2021 WSMO Speed Round

SMO Team

SR 1: Let $f^1(x) = (x-1)^2$, and let $f^n(x) = f^1(f^{n-1}(x))$. Find the value of $|f^7(2)|$.

SR 2: A square with side length of 4 units is rotated around one of its sides by $90°$. The volume the square sweeps out can be expressed as $m\pi$. Find m.

SR 3: Let $a@b = \frac{a^2-b^2}{a+b}$. Find the value of $1@(2@(\dots(2020@2021)\dots))$.

SR 4: A square $ABCD$ with side length 10 is placed inside of a right isosceles triangle XYZ with $\angle XYZ = 90°$ such that A and B are on XZ, C is on YZ, and D is on XY. Find the area of XYZ.

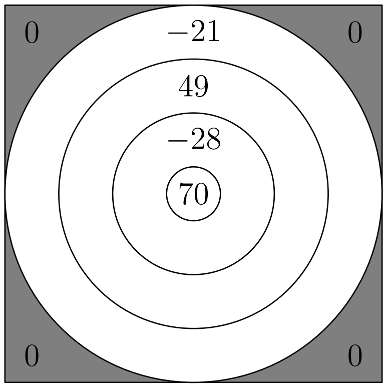
SR 5: The number of ways to arrange the characters in "delicious greenbeans" into two separate strings of letters can be expressed as $a \cdot b!$, where b is maximized and both a and b are positive integers. Find $a+b$. (A string of letters is defined as a group of consecutive letters with no spaces between them.)

SR 6: A bag weighs 1 pound and can hold 16 pounds of food at maximum. Danny buys 100 packages of tomatoes and 300 packages of potatoes. Tomatoes come in packages that are 12 ounces each and potatoes come in packages that are 24 ounces each. All of Danny's food must be packed into bags, and Danny will use only as many bags as he needs. What is the minimum possible weight, in pounds, of Danny's luggage, including the bags?

SR 7: Consider triangle ABC with side lengths $AB = 13, AC = 14, BC = 15$ and incircle ω. A second circle ω_2 is drawn which is tangent to AB, AC and externally tangent to ω. The radius of ω_2 can be expressed as $\frac{a-b\sqrt{c}}{d}$, where c is a squarefree positive integer and a, b, d are relatively prime positive integers. Find $a+b+c+d$.

SR 8: Let n be the number of ways to seat 12 distinguishable people around a hexagon with two people seated on each side, where the order of the two people on each side does not matter and rotations are disregarded. Find the number of divisors of n.

SR 9: Bobby is going to throw 20 darts at the dartboard shown below. It is formed by 4 concentric circles, with radii of $1, 3, 5,$ and 7, with the largest circle being inscribed in a square. Each point on the dartboard has an equally likely chance of being hit by a dart, and Bobby is guaranteed to hit the dartboard. Each region is labeled with its point value (the number of points Bobby will get if he hits that region). The expected number of points Bobby will get after throwing the 20 darts can be expressed as $\frac{m}{n}\pi$, where m and n are relatively prime positive integers. Find $m+n$.



SR 10: Find the remainder when $\underbrace{2021^{2022^{\cdots^{2022^{2021}}}}}_{2021\ 2021\text{'}s} \cdot \underbrace{2022^{2021^{\cdots^{2021^{2022}}}}}_{2022\ 2022\text{'}s}$ is divided by 11.