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COURSE: MAT 251

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① prove that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Solution

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

Differentiate

$$\lim_{x \rightarrow 0} \frac{\frac{d}{dx} \sin x}{\frac{d}{dx} x}$$
$$\lim_{x \rightarrow 0} \frac{\cos x}{1}$$
$$\lim_{x \rightarrow 0} \frac{\cos(0)}{1} = \frac{1}{1} = 1$$

② Show that $\begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous but not differentiable at the origin

Solution

to solve $\lim_{x \rightarrow 0} x \sin \left(\frac{1}{x} \right)$

Since sine function oscillate between -1 and 1 we can say $-1 \leq \sin f(x) \leq 1$ also $-1 \leq \sin \left(\frac{1}{x} \right) \leq 1$

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Multiply through by x to get
 $-x \leq x \sin(1/x) \leq x$

Considering $\lim_{x \rightarrow 0} x \sin(1/x)$

$$\lim_{x \rightarrow 0} (-x) \leq \lim_{x \rightarrow 0} x \sin(1/x) \leq \lim_{x \rightarrow 0} (x)$$

$$0 \leq \lim_{x \rightarrow 0} x \sin(1/x) \leq 0$$

Using Squeeze theorem

$$\lim_{x \rightarrow 0} x \sin(1/x) = 0$$

Since $\lim_{x \rightarrow 0} x \sin(1/x) = f(0) = 0$ (are equal)

hence this shows that function $f(x)$ defined as $x \sin 1/x$ is continuous at $x = 0$.

Same principle apply with: 0; $x = 0$ such that
 $\lim_{x \rightarrow 0} [0] = f(0) = 0$ which shows that $f(x)$ is

continuous

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Now to Show that the function is not differentiable at the origin i.e. $x = x_0 = 0$

$$f'(x) = \lim_{x \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

then it follows that at the origin where $x = 0$
hence $f'(0) = \lim_{x \rightarrow 0} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$$

recall $f(x) = x \sin(1/x)$ hence $f(h) = h \sin(1/h)$

$$\lim_{h \rightarrow 0} \frac{h \sin(1/h) + 0 \sin(1/0)}{h}$$

$$\lim_{h \rightarrow 0} \frac{h \sin(1/h)}{h} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \sin(1/h)$$

as h approach 0, the expression $\sin(1/h)$ oscillate infinitely between -1 and 1, therefore

$\lim_{x \rightarrow 0} \sin(1/h)$ does not exist

Since it does not exist,

hence $f(x)$ is not differentiable

(3) Use the mean value theorem to prove that
if $x > 0$, then $\ln(1+x) < x$

Solution

$$\ln(1+x) < x$$

$$\ln(1+x) - x < 0$$

Let $f(x) = \ln(1+x) - x$ Such that

$$f(0) = \ln(1+0) - 0 = 0$$

$$\text{Since } f(0) = \ln(1+0) - 0 = \ln(1) - 0 = 0 - 0 = 0$$

$$\text{hence } f'(c) = \frac{f(x) - f(0)}{x - 0} = \frac{f(x) - 0}{x}$$

$$f'(c) = \frac{f(x)}{x}$$

$$\text{Recall: } f(x) = \ln(1+x) - x$$

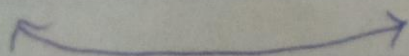
$$f'(x) = \frac{1}{1+x} - 1 = \frac{1-1-x}{1+x} = \frac{-x}{1+x}$$

$$f'(c) = \frac{-x}{1+x}$$

$$f'(c) = \frac{-c}{1+c} = f'(x)$$

$$f'(c) = \frac{f(x)}{x} = f'(x)$$

$$\frac{-c}{1+c} = \frac{f(x)}{x} = \frac{-x}{1+x}$$



Where $f'(c) < f'(x)$

$$\frac{-c}{1+c} = \frac{-x}{1+x}$$

$$\frac{-c}{1+c} < \frac{-x}{1+x}$$

$$-c < \frac{-x}{1+x} (1+c)$$

$$-c < \frac{-x}{1+x} - \frac{xc}{1+x}$$

$$-c(1+x) < -x - xc$$

$$-c - cx < -x - xc$$

$$-c < -x$$

$$c < x$$

hence $f'(c) < 0 \quad \forall x > 0$,

we have shown that $f(x)$ is a decreasing function on the interval $(0, x)$ since $f'(c) < 0$ then $f(x) < 0$

where $f(x) = \ln(1+x) - x \Rightarrow \ln(1+x) - x < 0$

$$\Rightarrow \underline{\ln(1+x) < x}$$