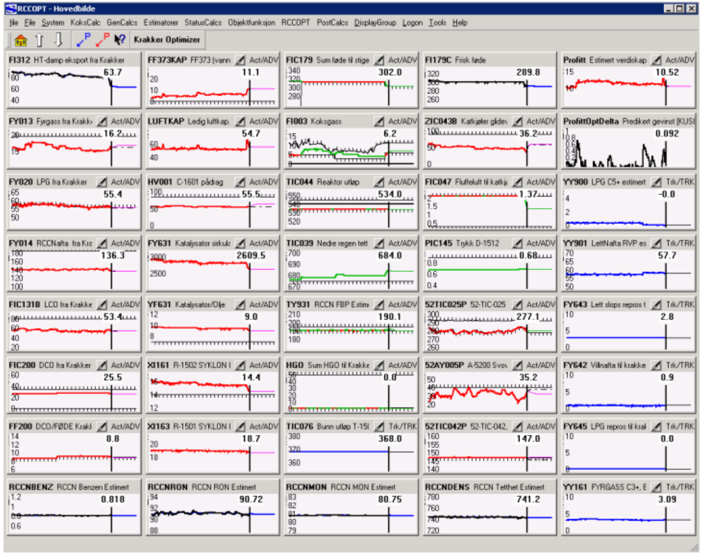
- Search direction from quadratic approximation
- Line search
- Update Hessian using BFGS

Model predictive control



A model of the process is used to compute the control signals (inputs) that optimize predicted future process behavior

SEPTIC
~~Statoil~~ Estimation and Prediction Tool for Identification and Control



equinor

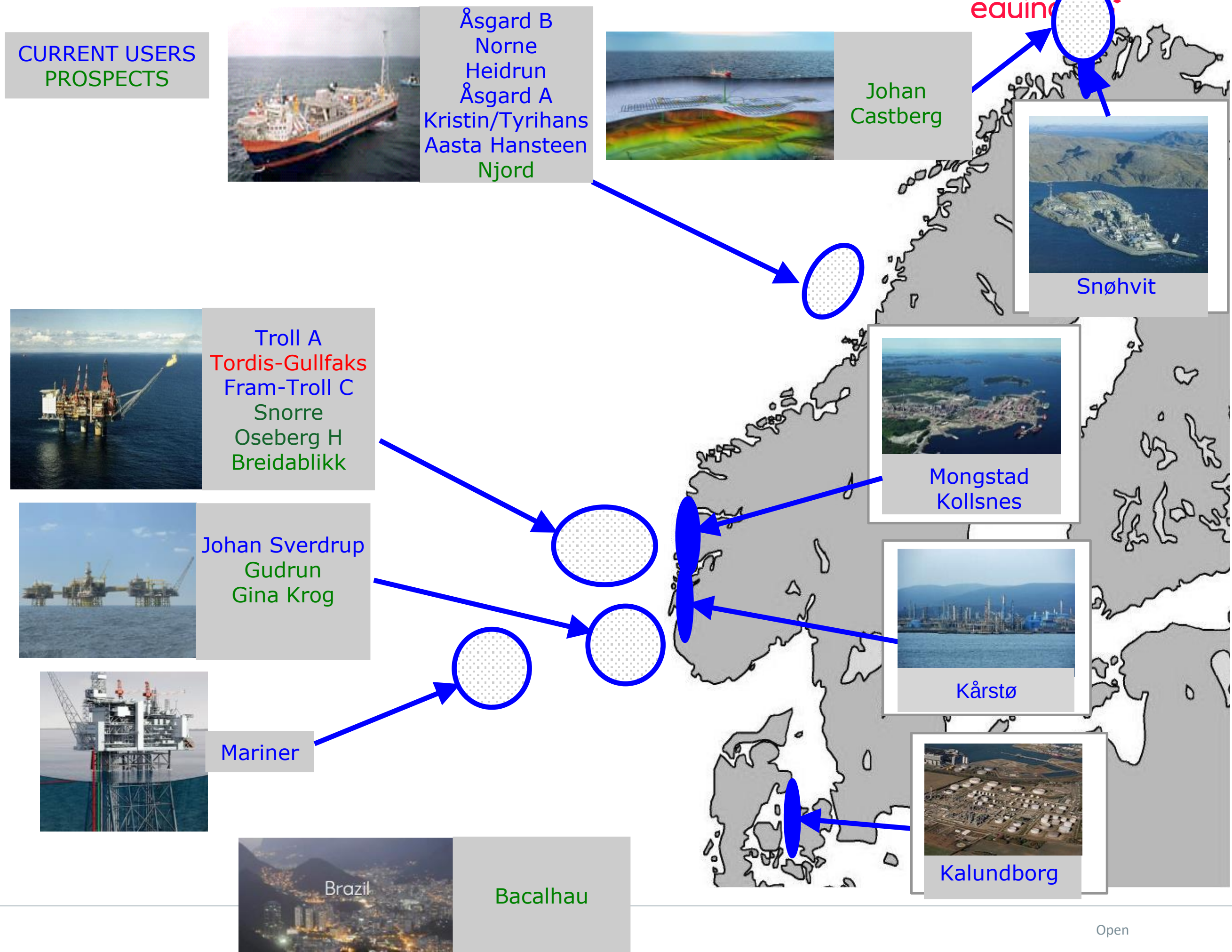
MPC in Equinor

Morten Fredriksen & John-Morten Godhavn
26/1-23

SEPTIC

~~Statoil~~ Estimation and Prediction Tool for Identification and Control

- Equinor inhouse software for MPC (Model Predictive Control)
- First implementation in 1997
- Available for all Equinor assets
- Widely used downstream & recently also upstream
- Typical BC: 1-5% increased production / reduced energy
- ~11 full time equivalents with cybernetics/automation/process control background
  - R&D: Rotvoll/Sandsli/Forus
  - Ops: Sandsli
  - Local: Mongstad, Kårstø, Snøhvit



When to use SEPTIC in Equinor -
alternative to complex control logic

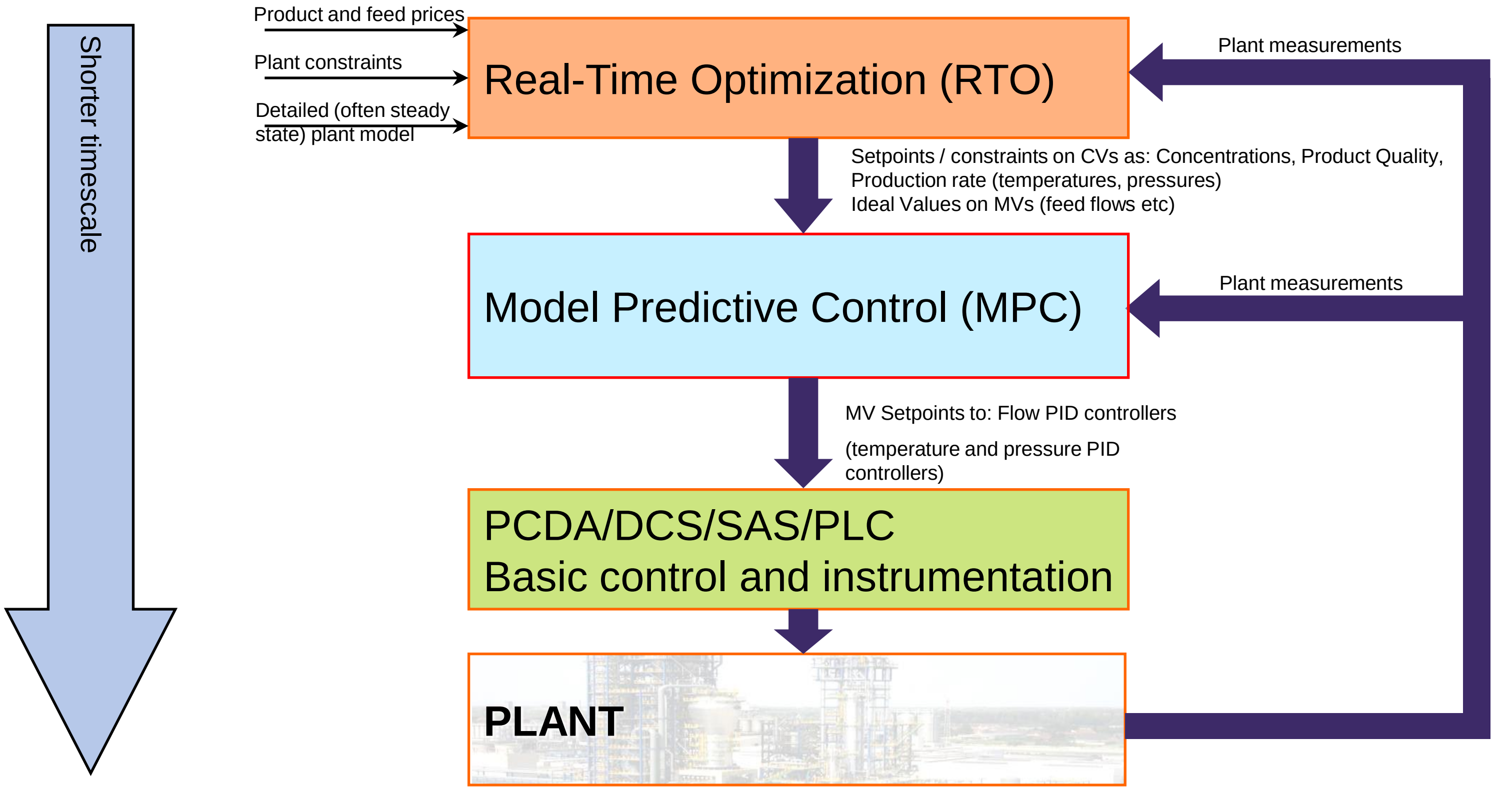
Properties:

- Multivariable: many to many
- Constraints: hard/soft
- Nonlinearities: step chokes, dead band, choke gain
- Robustify: good practice to QA «special» measurements
    - Rate estimator/multiphase meters («PC in the loop»)
    - Freeze detect bottom hole pressure gauge
- SEPTIC as alternative to complex SAS logic (min select, priorities, cascade control, feedforward, varying conditions):
    - Easier to maintain
    - SEPTIC engineer support

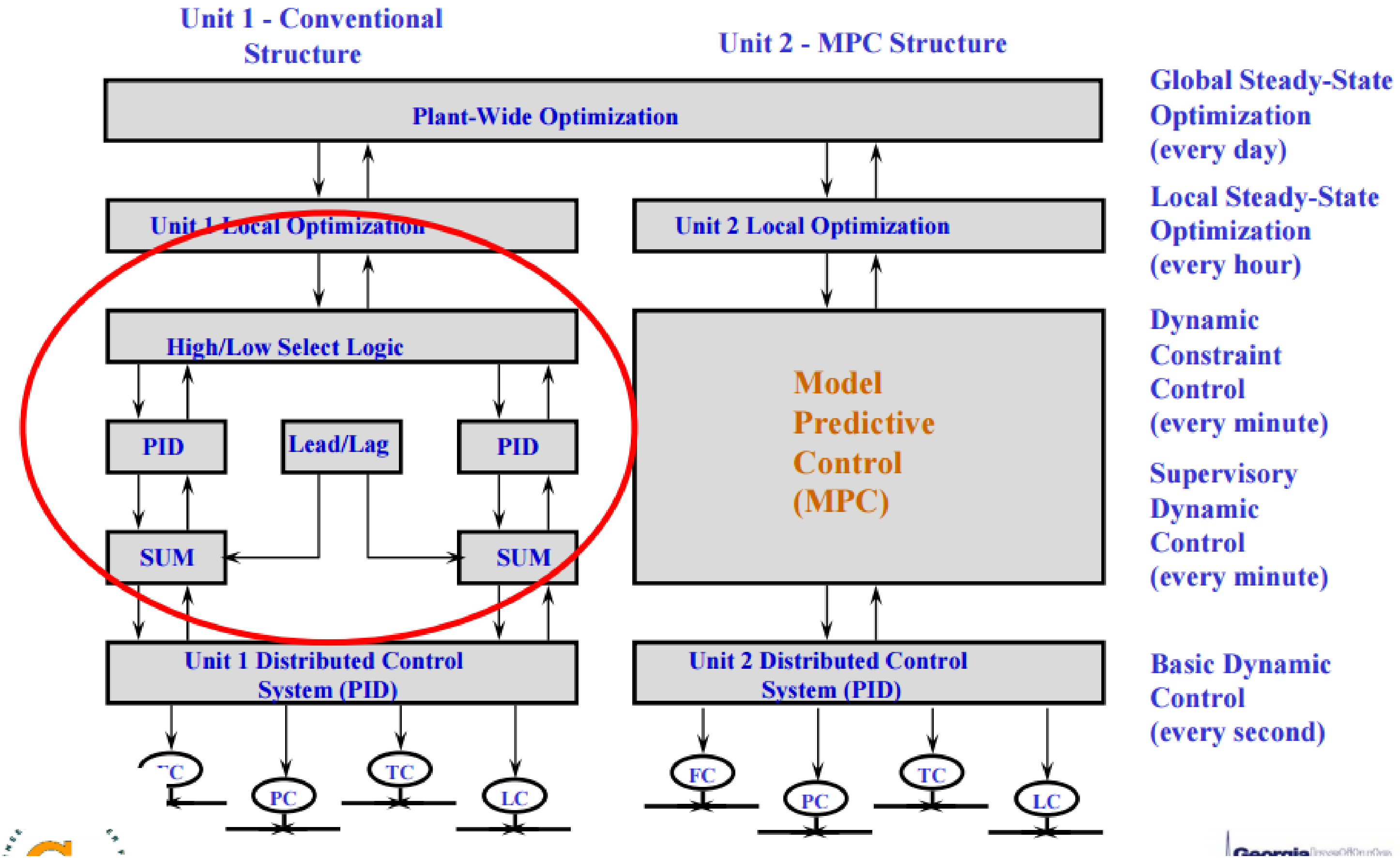
Considerations:

- SEPTIC server installed already?
- Time constant: 10 sec – 12 hours
- Not safety critical (e.g. safe injection)
- Stable process

MPC in the control hierarchy



Process control hierarchy before and after MPC



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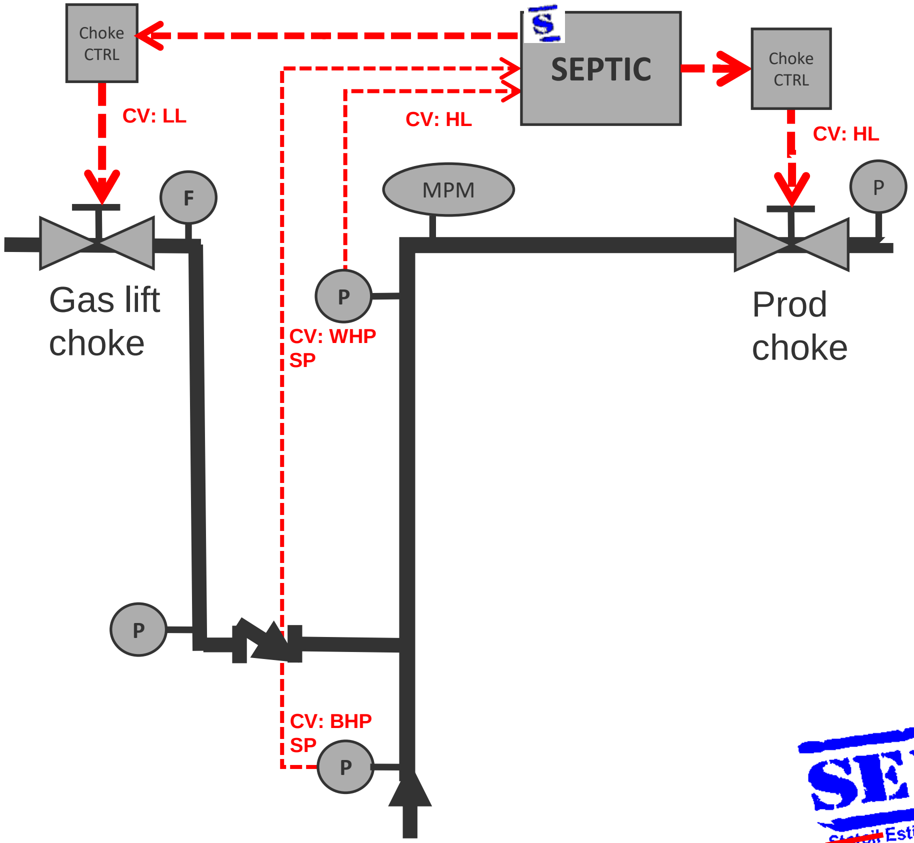
Offshore prodction applications

AUTO WELL	Single well automatic control (oil rate target; pressure constraints; smooth startup)
AUTO SAFE INJECTION	Pressure and rate control of injectors
AUTO SAFE GAS LIFT	Gas lift pressure control.
LINE CAP	Ensuring total line capacity is utilized at all times; coordinated start-up of lines.
OIL CAP	Ensuring total oil production process capacity is utilized at all times; coordinated start-up of wells.
GAS CAP	Ensuring total gas processing capacity is utilized and minimizing flaring.
WICC	Water injection capacity control
WEAK WELLS	Ensuring valid conditions to keep weak wells in operation.
TVP	Automatic control of export oil's TVP or RVP
Other	MEG regeneration; Liquid build-up in flowlines; Compressor optimization; Automated Well Testing; +++

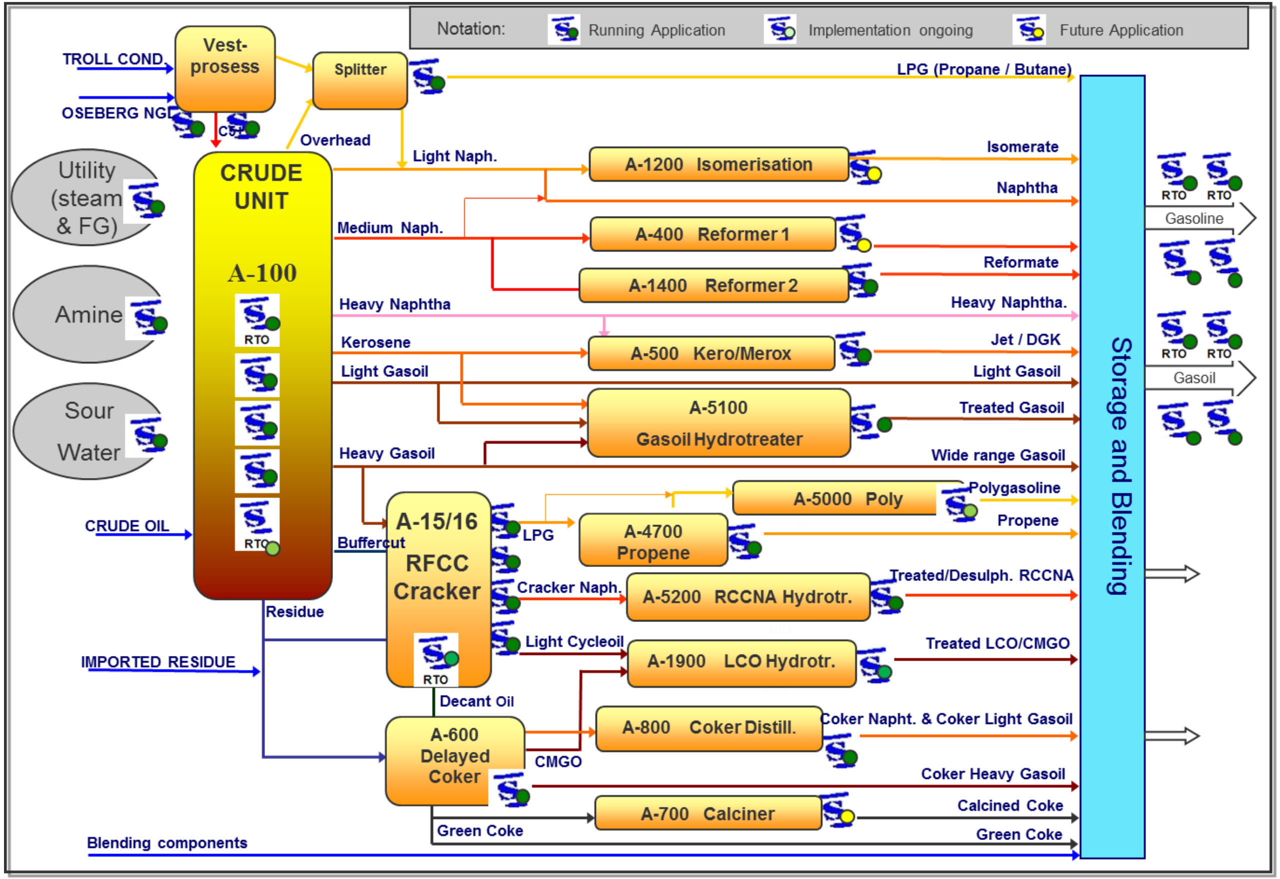
SEPTIC for automatic choke control of production well

equinor

- Manipulated variables:
    - Production choke position
    - Gas lift choke position
- Controlled variables:
    - Bottom hole pressure (BHP) – setpoint
    - Well head pressure (WHP) – setpoint
- Goal:
    - BHP-trajectory at start-up -> soft start-up by precise draw down control
    - BHP -> fast and accurate production target
    - WHP –> sufficient gas-lift

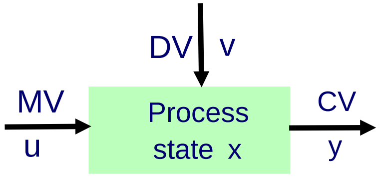


Mongstad Refinery - MPC & (D)RTO



MPC in SEPTIC

$$\min_{\Delta u,\alpha}\left[(y-y_{ref})^T Q_y (y-y_{ref}) + (u-u_{iv})^T Q_u (u-u_{iv}) + \Delta u^T P \Delta u + \alpha^T R \alpha\right]$$

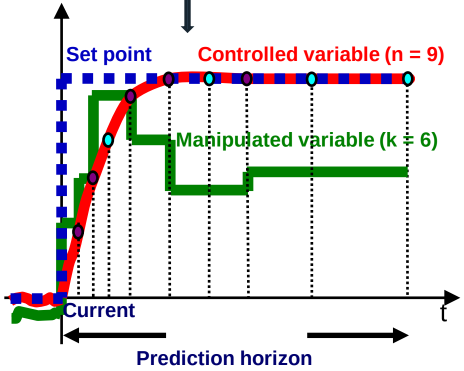


model: $\dot{x} = f(x,u,v)$ $\quad y = g(x,u)$

constraints: $y_{\min} - \alpha_l \leq y \leq y_{\max} + \alpha_h$ $\quad \Delta u_{\min} \leq \Delta u \leq \Delta u_{max}$ $\quad u_{\min} \leq u \leq u_{max}$

$0 \leq \alpha_l \leq \alpha_{l,\max}$ $\quad 0 \leq \alpha_h \leq \alpha_{h,\max}$ $\quad \alpha = [\alpha_l^T \alpha_h^T]^T$

discretization: $u = [u_1 u_2 \ldots u_k]$ $\quad \Delta u = [\Delta u_1 \Delta u_2 \ldots \Delta u_k]$ $\quad \Delta u_i = u_i - u_{i-1}$ $\quad y = [y_1 y_2 \ldots y_n]$



- MV blocking → size reduction
- CV evaluation points → size reduction
- CV reference specifications → tuning flexibility set point changes / disturbance rejection
- Soft constraints and priority levels → feasibility and tuning flexibility

Explicit control priorities - solved before the dynamic problem

1. MV rate of change limits
2. MV high/low Limits
3. CV hard constraints ("never" used)
4. CV soft constraints, CV set points, MV ideal values: Priority level 1
5. CV soft constraints, CV set points, MV ideal values: Priority level 2
6. CV soft constraints, CV set points, MV ideal values: Priority level n
7. CV soft constraints, CV set points, MV ideal values: Priority level 99

Sequence of steady-state QP solutions to solve 2 – 7 (or NLP if nonlinear models)

Then a single dynamic QP to meet the adjusted and feasible steady-state goals (or iterated QP if nonlinear models)

Equinor MPC strategy

SEPTIC
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- Use internal resources, only: develop competence, ownership, shared goals, and cooperation
- Prioritize maintenance of running applications
  - Always things to be improved, if you don't, the application will degrade, and die
  - Return of investment depends on the application performance
  - Maintenance and good performance make confidence
- Keep it as simple as possible
  - Scope based on a system analysis, include what we think is needed (can be extended later)
  - Modelling: we keep in mind that the system should solve
  - Use MPC, everyone is familiar with that, it is accepted, and it can prioritize between objectives
  - Optimized solution can be understood
  - Fast execution of project and running application
- RTO
  - Use dynamic models
  - When disturbances with economic impact (as most of them have) are frequent relative to the process dynamics
  - When model updating during transients is important to get satisfactory execution cycle

MPC in Equinor – way forward

- Continue work with onshore plants, particularly Mongstad
- Maintain and further develop running applications offshore
- Develop solutions for new field developments: Johan Castberg, Bacalhau, Bay de Nord, +++
- New type of applications:
    - Energy/CO2 optimization
    - Energy management: combining wind, turbines, solar, batteries, H2, etc
    - CCS

- MPC technology (MSc 2023)
    - Mixed integer optimization
    - Genetic algorithms
    - Neural networks

Summary MPC in Equinor

- MPC group 10-15 people with 100+ applications running, first in 1997
    - Build competence over time, learn from each other and operations
    - Used for different processes on- and offshore
    - New market: renewables, CCS
- Very good business case for many applications (1-5% of production)
- Inhouse software – add functionality on demand
- Mainly experimental step response models, but also nonlinear models
- Priority hierarchy implemented
- Flexible tool
- Maintenance and close follow up is the most important success factor

SEPTIC

~~Statoil~~ Estimation and Prediction Tool for Identification and Control

Embedded Model Predictive Control

PhD project Giorgio Kufoalor

Traditional MPC

- Successful in process industries
- Sampling times of minutes
- Powerful computing platforms

(M. Morari, 2013)

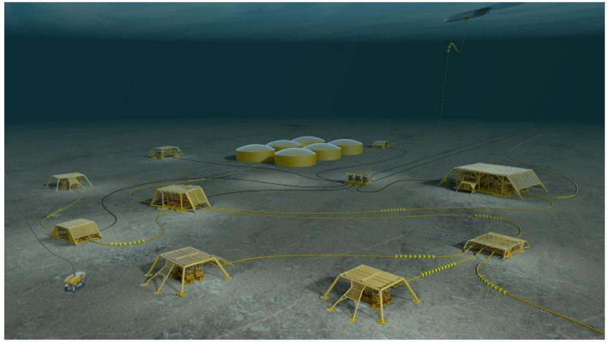
MIND THE GAP

Embedded MPC

- Small, high performance plants
- Sampling times of ms to ns
- Limited embedded platform

(M. Morari, 2013)

Embedded MPC for new industrial applications



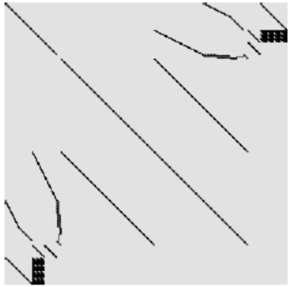
(Statoil subsea factory)

Main contributions to fill the gap

PhD project Giorgio Kufoalor

- Step-response MPC on ultra-reliable, resource constrained, industrial hardware

- Detailed study on *MPC formulations* and *solver methods* to achieve fast and reliable solutions
  - *Achieve significant savings*, both in computations and memory usage
  - Exploiting problem structure and specifics of computing platform
  - Automatic code generation (almost…)



- Development of new multistage QP framework, tailored to step-response MPC

- All extensively tested on a realistic subsea compact separation simulator using *hardware-in-the-loop* testing

D. K. Kufoalor, G. Frison, L. Imsland, T. A. Johansen, J. B. Jørgensen, **Block Factorization of Step Response Model Predictive Control Problems**, J. Process Control, Vol. 53, May, pp. 1–14, 2017;
D. K. M. Kufoalor, S. Richter, L. Imsland, T. A. Johansen, **Enabling Full Structure Exploitation of Practical MPC Formulations for Speeding up First-Order Methods**, 56th IEEE Conference on Decision and Control, 2017
D. K. M. Kufoalor, L. S. Imsland, T. A. Johansen, **Efficient Implementation of Step Response Models for Embedded Model Predictive Control**, Computers & Chemical Engineering, Volume 90, July, Pages 121–135, 2016
D. K. M. Kufoalor, V. Aaker, T. A. Johansen, L. Imsland, G. O. Eikrem, **Automatically Generated Embedded Model Predictive Control: Moving an Industrial PC-based MPC to an Embedded Platform**, J. Optimal Control - Applications and Methods, vol. 36, pp. 705–727, 2015

Nonlinear MPC

The three ways of solving NMPC optimization problem

- Sequential methods, aka single shooting
  - Only inputs as optimization variables, simulate to calculate objective and state constraints (and gradients)
  - “Small” optimization problem with no structure
  - Standard SQP methods are suitable

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- Simultaneous methods, aka “collocation”
  - Both inputs and states as optimization variables, include model as equality constraints
  - Huge optimization problem, but constraints and gradients are very structured (“sparse”, a lot of zeros)
  - Must use solvers that exploit this structure (usually interior point, e.g. IPOPT)

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- In-between method: Multiple shooting
  - Divide horizon into “sub-horizons”, use single-shooting on each sub-horizon and add equality constraints to “glue” each sub-horizon together
  - Results in medium-sized “block-structured” optimization problem
  - Ideally use SQP solvers that exploit this structure (but not many exists)

- What is best? Depends…

Why nonlinear MPC?

Nonlinear MPC:

- (Much) more effort in *modeling*, *estimation* and *optimization*

Why?

- Your plant is really nonlinear – linear MPC is not enough
  - E.g. batch process, grade-change, start-up, …
- The performance gain with nonlinear MPC is significant
  - E.g. in mass-produced units with embedded control solutions
- You already have a (nonlinear) model and want to exploit it

Good news:

- Tools for developing nonlinear optimization solutions has become much more available last ~5 years

NMPC example: van der Pol

- Controlled van der Pol oscillator

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + e(1 - x_1^2)x_2 + u$$

- Discretization of ODE (here: Euler)

- Single-shooting implementation

- Stability dependent on horizon length
- Importance of “warm-start”

Implementation: Tor Aksel Heirung

Output feedback MPC

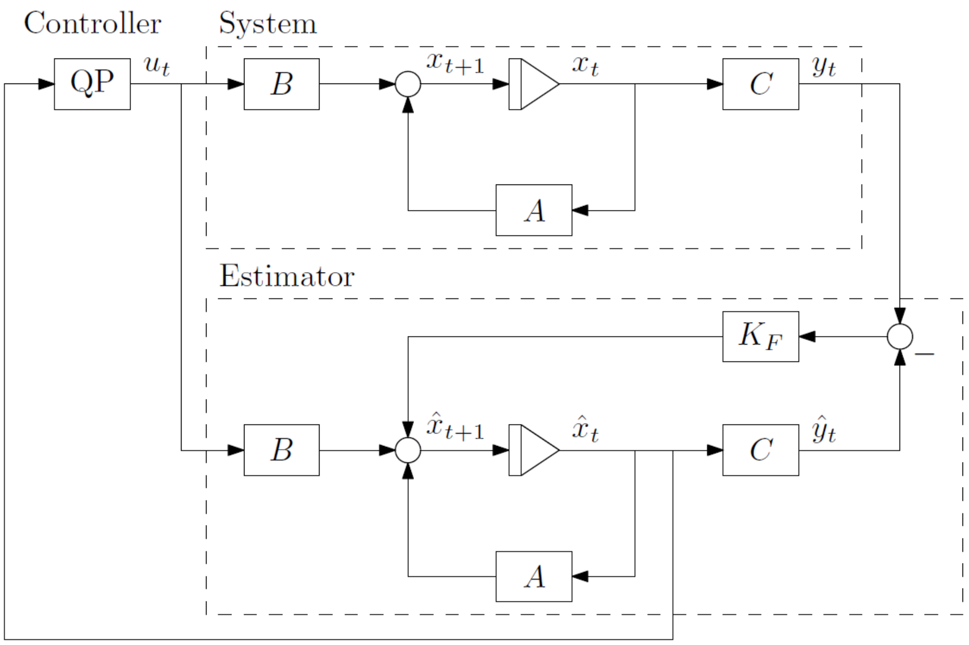
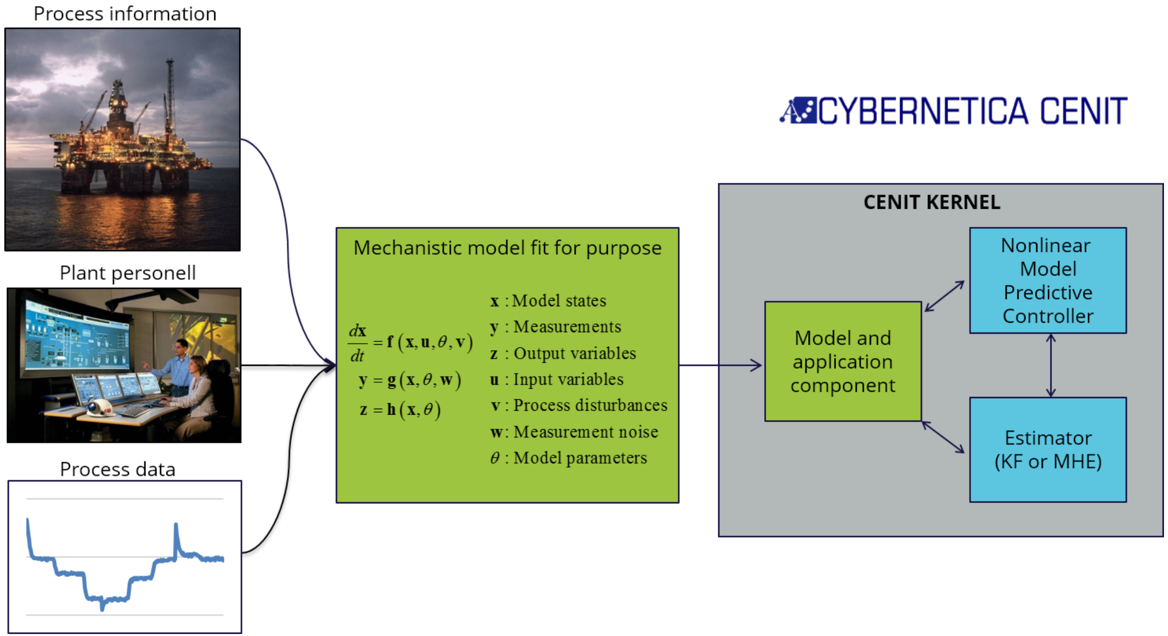


Figure 4.3: The structure of an output feedback linear MPC.

Output feedback NMPC

Cybernetica

- Cybernetica provides advanced model-based control systems for the process industry
  - Based on non-linear first principles (mechanistic) models
  - Nonlinear state- and parameter estimation (EKF, MHE)
  - Online dynamic optimization (nonlinear model predictive control, NMPC)



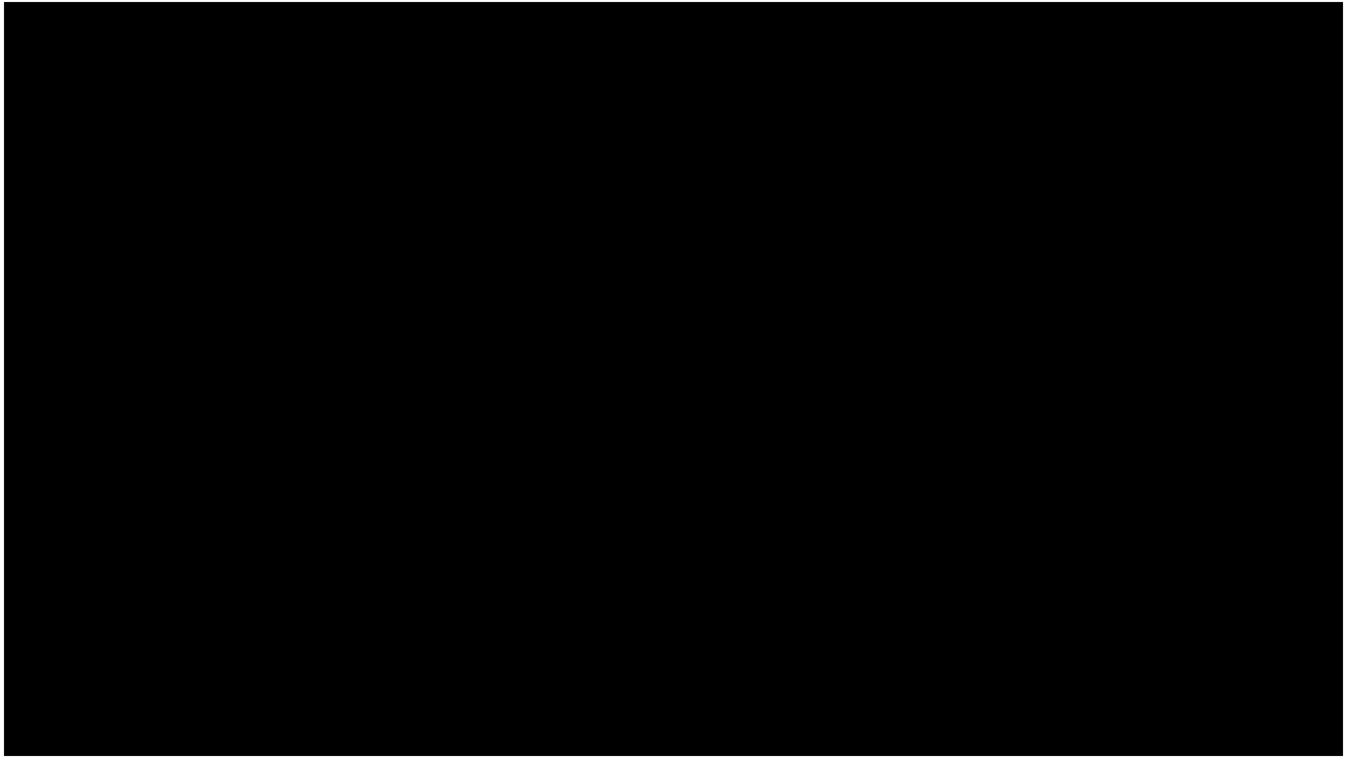
Other MPC providers in Norway…

- ABB, Siemens, Honeywell, Aspen, …

- Mostly using linear models
  - …and mostly these linear models are "step response"-type models, found from data

SOME EXAMPLES

Applied LQR



Source: https://doi.org/10.1109/HUMANOIDS.2014.7041387

Applied LQR

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = S^T\tau + J_c(q)^T f_c$$

1. Modify dynamics to null-space of the contact constraint
2. Linearize the model around desired pose $(q_0, \dot{q}_0, \tau_0)$

$$\dot{x} = Ax + Bu, \qquad x^T = [\Delta q^T, \Delta\dot{q}^T], \qquad u^T = [\Delta\tau]$$

3. Calculate the infinite horizon linear quadratic regulator

$$J = \int_0^\infty x^T Q x + u^T R u$$

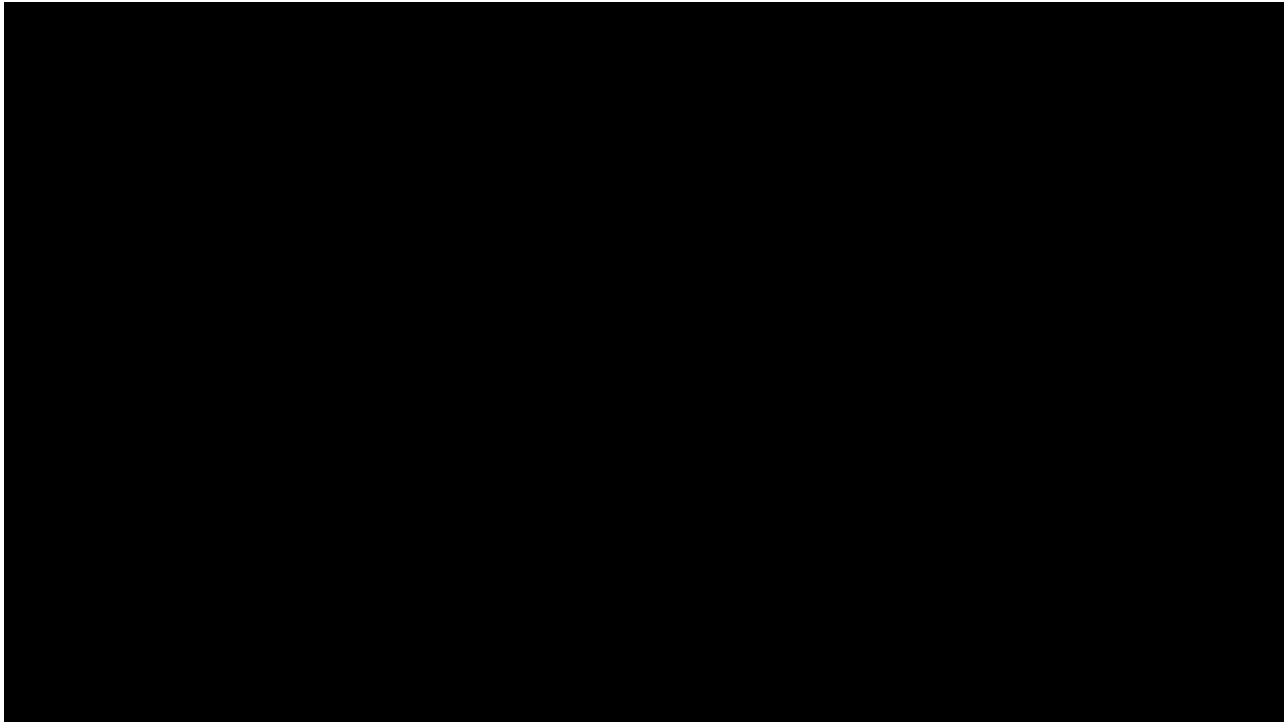
4. Apply the input

$$\tau = \tau_0 - Kx$$

Fig. 1: Hydraulically actuated torque controlled Sarcos humanoid used for experiments.

Source: https://doi.org/10.1109/HUMANOIDS.2014.7041387

Applied Open-Loop Dynamic Optimization



Source: https://arxiv.org/pdf/2003.04882.pdf

Applied Open-Loop Dynamic Optimization

1. Split the problem:
   - Lap time offline
   - Tracking online
2. Solve a “periodic” problem

$$
\begin{aligned}
\min_{\mathbf{X},\mathbf{U}} \quad & \sum_{k=0}^{N} j_{\text{LTO}}(\mathbf{x}_k, \mathbf{u}_k) \\
\text{s.t.} \quad & \mathbf{x}_{k+1} = f_s^d(\mathbf{x}_k, \mathbf{u}_k) \\
& f_s^d(\mathbf{x}_N, \mathbf{u}_N) = \mathbf{x}_0
\end{aligned}
$$

3. Apply NMPC for closed-loop

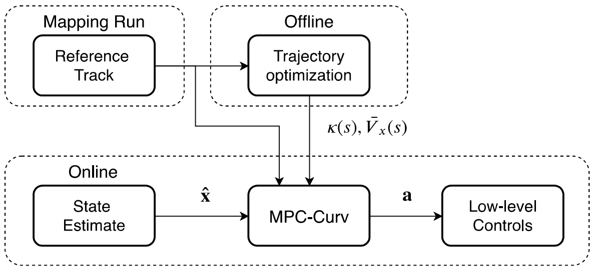
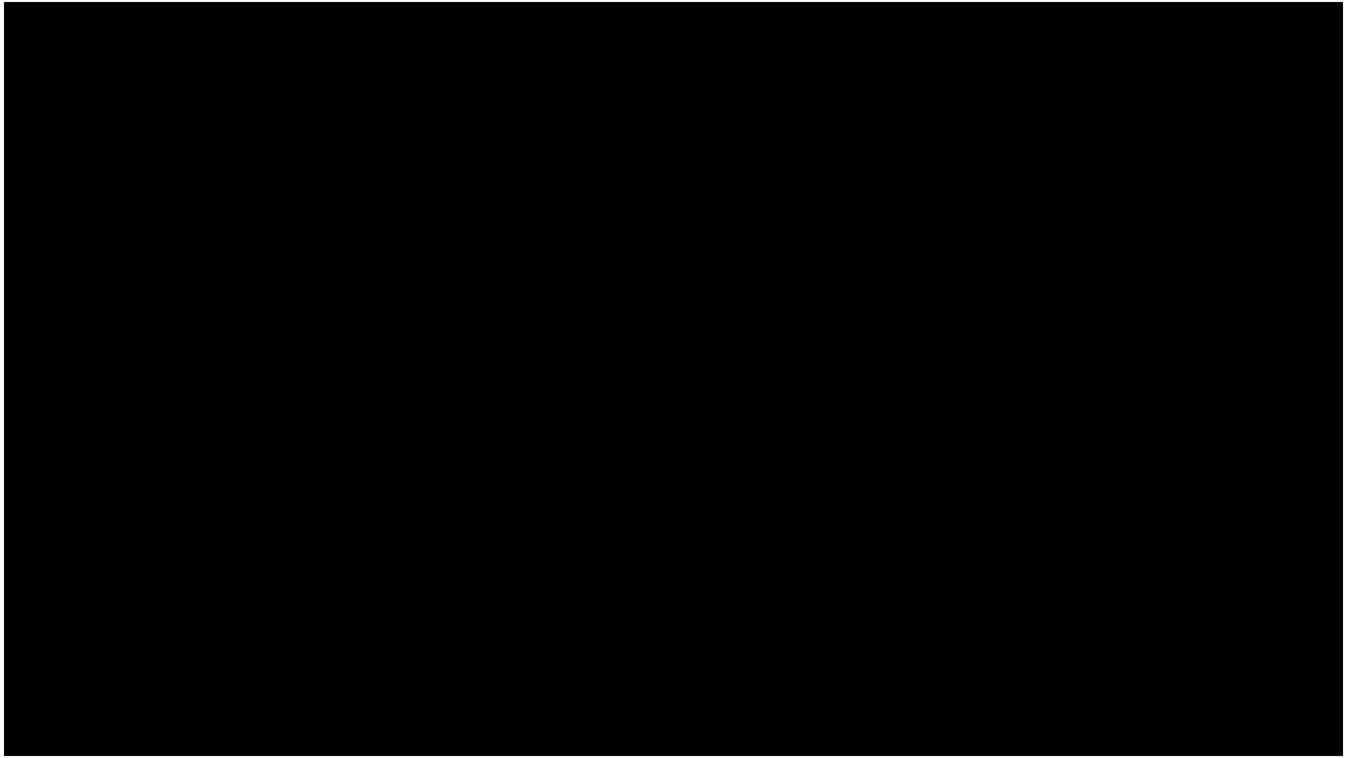


Fig. 3. The hierarchical controller uses the reference track in both stages of the hierarchical controller. First the lap time optimization (LTO) problem computes a reference path, whose curvature $\kappa(s)$ and speed profile $\bar{V}_x(s)$ are later used by the NMPC.

MPC using distributed SQP



Source: http://doi.org/10.1109/TCST.2018.2871397

MPC using distributed SQP

Algorithm 1 Schematic description of the Distributed SQP. The arguments of the functions are removed for brevity.

Coordinator initializes the problem.
while exit conditions not fulfilled **do**
    Coordinator broadcasts $\boldsymbol{T}$.
    Each vehicle solves (8)
    Each vehicle returns $\nabla V_i, \nabla^2 V_i, g_i, \nabla g_i, \nabla^2 g_i$.
    Coordinator solves the SQP sub-problem (21).
    Coordinator and vehicles compute α.
    Coordinator takes step (20).

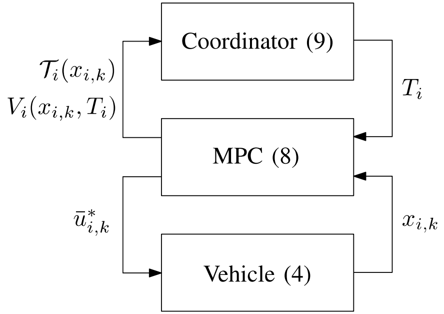
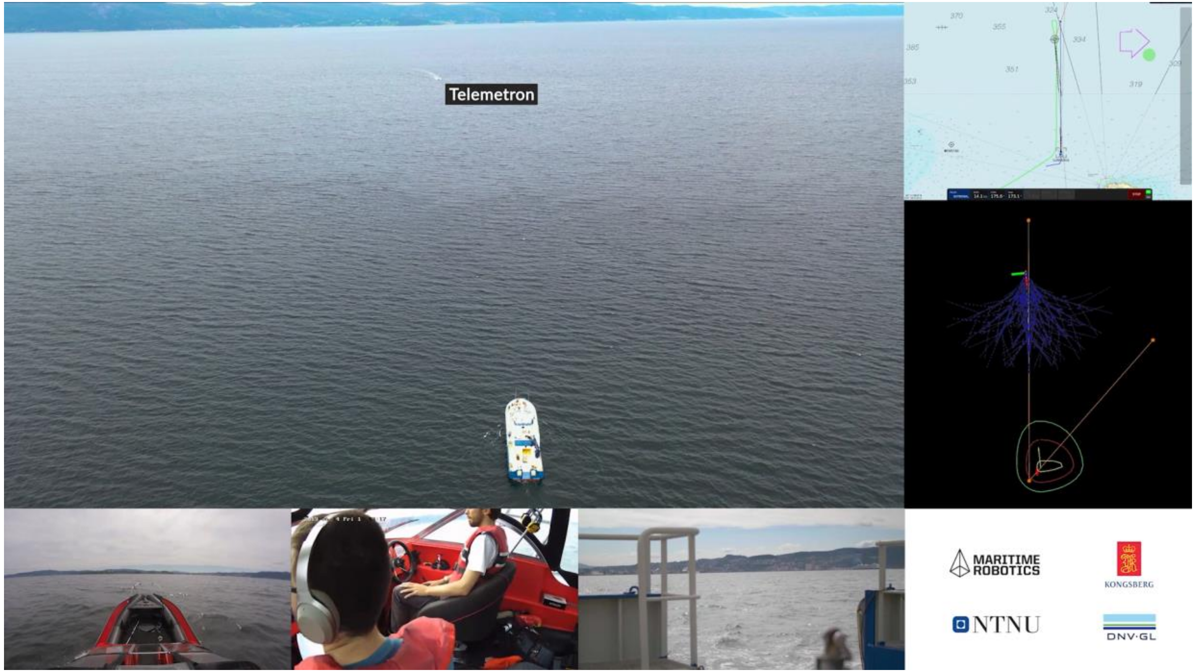


Fig. 2: Schematic illustration of the bi-level control structure for one vehicle. The coordinator is in closed-loop with all vehicles in the same way.

Branching Course MPC



Source: Bjørn-Olav Holtung Eriksen
https://doi.org/10.1002/rob.21900

Branching Course MPC

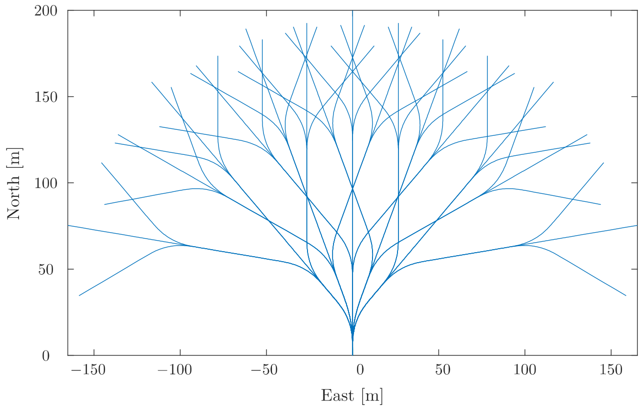
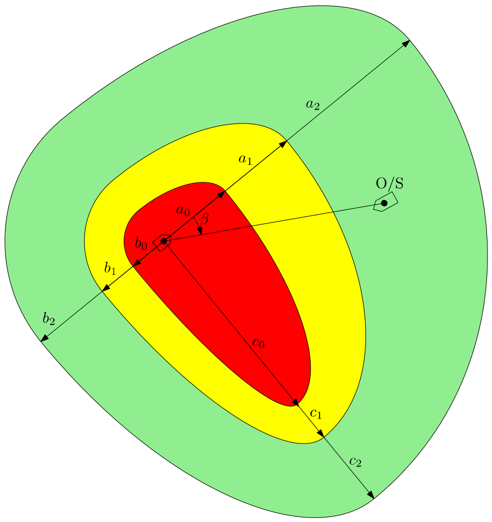


Figure 10: A set of predicted pose trajectories with 3 levels.



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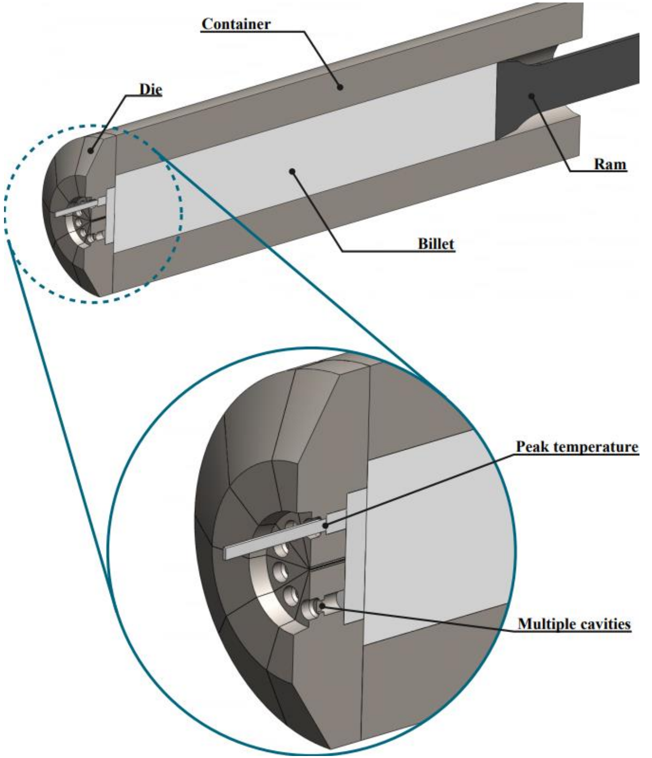
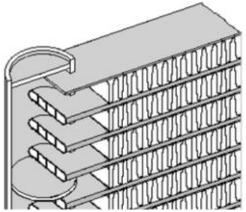
Ongoing project: ExtruTech

Optimization of extrusion cycle

- Partners: Cybernetica, Hydro, SINTEF
- PhD candidate: Trym Arve Gabrielsen





NTNU | Norwegian University of Science and Technology

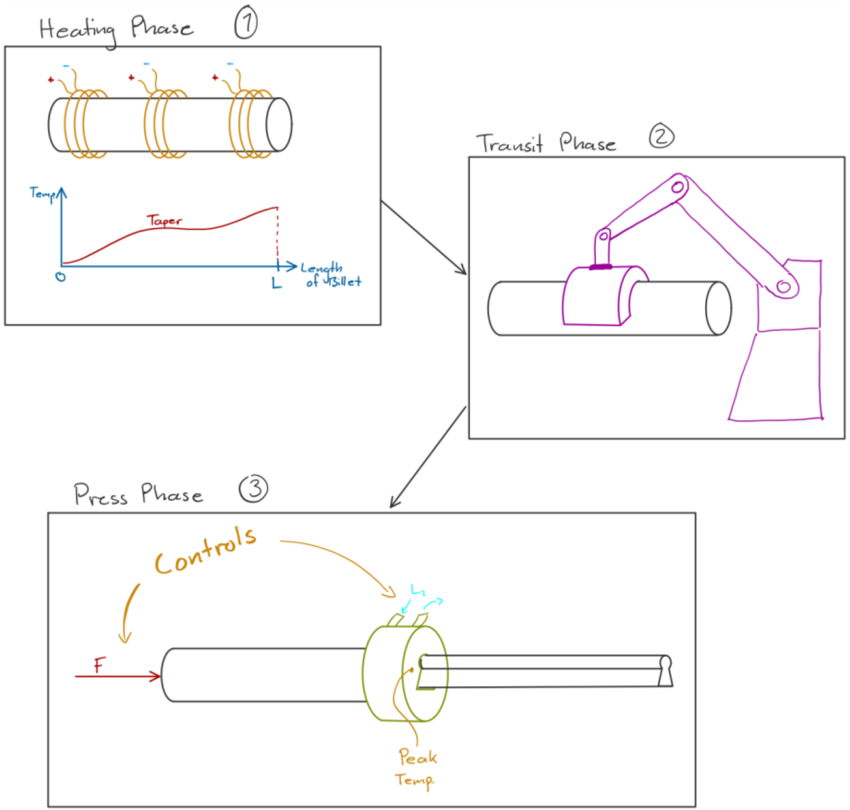
Optimizing extrusion cycle

- Goal: Optimize the Extrusion Cycle Time
- Simplified to: Optimize Extrusion Phase Time

- Extrusion Time is in the range of several minutes → The temperature control (NMPC Single Shooting) is not suited for integrating over the entire extrusion phase

- By using a progressor transformation for time to extrusion length, we are able to implement simultaneous methods instead (Direct Collocation). → Great integration properties!

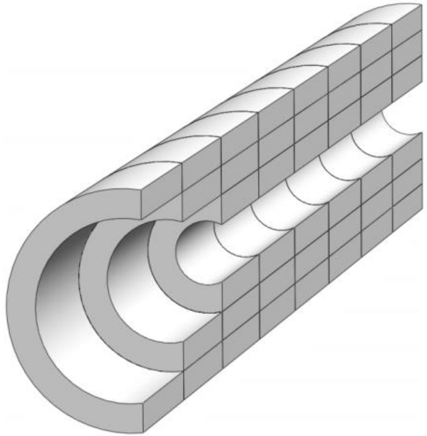
- There are also other benefits to this trick!



Model Description

- Continuous model (PDE): $\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x} + \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}\right) + \Phi(T; v) + \Psi(T; v)$

- Discretized in space (ODE's):



Model Description

- Finite difference scheme:

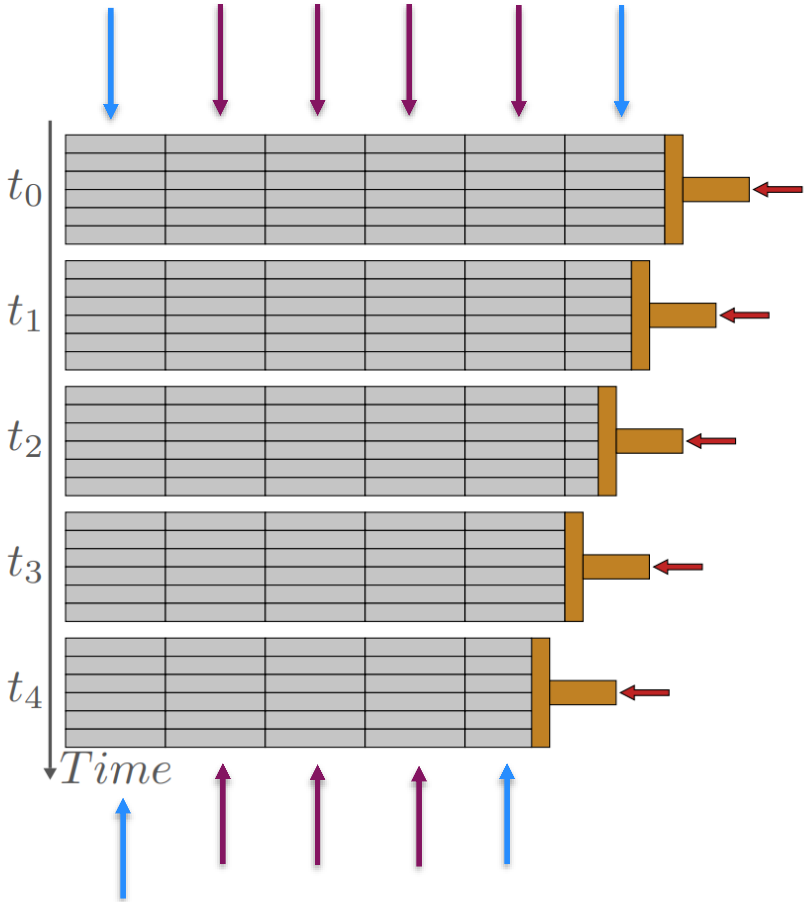
$$\left.\frac{\partial T}{\partial r}\right|_{r,x} \approx \frac{T(r+1,x)-T(r-1,x)}{2\Delta r}$$

$$\left.\frac{\partial T}{\partial x}\right|_{r,x} \approx \frac{T(r,x+1)-T(r,x)}{\Delta x}$$

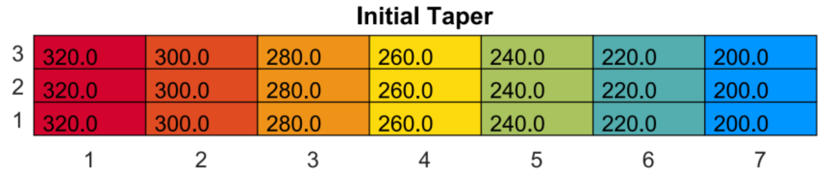
$$\left.\frac{\partial^2 T}{\partial r^2}\right|_{r,x} \approx \frac{T(r+1,x)-2T(r,x)+T(r-1,x)}{\Delta r^2}$$

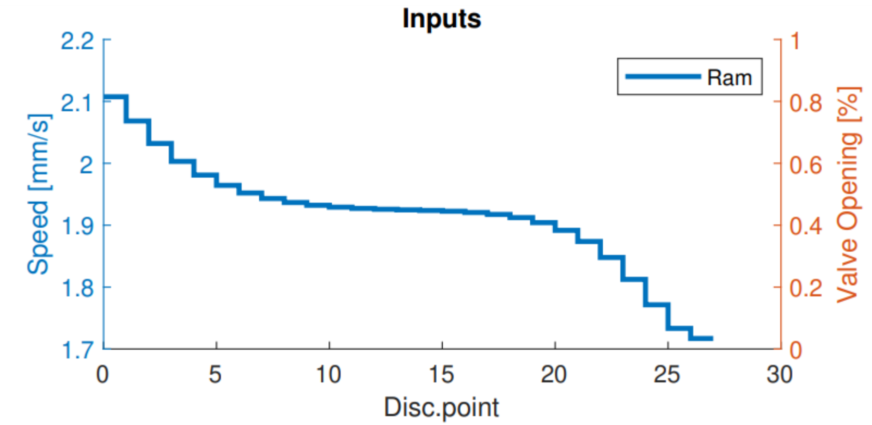
$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{r,x} \approx \frac{T(r,x+1)-2T(r,x)+T(r,x-1)}{\Delta x^2}$$

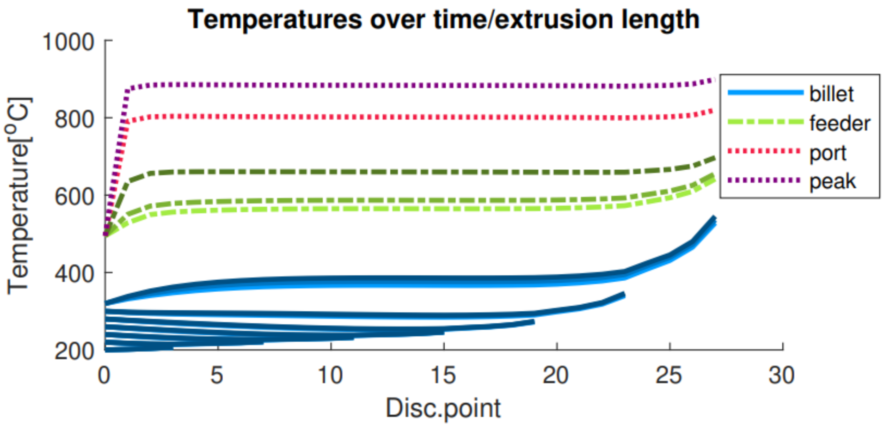
- Model changes as billet is extruded



Results


Initial Taper
3
2
1
320.0
300.0
280.0
260.0
240.0
220.0
200.0
1
2
3
4
5
6
7


Inputs
Ram
Speed [mm/s]
Valve Opening [%]
Disc.point
2.2
2.1
2
1.9
1.8
1.7
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30


Temperatures over time/extrusion length
billet
feeder
port
peak
Temperature[°C]
Disc.point
1000
800
600
400
200
0
5
10
15
20
25
30