

**1:**

$$\int_{-1}^2 \frac{x}{\sqrt{x^2+1}} dx$$

Need to use  $u$ -substitution

$$u = x^2 + 1$$

$$f(x) = \frac{1}{\sqrt{g(x)}}$$

$$du = 2x dx$$

$$x dx = \frac{du}{2}$$

$$\int \frac{1}{2\sqrt{u}} du$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{2} \int u^{-1/2} du$$

$$= \frac{1}{2} \cdot 2\sqrt{u}$$

$$= \sqrt{u}$$

$$\sqrt{u} = \sqrt{x^2 + 1}$$

$$\int \frac{x}{\sqrt{x^2+1}} dx = \sqrt{x^2+1} (+C)$$

$$\text{Between -1 and 2: } = \sqrt{5} - \sqrt{2}$$

**2:**

$$\int x e^{-4x} dx$$

$$u = e^{-4x}, v = \frac{x^2}{2}$$

$$u' = -4e^{-4x}, v' = x$$

$$uv - \int v u'$$

$$\frac{x^2 e^{-4x}}{2} - \int \frac{-4x^2 e^{-4x}}{2}$$

$$= \frac{x^2 e^{-4x}}{2} - \frac{(8x^2 + 4x + 1) \cdot e^{-4x}}{16}$$

$$= \frac{-(4x+1) \cdot e^{-4x}}{16} + C$$

**3:**

$$\int \frac{14}{(2x-1)(x+3)} dx$$

$$\frac{14}{(2x-1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+3}$$

$$14 = A(x+3) + B(2x-1)$$

To remove A:

$$x = -3$$

$$14 = A(0) = B(-7)$$

$$-14 = 7B$$

$$B = -2$$

To remove B:

$$x = \frac{1}{2}$$

$$14 = A\left(\frac{7}{2}\right) + B(0) \quad 14 = \frac{7A}{2}$$

$$7A = 28$$

$$A = 4$$

$$A = 4, B = -2$$

$$\begin{aligned}\int \frac{14}{(2x-1)(x+3)} dx &= \int \frac{4}{2x-1} dx + \int \frac{-2}{x+3} dx \\ &= 2 \ln |2x-1| + -2 \ln |x+3|\end{aligned}$$

**4:**

$$\int_1^3 \ln x dx$$

Int by parts needed.

$$\int uv' = uv - \int vu'$$

$$u = \ln x, v = x$$

$$u' = \frac{1}{x}, v' = 1$$

$$x \ln x - \int 1$$

$$= x \ln x - x$$

Between 1 and 3

$$(3 \ln 3 - 3) - (\ln 1 - 1)$$

$$= 3 \ln 3 - 2$$

**5:**

$$\int \frac{1}{\sqrt{81-x^2}} dx$$

$$x = 9 \sin u$$

$$dx = 9 \cos u du$$

$$= \frac{1}{\sqrt{-81 \sin^2 u + 81}}$$

$$= \frac{1}{9 \sqrt{-\sin^2 u + 1}}$$

$$= \frac{1}{9 \sqrt{\cos^2 u}}$$

$$= \frac{1}{9 \cos u} u = \arcsin \frac{x}{9}$$

$$\int x \ln x^2 dx = u$$

$$= \arcsin \frac{x}{9}$$

**6:**

$$\int x \ln x^2 dx$$

U-sub

$$u = x^2$$

$$\frac{du}{2} = x dx$$

$$= \int \frac{\ln u}{2} du$$

Int by parts:

$$\int ab' = ab - \int ba'$$

$$\begin{aligned}
 a &= \ln u, b = \frac{u}{2} \\
 a' &= \frac{1}{u}, b' = \frac{1}{2} \\
 &= \frac{u \ln u}{2} - \int \frac{1}{u} \frac{u}{2} \\
 &= \frac{u \ln u - u}{2} \\
 &= \frac{x^2 (\ln x^2 - 1)}{2}
 \end{aligned}$$

**7:**

$$\int \arctan x \, dx$$

$$u = \arctan x, v = x$$

$$u' = \frac{1}{x^2+1}, v' = 1$$

$$x \arctan x - \int \frac{x}{x^2+1}$$

$$\text{U-sub: } u = x^2 + 1$$

$$du = 2x dx$$

$$x dx = \frac{du}{2}$$

$$x \arctan x - \frac{1}{2} \int \frac{1}{u} du$$

$$x \arctan x - \frac{1}{2} \ln |u| = x \arctan x - \frac{1}{2} \ln |x^2 + 1|$$

$$= x \arctan x - \frac{1}{2} \ln |x^2 + 1| + C$$

**8:**

$$\int \frac{1}{(4x^2+9)^2}$$

$$\text{Let } x = \frac{3}{2} \tan \theta$$

$$d\theta =$$

**9:**

$$\int \sin^2 \theta \cos \theta \, d\theta$$