

# 2025 SSMO Relay Round 4

SMO Team

**RR 4 Part 1:** Call a positive integer *chuzzed* if the sum of the digits in its binary representation is equal to the units digit of its base-10 representation. Similarly, call a positive integer *chopped* if its binary representation does not contain two consecutive ones. Find the number of positive integers less than 128 that are chuzzed and chopped.

**RR 4 Part 2:** Let  $T = TNYWR$ . Jonathan and Kate are playing a game with  $n$  sticks. On each turn, a player may remove 1, 2, or 3 sticks. The player who picks up the last stick loses. Kate is first to remove sticks, and both players play optimally. For how many values of  $n$  in the range  $[T^3, 2T^3]$  does Kate have a winning strategy?

**RR 4 Part 3:** Let  $T = TNYWR$ . A particle moves in the coordinate plane such that at any time  $t$ , its position is

$$\left( \sum_{a=1}^{T-1} \cos(at), \sum_{a=1}^{T-1} \sin(at) \right).$$

Over the time interval  $t \in (0, k]$ , the particle lies on at least one coordinate axes  $T$  times. If the minimal value of  $k$  can be written as  $\frac{m\pi}{n}$  for relatively prime positive integers  $m$  and  $n$ , find  $m + n$ .

