2024 SSMO Speed Round

SMO Team

SR 1: Find the sum of the distinct prime factors of $2024^2 - 1$.

SR 2: Gracie's students play with some toys. When 4 or 5 students are present, the toys can be equally distributed to everyone. However, when there are only 3 students, there is one toy leftover after giving everyone the same number of toys. What is the least possible number of toys that Gracie could have?

SR 3: The polynomial $x^3 - 15x^2 + 4x + 4$ has distinct real roots r, s, and t. Find the value of

$$\left|(r^2 + s^2 + t^2)(rst)\right|.$$

SR 4: Sam wants to read the *Harry Potter* and *Warriors* books. There are 7 *Harry Potter* books that must be read in a specific order, and there are 6 *Warriors* books that also must be read in a specific order; however, he can read the two series at the same time. For example, he could read the first three *Harry Potter* books, then the first five *Warriors* books, then the remaining *Harry Potter* books, and finally the last *Warriors* book. In how many unique orders can Sam read the books?

SR 5: Let $\triangle ABC$ and $\triangle ADC$ be right triangles, such that $\angle ABC = \angle ADC = 90^\circ$. Given that $\angle ACB = 30^\circ$ and $BC = 3\sqrt{3}$, find the maximum possible length of BD.

SR 6: There are 4 people and 4 houses. Each person independently randomly chooses a house to live in. The expected number of inhabited houses can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 7: Let S denote the set of all ellipses centered at the origin and with axes AB and CD where $A = (-x, 0), B = (x, 0), C = (0, -y)$, and $D = (0, y)$, for $2 \mid x + y$ and $0 \leq x, y \leq 10$. Let T denote the set of similar ellipses centered at the origin and passing through (x, y) for $2 \nmid x + y$ and $0 \leq x, y, \leq 10$. If the positive difference between the sum of the areas of all ellipses in T and the sum of the areas of all the ellipses in S is $m\pi$, find m.

SR 8: Bob has two coins; one is fair, and one lands on heads with a probability of $\frac{2}{3}$. Bob chooses a random coin and flips it twice. Alice watches the two coin flips and guesses whether Bob flipped the fair or rigged coin. Given that Alice is a good mathematician and guesses the more likely option (guessing randomly when they are equally likely), the probability she guesses right can be expressed as $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m + n$.

SR 9: Let a, b, c, and d be positive integers such that $abcd = a + b + c + d$. Find the maximum possible value of a.

SR 10: Let $a_1, a_2, \ldots a_{14}$ be the roots of $(x^7 - x^3 + 2)^2 = 0$. Find the value of $\prod_{i=1}^{14}(a_i^7 + 1)$.