

# ARIA 양자회로 구현

<https://youtu.be/Pcl97yxT-IU>

# S-box

$$S_1(\alpha) := \mathbf{A}.\alpha^{-1} + \mathbf{a}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$S_1^{-1}(\alpha) := (\mathbf{A}^{-1} . (\alpha + \mathbf{a}))^{-1}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

AES와 동일

$$S_2(\alpha) := \mathbf{B}.\alpha^{247} + \mathbf{b}$$

$$\begin{aligned} S_2(\alpha) &:= \mathbf{B} . (\alpha^{-1})^8 + \mathbf{b} = \mathbf{B} . \mathbf{C} . \alpha^{-1} + \mathbf{b} \\ &= \mathbf{D} . \alpha^{-1} + \mathbf{b} \end{aligned}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$S_2^{-1}(\alpha) = (\mathbf{D}^{-1} . (\alpha + \mathbf{b}))^{-1}$$

$$\mathbf{D}^{-1} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

# S-box

- S-box ( $S_1$ )
  - Boyar and Peralta
    - Bit-slicing 기법을 AES S-box에 적용
    - $S(x) = A \cdot x^{-1} + [11000110]^T = B \cdot F(U \cdot x) + [11000110]^T$
    - 3단계로 구성 → **Top linear Layer ( $U$ ), a middle non-linear Layer, bottom linear layer ( $B$ )**

|                                |                                       |                                       |                                 |                                 |
|--------------------------------|---------------------------------------|---------------------------------------|---------------------------------|---------------------------------|
| Top Linear Part:               |                                       |                                       |                                 |                                 |
| $T_1 = U_0 + U_3$              | $T_2 = U_0 + U_5$                     | $T_3 = U_0 + U_6$                     | $T_4 = U_3 + U_5$               | $T_5 = U_4 + U_6$               |
| $T_6 = T_1 + T_5$              | $T_7 = U_1 + U_2$                     | $T_8 = U_7 + T_6$                     | $T_9 = U_7 + T_7$               | $T_{10} = T_6 + T_7$            |
| $T_{11} = U_1 + U_5$           | $T_{12} = U_2 + U_5$                  | $T_{13} = T_3 + T_4$                  | $T_{14} = T_6 + T_{11}$         | $T_{15} = T_5 + T_{11}$         |
| $T_{16} = T_5 + T_{12}$        | $T_{17} = T_9 + T_{16}$               | $T_{18} = U_3 + U_7$                  | $T_{19} = T_7 + T_{18}$         | $T_{20} = T_1 + T_{19}$         |
| $T_{21} = U_6 + U_7$           | $T_{22} = T_7 + T_{21}$               | $T_{23} = T_2 + T_{22}$               | $T_{24} = T_2 + T_{10}$         | $T_{25} = T_{20} + T_{17}$      |
| $T_{26} = T_3 + T_{16}$        | $T_{27} = T_1 + T_{12}$               |                                       |                                 |                                 |
| Nonlinear Part:                |                                       |                                       |                                 |                                 |
| $M_1 = T_{13} \cdot T_6$       | $M_2 = T_{23} \cdot T_8$              | $M_3 = T_{14} + M_1$                  | $M_4 = T_{19} \cdot U_7$        | $M_5 = M_4 + M_1$               |
| $M_6 = T_3 \cdot T_{16}$       | $M_7 = T_{22} \cdot T_9$              | $M_8 = T_{26} + M_6$                  | $M_9 = T_{20} \cdot T_{17}$     | $M_{10} = M_9 + M_6$            |
| $M_{11} = T_1 \cdot T_{15}$    | $M_{12} = T_4 \cdot T_{27}$           | $M_{13} = M_{12} + M_{11}$            | $M_{14} = T_2 \cdot T_{10}$     | $M_{15} = M_{14} + M_{11}$      |
| $M_{16} = M_3 + M_2$           | $M_{17} = M_5 + T_{24}$               | $M_{18} = M_8 + M_7$                  | $M_{19} = M_{10} + M_{15}$      | $M_{20} = M_{16} + M_{13}$      |
| $M_{21} = M_{17} + M_{15}$     | $M_{22} = M_{18} + M_{13}$            | $M_{23} = M_{19} + T_{25}$            | $M_{24} = M_{22} + M_{23}$      | $M_{25} = M_{22} \cdot M_{20}$  |
| $M_{26} = M_{21} + M_{25}$     | $M_{27} = M_{20} + M_{21}$            | $M_{28} = M_{23} + M_{25}$            | $M_{29} = M_{28} \cdot M_{27}$  | $M_{30} = M_{26} \cdot M_{24}$  |
| $M_{31} = M_{20} \cdot M_{23}$ | $M_{32} = M_{27} \cdot M_{31}$        | $M_{33} = M_{27} + M_{25}$            | $M_{34} = M_{21} \cdot M_{22}$  | $M_{35} = M_{24} \cdot M_{34}$  |
| $M_{36} = M_{24} + M_{25}$     | $M_{37} = M_{21} + M_{29}$            | $M_{38} = M_{32} + M_{33}$            | $M_{39} = M_{23} + M_{30}$      | $M_{40} = M_{35} + M_{36}$      |
| $M_{41} = M_{38} + M_{40}$     | $M_{42} = M_{37} + M_{39}$            | $M_{43} = M_{37} + M_{38}$            | $M_{44} = M_{39} + M_{40}$      | $M_{45} = M_{42} + M_{41}$      |
| $M_{46} = M_{44} \cdot T_6$    | $M_{47} = M_{40} \cdot T_8$           | $M_{48} = M_{39} \cdot U_7$           | $M_{49} = M_{43} \cdot T_{16}$  | $M_{50} = M_{38} \cdot T_9$     |
| $M_{51} = M_{37} \cdot T_{17}$ | $M_{52} = M_{42} \cdot T_{15}$        | $M_{53} = M_{45} \cdot T_{27}$        | $M_{54} = M_{41} \cdot T_{10}$  | $M_{55} = M_{44} \cdot T_{13}$  |
| $M_{56} = M_{40} \cdot T_{23}$ | $M_{57} = M_{39} \cdot T_{19}$        | $M_{58} = M_{43} \cdot T_3$           | $M_{59} = M_{38} \cdot T_{22}$  | $M_{60} = M_{37} \cdot T_{20}$  |
| $M_{61} = M_{42} \cdot T_1$    | $M_{62} = M_{45} \cdot T_4$           | $M_{63} = M_{41} \cdot T_2$           |                                 |                                 |
| Bottom Linear Part:            |                                       |                                       |                                 |                                 |
| $L_0 = M_{61} \oplus M_{62}$   | $L_1 = M_{50} \oplus M_{56}$          | $L_2 = M_{46} \oplus M_{48}$          | $L_3 = M_{47} \oplus M_{55}$    | $L_4 = M_{54} \oplus M_{58}$    |
| $L_5 = M_{49} \oplus M_{61}$   | $L_6 = M_{62} \oplus L_5$             | $L_7 = M_{46} \oplus L_3$             | $L_8 = M_{51} \oplus M_{59}$    | $L_9 = M_{52} \oplus M_{53}$    |
| $L_{10} = M_{53} \oplus L_4$   | $L_{11} = M_{60} \oplus L_2$          | $L_{12} = M_{48} \oplus M_{51}$       | $L_{13} = M_{50} \oplus L_0$    | $L_{14} = M_{52} \oplus M_{61}$ |
| $L_{15} = M_{55} \oplus L_1$   | $L_{16} = M_{56} \oplus L_0$          | $L_{17} = M_{57} \oplus L_1$          | $L_{18} = M_{58} \oplus L_8$    | $L_{19} = M_{63} \oplus L_4$    |
| $L_{20} = L_0 \oplus L_1$      | $L_{21} = L_1 \oplus L_7$             | $L_{22} = L_3 \oplus L_{12}$          | $L_{23} = L_{18} \oplus L_2$    | $L_{24} = L_{15} \oplus L_9$    |
| $L_{25} = L_6 \oplus L_{10}$   | $L_{26} = L_7 \oplus L_9$             | $L_{27} = L_8 \oplus L_{10}$          | $L_{28} = L_{11} \oplus L_{14}$ | $L_{29} = L_{11} \oplus L_{17}$ |
| $S_0 = L_6 \oplus L_{24}$      | $S_1 = L_{16} \oplus L_{26} \oplus 1$ | $S_2 = L_{19} \oplus L_{28} \oplus 1$ | $S_3 = L_6 \oplus L_{21}$       | $S_4 = L_{20} \oplus L_{22}$    |
| $S_5 = L_{25} \oplus L_{29}$   | $S_6 = L_{13} \oplus L_{27} \oplus 1$ | $S_7 = L_6 \oplus L_{23} \oplus 1$    |                                 |                                 |

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

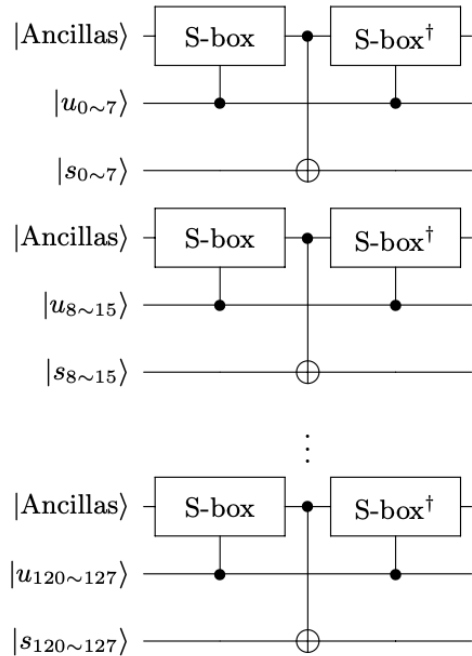
$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

# S-box

- S. Jaques, M. Naehrig, M. Roetteler, and F. Virdia, “**Implementing Grover Oracles for quantum key search on AES and LowMC,**” in *Advances in Cryptology - EUROCRYPT 2020 - 39th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Zagreb, Croatia, May 10-14, 2020, Proceedings, Part II*, ser. Lecture Notes in Computer Science, A. Canteaut and Y. Ishai, Eds., vol. 12106. Springer, 2020, pp. 280–310. [Online]. Available: [https://doi.org/10.1007/978-3-030-45724-2\\_10](https://doi.org/10.1007/978-3-030-45724-2_10)
- K. Jang, A. Baksi, H. Kim, G. Song, H. Seo, and A. Chattopadhyay, “**Quantum analysis of AES,**” Cryptology ePrint Archive, Paper 2022/683, 2022, <https://eprint.iacr.org/2022/683>.
- B. Langenberg, H. Pham, and R. Steinwandt, “**Reducing the cost of implementing the advanced encryption standard as a quantum circuit,**” *IEEE Transactions on Quantum Engineering*, vol. 1, pp. 1–12, 01 2020.
- J. Zou, Z. Wei, S. Sun, X. Liu, and W. Wu, “**Quantum circuit implementations of aes with fewer qubits,**” in *Advances in Cryptology – ASIACRYPT 2020*, S. Moriai and H. Wang, Eds. Cham: Springer, International Publishing, 2020, pp. 697–726.
- Z. Huang and S. Sun, “**Synthesizing quantum circuits of AES with lower T-depth and less qubits,**” Cryptology ePrint Archive, Report 2022/620, 2022, <https://eprint.iacr.org/2022/620>.

# S-box

- S-box ( $S_1$ )
  - “Quantum analysis of AES” 논문에서 사용한 기법 적용
  - $S\text{-box}^\dagger$  사용 X
  - 매번 S-box에 ancilla qubits(68개) 할당



(b) Using multiple ancilla sets.

**Table 3:** Comparison of quantum implementations of AES S-box.

| Method           | #CNOT<br>✱ | #1qCliff<br>⚡ | #T<br>✚ | TD<br>⚡ | M<br>⚡ | Full depth<br>✱ |     |
|------------------|------------|---------------|---------|---------|--------|-----------------|-----|
| S-box [32]       | 1818       | 124           | 1792    | 88      | 40     | 951             |     |
| S-box [16]       | 358        | 68            | 224     | 8       | 123    | 104             |     |
| S-box [17] ✚     | 392        | 72            | 238     | 6       | 136    | 85              |     |
| S-box [49]       | 628        | 98            | 367     | 40      | 32     | 514             |     |
| S-box [77]       | 437        | 72            | 245     | 55      | 22     | 339             |     |
| S-box [21, 22] { | 391 lines  | 1470          | 670     | 1218    | 66     | 399             | 640 |
|                  | 406 lines  | 1507          | 548     | 1245    | 74     | 414             | 709 |
|                  | 413 lines  | 1484          | 561     | 1169    | 62     | 421             | 591 |
|                  | 409 lines  | 1483          | 574     | 1190    | 74     | 416             | 693 |
|                  | 400 lines  | 2244          | 1006    | 2254    | 111    | 408             | 998 |
| S-box [36] {     | 418        | 72            | 238     | 4       | 136    | 72              |     |
|                  | 824        | 160           | 546     | 3       | 198    | 69              |     |
| S-box [51]       | .          | .             | .       | 32      | 20     | .               |     |
| S-box [52] {     | .          | .             | .       | 24      | 21     | .               |     |
|                  | .          | .             | .       | 22      | 22     | .               |     |
| S-box [54]       | 372        | 72            | 238     | 4       | 90     | 69              |     |
|                  | 418        | 72            | 238     | 4       | 136    | 61              |     |
| S-box            | ✚          | 366           | 72      | 238     | 4      | 84              | 58  |
|                  | ⚡          | 781           | 160     | 546     | 3      | 152             | 56  |

✚: Reused in this work to fix [44] ✚.

✚: Used in this work (Toffoli depth 4).

✚: Used in this work (Toffoli depth 3).

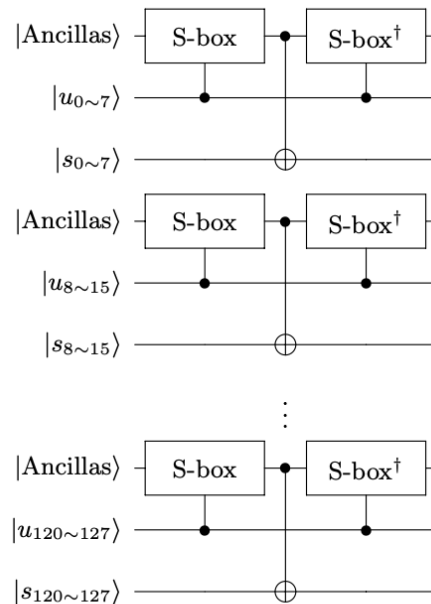
# S-box

- S-box ( $S_1$ )

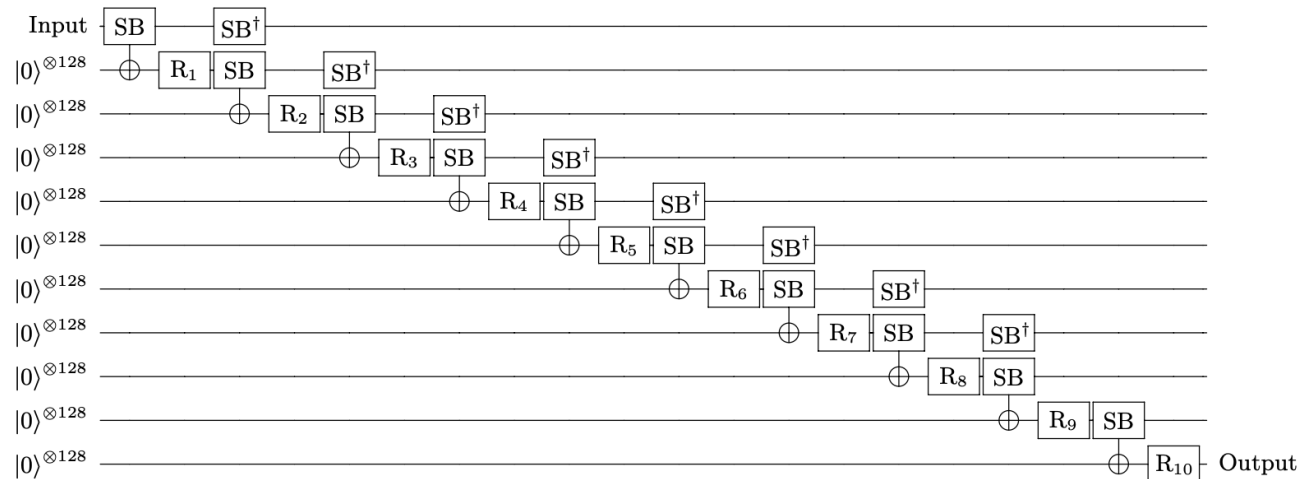
- “Quantum analysis of AES” 논문에서 사용한 기법 적용
- $S\text{-box}^\dagger$  사용  $x$ , 매번 S-box에 ancilla qubits(68개) 할당  
 → 이전 연구에 비해 큐비트 측면에서도 최적화  
 →  $S\text{-box}^\dagger$  비용이 들지 않는 pipeline 구조로 구현하기 복잡

| Method        | Source      | #CNOT | #X | #Toffoli | Toffoli depth | #Qubit | depth |
|---------------|-------------|-------|----|----------|---------------|--------|-------|
|               | [11]        | 569   | 4  | 448      | 196           | 40     | -     |
| Itoh-Tsujii   | [13]        | 1114  | 4  | 108      | 4             | 162    | 151   |
|               | <b>Ours</b> | 1106  | 4  | 108      | 4             | 170    | 137   |
| Boyar-Peralta | <b>Ours</b> | 162   | 4  | 34       | 4             | 84     | 33    |

38개의 ancilla qubit 재사용  
 garbage = 재사용 큐비트, Input, output 뿐 나머지 108개



(b) Using multiple ancilla sets.



(b) Shallow and shallow/low depth versions (Ours).

# S-box

- $S\text{-box}^{-1} (S_1^{-1})$ 
  - “Quantum analysis of AES” + “Synthesizing quantum circuits of AES with lower T-depth and less qubits”
  - “Synthesizing quantum circuits of AES with lower T-depth and less qubits” 해당 논문에서  $S\text{-box}^{-1}$  사용
  - $S\text{-box}^{-1}$  내의 S-box는 “Quantum analysis of AES” 기법 사용
  - $S\text{-box} = LS_0(x) + c = B \cdot F(U \cdot x) + [11000110]^T$ ,  
( $L = \text{linear function}, S_0(x) = \text{inversion}$ )
  - $x = S_0^{-1}L^{-1}(y + c) = S_0L^{-1}(y + c) = L^{-1}(LS_0)L^{-1}(y + c)$

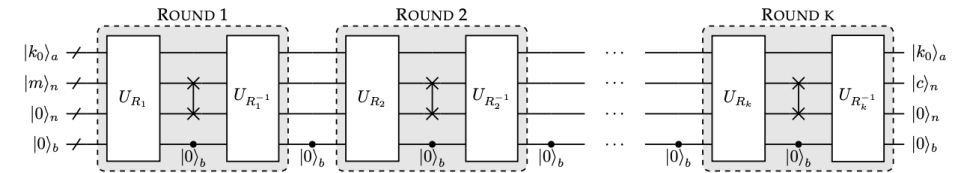


Fig. 5. The OP-based round-in-place structure

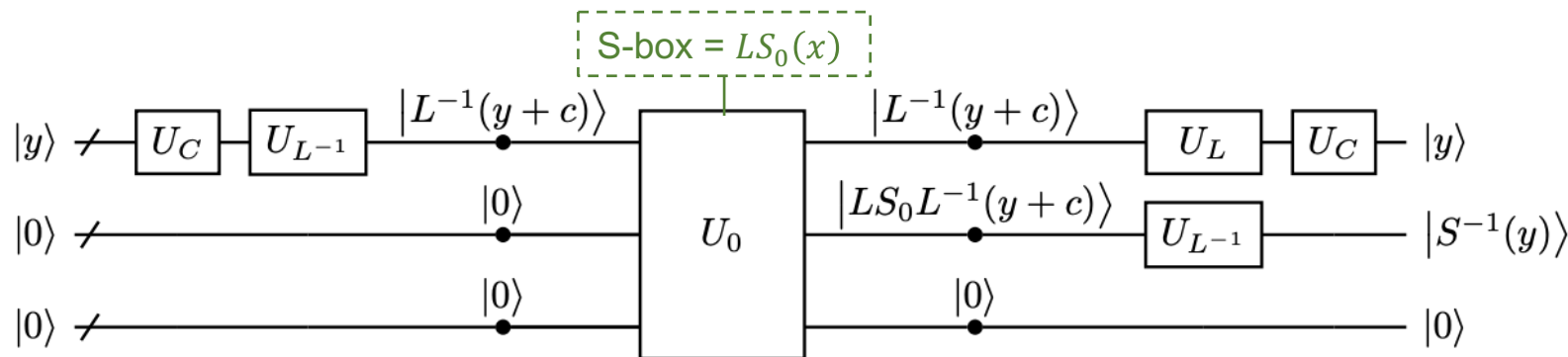


Fig. 15. The circuit for implementing the  $S\text{-box}^{-1}$  of AES

# S-box

- S-box ( $S_2$ )
  - Itoh-Tsujii algorithm
    - 곱셈과 제곱으로 이루어진 연산

$$\alpha^{-1} = \alpha^{254} = ((\alpha.\alpha^2).(\alpha.\alpha^2)^4.(\alpha.\alpha^2)^{16}.\alpha^{64})^2$$

- Squaring (제곱기)

- **XZLBZ** 사용

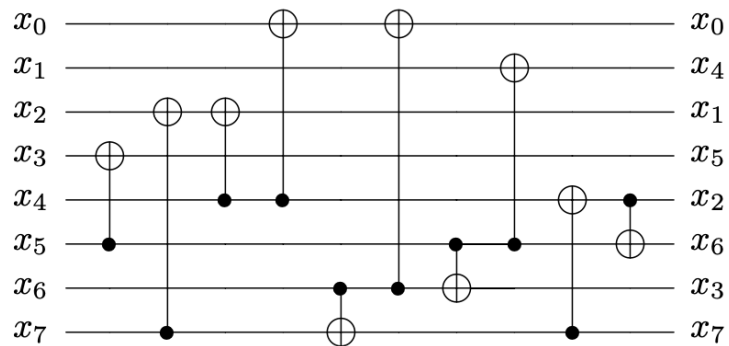


Fig. 4: Squaring in  $\mathbb{F}_{2^8}/(x^8 + x^4 + x^3 + x + 1)$  using XZLBZ

CNOT gate: 10

Depth : 7

- **PLU** 사용

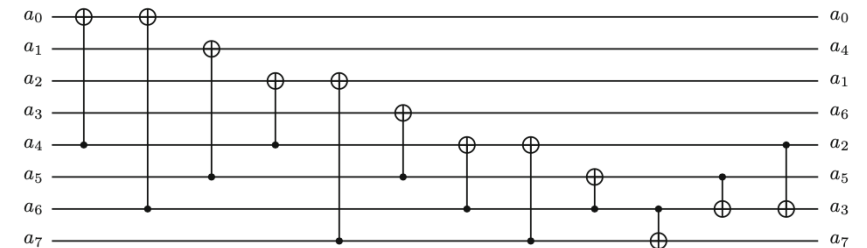


Fig. 1. Circuit for squaring in  $\mathbb{F}_2[x]/(x^8 + x^4 + x^3 + x + 1)$ .

CNOT gate: 12

Depth : 7



# S-box

- S-box ( $S_2$ )
  - Multiplication (곱셈기)
    - Karatsuba Multiplication (Jang.et.al)
      - 카라추바 알고리즘을 재귀적으로 사용하여 Toffoli depth가 1인 곱셈 (81개 중 38개의 ancilla qubit 재사용)

Table 1: Quantum resources required for multiplication.

| Source   | #Clifford | #T  | Toffoli depth | Full depth |
|----------|-----------|-----|---------------|------------|
| CMMP [2] | 435       | 448 | 28            | 195        |
| J++ [11] | 390       | 189 | 1             | 28         |

※: The multiplication size  $n$  is 8.

- Affine function
  - 결과 큐비트를 할당하여 **out-of-place** 연산

$$\begin{aligned} S_2(\alpha) &:= \mathbf{B} \cdot (\alpha^{-1})^8 + \mathbf{b} = \mathbf{B} \cdot \mathbf{C} \cdot \alpha^{-1} + \mathbf{b} \\ &= \mathbf{D} \cdot \alpha^{-1} + \mathbf{b} \end{aligned}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

# S-box

- S-box 양자 자원 비교
  - $S_1 \leftarrow$  Boyar-Peralta,  $S_2 \leftarrow$  Itoh-Tsujii
  - 다만, 비교를 위해 Itoh-Tsujii 기법을  $S_1$ 에 적용하여 비교

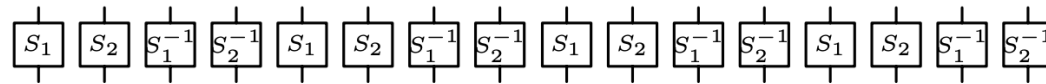
| Method        | Source      | #CNOT | #X | #Toffoli | Toffoli depth | #Qubit | depth |
|---------------|-------------|-------|----|----------|---------------|--------|-------|
| Itoh-Tsujii   | [11]        | 569   | 4  | 448      | 196           | 40     | -     |
|               | [13]        | 1114  | 4  | 108      | 4             | 162    | 151   |
|               | <b>Ours</b> | 1106  | 4  | 108      | 4             | 170    | 137   |
| Boyar-Peralta | <b>Ours</b> | 162   | 4  | 34       | 4             | 84     | 33    |

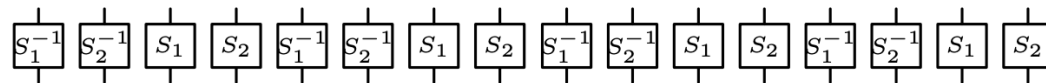
|               |           |      |   |     |   |     |     |
|---------------|-----------|------|---|-----|---|-----|-----|
| Itoh-Tsujii   | XZLBZ     | 1080 | 4 | 108 | 4 | 162 | 141 |
| Boyar-Peralta | Inversion | 190  | 4 | 34  | 4 | 84  | 55  |

# Substitution Layer

- Substitution Layer
  - 총 16개의 S-box가 병렬적으로 사용 → 순차 연산에 비해 depth 감소
  - [13] → 초기에 608 (38 x 16) ancilla qubit (재사용 가능) 할당
  - 초기에 **304 (38 x 8)** ancilla qubit (재사용 가능) 할당
    - $S_2, S_2^{-1}$ 에만 필요
  - $S_1$ 에 적용한 기술(Boyar-Peralta)은 큐비트 수는 감소하지만  
**병렬처리로 인해 depth 측면에서는 이득이 없음**
    - $S_2$ 의 depth가  $S_1$ 에 비해 높아 **depth가  $S_2$ 에 의해 측정**



(a) S-box layer type 1



(b) S-box layer type 2

# Diffusion Layer

- Diffusion Layer

$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \\ y_{14} \\ y_{15} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \end{pmatrix}$$

- 16 x 16 이진 행렬 곱셈
- 128 개의 결과 큐비트를 매 라운드마다 할당하여 **out-of-place** 연산 → **depth 최적화**

---

**Algorithm 1:** Quantum circuit implementation of ARIA Diffusion Layer using out-of-place.

---

**Input:**  $x, M$

**Output:**  $result$

```

0: Allocate result qubit →  $result[16][8]$ 
0: for  $0 \leq i \leq 16$  do
0:   for  $0 \leq j \leq 16$  do
0:     if  $M[16 + j] == 1$  then
0:       CNOT8bit( $x, j, result, i$ )
0: return  $result == 0$ 

```

---

| Method              | #CNOT | #Qubit | depth    |
|---------------------|-------|--------|----------|
| PLU                 | 768   | 128    | 31       |
| XZLBZ               | 376   | 128    | 17       |
| <b>Out-of-place</b> | 896   | 256    | <b>7</b> |

# Quantum resource estimation

- ARIA 양자 자원 추정
  - [11]에 비해 **Depth** 측면에서 최적화
  - [13]에 비해 **Depth, Qubit** 측면에서 모두 최적화
- ※ [13]에서 잘못된 추정 결과 발견

NCT Level

| Cipher   | Source    | #X    | #CNOT   | #Toffoli | Toffoli depth | #Qubit | Depth        |
|----------|-----------|-------|---------|----------|---------------|--------|--------------|
| ARIA-128 | [11]      | 1,595 | 231,124 | 157,696  | 4,312         | 1,560  | 9,260        |
|          | [13]      | 1,408 | 285,784 | 25,920   | 60            | 29,216 | 3,500        |
|          | This work | 1,408 | 173,652 | 17,040   | 60            | 26,864 | <b>2,187</b> |
| ARIA-192 | [11]      | 1,851 | 273,264 | 183,368  | 5,096         | 1,560  | 10,948       |
|          | [13]      | 1,624 | 324,136 | 29,376   | 68            | 32,928 | 3,978        |
|          | This work | 1,624 | 197,036 | 19,312   | 68            | 30,320 | <b>2,480</b> |
| ARIA-256 | [11]      | 2,171 | 325,352 | 222,208  | 6,076         | 1,688  | 13,054       |
|          | [13]      | 1,856 | 362,488 | 32,832   | 76            | 36,640 | 4,455        |
|          | This work | 1,856 | 220,420 | 21,584   | 76            | 33,776 | <b>2,772</b> |

Clifford + T Level

| Cipher   | Source    | #Clifford | #T        | T-depth | #Qubit | Full depth   |
|----------|-----------|-----------|-----------|---------|--------|--------------|
| ARIA-128 | [11]      | 1,494,287 | 1,103,872 | 17,248  | 1,560  | 37,882       |
|          | [13]      | 494,552   | 181,440   | 240     | 29,216 | 4,650        |
|          | This work | 311,380   | 119,280   | 240     | 26,864 | <b>2,952</b> |
| ARIA-192 | [11]      | 1,742,059 | 1,283,576 | 20,376  | 1,560  | 44,774       |
|          | [13]      | 560,768   | 205,632   | 272     | 32,928 | 5,285        |
|          | This work | 353,156   | 135,184   | 272     | 30,320 | <b>3,347</b> |
| ARIA-256 | [11]      | 2,105,187 | 1,555,456 | 24,304  | 1,688  | 51,666       |
|          | [13]      | 627,000   | 229,824   | 304     | 36,640 | 5,919        |
|          | This work | 394,948   | 151,088   | 304     | 33,776 | <b>3,741</b> |

[11] A. K. Chauhan and S. K. Sanadhya, "Quantum resource estimates of grover's key search on aria," in *Security, Privacy, and Applied Cryptography Engineering: 10th International Conference, SPACE 2020, Kolkata, India, December 17–21, 2020, Proceedings 10*. Springer, 2020, pp. 238–258.

[13] Y. Yang, K. Jang, Y. Oh, and H. Seo, "Depth-optimized quantum implementation of aria," *Cryptology ePrint Archive*, 2023.

# Quantum resource estimation

- ARIA 양자 자원 추정 (추가, 논문 x)
    - [11]에 비해 **Depth** 측면에서 최적화
    - [13]에 비해 **Depth, Qubit** 측면에서 모두 최적화
- ※ [13]에서 잘못된 추정 결과 발견

|                     | CNOT      | 1qClifford | T         | T-depth | Qubit  | Full depth    |
|---------------------|-----------|------------|-----------|---------|--------|---------------|
| [11]                | 1,494,287 |            | 1,103,872 | 17,248  | 1,560  | 37,882        |
| e_print [13]        | 441,560   | 53,248     | 181,440   | 240     | 29,216 | 4,650 (3,545) |
| ICISC               | 427,912   | 53,248     | 181,440   | 240     | 29,216 | 4,241 (3,158) |
| S-box만 변환           | 266,152   | 35,488     | 119,280   | 240     | 24,112 | 3,158         |
| DL 변환(out-of_place) | 273,432   | 35,488     | 119,280   | 240     | 25,904 | 3,028         |
| This work           | 275,892   | 35,488     | 119,280   | 240     | 26,864 | 2,952         |

[11] A. K. Chauhan and S. K. Sanadhya, "Quantum resource estimates of grover's key search on aria," in *Security, Privacy, and Applied Cryptography Engineering: 10th International Conference, SPACE 2020, Kolkata, India, December 17–21, 2020, Proceedings 10*. Springer, 2020, pp. 238–258.

[13] Y. Yang, K. Jang, Y. Oh, and H. Seo, "Depth-optimized quantum implementation of aria," *Cryptology ePrint Archive*, 2023.

# Grover's key search

- ARIA Grover 공격 비용 추정
  - Grover 공격 최적 iteration  $\lceil \frac{\pi}{4} \sqrt{2^k} \rceil$
  - Oracle에는 2개의 회로 필요  $\rightarrow 2 \times \lceil \frac{\pi}{4} \sqrt{2^k} \rceil \times \text{quantum resources}$
  - $r = \lceil \text{key size} / \text{block size} \rceil$ 개의 평문-암호문 쌍을 얻는 것이 고유한 키를 식별할 수 있음.

**$\rightarrow \text{Grover 공격 비용} : 2 \times r \times \lceil \frac{\pi}{4} \sqrt{2^k} \rceil \times \text{quantum resource}$**

- ARIA 는 NIST Level 1, 3, 5를 달성

| Cipher   | Source    | Total gates                             | Total depth                             | Cost<br>(complexity)                    | #Qubit        | NIST security |
|----------|-----------|---|---|---|---------------|---------------|
| ARIA-128 | [11]      | $1.998 \cdot 2^{85}$                    | $1.816 \cdot 2^{79}$                    | $1.814 \cdot 2^{165}$                   | 1,561         | Level 1       |
|          | [13]      | $1.117 \cdot 2^{84}$                    | $1.783 \cdot 2^{76}$                    | $1.991 \cdot 2^{160}$                   | 29,217        |               |
|          | This work | <b><math>1.296 \cdot 2^{83}</math></b>  | <b><math>1.132 \cdot 2^{76}</math></b>  | <b><math>1.468 \cdot 2^{159}</math></b> | <b>26,865</b> |               |
| ARIA-192 | [11]      | $1.146 \cdot 2^{119}$                   | $1.073 \cdot 2^{112}$                   | $1.23 \cdot 2^{231}$                    | 3,121         | Level 3       |
|          | [13]      | $1.2 \cdot 2^{117}$                     | $1.013 \cdot 2^{109}$                   | $1.216 \cdot 2^{226}$                   | 65,857        |               |
|          | This work | <b><math>1.469 \cdot 2^{116}</math></b> | <b><math>1.284 \cdot 2^{108}</math></b> | <b><math>1.886 \cdot 2^{224}</math></b> | <b>60,449</b> |               |
| ARIA-256 | [11]      | $1.384 \cdot 2^{151}$                   | $1.238 \cdot 2^{144}$                   | $1.714 \cdot 2^{295}$                   | 3,377         | Level 5       |
|          | [13]      | $1.336 \cdot 2^{149}$                   | $1.135 \cdot 2^{141}$                   | $1.516 \cdot 2^{290}$                   | 72,081        |               |
|          | This work | <b><math>1.642 \cdot 2^{148}</math></b> | <b><math>1.435 \cdot 2^{140}</math></b> | <b><math>1.178 \cdot 2^{289}</math></b> | <b>67,553</b> |               |

# Conclusion

- 이전 연구에 비해 depth, qubit 측면에서 모두 최적화
- ARIA-128, 192, 256 은 각각 NIST Level 1, 3, 5를 달성
- S-box에서 depth를 줄인 것은 전체 depth에 영향을 미치지 못함.
- 이후, 모든 S-box에 Boyar-Peralta 기법을 찾아서 구현할 예정



Q & A