2021 WSMO Accuracy Round

SMO Team

AR 1: Find the sum of all the positive integers n such that n is $\frac{2n^2-5n+5}{n-5}$ an integer.

AR 2: A fair 20-sided die has faces labeled with the numbers $1, 3, 6, \ldots, 210$. Find the expected value of a single roll of this die.

AR 3: Suppose f is a monic polynomial of minimal degree with rational coefficients satisfying $f\left(3+\sqrt{5}\right)=0$ and $f\left(4-\sqrt{7}\right)=0$. Find the value of $|f(1)|$.

AR 4: A 12-hour clock has a minute hand that is the same length as the second hand, and an hour hand half the length of the minute hand. In a day, the tip of the minute hand travels a distance of m, the tip of the second hand travels a distance of s, and the tip of the hour hand travels a distance of h. The value of $\frac{m^2}{hs}$ can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

AR 5: Suppose regular octagon $ABCDEFGH$ has side length 5. The distance from the center of the octagon to one of the sides can be expressed as $\frac{a+b\sqrt{c}}{d}$, where c is a squarefree positive integer and a, b, d are relatively prime positive integers. Find $a+b+c+d$.

AR 6: Roy is baking a circular three tier cake. All of the tiers are centered around the same point. Each tier's radius is $\frac{3}{4}$ of the radius of the tier below it, but the height of each tier stays constant. Roy wants to ice the cake, but only on the curved surfaces of the cake and the top of the smallest tier. The diameter of the lowest tier is 128 centimeters and its height is 10 centimeters. The surface area that is iced can be expressed as $m\pi$. Find m.

AR 7: Find the value of $\displaystyle\sum_{n=1}^{100}\left(\sum_{i=1}^{n} r_i\right)$, where r_i is the remainder when 2^i+3^i is divided by 10.

AR 8: 20 unit spheres are stacked in a triangular pyramid formation, such that the first layer has 1 sphere, the second layer has 3 spheres, the third layer has 6 spheres, and the fourth layer has 10 spheres. The radius of the smallest sphere that fully contains all of these spheres is $\frac{a+b\sqrt{c}}{d}$, where c is a squarefree positive integer and a, b, d are relatively prime positive integers. Find $a+b+c+d$.

AR 9: Let $x = 1+\frac{5}{2+\frac{3}{2+\frac{3}{2+\ldots}}}$. The value of $\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}$ can be written as $\frac{a+\sqrt{b}}{c}$, where a, b, c are positive integers with b squarefree. Find $a+b+c$.

AR 10: The largest value of x that satisfies the equation $5x^2-7\lfloor x\rfloor\{x\}=\frac{26\lfloor x\rfloor^2}{5}$ can be expressed as $\frac{a+b\sqrt{c}}{d}$, where c is a squarefree positive integer and a, b, d are relatively prime positive integers. Find $a+b+c+d$.