Solomon Greenberg,
2017/09/06,
Homework 1, Section 1.1,
Problems: 7, 11, 12, 13, 16, 24, 26, 39,

Problem 7

Solve:

$$\left| \begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 3 \\ x + 4y + 5z &= 4 \end{aligned} \right|$$

Solution:

$$\begin{matrix} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{matrix} \left| \begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 3 \\ x + 4y + 5z &= 4 \end{aligned} \right|$$

$$\left| \begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 3 \\ x + 4y + 5z &= 4 \end{aligned} \right| \begin{matrix} \longrightarrow \\ -\mathrm{I} \\ -\mathrm{I} \end{matrix} \left| \begin{aligned} x + 2y + 3z &= 1 \\ y + 4z &= 2 \\ 2y + 2z &= 3 \end{aligned} \right|$$

$$\left| \begin{aligned} x + 2y + 3z &= 1 \\ y + 4z &= 2 \\ 2y + 2z &= 3 \end{aligned} \right| \begin{matrix} \longrightarrow \\ -2\mathrm{III} \\ -2\mathrm{II} \end{matrix} \left| \begin{aligned} x + 2y + 3z &= 1 \\ -3y \qquad &= -4 \\ -6z &= -1 \end{aligned} \right|$$

$$\left| \begin{aligned} x + 2y + 3z &= 1 \\ -3y \qquad &= -4 \\ -6z &= -1 \end{aligned} \right| \begin{matrix} \longrightarrow \\ \cdot -3 \\ \cdot -6 \end{matrix} \left| \begin{aligned} x + 2y + 3z &= 1 \\ y \qquad &= 12 \\ z &= 6 \end{aligned} \right|$$

$$\left| \begin{aligned} x + 2y + 3z &= 1 \\ y \qquad &= 12 \\ z &= 6 \end{aligned} \right| \begin{matrix} -(2\mathrm{II}+3\mathrm{III}) \\ \longrightarrow \\ \longrightarrow \end{matrix} \left| \begin{aligned} x \qquad &= -41 \\ y \quad &= 12 \\ z &= 6 \end{aligned} \right|$$

$$\left| \begin{aligned} x \qquad &= -41 \\ y \quad &= 12 \\ z &= 6 \end{aligned} \right|$$

Problem 11

Solve and graph:

$$\left|\begin{array}{r} x-2y=2 \\ 3x+5y=17 \end{array}\right|$$

Solution:

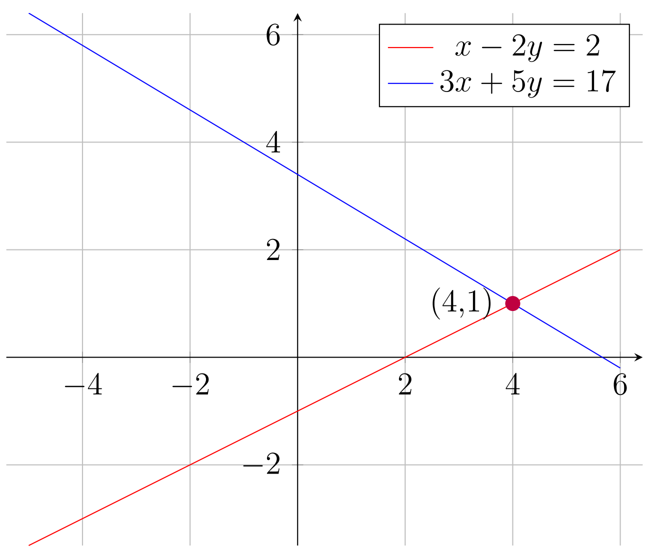
$$\begin{array}{c} \mathrm{I} \\ \mathrm{II} \end{array}\left|\begin{array}{r} x-2y=2 \\ 3x+5y=17 \end{array}\right|$$

$$\left|\begin{array}{r} x-2y=2 \\ 3x+5y=17 \end{array}\right| \xrightarrow[-3\mathrm{I}]{} \left|\begin{array}{lr} x- & 2y=2 \\ & 11y=11 \end{array}\right|$$

$$\left|\begin{array}{lr} x- & 2y=2 \\ & 11y=11 \end{array}\right| \xrightarrow[\cdot\frac{1}{11}]{} \left|\begin{array}{r} x-2y=2 \\ y=1 \end{array}\right|$$

$$\left|\begin{array}{r} x-2y=2 \\ y=1 \end{array}\right| \xrightarrow{+2\mathrm{II}} \left|\begin{array}{lr} x & =4 \\ & y=1 \end{array}\right|$$

$$\left|\begin{array}{lr} x & =4 \\ & y=1 \end{array}\right|$$



Problem 12

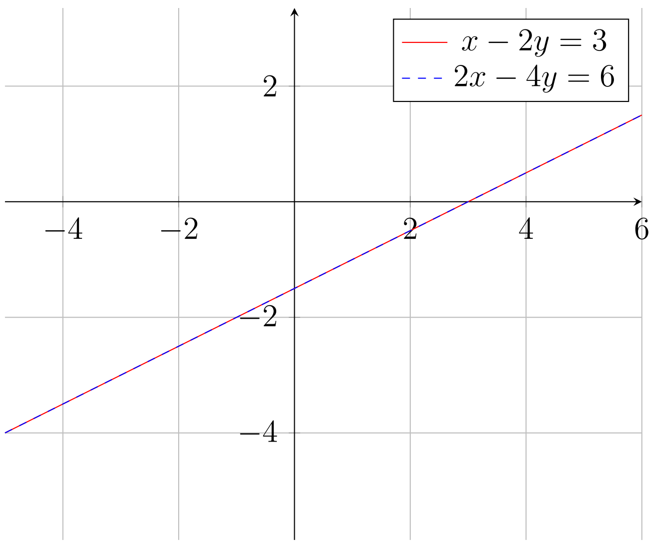
Solve:

$$\begin{vmatrix} x - 2y = 3 \\ 2x - 4y = 6 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} x - 2y = 3 \\ 2x - 4y = 6 \end{vmatrix} \begin{matrix} -\mathrm{II} \\ -\mathrm{I} \end{matrix} \begin{vmatrix} 0x \quad = 0 \\ 0y = 0 \end{vmatrix}$$

Note: Infinite solutions



Problem 13

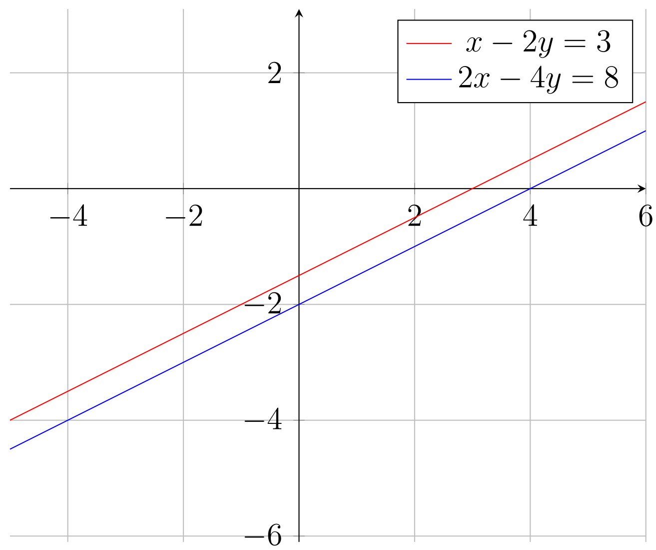
Solve:

$$\begin{vmatrix} x - 2y = 3 \\ 2x - 4y = 8 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} x - 2y = 3 \\ 2x - 4y = 8 \end{vmatrix} \xrightarrow[-2\mathrm{I}]{} \begin{vmatrix} x - 2y \quad = 3 \\ +0 = 2 \end{vmatrix}$$

Note: No solutions



Problem 16

Solve in terms of intersecting planes:

$$\left|\begin{array}{r} x + 4y + z = 0 \\ 4x + 13y + 7z = 0 \\ 7x + 22y + 13z = 0 \end{array}\right|$$

Solution:

$$\left|\begin{array}{r} x + 4y + z = 0 \\ 4x + 13y + 7z = 0 \\ 7x + 22y + 13z = 0 \end{array}\right| \begin{array}{c} \longrightarrow \\ -7\mathrm{I} \\ -7\mathrm{I} \end{array} \left|\begin{array}{r} x + 4y + z = 0 \\ -3x - 15y \quad = 0 \\ - 6y + 6z = 0 \end{array}\right|$$

$$\left|\begin{array}{r} x + 4y + z = 0 \\ -3x - 15y \quad = 0 \\ - 6y + 6z = 0 \end{array}\right| \begin{array}{c} \longrightarrow \\ \cdot\frac{-1}{3} \\ \cdot\frac{-1}{6} \end{array} \left|\begin{array}{r} x + 4y + z = 0 \\ x + 5y \quad = 0 \\ y - z = 0 \end{array}\right|$$

$$\left|\begin{array}{r} x + 4y + z = 0 \\ x + 5y \quad = 0 \\ y - z = 0 \end{array}\right| \begin{array}{c} \longrightarrow \\ -\mathrm{III} \\ \longrightarrow \end{array} \left|\begin{array}{r} x + 4y + z = 0 \\ x + 4y + z = 0 \\ y - z = 0 \end{array}\right|$$

$$\left|\begin{array}{r} x + 4y + z = 0 \\ x + 4y + z = 0 \\ y - z = 0 \end{array}\right| \begin{array}{c} \longrightarrow \\ -\mathrm{I} \\ \longrightarrow \end{array} \left|\begin{array}{r} x + 4y + z = 0 \\ 0x + 0y + 0z = 0 \\ y - z = 0 \end{array}\right|$$

$$\begin{vmatrix} x + 4y + z & = 0 \\ +0 & = 0 \\ y - z & = 0 \end{vmatrix}$$

$$\begin{cases} x & = -5y \\ +0 & = 0 \\ z & = y \end{cases}$$

$$\begin{vmatrix} x & = -5y \\ y & = y \\ z & = y \end{vmatrix}$$

$$\begin{vmatrix} x & = -5t \\ y & = t \\ z & = t \end{vmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5t \\ t \\ t \end{bmatrix}$$

Problem 24

Setup:

Let D_a represent the yearly demand for product A, in millions of dollars
Let D_b represent the yearly demand for product B, in millions of dollars
Let R_a represent the required \$ of product A to produce \$1 of B
Let R_b represent the required \$ of product B to produce \$1 of A

Solve:

$$\begin{vmatrix} a - R_b b = D_a \\ -R_a a + b = D_b \end{vmatrix}$$

Solution:

$$\begin{vmatrix} a - 0.1b = 1000 \\ -0.2a + b = 780 \end{vmatrix} \xrightarrow{\cdot 10} \begin{vmatrix} 10a - b = 10000 \\ -0.2a + b = 780 \end{vmatrix}$$

$$\begin{vmatrix} 10a - b = 10000 \\ -0.2a + b = 780 \end{vmatrix} \xrightarrow{+\text{II}} \begin{vmatrix} 9.8a = 10780 \\ -0.2a + b = 780 \end{vmatrix}$$

$$\begin{vmatrix} 9.8a = 10780 \\ -0.2a + b = 780 \end{vmatrix} \begin{matrix} \cdot\frac{1}{9.8} \\ \cdot 5 \end{matrix} \begin{vmatrix} a = 1100 \\ -a + 5b = 3900 \end{vmatrix}$$

$$\left|\begin{aligned} a &= 1100 \\ -a + 5b &= 3900 \end{aligned}\right| \underset{+\mathrm{I}}{\longrightarrow} \left|\begin{aligned} a &= 1100 \\ b &= 1000 \end{aligned}\right|$$

$$\left|\begin{aligned} a &= 1100 \\ b &= 1000 \end{aligned}\right|$$

Required output of product A: $1100 million/year
Required output of product B: $1000 million/year

Problem 26

Solve for a and b, then graph:

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - x = \cos t$$
$$x(t) = a\sin t + b\cos t$$

Solution:

$$x(t) = x''(t) - x'(t) - x(t)$$
$$x'(t) = a\cos t - b\sin t$$
$$x''(t) = -a\sin t + b\cos t$$
$$(-a\sin t + b\cos t) - (a\cos t - b\sin t) - (a\sin t + b\cos t) = \cos t$$
$$-2a\sin t - a\cos t + b\sin t - 2b\cos t$$
$$(b-2a)\sin t + (-a-2b)\cos t = \cos t$$

$$\begin{vmatrix} -2a + b = 0 \\ -a - 2b = 1 \end{vmatrix}$$
$$\begin{vmatrix} a \ = \frac{-1}{5} \\ b = \frac{-2}{5} \end{vmatrix}$$

Problem 39

Find the circle that runs through the points $(5,5)$, $(4,6)$, and $(6,2)$

Note: Write solution in form $x^2 + y^2 + a\cdot x + b\cdot y + c = 0$.

$$x^2 + y^2 + a\cdot x + b\cdot y + c = 0$$

$$\begin{vmatrix} 5a + 5b + c + 50 = 0 \\ 4a + 6b + c + 52 = 0 \\ 6a + 2b + c + 40 = 0 \end{vmatrix}$$

$$\begin{vmatrix} 5a + 5b + c + 50 = 0 \\ 4a + 6b + c + 52 = 0 \\ 6a + 2b + c + 40 = 0 \end{vmatrix} \begin{matrix} \cdot\frac{1}{5} \\ \longrightarrow \\ \longrightarrow \end{matrix} \begin{vmatrix} a + b + \frac{c}{5} + 10 = 0 \\ 4a + 6b + c + 52 = 0 \\ 6a + 2b + c + 40 = 0 \end{vmatrix}$$

$$\begin{vmatrix} a + b + \frac{c}{5} + 10 = 0 \\ 4a + 6b + c + 52 = 0 \\ 6a + 2b + c + 40 = 0 \end{vmatrix} \begin{matrix} \longrightarrow \\ -4\mathrm{I} \\ -6\mathrm{I} \end{matrix} \begin{vmatrix} a + b + \frac{c}{5} + 10 = 0 \\ 2b + \frac{c}{5} + 12 = 0 \\ -4b + \frac{-c}{5} - 20 = 0 \end{vmatrix}$$

$$\left|\begin{array}{rrrl} a+ & b+ & \frac{c}{5} & +10=0 \\ & 2b+ & \frac{c}{5} & +12=0 \\ & -4b+ & \frac{-c}{5} & -20=0 \end{array}\right| \begin{array}{c} \longrightarrow \\ \cdot 12 \\ \longrightarrow \end{array} \left|\begin{array}{rrrl} a+ & b+ & \frac{c}{5} & +10=0 \\ & b+ & \frac{c}{10}+ & 6=0 \\ & -4b+ & \frac{-c}{5} & -20=0 \end{array}\right|$$

$$\left|\begin{array}{rrrl} a+ & b+ & \frac{c}{5} & +10=0 \\ & b+ & \frac{c}{10}+ & 6=0 \\ & -4b+ & \frac{-c}{5} & -20=0 \end{array}\right| \begin{array}{c} -\text{II} \\ \longrightarrow \\ +4\text{II} \end{array} \left|\begin{array}{rrl} a & +\frac{c}{10} & +4=0 \\ b+ & \frac{c}{10} & +6=0 \\ & \frac{c}{5} & +4=0 \end{array}\right|$$

$$\left|\begin{array}{rrl} a & +\frac{c}{10} & +4=0 \\ b+ & \frac{c}{10} & +6=0 \\ & \frac{c}{5} & +4=0 \end{array}\right| \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \cdot 5 \end{array} \left|\begin{array}{rrl} a & +\frac{c}{10}+ & 4=0 \\ b+ & \frac{c}{10}+ & 6=0 \\ & c+ & 20=0 \end{array}\right|$$

$$\left|\begin{array}{rrl} a & +\frac{c}{10}+ & 4=0 \\ b+ & \frac{c}{10}+ & 6=0 \\ & c+ & 20=0 \end{array}\right| \begin{array}{c} -\frac{III}{10} \\ -\frac{III}{10} \\ \longrightarrow \end{array} \left|\begin{array}{rrl} a & + & 2=0 \\ b & + & 4=0 \\ c & + & 20=0 \end{array}\right|$$

$$\left|\begin{array}{rl} a & =-2 \\ b & =-4 \\ c & =-20 \end{array}\right|$$

$$-2x-4y-20-x^2+y^2=0$$

Center: (1, 2)
Radius: 5