1:
$\int_{-1}^{2} \frac{x}{\sqrt{x^2+1}}\, \mathrm{d}x$
Need to use u-substitution
$u = x^2 + 1$
$f(x) = \frac{1}{\sqrt{g(x)}}$
$\mathrm{d}u = 2x\mathrm{d}x$
$x\mathrm{d}x = \frac{\mathrm{d}u}{2}$
$\int \frac{1}{2\sqrt{u}}\, \mathrm{d}u$
$= \frac{1}{2} \int \frac{1}{2\sqrt{u}}\, \mathrm{d}u$
$= \frac{1}{2} \int u^{-1/2}\, \mathrm{d}u$
$= \frac{1}{2} \cdot 2\sqrt{u}$
$= \sqrt{u}$
$\sqrt{u} = \sqrt{x^2+1}$
$\int \frac{x}{\sqrt{x^2+1}} \mathrm{d}x = \sqrt{x^2+1}(+C)$
Between -1 and 2: $= \sqrt{5} - \sqrt{2}$

2:
$\int x\mathrm{e}^{-4x}\, \mathrm{d}x$
$u = \mathrm{e}^{-4x}, v = \frac{x^2}{2}$
$u' = -4\mathrm{e}^{-4x}, v' = x$
$uv - \int vu'$
$\frac{x^2\mathrm{e}^{-4x}}{2} - \int \frac{-4x^2\mathrm{e}^{-4x}}{2}$
$= \frac{x^2\mathrm{e}^{-4x}}{2} - \frac{(8x^2+4x+1)\cdot\mathrm{e}^{-4x}}{16}$
$= \frac{-(4x+1)\cdot\mathrm{e}^{-4x}}{16} + C$

3:
$\int \frac{14}{(2x-1)(x+3)}\, \mathrm{d}x$
$\frac{14}{(2x-1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+3}$
$14 = A(x+3) + B(2x-1)$

To remove A:
$x = -3$
$14 = A(0) = B(-7)$
$-14 = 7B$
$B = -2$

To remove B:
$x = \frac{1}{2}$
$14 = A(\frac{7}{2}) + B(0)\ 14 = \frac{7A}{2}$

$7A = 28$
$A = 4$
$A = 4, B = -2$

$\int \frac{14}{(2x-1)(x+3)}\,\mathrm{d}x = \int \frac{4}{2x-1}\,\mathrm{d}x + \int \frac{-2}{x+3}\,\mathrm{d}x$
$= 2\ln|2x-1| + -2\ln|x+3| + C$

4:
$\int_1^3 \ln x \mathrm{d}x$
Int by parts needed.
$\int uv' = uv - \int vu'$
$u = \ln x, v = x$
$u' = \frac{1}{x}, v' = 1$
$x \ln x - \int 1$
$= x\ln x - x$
Between 1 and 3
$(3\ln 3 - 3) - (\ln 1 - 1)$
$= 3\ln 3 - 2$

5:
$\int \frac{1}{\sqrt{81-x^2}}\,\mathrm{d}x$
$x = 9\sin u$
$\mathrm{d}x = 9\cos u \mathrm{d}u$
$= \frac{1}{\sqrt{-81\sin^2 + 81}}$
$= \frac{1}{9\sqrt{-\sin^2 u + 1}}$
$= \frac{1}{9\sqrt{\cos^2 u}}$
$= \frac{1}{9\cos u}$ $u = \arcsin\frac{x}{9}$
$\int x\ln x^2\,\mathrm{d}x = u$
$= \arcsin\frac{x}{9} + C$

6:
$\int x\ln x^2\,\mathrm{d}x$
U-sub
$u = x^2$
$\frac{\mathrm{d}u}{2} = x\mathrm{d}x$
$= \int \frac{\ln u}{2}\,\mathrm{d}u$
Int by parts:
$\int ab' = ab - \int ba'$

$a = \ln u, b = \frac{u}{2}$
$a' = \frac{1}{u}, b' = \frac{1}{2}$
$= \frac{u \ln u}{2} - \int \frac{1}{u} \frac{u}{2}$
$= \frac{u \ln u - u}{2}$
$= \frac{x^2(\ln x^2 - 1)}{2} + C$

7:
$\int \arctan x \, \mathrm{d}x$
$u = \arctan x, v = x$
$u' = \frac{1}{x^2+1}, v' = 1$
$x \arctan x - \int \frac{x}{x^2+1}$
U-sub:
$u = x^2 + 1$
$\mathrm{d}u = 2x\mathrm{d}x$
$x\mathrm{d}x = \frac{\mathrm{d}u}{2}$
$x \arctan x - \frac{1}{2} \int \frac{1}{u} \, \mathrm{d}u$
$x \arctan x - \frac{1}{2} \ln |u| = x \arctan x - \frac{1}{2} \ln |x^2 + 1|$
$= x \arctan x - \frac{1}{2} \ln |x^2 + 1| + C$

8:
$\int \frac{1}{(4x^2+9)^2}$
Let $x = \frac{3}{2} \tan \theta$
$\mathrm{d}\theta =$

9:
$\int \sin^2 \theta \cos \theta \, \mathrm{d}\theta$ $u = \sin x$
$\mathrm{d}u = \cos x \mathrm{d}x$
$\int u^2 \, \mathrm{d}u$
$= \frac{u^3}{3}$
$\frac{\sin^3 x}{3}$
$= \frac{\sin^3 x}{3} + C$

10:
$\int \frac{2x-1}{(x+1)(x^2+9)} \, \mathrm{d}x$
$\frac{1}{10} \int \frac{3x+17}{x^2+9} \, \mathrm{d}x - \int \frac{3}{10x+10} \, \mathrm{d}x$
$= \frac{1}{10} \int \frac{3x}{x^2+9} \, \mathrm{d}x - \int \frac{3}{10x+10} \, \mathrm{d}x + \frac{1}{10} \int \frac{17}{x^2+9} \mathrm{d}x$
U-sub:
$u = x^2 + 9, \mathrm{d}u = 2xdx, x\mathrm{d}x = \frac{\mathrm{d}u}{2}$
$= \frac{1}{10} \int \frac{3}{2u} \, \mathrm{d}x - \int \frac{3}{10x+10} \, \mathrm{d}x + \frac{1}{10} \int \frac{17}{x^2+9} \mathrm{d}x$

$= \frac{3}{20} \ln|x^2+9| - \int \frac{3}{10x+10}\,\mathrm{d}x + \frac{1}{10} \int \frac{17}{x^2+9} \mathrm{d}x$
$= \frac{3}{20} \ln|x^2+9| - \int \frac{3}{10x+10}\,\mathrm{d}x + \frac{17}{10} \int \frac{1}{x^2+9} \mathrm{d}x$
U-sub:
$x = 3u$
$\mathrm{d}x = 3\mathrm{d}u$
$u = \frac{x}{3}$
$= \frac{3}{20} \ln|x^2+9| - \int \frac{3}{10x+10}\,\mathrm{d}x + \frac{17}{30} \int \frac{1}{u^2+1} \mathrm{d}x$
$= \frac{3}{20} \ln|x^2+9| - \int \frac{3}{10x+10}\,\mathrm{d}x + \frac{17}{30} \arctan u$
$= \frac{3}{20} \ln|x^2+9| - \int \frac{3}{10x+10}\,\mathrm{d}x + \frac{17}{30} \arctan \frac{x}{3}$
$= \frac{3}{20} \ln|x^2+9| - \frac{3}{10} \int \frac{1}{x+1}\,\mathrm{d}x + \frac{17}{30} \arctan \frac{x}{3}$
U-sub:
$u = x+1$
$\mathrm{d}u = \mathrm{d}x$
$= \frac{3}{20} \ln|x^2+9| - \frac{3}{10} \ln|u| + \frac{17}{30} \arctan \frac{x}{3}$
$= \frac{3}{20} \ln|x^2+9| - \frac{3}{10} \ln|x+1| + \frac{17}{30} \arctan \frac{x}{3} + C$