Over 100x Faster Bootstrapping in Fully Homomorphic Encryption through Memory-centric Optimization with GPUs

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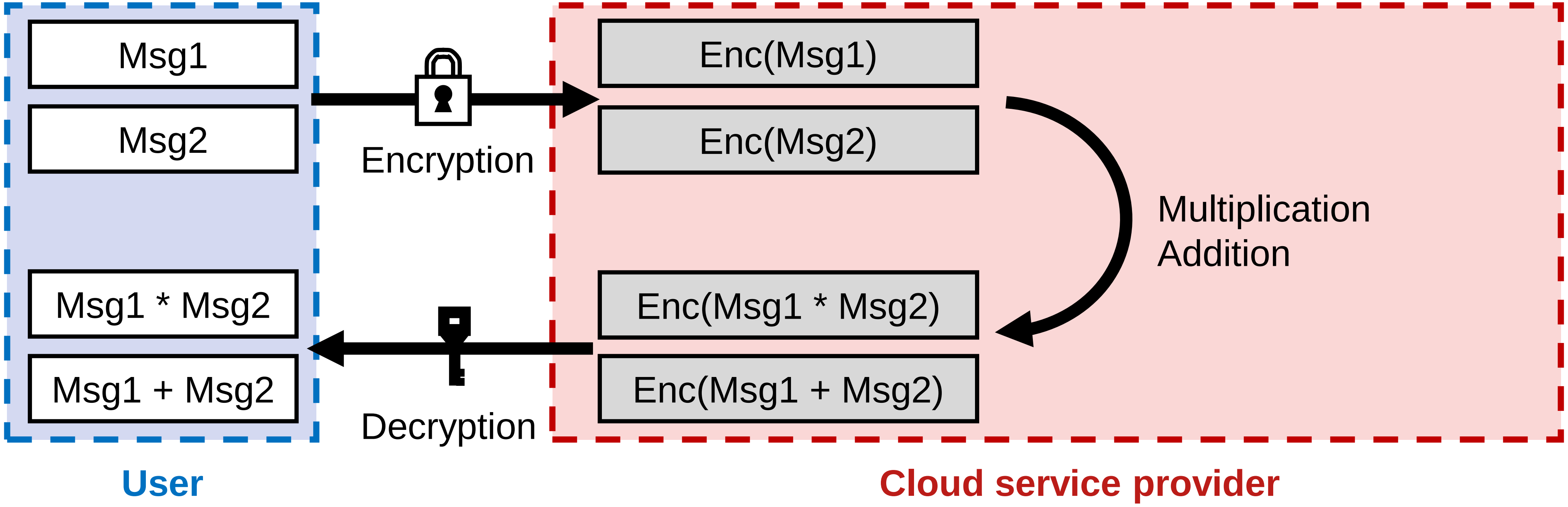
Thanks to Dr. Jung

Contents

Preliminary: Homomorphic encryption (HE) costs a lot

- HE is a cryptographic scheme that enables computation on encrypted messages (*ciphertexts*).
- Users send their data to a cloud service provider without privacy worries.
- Popular HE schemes
  - BFV (integers), TFHE (binary numbers), **CKKS** (fixed-point numbers)



Preliminary: Homomorphic encryption (HE) costs a lot

- Ciphertexts are large.
  - One ciphertext in CKKS = **10-100s of MBs**
- Slow HE operations.
  - (vs. native operations) 100x **–** 200x in addition
  - (vs. native operations) **1000x – 10000x** in multiplication
- *Bootstrapping* is even heavier.
  - Bootstrapping reduces the noise after HE operations on a ciphertext.
    - = It enables *Fully Homomorphic Encryption* (FHE).
  - It takes **10s of seconds** for a single bootstrapping in CKKS.
    - = (using a single-core of a latest CPU)

Preliminary: Homomorphic encryption (HE) costs a lot

- Each encrypted message is called ciphertext.
- Each ciphertext is represented as a pair of high-degree polynomial in a polynomial ring,

$$\textbf{ax1} = a_0 + a_1 x + a_2 x^2 + \cdots + a_{65535} x^{65535}$$

Ctxt1 = (**ax1**, **bx1**)

Ctxt2 = (**ax2**, **bx2**)

Ctxt3 = (**ax3**, **bx3**)

Multiplication

Preliminary: Homomorphic encryption (HE) costs a lot

- Each encrypted message is called ciphertext.
- Each ciphertext is represented as a pair of high-degree polynomial in a polynomial ring,
  - ...whose coefficients are big-integers.

1200-bit

ax1 = $a_0 + a_1 x + a_2 x^2 + \cdots + a_{65535} x^{65535} \in Z_q[X]/(X^N + 1)$

$q = 2^{1200}, N = 65536$

Ctxt1 = (**ax1**, **bx1**)

Ctxt2 = (**ax2**, **bx2**)

Multiplication

Ctxt3 = (**ax3**, **bx3**)

Multiplication between such polynomials is expensive.

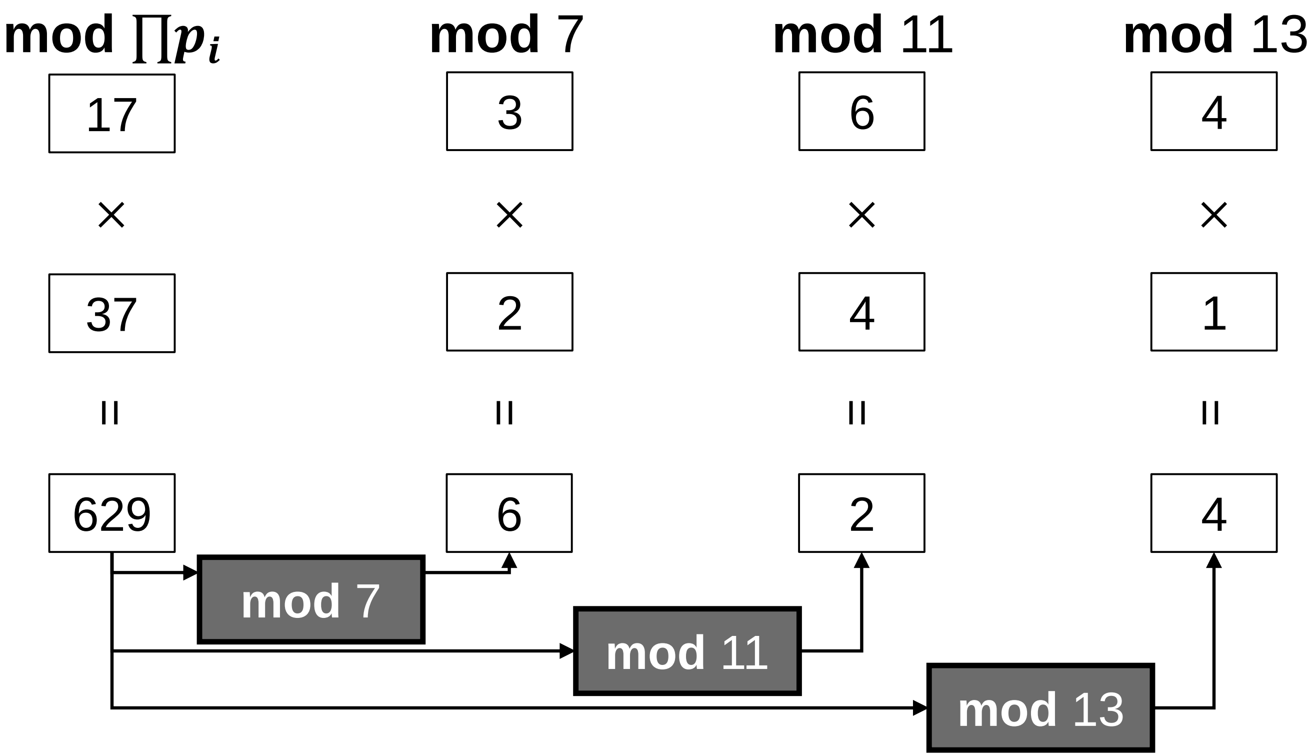
Preliminary: Massive parallelism in HE

- Fortunately, massive parallelism exists in polynomial multiplication:
  - Chinese Remainder Theorem (CRT) to exploit Residue Number System (RNS)
  - Number Theoretic Transform (NTT)

Preliminary: Chinese Remainder Theorem (CRT) in HE

- CRT reduces the computational complexity of multiplication of two big numbers.
  - Ex) multiplying 17 and 37 using coprimes $(p_1, p_2, p_3) = (7,11,13)$

(CRT moduli)



Preliminary: Chinese Remainder Theorem (CRT) in HE

- CRT reduces the computational complexity of multiplication of two big numbers.

  Ex) multiplying 17 and 37 using coprimes $(p_1, p_2, p_3) = (7,11,13)$

(CRT moduli)

mod $\prod p_i$	**mod** 7	**mod** 11	**mod** 13
17	3	6	4
$\times$	$\times$	$\times$	$\times$
37	2	4	1
$=$	$=$	$=$	$=$
$\boxed{629}$	6	2	4

Reconstruct (inverse CRT, iCRT)

Preliminary: Number Theoretic Transformation (NTT) in HE

- Each multiplication between big-integer coefficients can be solved by CRT, but still with O(N^2).

ax1 = $a_0 + a_1x + a_2x^2 + \cdots + a_{65535}x^{65535} \in Z_q[X]/(X^N + 1)$

$q = 2^{1200}, N = 65536$

Ctxt1 = (**ax1**, **bx1**)

Ctxt2 = (**ax2**, **bx2**)

CRT

$(a_0, a_1, \dots, a_{65536})\ mod\ p_1 \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_1$

$(a_0, a_1, \dots, a_{65536})\ mod\ p_2 \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_2$

$\dots \qquad \dots$

$(a_0, a_1, \dots, a_{65536})\ mod\ p_{np} \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_{np}$

Preliminary: Number Theoretic Transformation (NTT) in HE

- Each multiplication between big-integer coefficients can be solved by CRT, but still with $O(N^2)$.
- NTT, an integer-variant DFT, reduces $O(N^2)$ to $O(N\log N)$.

Point-wise mul, $\boldsymbol{O(N)}$

$$(A_0, A_1, \dots, A_{65536}) \quad \odot \quad (A'_0, A'_1, \dots, A'_{65536})$$

NTT, $\boldsymbol{O(NlogN)}$

$$(a_0, a_1, \dots, a_{65536})\ mod\ p_1 \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_1$$
$$(a_0, a_1, \dots, a_{65536})\ mod\ p_2 \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_2$$
$$\dots \qquad \dots$$
$$(a_0, a_1, \dots, a_{65536})\ mod\ p_{np} \quad * \quad (a'_0, a'_1, \dots, a'_{65536})\ mod\ p_{np}$$

Preliminary: Variants of Cheon-Kim-Kim-Song (CKKS)

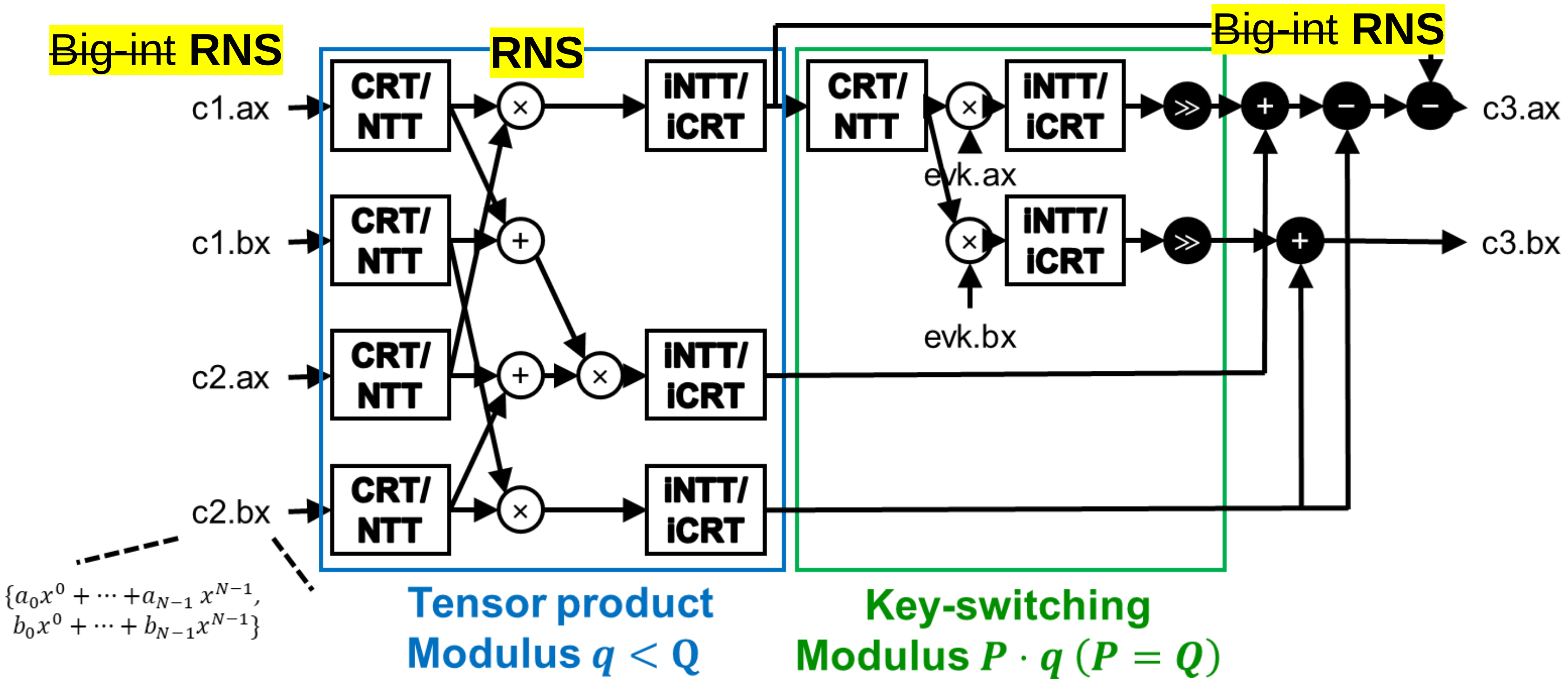
- Representation of plaintext and ciphertext in RLWE:
  - Plaintext $m(X) \in R_q = \mathbb{Z}_q[X]/(X^N + 1),\ q \leq Q(= ciphertext\ modulus\ )$
  - Ciphertext $ct(X) = (b(X), a(X)) \in R_q^2$
- Variants of CKKS
  - Binary CKKS [1]
    - = arbitrary q and Q
    - = needs expensive CRT & iCRT conversion.
  - RNS-CKKS [2]
    - = Fixed q and Q
    - = No (i)CRT; exploits *Fast base conversion*.
  - (Improved) RNS-CKKS [3]
    - = [2] + better algorithms (*key-switching* & bootstrapping)

[1] Cheon at el., Homomorphic encryption for arithmetic of approximate numbers, AsiaCrypt'17
[2] Cheon at el., A full RNS variant of approximate homomorphic encryption, SAC'18
[3] Han at el., Better bootstrapping for approximate homomorphic encryption, CT-RSA'20

Contents

Why Full-RNS CKKS?

- Polynomials are always in RNS domain.
  - no big-int computations



Multiplication of two ciphertexts (c1 * c2)

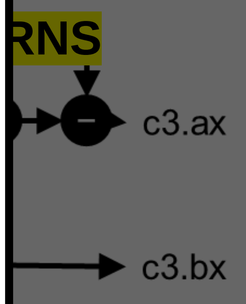
Why Full-RNS CKKS?

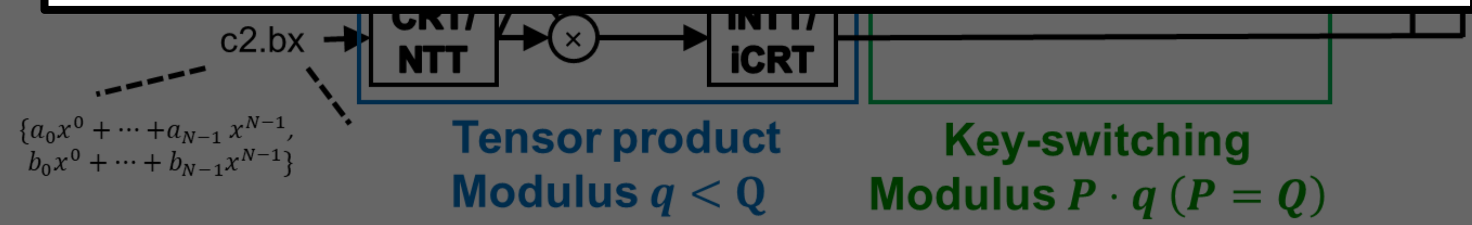
- Polynomials are always in RNS domain.
  - no big-int computations (iCRT+CRT ➔ *Fast base conversion*)

$$iCRT_{\{[x]_{p_i}\}\rightarrow BigInt} \quad \sum \left[r_i \cdot \left(\frac{\prod_j p_j}{p_i}\right)^{-1} \right]_{p_i} \cdot \left(\frac{\prod_j p_j}{p_i}\right) \bmod \prod_j p_J$$

$$Conv_{\{[x]_{p_i}\}\rightarrow\{[x]_{p'_k}\}} \quad \sum \left[r_i \cdot \left(\frac{\prod_j p_j}{p_i}\right)^{-1} \right]_{p_i} \cdot \left[\left(\frac{\prod_j p_j}{p_i}\right)\right]_{p'_k} \bmod p'_k$$

Similar to iCRT, but word-size
precomputed values and modulus





Multiplication of two ciphertexts (c1 * c2)

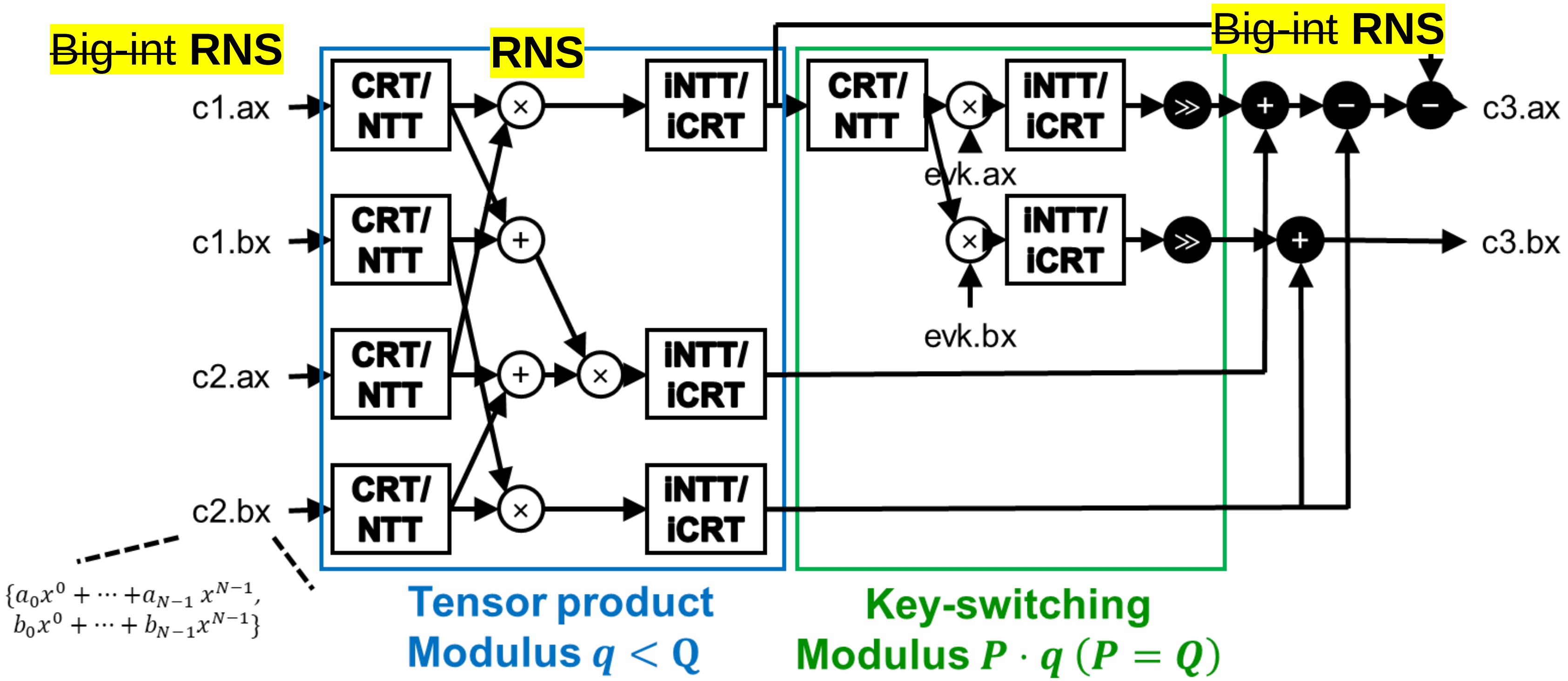
Why Full-RNS CKKS?

- Polynomials are always in RNS domain.
  - no big-int computations (iCRT+CRT ➔ *Fast base conversion*)
- Small *CRT modulus:* ciphertext modulus is a product of moduli ($Q = q_0 q_1 \ldots q_L,\ q = q_0 q_1 \ldots q_\ell$)

(In binary CKKS)
CRT moduli: q^2 and qQP

➔

(In Full-RNS)
q and $P \cdot q$



Multiplication of two ciphertexts (c1 * c2)

Accelerating Full-RNS CKKS

- The first GPU implementation of bootstrapping in the latest Full-RNS CKKS scheme.
- Analyze memory bandwidth bottleneck of the GPU implementation.
- Apply memory-centric optimizations, resulting in sub-second bootstrapping.

  = a speedup of 257x, compared to single-threaded CPU.

Background: a recent Full-RNS CKKS [HK20]

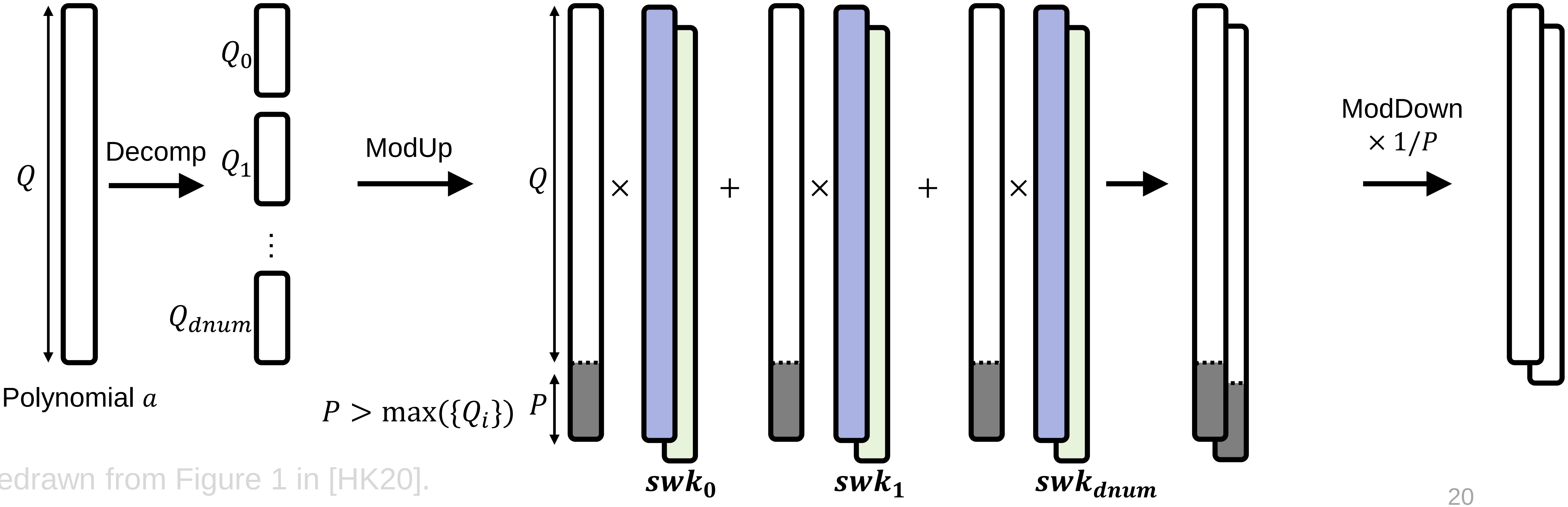
- Parameters
  - Multiplicative level L
  - Primes $q_0, \ldots, q_L, p_1, \ldots, p_k$
  - Modulus $Q = q_0 q_1 \ldots q_L$, $P = p_1 p_2 \ldots p_k$
- Plaintext representation:
  - (RNS form) $m(X) \in R_Q$
  - (NTT form) $m = NTT\big(m(X)\big)$
- Ciphertext representation:
  - (RNS form) $\boldsymbol{ct} = \big(b(X), a(X)\big) \in R_Q^2$
  - (NTT form) $\boldsymbol{ct} = (b, a)$

Background: a recent Full-RNS CKKS [HK20]

- Key operations
  - Plaintext multiplication
    - = $CMult(\boldsymbol{ct}, m) = (ma, mb)$
  - Ciphertext multiplication
    - = $Mult(\boldsymbol{ct_1}, \boldsymbol{ct_2})$
    1. computes a tensor product $(d_0, d_1, d_2) = (b_2 b_1, b_1 a_2 + b_2 a_1, a_1 a_2)$
    2. return $(d_0, d_1) + KeySwitch_{\boldsymbol{MultKey}}(d_2)$
  - Ciphertext rotation by a rotation index r
    - = $Rotate(\boldsymbol{ct}, r)$
    1. computes an automorphism $b'(X) = b\left(X^{5^r}\right), a'(X) = a\left(X^{5^r}\right)$
    2. return $(b', 0) + KeySwitch_{\boldsymbol{RotationKey_r}}(a')$
- [HK20] introduces a new key-switching method for CKKS.
  - Generalized key switching (a.k.a. hybrid key-switching)

Background: a recent Full-RNS CKKS [HK20]

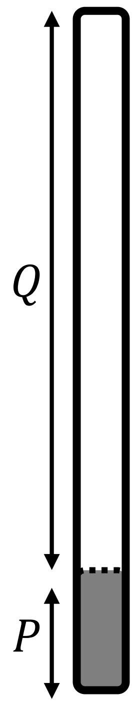
- $KeySwitch_{\boldsymbol{swk}}(a)$
  1. (*Decomp)* decompose into up to ***dnum*** parts, each having α moduli: $Q_0 = q_0 q_1 \cdots q_{\alpha-1}, Q_1 = q_\alpha q_{\alpha+1} \cdots$
  2. (*ModUp)* raise the modulus of each part to PQ.
  3. (*Inner-product)* do inner-product with the key-switching key $\boldsymbol{swk} = (\boldsymbol{swk_0}, \boldsymbol{swk_1}, \dots, \boldsymbol{swk_{dnum}})$.
  4. (*ModDown)* reduce the modulus to Q and multiply $1/P$.



Redrawn from Figure 1 in [HK20].

Background: Choosing a good *dnum* parameter

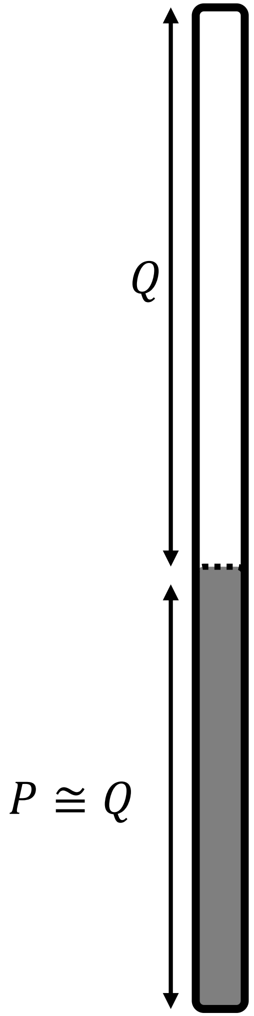
- $\uparrow logPQ$ = $\downarrow$ security
- The choice of **dnum** affects complexity, key size, and security.



(after ModUp)

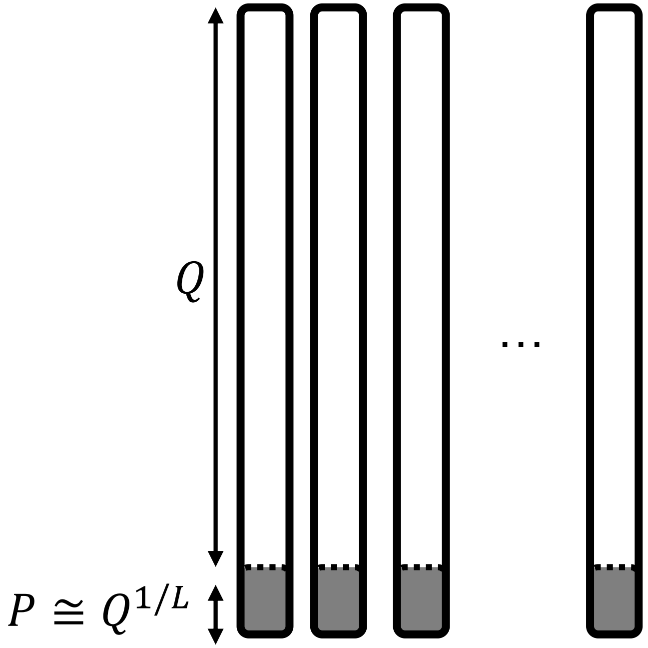
Background: Choosing a good *dnum* parameter

- $\uparrow logPQ$ = $\downarrow$ security
- The choice of **dnum** affects complexity, key size, and security.
- Given Q,
  - **dnum** = 1 (minimum)
    - $\uparrow P$
    - $\downarrow$ security
    - $\downarrow$ size of keys



Background: Choosing a good *dnum* parameter

- ↑ $logPQ$ = ↓ security
- The choice of **dnum** affects complexity, key size, and security.
- Given Q,
  - **dnum** = 1 (minimum)
    - ↑ P
    - ↓ security
    - ↓ size of keys
  - **dnum** = L (maximum)
    - ↓ P
    - ↑ security
    - ↑ size of keys



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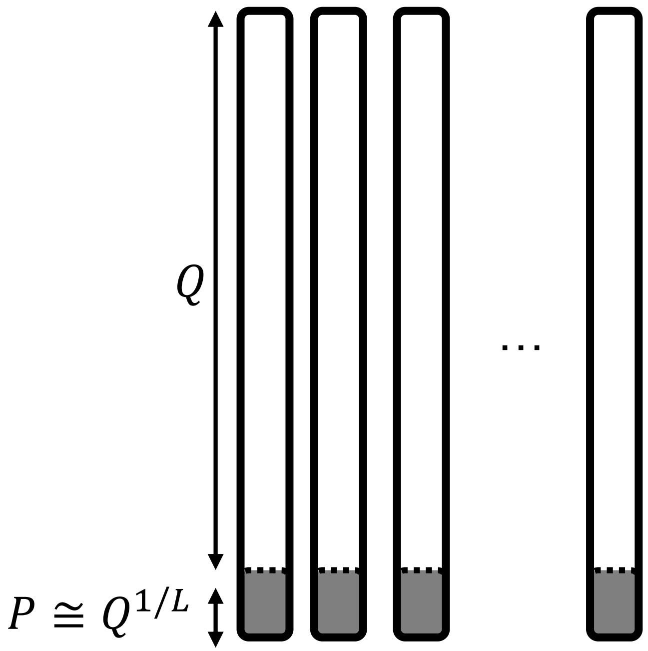
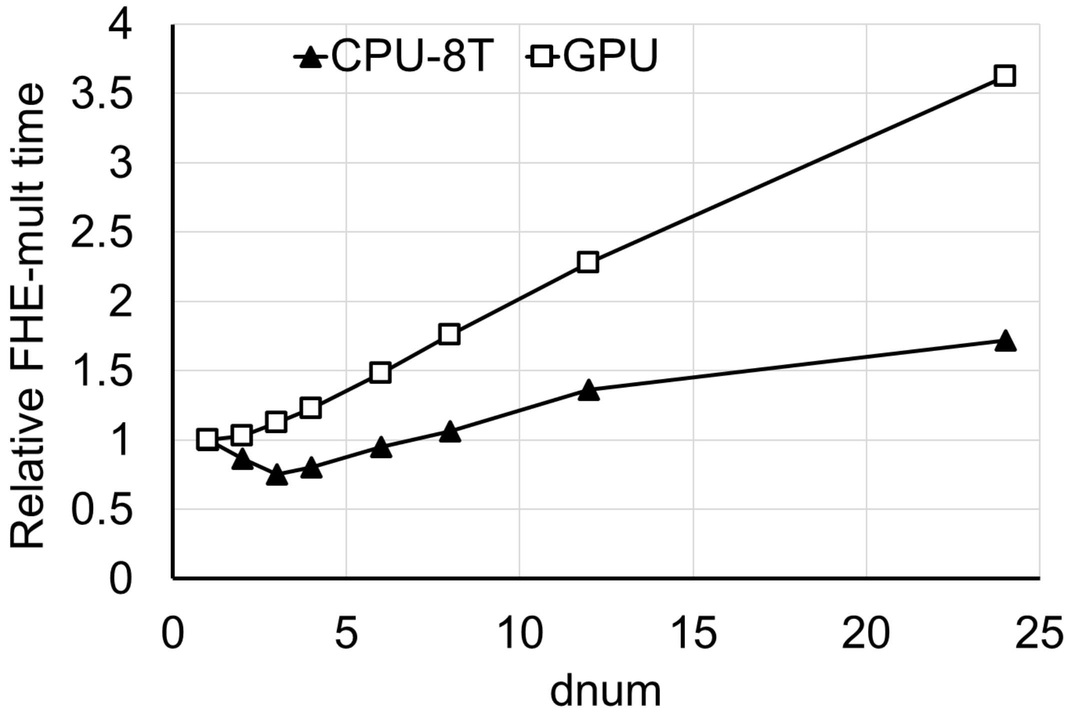
    - $\downarrow$ size of keys
  - **dnum** = L (maximum)
    - $\downarrow P$
    - $\uparrow$ security
    - $\uparrow$ size of keys
  - Computational complexity becomes minimal somewhere between them.
    - = [HK20] chooses that value.



Motivation: GPU implementations are more memory-bottlenecked

- Last-level cache size of a server class
  - CPU: ~256MB
  - GPU: ~6MB
    - = Hardly accommodates ciphertexts.



dnum vs. multiplication time for a fixed Q

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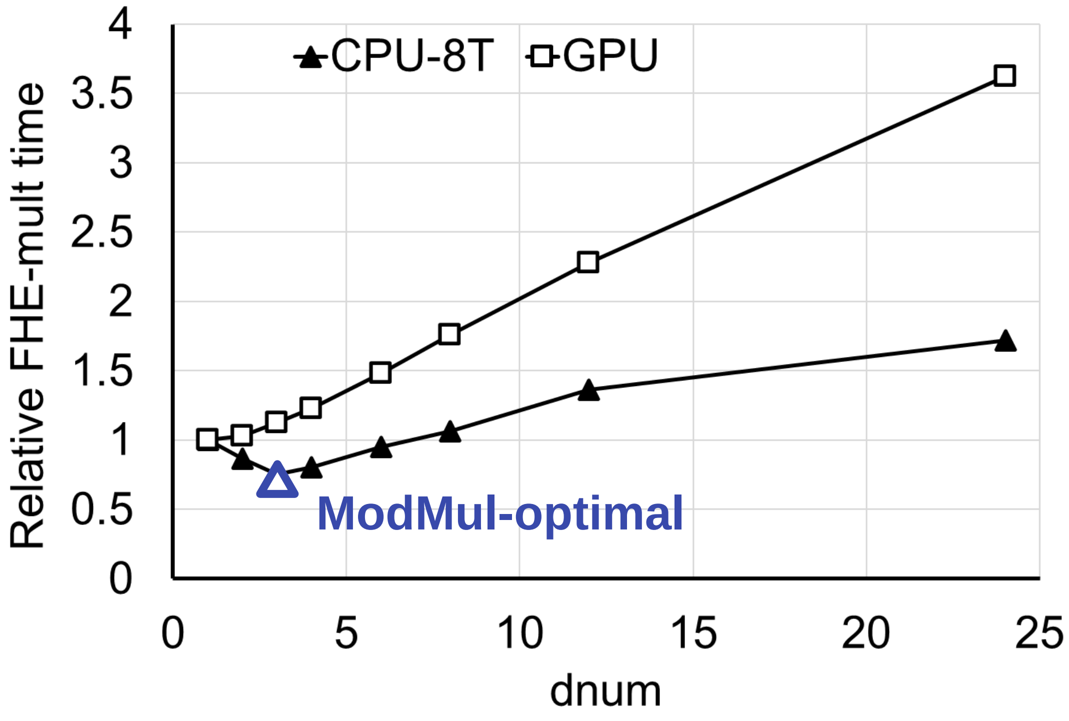

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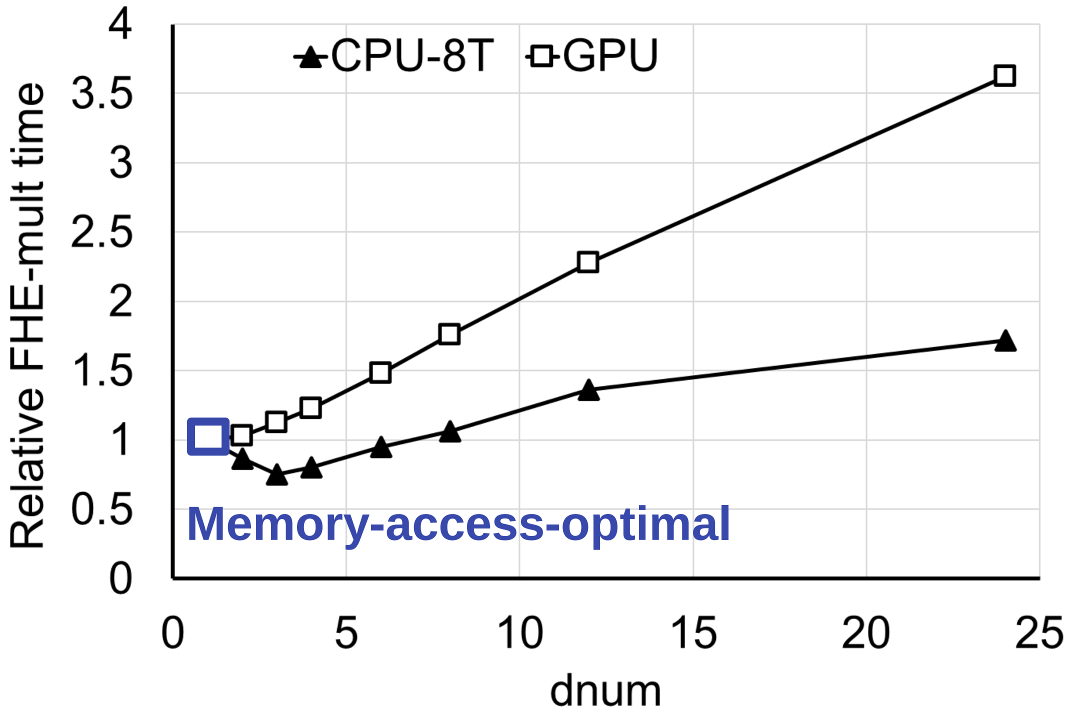

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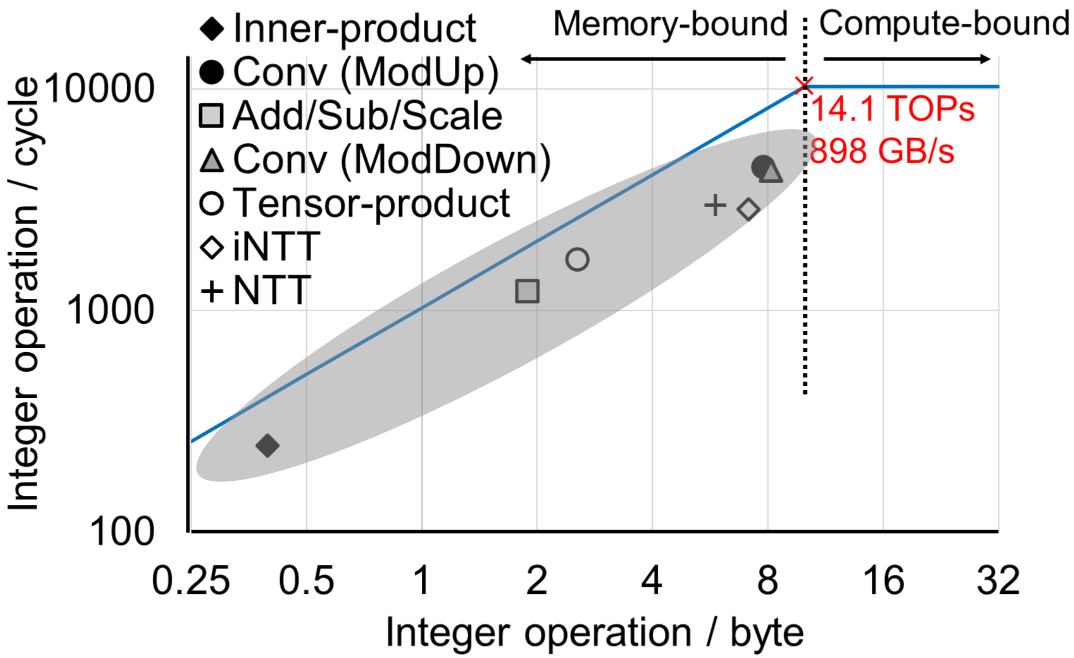


Roofline plot of a multiplication

HE operations are memory-bound, especially on GPUs.

Contents

Overview: Brief introduction to the programming model in GPUs

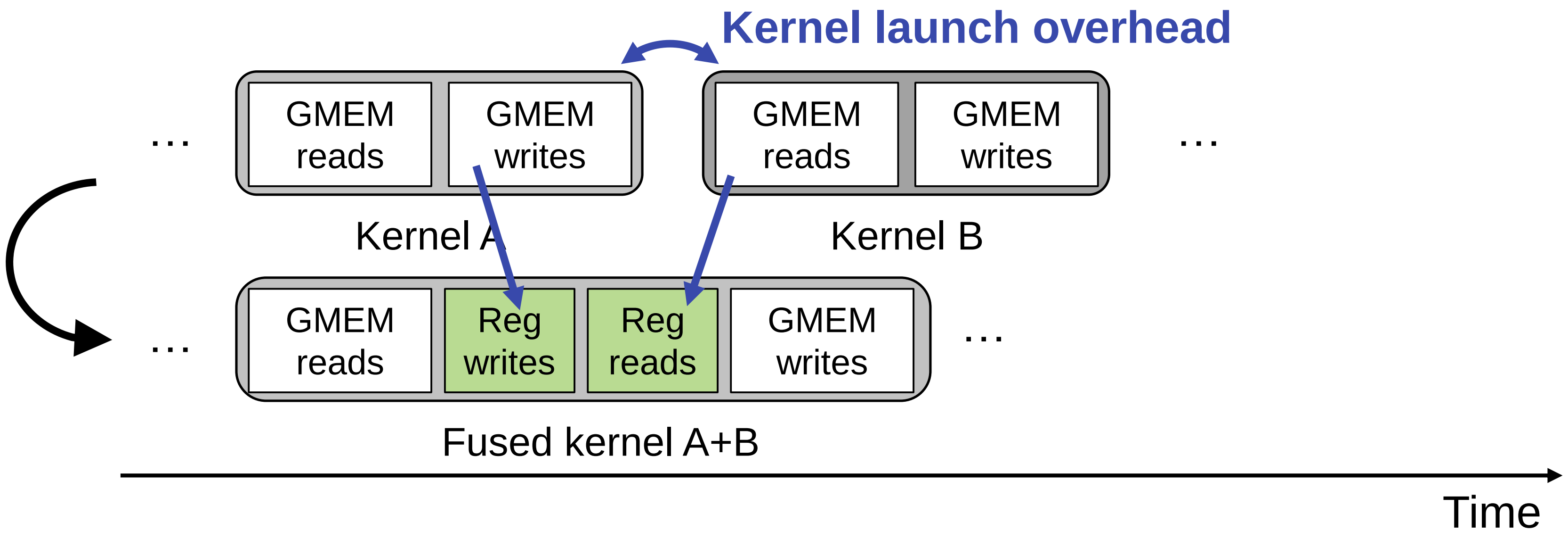
- A GPU has many *Streaming Multiprocessors (SM).*
  - Each SM manages and executes threads in groups of 32 parallel threads.
- A function executed in a GPU is called *Kernel.*
  - Many GPU threads run the same kernel in parallel in SMs.
- Each kernel is configured with
  - the number of threads running on the kernel
  - the amount of the *shared memory* it will use.
- Shared memory is a user-configurable scratch-pad memory.

Overview: Baseline GPU implementation

- Based on prior works:
  - NTT-friendly degree-first data layout [CLP17, HS14, CHK+19]
  - Executing $(L \times N)$ RNS operations in a kernel [BPA+19]
  - Fast NTT/iNTT implementations [KJPA20]
  - …

Overview: Kernel fusion - memory-centric optimization for GPUs

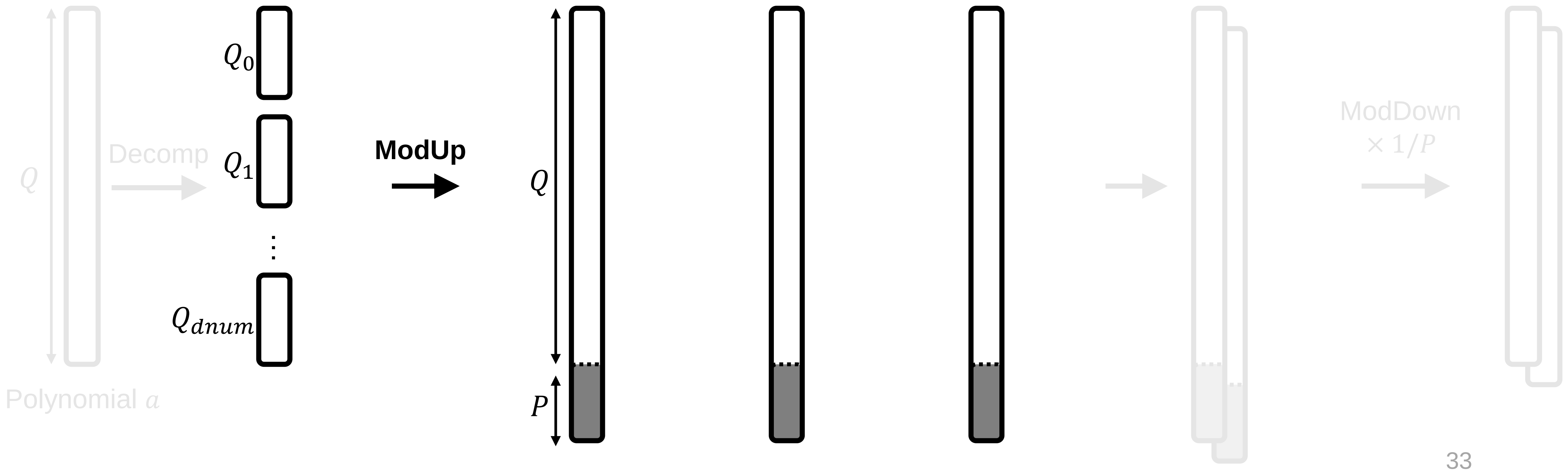
- Kernel fusion (operation fusion)
  – A common technique that fuses multiple GPU kernels into a single kernel
- Kernel fusion can
  – save global memory (DRAM) reads and writes by reusing the data in register file/shared memory.
    = Good for the kernels with low OP/B (memory-bound).
  – reduce *kernel launch overhead* by merging small kernels
- We found many kernel fusion opportunities in **intra- / inter-HE-operation** manner.



Kernel fusion reduces global memory (GMEM) accesses.

Intra-HE-operation Fusions

- *ModUp Fusion (MF)*



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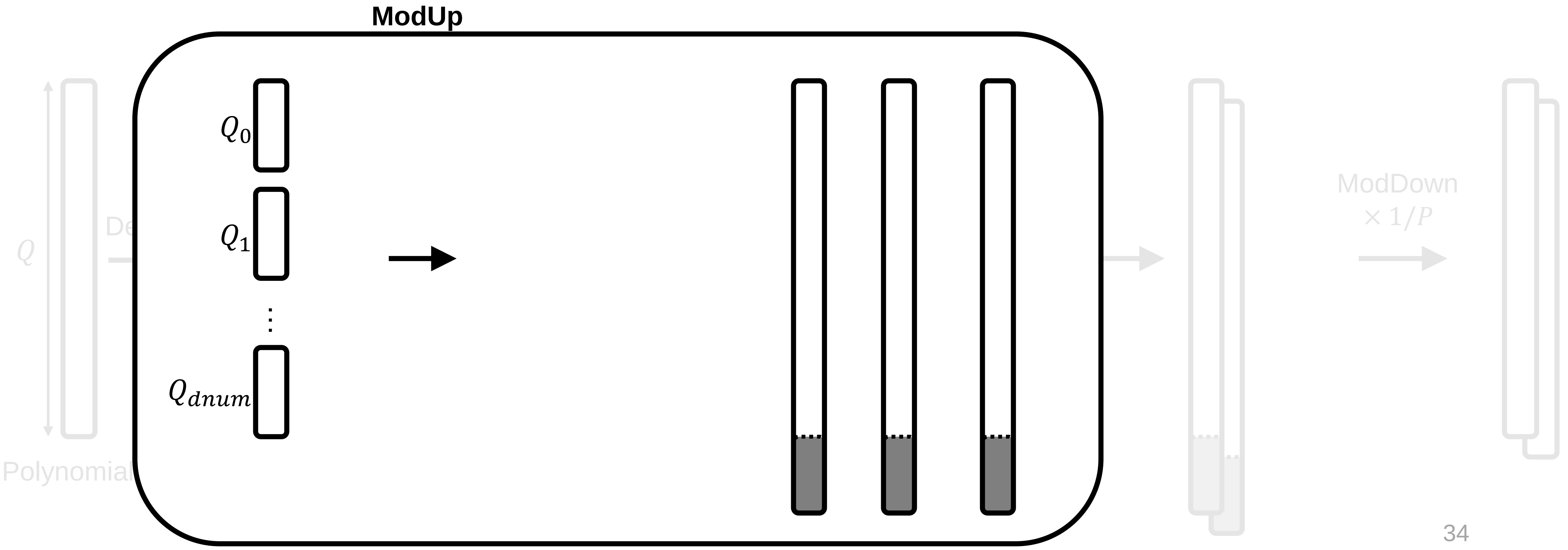

Intra-HE-operation Fusions

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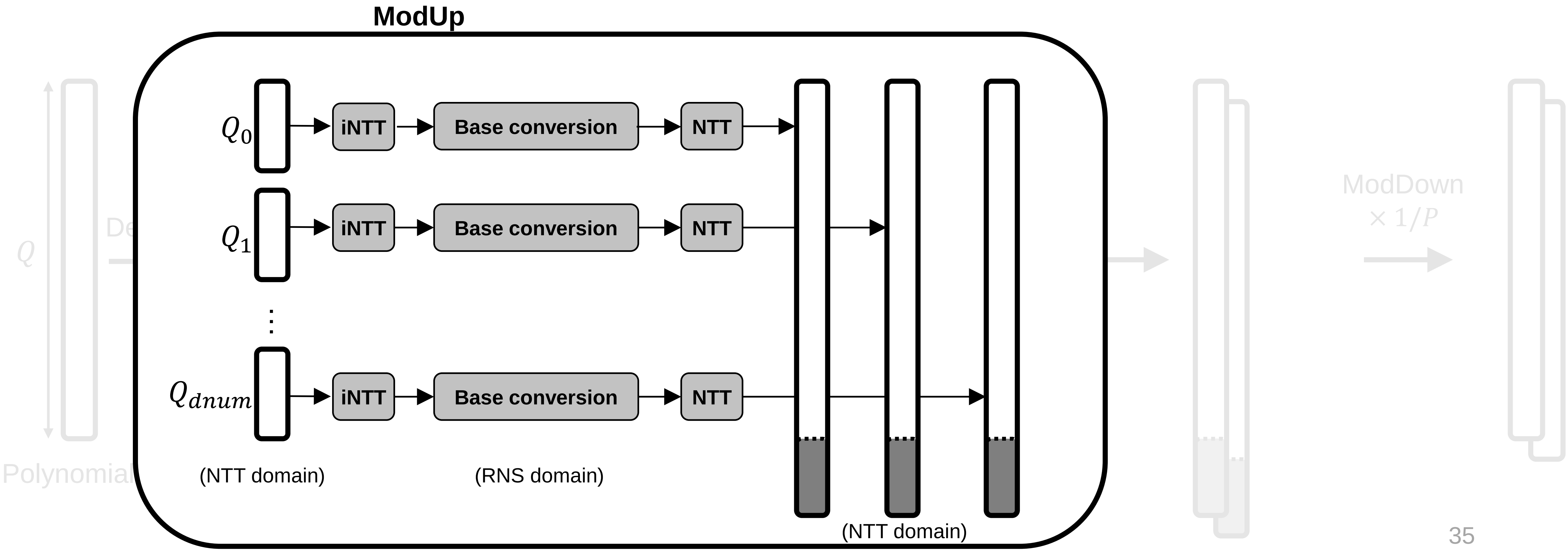
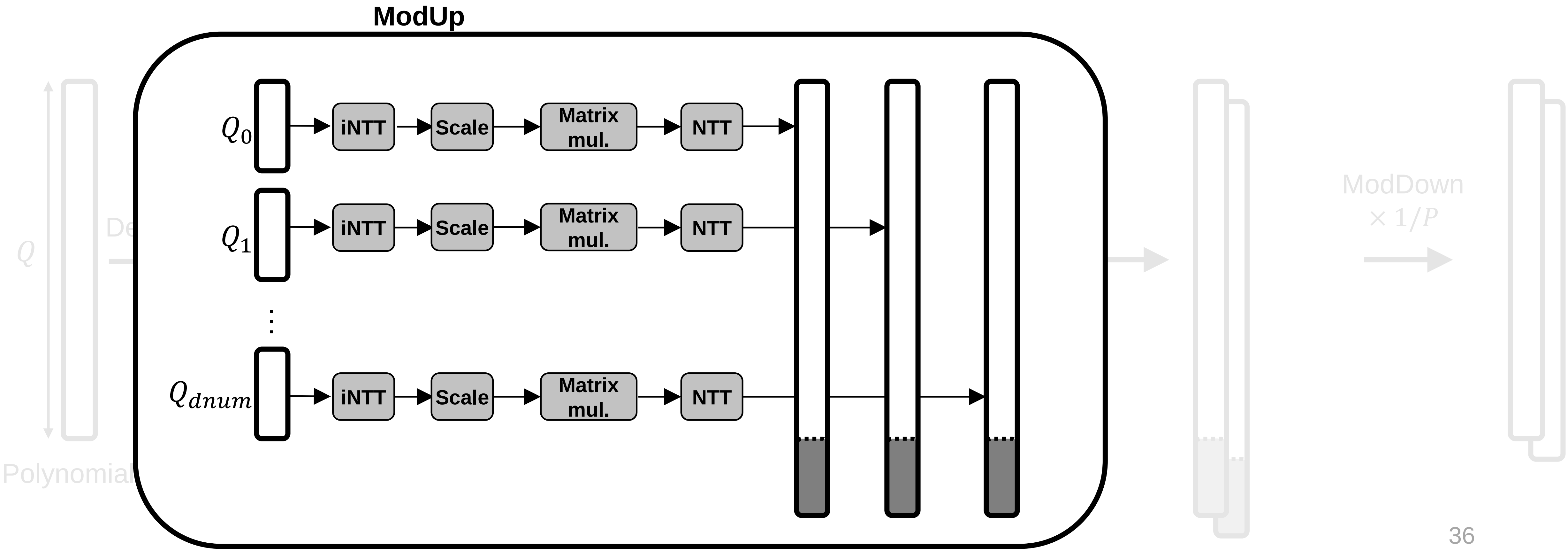


Intra-HE-operation Fusions

- *ModUp Fusion (MF)*
  1. fuses ‘small’ iNTT kernels into a large one.



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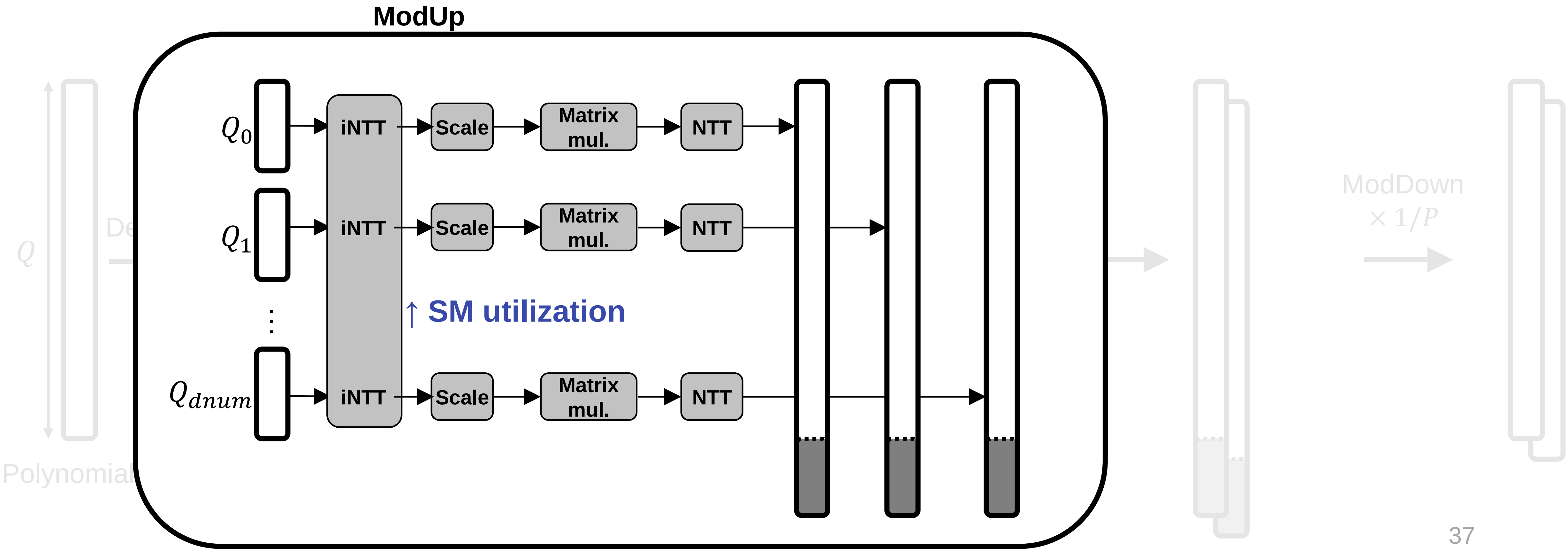
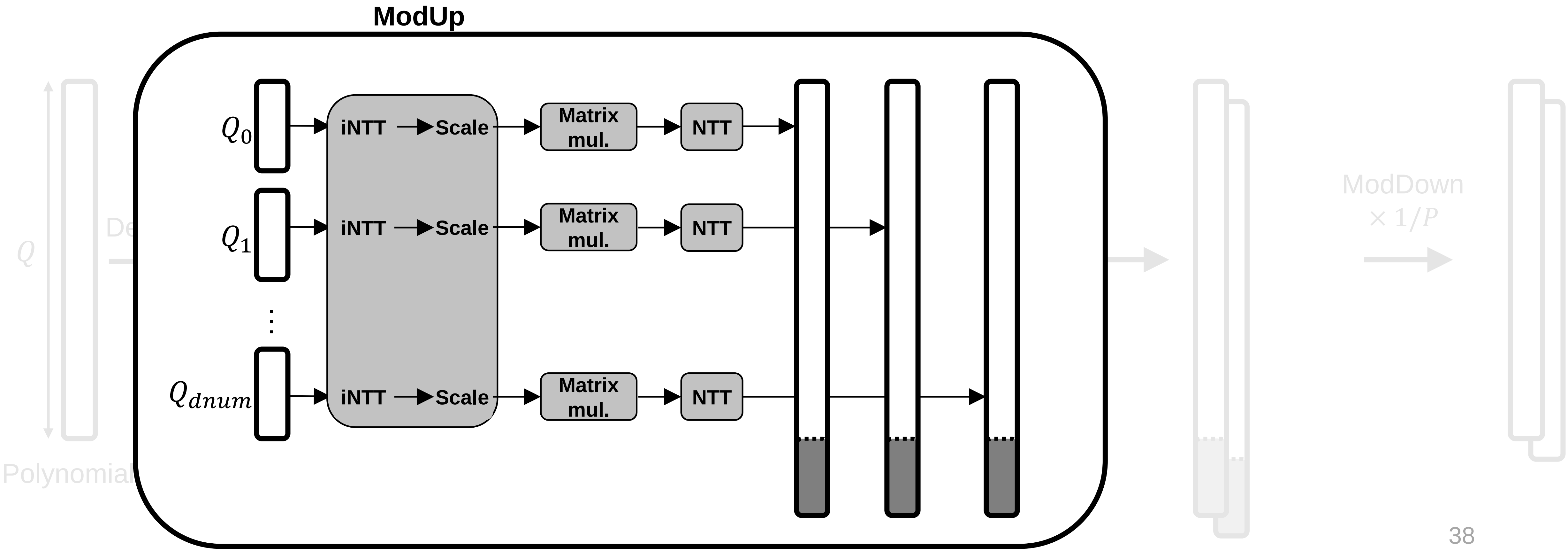


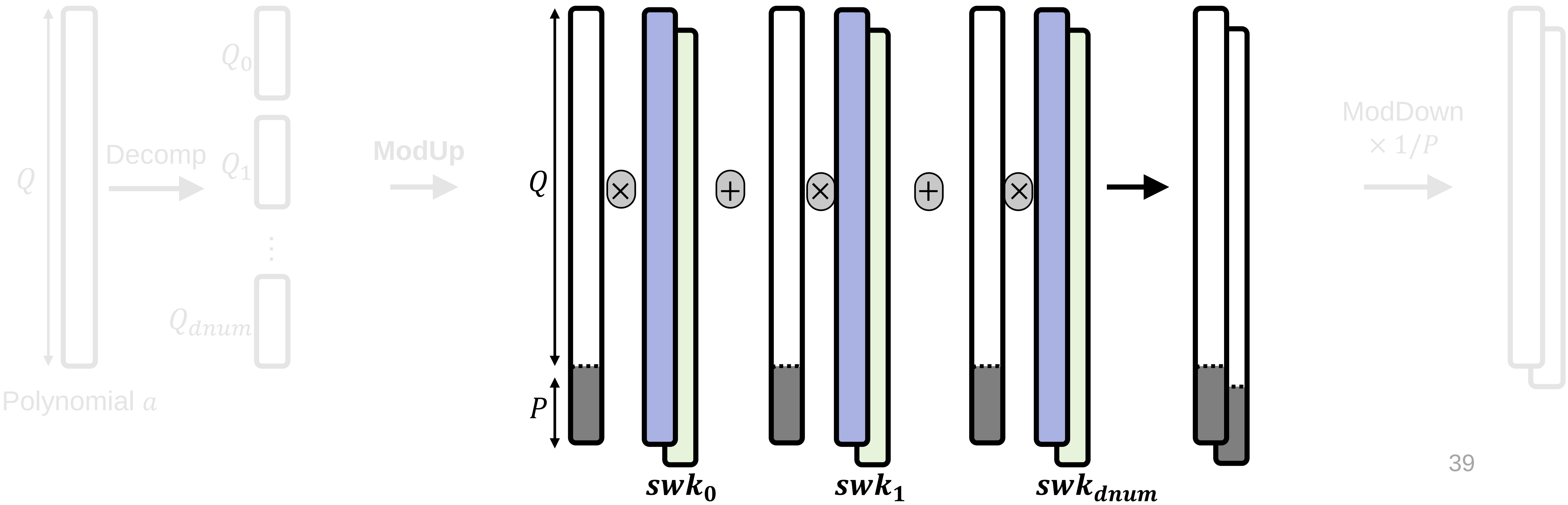
Intra-HE-operation Fusions

- *ModUp Fusion (MF)*
  1. fuses ‘small’ iNTT kernels into a large one.
  2. fuses the scaling kernels with iNTTs.



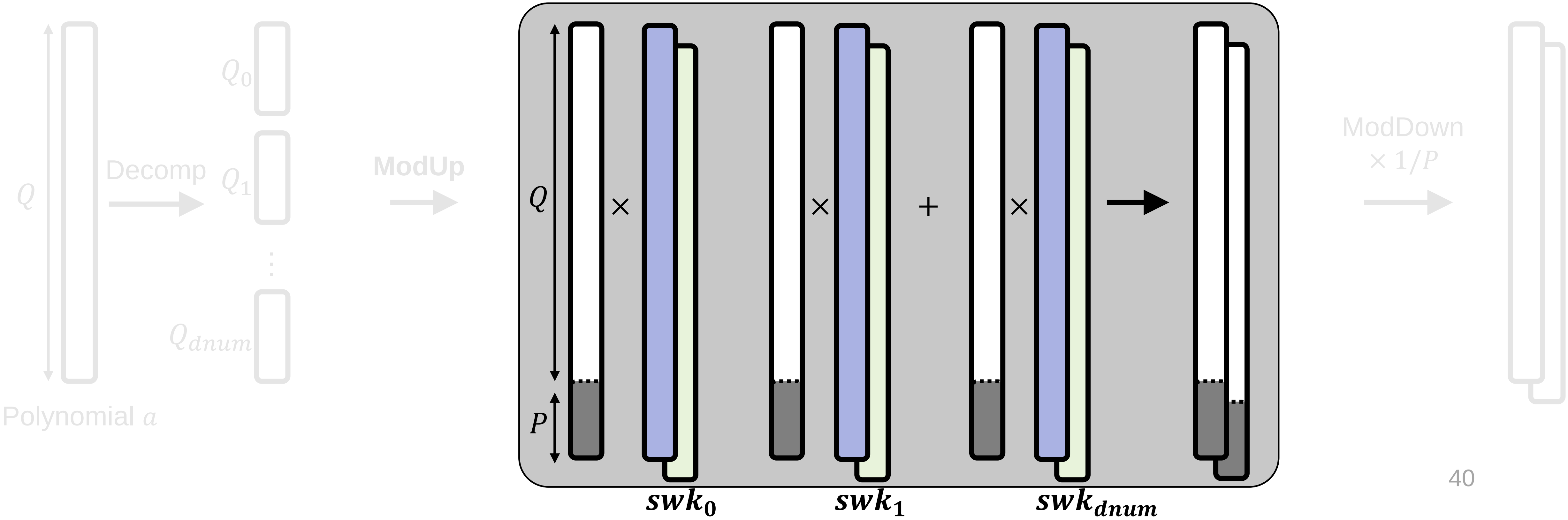
Intra-HE-operation Fusions

- *Inner-product Fusion (IF)*



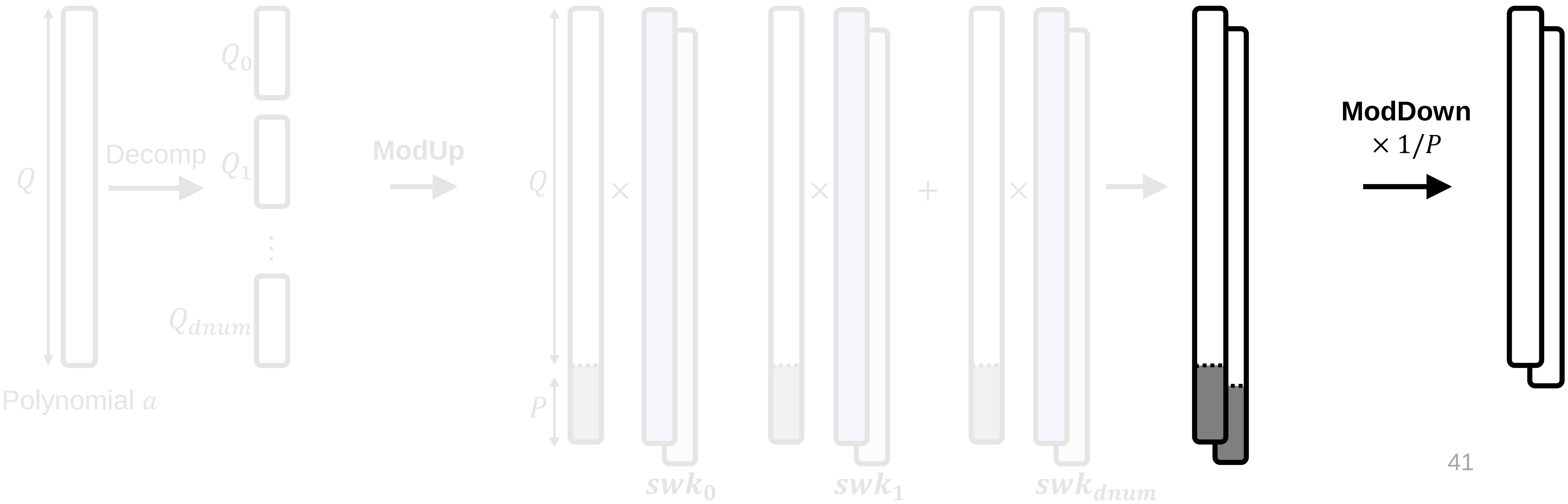
Intra-HE-operation Fusions

- *Inner-product Fusion (IF)*
  - fuses multiplications and additions into one kernel.



Intra-HE-operation Fusions

- *ModDown Fusion (MDF)*



Intra-HE-operation Fusions

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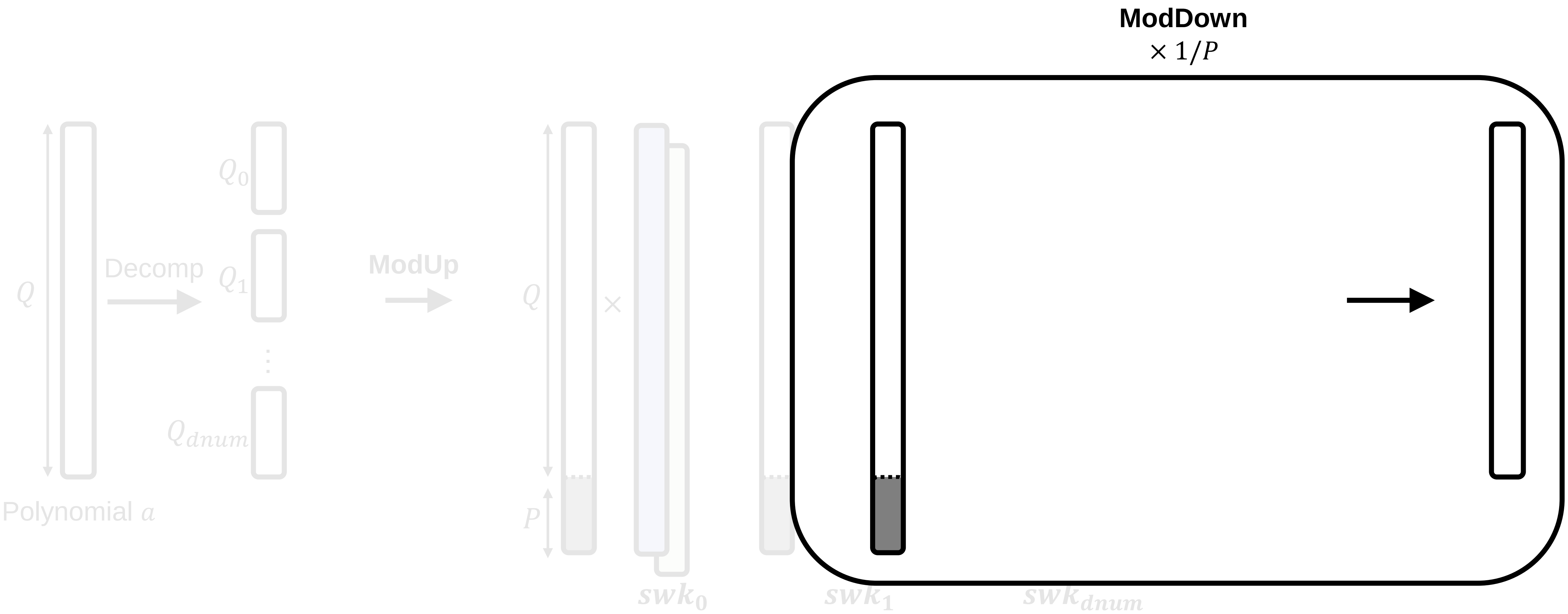

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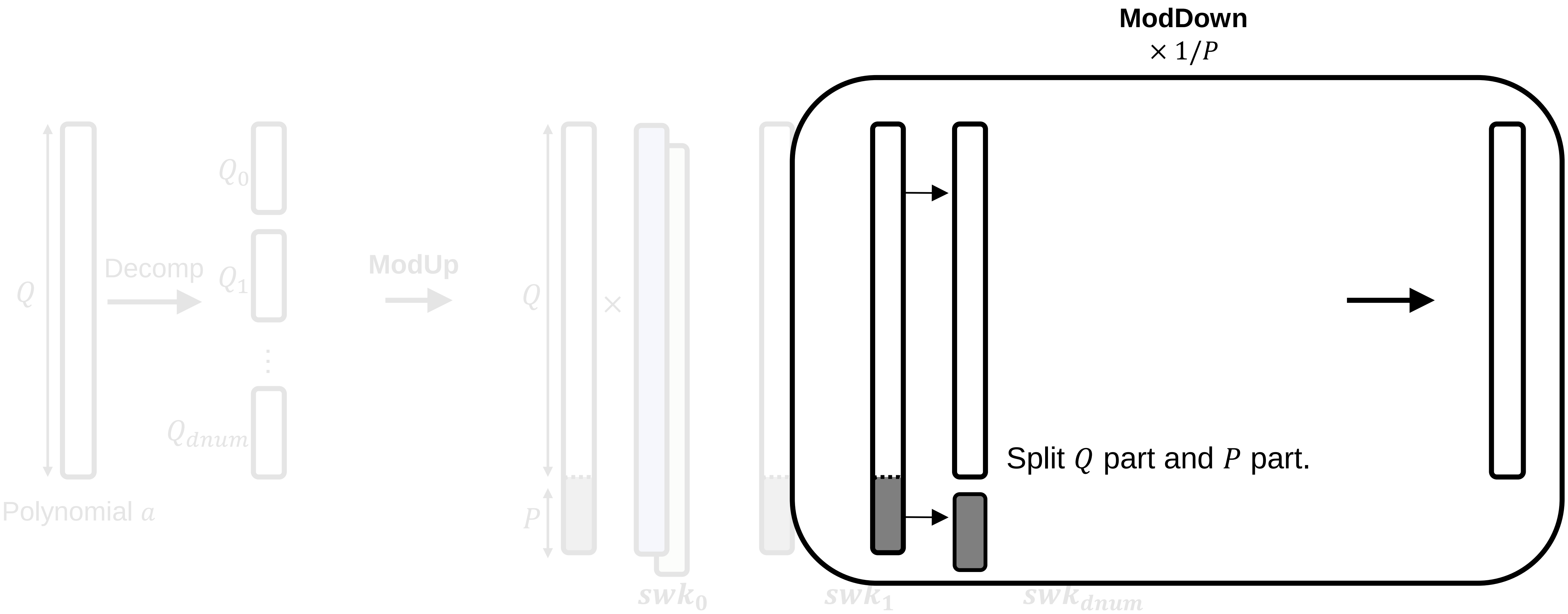

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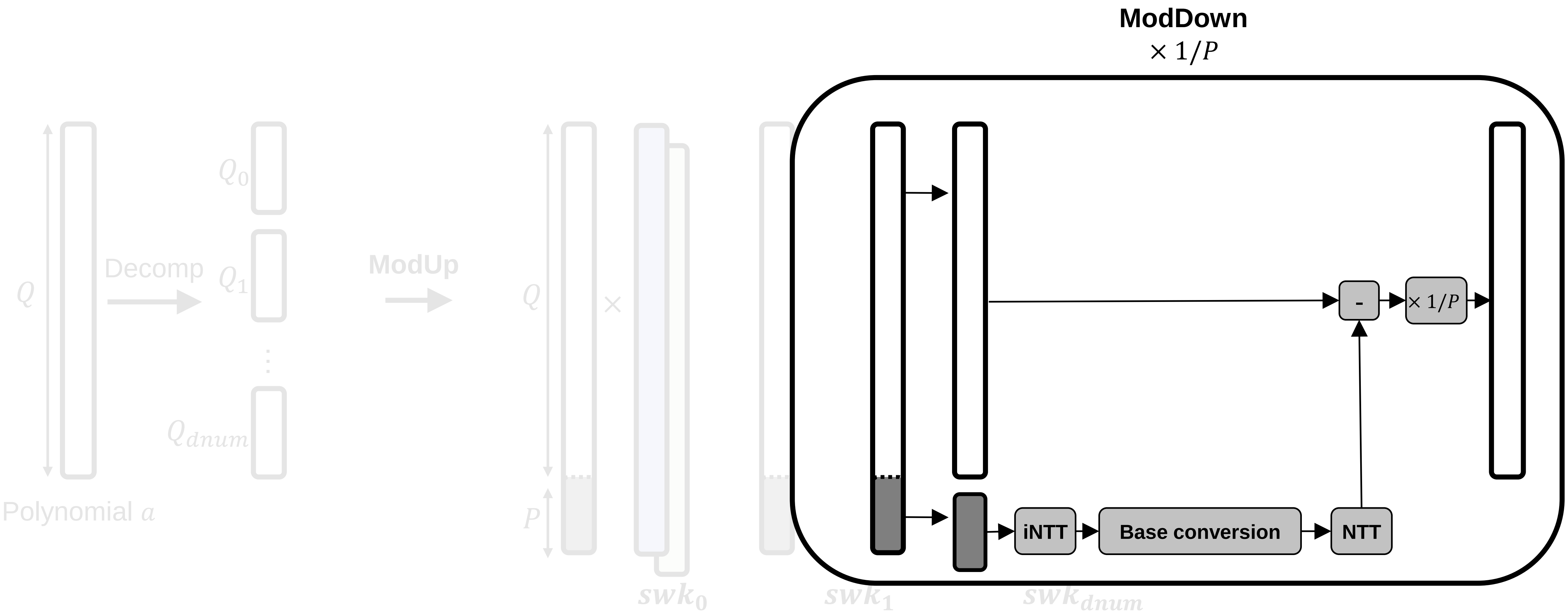
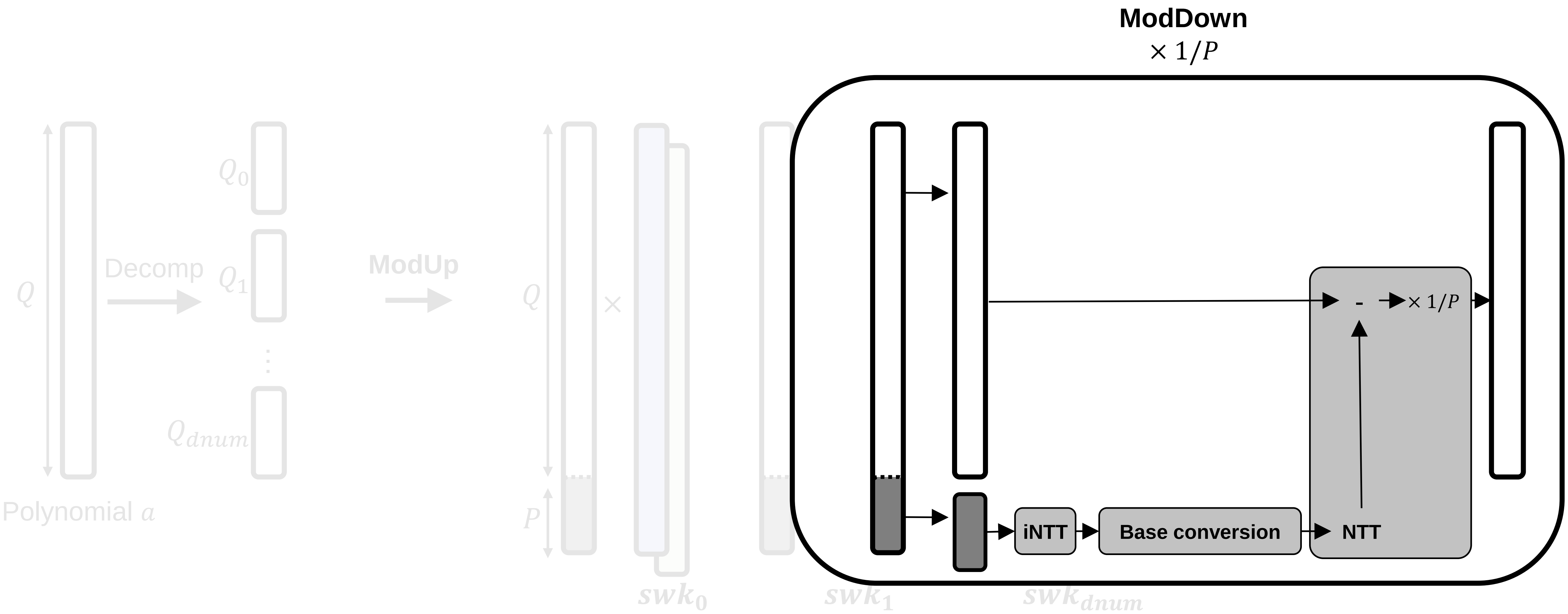


Intra-HE-operation Fusions

- *ModDown Fusion (MDF)*
  - fuses the element-wise, memory-bound kernels with NTTs.



Experiment results

- Latency of Mult and Rotate after the optimizations.

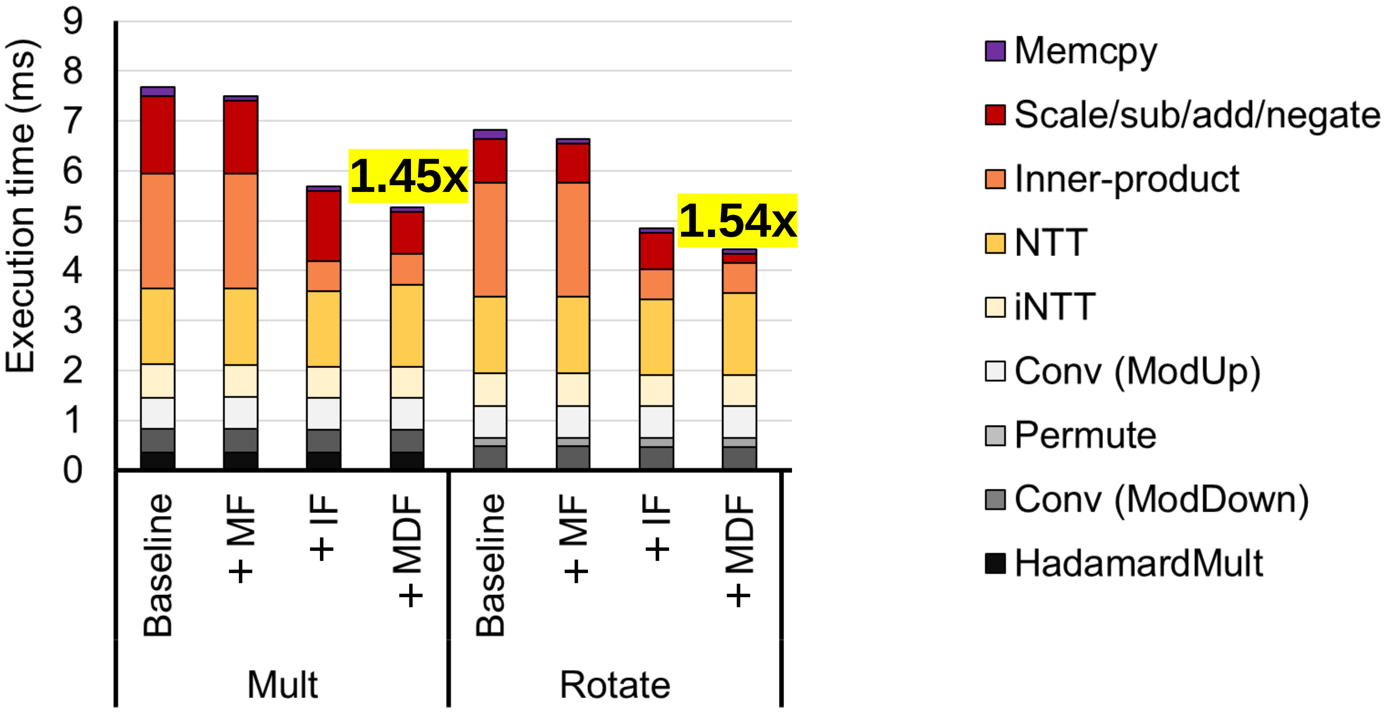
Latency with **dnum** = L (maximum)

$N = 2^{16}$, $\log Q = 2305$, $L = 44$. Used an NVIDIA V100.

Experiment results

- Latency of Mult and Rotate after the optimizations.
- **7.02x** (7.96 vs. 55.88 ms) faster Mult over a prior GPU impl. [BHM+20] which uses **dnum** = L
  - (same security & level)

Latency with **dnum** = L (maximum)

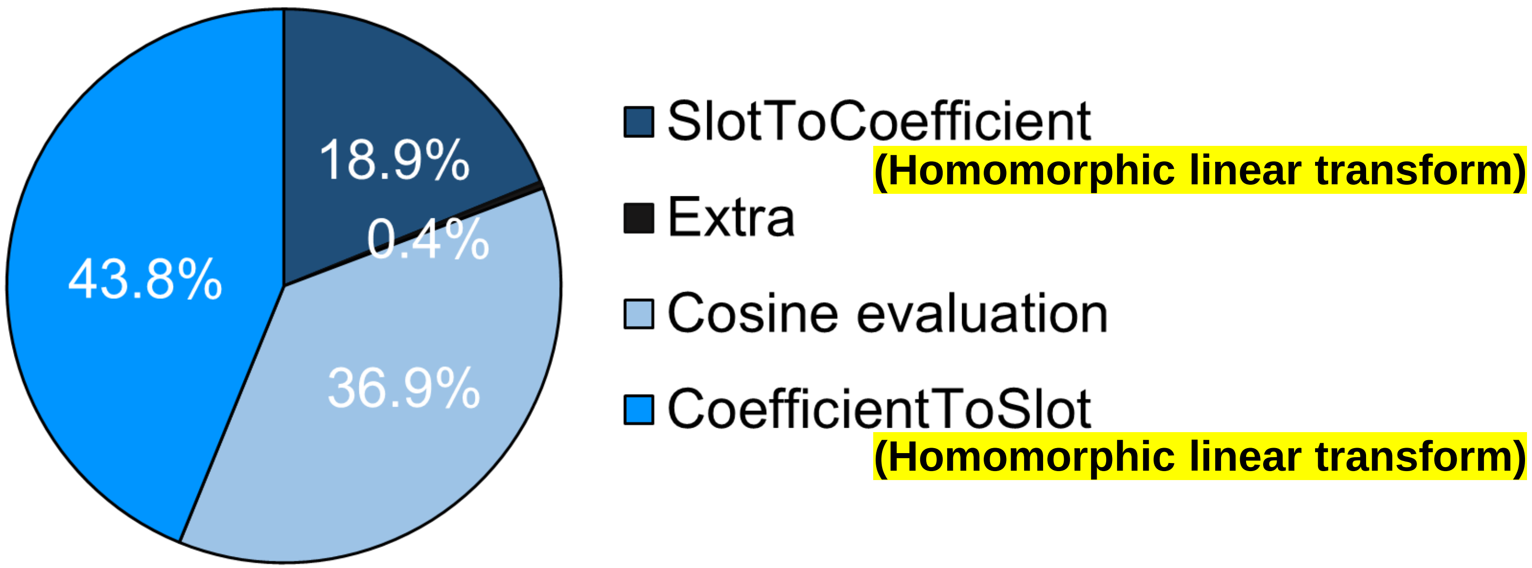


Latency with **dnum** = 3 [HK20]

$N = 2^{16}$, $\log Q = 2305$, $L = 44$. Used an NVIDIA V100.

Bootstrapping in CKKS

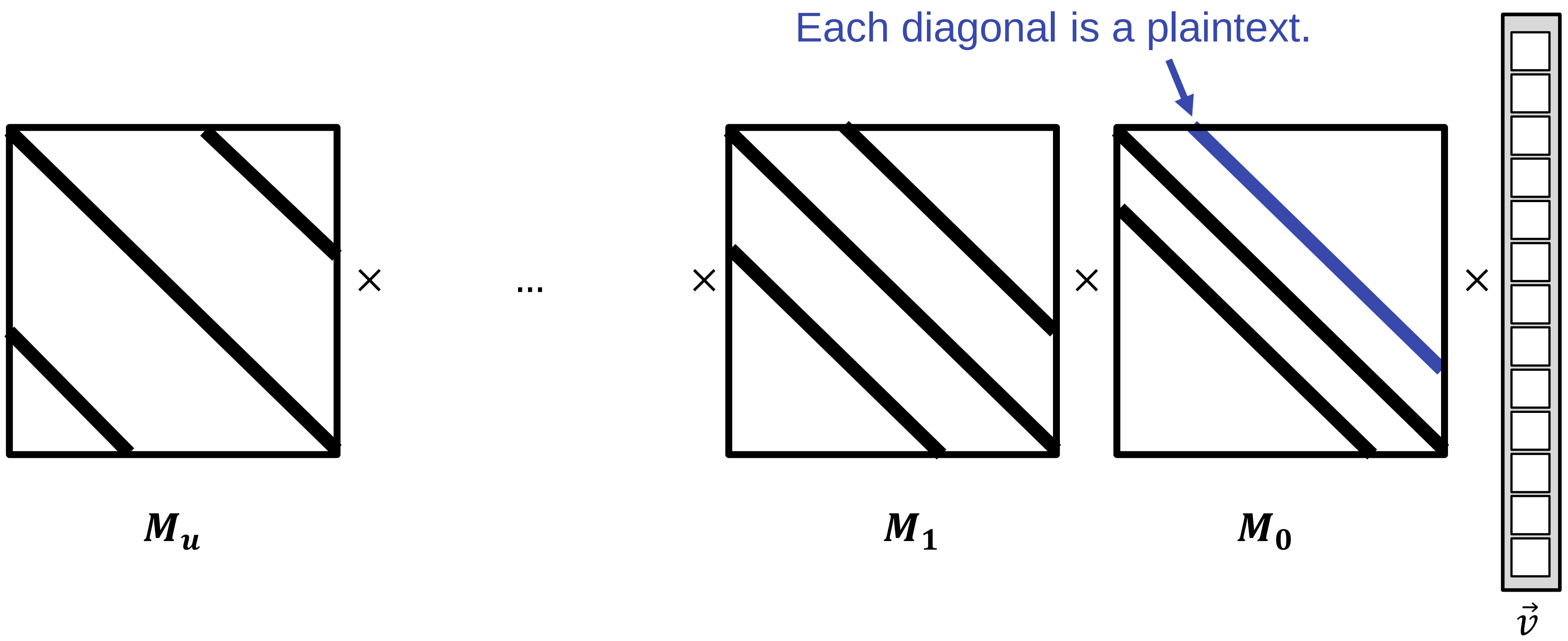
- Bootstrapping is an HE circuit, consisting of Mult, CMult, Rotate, ...
- Breakdown of a bootstrapping latency
  - Homomorphic linear transformations take up of 62.7%.



Bootstrapping latency breakdown on a GPU.

Homomorphic linear transformation in bootstrapping

- Represented as a series of matrix-vector multiplications, where
  - the matrix is sparse-diagonal matrices whose diagonals are plaintexts.
  - the vector is a ciphertext.
- How do we compute this?



ciphertext encrypting complex numbers.

Baby-step Giant-step (BSGS) algorithm

- A core algorithm for homomorphic linear transformation.
  - Used in matrix-vector multiplications in SlotToCoefficient/CoefficientToSlot
  - Setting $\ell, k \cong \sqrt{n}$, the # of rotations becomes $O(n)$➔$O(\sqrt{n})$.

The i-th diagonal element

Vector rotated by i

$$\boldsymbol{M} \cdot \vec{v} = \sum_{i=0}^{n} \mathsf{diag}_i(\boldsymbol{M}) \odot \mathsf{rot}_i(\vec{v})$$

$$= \sum_{i=0}^{\ell} \sum_{j=0}^{k} \mathsf{diag}_{ki+j}(\boldsymbol{M}) \odot \mathsf{rot}_{ki+j}(\vec{v})$$

$$= \sum_{i=0}^{\ell} \mathsf{rot}_{ki} \left(\sum_{j=0}^{k} \underbrace{\mathsf{rot}_{-ki}(\mathsf{diag}_{ki+j}(\boldsymbol{M}))}_{\text{Precomputed plaintext}} \odot \mathsf{rot}_j(\vec{v}) \right)$$

Ciphertext

Improved Homomorphic Discrete Fourier Transforms and FHE Bootstrapping, Kyoohyung Han, Minki Hhan, and Jung Hee Cheon, IEEE Access, 2019

Inter-HE-operation Fusion: Mult-and-add batching in BSGS

- Naïve multiplication-and-add requires multiple memory accesses on a temporal ciphertext.
  - Many reads and writes on $\boldsymbol{ct}$.

$$
\begin{aligned}
\boldsymbol{ct} &= m_1 \times \boldsymbol{ct_1} \\
\boldsymbol{ct} &= m_2 \times \boldsymbol{ct_2} + \boldsymbol{ct} \\
\boldsymbol{ct} &= m_3 \times \boldsymbol{ct_1} + \boldsymbol{ct} \\
&\dots
\end{aligned}
$$

The i-th diagonal element

Vector rotated by i

$$
\begin{aligned}
\boldsymbol{M} \cdot \vec{v} &= \sum_{i=0}^{n} \mathsf{diag}_i(\boldsymbol{M}) \odot \mathsf{rot}_i(\vec{v}) \\
&= \sum_{i=0}^{\ell} \sum_{j=0}^{k} \mathsf{diag}_{ki+j}(\boldsymbol{M}) \odot \mathsf{rot}_{ki+j}(\vec{v}) \\
&= \sum_{i=0}^{\ell} \mathsf{rot}_{ki} \left(\sum_{j=0}^{k} \underbrace{\mathsf{rot}_{-ki}(\mathsf{diag}_{ki+j}(\boldsymbol{M}))}_{\text{Precomputed plaintext}} \odot \overbrace{\mathsf{rot}_j(\vec{v})}^{\text{Ciphertext}} \right)
\end{aligned}
$$

Inter-HE-operation Fusion: Mult-and-add batching in BSGS

- Naïve multiplication-and-add requires multiple memory accesses on a temporal ciphertext.
  - Many reads and writes on $\boldsymbol{ct}$.
- Fusing multiple mul-and-add operations
  - removes most of read and writes on $\boldsymbol{ct}$.

$$\boldsymbol{ct} = m_1 \times \boldsymbol{ct_1}$$
$$\boldsymbol{ct} = m_2 \times \boldsymbol{ct_2} + \boldsymbol{ct}$$
$$\boldsymbol{ct} = m_3 \times \boldsymbol{ct_1} + \boldsymbol{ct}$$
$$\ldots$$

$$\boldsymbol{ct} = m_1 \times \boldsymbol{ct_1} + m_2 \times \boldsymbol{ct_2} + m_3 \times \boldsymbol{ct_1} + \cdots$$

(A single kernel)

The i-th diagonal element

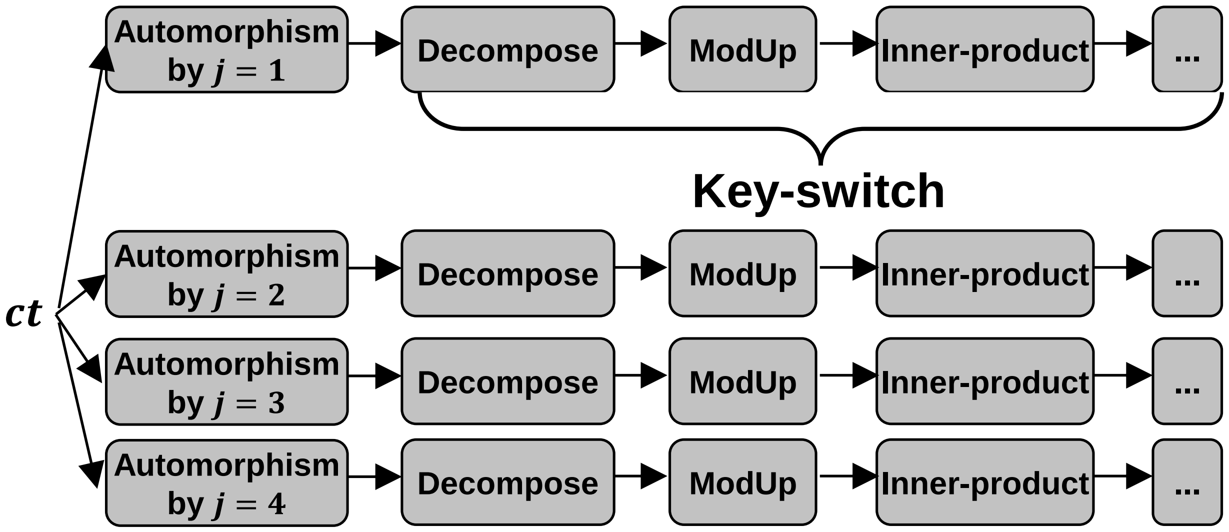
Vector rotated by i

$$\boldsymbol{M} \cdot \vec{v} = \sum_{i=0}^{n} \mathrm{diag}_i(\boldsymbol{M}) \odot \mathrm{rot}_i(\vec{v})$$
$$= \sum_{i=0}^{\ell} \sum_{j=0}^{k} \mathrm{diag}_{ki+j}(\boldsymbol{M}) \odot \mathrm{rot}_{ki+j}(\vec{v})$$
$$= \sum_{i=0}^{\ell} \mathrm{rot}_{ki} \left(\sum_{j=0}^{k} \underbrace{\mathrm{rot}_{-ki}(\mathrm{diag}_{ki+j}(\boldsymbol{M}))}_{\text{Precomputed plaintext}} \odot \mathrm{rot}_j(\vec{v}) \right)$$

Ciphertext

Hoisting [HS18]

- We also evaluated an optimization in [HS18]
- When rotating a single ciphertext multiple times, we save some ModUp computations.
  - Precomputes ModUp first, then permutes by each rotation index.



$$
\begin{aligned}
\boldsymbol{M} \cdot \vec{v} &= \sum_{i=0}^{n} \mathrm{diag}_i(\boldsymbol{M}) \odot \mathrm{rot}_i(\vec{v}) \\
&= \sum_{i=0}^{\ell} \sum_{j=0}^{k} \mathrm{diag}_{ki+j}(\boldsymbol{M}) \odot \mathrm{rot}_{ki+j}(\vec{v}) \\
&= \sum_{i=0}^{\ell} \mathrm{rot}_{ki} \left(\sum_{j=0}^{k} \mathrm{rot}_{-ki}(\mathrm{diag}_{ki+j}(\boldsymbol{M})) \odot \boxed{\mathrm{rot}_j(\vec{v})} \right)
\end{aligned}
$$

Performs rotations with different indices.

Experiment results

- Bootstrapping latency

	Latency (ms)				
Parameter set (N, L, dnum, λ)	Baseline	Intra-HE fusion	Mult-and-add Batching	Hoisting [HS18]	**Speedup (vs. single-thread CPU)**
$(2^{16}, 34, 5, 106)$	428.94	377.78	351.09	328.25	**242x**
$(2^{17}, 29, 3, 173)$	719.87	623.92	568.2	526.96	**257x**

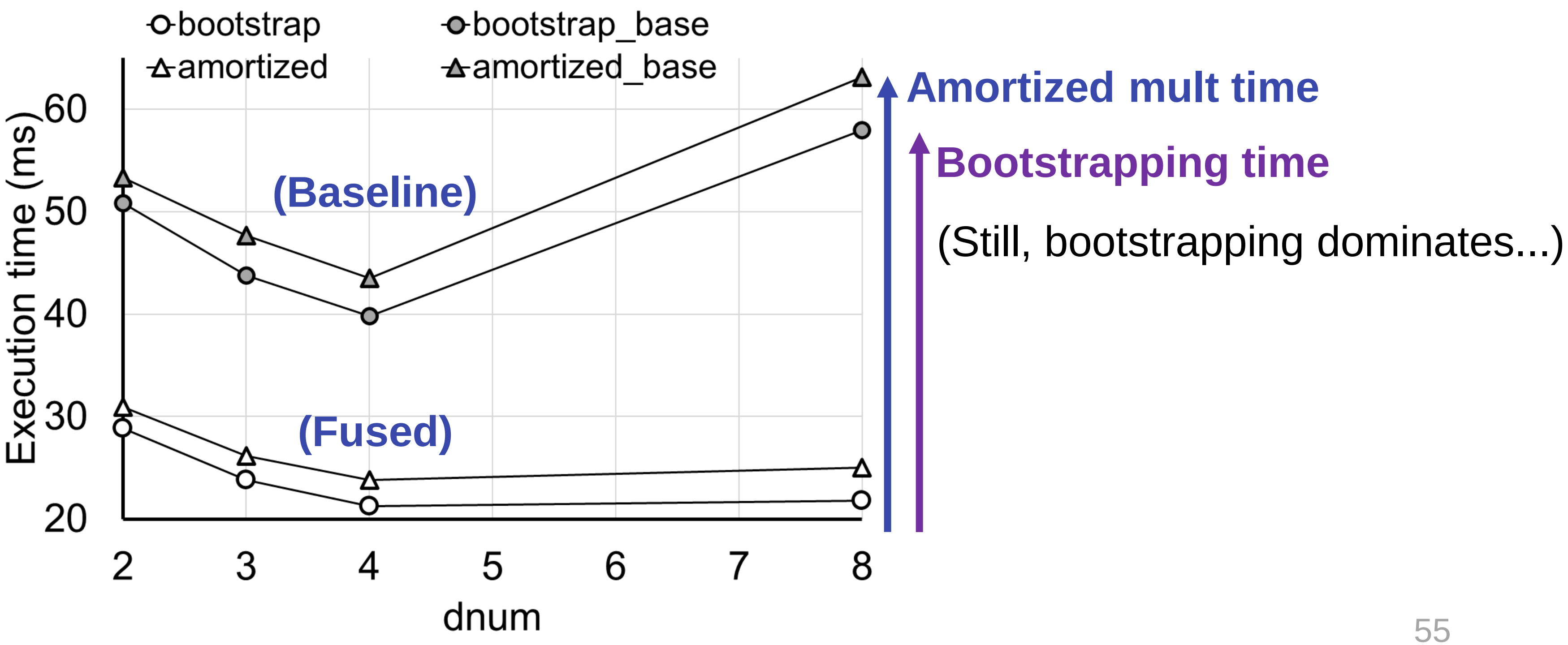
- End-to-end evaluation: training a binary classification (logistic regression) model
  - Compared to 8-threaded CPU case, GPU implementation exhibits a speedup of **40x** in total.

Amortized Mult-time

- We suggest a new metric for performance considering bootstrapping:

$$Amotized\ mult\ time = \frac{Bootstrapping\ time + multiplication\ times}{\#\ of\ multiplications\ after\ bootstrapping}$$

- Our optimizations largely amortize the memory bottleneck.



Summary

- Based on our performance analysis, we accelerated binary CKKS (HEAAN) with
  - AVX-512 & GPU implementation with microarchitecture-aware optimizations.
- Further, we accelerated the recent Full-RNS CKKS using GPU, including
  - the first implementation of bootstrapping
  - analysis on the severe memory-bandwidth bottlenecks
  - applying memory-centric optimizations, giving
    - = 257x of speedup (vs. single-threaded CPU),
    - = application-level speedups (40x vs. 8-threaded CPU in logistic regression)

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