Algorithm Assignment W2D1

Question 1: Design and Analysis of the algorithms

A. **Algorithm** TwoColor(A,n)

Input: Array A of n balls with 2 color

OutPut: Array A with all the balls sorted accouring to their color.

```
n←A.lengh
if n=0 || n=1
    return A
p←A[0]
i← 1
j← n
while i<=j
    if A[i]!=A[j]
        if p=A[j]
            SWAP(A[i],A[j])
            i++
            j--
    j--
return A
```

→My Algorithm is in place. And the time complicity is O(n).

B. **Algorithm** 4ColorAnd3Color(A,n)

Input: Array A of n balls with 3 or more Different Colors

Output: Array A sorted according to their color

```
n← A.length
if n=0 || n=1
    return A
p←A[0]
i← 1
j← n
while i!=n
    if A[i]!=A[j]
        if p = A[j]
            SWAP(A[i],A[j])
            i++

    if i>=j
        p=A[i]
        j=n
        i++
    j--
return A
```

➔ This Algorithm is working well for both B and C. Also it's in place. And the time complicity is O(n).

Question 2:

A. **A= {1, 2, 3, 4, 5, 6, 7, 8, 9}**
3(8)/4 = 6
pivot = A[6] = 7
L= {1,2,3,4,5,6} size which is equal to 6
→ So we can say this is Bad Call.
→we don't have to do the sorting its already sorted

B. **A={8, 7, 6, 5, 4, 3, 2, 1, 9}**
let's pick a p by using he median number= 8,9,4
p=4
{8,7,6,5,9,3,2,1,4} = A
{1,7,6,5,9,3,2,8,4}
{1,2,6,5,9,3,7,8,4}
{1,2,3,5,9,6,7,8,4}
{1,2,3,4,9,6,7,8,5}again choose a pivot using median: 5
{9,6,5,8,7}
{5,6,9,8,7}
{5,6,7,8,9}
→ Then join them up = {1,2,3,4,5,6,7,8,9}

C. {9, 1, 8, 2, 7, 3, 6, 4, 5}

Let's Select pivot

- Position A[0]+A[8]/2= 0+8/2=4

=>median of **9,7,5 is 7**

Pivot is 7

{9, 1, 8, 2, 7, 3, 6, 4, 5}

Swap(7,5)

- **{9, 1, 8, 2, 7, 3, 6, 4, 5}**
- **{4, 1, 6, 2, 5, 3}, 7, {9, 8}**
- **{2, 1, 6, 4, 5, 3}, 7, {9, 8}**
- **{2, 1,} 3 { 4, 5, 6}, 7, {9, 8}**
- **{1,2} 3 {4,5,6},7,{8,9}**
- **So the sorted will be {1,2,3, 4 ,5,6,7, 8 ,9}**

D. {5, 1, 4, 2, 3, 9, 7, 6, 8}

Let's Select pivot

- Position A[0]+A[8]/2= 0+8/2=4

=>median of **5,3,8 is 5**

- **{3,1,4,2,59,7,6,8}**
- **{3,1,4,2,8,9,7,6,5}**
- **{3,1,4,2}5{9,7,6,8}**
- **{1,2,4,3,5,6,7,8,9}**
- **So the sorted will be {1,2,3, 4 ,5,6,7, 8 ,9}**

Question 3:

N.B→ p=pivot

A. **A= {1, 2, 3, 4, 5, 6, 7, 8, 9}   k = 5**
   p= 3(8)/4=6 =>7
   E= {7}
   L={1,2,3,4,5,6} size is bigger than k so we have our number in here.
      p=3(5)/4=3 => 4
      E= {4}
      G={5,6} again pic a number for pivot
      p= 3(1)/4=1=>5
      E={5}=K=5
   → we found the number 5th smallest is 5.

B. **A= {8, 7, 6, 5, 4, 3, 2, 1, 9}   k = 3**
   p=3(8)/4= 6 => 2
   E= {2}
   L= {1} Size is 1 so definitely our number is in the G Array.
   G= {9,3,4,5,6,7,8}
      p = 3(6)/4 = 4 => 6
      E={6}
      L={3,4,5}
         p= 3(2)/4 = 1 => 3
         E={3} = k = 3
   → so we got the 3th smallest number is 3.

C. **A= {9, 1, 8, 2, 7, 3, 6, 4, 5} k = 8**

p=3(8)/4 = 6 => 6

E={6}

L={1,2,3,4,5}

G={7,8,9}

→so, k > |L|+|E|=8 > 5+1, that mean our number is in G in second place.

P= 3(2)/4 = 1 => 8

E= {8}

E=8=K=8

→so, the 8th smallest number is 8.

D. **A={5, 1, 4, 2, 3, 9, 7, 6, 8} k = 5**

p=3(8)/4 = 6 =>7

E={7}

L={1,2,3,4,5,6}

G={8,9}

→ So, K < |L|+|E|= 5 < 6+1, that mean our number is in L 5th place.

P= 3(5)/4 = 3 => 4

E={4}

L={1,2,3}

G={5,6}

→so, K > |L|+|E| = 5 > 3+1, that mean our number is in G 1st place.

Take the first pivot:

E={5} E=K=5

→ so, we find the 5th smallest number which is 5.

Question 4:

Good Self call: the size of L and G are each less than 2n/3

Bad Self Call: one of L and G has size greater than or equal to 2/3n

Since 2/3 is greater than ½ and less than ¾ we can say it probability is good self call

Let prove this.

The height of recursion tree is one less than the length of the descending sequence

That means $n, 2/3n, (2/3)^2 n, \ldots, 1, 0$

$n + 2/3n + (2/3)^2 n + (2/3)^3 n + \ldots + 1$

$n[1 + 2/3 + (2/3)^2 + (2/3)^3 + \ldots + 1]$

n[1+]]

n[1/1-x]

n[1/1-2/3]

n[1/3]

3n

T(n)=O(n)

At each level of the recursion tree, total processing time is O(n)

Therefore, total running time in the good case is O(nlog n)

And this 3n is less than 4n is better than 4/3n but since this is constant there is no difference b/n them.

And using master formula we can say also that 2/3n

Is O(n)

Let prove it

$T(n) \le T(2/3)n + Cn^k$

Therefore a=1

B= 3

K=1

B^k=3

That means a< b^k

Therefore, it will be Θ(n).