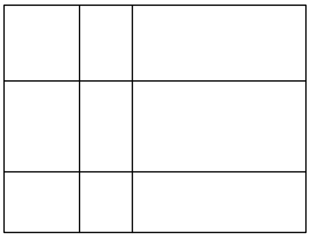
2025 SSMO Team Round

SMO Team

TR 1: There are N solutions (a, b, c) to $a + b^2 + c = 2025$, where a and b are positive integers and c is a nonnegative integer. Find the number of positive factors of N.

TR 2: Nonnegative integers a and b satisfy $a! + b^4 = 16a + 41$. Find the sum of all possible values of b.

TR 3: A rectangle is divided as shown into nine smaller rectangles. The areas of the five smallest rectangles are 1, 2, 3, 4, and 5. What is the largest possible area of the original rectangle?



TR 4: Let $P(x) = x^2 - ax + b$ be a quadratic with nonzero real coefficients. Given that $P(a)$ and $P(-b)$ are roots of $P(x)$, there exists a value of c such that $P(c)$ is constant for all possible $P(x)$. Find c.

TR 5: Bob rolls a fair six-sided die until he rolls an even number. What is the expected sum of all the numbers he rolled?

TR 6: The rhombus $PQRS$ has side length 3. The point X lies on segment PR such that line QX is perpendicular to line PS. Given $QX = 2$, the area of $PQRS$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

TR 7: Let n be the largest integer such that $20!^{25!} + 25!^{20!}$ is divisible by 2025^n. The value of n can be written in the form $a \cdot b!$, where a and b are positive integers and b is maximized. Find $a + b$.

TR 8: Adam has a fair coin with a 2 written on one side and a 3 written on the other. What is the expected number of times Adam needs to flip the coin for the sum of all his coin flips to be a multiple of 36?

TR 9: Pairwise distinct integers a, b, c, and d satisfy the system of equations

$$ab = cd$$
$$a + d = b + c$$
$$\tfrac{1}{a} + \tfrac{1}{b} - \tfrac{1}{5} = \tfrac{1}{c} + \tfrac{1}{d} + \tfrac{1}{5}.$$

What is the minimum possible value of $a^2 + b^2 + c^2 + d^2$?

TR 10: Anna has a three term arithmetic sequence of integers. She divides each term of her sequence by a positive integer $n > 1$, and tells Bob that the three resulting remainders are 20, 52, and R, in some order. For how many values of R is it possible for Bob to uniquely determine n?

TR 11: Squares s_1, s_2, and s_3 with side lengths 9, 17, and 10, respectively, lie inside of $\triangle ABC$ such that:

- s_1 has a side that lies on $\overline{AB}$, s_2 has a side that lies on $\overline{BC}$, and s_3 has a side that lies on $\overline{CA}$;
- each square shares exactly one vertex with each of the other two squares.

Find the perimeter of $\triangle ABC$.

TR 12: Let $a_n = (4 + 3\sqrt{2})^n$ for all nonnegative integers n. Let

$$\sum_{k=0}^{\infty} \frac{\lfloor a_k \rfloor}{10^k} = \frac{m}{n},$$

where m and n are relatively prime positive integers. Find $m + n$.

TR 13: Let S be the set of all ordered quintuples of integers $(x_1, x_2, x_3, x_4, x_5)$ satisfying $0 \le x_1 \le x_2 \le x_3 \le x_4 \le x_5 \le 5$. The *conjugate* of a quintuple in S is defined as $(y_1, y_2, y_3, y_4, y_5)$, where for each integer $1 \le i \le 5$, y_i is the number of indices $1 \le j \le 5$ satisfying $x_j \ge i$. A randomly chosen quintuple of S is a permutation of its conjugate with probability $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

TR 14: Find the number of ordered triples of positive integers (a, b, c) such that $a + b + c = 305$ and $ab + bc + ca$ is a multiple of 305.

TR 15: The circles ω_1 and ω_2 have radii 8 and 7, respectively, and intersect at points A and B. A line ℓ passing through A intersects ω_1 again at P and ω_2 again at Q, with $PQ = 24$. There exists a point T on line AB, with A between T and B, such that PT is tangent to ω_1 and QT is tangent to ω_2. The length of BT can be written as $\frac{m}{\sqrt{n}}$, where m and n are positive integers such that n is square-free. Find $m + n$.

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