2023 SSMO Speed Round

SMO Team

SR 1: Let $S_1 = \{2, 0, 3\}$ and $S_2 = \{2, 20, 202, 2023\}$. Find the last digit of

$$\sum_{a \in S_1, b \in S_2} a^b.$$

SR 2: Let A, B, C be independently chosen vertices lying in the square with coordinates $(-1,-1)$, $(-1,1)$, $(1,-1)$, and $(1,1)$. The probability that the centroid of triangle ABC lies in the first quadrant is $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m+n$.

SR 3: Pigs like to eat carrots. Suppose a pig randomly chooses 6 letters from the set $\{c, a, r, o, t\}$. Then, the pig randomly arranges the 6 letters to form a "word". If the 6 letters don't spell carrot, the pig gets frustrated and tries to spell it again (by rechoosing the 6 letters and respelling them). What is the expected number of tries it takes for the pig to spell "carrot"?

SR 4: Let $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$ be the Fibonacci numbers. If distinct positive integers $a_1, a_2, \ldots a_n$ satisfies $F_{a_1} + F_{a_2} + \cdots + F_{a_n} = 2023$, find the minimum possible value of $a_1 + a_2 + \cdots + a_n$.

SR 5: In a parallelogram $ABCD$ of dimensions 6×8, a point P is choosen such that $\angle APD + \angle BPC = 180°$. Find the sum of the maximum, M, and minimum values of $(PA)(PC) + (PB)(PD)$. If you think there is no maximum, let $M = 0$.

SR 6: Find the smallest odd prime that does not divide $2^{75!} - 1$.

SR 7: At FenZhu High School, 7th graders have a 60% of chance of having a dog and 8th graders have a 40% chance of having a dog. Suppose there is a classroom of 30 7th grader and 10 8th graders. If exactly one person owns a dog, then the probability that a 7th grader owns the dog is $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m+n$.

SR 8: Circle ω has chord AB of length 18. Point X lies on chord AB such that $AX = 4$. Circle ω_1 with radius r_1 and ω_2 with radius r_2 lie on two different sides of AB. Both ω_1 and ω_2 are tangent to AB at X and ω. If the sum of the maximum and minimum values of $r_1 r_2$ is $\frac{m}{n}$, find $m+n$.

SR 9: Find the sum of the maximum and minimum values of $8x^2 + 7xy + 5y^2$ under the constraint that $3x^2 + 5xy + 3y^2 = 88$.

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SR 10: In a circle centered at O with radius 7, we have non-intersecting chords AB and DC. O is outisde of quadrilateral $ABCD$ and $AB < CD$. Let $X = AO \cup CD$ and $Y = BO \cup CD$. Suppose that $XO+YO = 7$. If $YC-DX = 2$ and $XY = 3$, then $AB = \frac{a\sqrt{b}}{c}$ for $\gcd(a, c) = 1$ and squareless b. Find $a + b + c$.

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