(1) DATI

M

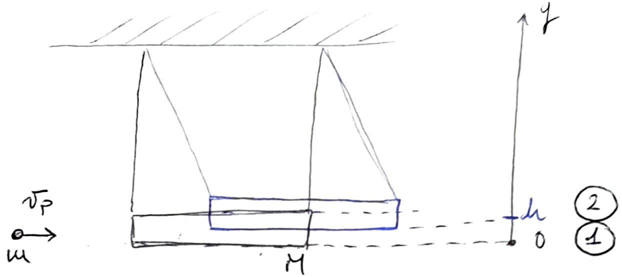
m

h

$(M >> m)$

RICHIESTA

$v_p = ?$



SOLUZIONE

$p_i = p_f$

- $p_i = m v_p$
- $p_f = (M+m) v_f$

$\rightarrow p_i = p_f \triangleright \dfrac{m v_p}{M+m} = \dfrac{(M+m) v_f}{M+m}$

$v_f = \dfrac{m}{M+m} v_p$

$(E = E_k + E_p)$

$E_1 = E_k^{(1)} + \cancel{E_p^{(1)}} = \dfrac{1}{2} m_{TOT} v_f^2$

$E_2 = \cancel{E_k^{(2)}} + E_p^{(2)} = m_{TOT}\, g\, h$

$\rightarrow E_1 = E_2 \triangleright 2 \cdot \dfrac{1}{2} \cancel{m_{TOT}} v_f^2 = \cancel{m_{TOT}}\, g\, h\, 2$

$v_f^2 = 2gh \triangleright v_f = \sqrt{2gh}$

$\dfrac{m}{M+m} v_p = \sqrt{2gh} \Rightarrow \boxed{v_p = \dfrac{(M+m)}{m}\sqrt{2gh}}$

$M >> m \Rightarrow M+m \rightarrow M \Rightarrow v_p = \dfrac{M}{m}\sqrt{2gh}$

MOTO ROTAZIONALE

- $\alpha = a/r$

$$S_f = S_0 + v_0 t + \frac{1}{2} a t^2 \quad - \quad v_f = v_0 + a t$$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad - \quad \omega_f = \omega_0 + \alpha t$$

- $\vec{\tau} = \vec{r} \times \vec{F}$

  $|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$

- $\vec{L} = \vec{r} \times \vec{p}$

  $|\vec{L}| = |\vec{r}||\vec{p}| \sin\theta$

▷ $L = I\omega \rightarrow I = \frac{L}{\omega} \rightarrow \frac{[m][Kg][m]}{[s][s]^{-1}}$

$[Kg \cdot m^2)$

- $\vec{F} = m\vec{a} \quad - \quad \vec{\tau} = I\vec{\alpha}$

- $K_R = \frac{1}{2} I \omega^2 \quad - \quad K = \frac{1}{2} m v^2$

(2) DATI

$m = 50\,kg$

$r = 180\,cm$

$F = 19.6\,N$

$\omega_0 = 0\,rad/s$

$t_f = 5\,s$

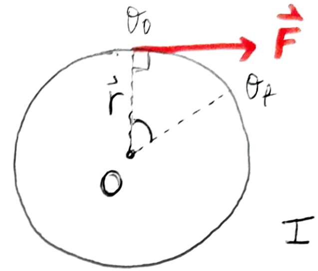
RICHIESTE

$\alpha = ?$

$\theta_f = ?$

$L = ?$

$K_R = ?$



$I = \frac{1}{2} m r^2$

SOLUZIONE

- $\vec{\tau} = I\vec{\alpha} \triangleright \alpha = \frac{\tau}{I}$
- $\vec{\tau} = \vec{F} \times \vec{r} = |\vec{F}||\vec{r}| \sin 90° = (19.6\,N)(1.8\,m) = 35.28\,N \cdot m$
- $I = \frac{1}{2} m r^2 = \frac{1}{2}(50\,kg)(1.8\,m)^2 = 81\,kg \cdot m^2$

$$\alpha = \frac{35.28\,N \cdot m}{81\,kg \cdot m^2} = 0.435\,rad/s^2$$

- $\theta_f = \cancel{\theta_0} + \cancel{\omega_0 t} + \frac{1}{2}\alpha t^2 \triangleright \theta_f = \frac{1}{2}\alpha t^2 = 5.4\,rad$
- $L = I\omega_f \qquad \omega_f = \cancel{\omega_0} + \alpha t_f \triangleright \omega_f = \alpha t_f = 2.18\,rad/s$

$$= (81\,kg \cdot m^2)(2.18\,rad/s) = 176.58\,\frac{kg\,m^2}{s}$$

- $K_R = \frac{1}{2} I \omega^2 = \frac{1}{2}(81\,kg \cdot m^2)(2.18\,rad/s)^2 = 192.47\,J$

③ **DATI**

$r = 40\ cm$

$F_{P,1} = 700\ N$

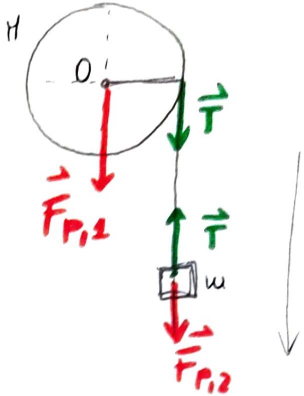
$\omega_0 = 0\ rad/s$

$F_{P,2} = 70\ N$

$t_f = 2\ s$

RICHIESTA

$\omega_f = ?$



SOLUZIONE

$$\vec{T} + \vec{F}_{P,2} = m\vec{a}$$

$$-T + F_{P,2} = ma \triangleright T = F_{P,2} - ma \triangleright T = mg - ma \triangleright T = m(g-a)$$

$$\vec{\tau} = I\vec{\alpha}$$

- $\vec{\tau} = \cancel{\vec{\tau}_{F_P}} + \vec{\tau}_T = |\vec{T}||\vec{r}| \sin 90° = Tr = mr(g-a)$
- $I = \frac{1}{2} m r^2$

$$\cancel{m}\cancel{r}(g-a) = \frac{1}{2}\cancel{m} r^{\cancel{2}} \alpha \triangleright \alpha = \frac{2(g-a)}{r} \qquad (a = \alpha r)$$

$$\alpha = \frac{2g}{r} - \frac{2a}{r} \triangleright \alpha = \frac{2g}{r} - \frac{2\alpha \cancel{r}}{\cancel{r}}$$

$$\alpha = \frac{2g}{r} - 2\alpha \triangleright 2\alpha + \alpha = \frac{2g}{r}$$

$$3\alpha = \frac{2g}{r}$$

$$\alpha = \frac{2g}{r} - \frac{2\alpha \cancel{r}}{\cancel{r}} \triangleright \frac{\cancel{3}\alpha}{\cancel{3}} = \frac{2g}{r3} \triangleright \alpha = \frac{2}{3}\cdot\frac{g}{r} = 16.35 \frac{rad}{s^2}$$

$$\omega_f = \cancel{\omega_0} + \alpha t_f \Rightarrow \omega_f = \alpha t_f = \left(16.35 \frac{rad}{s^2}\right)(2s) = 32.7\ rad/s$$

④ DATI

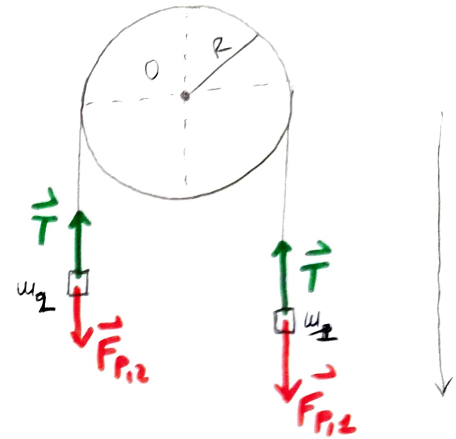
$R = 27\,cm$

$F_{P,1} = 133\,N$

$F_{P,2} = 89\,N$

RICHIESTA

$\alpha = ?$



SOLUZIONE

$$\begin{cases} \vec{T} + \vec{F}_{P,1} = m_1 \vec{a} & (I) \\ \vec{T} + \vec{F}_{P,2} = m_2 \vec{a} & (II) \end{cases}$$

$$\begin{cases} -T + F_{P,1} = m_1 a \\ +T - F_{P,2} = m_2 a \end{cases} \quad +$$

$$\cancel{-T} + F_{P,1} + \cancel{T} - F_{P,2} = m_1 a + m_2 a$$

$$F_{P,1} - F_{P,2} = a\left(\frac{m_1 g}{g} + \frac{m_2 g}{g}\right) \triangleright F_{P,1} - F_{P,2} = a\left(\frac{F_{P,1}}{g} + \frac{F_{P,2}}{g}\right)$$

$$g \cdot \frac{F_{P,1} - F_{P,2}}{F_{P,1} + F_{P,2}} = \frac{a}{\cancel{g}} \frac{\cancel{(F_{P,1} + F_{P,2})}}{\cancel{F_{P,1} + F_{P,2}}} \cdot \cancel{g} \triangleright a = g\,\frac{F_{P,1} - F_{P,2}}{F_{P,1} + F_{P,2}} = 1.94\,m/s^2$$

$$\frac{a}{R} = \frac{\alpha \cancel{R}}{\cancel{R}} \triangleright \alpha = \frac{a}{R} = \frac{1.94\,\cancel{m}/s^2}{27 \times 10^{-2}\,\cancel{m}} = 7.2\,rad/s^2$$

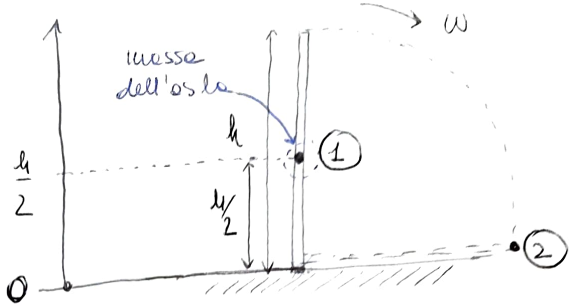
(5) **DATI**

$h = 1\,m$

$\omega_0 = 0\ rad/s$

RICHIESTA

$v_t = ?$



$$I = \underbrace{\frac{1}{3} m h^2}_{\text{momento d'inerzia di un'asta rigida}}$$

SOLUZIONE

$$E_1 = \cancel{E_k^{(1)}} + E_p^{(1)} = mg\frac{h}{2}$$

$$E_2 = E_k^{(2)} + \cancel{E_p^{(2)}} = K_R = \frac{1}{2} I \omega^2$$

$$E_1 = E_2 \Rightarrow mg\frac{h}{\cancel{2}} = \frac{1}{\cancel{2}} I \omega^2$$

$$3\,\frac{\cancel{m}gh}{h^2} = \frac{1}{3}\,\frac{\cancel{m}\cancel{h^2}\omega^2}{\cancel{h^2}} \cdot 3 \Rightarrow \omega^2 = \frac{3g}{h}$$

$$\omega = \sqrt{\frac{3g}{h}} \qquad \left(v = \omega h \Rightarrow \omega = v/h\right)$$

$$\frac{v_t}{h} = \sqrt{\frac{3g}{h}} \Rightarrow v_t = h\sqrt{\frac{3g}{h}}$$

$$v_t = \sqrt{\frac{3gh^2}{h}} \Rightarrow v_t = \sqrt{3gh} = 5.42\ m/s$$

(6) **DATI**

$M = 1 kg$

$m = 0.2 kg$

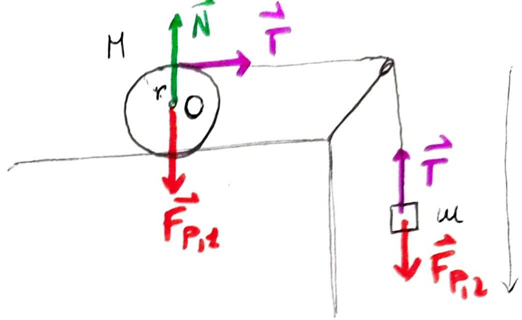
$r = 0.2 m$

RICHIESTE

$a = ?$

$\alpha = ?$

$T = ?$



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SOLUZIONE

cilindro $\quad \Sigma \vec{\tau} = I \vec{\alpha}$

- $\vec{\tau} = \vec{r} \times \vec{T} = |\vec{r}||\vec{T}| \sin\theta = T \cdot r$
- $I = \frac{1}{2} M r^2$

$$\frac{T \cdot \cancel{r}}{\cancel{r}} = \frac{1}{2} \frac{M r^{\cancel{2}} \alpha}{\cancel{r}} \triangleright T = \frac{1}{2} M r \alpha \quad (I)$$

massa $\quad \Sigma \vec{F} = m \vec{a} \triangleright \vec{T} + \vec{F}_{P,2} = m \vec{a} \triangleright -T + F_{P,2} = ma \quad (II)$

$$\begin{cases} T = \frac{1}{2} M r \alpha & (I) \\ -T + F_{P,2} = ma & (II) \end{cases} \quad \begin{cases} T = 1/2\, M r \alpha \\ -\frac{1}{2} M r \alpha + mg = ma \end{cases} \quad (\alpha = a/r)$$

$$\hookrightarrow -\frac{1}{2} M \cancel{r} \frac{a}{\cancel{r}} + mg = ma$$

$$ma + \frac{1}{2} M a = mg \rightarrow a\left(m + \frac{1}{2} M\right) = mg$$

$$\frac{\cancel{2} \cdot a}{2m+M} \left(\frac{2m+M}{\cancel{2}}\right) = \frac{mg \cdot 2}{2m+M} \rightarrow a = \frac{2m}{2m+M} g = 2.8 \, m/s^2$$

$$\alpha = \frac{a}{r} = \frac{2.8\ m/s^2}{0.2\ m} = 14\ rad/s^2$$

$$T = \frac{1}{2} M r \alpha = 1.4\ N$$