2024 SSMO Relay Round 5

SMO Team

RR 5 Part 1: Let the super factorial $!(n)$ be defined on positive integers as $\prod_{i=1}^{n} i!$. Find the largest positive integer k such that that there are exactly k positive integers n such that $!(n)$ has fewer than k trailing zeroes.

RR 5 Part 2: Let $T = TNYWR$. In the game of high and low, the computer chooses two integers without replacement from the set $\{1, 2, 3, \ldots, T\}$. The computer displays the first integer and asks the player if the second integer is higher or lower. Given that the player always plays optimally, the chances of guessing correctly is $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m + n$.

RR 5 Part 3: Let $T = TNYWR$. Let k be the maximum prime factor that divides T. How many values of x satisfy both $\sin x^2 + \cos x^2 = \sin^2 x + \cos^2 x$ and $-k \leq x \leq k$?