Day 4 Assignment

Question 1

a. The expected number of slots that need to be searched is < 4. So average-case running time in this case is O(4) = O(1), i.e. constant time.

   Reference page from slide :- page 4.

b. Z = {1, 2, 3, 4 …….,100}

   Z = i occurs if D is found in slot

since each of the possible slots in the array are equally likely to contain D, the events X = i, 1 ≤ i ≤ 100, are also all equally likely to occur

Therefore Pr(X = i) = 1 /100 , for each I

E[Z] = $\sum_{i=1}^{100} i$*Pr(Z = i)

= $\sum_{i=1}^{100} i$ * 1/100

=1*1/100 + 2 * 1/100 + …. + 100/100

=50.5

c. Z = {1, 2, 3, 4 …….,10,000}

   Therefore Pr(X = i) = 1 /10000, for each i

E[Z] = $\sum_{i=1}^{10000} i$*Pr(Z = i)

= $\sum_{i=1}^{100} i$ * 1/10000

=1*1/100 + 2 * 1/10000 + …. + 10000/10000

=5000.5

d. O(1)

Question 2.

a. The probability of each slot to fill with D is;

   $P = \frac{1}{4} = .25$

Let Y denote the random variable whose value is the number of trials needed to get a successful outcome.

average number of array locations to inspect to find 10 D's is

$E(Y) = 10/P$

$= 10/.25$

$= 40$

b. $E(Y) = K/P$ where K is the number of trial and P is the probability of each slot to fill with D.
c. The average time complexity to find k D's in an array is O(1).

Question 3.

Prove: $1 + 1/2 + 1/3 + \ldots + 1/n = O(\log n)$.

For $n = 7$

$1 + 1/2 + 1/3 + 1/4 + \ldots. + 1/7 < 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 = \log(7+1)$

For $n= 15$

$1 + 1/2 + 1/3 + 1/4 + \ldots. + 1/15 < 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + 1/8 * 8 = 4 = \log(15+1)$

For $n = 31$

$1 + 1/2 + 1/3 + 1/4 + \ldots. + 1/31 < 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + 1/8 * 8 + 1/16 * 16 = 5 = \log(31+1)$

There by deduction hypothesis

$1 + \frac{1}{2} + 1/3 + \ldots\ldots.. + 1/n < \log(n + 1)$

$= \log n + \log 1$

$= \log n \rightarrow O(\log n)$

Question 4,

$S = ½ + 2/4 + 3/8 + 4/16 + \ldots\ldots\ldots + n/2^n$

$S/2 = \quad 1/4 + 2/8 + 3/16 + \ldots\ldots\ldots. + n\text{-}1/2^n + n/2^{n+1}$

$S – S/2 = ½ + ¼ + 1/8 + 1/16 + \ldots\ldots..+ 1/2^n - n/2^{n+1}$

Since $½ + ¼ + 1/8 + 1/16 + \ldots\ldots..+ 1/2^n = 1$

$S/2 = 1 - n/2^{n+1}$

$S = 2 - 2n/2^{n+1}$

As n approaches to infinity $2n/2^{n+1} = 0$

S=2