

2022 SSMO Relay Round 5

SMO Team

RR 5 Part 1: Consider an 8×8 chessboard with a knight in one of the center squares. The knight may move in an L -shaped fashion, going two squares in one direction and one square in a perpendicular direction, but cannot go outside the chessboard. How many squares can the knight reach in exactly two moves?

RR 5 Part 2: Let $T = \text{TNYWR}$, and let $S = \{a_1, a_2, \dots, a_{2022}\}$ be a sequence of 2022 positive integers such that $a_1 \leq a_2 \leq \dots \leq a_{2022}$ and $\text{lcm}(a_1, a_2, \dots, a_{2022}) = 70T$. Also, $\gcd(a_i, a_j) = 1$ for all $1 \leq i < j \leq 2022$. Find the number of possible sequences S .

RR 5 Part 3: Let $T = \text{TNYWR}$, and let $a_k = \text{cis}\left(\frac{k\pi}{T+1}\right)$. Suppose that

$$\sum_{k=1}^{T+1} \frac{|a_{2k} + a_{2k+2} - a_{2k-1+T}|}{|a_{2k+1} - (a_{2k} + a_{2k+2})|}$$

can be expressed in the form of $a + b \cos\left(\frac{\pi}{c}\right)$, where $\text{cis}(x) = \cos(x) + i \sin(x)$. Find $a + b + c$.

