2025 WSMO Team Round

SMO Team

TR 1: Define $f(n)$ as the remainder when n is divided by 5. Compute

$$f(1) + f(2) + f(3) + \cdots + f(2025).$$

TR 2: Let a and b be positive reals such that $\frac{1}{a} + \frac{1}{b} = \frac{1}{6}$. Find the minimum possible value of $a + b$.

TR 3: How many distinct ways are there to arrange the letters in SOLSTICE such that no two vowels are adjacent and the resulting word does not begin or end with a vowel.

TR 4: For how many positive integers $n \leq 2025$ does n^6 divide evenly into 6^n?

TR 5: Anna picks two distinct prime numbers $p > q$ such that $n = pq(p - q) < 1000$. She tells Bob the value of n, but Bob is not able to uniquely determine the values of p and q. Find the largest possible value of n.

TR 6: Let $d(n)$ be a function from the positive integers to the integers defined by

$$d(n) = \sum_{m>0 \text{ divides } n} (-1)^{d(m)}.$$

Find the value of $d(30^9)$.

TR 7: Let $\{a_n\}_{n\geq 1}$ and $\{b_n\}_{n\geq 1}$ be sequences of real numbers with the property that for every positive integer n,

$$a_{n+1}a_n = b_{n+1}b_n + b_{n+1}a_n + a_{n+1}b_n.$$

Given that $a_1 = 25$, $b_1 = 20$, and $a_{100} = 2025$, find the value of b_{100}.

TR 8: Let $ABCD$ be a trapezoid with $AB \parallel CD$. Circles ω_1 and ω_2 have diameters AD and BC, respectively, and are tangent at point P. Given that $PA = 3$, $PB = 2$, and $PC = 4$, the value of PD^2 can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

TR 9: Let k be a positive integer such that the equation

$$\left\lfloor \frac{k}{\left\lfloor \frac{k}{n} \right\rfloor} \right\rfloor = n$$

has exactly 25 positive integer solutions n. Find the largest possible value of k.

TR 10: Let a, b, and c be complex numbers such that $|a| = |b| = |c| = 1$. Suppose that the three summations

$$\sum_{i=0}^{5} a^i b^{5-i}, \ \sum_{i=0}^{7} b^i c^{7-i}, \text{ and } \sum_{i=0}^{11} c^i a^{11-i}$$

are all nonzero real numbers. Find the number of possible ordered pairs (a, b, c).

TR 11: Let A, B, and C be collinear in that order such that $AB = 7$, $AC = 17$. Additionally, let B, M, and N be collinear in that order such that $BN = 10$, $MN = 3$. Let P be a point such that $NP \perp AN$ and $NP = MC$. As M and N vary, the set of all possible points P traces out a convex region with area $k\pi$. Find k.

TR 12: For a positive integer n, let S_n denote the sum of all positive integers m such that $\frac{n}{m-3}$ is a positive integer. Suppose k is a positive integer such that $S_{2k} = 1650$, $S_{4k} = 3492$, and $S_{8k} = 7158$. Find S_k.

TR 13: Let $ABCD$ be a rhombus. Circle ω passes through A and is tangent to sides BC and CD at points P and Q, respectively. Points R and S distinct from A are the intersections of ω with sides AB and DA, respectively. Given that $PQ = 2$ and $RS = 3$, the square of the area of $ABCD$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

TR 14: Determine the number of ways we can place a number in each cell of a 32×32 grid such that the following three conditions are met:

- Every positive integer less than or equal to 32^2 appears in the grid;
- For any 32 cells of the grid such that no two lie in the same row or column, the sum of the numbers in those cells is always the same;
- The numbers in each row increase from left to right, and the numbers in each column increase from bottom to top.

TR 15: Let $ABCD$ be a cyclic quadrilateral. Lines AB and CD intersect at point P. The angle bisector of $\angle BPC$ intersects sides BC and AD at points X and Y, respectively. Given that $AC = 5$, $BD = 7$, and $XY = \frac{14}{3}$, the area of $ABCD$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.