2025 WSMO Accuracy Round

SMO Team

AR 1: Real numbers x and y satisfy

$$x + y = 10 \quad \text{and} \quad x^2 + y^2 = 58.$$

Find $|x - y|$.

AR 2: Points A, B, C, D, and E are collinear such that $AB = 20$, $BC = 52$, $CD = 34$, and $DE = 25$. Find the minimum possible length of EA.

AR 3: On a string of holiday lights, there are 4 blue lights for every 3 green lights, 7 red lights for every 6 blue lights, and 20 white lights for every 13 colored lights. What is the smallest number of lights that can be on the string of lights?

AR 4: Evaluate:

$$\sum_{1 \le a,b \le 25} \left\lfloor \frac{a + b\sqrt{2}}{1 + \sqrt{2}} \right\rfloor.$$

AR 5: Let $\mathcal{E}$ be an ellipse. Points W, X, Y, and Z lie on $\mathcal{E}$ such that WY and XZ are parallel to the minor and major axes of $\mathcal{E}$, respectively. Suppose that segments WY and XZ intersect at point P. Given that $PW = 9$, $PX = 6$, $PY = 5$, and $PZ = 30$, the area of $\mathcal{E}$ can be expressed as $n\pi$. Find n.

AR 6: Let $t_1, t_2, \ldots, t_6$ be integers and $N < 100$ be a positive integer such that $Nt_1 < 60t_2 < 27t_3 < 8t_4 < 50t_5 < 45t_6$. Over all choices of $t_1, t_2, \ldots, t_6$, the minimum possible value of $45t_6 - Nt_1$ is 25. Find the sum of all possible values of N.

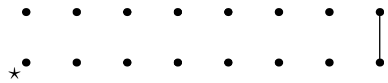
AR 7: Let S be the set of all non-congruent right triangles with integer side lengths such that the area of the triangle is equal to three times the perimeter of the triangle. Find the sum of the lengths of hypotenuses of all the right triangles in S.

AR 8: Nonnegative real numbers x and y satisfy the system of equations

$$16x^6 - 24x^4 + 9x^2 = \frac{y+1}{2}$$
$$8y^3 - 6y = 1.$$

The smallest possible value of $\arccos(x)$ can be expressed as $\frac{m}{n}^\circ$, where m and n are relatively prime positive integers. Find $m + n$.

AR 9: Amy the ant lives in the 8×2 rectangular grid of dots shown. From some dot D, she can move to a dot that is either horizontally adjacent to D, diagonally adjacent to D, or connected to D by a segment. In how many different ways can Amy visit every dot exactly once if she starts at the starred dot?



AR 10: Let $f : \{1, 2, \ldots, 20\} \to \{1, 2, \ldots, 20\}$ be a function such that for all $m, n \in \{1, 2, \ldots, 20\}$, we have

$$\gcd(m, f(n)) = \gcd(n, f(m)).$$

How many such functions f are there?