

2024 SSMO Relay Round 3

SMO Team

RR 3 Part 1: Alice and Bob are on the second floor of a building right next to an escalator moving upwards. Alice and Bob each run at 200 steps per minute and the escalator is a total of 225 steps. Alice chooses to use the escalator moving upwards, while Bob chooses to run to the escalator moving downwards, 300 steps away, and then ride the escalator down. If Alice and Bob hit the bottom floor at the exact same time, find the speed of the escalators in terms of steps per minute.

RR 3 Part 2: Let $T = TNYWR$. Find the greatest odd integer n for which $n^2 + (T - 1)n$ is a perfect square.

RR 3 Part 3: Let $T = TNYWR - 1$. Riley and Boris are playing a game on a $T \times T$ grid of dots. The game alternates turns and starts with Riley. Each turn, a player draws a line connected two different random dots, exactly 1 unit apart. The first person to complete the first unit square loses the game. Given that Riley plays optimally and Boris plays randomly, the probability that Riley wins can be expressed as P . Find the least positive integer a such that aP is an integer.

