2023 SSMO Relay Round 4

SMO Team

RR 4 Part 1: Martha starts out with the number 7 on her calculator, which has three buttons that multiply the current number by $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ respectively. She randomly presses one of the buttons four times. After these 4 presses, she cubes the number. The expected value of the final number is $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m+n$.

RR 4 Part 2: Let $T = TNYWR$. Let $n = \left\lfloor \sqrt{N} \right\rfloor$. Suppose that N points are chosen on the sides of a triangle with area 1 such that there is at least one point on each side. Let m be the area of the polygon formed by connecting the N points in counterclockwise order. Find the expected value of $\frac{30}{1-m}$ (Note that $\{x\} = x - \lfloor x \rfloor$)

RR 4 Part 3: Let $T = TNYWR$. $N+1$ numbers are chosen from the set $\{1, 2, 3, \ldots, N+1\}$ with replacement. If the probability that the median of these $N+1$ numbers is greater than $\frac{N+2}{2}$ is M, such that the decimal representation of $\frac{1}{M}$ has a 0's before the first nonzero digit of it, find n rounded to nearest multiple of 5.