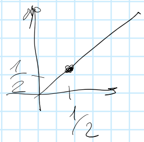
TUTORATO 14/03/23

① $R_3 = \{ (x,y) \in [0,1] \times [0,1] \mid x = y$ oppure $x + y = 1 \}$



- $x = x$ oppure $x + x = 1$ ✓ $\Rightarrow$ Riflessiva
- $\underbrace{x \, R_3 \, y}_{\Downarrow} \Rightarrow y \, R_3 \, x$?

$x = y \quad \vee \quad x + y = 1$

$\Downarrow \qquad \Downarrow$

$y = x \quad \vee \quad y + x = 1 \Rightarrow y \, R_3 \, x \Rightarrow$ Simmetrica

$\boxed{x = y \ e \ x + y = 1} \Rightarrow R_4$

$2x = 1$

$x = \frac{1}{2}$

$\begin{cases} x = y \\ x + y = 1 \end{cases}$

- $x \, R_3 \, y \Rightarrow x = y \ \vee \ x + y = 1$

$y \, R_3 \, z \Rightarrow y = z \ \vee \ y + z = 1$

$\Rightarrow \boxed{x + z = 1 \ \vee \ x = z}$

1) Se $x = y$ e $y = z \Rightarrow x = z$ ✓

2) Se $x = y$ e $y + z = 1 \Rightarrow x + z = 1$

3) Se $x + y = 1$ e $y = z \Rightarrow x + z = 1$ ✓

4) Se $x + y = 1$ e $y + z = 1 \Rightarrow \cancel{1} - x + z = \cancel{1} \Leftrightarrow x = z$ ✓ $\Rightarrow$ transitiva

$\rightarrow (0,1) \in R_3 \quad 0 = 1 \ \vee \ \boxed{0 + 1 = 1} \quad x + y = 1$

$\rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \in R_3 \quad \frac{1}{2} = \frac{1}{2} \ \vee \ \frac{1}{2} + \frac{1}{2} = 1$

$\rightarrow (0,0) \in R_3 \quad \boxed{0 = 0} \ \vee \ \cancel{0 + 0 = 1} \quad x = y$

$(0,0) \quad (0,1) \Rightarrow (0,1) \in R_3$

$x = y \quad x + y = 1$

$x + z$

$\Rightarrow R_3$ è rel. di equivalenza

② ES 4 Cap 1

$\vec{v} = (1, -1)$ per quali VALORI DI $a \in \mathbb{R}$ IL VETTORE $\vec{w} = (a, 3)$ è perpendicolare a $\vec{v}$?

$\vec{v} \perp \vec{w}$? $\quad \langle \vec{v}, \vec{w} \rangle = 0$?

$\vec{v} \cdot \vec{w} = 0$?

$\langle \vec{v}, \vec{w} \rangle = |\vec{v}| |\vec{w}| \boxed{\cos(\varphi)}$?

$= v_1 \cdot w_1 + v_2 \cdot w_2 = 1 \cdot a + (-1) \cdot 3 = a - 3 = 0$

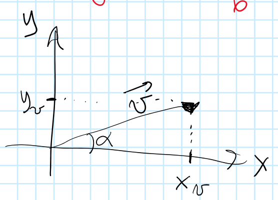
$\Updownarrow$

$a = 3 \qquad \vec{w} = (3,3)$

③ ES 5 CAP 1

Se $\vec{v}$ ha modulo 2 e forma con l'asse delle ascisse un angolo di $\frac{\pi}{6}$. Quali sono le sue proiezioni sugli assi?

un angolo di $\frac{\pi}{6}$. Quali sono le sue proiezioni sugli assi?



$\alpha = \frac{\pi}{6}$ $\qquad x_v = |\vec{v}| \cos\alpha = 2 \cdot \cos\frac{\pi}{6} = \cancel{2} \cdot \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}$

$|\vec{v}| = 2$ $\qquad y_v = |\vec{v}| \operatorname{sen}\alpha = 2 \cdot \operatorname{sen}\frac{\pi}{6} = \cancel{2} \cdot \frac{1}{\cancel{2}} = 1$

α in gradi : $180° = \alpha$ in radianti : π

$\alpha° : 180° = \frac{\pi}{6} : \pi \iff \alpha° = 30°$

30°, 45°, 60°, 90°, 180°, 360° $\rightarrow$ in radianti + sen/cos

④ Siano dati i seguenti vettori di $\mathbb{R}^3$ $\vec{v}_1 = (1, -1, 0)$ $\vec{v}_2 = (-1, -2, 4)$

Determinare:

ⓐ il versore di $\vec{v}_2$, detto $\vec{u}_2$ $\qquad \vec{u} = \frac{\vec{v}}{|\vec{v}|}$ ($\vec{u}$ versore di $\vec{v}$)

$$\vec{u}_2 = \frac{\vec{v}_2}{|\vec{v}_2|}$$

$$|\vec{v}_2| = \sqrt{(-1)^2 + (-2)^2 + (4)^2} = \sqrt{1 + 4 + 16} = \sqrt{21}$$

$$\vec{u}_2 = \frac{(-1, -2, 4)}{\sqrt{21}} = \left(-\frac{1}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}\right)$$

ⓑ la componente ortogonale α di $\boxed{\vec{v}_1}$ rispetto a una retta parallela e concorde con il versore di $\vec{v}_2$

$\cancel{\vec{v}'} = \alpha \cdot \frac{\vec{w}}{|\vec{w}|}$ $\qquad \boxed{\alpha = \langle \vec{v}, \frac{\vec{w}}{|\vec{w}|} \rangle}$

Proiezione ortogonale è vettore

Componente ortog. è SCALARE

$$\alpha = \langle \vec{v}_1, \vec{u}_2 \rangle = \langle (1, -1, 0), \left(-\frac{1}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}\right) \rangle =$$

$$= 1 \cdot \left(-\frac{1}{\sqrt{21}}\right) + (-1) \cdot \left(-\frac{2}{\sqrt{21}}\right) + 0 \cdot \frac{4}{\sqrt{21}} = -\frac{1}{\sqrt{21}} + \frac{2}{\sqrt{21}} = \frac{1}{\sqrt{21}}$$

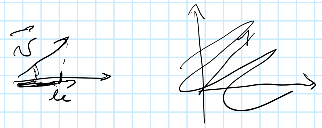
ⓒ la proiezione ortogonale $\vec{v}'$ di $\vec{v}_1$ sulla retta del punto ⓑ

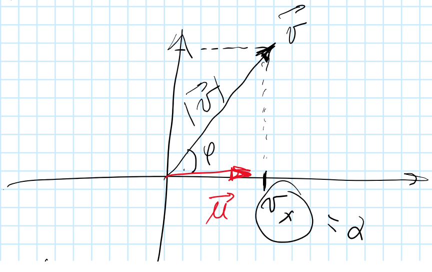
$$\vec{v}' = \langle \vec{v}_1, \vec{u}_2 \rangle \cdot \vec{u}_2 = \alpha \vec{u}_2$$

$$\alpha = \langle \vec{v}, \vec{u} \rangle = |\vec{v}| \overset{1}{|\vec{u}|} \cos\varphi$$

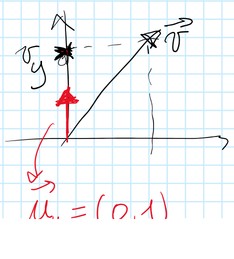
$\rightarrow \alpha = \langle \vec{v}, \vec{u} \rangle = \underbrace{|\vec{v}| \overset{1}{|\vec{u}|} \cos\varphi}$

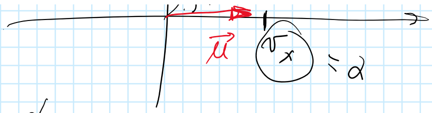
$\rightarrow \alpha = |\vec{v}| \cos\varphi$



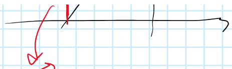


$(1, 0) = \vec{u}$



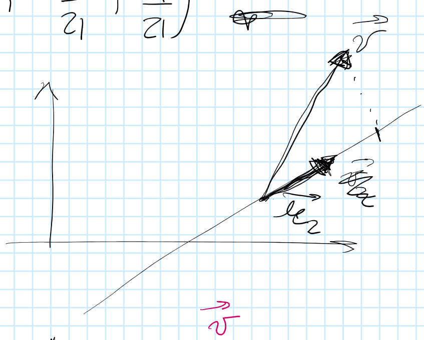


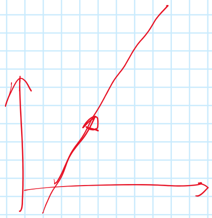
$(1,0) = \vec{i}$

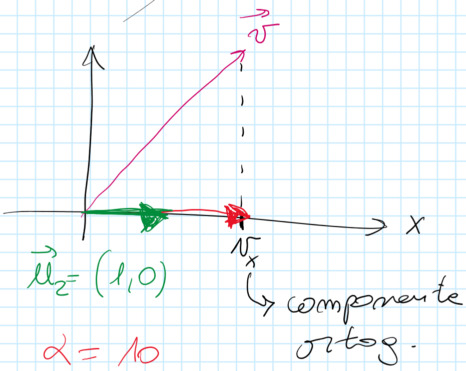


$$\boxed{\vec{v}'} = \alpha\, \vec{u}_2 = \underbrace{\frac{1}{\sqrt{21}}}^{\alpha} \left(-\frac{1}{\sqrt{21}}, -\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}\right) = \left(-\frac{1}{21}, -\frac{2}{21}, \frac{4}{21}\right)$$

② Trovare i coseni direttori







$\vec{u}_2 = (1,0)$

$\alpha = 10$

$\vec{v}' = \alpha \cdot \vec{u}_2 = 10 \cdot (1,0) = (10,0) \to v_x$

$\vec{v}' = \alpha \cdot (2,0,1000) =$

$\mathbb{R}^3 \ni \vec{v}_1 = (1, -1, 0)$ $\quad \vec{v} = (x, y, z)$

$$\cos(\theta_x) = \frac{\langle \vec{v}, \vec{i} \rangle}{|\vec{v}|} = \frac{x}{|\vec{v}|} \qquad \vec{v} = (v_1, v_2, v_3)$$

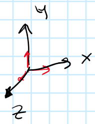
$$\cos(\theta_y) = \frac{\langle \vec{v}, \vec{j} \rangle}{|\vec{v}|} = \frac{y}{|\vec{v}|}$$

$$\cos(\theta_z) = \frac{\langle \vec{v}, \vec{k} \rangle}{|\vec{v}|} = \frac{z}{|\vec{v}|}$$

$\vec{i} = (1,0,0) = \vec{e}_1$

$\vec{j} = (0,1,0) = \vec{e}_2$

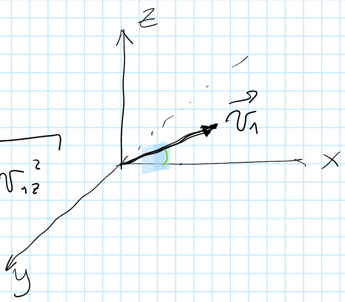
$\vec{k} = (0,0,1) = \vec{e}_3$



$$\cos\theta_x = \frac{\langle \vec{v}_1, i \rangle}{|\vec{v}_1|} = \frac{v_{1x}}{|\vec{v}_1|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\theta_y = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad |\vec{v}_1| = \sqrt{v_{1x}^2 + v_{1y}^2 + v_{1z}^2}$$

$$\cos\theta_z = 0$$



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① Stabilire se i vettori $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$

$\vec{w} = \frac{5}{3}\vec{i} - \frac{5}{2}\vec{j} + \frac{5}{6}\vec{k}$ sono paralleli / ortogonali / nessuno dei due

$$\langle \vec{v}, \vec{w} \rangle = 0 \iff \vec{v} \perp \vec{w}$$

$$\vec{v} \times \vec{w} = \vec{0} \iff \vec{v} \parallel \vec{w}$$

$$\langle \vec{v}, \vec{w} \rangle = 2 \cdot \frac{5}{3} - \frac{5}{2} \cdot (-3) + \frac{5}{6} = \frac{70}{6} \implies \vec{v} \not\perp \vec{w}$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ \frac{5}{3} & -\frac{5}{2} & \frac{5}{6} \end{vmatrix} = \vec{i}\left(-3 \cdot \frac{5}{6} - \left(-\frac{5}{2}\right) \cdot 1\right) - \vec{j}\left(2 \cdot \frac{5}{6} - 1 \cdot \frac{5}{3}\right) + \vec{k}\left(2 \cdot -\frac{5}{2} - (-3) \cdot \left(\frac{5}{3}\right)\right) =$$

$$= \vec{i}\left(-\frac{15}{6} + \frac{5}{2}\right) - \vec{j}\left(\frac{10}{6} - \frac{5}{3}\right) + \vec{k}\left(-\frac{10}{2} + \frac{15}{3}\right) =$$

$$= \vec{i}\left(-\frac{5}{2} + \frac{5}{2}\right) - \vec{j}\left(\frac{5}{3} - \frac{5}{3}\right) + \vec{k}\left(-5 + 5\right) =$$

$$= \vec{0} = (0,0,0) \implies \vec{v} \parallel \vec{w}$$

② Determinare $h_1, h_2 \in \mathbb{R}$ tali che $\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$ e $\vec{w} = \vec{i} + h_1\vec{j} + h_2\vec{k}$ risultino paralleli

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ 1 & h_1 & h_2 \end{vmatrix} = \vec{i}\,(h_2 + 3h_1) - \vec{j}\,(2h_2 + 3) + \vec{k}\,(2h_1 - 1) = \vec{0}$$

$$\begin{cases} h_2 + 3h_1 = 0 \\ -2h_2 - 3 = 0 \\ 2h_1 - 1 = 0 \end{cases} \iff \begin{cases} h_2 = -\frac{3}{2} \\ h_1 = \frac{1}{2} \end{cases} \qquad -\frac{3}{2} + 3 \cdot \frac{1}{2} = 0 \checkmark$$

③ $\vec{v}_1 = (1,0,1)$ $\quad \vec{v}_2 = (0,1,0)$ $\quad \vec{v}_3 = (1,1,2)$ $\in \mathbb{R}^3$

a) $\vec{v}_1, \vec{v}_2, \vec{v}_3$ sono complanari?

$$\langle \vec{v}_1, \vec{v}_2 \times \vec{v}_3 \rangle =$$

IL PRODOTTO VETTORIALE
NON E' COMMUTATIVO!

$$\vec{v}_1 = (1,0,0) \qquad \vec{v}_2 = (0,1,0)$$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \cancel{\vec{i}} - \cancel{\vec{j}} + \vec{k} \cdot 1 = (0,0,1)$$

$$\not\parallel$$

$$\vec{v}_2 \times \vec{v}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \end{vmatrix} = \cancel{\vec{i}} - \cancel{\vec{j}} - \vec{k} = -(0,0,1)$$

$$\vec{v}_2 \times \vec{v}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \cancel{\vec{i}} - \cancel{\vec{j}} - \vec{k} = -(0,0,1)$$

$$\langle \vec{v}_1 , \underbrace{\vec{v}_2 \times \vec{v}_3}_{\vec{v}_3 \times \vec{v}_2} \rangle = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 1 \cdot (1 \cdot 2 - 0 \cdot 1) + 1 \cdot (-1) = 1 \neq 0$$

$\Downarrow$

NON sono comp-

$\langle \vec{v}_2 , \vec{v}_1 \times \vec{v}_3 \rangle$