05

PQC 최적화 I

양자 컴퓨터 개발 현황 (Google vs IBM)

2019년 구글에서 양자 컴퓨터 supremacy 달성
(난수성 문제 증명: 10,000년 vs 3분 20초)


Article

Quantum supremacy using a programmable superconducting processor

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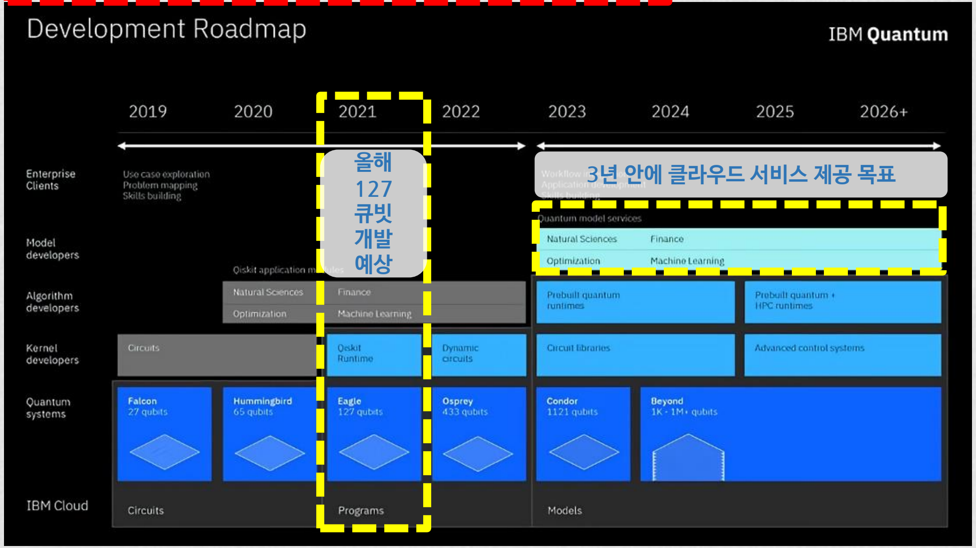
The promise of quantum computers is that certain computational tasks might be executed exponentially faster on a quantum processor than on a classical processor[1]. A fundamental challenge is to build a high-fidelity processor capable of running quantum algorithms in an exponentially large computational space. Here we report the use of a processor with programmable superconducting qubits[2–7] to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 2^{53} (about 10^{16}). Measurements from repeated experiments sample the resulting probability distribution, which we verify using classical simulations. Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years. This dramatic increase in speed compared to all known classical algorithms is an experimental realization of quantum supremacy[8–14] for this specific computational task, heralding a much-anticipated computing paradigm.

In the early 1980s, Richard Feynman proposed that a quantum computer would be an effective tool with which to solve problems in physics and chemistry, given that it is exponentially costly to simulate large quantum systems with classical computers[1]. Realizing Feynman's vision poses substantial experimental and theoretical challenges. First, can a quantum system be engineered to perform a computation in a large enough computational (Hilbert) space and with a low enough error rate to provide a quantum speedup? Second, can we formulate a problem that is hard for a classical computer but easy for a quantum computer? By computing such a benchmark task on our superconducting qubit processor, we tackle both questions. Our experiment achieves quantum supremacy, a milestone on the path to full-scale quantum computing[8–14].

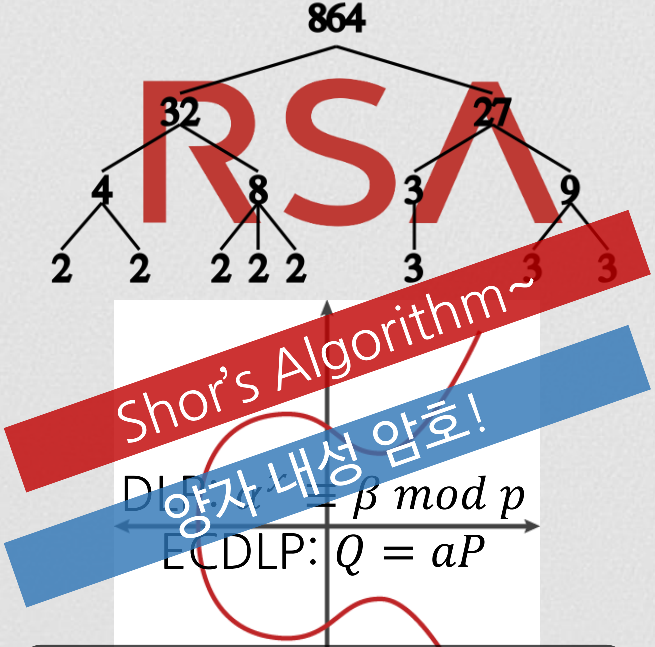
In reaching this milestone, we show that quantum speedup is achievable in a real-world system and is not precluded by any hidden physical laws. Quantum supremacy also heralds the era of noisy intermediate-scale quantum (NISQ) technologies[15]. The benchmark task we demonstrate has an immediate application in generating certifiable random numbers (S. Aaronson, manuscript in preparation); other initial uses for this new computational capability may include optimization[16,17], machine learning[18–21], materials science and chemistry[22–24]. However, realizing the full promise of quantum computing (using Shor's algorithm for factoring, for example) still requires technical leaps to engineer fault-tolerant logical qubits[25–29].

To achieve quantum supremacy, we made a number of technical advances which also pave the way towards error correction. We

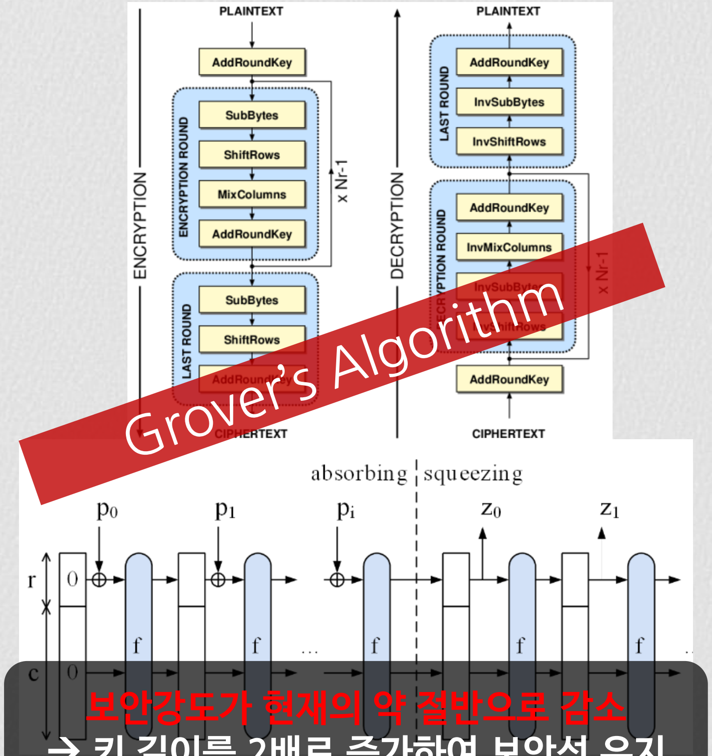
2025년 IBM에서 약 4~5천 큐빗 양자컴퓨터 개발 계획 발표
낮은 보안 강도를 가지는 RSA/ECC의 경우 실제 해킹 가능



현대 암호의 위기와 차세대 암호 알고리즘



보안 강도와 상관없이 취약성 존재
→ 다른 대안 암호로 교체 필요



보안강도가 현재의 약 절반으로 감소
→ 키 길이를 2배로 증가하여 보안성 유지

양자 내성 암호 공모전

양자 알고리즘과 양자 내성 암호

- 양자 알고리즘 → 현대 공개키 암호의 붕괴
- 새로운 공개키 암호의 필요 → 양자 내성 암호 공모전 (NIST)

양자 내성 암호 공모전

양자 내성 암호 표준화 일정

- 초기 계획과 달리 일정이 점차 앞당겨지고 있음

기존 일정	수정된 일정	내용
2017. 11	2017. 11	알고리즘 제안 마감
2018. 04	2018. 04	제안 알고리즘 소개
2018~2019	2018~2019	1차 평가분석 진행 (1차 후보 선정)
2019. 08	2019. 01	1차 선정 결과 발표
2020~2021	2019~2020	2차 평가분석 진행 (2차 후보 선정)
2022~	2020. 07	2차 선정 결과 발표
	2020~2022	3차 평가분석 진행(최종 후보 선정)
	2022~2024	최종 선정 결과 발표 및 표준화

양자 내성 암호 공모전 Round 3 (finalist, alternate)

격자 기반 암호가 가장 큰 관심을 받고 있음

- 현재 격자기반 암호 알고리즘이 가장 많이 후보군에 선정됨

종류	구분	Finalist	Alternate	장점	단점
격자	키교환	KYBER, NTRU, SABER	FrodoKEM, NTRU Prime	고속 구현	파라미터 설정 어려움
	서명	DILITHIUM, FALCON	-		
다변수 다항식	키교환	-	-	작은 서명, 고속 구현	큰 키 사이즈
	서명	Rainbow	GeMSS		
해시	키교환	-	-	안전성 증명 가능	큰 서명 사이즈
	서명	-	SPHINCS+		

양자 내성 암호 공모전 Round 3 (finalist, alternate)

종류	구분	Finalist	Alternate	장점	단점
아이소지니	키교환	–	SIKE	작은 키 사이즈	느린 속도
	서명	–	–		
코드	키교환	Classic McEliece	BIKE, HQC	고속 구현	큰 키 사이즈
	서명	–	–		
영지식	키교환	–	–	number-theoretic 혹은 structured hardness에 기반하지 않음	큰 서명 사이즈
	서명	–	Picnic		

타겟 프로세서

일정한 제한을 두고 양자내성암호 벤치마크를 수행 중에 있음

- STM32F4DISCOVERY
  - ARM Cortex-M4
  - 32-bit ARMv7E-M
  - 192KB RAM, 168 MHz

What NIST wants

2 라운드 주요 관심사

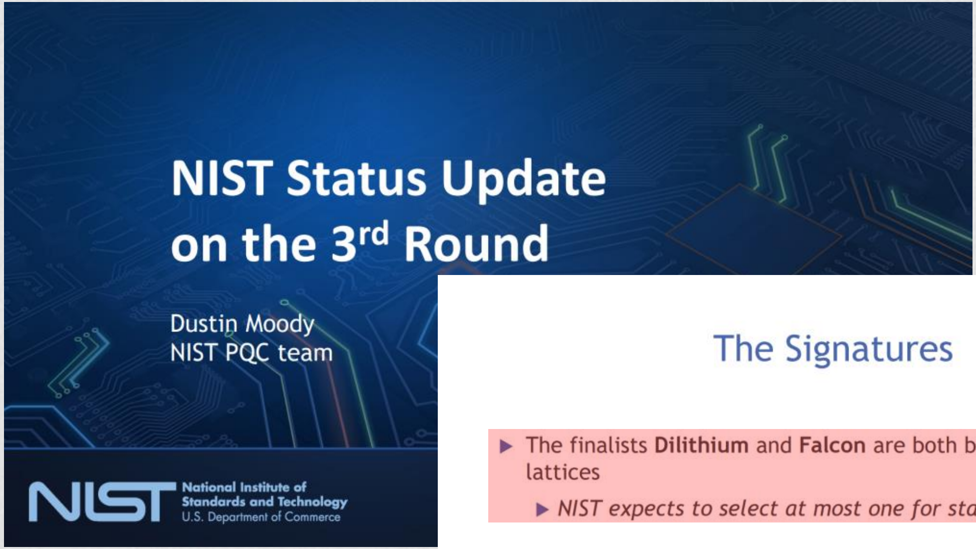
- Performance (hardware+software) will play more of a role
  - More benchmarks
  - For hardware, NIST asks to focus on Cortex M4 (with all options) and Artix-7
    - pqc-hardware-forum
- Continued research and analysis on **ALL** of the 2nd round candidates

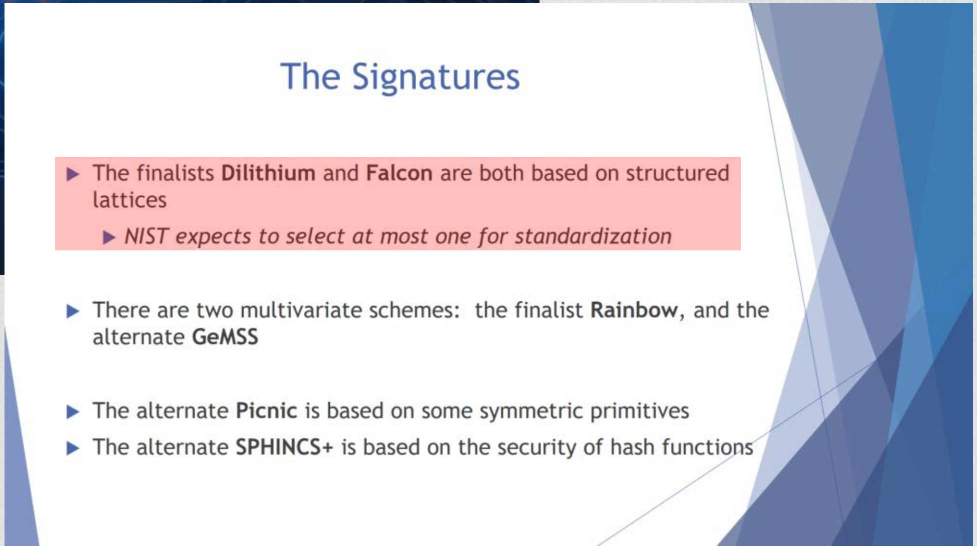
2019년, NIST에서는 임베디드 장비에 대한 벤치마크를 요구

- See how submissions fit into applications/procotols. Any constraints?

격자 기반 양자 내성 암호

격자 기반 서명 알고리즘인 Dilithium과 Falcon 중 하나 선정 예정





Dilithium 서명 알고리즘 (NIST 3 라운드 finalist)

- 작은 키와 서명 크기를 가지고 있음
- **Polynomial ring 상에서 연산:** $R_q = Z_q[X]/(X^{256}+1)$, $q = 8380417$
- **NTT 연산을 통해 최적화 구현:** $a \cdot b = INTT(NTT(a) \bullet NTT(b))$

알고리즘	NIST Level	Public Key [bytes]	Signature [bytes]	Rejection sampling의 예상 반복 횟수
Dilithium2	1	1,184	2,044	5.9
Dilithium3	2	1,472	2,701	6.6
Dilithium4	3	1,760	3,366	4.3

Algorithm 1 Dilithium key generation

Output: Secret key $sk = (\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0)$
Output: Public key $pk = (\rho, \mathbf{t}_1)$
$\rho \leftarrow \{0,1\}^{256}$
$K \leftarrow \{0,1\}^{256}$
$(\mathbf{s}_1, \mathbf{s}_2) \leftarrow S_\eta^\ell \times S_\eta^k$
$\hat{\mathbf{A}} \in R_q^{k\times\ell} := \mathtt{ExpandA}(\rho)$
$\mathbf{t} := \mathtt{NTT}^{-1}(\hat{\mathbf{A}} \circ \mathtt{NTT}(\mathbf{s}_1)) + \mathbf{s}_2$
$(\mathbf{t}_1, \mathbf{t}_0) := \mathtt{Power2Round}(\mathbf{t})$
$tr \in \{0,1\}^{384} := \mathcal{H}(\rho || \mathbf{t}_1)$

Algorithm 2 Dilithium signature generation

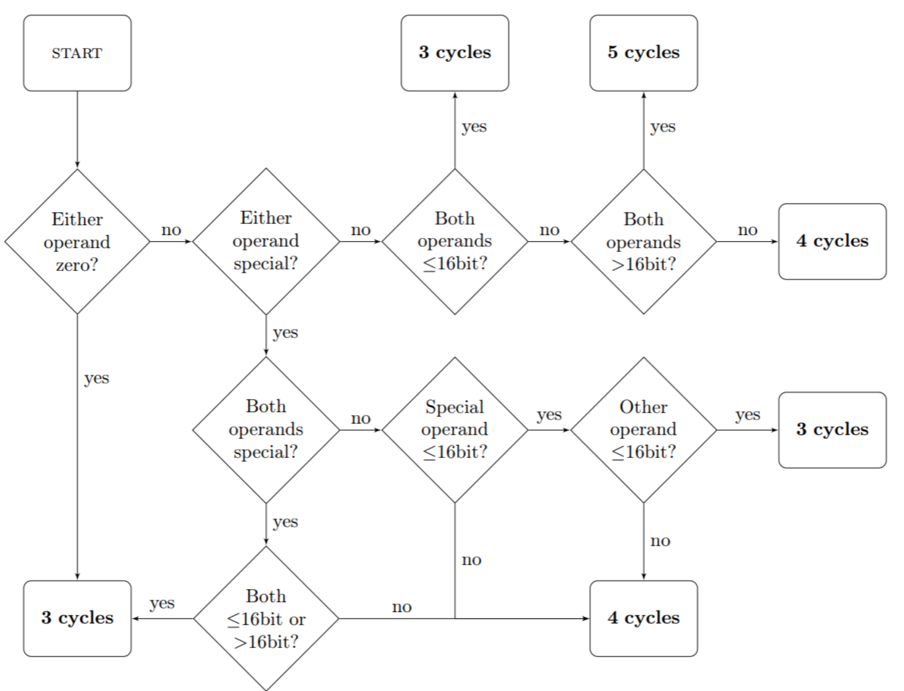
Input: Secret key $sk = (\rho, K, tr, \mathbf{s}_1, \mathbf{s}_2, \mathbf{t}_0)$
Input: Message $M \in \{0,1\}^*$
Output: Signature $\sigma = (\mathbf{z}, \mathbf{h}, c)$
$\hat{\mathbf{A}} \in R_q^{k\times\ell} := \mathtt{ExpandA}(\rho)$
$\mu \in \{0,1\}^{384} := \mathcal{H}(tr || M)$
$\kappa := 0; (\mathbf{z}, \mathbf{h}) = \perp$
$\rho' \in \{0,1\}^{384} := \mathcal{H}(K || \mu)$
$\hat{\mathbf{s}}_1 := \mathtt{NTT}(\mathbf{s}_1); \hat{\mathbf{s}}_2 := \mathtt{NTT}(\mathbf{s}_2); \hat{\mathbf{t}}_0 := \mathtt{NTT}(\mathbf{t}_0)$
while $(\mathbf{z}, \mathbf{h}) = \perp$ **do**
  $\mathbf{y} \in S_{\gamma_1 - 1}^\ell := \mathtt{ExpandMask}(\rho', \kappa)$
  $\mathbf{w} := \mathtt{NTT}^{-1}(\hat{\mathbf{A}} \circ \mathtt{NTT}(\mathbf{y}))$
  $\mathbf{w}_1 := \mathtt{HighBits}(\mathbf{w})$
  $\hat{c} := \mathtt{NTT}(\mathcal{H}_B(\mu || \mathbf{w}_1))$
  $\mathbf{z} := \mathbf{y} + \mathtt{NTT}^{-1}(\hat{c} \circ \hat{s}_1)$
  $(\mathbf{r}_1, \mathbf{r}_0) := \mathtt{Decompose}(\mathbf{w} - \mathtt{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_2))$
  if $||\mathbf{z}||_\infty \geq \gamma_1 - \beta$ **or** $||\mathbf{r}_0||_\infty \geq \gamma_2 - \beta$ **or** $\mathbf{r}_1 \neq \mathbf{w}_1$ **then**
    $(\mathbf{z}, \mathbf{h}) = \perp$
  else
    $\mathbf{h} := \mathtt{MakeHint}\left(-\mathtt{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0), \mathbf{w} - \mathtt{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{s}}_2) + \mathtt{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0)\right)$
    if $||\mathtt{NTT}^{-1}(\hat{c} \circ \hat{\mathbf{t}}_0)||_\infty \geq \gamma_2$ **or** $\#$ 1's in $\mathbf{h} > \omega$ **then**
      $(\mathbf{z}, \mathbf{h}) = \perp$

Algorithm 3 Dilithium verification

Input: Public key $pk = (\rho, \mathbf{t}_1)$
Input: Message $M \in \{0,1\}^*$
Input: Signature $\sigma = (\mathbf{z}, \mathbf{h}, c)$
Output: `Valid` or `Invalid`
$\hat{\mathbf{A}} \in R_q^{k\times\ell} := \mathtt{ExpandA}(\rho)$
$\mu \in \{0,1\}^{384} := \mathcal{H}(\mathcal{H}(\rho || \mathbf{t}_1) || M)$
$\mathbf{w}'_1 := \mathtt{UseHint}(\mathbf{h}, \mathtt{NTT}^{-1}(\hat{\mathbf{A}} \circ \mathtt{NTT}(\mathbf{z}) - \mathtt{NTT}(c) \circ \mathtt{NTT}(2^d \cdot \mathbf{t}_1)))$
if $c = \mathcal{H}_B(\mu || w'_1)$ **and** $||\mathbf{z}||_\infty < \gamma_1 - \beta$ **and** $\#$ 1's in $\mathbf{h} \leq \omega$ **then**
  return `Valid`
else
  return `Invalid`

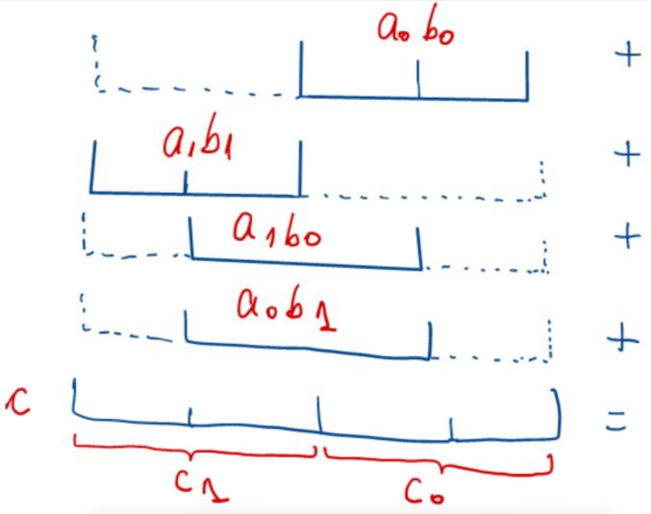
ARM Cortex-M3/M4 상에서의 Constant Timing 곱셈 구현

- UMULL RdHi, RdLo, Rm, Rs→ RdHi : RdLo := Rm * Rs
  - unsigned 64-비트 곱셈기 명령어
  - M4에서는 single cycle
  - M3에서는 경우에 따라 다름 → Timing Attack 가능



M3 상에서의 Constant Timing 구현

- **64-비트 곱셈기는 variable timing**
- **대신 32-비트 곱셈기는 constant timing**
  - Radix 2^{16}을 이용하여 64-비트 값 표현
  - $a = 2^{16}a_1 + a_0$ 그리고 $b = 2^{16}b_1 + b_0$
    $(0 \leq a_0, b_0 < 2^{16}, -2^{15} \leq a_1, b_1 < 2^{15})$
  - $ab = 2^{32}a_1b_1 + 2^{16}(a_0b_1 + a_1b_0) + a_0b_0 \; (-2^{31} \leq a_ib_j < 2^{31})$



M3/M4 상에서의 Constant Timing 구현

Listing 1 CT butterfly from [GKOS18]

```
; q=8380417, qinv=4236238847
; Input: p0, p1, twiddle
; Output: p0, p1
umull tmp0, tmp1, p1, twiddle
mul   pol1, tmp0, qinv
umlal tmp0, tmp1, p1, q
add   p1, p0, q, lsl#1
sub   p1, p1, tmp1
add   p0, p0, tmp1
```

Listing 2 Our CT butterfly

```
; q=8380417, qinv=4236238847
; Input: p0, p1, twiddle
; Output: p0, p1
smull tmp0, tmp1, p1, twiddle
mul   p1, tmp0, qinv
smlal tmp0, tmp1, p1, q
sub   p1, p0, tmp1
add   p0, p0, tmp1
```

Listing 3 GS butterfly in [GKOS18]

```
; q=8380417, qinv=4236238847
; Input: p0, p1, twiddle
; Output: p0, p1
add   tmp0, p0, q, lsl#8
sub   tmp0, tmp0, p1
add   p0, p0, p1
umull tmp1, p1, tmp0, twiddle
mul   tmp0, tmp1, qinv
umlal tmp1, p1, tmp0, q
```

Listing 4 Our GS butterfly

```
; q=8380417, qinv=4236238847
; Input: p0, p1, twiddle
; Output: p0, p1
sub   tmp0, p0, p1
add   p0, p0, p1
smull tmp1, p1, tmp0, twiddle
mul   tmp0, tmp1, qinv
smlal tmp1, p1, tmp0, q
```

기존 unsigned를 signed로 변경

Listing 5 Schoolbook SMULL (SBSMULL)

```
; Input:  a = a0 + a1*2^16
;         b = b0 + b1*2^16
; Output: c = a*b = c0 + c1*2^32
mul  c0, a0, b0
mul  c1, a1, b1
mul  tmp, a1, b0
mla  tmp, a0, b1, tmp
adds c0, c0, tmp, lsl #16
adc  c1, c1, tmp, asr #16
```

Listing 6 Schoolbook SMLAL (SBSMLAL)

```
; Input:  a = a0 + a1*2^16
;         b = b0 + b1*2^16
;         c = c0 + c1*2^32
; Output: c = c + a*b
;           = c0 + c1*2^32
mul  tmp, a0, b0
adds c0, c0, tmp
mul  tmp, a1, b1
adc  c1, c1, tmp
mul  tmp, a1, b0
mla  tmp, a0, b1, tmp
adds c0, c0, tmp, lsl #16
adc  c1, c1, tmp, asr #16
```

M3의 경우 smull/smlal이 variable timing

→ mul과 mla을 통해 constant timing으로 재 구현

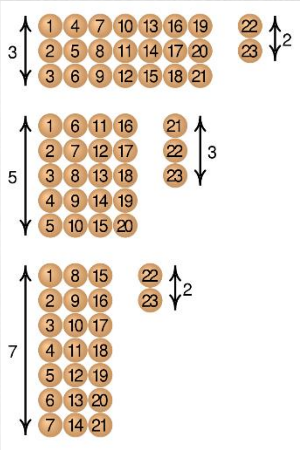
NTT

- **두 polynomial (*a* 그리고 *b*) 에 대한 연산 결과 (*c*):**
  1) 입력인자 (*a* 그리고 *b*) 를 *N*개의 degree-0 polynomials 으로 변환 (Forward transform; NTT).

  2) Pointwise multiplication 을 수행

  3) Chinese remainder theorem을 통해 본래의 값 복구 (*c*) (Backward transform; INTT).

Chinese remainder theorem 예시
$x \equiv 2 \ (mod\ 3) \equiv 3 \ (mod\ 5) \equiv 2 \ (mod\ 7)$
해답 $x = 23 + 105k$, k는 integer



NTT 수학 (1 / 2)

- **Case study (reduction polynomial을 나누기)**
  1) $R_q = Z_q[X]/(X^{256}+1)$
  2) $(X^{128}-\alpha)(X^{128}+\alpha) = (X^{256}+1)$

$$(X^{256}+1) = (X^{128}-\alpha)(X^{128}+\alpha)$$

$$(X^{256}+1) = X^{256} + (-\alpha+\alpha)X^{128} - \alpha^2$$

$$1 = -\alpha^2$$

$$\alpha^2 = -1$$

$$\alpha^4 = 1, \alpha = \sqrt[4]{1}$$

(Fourth primitive root of 1)

NTT 수학 (2 / 2)

- **Case study (reduction polynomial을 나누기)**

1) $R_q = Z_q[X]/(X^{256} + 1)$
2) $(X^{128} - \alpha)(X^{128} + \alpha) = (X^{256} + 1)$
3) $(X^{128} + \alpha) = (X^{64} - \gamma)(X^{64} + \gamma)$

$$(X^{128} + \alpha) = X^{128} + (-\gamma + \gamma)X^{64} - \gamma^2$$

$$\alpha = -\gamma^2$$

$$\gamma = \sqrt{-\alpha} = \sqrt{(-1) \cdot \alpha} = \sqrt{\alpha^2 \cdot \alpha} = (\sqrt{\alpha})^3$$
$$= (\sqrt{\zeta_4})^3 = \zeta_8^3$$

$$(X^{128} - \alpha) = (X^{64} - \beta)(X^{64} + \beta)$$

$$(X^{128} - \alpha) = X^{128} + (-\beta + \beta)X^{64} - \beta^2$$

$$-\alpha = -\beta^2$$

$$\beta = \sqrt{\alpha} = \sqrt{\zeta_4} = \zeta_8$$

ζ_k 는 k-th primitive root of 1, 혹은 수학식은 다음과 같음 ($\zeta_k = \sqrt[k]{1}$).

NTT 예시 (1 / 3)

- **Case study (reduction polynomial을 나누기)**

1) $R_q = Z_q[X]/(X^{256}+1)$

2) $(X^{128}-\alpha)(X^{128}+\alpha) = (X^{256}+1)$

3) $(X^{128}+\alpha) = (X^{64}-\gamma)(X^{64}+\gamma)$ $\quad (X^{128}-\alpha) = (X^{64}-\beta)(X^{64}+\beta)$

4) $(X^{256}+1) = (X^{64}-\zeta_8)(X^{64}+\zeta_8)(X^{64}-\zeta_8^3)(X^{64}+\zeta_8^3)$

Degree-0 polynomials에 도달할 때까지 나누기

5) $(X^{256}+1) = (X-\zeta_{512})(X+\zeta_{512})(X-\zeta_{512}^{129})(X+\zeta_{512}^{129})\cdots(X-\zeta_{512}^{127})(X+\zeta_{512}^{127})(X-\zeta_{512}^{255})(X+\zeta_{512}^{255})$

NTT 예시 (2 / 3)

- **Case study (reduction polynomial을 나누기)**

1) $a \in Z_q[X]/(X^{256}+1)$

$$(X^{128}-\alpha)(X^{128}+\alpha) = (X^{256}+1)$$

a_L a_R

2) $a_L \in Z_q[X]/(X^{128}-\alpha)$

$$X^{128} = \alpha$$

$$a_i\ X^i = a_i\ \alpha X^{i-128}$$

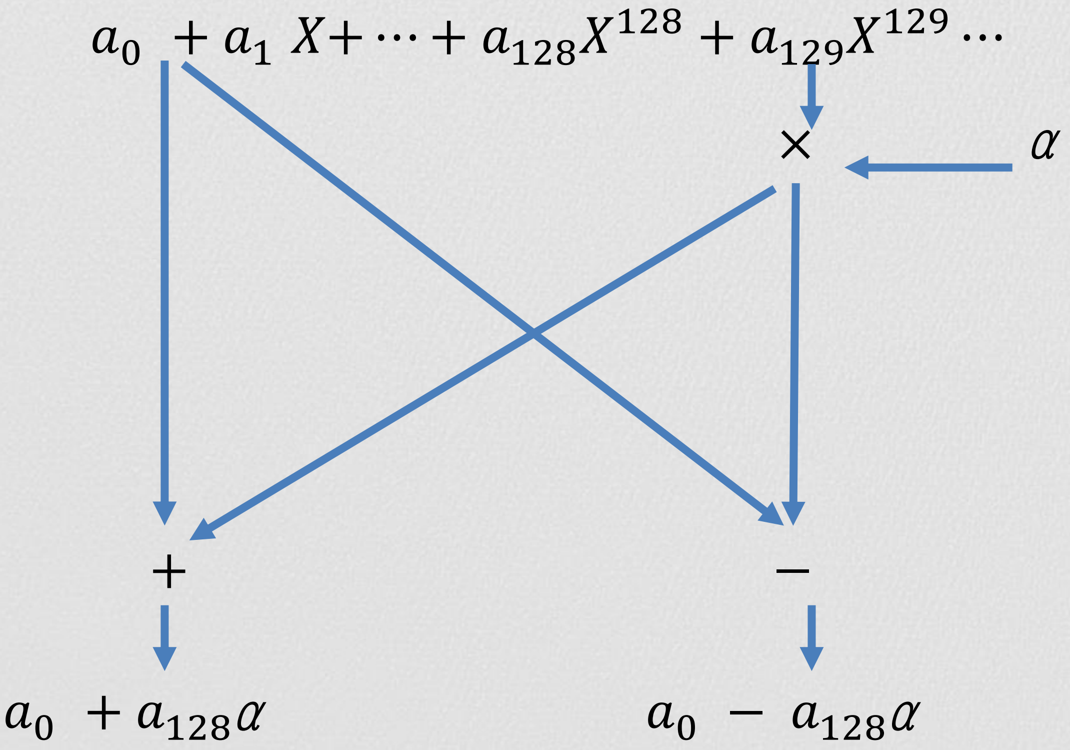
3) $a_L = (a_0 + a_{128}\alpha) + (a_1 + a_{129}\alpha)X + (a_2 + a_{130}\alpha)X^2 + \cdots$

유사하게 a_R 에 적용 가능~

4) $a_R = (a_0 - a_{128}\alpha) + (a_1 - a_{129}\alpha)X + (a_2 - a_{130}\alpha)X^2 + \cdots$

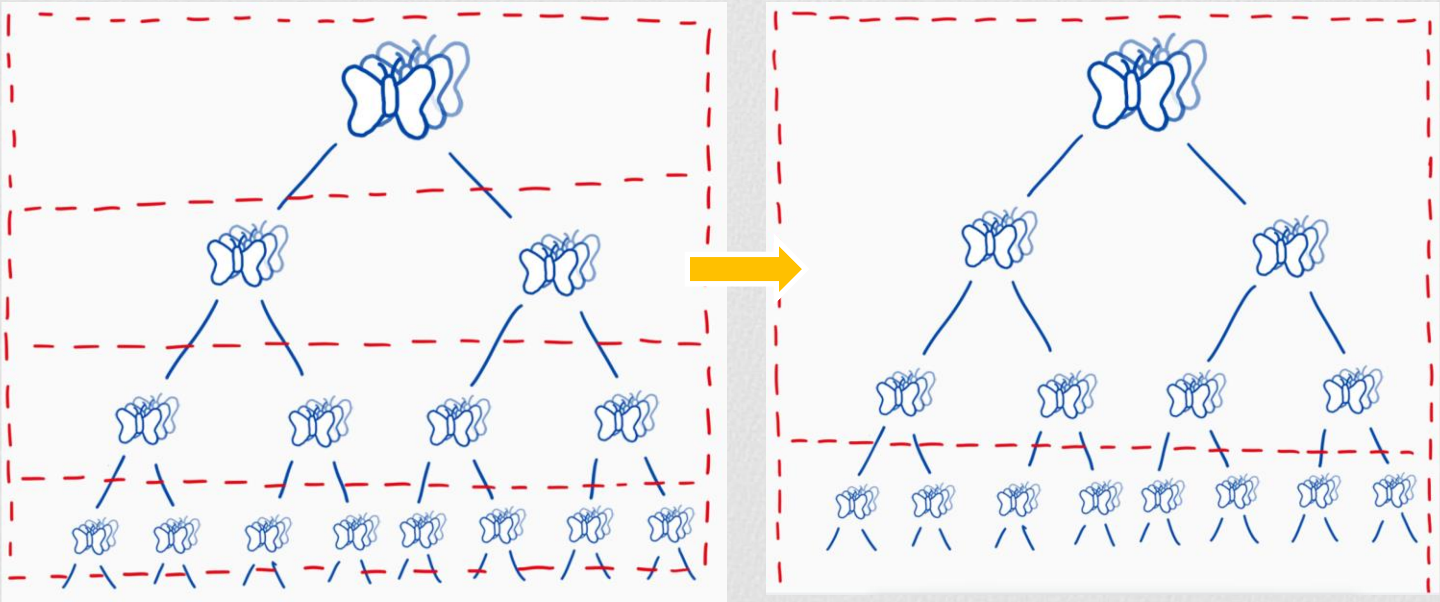
NTT 예시 (3 / 3)

- Case study (reduction polynomial을 나누기)
  - Butterfly diagram



Inverse NTT 는 NTT 연산과 유사!

Merging layers (NTT) for M3/M4



프로세서 별 constant-time 구현상세
- M4: 2 layers를 merging 함
- M3: layer들을 merging 하지 않음

Modular Operation: Barret reduction

$$\frac{1}{b} = \frac{(2^k)/b}{b \cdot (2^k)/b} = \frac{(2^k)/b}{2^k} \approx \frac{x}{2^k}$$

$$x = \lfloor 2^k/b \rfloor$$

$$k = \log_2(a)$$

근사치를 이용한 modular operation

Input: Two numbers a and b, parameter k, $x = \left\lfloor \frac{2^k}{b} \right\rfloor$

Output: $a \pmod b$

```
q ← (a × x) >> k;
r ← a − q × b;
if r ≥ b then
  |  r ← r − b
end
return r
```

Modular Operation: Montgomery reduction

- 2의 배수의 값으로 나누어지도록 숫자를 변환하여 계산
- z는 R^2 승수를 가지는 임의의 수를 의미
- 연산 복잡도는 n 워드 연산 시 $2n^2 + n$ 곱셈 필요

$$\tilde{x} = xR$$

$$Mont(\tilde{x}, \tilde{y}) = \tilde{x}\tilde{y}R^{-1}\, mod\, p = xyR\, mod\, p.$$

$$c \leftarrow \frac{z + (zp' mod\, R)p}{R}, p' = -p^{-1}\, mod\, R$$

만약 $c \geq p$ 라면 $c \leftarrow c - p$ 수행

z는 곱셈 결과값, p는 modulus 값, c는 reduction 이후 값

Modular Operation: *K*−Montgomery reduction

- $q = 12289 = 3 \cdot 2^{12} + 1$, 여기서 3을 k로 설정
- 모듈러가 $q = k \cdot 2^{m} + 1$이고 입력 인자 $0 \leq a, b \leq q$인 경우
- $C = a \cdot b \rightarrow 0 \leq C \leq q^2 = k^2 2^{2m} + k2^{m+1} + 1$
- $C = C_0 + 2^m C_1 \ (0 \leq C_0 \leq 2^m)$
- $0 \leq C_1 = \frac{C - C_0}{2^m} < k^2 2^m + 2k + \frac{1}{2^m} = kq + k + \frac{1}{2^m}$
- $kC \equiv kC_0 - C_1 \ (mod\ q), |kC_0 - C_1| < \left(k + \frac{1}{2^m}\right) q$

function K-RED(C)
    $C_0 \leftarrow C \bmod 2^m$
    $C_1 \leftarrow C / 2^m$
    return $kC_0 - C_1$
end function

Modular Operation: *K*–Montgomery reduction

- 몽고메리 reduction을 통해 격자기반 modulus에 대한 reduction 효율적으로 수행

- $\tilde{x} = x \cdot k^{-1}\ mod\ p\ , \tilde{y} = y \cdot k^{-1}\ mod\ p$

- $K - RED(\tilde{x} \cdot \tilde{y}) \equiv \tilde{x}\tilde{y}k \equiv xy \cdot k^{-1}\ (mod\ p)$

```
function K-RED(C)
    C_0 ← C mod 2^m
    C_1 ← C/2^m
    return kC_0 − C_1
end function
```