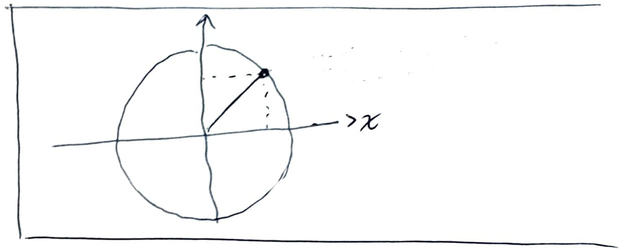
OSCILLATORE ARMONICO

- $x(t) = A\cos(\omega t + \varphi) \rightarrow \boxed{\omega = \frac{2\pi}{T} = 2\pi f}$ $\quad [T\,[s];\ f\,[s^{-1}] = [Hz]]$



$$v(t) = \frac{dx}{dt} = -A\omega \sin(\omega t + \varphi)$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \varphi) = -\omega^2 x(t)$$

Forza elastica $\vec{F} = -k\vec{x}$

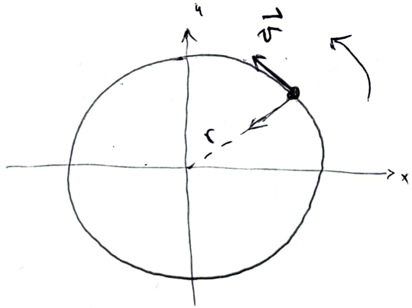
$$-kx = ma \rightarrow -\frac{k}{m}x = \cancel{\frac{m}{m}}\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt} + \boxed{\frac{k}{m}}x = 0 \qquad \omega^2 = \frac{k}{m} \Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \varphi\right)$$

MOTO CIRCOLARE UNIFORME

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{2\pi}{T}$$



$$\boxed{v = \omega r}$$

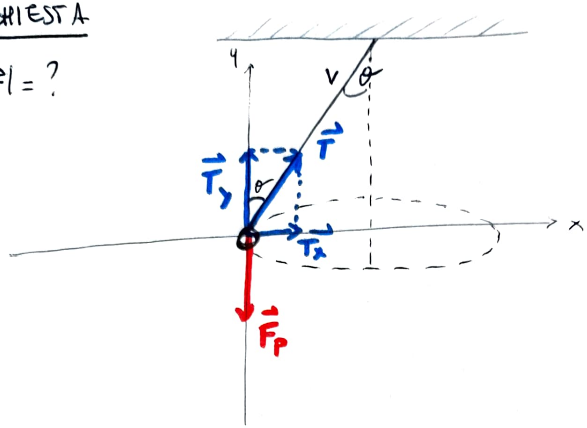
$$\boxed{a_c = \frac{v^2}{r}} = \frac{\omega^2 r^{\cancel{2}}}{\cancel{r}} \rightarrow \boxed{a_c = \omega^2 r}$$

① **DATI**

m
L
r
$|\vec{v}| = cost.$

RICHIESTA

$|\vec{v}| = ?$



SOLUZIONE

asse y $\quad \vec{T}_y + \vec{F}_P = 0 \;\triangleright\; +T_y - F_P = 0 \;\triangleright\; T_y = F_P \;\triangleright\; T\cos\theta = mg \quad (I)$

asse x $\quad \vec{T}_x = m\vec{a}_c \;\triangleright\; T_x = m\frac{v^2}{r} \;\triangleright\; T\sin\theta = m\frac{v^2}{r} \quad (II)$

$$\Rightarrow \frac{(II)}{(I)} \longmapsto \frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}v^2/r}{\cancel{m}g} \;\triangleright\; \tan\theta = \frac{v^2}{rg} \;\triangleright\; v^2 = rg\tan\theta$$

$$\Rightarrow \boxed{v = \sqrt{rg\tan\theta}}$$

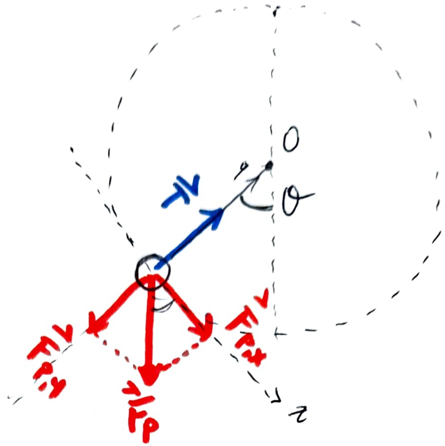
(2) DATI

R

v

RICHIESTA

$T = ?$



SOLUZIONE

asse y

$$\vec{T} + \vec{F}_{P,y} = m\vec{a}_r$$

$$+T - F_{P,y} = m\frac{v^2}{r}$$

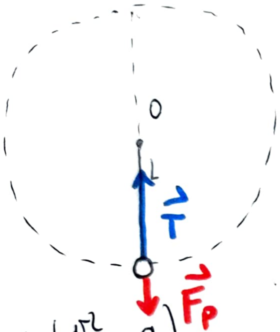
$$T = m\frac{v^2}{r} + F_{P,y} \triangleright T = m\frac{v^2}{r} + mg\cos\theta = m\left(\frac{v^2}{r} + g\cos\theta\right)$$

$$\boxed{T = m\left(\frac{v^2}{r} + g\cos\theta\right)}$$

$$\boxed{T = m\left(\frac{v^2}{r} + g\cos\theta\right)}$$

(VALE $\forall$ VALORE DI θ. ORA VEDIAMO 3 CASI PARTICOLARI)

1. $\theta = 0°$ $(\cos\theta = 1)$

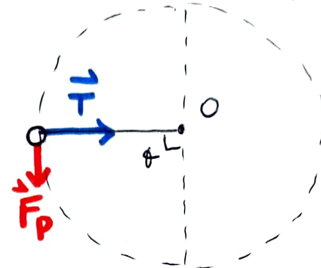


$$T = m\left(\frac{v^2}{r} + g\right)$$

$$T = m\frac{v^2}{r} + mg$$

$$T = F_C + F_P$$

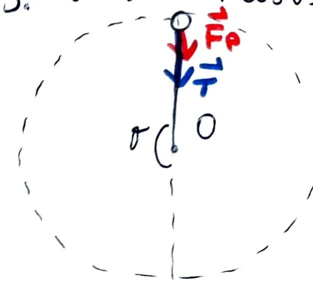
2. $\theta = 90°$ $(\cos\theta = 0)$



$$T = m\frac{v^2}{r}$$

$$T = F_C$$

3. $\theta = 180°$ $(\cos\theta = -1)$



$$T = m\left(\frac{v^2}{r} - g\right)$$

$$T = F_C - F_P$$

③ DATI

$m = 200\,g = 0.2\,kg$

$k = 5\ N/m$

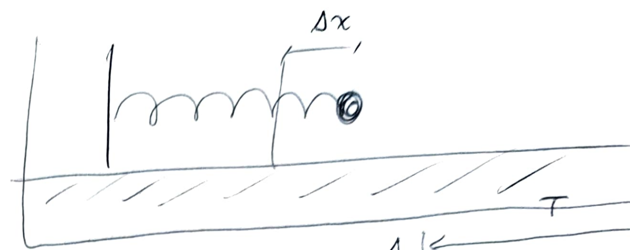
$\Delta x = 5\,cm = 5 \times 10^{-2}\,m$

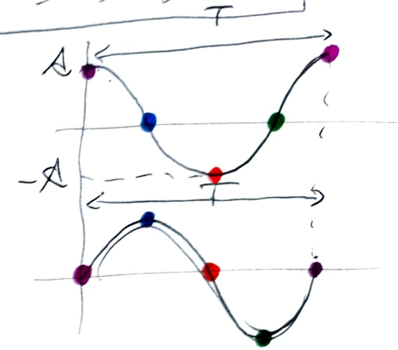
RICHIESTE

$T = ?$

$v_{max} = ?$

$a_{max} = ?$

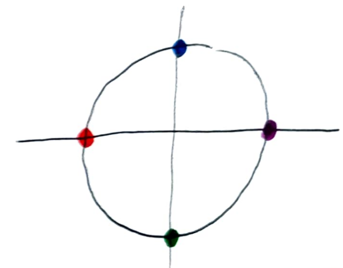




SOLUZIONE

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2\,kg}{5\ N/m}} = \boxed{1.26\,s}$$

$$\omega = \frac{2\pi}{T} = \cancel{0.25\,rad/s} = 4.98\,rad/s$$



$$x(t) = \Delta\cos\left(\sqrt{\frac{k}{m}}\,t + \varphi\right) = \Delta x\cos\left(\sqrt{\frac{k}{m}}\,t + \varphi\right)$$

$$v(t) = \frac{dx}{dt}(t) = -\sqrt{\frac{k}{m}}\,\Delta x\,\sin\left(\sqrt{\frac{k}{m}}\,t + \varphi\right) \triangleright |v_{max}(t)| = \sqrt{\frac{k}{m}}\,\Delta x = \boxed{0.25\,m/s}$$

$$a(t) = \frac{d^2x}{dt^2} = -\frac{k}{m}\cdot\Delta x\cos\left(\sqrt{\frac{k}{m}}\,t + \varphi\right) \triangleright |a_{max}(t)| = \frac{k}{m}\,\Delta x = \boxed{1.25\,m/s^2}$$

(4) **DATI**

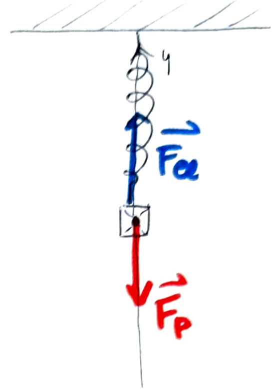
$m_1 = 50\,g = 50 \times 10^{-3}\,kg$

$\Delta x_1 = 1\,cm = 1 \times 10^{-2}\,m$

$T_2 = 1\,s$

RICHIESTE

$m_2 = ?$



$$\left(\vec{F} = -k \Delta \vec{x} \right)$$

SOLUZIONE

$$T_2 = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m_2}{k}}$$

$$\vec{F}_p + \vec{F}_{el} = 0 \triangleright -F_p + F_{el} = 0 \triangleright F_{el} = F_p$$

$$\frac{k\,\cancel{\Delta x_1}}{\cancel{\Delta x_1}} = \frac{m_1 g}{\Delta x_1} \triangleright k = \frac{m_1 g}{\Delta x_1} = 49\ N/m$$

$$\frac{T_2}{2\pi} = \sqrt{\frac{m_2}{k}} \triangleright k \cdot \left(\frac{T_2}{2\pi} \right)^2 = \frac{m_2}{\cancel{k}} \cdot \cancel{k} \triangleright m_2 = k \left(\frac{T_2}{2\pi} \right)^2 = \boxed{1.24\,kg}$$