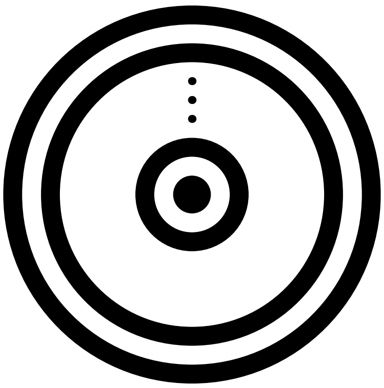
2023 WSMO Team Round

SMO Team

TR 1: Bob has a number of pencils. When Bob splits them into groups of 10, he has 3 left over. When he splits them into groups of 12, he has 5 left over. Find the smallest number of pencils Bob can have.

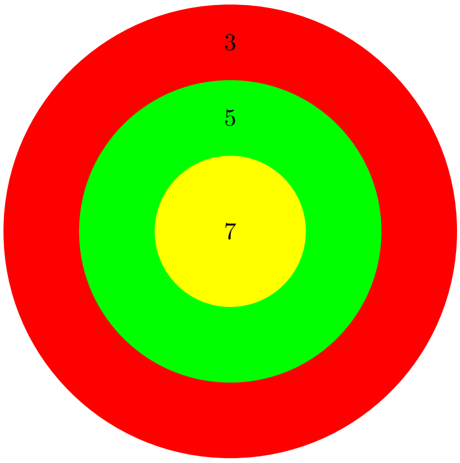
TR 2: Integers W, S, M, and O satisfy $W + S + M + O = 13$. Find the maximum possible value of $WSMO$.

TR 3: In the figure below, there are 1023 total circles. The area between circles alternate between shaded and non-shaded. If the area of the shaded region is $k\pi$, find the remainder when k is divided by 1000.



TR 4: Honko the hamster is in his cage. He wants to find the smallest distance needed to travel to reach four tennis balls. His current position is $(0, 0)$. The tennis balls are located at $(1, 1), (2, -2), (-3, -3)$, and $(-4, 4)$. The length of the shortest path can be expressed as $\sum_1^n \sqrt{a_i}$, where n is minimal. Find $\sum_1^n a_i$.

TR 5: A monkey is throwing darts at the dart board pictured below. The dart is equally likely to land anywhere on the board. Point values for the three regions are labeled and the radii the three circles are $1, 2, 3$, respectively. If the expected value of points the monkey gets from 5 dart throws is $\frac{m\pi}{n}$, for relatively prime positive integers m and n, find $m + n$.



TR 6: A quartic real polynomial $f(x)$ satisfying $f(3+2i)=0$ has 3 distinct roots. If the sum of the three roots is 12, find their product.

TR 7: In triangle ABC with $AB=13, AC=14$, and $BC=15$, a rectangle $WXYZ$ is inscribed such that the area of $WXYZ$ is maximized. If the minimum possible value of $\frac{WX}{XY}$ is $\frac{m}{n}$, for relatively prime positive integers m and n, find $m+n$.

TR 8: Let $f(x)=\sqrt{x-\sqrt{x-\sqrt{x-\ldots}}}$. Find the modulo 1000 on the minimum integer a such $f(f(f(f(f(a)))))$ is a positive integer.

TR 9: A circle has three chords of equal length, $4+2\sqrt{3}$ which intersect forming a triangle with side lengths 2, 2, and $2\sqrt{3}$. If the radius of the circle is r, then $r^2=a-b\sqrt{c}$, for positive integers a, b and squarefree c. Find $a+b+c$.

TR 10: Let square $ABCD$ be a square with side length 4. Define ellipse ω as the ellipse that is able to be inscribed inside $ABCD$ such that 2 of its vertices on its minor axis and 1 of its vertices on its major axis form an equilateral triangle. The largest possible area of ω is $m\pi\sqrt{n}$, for squarefree n. Find $m+n$.

TR 11: When $\frac{1}{7}$ is expressed in base k, the digits are $0.a_{k,1}a_{k,2}\ldots$, where $a_{k,1}a_{k,2},\ldots$ are decimal digits. Let $f(p)$ denote the minimum positive integer $x\geq 2$ such that $a_{p,1}=a_{p,x}$, for $k\not\equiv 0 \pmod 7$, and 1 for $k\equiv 0 \pmod 7$. Find $\sum_{i=2}^{100} f(i)$.

TR 12: Consider parabola $\mathcal{P}$ pointing upwards with vertex at the origin of the Cartesian plane. Denote the focus of it as F and the directrix of it as $\mathcal{L}$. Point P with x-coordinate 4 is selected in $\mathcal{P}$. The perpendicular bisector of FP meets $\mathcal{L}$ at Q. Given the x-coordinate of Q is 3, then $FP^2=\frac{m}{n}$, for relatively prime positive integers m and n. Find $m+n$.

TR 13: Gerry the Squirrel is at a corner of a 9×9 grid. 1 acorn A_1 is placed. There are four coins placed randomly on the 9×9 grid. Gerry will go the shortest path to A_1, and if there are multiple shortest paths, Gerry will pick one randomly. Given that Gerry does not start on top of a coin, the acorn is not on a coin, find the expected value of coins Gerry passes during his trip to A_1 is $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m+n$.

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TR 14: Consider quadrilateral $ABCD$ with circumcircle centered at O. Given $AB = 2 < CD$, the circumcircle of $\triangle ABO$ is tangent to the circumcircle of $\triangle COD$. The circumcircle of $\triangle AOD$ passes through the center of the circumcircle of $\triangle ABO$. Given the radius of the circumcircle of $ABCD$ is 6. Denote M, N as the area and perimeter of $ABCD$ respectively, compute $\frac{M}{N}$. The answer can be expressed in the simplest form of $\frac{p\sqrt{q}}{r}$. Find $p + q + r$

TR 15: On a number line labeled $0, 1, 2, 3, 4, 5$, and old man starts at 0 and tries to reach 5. Initially, he knows to walk right. However, he has dementia. On each move, there is a $\frac{1}{3}$ chance he forgets which direction he is supposed to go, resulting in him walking the opposite direction. If the probability the old man reaches 5 without dying is $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m + n$. (Note: if the old man tries to walk left when he is at 0, he falls off a cliff and dies.)

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