venerdì 12 maggio 2023 08:48

③ SIA DATA $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 2 & 1 & 3 \end{pmatrix}$

a) DETERMINARE LE EQ. DELL' APPLICAZIONE LINEARE $f: \mathbb{R}^3 \to \mathbb{R}^3$ AVENTE A COME MATRICE RISPETTO ALLA BASE CANONICA DI $\mathbb{R}^3$.

$$f(x,y,z) = (x+2y+3z, -y-z, 2x+y+3z)$$

b) DET. UNA BASE PER $Ker(f)$ e $dim(Imm(f))$

$$* \begin{cases} x+2y+3z=0 \\ -y-z=0 \\ 2x+y+3z=0 \end{cases} \iff \begin{cases} z=-y \\ x+2y-3y=0 \\ 2x+y+3z=0 \end{cases} \iff \begin{cases} z=-y \\ x=y \end{cases}$$

$(y, y, -y)$

$$Ker(f) \overset{*}{=} [(1,1,-1)] \Rightarrow dim(Ker(f)) = 1$$

$$dim(Imm(f)) = dim(\mathbb{R}^3) - dim(Ker(f)) \overset{!}{=} 3 - 1 = 2$$

c) SIANO $\vec{v} = (2,0,4)$ $\vec{w} = (-3,1,-3)$ VERIFICARE $Imm(f) = [\vec{v}, \vec{w}]$

$$Imm(f) = \left[\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix}\right] = \left[2\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, (-1)\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}\right] = [\vec{v}, \vec{w}]$$

d) DET. $dim(Imm(f) \cap Ker(f))$

$$dim(Imm(f) \cap Ker(f)) = dim(Imm(f)) + dim(Ker(f)) - dim(\mathbb{R}^3) \overset{!}{=} 2 + 1 - 3 = 0$$

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① SIA $f: \mathbb{R}^2 \to \mathbb{R}^3$ l'applicazione lineare definita da $f(x,y) \overset{*}{=} (x+y, x+2y, x+3y)$. Determinare la matrice A associata ad f rispetto alle basi $\mathcal{B} = \{(2,1)\ (1,-2)\}$ di $\mathbb{R}^2$ e $\mathcal{B}' = \{(1,0,0), (1,2,0), (1,2,3)\}$ di $\mathbb{R}^3$.

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$$

Ⓘ $f\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}^*$ $f\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1-2 \\ 1-4 \\ 1-6 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix}$ i vettori di $\mathcal{B}$ rispetto all' app. f

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + c\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{cases} x = a+b+c \\ y = 2b+2c \\ z = 3c \end{cases} \xrightarrow{\text{TROVIAMO } a, b \text{ e } c} \begin{cases} x = a+b+\frac{z}{3} \\ y = 2b + \frac{2}{3}z \\ c = \frac{z}{3} \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = a + b + \frac{z}{3} \\ b = \frac{y}{2} - \frac{z}{3} \\ c = \frac{z}{3} \end{cases} \qquad \Longleftrightarrow \begin{cases} a = x - \frac{y}{2} + \cancel{\frac{z}{3}} - \cancel{\frac{z}{3}} \\ b = \frac{y}{2} - \frac{z}{3} \\ c = \frac{z}{3} \end{cases}$$

VETTORE $(3,4,5)$

$$\begin{cases} a = 3 - \frac{4}{2} \\ b = \frac{4}{2} - \frac{5}{3} \\ c = \frac{5}{3} \end{cases} \Rightarrow \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}_C = \begin{pmatrix} 1 \\ 1/3 \\ 5/3 \end{pmatrix}_{B'}$$

VETTORE $(-1,-3,-5)$

$$\begin{cases} a = -1 - \left(\frac{-3}{2}\right) \\ b = \frac{-3}{2} + \frac{5}{3} \\ c = -5/3 \end{cases} \qquad \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix}_C = \begin{pmatrix} 1/2 \\ 1/6 \\ -5/3 \end{pmatrix}_{B'}$$

$$\Rightarrow A = \mathcal{M}^{B}_{B'}(f) = \begin{pmatrix} 1 & 1/2 \\ 1/3 & 1/6 \\ 5/3 & -5/3 \end{pmatrix}$$

$$\mathcal{M}^{B}_{B'}(f) = \mathcal{M}^{C}_{B'}\,\mathcal{M}^{C}_{C}\,\mathcal{M}^{B}_{C}$$

$$\begin{tikzcd} C \quad \mathbb{R}^2 \arrow[r, "\mathcal{M}^C_C(f)", "f"'] \arrow[d, leftrightarrow, "\mathcal{M}^C_B(i_{\mathbb{R}^2})"'] & \mathbb{R}^3 \quad C \arrow[d, leftrightarrow, "\mathcal{M}^C_{B'}(i_{\mathbb{R}^3})"] \\ B \quad \mathbb{R}^2 \arrow[r, dashed] & \mathbb{R}^3 \quad B' \end{tikzcd}$$

$$\mathcal{M}^{C}_{C}(f) = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$$

$$\mathcal{M}^{B}_{C}(i_{\mathbb{R}^2}) = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$$

$\mathcal{M}^{C}_{B'}(i_{\mathbb{R}^3}) \longrightarrow$ OCCORRE RISOLVERE IL SISTEMA $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x - \frac{y}{2} \\ \frac{y}{2} - \frac{z}{3} \\ \frac{z}{3} \end{pmatrix}$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_C = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{B'}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_C = \begin{pmatrix} -1/2 \\ 1/2 \\ 0 \end{pmatrix}_{B'} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_C = \begin{pmatrix} 0 \\ -1/3 \\ 1/3 \end{pmatrix}_{B'}$$

$$\mathcal{M}^{B}_{B'}(f) = \mathcal{M}^{C}_{B'}(i_{\mathbb{R}^3})\,\mathcal{M}^{C}_{C}(f)\,\mathcal{M}^{B}_{C}(i_{\mathbb{R}^2}) =$$

$$= \underset{3\times 3}{\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & -1/3 \\ 0 & 0 & 1/3 \end{pmatrix}} \underset{3\times 2}{\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}} = \dots = \begin{pmatrix} 1 & 1/2 \\ 1/3 & 1/6 \\ 5/3 & -5/3 \end{pmatrix}$$

② DETERMINARE L'APP. LIN. $f: \mathbb{R}^2 \to \mathbb{R}^3$ ASSOCIATA ALLA MATRICE $A = \begin{pmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{pmatrix}$ RISPETTO ALLA BASE CANONICA DI $\mathbb{R}^2$

$$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

E ALLA BASE $\mathcal{B}' = \{(1,0,0), \underbrace{(1,-1,0)}, (-1,-1,1)\}$ di $\mathbb{R}^3$.

$$\begin{tikzcd} C\ \mathbb{R}^2 \arrow[r, "f"', "A"] \arrow[dr, dashed] & \mathbb{R}^3\ \mathcal{B}' \arrow[d, "\mathcal{M}_C^{B'}"] \\ & \mathbb{R}^3\ C \end{tikzcd}$$

$$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ 3x+3y \\ 2x+y \end{pmatrix}_{B'} \overset{*}{=} (x+2y)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (3x+3y)\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + (2x+y)\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} =$$

2° METODO (PIU' RAPIDO)

$$\mathcal{M}_C^C = \mathcal{M}_C^{B'} A = \underset{3\times 3}{\begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}} \underset{3\times 2}{\begin{pmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{pmatrix}} = \begin{pmatrix} 2 & 4 \\ -5 & -4 \\ 2 & 1 \end{pmatrix}$$

$$* = (x+2y)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (3x+3y)\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] + (2x+y)\left[-\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right]$$

$$= (x+2y+3y+3x-2x-y)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-3x-3y-2x-y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + (2x+y)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= (2x+4y)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (-5x-4y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + (2x+y)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2x+4y \\ -5x-4y \\ 2x+y \end{pmatrix}_C$$

$$= (2x+4y)\vec{i} + (-5x-4y)\vec{j} + (2x+y)\vec{k}$$

OVE $\vec{i}, \vec{j}, \vec{k}$ SONO I VETTORI DELLA BASE CANONICA

$$\mathcal{M}_C^C = \begin{pmatrix} 2 & 4 \\ -5 & -4 \\ 2 & 1 \end{pmatrix}$$

③ DET. L'APP. LIN. $f: \mathbb{R}^3 \to \mathbb{R}^2$ ASSOCIATA ALLA MATRICE $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ RISPETTO ALLA BASE $\mathcal{B} = \{(1,0,1), (0,-1,2), (1,-1,0)\}$ DI $\mathbb{R}^3$ E ALLA BASE $\mathcal{B}' = \{(2,5),(0,3)\}$ DI $\mathbb{R}^2$

$$\begin{tikzcd} \mathcal{B}\ \mathbb{R}^3 \arrow[r, "A"] \arrow[d, leftrightarrow] & \mathbb{R}^2\ \mathcal{B}' \arrow[d, leftrightarrow] \\ C\ \mathbb{R}^3 \arrow[r, dashed] & \mathbb{R}^2\ C \end{tikzcd}$$

I METODO

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_C = a\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + b\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} + c\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \iff \begin{cases} x = a + c \\ y = -b - c \\ z = a + 2b \end{cases}$$

$$\iff \begin{cases} x = z - 2b + c \\ y = -b - c \\ a = z - 2b \end{cases} \iff \begin{cases} x = z \underbrace{-2b - b} - y \\ c = -b - y \\ a = z - 2b \end{cases} \iff$$

$$\iff \begin{cases} 3b = z - y - x \\ c = -y - b \\ a = z - 2b \end{cases} \iff \begin{cases} b = -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z \\ c = \frac{1}{3}x - \frac{2}{3}y - \frac{1}{3}z \\ a = z + \frac{2}{3}x + \frac{2}{3}y - \frac{2}{3}z \end{cases}$$

$$\iff \begin{cases} a = \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z \\ b = -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z \\ c = \frac{1}{3}x - \frac{2}{3}y - \frac{1}{3}z \end{cases} \qquad \begin{pmatrix} x \\ y \\ z \end{pmatrix}_C = \begin{pmatrix} \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z \\ -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z \\ \frac{1}{3}x - \frac{2}{3}y - \frac{1}{3}z \end{pmatrix}_B$$

cioè
$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_C = \begin{pmatrix} 2/3 \\ -1/3 \\ 1/3 \end{pmatrix}_B \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_C = \begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}_B \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_C = \begin{pmatrix} 1/3 \\ 1/3 \\ -\frac{1}{3} \end{pmatrix}_B$$

ora
$$f\left(\begin{pmatrix} 2/3 \\ -1/3 \\ 1/3 \end{pmatrix}_B\right) = A\begin{pmatrix} 2/3 \\ -1/3 \\ 1/3 \end{pmatrix}_B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 2/3 \\ -1/3 \\ 1/3 \end{pmatrix}_B = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}_{B'}$$

$$f\left(\begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}_B\right) = A\begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}_B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}_B = \begin{pmatrix} 1/3 \\ -1/3 \end{pmatrix}_{B'}$$

$$f\left(\begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix}_B\right) = A\begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix}_B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix}_B = \begin{pmatrix} 2/3 \\ 4/3 \end{pmatrix}_{B'}$$

SE VOLETE LASCIARE INDICATI x, y, z POTETE FARE COSÌ:

$$f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}_C\right) = f\left(\begin{pmatrix} \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z \\ -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z \\ \frac{1}{3}x - \frac{2}{3}y - \frac{1}{3}z \end{pmatrix}_B\right) = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z \\ -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z \\ \frac{1}{3}x - \frac{2}{3}y - \frac{1}{3}z \end{pmatrix}$$

$$= \begin{pmatrix} \boxed{\frac{1}{3}x + \frac{1}{3}y + \frac{2}{3}z} \\ \frac{2}{3}x - \frac{1}{3}y + \frac{4}{3}z \end{pmatrix}_{B'}$$

SFRUTTANDO I VETTORI DELLA BASE B' HO CHE

$$\begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}_{B'} = \frac{1}{3}\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{5}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{6}{3} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{11}{3} \end{pmatrix}_C$$

$$\begin{pmatrix} \frac{1}{3} \\ -1/3 \end{pmatrix}_{B'} = \frac{1}{3}\begin{pmatrix} 2 \\ 5 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{5}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}_C$$

$$\begin{pmatrix} \frac{2}{3} \\ \frac{4}{3} \end{pmatrix}_{B'} = \frac{2}{3}\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \frac{4}{3}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{3} \\ \frac{10}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{12}{3} \end{pmatrix} = \begin{pmatrix} \frac{4}{3} \\ \frac{22}{3} \end{pmatrix}_C$$

SE INVECE VOLETE LASCIARE INDICATI x, y e z POTETE FARE COSÌ:

$$\left(\frac{1}{3}x + \frac{1}{3}y + \frac{2}{3}z\right)\cdot\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \left(\frac{2}{3}x - \frac{1}{3}y + \frac{4}{3}z\right)\begin{pmatrix} 0 \\ 3 \end{pmatrix} =$$

$$= \left(\frac{1}{3}x + \frac{1}{3}y + \frac{2}{3}z\right)\left(2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 5\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) + \left(\frac{2}{3}x - \frac{1}{3}y + \frac{4}{3}z\right)\left(0\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) =$$

$$= \left(\frac{2}{3}x + \frac{2}{3}y + \frac{4}{3}z\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(\frac{5}{3}x + \frac{5}{3}y + \frac{10}{3}z\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} + (2x - y + 4z)\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \left(\frac{2}{3}x + \frac{2}{3}y + \frac{4}{3}z\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(\frac{11}{3}x + \frac{2}{3}y + \frac{22}{3}z\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3}x + \frac{2}{3}y + \frac{4}{3}z \\ \frac{11}{3}x + \frac{2}{3}y + \frac{22}{3}z \end{pmatrix}_C$$

ALLA FINE E' COME SCRIVERE $\left(\frac{2}{3}x + \frac{2}{3}y + \frac{4}{3}z\right)\vec{i} + \left(\frac{11}{3}x + \frac{2}{3}y + \frac{22}{3}z\right)\vec{j}$

QUINDI VA BENE COME NOTAZIONE IL +

II METODO

$$\mathcal{M}_C^C(f) = \mathcal{M}_C^{B'}(i_{\mathbb{R}^2}) \cdot \underbrace{\mathcal{M}_{B'}^{B}(f)}_{A}\, \mathcal{M}_B^C(i_{\mathbb{R}^3})$$

$$\begin{tikzcd} B & \mathbb{R}^3 \arrow[r, "A"] \arrow[d, leftrightarrow] & \mathbb{R}^2 \arrow[d, leftrightarrow] & B' \\ C & \mathbb{R}^3 \arrow[r, dashed] & \mathbb{R}^2 & C \end{tikzcd}$$

$$\mathcal{M}_C^{B'}(i_{\mathbb{R}^2}) = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}$$

$$\mathcal{M}_B^C(i_{\mathbb{R}^3}) \underset{\text{DAL SISTEMA}}{=} \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$

$$\mathcal{M}_C^C(f) = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 2/3 & 2/3 & 1/3 \\ -1/3 & -1/3 & 1/3 \\ 1/3 & -2/3 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{11}{3} & \frac{2}{3} & \frac{22}{3} \end{pmatrix}$$

$$\boxed{f(x,y,z) = \left(\frac{2}{3}x + \frac{2}{3}y + \frac{4}{3}z,\ \frac{11}{3}x + \frac{2}{3}y + \frac{22}{3}z\right)}$$ NB

④ Siano $f: \mathbb{R}^2 \to \mathbb{R}^4$ $g: \mathbb{R}^4 \to \mathbb{R}^2$ due app. lin. definite da

$$f(x_1, x_2) = (x_1+x_2,\ x_1-x_2,\ 2x_1,\ x_2)$$

$$g(y_1, y_2, y_3, y_4) = \underbrace{(y_1-y_2-y_3+y_4,\ 2y_4)}$$

a) Scrivere $\mathcal{M}^C_C(g\circ f)$ associata a $g\circ f$ rispetto alla base canonica C di $\mathbb{R}^2$

$$g\circ f: \mathbb{R}^2 \xrightarrow[f]{} \mathbb{R}^4 \xrightarrow[g]{} \mathbb{R}^2$$

$$\mathcal{M}^C_C(g\circ f) = \mathcal{M}^C_C(g)\ \mathcal{M}^C_C(f) = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -2 & 3 \\ 0 & 2 \end{pmatrix}$$

b) Scrivere $\mathcal{M}^B_B(g\circ f)$ associata a $g\circ f$ rispetto alla base $B = \{(2, 3), (1,2)\}$, utilizzando la matrice del cambiamento di base da B a C

PROVARE PER CASA

$$\begin{tikzcd} C & \mathbb{R}^2 \arrow[r, "g\circ f"] \arrow[d, leftrightarrow] & \mathbb{R}^2 \arrow[d, leftrightarrow] & C \\ B & \mathbb{R}^2 \arrow[r, dashed] & \mathbb{R}^2 & B \end{tikzcd}$$

$$\mathcal{M}^B_B(g\circ f) = \mathcal{M}^C_B(i_{\mathbb{R}^2}) \cdot \mathcal{M}^C_C(g\circ f) \cdot \mathcal{M}^B_C(i_{\mathbb{R}^2})$$

$$\mathcal{M}^B_C(i_{\mathbb{R}^2}) = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$