2025 SSMO Relay Round 5

SMO Team

RR 5 Part 1: The numbers 17 and 25 are written on a blackboard. Every second, if the number n is currently written on the blackboard, it is replaced by $3n$ with probability p and by $2n + 3$ with probability $1 - p$. For example, after one second, the two numbers on the blackboard may be $3 \cdot 17 = 51$ and $2 \cdot 25 + 3 = 53$. Given that the expected positive difference between the two numbers on the blackboard after six seconds is 1000, the value of p can be expressed as $\sqrt{a} - b$, where a and b are positive integers. Find $a + b$.

RR 5 Part 2: Let $T = TNYWR$. Ethan places T red cards and T blue cards in a row so that the leftmost card is red and every second card is red. He performs a sequence of operations called *replacements*: in each replacement, he randomly selects a card other than the leftmost one, discards it, and moves the leftmost card to the discarded card's position. This process is repeated until only one card remains. The probability that the final remaining card is red is $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

RR 5 Part 3: Let $T = TNYWR$. How many positive integers n satisfy

$$\mathrm{lcm}(\gcd(n, n^2 + T^2), \gcd(T, n^2 + T^2)) = \mathrm{lcm}(n, T)?$$