2022 SSMO Relay Round 3

SMO Team

RR 3 Part 1: Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function such that $f(0) = 0$ and $f\left(|x^2 - 4|\right) = 0$ if $f(x) = 0$. Moreover, $|f(x + 1) - f(x)| = 1$ for all $x \in \mathbb{Z}$. Let N be the number of possible sequences $\{f(1), f(2), \ldots, f(21)\}$. Find the remainder when N is divided by 1000.

RR 3 Part 2: Let T = TNYWR. In cyclic quadrilateral $ABCD$, $\angle BAD = 60°$, and $BC = CD = T$. If AB is a positive integer, find twice the median of all (not necessarily distinct) possible values of AB.

RR 3 Part 3: Let T = TNYWR. Let $f(x)$ be a polynomial of degree 10, such that $f(i) = i$ for all $i = 1, 2, \ldots, 10$ and $f(11) = T$. Find the remainder when $f(13)$ is divided by 1000.