2023 SSMO Tiebreaker Round

SMO Team

Tiebreaker Round Problem 1: From the phrase "Summer Solstice", how many ways are there to make a 4 letter "word" such that the second and third letters aren't spaces and the first letter is capital. Note: You can only use a letter twice if it appears in "Summer Solstice" twice.

Tiebreaker Round Problem 2: Let $P(x) = x^3+3ax^2+3bx+(a+b)$ be a real polynomial with nonnegative and nonzero real roots p, q, r. Suppose that

$$(p+1)^3 + (q+1)^3 + (r+1)^3 + 3P(-1) = 0$$

If $P(1) = a_1 + b_1\sqrt{c_1}$, for squarefree c_1, find $a_1 + b_1 + c_1$.

Tiebreaker Round Problem 3: For $n \geq 4$, let a_n be the maximum possible value of $P(n+1)$ given that $P(x)$ is a n degree monic polynomial that satisfies $P(i) \in \{1, 2, 3 \ldots, n\}$ for $1 \leq i \leq n$. If $\frac{m}{n} = \sum_{n=4}^{\infty} \frac{a_n - n!}{3^n}$, for relatively prime positive integers m and n, find $m+n$.