Binary Heaps: Homework

17/3/2020

1. Implement the array-based representation of binary heap together with the functions `HEAP_MIN`, `REMOVE_MIN`, `HEAPIFY`, `BUILD_HEAP`, `DECREASE_KEY`, and `INSERT_VALUE`.

2. Implement an iterative version of `HEAPIFY`.

3. Test the implementation on a set of instances of the problem and evaluate the execution time.

4. (Ex. 6.1-7 in [1]) Show that, with the array representation, the leaves of a binary heap containing n nodes are indexed by $\lfloor n/2 \rfloor + 1, \lfloor n/2 \rfloor + 2, \ldots n$.

5. (Ex. 6.2-6 in [1]) Show that the worst-case running time of `HEAPIFY` on a binary heap of size n is $\Omega(\log n)$. (Hint: For a heap with n nodes, give node values that cause `HEAPIFY` to be called recursively at every node on a simple path from the root down to a leaf.)

6. (Ex. 6.3-3 in [1]) Show that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any n-element binary heap.

References

[1] T.H. Cormen, C.E. Leiserson, R.L. Rivest, and C. Stein. *Introduction to Algorithms*. The MIT Press. MIT Press, 2009.