Using apps for teaching and learning mathematics: A socio-material assemblage

How Might Apps Reshape the Mathematical Learning Experience?

Nigel Calder and Carol Murphy

Abstract This chapter reports on how the use of mathematics apps has the potential to reshape the *learning experience*, a particular aspect of learning with apps that emerged from a research project examining the ways mobile technologies are used in primary-school mathematics. The chapter will consider a number of key themes related to student learning that have emerged through the research. When using some of the apps in the study, students used different digital tools within the app to solve word problems, while the affordances, including multi-representation, dynamic and haptic, made the learning experience different from when using pencil-and-paper technology. Other themes that were identified included: collaboration, socio-material assemblages, and personalisation. All of these appeared influential in the development of mathematical thinking. While the affordances of the mobile technologies are important, the teacher's pedagogical approach and the dialogue that the apps evoked were central in the learning.

Keywords Affordances · Collaboration · Differentiation · Engagement
Mathematical thinking · Primary-school mathematics · Assemblages
TPACK · Video analysis

N. Calder (✉) · C. Murphy
University of Waikato, Hamilton, New Zealand
e-mail: nigel.calder@waikato.ac.nz

C. Murphy
e-mail: carol.murphy@utas.edu.au

N. Calder et al. (eds.), *Using Mobile Technologies in the Teaching and Learning of Mathematics*, Mathematics Education in the Digital Era 12,
https://doi.org/10.1007/978-3-319-90179-4_3

Introduction

Mobile technologies are everywhere! The use of mobile technologies (MT), especially smart-phones, has grown markedly, as has the availability of WiFi, both in educative settings and in community settings. The accompanying growth in the number of apps has likewise been recognised (Larkin, 2015). There is a lot of excitement regarding their potential to transform the learning experience and enhance mathematics learning opportunities. Their ease of operation, allied with the students' interaction being focused primarily on touch and sight, can make their use intuitive for learners (Calder & Campbell, 2016). While concerns have been raised regarding the suitability of the pedagogical approaches utilized through and with apps (e.g., Philip & Garcia, 2014) other research has highlighted their effectiveness in various aspects of mathematics learning (Attard, 2015; Carr, 2012). Also, as MT have become a more enduring element of the evolving digital world, we need to consider their potential for learning. This chapter reports on an aspect of a research project examining the ways tablets, as examples of MT, are used in primary-school mathematics. The project considers the pedagogy that might best facilitate the learning with students (ages 5–11) when engaging in mathematics using MT. One aim of the research was to identify aspects of the learning process that influenced the mathematical learning, when students engaged with mathematics using apps. What were features of the learning that emerged through using MT, apps in particular? The chapter reports on the themes related to pedagogy that emerged from the research and how they might be seen to reshape learning experiences in primary mathematics.

In the chapter, we will concentrate on the emerging themes related to the ways that teachers are using MT in their classrooms and mathematics programmes. The themes and framework emerged from an iterative process of co-construction by the research team, including 12 teacher researchers. The themes are: affordances, collaboration, socio-material assemblages, and personalisation/differentiation. The ways that mathematical thinking is hinged to each of these themes will be inherent in the discussion of each, as will the interconnectedness and relationships between the themes. Each theme will be considered in a separate section, prefaced by some informative, and influential, theoretical and research perspectives. The chapter will conclude with how some themes might overlay and influence each other, as well as some perspectives that emerged from the project overall.

Methodology

The research project used an interpretive methodology that relates to building knowledge and developing research capability through collaborative analysis and critical reflection of classroom practice and student learning. The research design was aligned with teacher and researcher co-inquiry whereby the university researchers and practicing teachers work as co-inquirers and co-learners (Hennessy, 2014). Allied to this is an emphasis on collaborative knowledge building. This research method is based on a transformational partnership arrangement that generates new professional knowledge for both academic researchers and teachers (Groundwater-Smith et al., 2013). There is joint scrutiny of the reflections and evaluations, and hence joint scrutiny of an educational practice. This scrutiny informs new forms of awareness for teachers and researchers (Hennessy, 2014). Three teachers, all experienced with using MT in their programmes, were involved in the first year of the study. One teacher taught a year-4 class in a school using a approach, while the other two teachers team-taught in a year-5 & 6 class, in a school with 1-1 iPad provision. The data were analysed using NVivo. The themes developed from the observed use of MT in classes, with data collected by video, teacher semi-structured interviews and student blogs, and from collaboration in research meetings with teachers viewing one or two extracts of video each time. The video extracts were of the students working in class during their mathematics lessons. Ethical approval was obtained and pseudonyms are used for participants.

In the second year of the project, nine other teachers joined the research team. These teachers were across a range of year levels (years 1–6) and experience with using apps in their mathematics programmes (from using apps for students to practice a particular skill, to using apps such as *Math Shake* with screen-casting ability, for students to explain their strategies and solutions). See Table 1 for demographic information of teachers discussed in this chapter. The themes from the first year were carried forward into year two. Refinement of the identified themes occurred through joint critical reflection between the teacher practitioners and academic researchers in research meetings. As the chapter focuses on the themes, the teacher data from both years were considered, while the student data is only from the first year of the project.

Table 1 The teachers, their context, and experience with MT at the start of the project

Teacher	School	Year level	Experience with MT in mathematics programme
Anna	1-1 iPad	Y 2	Uses apps to support skill development
Sarah	BYOD	Y 5	Very experienced, but uses apps to support skill development
Brad	1-1 iPad	Y 5 & 6	Integrates MT into many aspects of programme including coding. Uses screen-casting effectively
Jane	BYOD	Y 4	Integrates MT into many aspects of programme. Uses screen-casting effectively
Alan	1-1 iPad	Y 6	Limited, uses apps to support skill development
Trish	1-1 iPad	Y 5 & 6	Integrates MT into many aspects of programme. Uses screen-casting effectively
Joy	1-1 iPad	Y 1	Limited, uses apps to support skill development

In this chapter we present extracts of data from the study that were considered in relation to the emerging themes of the project. The following sections consider some theoretical perspectives of each theme, along with data from the research project that illustrate how engaging with the mathematics through the apps, might reshape the mathematical learning.

Affordances

In relation to Gibson's (1977) notion of affordance as the complementarity of the learner and the environment, the affordances of MT, including visual, haptic and dynamic, may be seen to fashion the learning experience in distinctive ways. This gives opportunity to reposition students' engagement with mathematics. Affordances can be thought of as the potential opportunities and constraints in the relationship between the digital object and the user (Calder, 2011).

An affordance frequently associated with digital environments is the notion of multiple representations. The ability to link and simultaneously interact with visual, symbolic, and numerical representations in a dynamic way has been acknowledged extensively in research (e.g., Calder, 2011). In a similar way, various studies involving dynamic geometry software, report that the dynamic, visual representations enhanced mathematical understanding (e.g., Falcade, Laborde, & Mariotti, 2007). This dynamic affordance, coupled with the instant feedback to input, opens opportunity for reshaping the learning experience.

Virtual manipulatives (VM) are frequently part of mathematical apps. They are described as interactive, web-based visual representations of dynamic objects (Moyer, Bolyard, & Spikell, 2002) that might afford opportunities for mathematical thinking. VM offer potential to extend the learning experiences with representations beyond those with pencil-and-paper medium (Arcavi & Hadas, 2000). In *Math Shake*, for example, word problems are generated at various levels, and it provides a range of digital pedagogical tools (e.g., empty number lines, counters, ten frames), that students can select to help with their solutions.

Moyer-Packenham and Westenskow (2013) identified the affordances of focused constraint, creative variation, simultaneous linking, efficient precision, and motivation when students used apps in their mathematical learning. While these affordances interact, and appear to be mutually influential of each other, three of them resonate with the other emerging themes. These are: focused restraint, where the app might focus students' attention on particular mathematical concepts or processes; creative variation, where the app might encourage creativity, hence evoking a range of student approaches and potential solutions; and simultaneous linking, where the app might link representations simultaneously and connect them to student activity (Moyer-Packenham & Westenskow, 2013).

The interface of an iPad offers a further affordance through touch. Student interaction is more direct and tactile than when working on desktops, further enhancing the relatively high agency of the medium. There is direct interaction with the phenomena, rather than being mediated through a mouse or keyboard, making the iPad more suitable for young children than desktop computers (Sinclair & Heyd-Metzuyanim, 2014). Some apps make use of this haptic affordance (e.g., with *Multiplier*, where within the task, the student drags out the visual area matrix associated with multiplication facts). This app also evokes multi-touch functionality, enabling students to make sense of individual effects of particular screen touches (Hegedus, 2013; Jackiw, 2013), and to create personal explanations of their thinking (e.g., making a screencast of their problem solving strategy). This is similar to the simultaneous linking and creative variation that Moyer-Packenham and Westenskow (2013) identified (Fig. 1).

Much of the discussion regarding the ways iPads and apps might transform the learning experience is centred on the notion of student engagement (e.g., Attard, 2015, also see chapter in this book). Meanwhile other researchers have reported improved high-level reasoning and problem solving linked to learners' investigations in digital environments (Sandholtz, Ringstaff, & Dwyer, 1997). Many apps provide affordances of interactivity and non-threatening instantaneous feedback that foster the learner's willingness to experiment and take cognitive risks with their learning (Calder & Campbell, 2016). These types of apps allow students to model in a dynamic, reflective way. Students in the study took risks while using *Multiplier* by trying different arrays. They would try a number of possibilities, some of which were unconventional and sometimes incorrect, before settling on their preferred option.

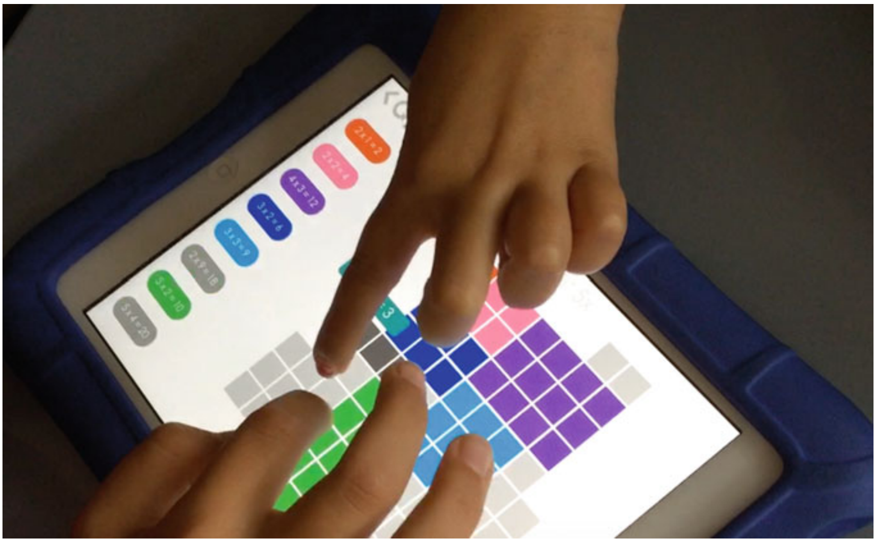

Fig. 1 Image from video data illustrating the visual, haptic and interactive affordances of *Multiplier*

Apps such as *Explain Everything* and *Math Shake* allow students to record individual or group presentations of mathematical processes, strategies and solutions. The screen-casting feature of the app, and the simplicity with which it is enabled on an iPad, opens up other learning opportunities that would not be possible with pencil-and-paper technology. Such apps introduce a further multi-representational affordance whereby students can create an aural representation that other students can listen to.

One teacher, Anna, commented on the direct interface of the iPad screen, suggesting that the students were interacting more directly with the content of the mathematics—"Like a physical object that they're interacting with." This suggests the haptic affordance and focused restraint as the teacher perceived the app facilitating more direct interaction with mathematical content. Some students commented how the feedback and opportunity to record their solutions had helped their learning in mathematics:

> Jake: *We can write things down and answer questions to see if we are right or wrong.*

Sometimes this feedback was directly from the app, and at times it would be from other students, or the teacher, after they had viewed the screencasts in *Google classroom*. One student comment identified the range of digital tools, such as those that enabled screencasting, as being beneficial for learning.

> Josh: *The most helpful app for me is* Explain Everything *as it has lots of tools and options to help learning rather than doing it on paper with a pencil.*

Teacher comment likewise indicated that the features of the MT medium afforded particular teaching and learning opportunities:

> Sarah: *One to help me as a teacher… a teaching tool… to explain things or to use a learning object like an interactive number line or arrow cards or voice recordings as a teacher tool.*

> Brad: *They had to use an app called* Tickle *to program some robots to draw those same shapes on the map in real life – which was really cool because there's quite a little bit of shift in the mathematics thinking because you couldn't just use the internal angles, you had to convert from how much the robot has to turn using the internal angles as a reference point … and the kids had to work out why that worked and what to do to get that to work.*

This also involved the students working together, trying ideas out in practice and negotiating possible solutions. Hence, the collaboration theme was identified.

Collaboration

Simply put, collaborative learning may occur when two or more students are engaged in an activity and learning together (Dillenbourg, 1999). Such a perspective on learning in mathematics shifts from individual acquisition to participation in a social practice (e.g., Cobb & Bowers, 1999; Sfard, 1998). Educational engagement and collaboration associated with joint problem solving has been connected to academic success. For example, Mercer and Sams (2006) showed how collaboration with students engaged in an online task supported learning outcomes in mathematics. More recently Mercier and Higgins' (2013) study has shown how the collaborative use of digital technologies can support students in developing more flexible approaches to problem solving.

The ability of iPads to support collaboration would seem a key aspect of reshaping the learning of students in mathematics. The iPads potential for social computing has been acknowledged (della Cava, 2010), but this potential is still to be fully explored in the mathematics classroom. Zurita and Nussbaum (2004) noted how the flexibility of MT allows "students to engage in highly collaborative activities anywhere, at any time" (p. 293), and Fisher, Lucas, and Galstyan (2013) indicated how the portability and tactile interface of the iPad allows students to work both privately and publicly and to transition easily between the two.

In the same way that Fisher et al. (2013) noted the transition between private and public use, Looi et al. (2009) noted how the mobility of iPads allows students to not only make choices regarding where they are working, but also whom they may wish to work with. For example, there is the ability for a student to easily "…swivel and show…" another student what they are doing or share what they had recorded earlier (Looi et al., 2009), and hence share their thinking on the iPad with one or more of their fellow students. In this way the use of iPads encouraged incidental collaboration between the students in the classrooms we studied (see Fig. 2).

Fig. 2 Students collaborating on volume models related to *Minecraft* activity

In the interviews, the teachers also reported that the flexible learning environment encouraged collaboration:

> Jane: *I was surprised at how much collaboration went on because they were allowed to sit anywhere they wanted ... they would just ask their neighbour something and then there was this little conversation.*

While this might occur in a learning situation without MT, having the MT as the medium for learning enhanced student collaboration in the classroom setting.

Zurita and Nussbaum (2004) described different collaboration areas including the use of MT devices to coordinate task activities, where the mobility of the device allowed students to move with the device and work with other students at different locations. The joint coordination of a task enables students to communicate and negotiate in order to support decision-making (Zurita & Nussbaum, 2004), and, as such, they are involved in "a coordinated joint commitment to a shared goal" (Mercer & Littleton, 2007, p. 23). Teachers used iPads directly for joint collaborative activities with their students, for example when students worked together to create a screen cast together.

Alan set a collaborative task in relation to students' calculation strategies and noted:

> *That's another thing we did – we sent them off in groups to work on a strategy – they each used a different strategy, video recorded their thinking, came back together, argued about which strategy was the best by watching the videos and then decided.*

Here, the screencasting feature of the app appeared to better facilitate the ease with which the students could video their explanations, review them as a group, and then debate the merits of each before collaborating to decide the content of their group's final screencast. Two groups of students working on a problem using *Minecraft,* on a single iPad, suggested two aspects of collaboration. The first related to the contestation of ideas and processes:

Aaron	*Okay, 5 lots of 5 blocks*
Zac	*Yep, 5 blocks*
Don	*Shall we use a line?* (He indicated where the 5 blocks might go on the screen)
Zac	*No, not 5 blocks up!*
Aaron	*Yes, you need to use it there*
Don	*Yeah, there*
Zac	*Is it? No this one* (pointed to the screen)
Aaron	*You need the 5 blocks across and going up* (indicated on the screen)
Zac	*Oh yeah, yeah now I see.*

Here, Zac's understanding of the solution and the process changes through the discussion related to the group's direct interaction with the iPad screen. It was the visual tension evoked from touching the screen, and the immediate impact from that action, that initiated the dialogue and also enabled, in conjunction with the dialogue, the transition in Zac's understanding. The second excerpt relates to the sharing of knowledge and ideas:

Ali	*You can fly too, you realize* (demonstrates by touching the arrows on the screen). *Double click the jump button.* (Whetu double clicks the button)
Ali	*That isn't the jump button!* (Ali demonstrates the button and how to fly again. Whetu takes over)
Whetu	*I can fly, fly high in the sky!*
Ali	*So you can control your flight with that one* (Whetu takes over the arrow controls)
Whetu	*Going up! Weee! Now I want to go down now.*
Ali	*Use the other one then* (points to the screen). (Whetu changes buttons and brings the "flight" down).

Ali peer shares her knowledge of the app (how to fly in that particular digital environment) and then peer teaches Whetu so that her understanding of the process and potential of the app is enhanced. With both excerpts, the particular visual, interactive affordances of the app, coupled with the instantaneous visual feedback to their input, influenced the dialogue and the interaction with the task. This enabled the collaboration and learning process to evolve in a manner that is distinctive from pencil-and-paper approaches.

Mercer and Littleton's (2007) definition of collaborative learning goes beyond the sharing of ideas and task coordination to "reciprocity, mutuality and the continual (re)negotiation of meaning" (p. 23). Collaborative learning in line with this definition was identified by one student, Tui, in commenting on how the MT facilitated collaboration through the utilization of individual understandings and expertise:

> Tui: *... we can work through it together because I might be smarter with the device and I can help you with the device but you can help me with my maths so when we ... we can go and work together and solve things.*

Hence, it was noted that apps enabled collaborative approaches to learning when a MT was being used in a jointly coordinated shared task, as well as incidentally when students were working individually and informal opportunities to share arose. In both instances, the use of apps initiated discussion, with the potential to renegotiate thinking, which in turn initiated further engagement with the MT (Fig. 3). In this way the learning through apps took place within interconnected groupings of digital elements and the social aspects that they evoked. We have identified this relationship as socio-material assemblages.

Fig. 3 Incidental collaboration on an area task using *Brainpop*

Socio-material Assemblages

It has been suggested that MT offer a socio-material bricolage for learning (Meyer, 2015). Drawing on Fenwick and Edwards' (2012) notion of socio-material approaches to learning, Meyer envisaged interconnected systems where resources interact with knowledge that is socially distributed. A range of people, communities and sites of practice might be influential in assisting student learning. Meyer (2015) used the term socio-material bricolage to describe the "ecological entanglement of material and social aspects of teaching and learning with technology" (p. 28).

The notion of bricolage suggests that there is a mutually influential collective of tools and users affecting the dialogue, learning experience, and mathematical thinking, in particular and personalised ways. For example, when students collaborate on a task, they incorporate input from the wider class, school and 'home' communities, while also drawing from the broader underlying discourses, such as political or socio-cultural elements that influence their pre-conceptions about the task and mathematical activity. De Freitas and Sinclair (2014) discussed 'thought' as being distributed across both social and physical environs and influencers. We consider that thought evolves in a complex material and social milieu. When screencasting their strategy and solution(s) the students might incorporate a range of digital, visual, and concrete material resources in mutually interdependent ways. All of this activity has associated social elements, both immediate interaction as well as the drawing forth of the underlying discourses. The resulting process is not just the accumulation of the various 'bits', but also a new mesh of the social and material elements.

Johri (2011) argues that in learning, the education participants often make do with the tools available to them, with what is at hand, rather than following planned approaches with tools not immediately available. He contends that a socio-material bricolage supports the interwoven social and material nature of human practices. Sandholtz et al. (1997) indicated that affordances of digital technologies, together with the associated dialogue and social interaction, may lead to students exploring powerful ideas in mathematics, learning to pose problems, and creating explanations of their own. Various aspects overlap and interlace. Students are seen to have a choice in how they imbricate their perceptions with the material, related to the features of the iPad and the app, within the context of a mathematical problem. In turn, the material has the potential to influence the imbrication. The data were relatively cohesive, in terms of being influential in the learning process, regarding the connection between the use of the apps, other technologies such as concrete materials, and the dialogue and social interaction that engagement with them evoked (Meyer, 2015). For instance, Trish commented:

> *They used the iPad to watch a video and they took a brainstorm on a piece of paper about what a triangle is and different types of triangles – what internal angles are and external angles and things like that and then we… the kids used that information to create some triangles.*

Here, we see the use of different technologies (including paper) yet it is the interconnected, mutually influential social elements, such as, brainstorming and using the information to create, which become part of the socio-material assemblage. For example, students were observed using the iPad to investigate a problem in context, then using counters and rods, all the time interacting with each other and the range of tools. They used an empty number line in the app and a white table for story boarding the screencast of their strategy and solution. This was then loaded into a *Google classroom* site that the teacher could access for review and feedback.

One teacher, Brad, saw this tapestry of material and social elements as an ecosystem:

> Brad discussing *Hopscotch*: *There's a really big app eco system – I don't think there's many other devices that you can program on the iPad and then program robots and record your voice and make videos and all that stuff – it's a very rich ecosystem.*

There were also instances where concrete materials were used in conjunction with apps. For example, Joy talks mainly of an assemblage of material elements, with the associated social aspects, including the relationships and interaction between students, teacher, school community and the broader societal discourses inherent in the activity described:

> *You might do something with those Cuisenaire rods… those plastic things… there's also an app that would do it as a lesson and then there's an app that actually has the rods in it so the kids can go away and practice moving them around the screen after they've done it with you physically… so there's a nice connection.*

The students recognized the same potential for using a mixture of technologies at the appropriate time for their learning:

Fig. 4 Students solving a number problem in pairs using a mixture of writing and digital activity

> Tim: *Sometimes I make a plan (on paper) to work out my word problem, then I can put the pictures on and record my answer on the iPad.*

> Whitu: *So sometimes things are better to work out on paper, but other things are better on my iPad.*

These student blog data were examples of the students integrating different technologies to best investigate and solve a problem (Fig. 4).

In our observations we saw that students moved relatively seamlessly between pencil-and-paper and digital technology and utilized the type that they found best facilitated the learning process for them. This indicates that they chose the technology, and the way that they used it, to suit their individual or group requirements—a form of personalisation. The next section considers this theme.

Personalisation/Differentiation

Current perspectives on, and manifestations of, personalisation vary markedly, often in conflicting ways. Some advocate that student choice is paramount, while in other perspectives personalisation is something the teacher directs, with no student input. An and Reigeluth (2011) note that personalised learning involves teachers paying close attention to individual student's knowledge and skills and using this knowledge to provide personalised experiences and support in learning. Contrastingly, Leadbetter (2005) contends that personalised learning is being focused on motivating students to become engaged in their own learning by allowing them to make personal choices about it. In this view the teacher's aim would be to create an environment where students are empowered to make decisions.

Waldrip et al. (2014) note that personalised learning relates to structured activities that students engage in with scaffolding from their teachers such as "… modeling, guidance in goal-setting and timely feedback" (p. 357). Using Tomlinson's (2009) model of student variance as interest, readiness, and learning profile, the use of apps such as *Explain Everything* can be examined in relation to socio-material assemblages. The entanglement of the social (students' interests, readiness and learning profile) and the material (hardware and software) suggests evidence of reconfiguration related to human and material agency. Interestingly, Tomlinson (2009) used the term differentiation to describe this sort of concept and concluded that teaching with an emphasis on student variance and choice should elicit conceptual understanding. The use of apps such as *Explain Everything* can be examined while also enhancing student efficacy and ownership of learning. Others contend that MT can provide new forms of personal ownership (e.g., Meyer, 2015) that in turn supports learners' personal understanding and conceptual frames (Melhuish & Falloon, 2010). Whatever the definition of personalised learning, a key tenet is that students choose the tools and the contexts for the task; for instance, students' personal use of images and recordings when using the *Explain Everything* app. iPads, as a type of mobile technology, have been identified as having the potential to enhance personalised learning due to two key characteristics, the mobility of the devices, and their ability to continually change contexts (Looi et al., 2013).

The feature of mobility suggests that iPads can be used anytime and anywhere. When used in education this serves to change the definition of what is considered a learning space. Learning is no longer in a particular place or time, but can be anytime and anywhere (Melhuish & Falloon, 2010). This mobility extends beyond the classroom as the iPads can be used seamlessly, between school, home and further afield such as on field trips (Calder & Campbell, 2016) (Fig. 5).

Fig. 5 Students in the study used a variety of work spaces

Another way in which different contexts are created is through the ability of apps to shape experiences to meet specific needs. There are specific apps (e.g., *MathsBlaster*) designed to meet various learning stages and steps, meaning that the selection of apps can be personalised to meet a range of students' needs (Clark & Luckin, 2013). Furthermore, some apps provide specialized features that enable specific learning and instruction within the app (e.g., *Brainpop*). This means that not only can the choice of apps be personalised, but also what happens within them can be modified, enabling interaction with the apps to be personalised to a student's specific learning needs, perhaps through the type or level of question (Calder & Campbell, 2016; O'Malley et al., 2013). Teacher-initiated personalisation resonates with Cutler, Waine and Brehony's (2007) contention that personalisation is about the raising of achievement. In the classroom situation, the teacher's ability to choose apps, and levels and tasks within apps to suit specific learning needs, has facilitated differentiation of the learning for specific students (Clark & Luckin, 2013; O'Malley et al., 2013).

Changing contexts can also be associated with personalising the features of the iPad working environment, such as the font and colour in their presentations (Robinson & Sebba, 2010). However, within this customization, there are concerns with Looi et al., (2009) noting that the endless customization features of mobile technologies led a student in their study to spend an excessive amount of time on the aesthetic features, rather than focusing on the intended learning. A blog post from a student indicated the impact of customizing features:

> Ella: *You can make math more interesting by changing the colour, font and size, and you can use pictures from the internet.*

Another identified the features in *Explain Everything* as being both motivational and helpful for the learning:

> Kate: *I use* Explain Everything *with my Thinking Boards. I use the voice recorder, the drawing pen, different colours, I can pick the size of my pictures, duplicate things. I can move things to show my thinking.*

A sense of ownership and individuality can also be expressed in the images the students used as screen-savers on their iPads, which might lead to an emotional attachment. For instance, one boy commented when asked to leave his iPad in the classroom "Goodbye my darling," as he hugged it goodbye! Importantly, the apps can facilitate the differentiation of the learning associated with cognitive understanding, either for individuals or for groups, sometimes linked to accuracy and speed. Student blog data was illustrative of this:

> Ethan: *I sometimes do* Skoolbo *and I have* Maths Sums *where I get to choose what sum you wanna do and you get to choose the level, and you have to unlock the level, and you get 20 seconds to answer the question.*

> Julie: Math Shake *is a great learning tool because it can help you with your problem solving. So you can choose a level for you, so just say you were genius or easy or confident or even beginner, there are a lot of levels to choose from. And there are also some amazing*

tools to help you solve your word problem for instance number lines, fractions, counters, and there is also different coloured pencils that you have to earn.

Teresa: *In* Money Mind NZ*, I like to go shopping and I have to work out how much I have to pay (I like choosing my items to buy), and I like getting it right.*

Teacher comment also indicated that in teacher directed differentiation apps could be selected based on the basis of their suitability for particular levels of learning. The teacher could shape the learning experience based on their knowledge of the students, including their conceptual and technological understanding (Fig. 6).

Two teacher comments were particularly indicative of this, with the first related to teaching a group of high achieving mathematics students and the second teaching a group needing more support with their mathematics learning:

Brad: *An extension app that I love to use which is a web app is called* Lure of the Labyrinth *which has been really good at high end critical thinking and things like… the kids, we were converting like between base 10 numbers… base the total, like base 6, 7, 8 and 9 numbers and binary to solve puzzles.*

Trish: *They've made stop motion animations on polygons – this is the lower group – like what is a hexagon, what is an octagon, what is a triangle and they use little stop motion animation and match sticks to make those shapes and animate them then talk about them.*

While there is some fluidity in use and meaning associated with personalisation and differentiation of the learning experience, in this chapter we have considered

Fig. 6 Solving a problem using a personal workspace and images

two versions in particular. One that involves teachers paying close attention to individual student's knowledge and skills and using this knowledge to provide personalised experiences and support in learning (An & Reigeluth, 2011). Meanwhile, the second advocates that the teacher focus on motivating students to become engaged in their own learning by allowing them to make personal choices about it (Leadbetter, 2005). There are areas of convergence and contrast in these two perspectives, with a key element of both being the teacher trying to optimize the students' engagement with, and understanding of, the mathematics. The intention is to differentiate the learning experience to best facilitate mathematical learning for the individual or group of students.

Having a classroom culture and mathematical activities that promote individual student choice is intended to engage and motivate the students through a sense of ownership of the learning so that they might be more receptive to the mathematics learning. These two perspectives are not distinct, however. They may operate in tandem, and there is a continuum of the possible inter-relatedness of both in the learning experience. In a similar way, the themes identified in this research can overlap and be mutually influential in the mathematics learning. The next section draws together the four themes to consider the ways that they might facilitate the mathematics learning.

The Weaving of Themes

Although the four themes are different and influence the mathematical learning in varying ways, they are not discrete or necessarily independent. While the personalisation of the working environment seemed to motivate the students, the affordances of the apps coupled with the pedagogical approach and culture of the classroom, appeared to be influential in personalising the learning experience, and for differentiating the individual learning needs and preferences. Jane's comment is indicative of other teacher comments:

> *Students were asked to explore a mathematics strategy with a buddy, creating a video in* Explain Everything *to explain how their selected strategy worked, and what it was good for. Students were free to select any strategy they liked, and engage in their learning how they wanted. They were observed exploring various strategies, such as equal addition and reversibility, with their methods and recording occurring in a multitude of ways, such as through the use of whiteboards, calculators and discussions.*

This excerpt of data also suggests other themes and indicates their inter-connectedness. The students use the affordance of the MT, they collaborate, and there is reference to the use of a multiple of ways, including whiteboards, calculators and discussion—a socio-material assemblage.

Brad's comment below focused on developing the cognitive understanding of triangles and exploring the relationships in their properties. It identified the use of the affordances of the apps, collaboration (both between students and with the teacher), and socio-technical assemblages with an app, a concrete resource (*Sphero)* and social aspects being integrated. Evident is the student choice and potential for differentiated learning, while the key focus throughout is the students' conceptual understanding of triangles:

> *The app called* Tickle *was used whilst trying to program the* Spheros *(little robotic balls) to move in triangles for our project. This helped by showing us the way triangles were made, and improved our patience when the programming didn't work.* Tickle *is the actual app used for the programming.* Hopscotch*, is another programming app, but used to program a virtual character of your choice. It is the same, but it is different to use, different commands. This is helping by helping us discover the degrees and angles of the triangles.*

Concluding Comments

The characteristics of learning mathematics through MT, including apps are important. There are some that better facilitate individual mathematical thinking and understanding, while others reshape the nature of the learning experience through the affordances, such as dynamic, visual and haptic experiences, not easily obtained with other pedagogical media. Others offer non-threatening, instantaneous feedback that enable better opportunity for investigative approaches and differentiation of the learning. However, the research project on which this chapter is based suggests that it is more than the qualities of the app that are influential in optimizing the learning opportunities. The themes and the associated data were relatively coherent that the expertise and experience of the teacher, manifested through their technological pedagogical and content knowledge, were vital elements of the learning process.

Other understandings that are beginning to emerge for the project are the importance of pedagogy over app quality. This is in relation to student engagement and learning. Another finding is the ways that apps and other technologies (e.g., equipment and concrete materials) can be integrated effectively, with the transition of students between them, seeming to help build relational understanding of mathematical concepts. A key finding identified by the extended research group relates to the ways that these groupings of technologies become part of socio-material assemblages through evoking social engagement and dialogue. While these findings need further research, and their connection to student mathematical understanding better understood and articulated, they nevertheless indicate the potential of MT to transform the mathematics experience. This, in turn, will enhance both the engagement and mathematical thinking of primary and secondary school students.

References

Attard, C. (2015). Introducing iPads into primary mathematics classrooms: Teachers' experiences and pedagogies. In M. Meletiou-Mavrotheris, K. Mavrou, & E. Paparistodemou (Eds.), *Integrating touch enabled and mobile devices into contemporary mathematics education* (pp. 197–217). Hershey, PA: IGI Global.

An, Y.-J., & Reigeluth, C. (2011). Creating technology-enhanced, learner-centered classrooms: K-12 teachers' beliefs, perceptions, barriers, and support needs. *Journal of Digital Learning in Teacher Education, 28*(2), 54–62.

Arcavi, A., & Hadas, N. (2000). Computer mediated learning: An example of an approach. *International Journal for Computer for Mathematics Learning, 5,* 25–45.

Calder, N. S. (2011). *Processing mathematics through digital technologies: The primary years*. Rotterdam, The Netherlands: Sense.

Calder, N. S., & Campbell, A. (2016). Using mathematical apps with reluctant learners. *Digital Experiences in Mathematics Education.* https://doi.org/10.1007/s40751-016-0011-y.

Carr, J. (2012). Does math achievement "h'APP'en" when iPads and game-based learning are incorporated into fifth-grade mathematics instruction? *Journal of Information Technology Education, 11,* 269–286.

Clark, W., & Luckin, R. (2013). *What the research says: iPads in the classroom*. University of London, UK: Institute of Education.

Cobb, P., & Bowers, J. S. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher, 28*(2), 4–15.

Cutler, T., Waine, B., & Brehony, K. (2007). A new epoch of individualization? Problems with the "personalization" of public sector services. *Public Administration, 85*(3), 847–855.

della Cava, M. R. (2010). Does the iPad have the magic to bring people together? *USA Today*. Retrieved from http://www.usatoday.com/life/lifestyle/2010-06-07-ipadculture07_CV_N.htm.

De Freitas, E. & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. New York, NY: Cambridge University Press.

Dillenbourg, P. (1999) *Collaborative learning: Cognitive and computational approaches.* Advances in learning and instruction series. New York, NY: Elsevier Science, Inc.

Falcade, R., Laborde, C., & Mariotti, M. A. (2007). Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics, 66*(3), 317–333.

Fenwick, T., & Edwards, R. (Eds.). (2012). *Researching education through actor network theory*. Sussex: Wiley-Blackwell.

Fisher, B., Lucas, T., & Galstyan, A. (2013). The role of iPads in constructing collaborative learning spaces. *Technology, Knowledge, and Learning, 18*(3), 165–178.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, Acting, and Knowing: Toward an Ecological Psychology* (pp. 67–82). Hillsdale, NJ: Lawrence Erlbaum.

Groundwater-Smith, S., Mitchell, J., Mockler, N., Ponte, P., & Ronnerman, K. (2013). *Facilitating practitioner research*. London, England: Routledge.

Hegedus, S. (2013). Young children investigating advanced mathematical concepts with haptic technologies: Future design perspectives. *The Mathematics Educator, 10*(1&2), 87–107.

Hennessy, S. (2014). *Bridging between research and practice: Supporting professional development through collaborative studies of classroom teaching with technology*. Rotterdam, The Netherlands: Sense.

Jackiw, N. (2013). Touch & multitouch in dynamic geometry: Sketchpad explorer and "digital" mathematics, In *Proceedings of ICTMT12* (pp. 149–155), Bari, Italy.

Johri, A. (2011). The socio-materiality of learning practices and implications for the field of learning technology. *Research in Learning Technology, 19*(3), 207–217.

Larkin, K. (2015). "An App! An App! My Kingdom for An App": An 18-month quest to determine whether apps support mathematical knowledge building. In T. Lowrie & R. Jorgensen (Eds.), *Digital Games and Mathematics Learning: Potential, Promises and Pitfalls*. (Vol. 4, pp. 251–276). The Netherlands: Springer.

Leadbetter, C. (2005). The shape of things to come: Personalised learning through collaboration. In *Department for education and skills*. Retrieved from http://www.standards.dfes.go.uk/innovation-uni.

Looi, C.-K., Wong, L.-H., So, H.-J., Seow, P., Toh, Y., Chen, W., … Soloway, E. (2009). Anatomy of a mobilized lesson: Learning my way. *Computers & Education*, *53*(4), 1120–1132. http://doi.org/10.1016/j.compedu.2009.05.021.

Looi, C.-K., Wong, L.-H., & Song, Y. (2013). Mobile computer-supported collaborative learning. In C. E. Hmelo-Silver, C. A. Chinn, & C. Chan (Eds.), *The international handbook of collaborative learning* (pp. 420–436). Florence, KY, USA: Routledge.

Melhuish, K., & Falloon, G. (2010). Looking to the future: M-learning with the iPad. *Computers in New Zealand Schools: Learning, Leading. Technology, 22*(3), 1–15.

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London, UK: Routledge.

Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve mathematics problems. *Language and Education, 20*(6), 507–528.

Mercier, E., & Higgins, S. (2013). Collaborative learning with multi-touch technology: Developing adaptive expertise. *Learning and Instruction, 25,* 13–23.

Meyer, B. (2015). iPads in inclusive classrooms: Ecologies of learning. In P. Isaias, J. M. Spector, & D. Ifenthaler (Eds.), *E-learning systems, environments and approaches theory and implementation*. Dordrecht: Springer International Publishing.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372–377.

Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. *International Journal of Virtual and Personal Learning Environments, 4*(3), 35–50.

O'Malley, P., Jenkins, S., Wesley, B., Donehower, C., Rabuck, D., & Lewis, M. E. B. (2013). Effectiveness of using iPads to build math fluency. In *Presented at the Council for Effectiveness of using iPads to build math fluency, San Antonio. Texas, USA*. Retrieved from http://eric.ed.gov/?id=ED541158.

Philip, T. M., & Garcia, A. (2014). Schooling mobile phones: Assumptions about proximal benefits, the challenges of shifting meanings, and the politics of teaching. *Educational Policy, 29*(4), 676–707.

Robinson, C., & Sebba, J. (2010). Personalising learning through the use of technology. *Computers & Education, 54*(3), 767–775.

Sandholtz, J. H., Ringstaff, C., & Dwyer, D. C. (1997). *Teaching with technology: Creating a student centred classroom*. New York: Teachers' College Press.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher, 27*(2), 4–13.

Sinclair, N., & Heyd-Metzuyanim, E. (2014). Learning number with *TouchCounts:* The role of emotions and the body in mathematical communication. *Technology, Knowledge and Learning, 19*(1), 81–99.
Tomlinson, C. (2009). The Goals of Differentiation. In M. Scherer (Ed.), *Supporting the whole child: Reflections on best practices in learning, teaching, and leadership* (pp. 3–11). Alexandria, Virginia, USA: ASCD.
Waldrip, B., Cox, P., Deed, C., Dorman, J., Edwards, D., Farrelly, C., … Yager, Z. (2014). Student perceptions of personalised learning: development and validation of a questionnaire with regional secondary students. *Learning Environments Research*, *17*(3), 355–370.
Zurita, G., & Nussbaum, M. (2004). Computer supported collaborative learning using wirelessly interconnected handheld computers. *Computers & Education, 42,* 289–314.

Dr. Nigel Calder is an Associate Professor at the University of Waikato, Tauranga. He has taught in both primary and secondary schools. His research interests are predominantly in the use of digital technologies in mathematics education. His present and recent research projects are: The influence of apps on primary-school mathematics teaching and learning; The influence of mobile technologies on the engagement of 16–18 year-old reluctant learners; Connecting science and mathematics instruction through exploratory talk and inquiry; and How mathematical thinking emerges through student-centred inquiry learning in secondary schools. He wrote the book: *Processing Mathematics Through Digital Technologies: ThePrimaryYears* and has published numerous resources (including Figure It Out books), book chapters and journal articles.

Carol Murphy previously at the University of Waikato, is currently a Senior Lecturer in Mathematics Education at the University of Tasmania, Australia. Her interests in student learning in mathematics stemmed from her background as a primary school teacher in England. Now, as a primarymathematics education lecturer and researcher, her interest into children's mathematical thinking has led her to focus on the teaching of calculation strategies as well as on collaboration and use of talk. Carol has also written on teacher subject knowledge and changes in teacher practice. These research interests have informed her recent investigations into teachers' practices and student learning in the use of iPads in New Zealand primary mathematics classrooms.