**Day 4 Assignment**

**Question 1**

1. The expected number of slots that need to be searched is < 4. So average-case running time in this case is O(4) = O(1), i.e. constant time.

Reference page from slide :- page 4.

1. Z = {1, 2, 3, 4 …....,100}

Z = i occurs if D is found in slot

since each of the possible slots in the array are equally likely to contain D, the events X = i, 1 ≤ i ≤ 100, are also all equally likely to occur

Therefore Pr(X = i) = 1 /100 , for each I

E[Z] = \*Pr(Z = i)

= \* 1/100

=1\*1/100 + 2 \* 1/100 + …. + 100/100

=50.5

1. Z = {1, 2, 3, 4 …....,10,000}

Therefore Pr(X = i) = 1 /10000, for each i

E[Z] = \*Pr(Z = i)

= \* 1/10000

=1\*1/100 + 2 \* 1/10000 + …. + 10000/10000

=5000.5

1. O(1)

Question 2.

* 1. The probability of each slot to fill with D is;

P = ¼ = .25

Let Y denote the random variable whose value is the number of trials needed to get a successful outcome.

average number of array locations to inspect to find 10 D’s is

E(Y) = 10/P

= 10/.25

= 40

* 1. E(Y) = K/P where K is the number of trial and P is the probability of each slot to fill with D.
  2. The average time complexity to find k D’s in an array is O(1).

Question 3.

Prove: 1 + 1/2 + 1/3 + …+ 1/n = O(log n) .

For n = 7

1 + 1/2 + 1/3 +1/4 + …. + 1/7 < 1+ ½ + ½ + ¼ + ¼ + ¼ + ¼ = 3 = log(7+1)

For n= 15

1 + 1/2 + 1/3 +1/4 + …. + 1/15 < 1+ ½ + ½ + ¼ + ¼ + ¼ + ¼ +1/8 \*8 = 4 = log(15+1)

For n =31

1 + 1/2 + 1/3 +1/4 + …. + 1/31 < 1+ ½ + ½ + ¼ + ¼ + ¼ + ¼ +1/8 \*8 + 1/16 \*16= 5 = log(31+1)

There by deduction hypothesis

1 + ½ + 1/3 + ……..+1/n < log(n + 1)

= log n + log1

=log n 🡪O(log n)

Question 4,

S = ½ + 2/4 + 3/8 + 4/16 + ……… + n/2n

S/2 = 1/4 + 2/8 + 3/16 + ………. + n-1/2n + n/2n+1

S – S/2 = ½ + ¼ + 1/8 + 1/16 + ……..+ 1/2n - n/2n+1

Since ½ + ¼ + 1/8 + 1/16 + ……..+ 1/2n= 1

S/2 = 1 - n/2n+1

S = 2 - 2n/2n+1

As n approaches to infinity 2n/2n+1= 0

S=2