2025 SSMO Speed Round

SMO Team

SR 1: Define

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ x^2+1 & \text{if } x \geq 0 \end{cases}$$

and let $x_0 = -3$. Define $x_{n+1} = f(x_n)$. Find the least n such that $x_n > 100$.

SR 2: Let A and B be points such that $AB = 50$. Points M and N lie on $\overline{AB}$ such that M lies between points A and N and N lies between points B and M. Given that $MN = 20$ and $AN \cdot BM$ is maximized, find the length of AM.

SR 3: Anna is buying different types of cheese from the local supermarket. Let x, y, and z be the number of pieces of blue, cheddar, and mozzarella cheese, respectively, that Anna buys. She can buy any nonnegative integer number of each type, but the total number of pieces must be at most 12. How many different combinations (x, y, z) of cheese can Anna buy? (Anna is allowed to buy 0 pieces of cheese.)

SR 4: In rectangle $ABCD$, let $AB = 8, BC = 15, \omega$ be the circumcircle of $ABCD$, ℓ be the line through B parallel to AC, and $X \neq B$ be the intersection of ℓ and ω. Suppose the value of BX can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SR 5: Let $N = 101112\cdots9899$ be the number formed when all the two-digit positive integers are concatenated in increasing order. How many ordered triples of digits (a, b, c) are there such that a, b, and c appear as consecutive digits (in that order) in the decimal representation of N?

SR 6: The centroid G of $\triangle ABC$ has distances 21, 60, and 28 from sides AB, BC, and CA, respectively. Find the perimeter of $\triangle ABC$.

SR 7: Positive integers a and b satisfy $63a = 40b$. The sum of all possible values of $\frac{\varphi(a)}{\varphi(b)}$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SR 8: Let S be the set of all ordered pairs (P, Q), where P and Q are subsets of $\{1, 2, \ldots, 25\}$ satisfying $|P \cup Q| = 17$ and $|(P \cap Q) \cap \{20, 25\}| \geq 1$. If an ordered pair (A, B) is chosen randomly from S, the expected value of $|A \cap B|$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SR 9: Let ABC be a triangle. The point P lies on side BC, the point Q lies on side AB, and the point R lies on side AC such that $PQ = BQ$, $CR = PR$, and $\angle APB < 90^\circ$. Let H be the foot of the altitude from A to BC. Given that $BP = 3$, $CP = 5$, and $[AQPR] = \frac{3}{7} \cdot [ABC]$, the value of $BH \cdot CH$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 10: Let p be a quadratic with a positive leading coefficient, and let r be a real number satisfying $r < 1 < \frac{5}{2r} < 5$. Given that $p(p(x)) = x$ for $x \in \{r, 1, \frac{5}{2r}, 5\}$, find $p(12)$.

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