venerdì 24 marzo 2023 15:43

1) DISCUTERE AL VARIARE DI $k \in \mathbb{R}$ LA DIAGONALIZZABILITÀ DI $A = \begin{pmatrix} -9 & k & 3 \\ 0 & k & 0 \\ 3 & 0 & -1 \end{pmatrix}$

$$|\lambda I - A| = \begin{vmatrix} \lambda+9 & -k & -3 \\ 0 & \lambda-k & 0 \\ -3 & 0 & \lambda+1 \end{vmatrix} = (\lambda-k)\begin{vmatrix} \lambda+9 & -3 \\ -3 & \lambda+1 \end{vmatrix} =$$

$$= (\lambda-k)\left((\lambda+9)(\lambda+1)-9\right) = (\lambda-k)(\lambda^2+\lambda+9\lambda+9-9) =$$

$$= (\lambda-k)(\lambda^2+10\lambda) = \lambda(\lambda-k)(\lambda+10)$$

$$|\lambda I - A| = 0 \iff \lambda=0 \lor \lambda=k \lor \lambda=-10$$

- Se $k \neq 0$ e $k \neq -10 \Rightarrow$ tutti gli autovalori hanno m.a pari a 1 $\Rightarrow$ anche m.g $(\lambda_i) = 1 \quad \forall i \Rightarrow A$ e' diag.

- Se $k=0 \Rightarrow \lambda_1 = 0 \quad m.a(\lambda_1) = 2$

$$\begin{pmatrix} 9 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \begin{cases} 9x - 3z = 0 \\ -3x + z = 0 \end{cases} \iff z = 3x$$

$$\Rightarrow (x, y, 3x) = x(1,0,3) + y(0,1,0)$$

$$\Rightarrow V_{\lambda_1} = [(1,0,3),(0,1,0)] \Rightarrow m.g(\lambda_1) = 2$$

$\lambda_2 = -10 \quad m.a(\lambda_2) = 1$

$$\begin{pmatrix} -1 & 0 & -3 \\ 0 & -10 & 0 \\ -3 & 0 & -9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \begin{cases} -x - 3z = 0 \\ -10y = 0 \\ -3x - 9z = 0 \end{cases} \Rightarrow \begin{cases} x = -3z \\ y = 0 \end{cases}$$

$$(-3z, 0, z) = z(-3,0,1)$$

$$\Rightarrow V_{\lambda_2} = [(-3,0,1)] \Rightarrow m.g(\lambda_2) = 1$$

$\Rightarrow$ Per $k=0$ la matrice A e' diagonalizzabile

- Se $k=-10 \Rightarrow \lambda_1 = 0 \quad m.a.(\lambda_1) = 1 \Rightarrow m.g(\lambda_1) = 1$

  $\lambda_2 = -10 \quad m.a(\lambda_2) = 2$

$$\lambda_1 = 0 \Rightarrow \begin{pmatrix} 9 & 10 & -3 \\ 0 & 10 & 0 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \begin{cases} 9x + 10y - 3z = 0 \\ 10y = 0 \\ -3x + z = 0 \end{cases} \iff \begin{cases} y = 0 \\ z = 3x \end{cases}$$

$$\Rightarrow V_{\lambda_1} = [(1,0,3)] \Rightarrow m.g(\lambda_1) = 1$$

$$\lambda_2 = -10 \Rightarrow \begin{pmatrix} -1 & 10 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \begin{cases} -x + 10y - 3z = 0 \\ -3x + 9z = 0 \end{cases}$$

$$\iff \begin{cases} x = -3z \\ 3z + 10y - 3z = 0 \end{cases} \iff \begin{cases} x = -3z \\ y = 0 \end{cases}$$

$V_{\lambda_2} = [(-3,0,1)] \Rightarrow m.g(\lambda_2) = 1 \neq m.a(\lambda_2) = 2 \Rightarrow$ A NON è diag per $k=-10$

② SIA $\beta = \{(1,1,0),(0,1,1),(0,0,1)\}$ UNA BASE DI $\mathbb{R}^3$.

a) COSTRUIRE A PARTIRE DA β UNA BASE ORTOGONALE β_1 PER $\mathbb{R}^3$

Pongo $v_1' = v_1 = (1,1,0)^T$

$$v_2' = v_2 - \frac{\langle v_2, v_1'\rangle}{\langle v_1', v_1'\rangle} v_1' = (0,1,1) - \frac{\langle (0,1,1),(1,1,0)\rangle}{\langle (1,1,0),(1,1,0)\rangle}(1,1,0) =$$

$$= (0,1,1) - \frac{1}{2}(1,1,0) = \left(-\frac{1}{2}, \frac{1}{2}, 1\right)$$

$$v_3' = v_3 - \frac{\langle v_3, v_1'\rangle}{\langle v_1', v_1'\rangle} v_1' - \frac{\langle v_3, v_2'\rangle}{\langle v_2', v_2'\rangle} v_2' = \quad =$$

$$= (0,0,1) - \frac{\langle (0,0,1),(1,1,0)\rangle}{\langle (1,1,0),(1,1,0)\rangle}(1,1,0) - \frac{\langle (0,0,1),\left(-\frac{1}{2},\frac{1}{2},1\right)\rangle}{\langle \left(-\frac{1}{2},\frac{1}{2},1\right),\left(-\frac{1}{2},\frac{1}{2},1\right)\rangle}\left(-\frac{1}{2},\frac{1}{2},1\right) =$$

$$= (0,0,1) - \frac{1}{\frac{1}{4}+\frac{1}{4}+1}\left(-\frac{1}{2},\frac{1}{2},1\right) =$$

$$= (0,0,1) - \frac{2}{3}\left(-\frac{1}{2},\frac{1}{2},1\right) = \left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right)$$

$$\Rightarrow \beta_1 = \left\{(1,1,0), \left(-\frac{1}{2},\frac{1}{2},1\right), \left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)\right\}$$

b) COSTRUIRE A PARTIRE DA β_1 UNA BASE ORTONORMALE β_2 PER $\mathbb{R}^3$

$\Downarrow$

BASTA NORMALIZZARE LA BASE β_1 CHE ABBIAMO APPENA TROVATO

$$v_1'' = \frac{v_1'}{|v_1'|} = \frac{(1,1,0)}{\sqrt{1+1}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$v_2'' = \frac{v_2'}{|v_2'|} = \frac{\left(-\frac{1}{2},\frac{1}{2},1\right)}{\sqrt{\frac{1}{4}+\frac{1}{4}+1}} = \frac{\sqrt{2}}{\sqrt{3}}\left(-\frac{1}{2},\frac{1}{2},1\right) = \left(-\frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$v_3'' = \frac{v_3'}{|v_3'|} = \frac{\left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)}{\sqrt{\frac{1}{9}+\frac{1}{3}+\frac{1}{9}}} \quad \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

$$\Rightarrow \beta_2 = \left\{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right), \left(-\frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right), \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)\right\}$$

c) DATO IL VETTORE $\vec{v} = (2,1,4)$, SCRIVERE LE COMPONENTI DI $\vec{v}$ RISPETTO A β_1 E β_2

$$a_i = \frac{\langle v, v_i\rangle}{\langle v_i, v_i\rangle} = \frac{\langle v, v_i\rangle}{|v_i|} \longrightarrow \text{RISPETTO ALLA BASE ORTOGONALE}$$

$$a_1 = \frac{\langle v, v_1\rangle}{\langle v_1, v_1\rangle} = \frac{\langle (2,1,4),(1,1,0)\rangle}{\langle (1,1,0),(1,1,0)\rangle} = \frac{3}{2}$$

$$a_2 = \frac{\langle v, v_2\rangle}{\langle v_2, v_2\rangle} = \frac{\langle (2,1,4),\left(-\frac{1}{2},\frac{1}{2},1\right)\rangle}{\langle \left(-\frac{1}{2},\frac{1}{2},1\right),\left(-\frac{1}{2},\frac{1}{2},1\right)\rangle} = \frac{-1+\frac{1}{2}+4}{\frac{1}{4}+\frac{1}{4}+1} = \frac{\frac{7}{2}}{\frac{3}{2}} = \frac{7}{3}$$

$$a_3 = \frac{\langle v, v_3\rangle}{\langle v_3, v_3\rangle} = \frac{\langle (2,1,4),\left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)\rangle}{\langle \left(\frac{1}{3},-\frac{1}{3},\frac{1}{2}\right),\left(\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)\rangle} = \frac{\frac{2}{3}-\frac{1}{3}+\frac{4}{3}}{\frac{1}{9}+\frac{1}{9}+\frac{1}{9}} = \frac{\frac{5}{3}}{\frac{1}{3}} = 5$$

LE COORDINATE DI $\vec{v}$ RISPETTO ALLA BASE $\mathcal{B}_1$ SONO $\left(\frac{3}{2}, \frac{7}{3}, 5\right)$

$a_i = \langle v, v_i \rangle \rightarrow$ RISPETTO ALLA ~~BASE~~ ORTONORMALE

$$a_1 = \left\langle (2,1,4), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \right\rangle = \frac{3}{\sqrt{2}}$$

$$a_2 = \left\langle (2,1,4), \left(-\frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right) \right\rangle = -\frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{2}}{2\sqrt{3}} + \frac{4\sqrt{2}}{\sqrt{3}} = \frac{-2\sqrt{2}+\sqrt{2}+8\sqrt{2}}{2\sqrt{3}} = \frac{7}{\sqrt{6}}$$

$$a_3 = \left\langle (2,1,4), \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) \right\rangle = \frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} = \frac{5\sqrt{3}}{3} = \frac{5}{\sqrt{3}}$$

$\Rightarrow$ LE COORDINATE DI $\vec{v}$ RISPETTO ALLA BASE $\mathcal{B}_2$ SONO $\left(\frac{3}{\sqrt{2}}, \frac{7}{\sqrt{6}}, \frac{5}{\sqrt{3}}\right)$

RICORDO: LE COORDINATE DI UN VETTORE RISPETTO A UNA BASE ORTOGONALE SI DICONO COEFFICIENTI DI FOURIER DI $\vec{v}$ RISPETTO AGLI ELEMENTI DELLA BASE

3 SIA $W = [(1,1,0,1), (1,-2,0,0), (1,0,-1,2)]$ SOTTOSPAZIO DI $\mathbb{R}^4$.
TROVARE UNA BASE PER IL COMPLEMENTO ORTOGONALE $W^\perp$

DEF SIA V UNO SPAZIO EUCLIDEO REALE E SIA W SOTTOINSIEME DI V. $W^\perp := \{ v \in V : \langle v, w \rangle = 0 \quad \forall\, w \in W \}$ E' IL COMPLEMENTO ORTOGONALE DI W, OSSIA L'INSIEME DI TUTTI I VETTORI DI V ORTOGONALI A TUTTI I VETTORI DI W.

$$\overset{DEF}{\Rightarrow} \begin{cases} \langle (x,y,z,t), (1,1,0,1) \rangle = 0 \\ \langle (x,y,z,t), (1,-2,0,0) \rangle = 0 \\ \langle (x,y,z,t), (1,0,-1,2) \rangle = 0 \end{cases} \Longleftrightarrow \begin{cases} x+y+t=0 \\ x-2y=0 \\ x-z+2t=0 \end{cases} \Longleftrightarrow \begin{cases} t=-3y \\ x=2y \\ z=-4y \end{cases}$$

$$\Rightarrow W^\perp = \{(2y, y, -4y, -3y) : y \in \mathbb{R}\} = [(2,1,-4,-3)]$$

4 SIA $U = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 0\} \subseteq \mathbb{R}^3$
a) SI DETERMINI UNA BASE DI U CHE SIA ORTONORMALE.

$$U = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_1 = -2x_2 - 3x_3\}$$
$$= \{(-2x_2 - 3x_3, x_2, x_3) : x_2, x_3 \in \mathbb{R}\} = [(-2,1,0), (-3,0,1)]$$

Sono ortogonali? $\langle(-2,1,0),(-3,0,1)\rangle = 6 \neq 0 \Rightarrow$ NO

$\Rightarrow$ USO ORTOGONALIZZAZIONE DI GRAM-SCHMIDT

$$v_1' = \frac{v_1}{|v_1|} = (-2,1,0)$$

$$v_2' = v_2 - \frac{\langle v_2, v_1' \rangle}{\langle v_1', v_1' \rangle} v_1' = (-3,0,1) - \frac{\langle(-3,0,1),(-2,1,0)\rangle}{\langle(-2,1,0),(-2,1,0)\rangle}(-2,1,0) =$$

$$= (-3,0,1) - \frac{6}{5}(-2,1,0) = \left(-\frac{3}{5}, -\frac{6}{5}, 1\right)$$ COSÌ SI OTTIENE UNA BASE ORTOGONALE

$$\Rightarrow \text{NORMALIZZO}: \quad v_1'' = \frac{v_1'}{|v_1'|} = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$$

$$v_2'' = \frac{v_2'}{|v_2'|} = \frac{5}{\sqrt{70}}\left(-\frac{3}{5}, -\frac{6}{5}, 1\right) = \left(-\frac{3}{\sqrt{70}}, -\frac{6}{\sqrt{70}}, \frac{5}{\sqrt{70}}\right)$$

$$|v_2'| = \sqrt{\frac{9}{25}+\frac{36}{25}+1} = \sqrt{\frac{70}{25}} = \frac{\sqrt{70}}{5}$$

$$\Rightarrow \beta = \left\{ \left(-\frac{3}{5}, -\frac{6}{5}, 1\right), \left(-\frac{3}{\sqrt{70}}, -\frac{6}{\sqrt{70}}, \frac{5}{\sqrt{70}}\right) \right\}$$

b) COMPLETARE β AD UNA BASE ORTONORMALE β' DI $\mathbb{R}^3$

$$\begin{cases} \left\langle (x,y,z), \left(-\frac{2}{\sqrt5}, \frac{1}{\sqrt5}, 0\right) \right\rangle = 0 \\ \left\langle (x,y,z), \left(-\frac{3}{\sqrt{70}}, -\frac{6}{\sqrt{70}}, \frac{5}{\sqrt{70}}\right) \right\rangle = 0 \end{cases} \iff \begin{cases} -\frac{2}{\sqrt5}x + \frac{1}{\sqrt5}y = 0 \\ -\frac{3}{\sqrt{70}}x - \frac{6}{\sqrt{70}}y + \frac{5}{\sqrt{70}}z = 0 \end{cases}$$

$$\iff \begin{cases} -2x+y=0 \\ -3x-6y+5z=0 \end{cases} \overset{\text{NOTAZIONE}}{\iff} \begin{cases} -2x_1+x_2=0 \\ -3x_1-6x_2+5x_3=0 \end{cases}$$

$\Rightarrow$ CONTINUA NEL TUTORATO T13

(5) SIA $W = \{(x,y,z) \in \mathbb{R}^3 : x = z+2y\}$ SOTTOSPAZIO VETTORIALE DI $\mathbb{R}^3$. SI CHIEDE DI:

a) SCRIVERE UNA BASE ORTONORMALE PER W

$$W = \{(z+2y, y, z) \in \mathbb{R}^3 : y,z \in \mathbb{R}\} = [(1,0,1),(2,1,0)]$$

$$(z+2y, y, z) = (z,0,z) + (2y,y,0) = z(1,0,1) + y(2,1,0)$$

I GENERATORI DI W SONO ORTOGONALI?

$$\langle (1,0,1),(2,1,0)\rangle = 2 \neq 0 \Rightarrow \text{NO} \Rightarrow \text{USO GRAHM-SCHMIDT}$$

$$\Rightarrow v_1' = (1,0,1)$$

$$v_2' = v_2 - \frac{\langle v_2, v_1'\rangle}{\langle v_1', v_1'\rangle} v_1' = (2,1,0) - \frac{\langle (2,1,0),(1,0,1)\rangle}{\langle (1,0,1),(1,0,1)\rangle}(1,0,1) =$$

$$= (2,1,0) - \frac{2}{2}(1,0,1) = (1,1,-1)$$

$$\langle v_1', v_2'\rangle = \langle (1,0,1),(1,1,-1)\rangle = 1-1 = 0 \ \checkmark$$

$\Rightarrow$ NORMALIZZO v_1' e v_2'

$$v_1'' = \frac{v_1'}{|v_1'|} = \frac{(1,0,1)}{\sqrt2} = \left(\frac{1}{\sqrt2}, 0, \frac{1}{\sqrt2}\right)$$

$$\Rightarrow v_2'' = \frac{v_2'}{|v_2'|} = \frac{(1,1,-1)}{\sqrt3} = \left(\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, -\frac{1}{\sqrt3}\right) \qquad \Rightarrow \mathcal{B} = \{v_1'', v_2''\}$$

b) COMPLETARE $\mathcal{B}$ A UNA BASE ORTONORMALE $\mathcal{B}'$ DI $\mathbb{R}^3$.

DETERMINO UN VETTORE ORTOGONALE A v_1'' E v_2''

$$\Rightarrow \begin{cases} \left\langle (x,y,z), \left(\frac{1}{\sqrt2}, 0, \frac{1}{\sqrt2}\right)\right\rangle = 0 \\ \left\langle (x,y,z), \left(\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, -\frac{1}{\sqrt3}\right)\right\rangle = 0 \end{cases}$$

$$\iff \begin{cases} \frac{x}{\sqrt2} + \frac{z}{\sqrt2} = 0 \\ \frac{x}{\sqrt3} + \frac{y}{\sqrt3} - \frac{z}{\sqrt3} = 0 \end{cases} \iff \begin{cases} x = -z \\ \frac{y}{\sqrt3} = \frac{z}{\sqrt3} + \frac{z}{\sqrt3} \end{cases} \iff \begin{cases} x = -z \\ y = 2z \end{cases}$$

$$(-z, 2z, z)$$

$$\Rightarrow v_3' = (-1,2,1) \Rightarrow v_3'' = \frac{v_3'}{|v_3'|} = \frac{(-1,2,1)}{\sqrt6} = \left(-\frac{1}{\sqrt6}, \frac{\sqrt2}{\sqrt3}, \frac{1}{\sqrt6}\right)$$

$$\Rightarrow \mathcal{B}' = \{v_1'', v_2'', v_3''\}$$

⑥ SIANO DATI $\vec{v}_1=(3,4,0)$, $\vec{v}_2=(-1,2,0)$, $\vec{v}_3=(1,-2,-1)$ VETTORI IN $\mathbb{R}^3$.

a) MOSTRARE CHE $\mathcal{B}=\{\vec{v}_1,\vec{v}_2,\vec{v}_3\}$ E' UNA BASE DI $\mathbb{R}^3$. A PARTIRE DA ESSA TROVARE UNA BASE ORTOGONALE E UNA ORTONORMALE

$$\begin{vmatrix} 3 & -1 & 1 \\ 4 & 2 & -2 \\ 0 & 0 & -1 \end{vmatrix} = -\begin{vmatrix} 3 & -1 \\ 4 & 2 \end{vmatrix} = -(6+4) = -10 \neq 0$$

$$v_1' = v_1 = (3,4,0)$$

$$v_2' = v_2 - \frac{\langle v_2, v_1'\rangle}{\langle v_1', v_1'\rangle} v_1' = (-1,2,0) - \frac{\langle(-1,2,0),(3,4,0)\rangle}{\langle(3,4,0),(3,4,0)\rangle}(3,4,0) =$$

$$= (-1,2,0) - \frac{5}{25}(3,4,0) = (-1,2,0) - \left(\frac{3}{5},\frac{4}{5},0\right) = \left(-\frac{8}{5},\frac{6}{5},0\right)$$

$$v_3' = v_3 - \frac{\langle v_3, v_2'\rangle}{\langle v_2', v_2'\rangle} v_2' - \frac{\langle v_3, v_1'\rangle}{\langle v_1', v_1'\rangle} v_1' =$$

$$= (1,-2,-1) - \frac{\langle(1,-2,-1),\left(-\frac{8}{5},\frac{6}{5},0\right)\rangle}{\langle\left(-\frac{8}{5},\frac{6}{5},0\right),\left(-\frac{8}{5},\frac{6}{5},0\right)\rangle}\left(-\frac{8}{5},\frac{6}{5},0\right) +$$

$$- \frac{\langle(1,-2,-1),(3,4,0)\rangle}{\langle(3,4,0),(3,4,0)\rangle}(3,4,0) =$$

$$= (1,-2,-1) - \left(\frac{-\frac{8}{5}-\frac{12}{5}}{\frac{64}{25}+\frac{36}{25}}\right)\left(-\frac{8}{5},\frac{6}{5},0\right) - \frac{3-8}{9+16}(3,4,0) =$$

$$= (1,-2,-1) + \left(-\frac{8}{5},\frac{6}{5},0\right) + \frac{1}{5}(3,4,0) =$$

$$= \left(1-\frac{8}{5}+\frac{3}{5}, -2+\frac{6}{5}+\frac{4}{5}, -1\right) = (0,0,-1)$$

$\Rightarrow \mathcal{B} = \{v_1', v_2', v_3'\}$ E' UNA BASE ORTOGONALE

PER OTTENERE UNA BASE ORTONORMALE, OCCORRE NORMALIZZARE

$$v_1'' = \frac{v_1'}{|v_1'|} = \frac{(3,4,0)}{\sqrt{25}} = \left(\frac{3}{5},\frac{4}{5},0\right)$$

$$v_2'' = \frac{v_2''}{|v_2''|} = \frac{\left(-\frac{8}{5},\frac{6}{5},0\right)}{\sqrt{\frac{64}{25}+\frac{36}{25}}} = \left(-\frac{4}{5},\frac{3}{5},0\right) \qquad v_3'' = v_3' = (0,0,1)$$

$\Rightarrow \mathcal{B}_2 = \{v_1'', v_2'', v_3''\}$

b) SIA $W=[v_1,v_2]$. CALCOLARE UNA BASE PER $W^\perp$

$v_1 = (3,4,0)$ $\quad v_2 = (-1,2,0)$

$$\begin{cases} \langle v, v_1\rangle = 0 \\ \langle v, v_2\rangle = 0 \end{cases} \iff \begin{cases} 3x+4y=0 \\ -x+2y=0 \end{cases} \iff \begin{cases} 10y=0 \\ x=2y \end{cases} \iff \begin{cases} y=0 \\ x=0 \end{cases}$$

$\Rightarrow W^\perp = [(0,0,1)] \Rightarrow \mathcal{B}_3 = \{(0,0,1)\}$.

OSS. SE CAMBIATE L'ORDINE DEI VETTORI NEL PROCEDIMENTO DI GRAM-SCHMIDT OTTENETE COMUNQUE UNA BASE CHE VA BENE!

(OVVIAMENTE SE FATE I CONTI CORRETTAMENTE)

GLI ELEMENTI DELLA BASE, SE PARTITE DA $\vec{v}_1$, SARANNO DIVERSI DA QUELLI CHE OTTENETE SCEGLIENDO COME PRIMO VETTORE $\vec{v}_2$, MA SEMPRE UNA BASE ORTOGONALE OTTERRETE