

# 논문 리뷰

Quantum circuits of T-depth one

발표자: 양유진

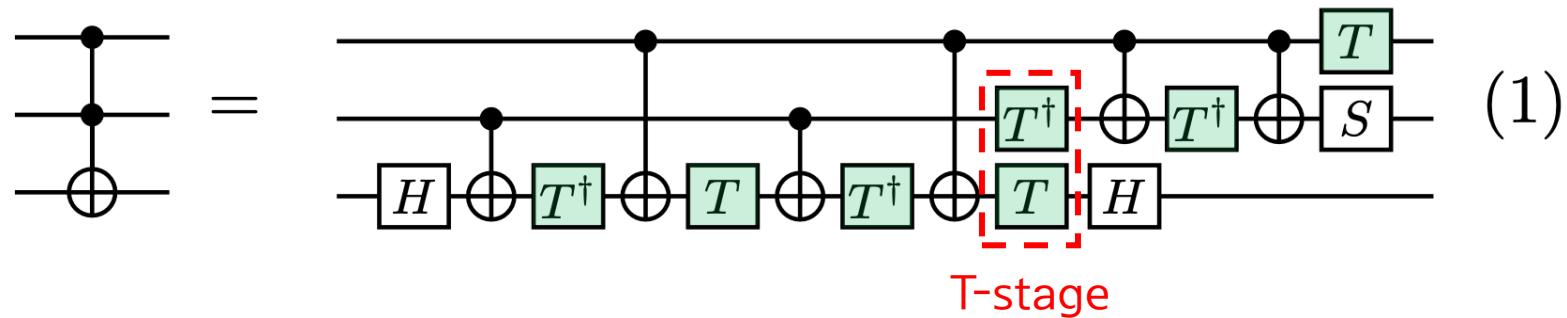
링크: <https://youtu.be/ellmnduDeAY>

# 1. Introduction

- 비용 함수로는 “gate 수”, “T-gate 수”, “circuit depth”, “사용된 ancillas 수” 가 있음.
- Amyet. al.<sup>[1]</sup>은 새로운 비용 함수로 T-depth를 제안함.

## T-stage

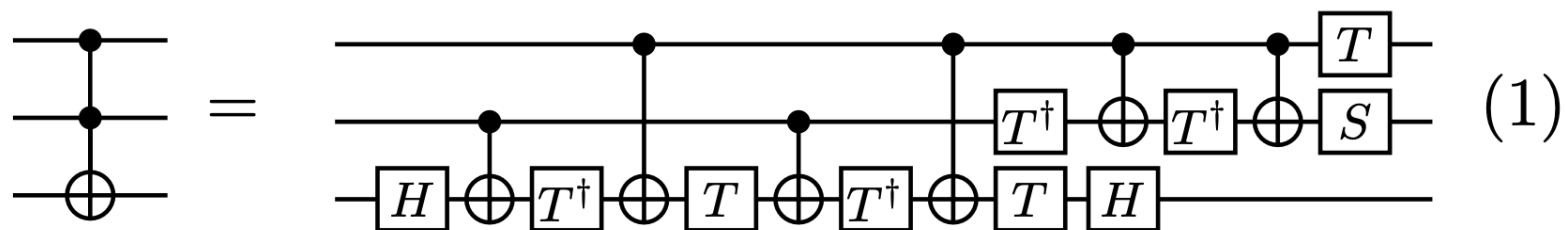
동시에 수행할 수 있는 하나 이상의  $T/T^\dagger$ -gate 그룹.



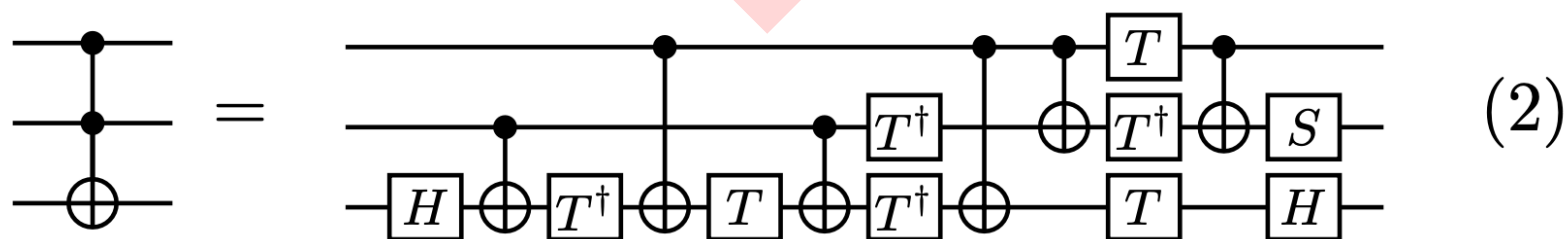
T-count: 7

T-depth: 6

# 1. Introduction



trivial communication 이용

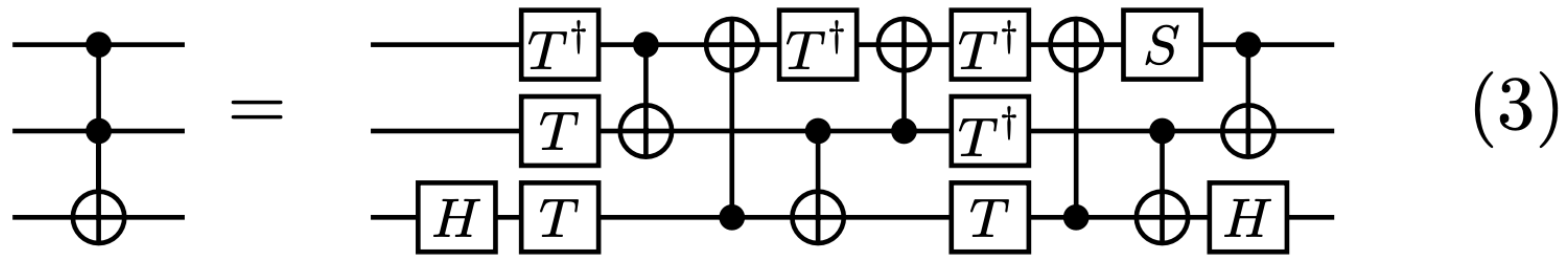


T-count: 7

T-depth: 6  $\rightarrow$  4

# 1. Introduction

- Amy et. al.<sup>[1]</sup> 은 Toffoli gate의 T-depth를 추가 개선하였음.
- ancillas를 사용하지 않은 회로 중 해당 회로가 가장 최적이라고 추측함.
- T-gate는 비싸고, ancillas는 저렴함.



T-count: 7

T-depth:  $4 \rightarrow 3$

→ [목적] Ancillas를 사용하여 Toffoli gate 및 많은 회로의 T-depth를 1로 줄일 수 있음을 보이는 것.

## 2. A T-depth on representation of the Toffoli gate

- Clifford Group은 unit scalar와 H, S, CNOT 게이트에 의해 생성됨.

$$S = T^2$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix},$$

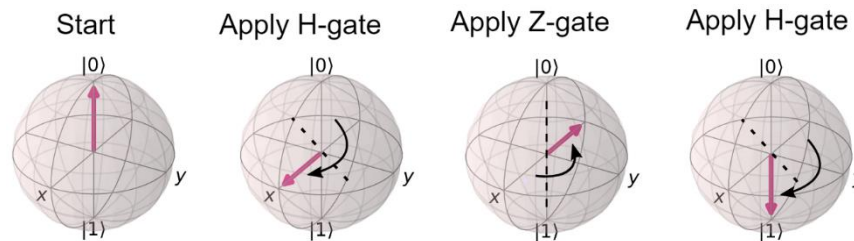
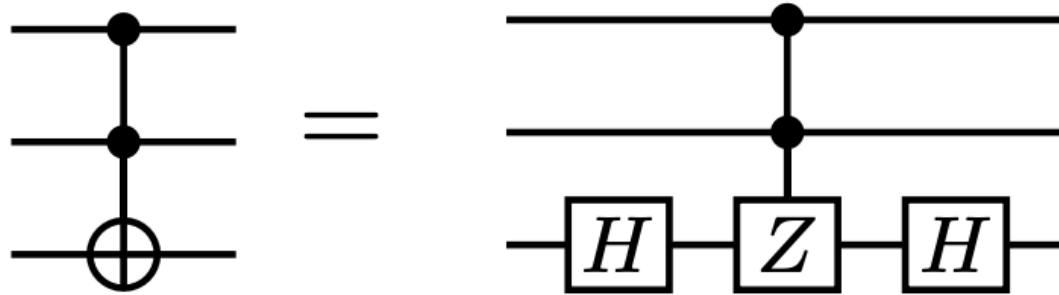
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli operation에 대한 게이트 작성

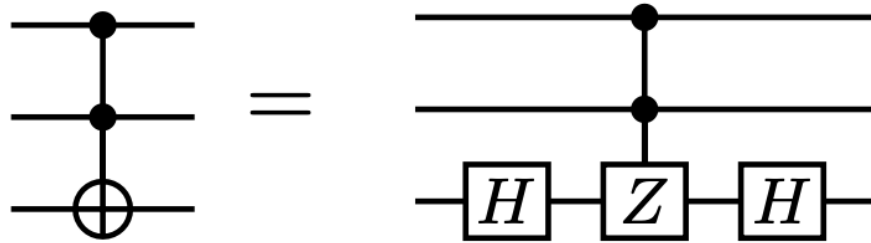
## 2-1. doubly-controlled Z-gate

- Toffoli gate는 기본변경을 통해 doubly-controlled Z-gate로 표현할 수 있음

$$|xyz\rangle \mapsto (-1)^{xyz}|xyz\rangle$$



## 2-1. doubly-controlled Z-gate



$$|xyz\rangle \mapsto (-1)^{xyz} |xyz\rangle$$

inclusion-exclusion style formula ( $x, y, z \in \{0, 1\}$ )

증명가능

$$4xyz = x + y + z - (x \oplus y) - (y \oplus z) - (x \oplus z) + (x \oplus y \oplus z). \quad (5) \quad \leftarrow x \oplus y = x + y - 2xy$$

[가정]  $\omega = (-1)^{1/4} = e^{i\pi/4}$

$$(-1)^{xyz} = \omega^{4xyz}$$

$$= \omega^x \omega^y \omega^z (\omega^\dagger)^{x \oplus y} (\omega^\dagger)^{y \oplus z} (\omega^\dagger)^{x \oplus z} \omega^{x \oplus y \oplus z}.$$

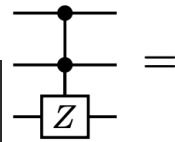
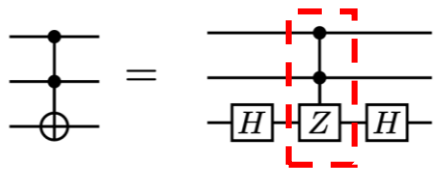
(6) doubly controlled Z-gate

$$T|x\rangle = \omega^x |x\rangle$$

doubly controlled Z-gate는 T/T<sup>†</sup>-gate를 이용하여 구현할 수 있음.

## 2-2. doubly-controlled Z-gate (T-depth = 1)

- doubly-controlled Z-gate는 4개의 ancillas와 T/T<sup>†</sup>-gate를 이용하여 다음과 같이 구현할 수 있음.
- 임의의 순서로 변경하고 병렬로 수행함으로써 T-depth=1로 줄일 수 있음.



```
Gate class counts:
  AllocateQubitGate : 7
  CXGate : 16
  DaggeredGate : 3
  DeallocateQubitGate : 7
  HGate : 2
  TGate : 4
```

```
Gate counts:
  Allocate : 7
  CX : 16
  Deallocate : 7
  H : 2
  T : 4
  T^\dagger : 3
```

```
Depth : 7.
```

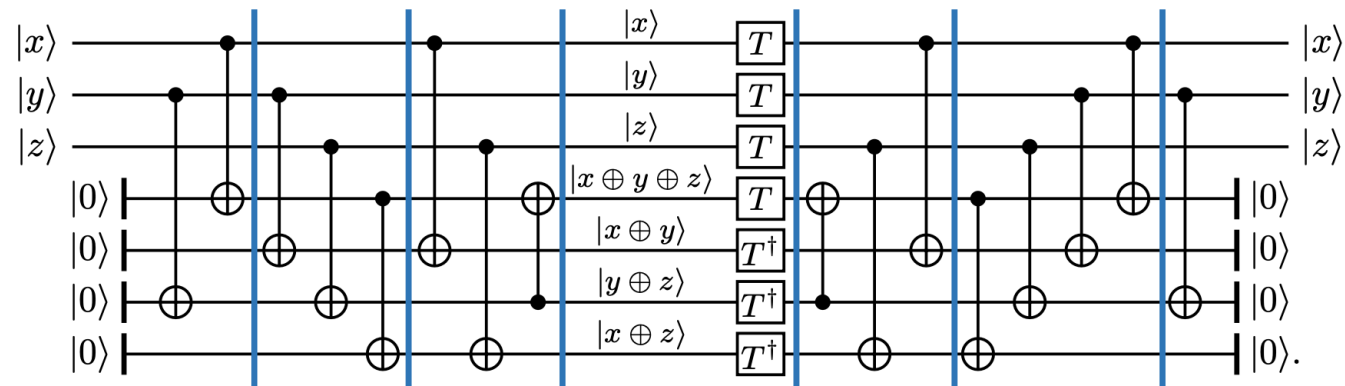


Figure 1:  $T$ -depth 1 representation of the Toffoli gate

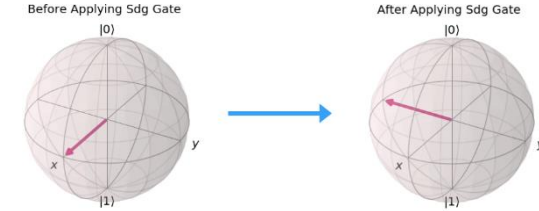
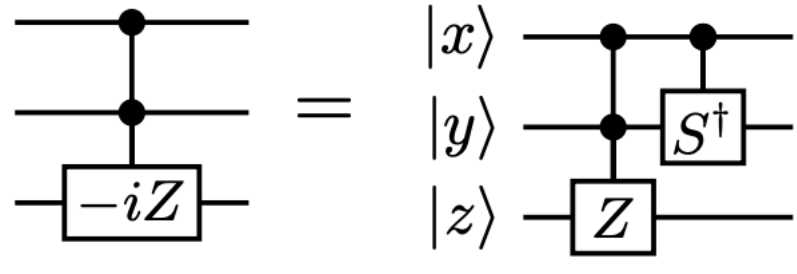
T-depth: 1

total depth: 7



### 3. An application to multiply-controlled gates

doubly-controlled  $(-iZ)$ -gate



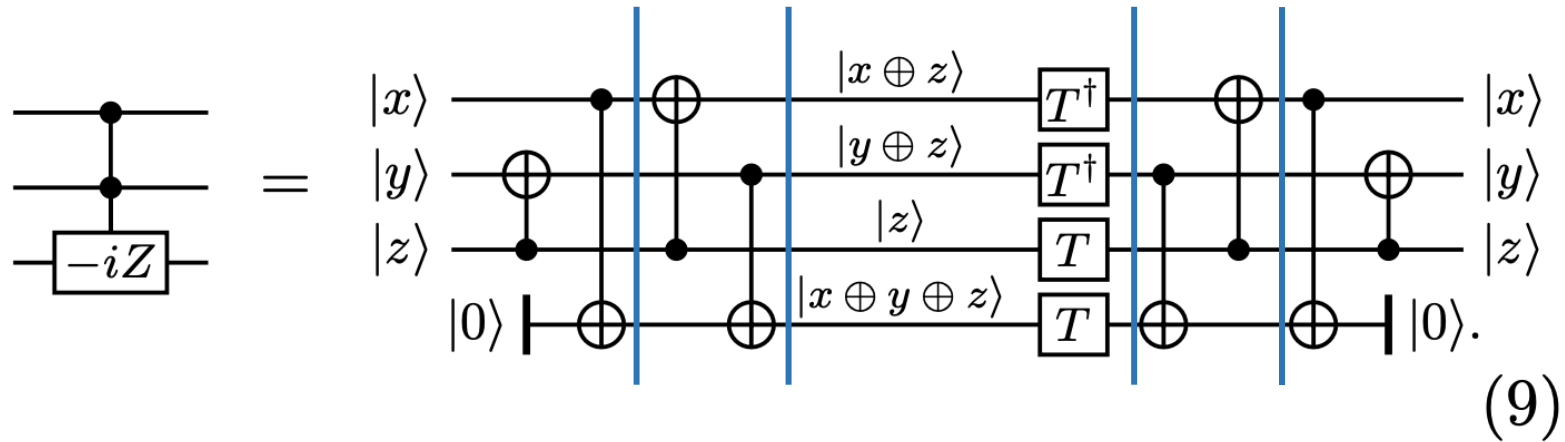
(7)

- doubly-controlled  $(-iZ)$ -gate는 doubly-controlled Z-gate와 controlled  $S^\dagger$ -gate의 조합으로 구성됨.

$$\begin{cases} (-1)^{xyz} = \omega^x \omega^y \omega^z (\omega^\dagger)^{x \oplus y} (\omega^\dagger)^{y \oplus z} (\omega^\dagger)^{x \oplus z} \omega^{x \oplus y \oplus z} \\ (-i)^{xy} = (\omega^\dagger)^x (\omega^\dagger)^y \omega^{x \oplus y} \end{cases}$$

$$(-1)^{xyz} (-i)^{xy} = \omega^z (\omega^\dagger)^{y \oplus z} (\omega^\dagger)^{x \oplus z} \omega^{x \oplus y \oplus z}, \quad (8)$$

## 3-1. doubly-controlled $(-iZ)$ -gate (1)



T-count: 4

T-depth: 1

the number of gates: 12

the number of ancillas: 1

total depth: 5

```
Gate class counts:
  AllocateQubitGate : 4
  CXGate : 8
  DaggeredGate : 2
  DeallocateQubitGate : 4
  TGate : 2
```

Gate counts:

Allocate : 4

CX : 8

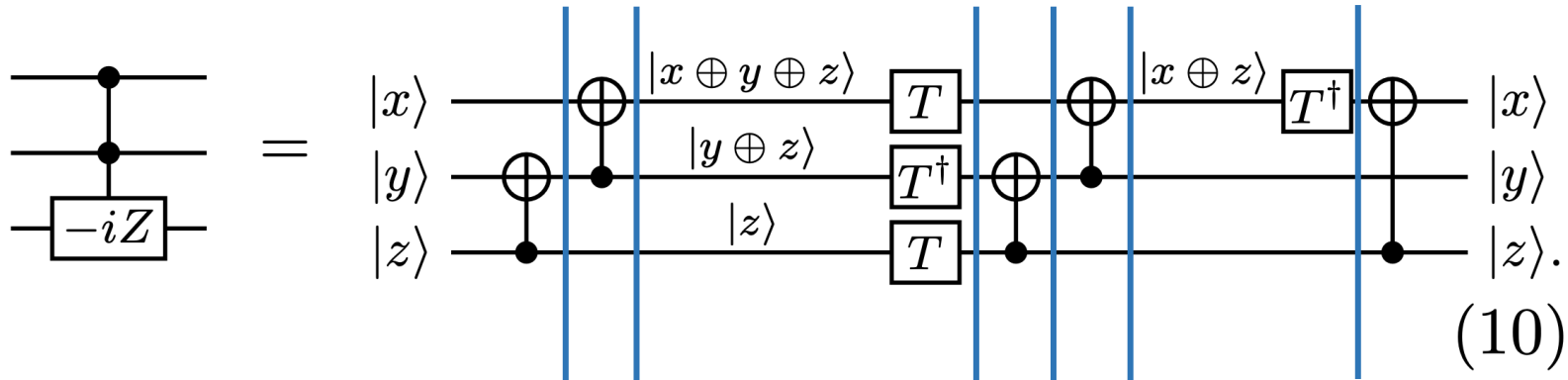
Deallocate : 4

T : 2

$T^\dagger$  : 2

Depth : 5.

## 3-2. doubly-controlled $(-iZ)$ -gate (2)



T-count: 4

T-depth: 2

the number of gates: 9

the number of ancillas: 0

total depth: 7

Gate class counts:

AllocateQubitGate : 3

CXGate : 5

DaggeredGate : 2

DeallocateQubitGate : 3

TGate : 2

Gate counts:

Allocate : 3

CX : 5

Deallocate : 3

T : 2

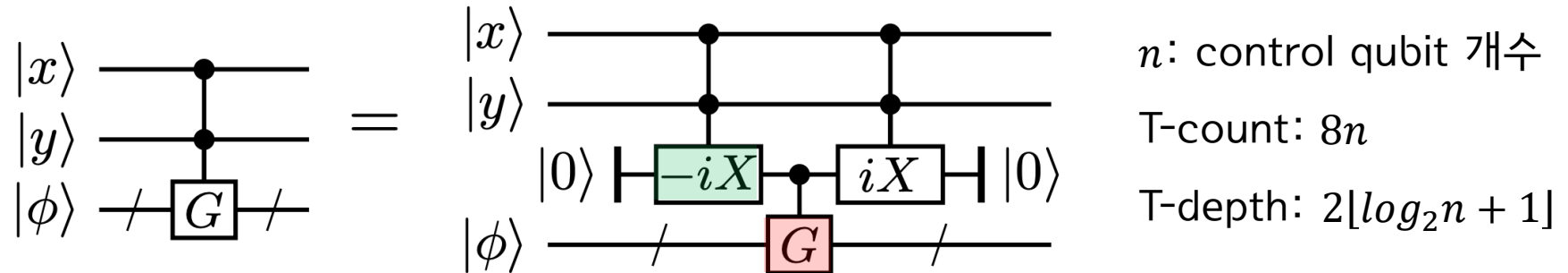
$T^\dagger$  : 2

Depth : 7.

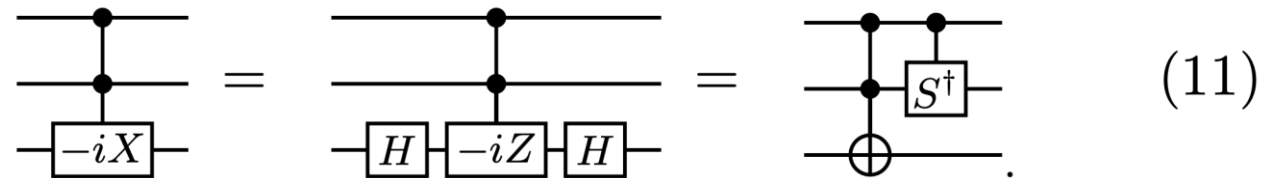
### 3-3. doubly-controlled $(-iX)$ -gate, doubly-controlled $G$ -gate

controlled quantum gate  $G$  의 Clifford+T 표현이 있다고 가정하였을 때,

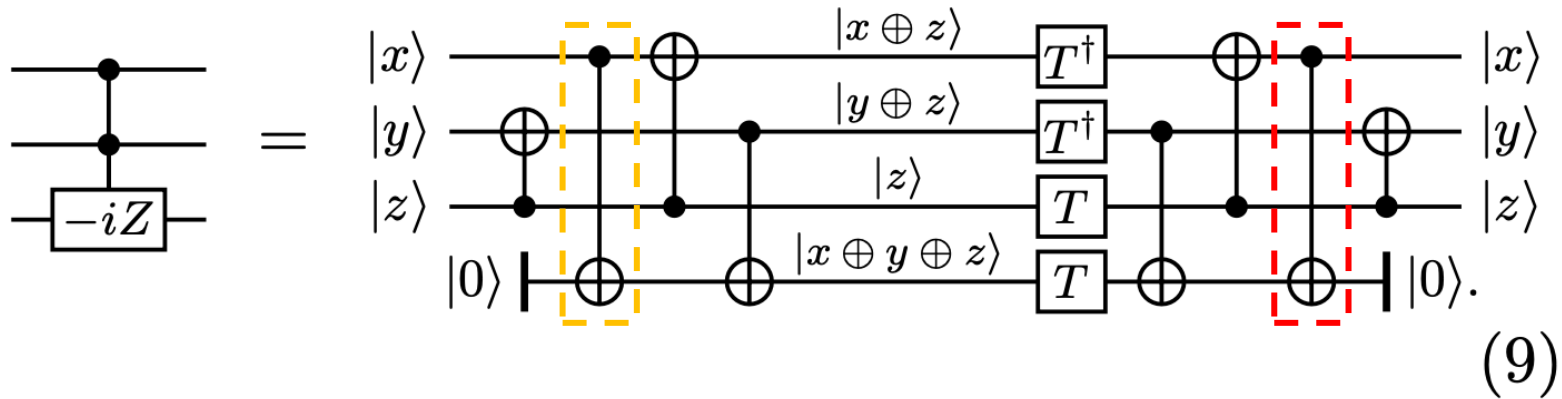
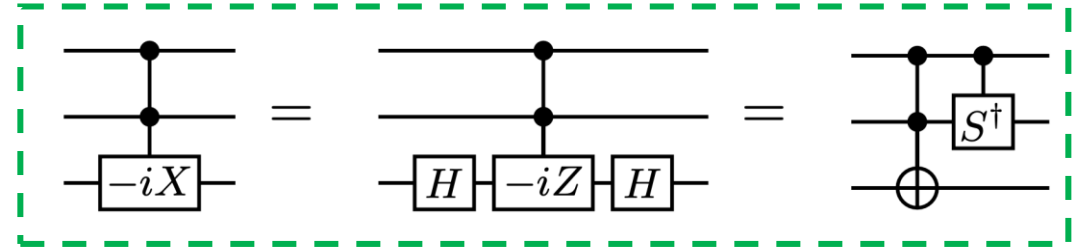
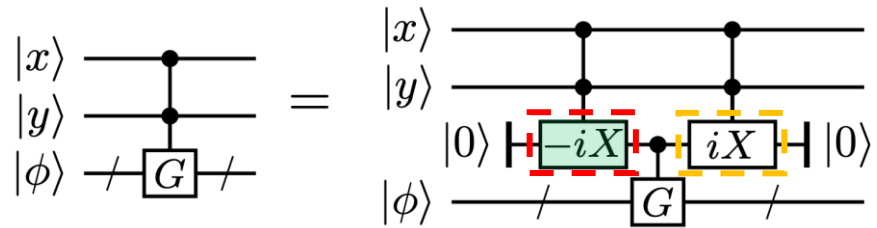
doubly-controlled  $G$ -gate



doubly-controlled  $(-iX)$ -gate

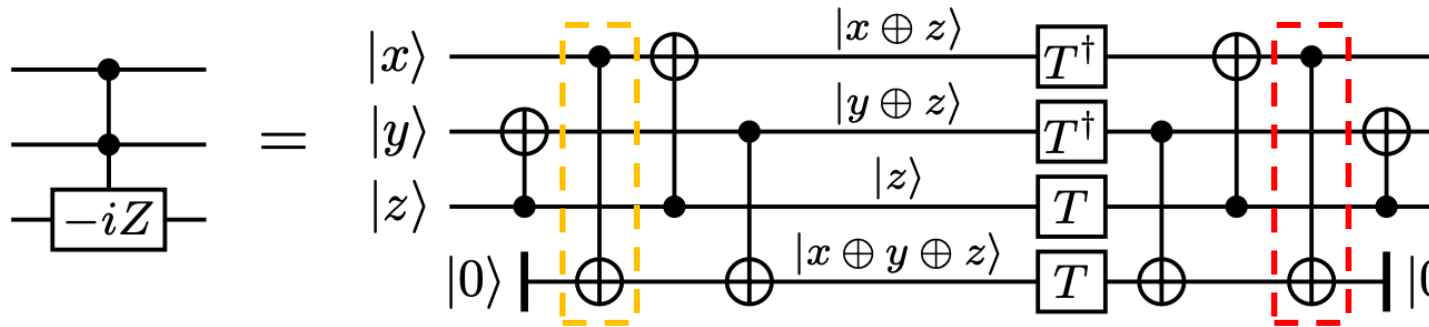
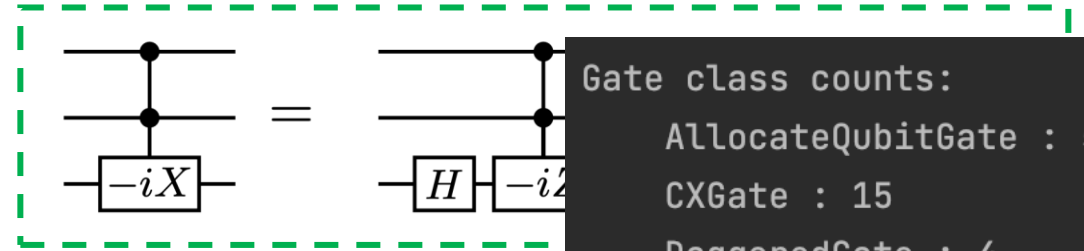
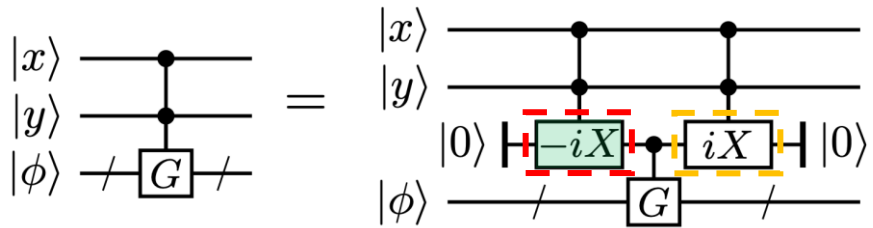


## 3-4. doubly-controlled $G$ -gate (1)



- ( (9) 의 게이트 12 + H 게이트 2 ) x 2 = 28
- $|x\rangle$  상태에 있는 ancilla를 clean하지 않고 유지시킴으로써 CNOT 게이트 2개 절약 (28 -> 26)
- T-depth: 2, total depth: 15
- G gate를 계산하는 동안 ancilla를 그대로 가져가야 하기 때문에 trade-off가 존재함.

## 3-4. doubly-controlled $G$ -gate (1)



- ( (9) 의 게이트 12 + H 게이트 2 ) x 2 = 28
- $|x\rangle$  상태에 있는 ancilla를 clean하지 않고 유지시킴으로써 CNOT 게이트 2개
- T-depth: 2, total depth: 15
- G gate를 계산하는 동안 ancilla를 그대로 가져가야 하기 때문에 trade-off가

Gate class counts:

AllocateQubitGate : 5

CXGate : 15

DaggeredGate : 4

DeallocateQubitGate : 5

HGate : 4

TGate : 4

Gate counts:

Allocate : 5

CX : 15

Deallocate : 5

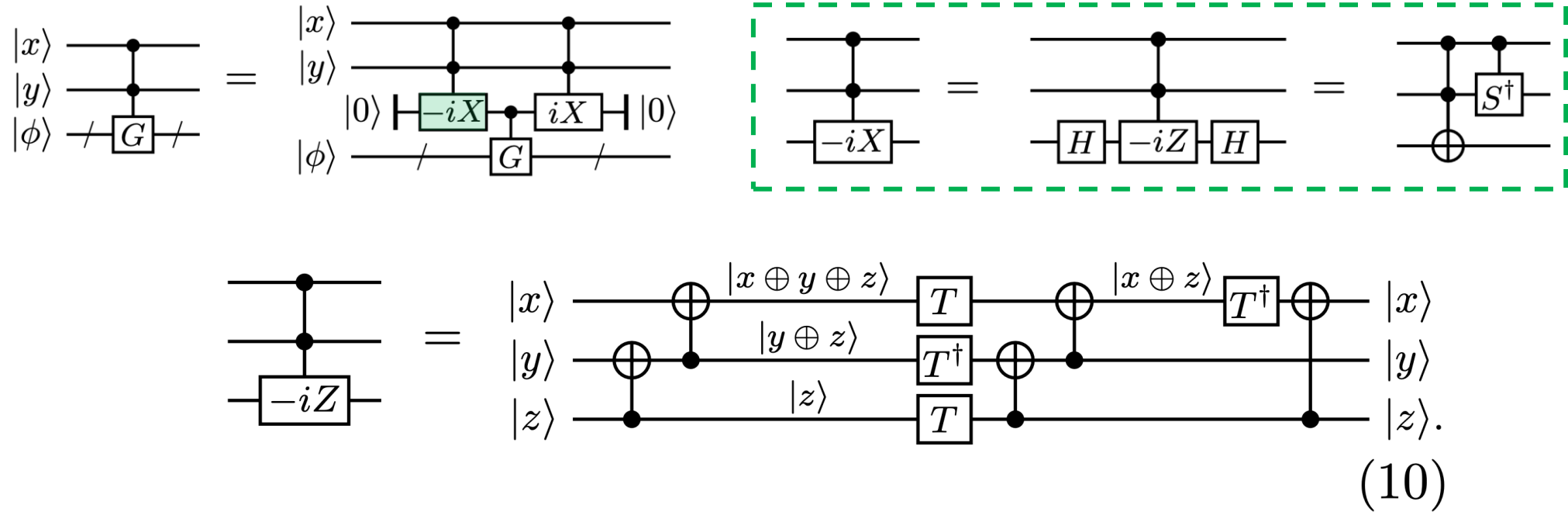
H : 4

T : 4

T<sup>†</sup> : 4

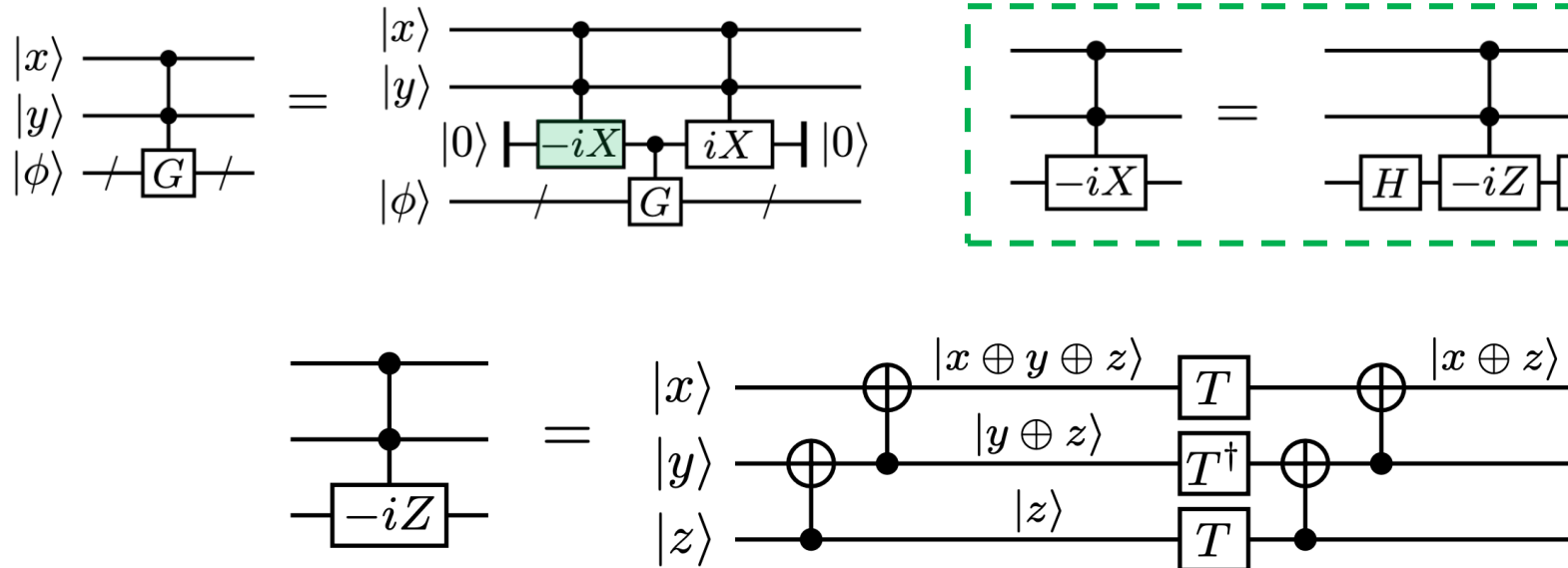
Depth : 15.

## 3-5. doubly-controlled $G$ -gate (2)



- ( (10) 의 게이트 9 + H 게이트 2 )  $\times$  2 = 22
- 1개의 ancilla qubit 사용됨.
- T-depth와 total depth가 각각 증가함. ( $2 \rightarrow 4$ ,  $15 \rightarrow 18$ )

## 3-5. doubly-controlled $G$ -gate (2)



- ( (10) 의 게이트 9 + H 게이트 2 )  $\times$  2 = 22
- 1개의 ancilla qubit 사용됨.
- T-depth와 total depth가 각각 증가함. ( $2 \rightarrow 4$ ,  $15 \rightarrow 18$ )

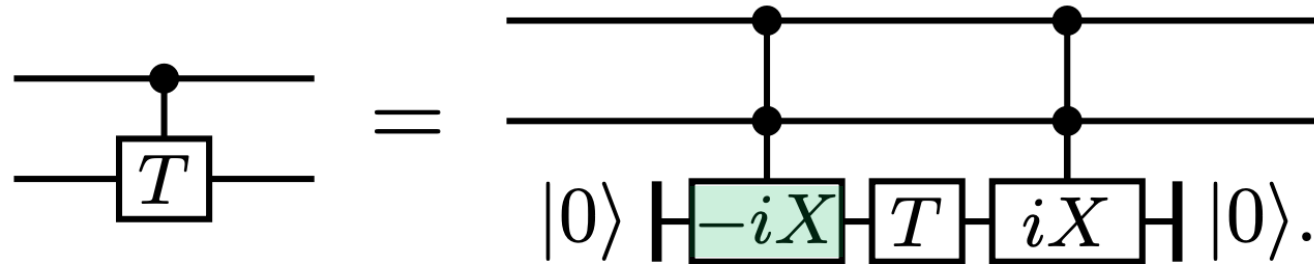
```
Gate class counts:
  AllocateQubitGate : 4
  CXGate : 11
  DaggeredGate : 4
  DeallocateQubitGate : 4
  HGate : 4
  TGate : 4
```

```
Gate counts:
  Allocate : 4
  CX : 11
  Deallocate : 4
  H : 4
  T : 4
  T^\dagger : 4
```

```
Depth : 19.
```



## 3-6. controlled $T$ -gate



```
Gate class counts:
  AllocateQubitGate : 4
  CXGate : 14
  DaggeredGate : 4
  DeallocateQubitGate : 4
  HGate : 4
  TGate : 5

Gate counts:
  Allocate : 4
  CX : 14
  Deallocate : 4
  H : 4
  T : 5
  T^\dagger : 4

Depth : 15.
```

<(9) 회로 사용>

T-count: 9

T-depth: 3

the number of gates: 29 -> 27

total depth: 15

the number of ancillas: 2

<(10) 회로 사용>

T-count: 9

T-depth: 5

the number of gates: ~~27~~ 23

total depth: 19

the number of ancillas: 1

```
Gate class counts:
  AllocateQubitGate : 3
  CXGate : 10
  DaggeredGate : 4
  DeallocateQubitGate : 3
  HGate : 4
  TGate : 5

Gate counts:
  Allocate : 3
  CX : 10
  Deallocate : 3
  H : 4
  T : 5
  T^\dagger : 4

Depth : 19.
```

## 4. Conclusion

- 본 논문에서는 ancillas를 충분히 사용하여 T-depth를 1로 줄일 수 있는 회로를 찾아 제시하였음.
- 일부 회로는 T-depth를 1로 만들 수 없음을 보였음.
  - 2개의 T-stage가 *almost classical*\* gate에 의해서만 분리가 된다면 단일 T-stage로 결합이 가능함.
  - 그러나, 단일큐비트 연산자 THT는  $|0\rangle$ 로 초기화된 임의의 ancillas를 사용하여 T-depth 1인 Clifford+T 회로를 구현할 수 없음.
  - 계산이 끝날 때, ancilla가 초기상태로 돌아갈 필요가 없는 경우에도 구현 불가능.
- Clifford+T 회로의 최소 T-depth와 T-count를 결정하는 방법은 아직 열린 문제로 남아있음.

\*classical operator x diagonal operator 형태로 표현할 수 있는 연산자를 almost classical 하다고 말함.

감사합니다