From error-correction coding to cryptography for resisting quantum computers

Marco Baldi

Università Politecnica delle Marche
Ancona, Italy
m.baldi@univpm.it

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Quantum computing

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- Theorized by Richard Feynman and Yuri Manin in the early 1980s.
- Shor's algorithm (1994):
  - factorizes integers on a quantum computer,
  - given an integer N, it factors it in a time polynomial in $\log(N)$,
  - on a classic computer the time is exponential in N.
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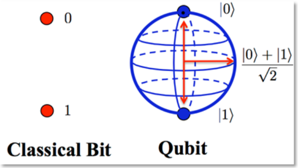

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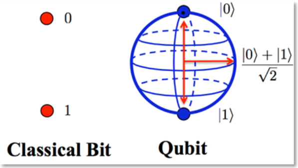

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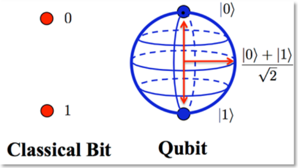

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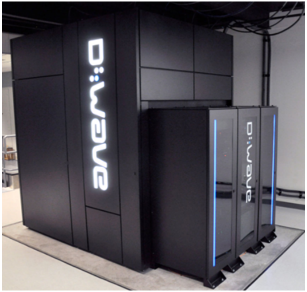
Towards practical quantum computers

- **October 2011**: First academic center of quantum computing (USC, Lockheed Martin and D-Wave Systems).
- **January 2012**: D-Wave announces a quantum computer with 84 qubits.
- **Spring 2013**: Quantum D-Wave Two™ computer installed at the NASA Advanced Supercomputing (NAS) center of the Ames Research Center.



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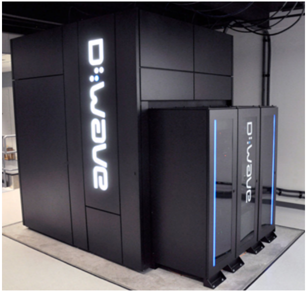

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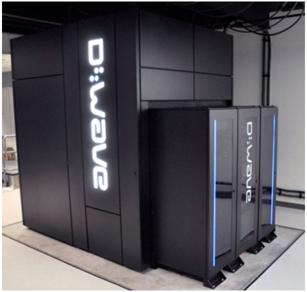

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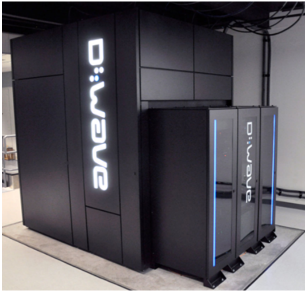

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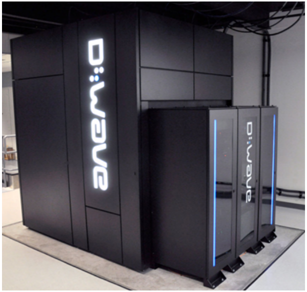

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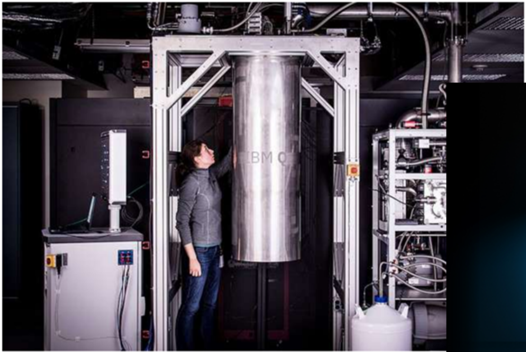
Towards practical quantum computers (2)

IBM builds its most powerful universal quantum computing processors

May 17, 2017



IBM Research Staff Member Katie Pooley, a Physics PhD from Harvard who recently joined IBM, pictured
Center, working on a new prototype of a commercial quantum processor, which will be the core for the firs

Towards practical quantum computers (3)

- On January 2019 IBM announced Q System One, the first commercial quantum computer.
- It has 20 qubits (50 qubits are deemed necessary to compete with classic computers).
- It exploits quantum superposition.
- It must be kept at a very low temperature and isolated from any form of electromagnetic noise.
- Quantum equivalent of the first computers of the 1950s and 1960s.
- Simulators and software models available for programming.

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Quantum supremacy

- Google and IBM are competing towards achieving quantum supremacy.
- The 72-qubit system that Google was developing in 2017 proved too difficult to control.
- Google then started the development of a 53-qubit system called Sycamore.
- In October 2019, Google claimed that the Sycamore processor was able to perform a calculation in 200 seconds that would have taken the world's most powerful supercomputer 10,000 years.
- IBM disclaimed this, stating that Google's system is specialized to solve a single problem, differently from IBM's general-purpose quantum computer.

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Quantum-vulnerable cryptography

The most widespread cryptographic systems today are based on mathematical problems that can be solved with Shor's algorithm:

- **RSA**: public key cryptosystem based on integer factorization (used in SSL/TLS, online banking, ATM, ...).
- **ElGamal**: public key cryptosystem based on discrete logarithm (used in SSL/TLS, ...).
- **DSA**: digital signature algorithm based on discrete logarithm (used in SSL/TLS, ...).
- **Diffie-Hellman**: key exchange protocol based on discrete logarithm (used in SSL/TLS, NFC, contactless payments, ...).
- **ECDH**: Elliptic-curve Diffie–Hellman, used for end-to-end encryption (Signal, WhatsApp, Facebook Messenger, Skype, ...).
- **ECDSA**: Elliptic-curve digital signature algorithm (used in Bitcoin (secp256k1), Ethereum, ...).

Post-quantum cryptography

Asymmetric schemes:

- Based on lattices
- Based on codes
- Based on multivariate polynomials
- Based on hash functions
- Others (isogenies ...)

Symmetric schemes:

- Symmetric encryption schemes (AES ...)
- Hash functions (SHA ...)
- Can still be used as long as Grover's algorithm is taken into account

NIST PQcrypto Project

- **NIST** has initiated a process for the development and standardization of one or more public-key cryptographic algorithms to enrich:

  - Recommendation FIPS 186-4 (Digital Signature Standard - DSS)
  - Special publication SP 800-56A Rev 2 (key establishment systems based on discrete logarithm)
  - Special publication SP 800-56B (key establishment systems based on integer factorization)

NIST PQcrypto call timeline

- **2-3 April 2015**: NIST Workshop on Cybersecurity in a Post-Quantum World
- **24-26 February 2016**: Announcement and description of the NIST call
- **28 April 2016**: NISTIR 8105 report on post-quantum cryptography released
- **20 December 2016**: Official publication of the call
- **30 November 2017**: Deadline for submission of candidates

NIST PQcrypto requirements

Public-key encryption

Shall include algorithms for key generation, encryption, and decryption. At a minimum, the scheme shall support the encryption and decryption of messages that contain symmetric keys of length at least 256 bits.

Key encapsulation mechanism (KEM)

Shall include algorithms for key generation, encapsulation, and decapsulation. At a minimum, the KEM functionality shall support the establishment of shared keys of length at least 256 bits.

Digital signature

Shall include algorithms for key generation, signature generation and signature verification. The scheme shall be capable of supporting a message size up to 2^{63} bits.

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NIST PQcrypto security categories

Any attack that breaks the relevant security definition must require computational resources comparable to or greater than those required for:

1. Key search on a block cipher with a **128-bit key** (e.g. AES128)
2. Collision search on a **256-bit hash** function (e.g. SHA256/ SHA3-256)
3. Key search on a block cipher with a **192-bit key** (e.g. AES192)
4. Collision search on a **384-bit hash** function (e.g. SHA384/ SHA3-384)
5. Key search on a block cipher with a **256-bit key** (e.g. AES 256)

NIST PQcrypto 1st round candidates

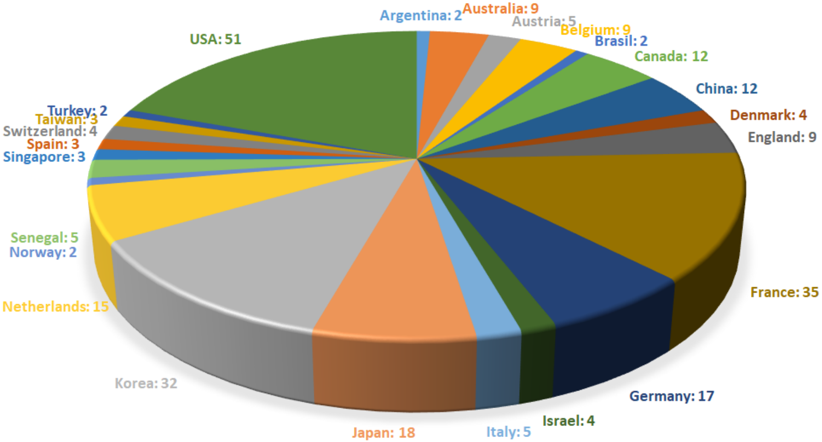
FINAL SUBMISSIONS RECEIVED

- The deadline is past – no more submissions
- 82 total submissions received
  - 23 signature schemes
  - 59 Encryption/KEM schemes

	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	**23**	**59**	**82**

NIST PQcrypto 1st round - Countries involved

263 researchers from **24** Countries



NIST PQcrypto selection steps

- **21 December 2017**: Round 1 algorithms announced (69 submissions accepted as "complete and proper")
- Candidates analyzed for over a year by NIST and international community
- Security has been the main criterion for the first round
- **11-13 April 2018**: First PQC Standardization Conference
- **30 January 2019**: Second round candidates announced (26 algorithms)
- **22-24 August 2019**: Second PQC Standardization Conference
- **2020/2021**: Round 3 begins
- **2022/2024**: Draft standards available

NIST PQcrypto 2nd round KEM/PKC candidates

Code-based

- BIKE
- Classic McEliece
- HQC
- LEDAcrypt
- NTS-KEM
- ROLLO
- RQC

Isogeny-based

- SIKE

Lattice-based

- CRYSTALS-KYBER
- FrodoKEM
- LAC
- NewHope
- NTRU
- NTRU Prime
- Round5
- SABER
- Three Bears

NIST PQcrypto 2nd round digital signature candidates

Lattice-based

- CRYSTALS-DILITHIUM
- FALCON
- qTESLA

Hash-based+

- Picnic
- SPHINCS+

Multivariate

- GeMSS
- LUOV
- MQDSS
- Rainbow

Lattice-based cryptography

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- Regev has shown that the problem of **learning with errors** (LWE) is as difficult to solve as several lattice problems in their worst case.
- **2009**: Craig Gentry introduces the first fully homomorphic lattice-based cryptosystem.

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- The most important computational problem defined on lattices is the Shortest Vector Problem (SVP).
- It consists in finding the non-zero vector with the minimum (Euclidean) length inside a lattice.
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Multivariate cryptography

- Solving multivariate polynomial equation systems is a computationally intensive problem.
- **1988**: Matsumoto and Imai presented the C* scheme, which was broken in 1995.
- **1996**: Jacques Patarin introduced the Hidden Field Equations (HFE) scheme.
- The original scheme is now considered weak, but some of its variants are considered secure.
- These schemes are particularly efficient for the construction of digital signature primitives.

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- Introduced by Robert McEliece in **1978** for asymmetric encryption
- Security reduced to the problem of decoding a random(-looking) linear code
- Resisted 40+ years of cryptanalysis
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Trapdoors from decoding

First ingredient for a trapdoor

The problem of decoding a random linear code cannot be solved in polynomial time.

Second ingredient for a trapdoor

Many families of non-random (Goppa, GRS, convolutional) and quasi-random (LDPC, MDPC) linear codes admit polynomial-time decoding algorithms.

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McEliece cryptosystem

- Proposed by Robert McEliece in **1978**.
- Irreducible Goppa codes were used in the original proposal.
- Secret irreducible Goppa code:
  - based on an irreducible polynomial of degree t over $GF(2^m)$,
  - length (maximum): $n = 2^m$,
  - dimension: $k \geq n - t \cdot m$,
  - correction capability: t errors.

Rationale

1. The probability that a random polynomial is irreducible is $\approx 1/t$, and a fast algorithm exists for testing irreducibility.
2. The number of irreducible polynomials of degree t over $GF(n)$ is $\approx n^t/t$.

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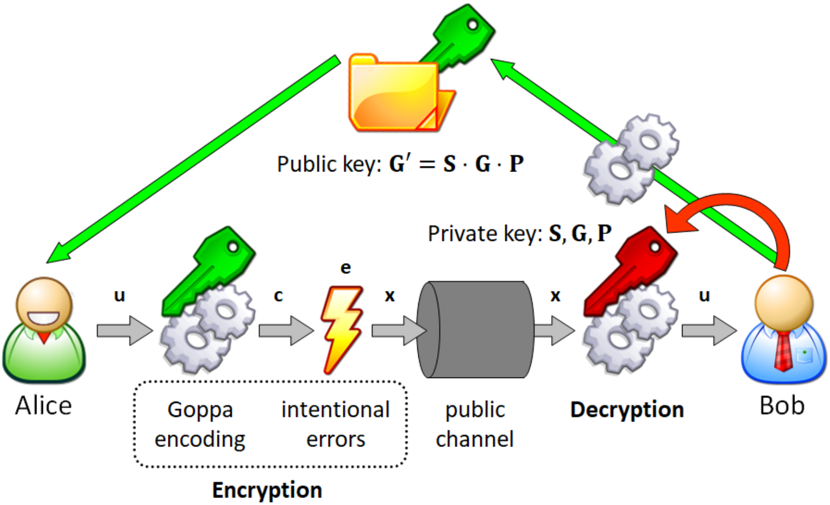
McEliece cryptosystem (2)



McEliece cryptosystem - key generation

Private key

- $k \times n$ generator matrix $\mathbf{G}$ of a secret Goppa code,
- random dense $k \times k$ non-singular "scrambling" matrix $\mathbf{S}$,
- random $n \times n$ permutation matrix $\mathbf{P}$.

Public key

$$\mathbf{G}' = \mathbf{S} \cdot \mathbf{G} \cdot \mathbf{P}$$

- The public code is permutation equivalent to the secret code.
- Is the secret Goppa code matrix $\mathbf{G}$ well disguised enough to make $\mathbf{G}'$ look like the generator matrix of a random linear code?

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1. Alice gets Bob's public key $\mathbf{G}'$.
2. She generates a random error vector of length n and weight t.
3. She encrypts any k-bit block $\mathbf{u}$ as

$$\mathbf{x} = \mathbf{u} \cdot \mathbf{G}' + \mathbf{e} = \mathbf{c} + \mathbf{e}$$

Alert

This only provides semantic security!

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McEliece cryptosystem - decryption

1. Bob computes

$$
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\mathbf{x}' = \mathbf{x} \cdot \mathbf{P}^{-1} = \\
= (\mathbf{u} \cdot \mathbf{S} \cdot \mathbf{G} \cdot \mathbf{P} + \mathbf{e}) \cdot \mathbf{P}^{-1} = \\
= \mathbf{u} \cdot \mathbf{S} \cdot \mathbf{G} + \mathbf{e} \cdot \mathbf{P}^{-1}
\end{aligned}
$$

2. Bob decodes the secret code and obtains

$$\mathbf{u}' = \mathbf{u} \cdot \mathbf{S}$$

3. Bob computes $\mathbf{u} = \mathbf{u}' \cdot \mathbf{S}^{-1}$.

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Attacks against McEliece/Niederreiter

General attacks

General attacks against McEliece/Niederreiter are those aimed at decoding the random-like public code.

Code-specific attacks

Specific attacks are those tailored to each code family (Goppa, GRS, convolutional, LDPC, MDPC, ...).

Attacks against McEliece/Niederreiter

Decryption attacks

Aimed at decrypting one or more ciphertexts without knowing the private key.

Key recovery attacks

Aimed at recovering the private key from the public key.

Decoding attacks

- The most dangerous decoding attacks (DAs) exploit information set decoding (ISD).
- The ISD principle was introduced by Prange in 1962.
- Improved variants were introduced by Lee-Brickell and Leon-Stern in 1988/89.
- A great research effort has been devoted to improving these techniques in recent years.

- E. Prange, "The use of information sets in decoding cyclic codes," Information Theory," IRE Transactions on, vol. 8, no. 5, pp. 5–9, 1962.
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Classical information set decoding

Rationale

For some ciphertext, random errors may not affect a (randomly chosen) information set of the code.

$$\mathbf{x}_{\mathcal{K}} = \mathbf{u} \cdot \mathbf{G}'_{\mathcal{K}} + \mathbf{e}_{\mathcal{K}}$$

- If $\mathcal{K}$ represents an information set, then $\mathbf{G}'_{\mathcal{K}}$ is invertible.
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Decoding attacks - Goppa codes with rate $\approx 1/2$

n	k	m	t	Lee-Brickell	Stern	Peters	Becker et al.
512	260	9	28	45.39	41.57	40.44	33.10
1024	524	10	50	70.56	63.54	62.34	53.05
2048	1036	11	92	114.97	104.83	103.61	94.10
4096	2056	12	170	195.79	182.46	180.63	171.36
8192	4110	13	314	345.37	328.31	325.94	316.74
16384	8208	14	584	623.24	601.40	596.69	590.36

Work factor ($\log_2$) of decoding attacks against McEliece/Niederreiter cryptosystems using Goppa codes with rate about $1/2$.

Decoding attacks - Goppa codes with rate $\approx 2/3$

n	k	m	t	Lee-Brickell	Stern	Peters	Becker et al.
512	350	9	18	46.94	41.24	39.66	33.13
1024	684	10	34	73.11	64.37	62.80	54.16
2048	1366	11	62	119.79	107.88	106.29	97.47
4096	2752	12	112	204.65	189.46	187.56	178.37
8192	5462	13	210	362.10	342.84	339.86	331.63
16384	10924	14	390	654.48	630.37	623.88	619.72

Work factor ($\log_2$) of decoding attacks against McEliece/Niederreiter cryptosystems using Goppa codes with rate about 2/3.

Decoding attacks - Goppa codes with rate $\approx 3/4$

n	k	m	t	Lee-Brickell	Stern	Peters	Becker et al.
512	386	9	14	45.40	39.18	37.50	31.30
1024	784	10	24	69.15	59.61	57.92	49.58
2048	1542	11	46	113.93	101.30	99.62	91.03
4096	3088	12	84	193.91	178.04	176.05	166.91
8192	6164	13	156	342.25	322.02	318.63	311.00
16384	12296	14	292	619.00	593.92	586.76	583.47

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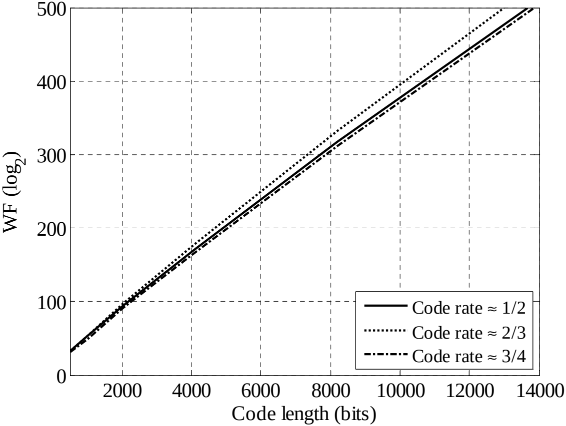
Complexity of BJMM against Goppa codes



- Y. Hamdaoui, N. Sendrier, "A non asymptotic analysis of information set decoding," IACR Cryptology ePrint Archive, Report 2013/162.

Pre-quantum VS post-quantum

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- For searching one entry of an unsorted list of n entries,
  - Grover's algorithm requires $\pi/4\sqrt{n}$ steps using $\log_2(n)$ qubits.
  - The best classical algorithm requires $n/2$ steps on average.
- Grover' algorithm reduces the number of iterations but does not reduce the cost per iteration.
- However, it somehow impacts the work factor of ISD.

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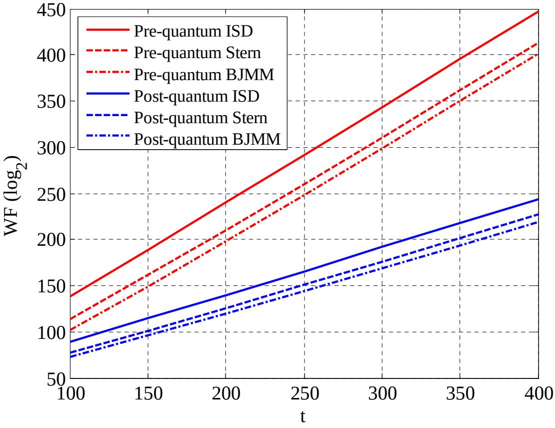
Pre- and post-quantum WF of some ISD algorithms versus t, for codes with $n = 12000$, $k = 6000$.



CCA2 secure conversions

- McEliece/Niederreteir cannot be used naively:
  - Ciphertexts are malleable.
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- Some conversions of these systems exist that achieve CCA2 security.
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Alternatives to Goppa codes (Hamming metric)

Goppa codes
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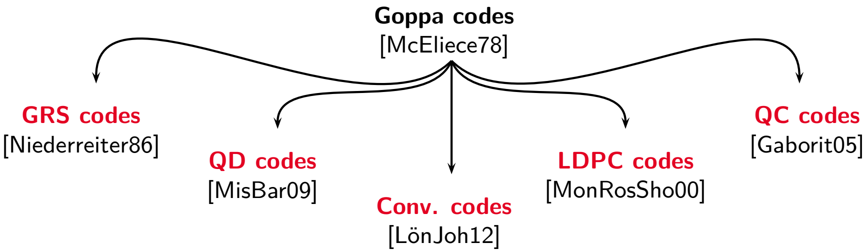

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Goppa codes
[McEliece78]

GRS codes
[Niederreiter86]

QD codes
[MisBar09]

Conv. codes
[LönJoh12]

LDPC codes
[MonRosSho00]

QC codes
[Gaborit05]

GRS subcodes
[BerLoi05]

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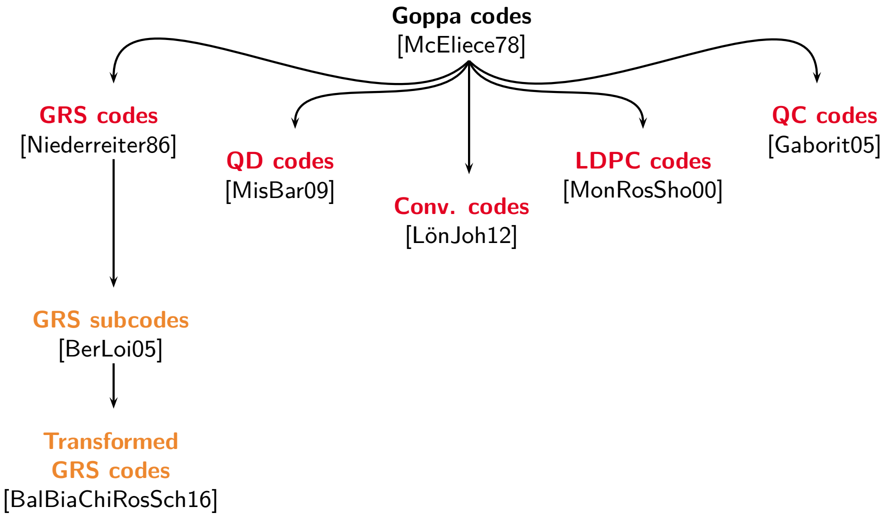

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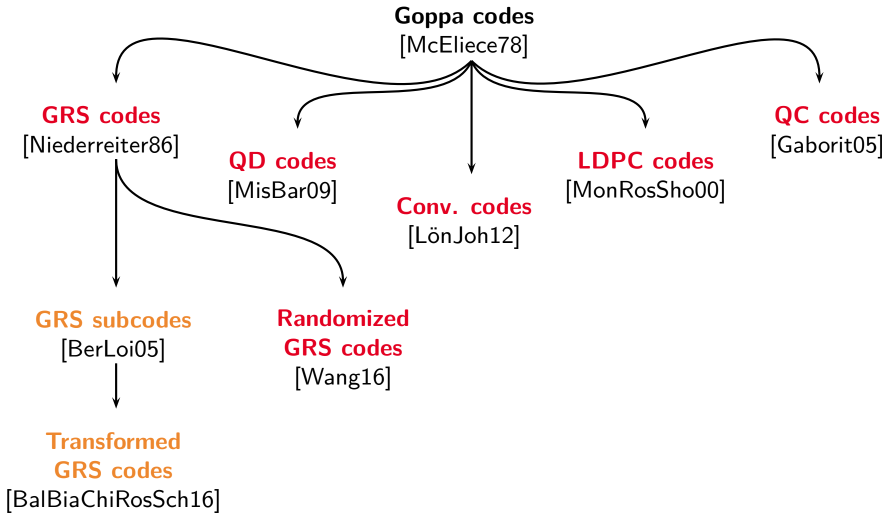

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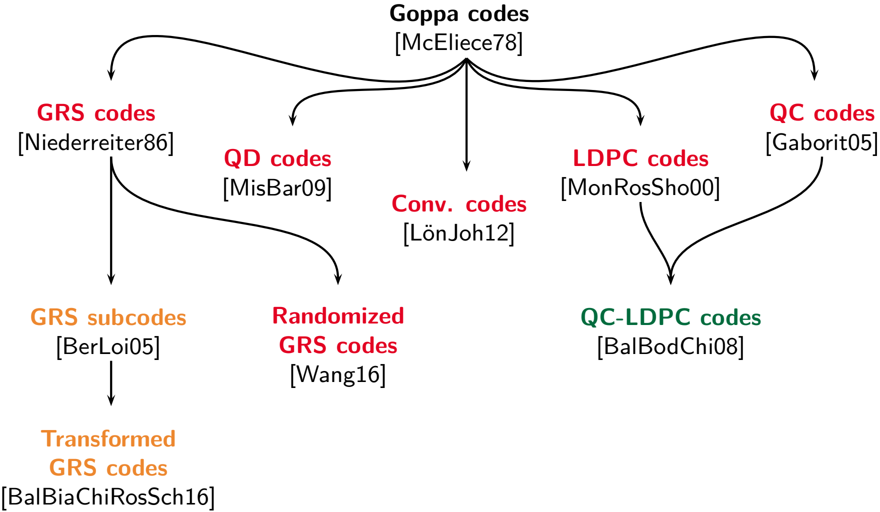

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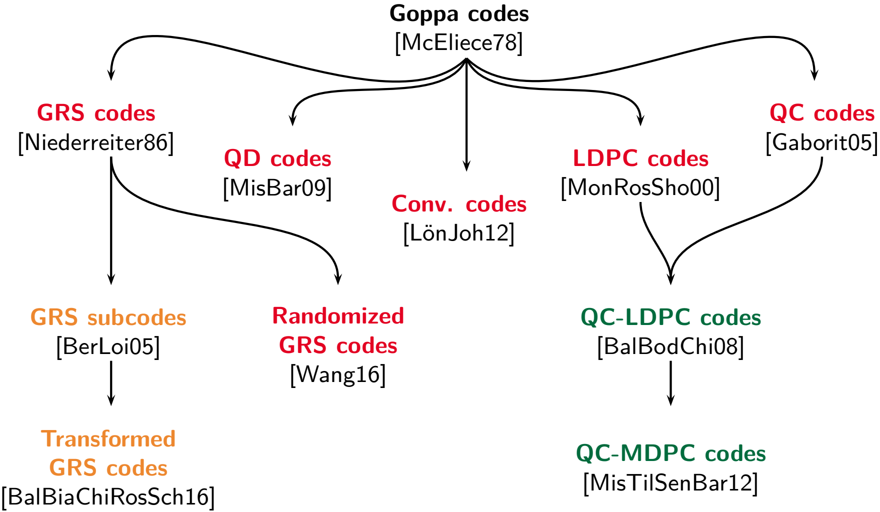

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- Official website (https://www.ledacrypt.org/):
  - First and second round specifications.
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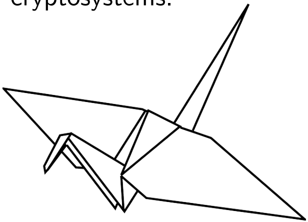

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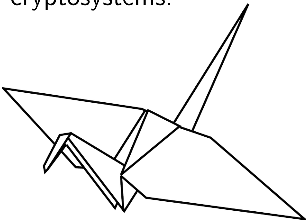

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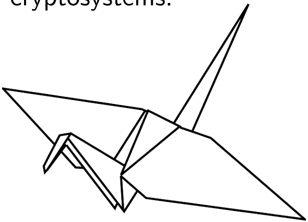

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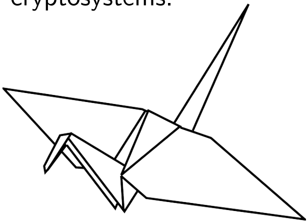

LEDAcrypt features

1. Both KEM and PKC modes.
2. Closed-form upper bound on the decoding failure rate (DFR).
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4. Instances with:
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Performance of LEDAcrypt KEM (ephemeral)

Software running on an Intel i5-6500, 3.2 GHz

NIST Category	n_0	KeyGen (ms)	Encap. (ms)	Decap. (ms)	Total exec. time (ms)	Ctx+kpub Size (kiB)
1	2	1.32	0.06	0.24	1.62	3.65
	3	0.50	0.03	0.23	0.77	3.04
	4	0.47	0.02	0.26	0.76	3.68
3	2	3.63	0.12	0.61	4.37	6.28
	3	1.72	0.07	0.54	2.33	5.91
	4	1.50	0.07	0.69	2.27	7.03
5	2	7.18	0.20	0.95	8.35	9.01
	3	4.64	0.16	1.05	5.86	10.05
	4	3.83	0.13	1.05	5.02	11.09

Code-based digital signatures

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- Only a few replacements are available up to now (like hash-based signatures).
- Code-based digital signatures are post-quantum.
- But finding efficient code-based solutions is still a challenge.
- Two historical proposals: Kabatianskii-Krouk-Smeets (KKS) and Courtois-Finiasz-Sendrier (CFS) schemes.

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  - statistical attacks exploiting key leakage, only one-time or few-times signatures
- **2018**: Debris-Alazard-Sendrier-Tillich: Wave scheme
  - based on generalized $(U; U+V)$ codes
  - degraded version cryptanalyzed on hundreds of signatures
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Classic code-based digital signature schemes

- In KKS, two codes with different size are used to create the trapdoor, one selecting the subset support of the other.
- An important weakness of this system has been pointed out.
- There are still some parameter choices which make it secure.
- In any case, KKS can only be used as a one-time or few-times signature scheme.
- The CFS scheme instead implements a hash-and-sign scheme exploiting only one secret Goppa code.
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CFS scheme

- $\mathcal{H}$: public hash algorithm with r-bit digest.
- $\mathcal{F}$: function able to transform (in a reasonable time) any hash value computed through $\mathcal{H}$ into a correctable syndrome through C.

Private key

$$\{\boldsymbol{S}, \boldsymbol{H}\}$$

- $\boldsymbol{H}$: parity-check matrix of a secret t-error correcting Goppa code $C(n, k)$.
- $\boldsymbol{S}$: $n \times n$ non-singular random matrix.

Public key

$$\boldsymbol{H}' = \boldsymbol{S} \cdot \boldsymbol{H}$$

CFS scheme

- $\mathcal{H}$: public hash algorithm with r-bit digest.
- $\mathcal{F}$: function able to transform (in a reasonable time) any hash value computed through $\mathcal{H}$ into a correctable syndrome through C.

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Signature generation for a file D:

1. The signer computes $\boldsymbol{h} = \mathcal{H}(D)$.
2. The signer computes $\boldsymbol{s} = \mathcal{F}(\boldsymbol{h})$ such that $\boldsymbol{s}' = \boldsymbol{S}^{-1} \cdot \boldsymbol{s}$ is a correctable syndrome (the parameters to be used in $\mathcal{F}$ are made public).
3. Through syndrome decoding, the signer finds $\boldsymbol{e}$ with weight $\leq t$ such that $\boldsymbol{s}' = \boldsymbol{H} \cdot \boldsymbol{e}$.
4. The signature of D is $\boldsymbol{e}$.

Verification of the signature of D:

1. The verifier receives the signed $\widehat{D}$ and computes $\boldsymbol{H}' \cdot \boldsymbol{e} = \boldsymbol{S} \cdot \boldsymbol{H} \cdot \boldsymbol{e} = \boldsymbol{S} \cdot \boldsymbol{s}' = \boldsymbol{s}$.
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CFS scheme (3)

Main limitation of the CFS scheme

It is very hard to find a function $\mathcal{F}$ that quickly transforms an arbitrary hash vector into a correctable syndrome.

- Two possible solutions:
  1. appending a counter to the message,
  2. performing complete decoding.

Drawbacks

- Codes with very high rate and very small error correction capability are needed.
- This has exposed the cryptosystem to attacks based on the generalized birthday algorithm and Goppa codes distinguishers.

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- For 80-bit security, the original CFS system needs a Goppa code with $n = 2^{21}$ and $r = 210$, which gives a key size of 52.5 MiB.
- By using the parallel CFS, the same security level is obtained with key sizes between 1.25 MiB and 20 MiB.

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Sparse code-based signatures

Main differences with CFS

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Attacks against sparse code-based signatures

- A statistical attack against this scheme has been devised.
- It is based on the observation of a large number of signatures.
- It exploits some residual correlation among signature bits to recover an equivalent secret key.
- For parameter sets with 80-bit security, it was successful after the observation of 100′000 signatures originating from the same secret key.
- The scheme can still be used as a few times signature scheme.
- Moving to metrics other than Hamming may help countering this attack.

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System examples - key size

Category	ID	Private Key Size At rest (B)	Private Key Size In memory (kiB)	Public Key size (kiB)	Signature size (kiB)
1	a_3	56	53.66	315.67	3.55
	a_6	56	21.89	540.80	6.52
	α_3	56	32.54	828.81	9.32
2–3	b_3	64	76.29	1364.28	9.16
	b_6	80	40.30	3160.47	27.98
	β_3	64	55.77	3619.48	35.15
4–5	c_3	88	86.03	2818.20	18.92
	c_6	88	69.79	11661.05	89.02
	γ_3	88	159.01	15590.80	112.17

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System examples - speed

Category	ID	KeyGen (ms)	Sign (ms)	Sign+Decomp. (ms)	Verify (ms)
1	a_3	35.51	0.29	1.96	28.71
	a_6	27.23	0.14	1.06	31.18
	α_3	43.45	0.28	1.52	51.10
2–3	b_3	154.49	0.27	2.29	97.83
	b_6	227.14	0.55	2.30	179.89
	β_3	249.69	1.11	2.62	212.19
4–5	c_3	290.95	0.71	5.97	186.30
	c_6	840.74	2.59	3.81	650.78
	γ_3	1714.01	4.27	9.16	926.35

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End of presentation

Thank you!

`www.univpm.it/marco.baldi`

`m.baldi@univpm.it`

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