1 DATI

$\mu_d = 0.2$

$\mu_s = 0.3$

$m_1 = 1\,kg$

$m_2 = 3\,kg$

$\therefore$

$T_{max} = 20\,N$

RICHIESTE

$a, T = ?$

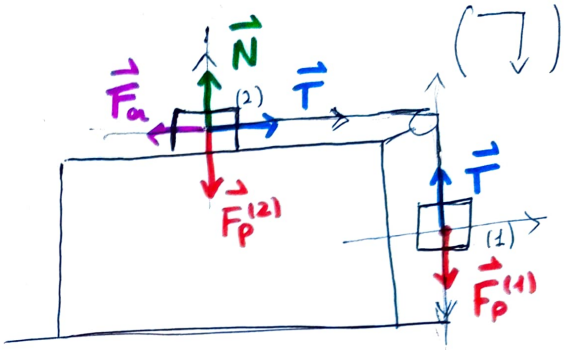
$\rightarrow m_1 >> m_2$

$a' = ?$

$T' = ?$

$m_{max}^{(1)} = ?$

Condizione di equilibrio = ?



SOLUZIONE

CORPO 1 $\quad \sum \vec{F}_{(1)} = m_1 \vec{a}_1 \rightarrow \vec{T} + \vec{F}_p^{(1)} = m_1 \vec{a}_1$

$$\boxed{-T + F_p^{(1)} = m_1 a_1} \quad (I)$$

CORPO 2 $\quad \sum \vec{F}_{(2)} = m_2 \vec{a}_2$

asse y : $|\vec{F}_p^{(2)}| = |\vec{N}| \Rightarrow$ equilibrio

asse x : $\vec{F}_a + \vec{T} = m_2 \vec{a}_2$

$$\boxed{-F_a + T = m_2 a_2} \quad (II)$$

$(a_1 = a_2 = a)$

$$\begin{cases} m_1 a = -T + F_p^{(1)} \\ m_2 a = -Fa + T \end{cases} \longrightarrow \begin{cases} a(m_1+m_2) = -T + F_p^{(1)} - Fa + T \\ T = m_2 a + Fa \end{cases}$$

$$\begin{cases} a = \dfrac{F_p^{(1)} - Fa}{m_1 + m_2} \\ T = m_2 a + Fa \end{cases}$$

$$\rightarrow F_p^{(1)} = m_1 g$$

$$\rightarrow Fa = \mu_d m_2 g$$

$$a = \frac{m_1 g - \mu_d m_2 g}{m_1 + m_2} = g\,\frac{m_1 - \mu_d m_2}{m_1 + m_2} \longrightarrow \boxed{a = g\,\frac{m_1 - \mu_d m_2}{m_1 + m_2}}$$

$$T = m_2 g\,\frac{m_1 - \mu_d m_2}{m_1 + m_2} + \mu_d m_2 g = m_2 g\left[\frac{m_1 - \mu_d m_2}{m_1 + m_2} + \mu_d\right]$$

$$= m_2 g\left[\frac{m_1 - \mu_d m_2 + \mu_d (m_1 + m_2)}{m_1 + m_2}\right] = m_2 g\left[\frac{m_1 - \mu_d m_2 + \mu_d m_1 + \mu_d m_2}{m_1 + m_2}\right]$$

$$= m_2 g\left[\frac{m_1(1+\mu_d)}{m_1 + m_2}\right] \longrightarrow \boxed{T = \frac{m_1 m_2 g (1+\mu_d)}{m_1 + m_2}}$$

- $m_1 >> m_2$

$$a = g\frac{m_1 - \mu_d m_2}{m_1 + m_2} \longrightarrow a = g\frac{m_1}{m_1} \quad *$$

$$T = \frac{m_1 m_2 g(1+\mu_d)}{m_1 + m_2} \longrightarrow T = \frac{m_1 m_2 g(1+\mu_d)}{m_1} = m_2 g(1+\mu_d) \quad *$$

- $m_1 = 1\,kg, \quad m_2 = 3\,kg$

$$a = \left(9.81\,\frac{N}{kg}\right)\frac{1\,kg - 0.23\,kg}{1\,kg + 3\,kg} = 0.98\,\frac{N}{kg} = 0.98\,\frac{m}{s^2}$$

$$T = [\dots] = 8.83\,N$$

- $T_{max} = \dfrac{m_1^{max} m_2 g(1+\mu_d)}{m_1^{max} + m_2} \longrightarrow T_{max}(m_1^{max} + m_2) = m_1^{max} m_2 g(1+\mu_d)$

$$[\dots] \quad m_1^{max} = \frac{T_{max} m_2}{g m_2(1+\mu_d) - T_{max}} [\dots] = 3.92\,kg$$

- condizione di equilibrio

$$\begin{cases} m_1 g - T = 0 \\ -F_a + T = 0 \end{cases} \quad (+)$$

$$m_1 g - T - F_a + T = 0$$

$$F_a = m_1 g \rightarrow \boxed{F_a \leq F_{s,max}}$$

$$F_{s,max} = \mu_s m_2 g$$

$$m_1 g \leq \mu_s m_2 g$$

$$m_1 \leq \mu_s m_2$$

$$\left(\vec{F}_{s,max} = \mu_s \vec{F}_1 \rightarrow \vec{F}_s \leq \vec{F}_{s,max} \right)$$

3.1

2 DATI

$\theta = 30°$

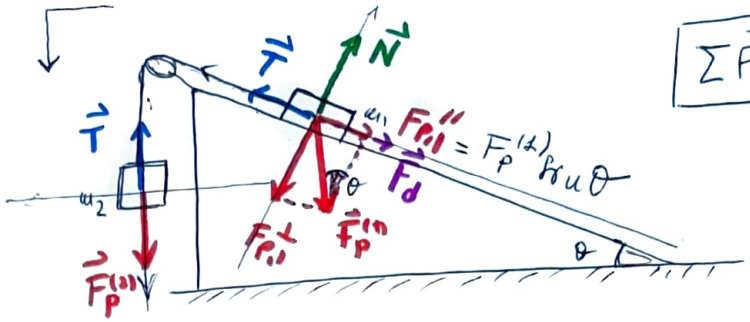
$m_1 = 10\ kg$

$m_2 = 6\ kg$

$\mu_d = 0.1$

RICHIESTE

$a = ?$



$\Sigma \vec{F} = m\vec{a}$

SOLUZIONE

Corpo 1 $\quad \Sigma \vec{F}_{(1)} = m_1 \vec{a}$

asse y : $|\vec{F}_{P,1}^{\perp}| = |\vec{N}| \Rightarrow a_y^{(1)} = 0 \Rightarrow$ equilibrio

asse x : $\vec{T} + \vec{F}_d + \vec{F}_{P,1}'' = m_1 \vec{a}$

$$\boxed{T - F_d - F_{P,1}'' = m_1 a}$$

Corpo 2 $\quad \Sigma \vec{F}_{(2)} = m_2 \vec{a} \rightarrow \vec{F}_P^{(2)} + \vec{T} = m_2 \vec{a} \rightarrow \boxed{F_P^{(2)} - T = m_2 a}$

(// ⊥)

$$\begin{cases} T - F_d - F_{P,1}'' = m_1 a & (+) \\ F_P^{(2)} - T = m_2 a \end{cases}$$

$$T - F_d - F_{P,1}'' + F_P^{(2)} - T = m_1 a + m_2 a \rightarrow a(m_1 + m_2) = F_P^{(2)} - F_{P,1}'' - F_d$$

$$a = \frac{F_P^{(2)} - F_{P,1}'' - F_d}{m_1 + m_2}$$

- $F_P^{(2)} = m_2 g$
- $F_P^{(1)} = m_1 g$
- $F_{P,1}'' = F_P^{(1)} \sin\theta$
- $\boxed{F_d = \mu_d F_{P,1}^{\perp}} = \mu_d F_P^{(1)} \cos\theta$

$$a = \frac{F_P^{(2)} - F_{P,1}'' - F_d}{m_1 + m_2} = \frac{m_2 g - m_1 g \sin\theta - \mu_d m_1 g \cos\theta}{m_1 + m_2} = [\dots] = 0.47\ m/s^2$$

(3) DATI

$M = 10\ kg$

$(\sum \vec{F} = 0)$

$\theta = 30°$

$\mu_s = 0.3$

$k = 300\ N/m$

$\Delta x = 10\ cm = 0.1\ m$

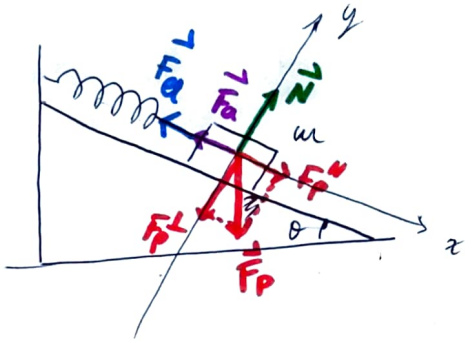
$\beta = 45°$

$\mu_d = 0.2$

RICHIESTE

$\vec{F}_s = ?$

$a = ?$



SOLUZIONE

$\Sigma \vec{F} = 0$

ASSE Y : $\vec{F}_p^{\perp} + \vec{N} = 0 \rightarrow -F_p^{\perp} + N = 0 \rightarrow F_p^{\perp} = N$

ASSE X : $\vec{F}_{el} + \vec{F}_{attr} + \vec{F}_p'' = 0$

$-F_{el} - F_{attr} + F_p'' = 0$

~~$F_s = F_{el} - F_p''$~~ $F_s = F_p'' - F_{el} = F_p \cdot \sin\theta - k\Delta x = M \cdot g \sin\theta - k\Delta x$

$F_s = (10\ kg)(9.81\ \frac{N}{kg})\sin\theta - 300\ \frac{N}{m}\ 0.1\ m = 19.05\ N$

$$\sum \vec{F} = M\vec{a} \longrightarrow \vec{F}_{el} + \vec{F}_d + \vec{F}_p'' = M\vec{a}$$

$$-F_{el} - F_d + F_p'' = Ma$$

$F_s \leq F_s^{max}$

$F_d = \mu_d F_\perp$

$$a = \frac{-F_{el} - F_d + F_p''}{M} = \frac{-k\Delta x - \mu_d Mg\cos\beta + Mg\,\mathrm{sen}\beta}{M} \quad [\dots] = 2.55\ m/s^2$$

$F_p'' = F_p \,\mathrm{sen}\beta$

$F_p^\perp = F_p \cos\beta$

4 DATI

$m_1 = 2\,kg$

$m_2 = 5\,kg$

$\theta_1 = 45°$

$\theta_2 = 30°$

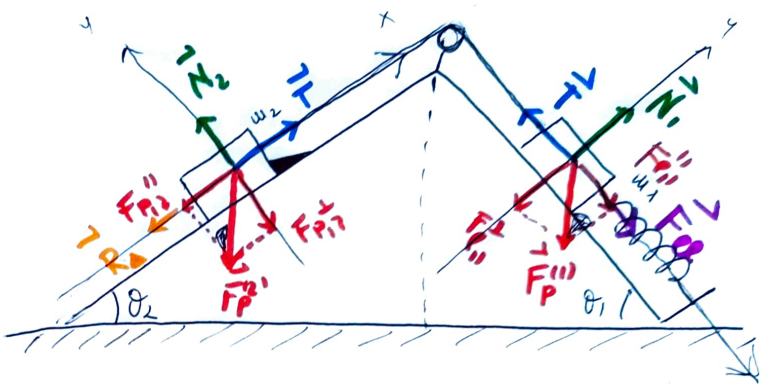
$k = 300\,N/m$

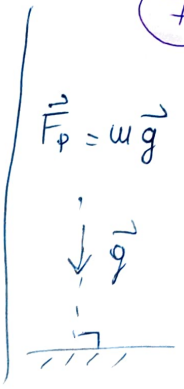
$\Delta l = 30\,cm = 0.3\,m$

RICHIESTE

$|\vec{R}_{V,2}| = ?$

$a = ?$





SOLUZIONE

$$|\vec{R}_{V,2}| = \sqrt{N_2^2 + R_\Delta^2}$$

(↑ N_2, ↑ R_Δ)

corpo 1 $\quad \vec{T} + \vec{F}''_{P,1} + \vec{F}_{el} = 0$

$$\boxed{-T + F''_{P,1} + F_{el} = 0}$$

corpo 2 $\quad \vec{R}_\Delta + \vec{F}''_{P,2} + \vec{T} = 0$

$$\boxed{-R_\Delta - F''_{P,2} + T = 0}$$

$$N_2 = m_2 g \cos\theta_2 \cong 42.4\,N$$

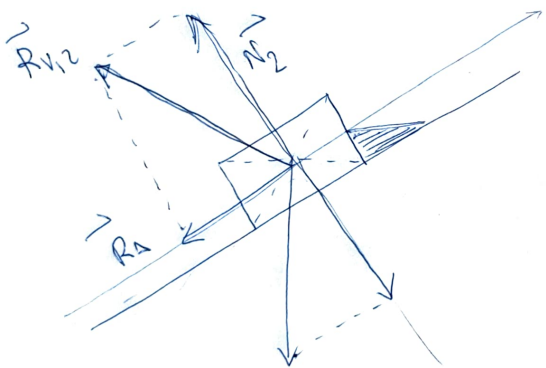
$$\begin{cases} -T + F''_{P,1} + F_{el} = 0 \\ -R_\Delta - F''_{P,2} + T = 0 \end{cases} \rightarrow \begin{cases} T = F''_{P,1} + F_{el} \\ -R_\Delta - F''_{P,2} + F''_{P,1} + F_{el} = 0 \end{cases}$$

$$R_\Delta = -F''_{P,2} + F''_{P,1} + F_{el}$$

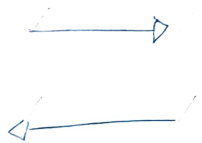
- $F''_{P,2} = F_{P,2} \sin\theta_2$
- $F''_{P,1} = F_{P,1} \sin\theta_1$
- $F_{el} = k\Delta\ell$

$$R_\Delta = -m_2 g \sin\theta_2 + m_1 g \sin\theta_1 + k\Delta\ell = [\dots] \simeq 79.4\ N$$

$$|\vec{R}_{V,2}| = \sqrt{N_2^2 + R_\Delta^2} \simeq 90\ N$$



$$\vec{F}_{el} = -k\Delta\vec{x}$$



Corpo 1 $\quad \vec{T} + \vec{F}''_{p,1} + \vec{F}_{el} = m_1 \vec{a} \longrightarrow \begin{cases} -T + F''_{p,1} + F_{el} = m_1 a \\ -F''_{p,2} + T = m_2 a \end{cases}$ (+)

Corpo 2 $\quad \vec{F}''_{p,2} + \vec{T} = m_2 \vec{a}$

$$-T + F''_{p,1} + F_{el} - F''_{p,2} + T = m_1 a + m_2 a$$

$$a(m_1 + m_2) = F''_{p,1} + F_{el} - F''_{p,2} \quad \triangleright \quad a = \frac{F''_{p,1} + F_{el} - F''_{p,2}}{m_1 + m_2}$$

$$a = \frac{m_1 g \sin\theta_1 + k\Delta l - m_2 g \sin\theta_2}{m_1 + m_2} = [\dots] \cong 11.3 \ m/s^2$$