(28)

$$\begin{vmatrix} 1 & -1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & -1 & -1 & -2 \\ 0 & 1 & 1 & 2 \end{vmatrix}$$

$$= \overset{1+1}{1}\begin{vmatrix} 0 & 1 & 2 \\ -1 & -1 & -2 \\ 1 & 1 & 2 \end{vmatrix} + \overset{1+2}{1}\begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 2 \end{vmatrix} + \overset{1+3}{2}\begin{vmatrix} 1 & 0 & 2 \\ 3 & -1 & -2 \\ 0 & 1 & 2 \end{vmatrix}$$

$$\overset{1+4}{-3}\begin{vmatrix} 1 & 0 & 1 \\ 3 & -1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= -1\begin{vmatrix} -1 & -2 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} -1 & -2 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 3 & -2 \\ 0 & 2 \end{vmatrix}$$

$$+ 2\begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} + 2\begin{vmatrix} -1 & -2 \\ 1 & 2 \end{vmatrix} + 4\begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} +$$

$$-3\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= 0 + 0 + 0 - 6 + 6 + 0 + 12 - 9 = 3$$

28.

$$\begin{vmatrix} 1 & -1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & -1 & -1 & -2 \\ 0 & 1 & 1 & 2 \end{vmatrix} = \begin{array}{c} \begin{array}{cccc} & A^2+A^1 & A^3-2A^1 & A^4-3A^1 \end{array} \\ \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 2-3 \\ 3 & 2 & -1-6 & -2-9 \\ 0 & 1 & 1 & 2 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 \\ 3 & 2 & -7 & -11 \\ 0 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -1 \\ 2 & -7 & -11 \\ 1 & 1 & 2 \end{vmatrix} = \begin{array}{c} \begin{array}{ccc} & A^2+A^1 & A^3+A^1 \end{array} \\ \begin{vmatrix} 1 & 0 & 0 \\ 2 & -5 & -9 \\ 1 & 2 & 3 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} -5 & -9 \\ 2 & 3 \end{vmatrix} = -15+18 = 3$$

29.

$$\begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 3 & 2 \\ 1 & 2 & 3 & -1 \\ 2 & 3 & 1 & -1 \end{vmatrix} = \begin{array}{c} \begin{array}{cccc} & A^2-A^1 & A^3-2A^1 & A^4-A^1 \end{array} \\ \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 3-2 & 1 \\ 1 & 1 & 3-2 & -2 \\ 2 & 1 & 1-4 & -3 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -3 & -3 \end{vmatrix} = \begin{array}{c} \begin{array}{ccc} & A^2-A^3 & \end{array} \\ \begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & -2 \\ 1 & 0 & -3 \end{vmatrix} \end{array} = \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -3$$

31. $\det(tA) = t^n \det A$

$$\left| tA^1 \; tA^2 \; \ldots \; tA^n \right| = \underbrace{t.t. \ldots t.}_{n \text{ volte}} \left| A^1 \; \ldots \; A^n \right| = t^n \det A$$

32. $A = v v^T$

$$A = \begin{vmatrix} v_1^2 & v_1 v_2 & \ldots & v_n v_1 \\ v_1 v_2 & v_2^2 & \ldots & v_n v_2 \\ \vdots & & & \\ v_1 v_n & v_2 v_n & \ldots & v_n^2 \end{vmatrix}$$

$$|A| = v_1 \begin{vmatrix} v_1 & v_1 v_2 & \ldots & v_n v_1 \\ \vdots & \vdots & & \\ v_n & v_2 v_n & \ldots & v_n^2 \end{vmatrix} =$$

$$= v_1 v_2 \begin{vmatrix} v_1 & v_{\cancel{1}} & \ldots & v_1 v_n \\ \vdots & & & \\ \underbrace{v_n \quad v_n}_{y} & & & v_n^2 \end{vmatrix}$$

$$= 0$$

30 $$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 2 & 4 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2.3 \\ 2 & 0 & 1.2 \\ 3 & 1 & 4.2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 2 & 0 & 2 \\ 3 & 1 & 8 \end{vmatrix} = \frac{1}{5} \begin{vmatrix} 1 & 5 & 6 \\ 2 & 0 & 2 \\ 3 & 5 & 8 \end{vmatrix} = 0 \qquad A^3 = A^1 + A^2$$

29.

$$\begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 3 & 2 \\ 1 & 2 & 3 & -1 \\ 2 & 3 & 1 & -1 \end{vmatrix} = \begin{array}{c} \begin{vmatrix} 1 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & -3 & -3 \end{vmatrix} \end{array} \begin{array}{l} \\ A_2 - A_1 \\ A_3 - A_1 \\ A_4 - 2A_1 \end{array}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -3 & -3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & -2 \\ 0 & -4 & -1 \end{vmatrix} \begin{array}{l} \\ \\ A_3 - A_2 \end{array}$$

$$= (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -(-1+4) = -3$$