2022 SSMO Speed Round

SMO Team

SR 1: Bobby is bored one day and flips a fair coin until it lands on tails. Bobby wins if the coin lands on heads a positive even number of times in the sequence of tosses. Then the probability that Bobby wins can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SR 2: A bag is big enough to hold exactly 6 large pencils, 12 medium pencils, or 30 small pencils, with no space left over. Given that there is 1 large pencil and 3 medium pencils currently in the bag, what is the maximum number of small pencils that may be added to the bag? Note that there may still be space left over in the bag.

SR 3: Let $ABCD$ be a parallelogram such that E is a point on CD such that $\frac{CE}{DE} = \frac{2}{3}$. Suppose that BE and AC intersect at F. If the area of triangle AEF is 15, find the area of $ABCD$.

SR 4: Consider a quadrilateral $ABCD$ with area 120 and satisfying $AB + CD = AD + BC = 24$. There exists a point P in 3D space such that the distances from P to AB, BC, CD, and DA are all equal to 13. Find the volume of $PABCD$.

SR 5: Let $ABCD$ be a square such that E is on AD and F is on CD. If $AE = DF$ and $\frac{[BEF]}{[ABCD]} = \frac{7}{18}$, then the value of $\frac{EF^2}{BC^2}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SR 6: At the beginning of day 1, there is a single bacterium in a petri dish. During each day, each bacterium in the petri dish divides into $a > 1$ new bacteria, and $b \geq 1$ bacteria are added to the petri dish (these bacteria do not divide on the day they were added). For example, at the end of day 1, there are $a+b$ bacteria in the petri dish. If, at the end of day 4, the number of bacteria in the petri dish is a multiple of 48, find the minimum possible value of $a+b$.

SR 7: Let $A_1 = (1, 0)$. Define A_{n+1} as the image of A_n under a rotation of either $45°$, $90°$, or $135°$ clockwise about the origin, with each choice having a $\frac{1}{3}$ chance of being selected. Find the expected value of the smallest positive integer $n > 1$ such that A_n coincides with A_1.

SR 8: How many positive integers cannot be written as $7a + 19b + 28c$, where a, b, and c are positive integers (not necessarily distinct)?

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SR 9: Consider a triangle ABC such that $AB = 13$, $BC = 14$, $CA = 15$ and a square $WXYZ$ such that Y and Z lie on $\overleftrightarrow{BC}$, W lies on $\overleftrightarrow{AB}$, and X lies on $\overleftrightarrow{CA}$. Suppose further that W, X, Y, and Z are distinct from A, B, and C. Let O be the center of $WXYZ$. If AO intersects BC at P, then the sum of all values of $\frac{BP}{CP}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 10: Let $S = \{2, 7, 15, 26, ...\}$ be the set of all numbers for which the i^{th} element in S is the sum of the i^{th} triangular number and the i^{th} positive perfect square. Let T be the set which contains all unique remainders when the elements in S are divided by 2022. Find the number of elements in T.