2023 SSMO Relay Round 3

SMO Team

RR 3 Part 1: In triangle ABC with $AB = 13, AC = 14, BC = 15$, circles ω_1, ω_2, and ω_3 are drawn, centered at A, B and $, C$, respectively. Each of the three circles are externally tangent to the two other circles. If the radius of a circle ω such that ω is internally tangent to ω_1, ω_2, and ω_3 is $\frac{m}{n}$, for relatively prime positive integers m and n, find $m + n$.

RR 3 Part 2: Let $T = TNYWR$. In triangle ABC with circumradius and inradius having lengths R and r, respectively. Given that

$$\sin \angle A + \sin \angle B + \sin \angle C = \left\{ \sqrt{N} \right\}$$

the maximum value of

$$8 \sin \angle A \sin \angle B \sin \angle C$$

is $b + c\sqrt{a}$, for squarefree a, find $|a + b + c|$. (Note that $\{x\} = x - \lfloor x \rfloor$)

RR 3 Part 3: Let $T = TNYWR$. Let $n = N + 1$. A spray painter has a paint gun that paints all areas within a radius of 2. The spray painter walks in the following locations, where red lines indicate red paint coming out of the gun and blue lines indicate blue paint coming out of the gun. The spray painter starts from the outermost square and works his way inwards, where in the end. The positive difference between the area of the blue-painted region and the area of the red-painted region is $a + b\pi$. Find $a + b$. (Note: if a spray painter paints an area with multiple colors, only the last color will be showing).