Note sheet: FTC 1: if f is contonuous on $[a, b]$, then the function g defined by
$g(x) = \int_a^x f(t)\,\mathrm{d}t \quad a \le x \le b$
is an antiderivitive of f, that is, $g'(x) = f(x)$ for $a < x < b$

Substitution Rule:
$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$
$\int x^3 \cos(x^4 + 2)\,\mathrm{d}x$
$u = x^4 + 2$
$\mathrm{d}u = 4x^3\mathrm{d}x$
$\frac{1}{4}\mathrm{d}u = x^3\mathrm{d}x$

Integration by Parts:
$\int f(x)g'(x)\,\mathrm{d}x = f(x)g(x) - \int g(x)f'(x)\,\mathrm{d}x$
$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$
$\int x \sin x\,\mathrm{d}x$
$u = x, u' = 1$
$v = -\cos(x), v' = \sin(x)$
$= -x\cos(x) - \int -\cos(x)\,\mathrm{d}x$
$= -x\cos(x) + \sin(x) + C$

FTC 2:
$F(x) = \int_a^x f(t)\,\mathrm{d}t$

Summation:
$\Delta x = \frac{b-a}{N}$
$x_i = a + i\Delta x$
$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$
$R_n = \sum_{i=1}^{N} f(x_i) \cdot \Delta x$
$L_n = \sum_{i=0}^{N-1} f(x_i) \cdot \Delta x \;\; M_n = \sum_{i=1}^{N} f(\bar{x}_i)\Delta x$
$T_n = \frac{1}{2}(\sum_{i=1}^{N}(f(x_{i-1})\Delta x) + \sum_{i=1}^{N}(f(x_i)\Delta x))$
$T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$
$S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

Error:
$|f''(x)| \le K$ for $a \le x \le b$
$|E_T| \le \frac{K(b-a)^3}{12N^2}$
$|E_M| \le \frac{K(b-a)^3}{24N^2}$
$|E_S| \le \frac{K(b-a)^5}{180N^4}$
$\sin x = \frac{1}{\csc x}, \cos x = \frac{1}{\sec x}, \csc x = \frac{1}{\sin x}, \sec x = \frac{1}{\cos x}, \cot x = \frac{1}{\tan x}$
$\sin^2 x = \cos^2 x = 1 \;\; 1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$
$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x}$

Convergence/Divergence (improper integrals):
$\int_1 \infty \frac{1}{x^2}\,\mathrm{d}x$, replace ∞ with R
$|_1^R$

Arc length:
$f(x)_{length} = \int_a^b \sqrt{f'(x)^2 + 1}\,\mathrm{d}x$
$f(y)_{length} = \int_a^b \sqrt{f'(y)^2 + 1}\,\mathrm{d}x$
$f(t)_{length} = \int_a^b \sqrt{\frac{dy}{dt}^2 + \frac{dx}{dt}^2}\,\mathrm{d}x$

Volume of solids of rotation:
Rings:
$A = \pi \int_a^b outerRadius^2 - innerRadius^2\,\mathrm{d}x$
$A = \pi \int_a^b r_{inner}^2 - r_{inner}^2\,\mathrm{d}x$