

2025 SSMO Relay Round 4

SMO Team

RR 4 Part 1: Call a positive integer *chuzzed* if the sum of the digits in its binary representation is equal to the units digit of its base-10 representation. Similarly, call a positive integer *chopped* if its binary representation does not contain two consecutive ones. Find the number of positive integers less than 128 that are chuzzed and chopped.

RR 4 Part 2: Let $T = TNYWR$. Jonathan and Kate are playing a game with n sticks. On each turn, a player may remove 1, 2, or 3 sticks. The player who picks up the last stick loses. Kate is first to remove sticks, and both players play optimally. For how many values of n in the range $[T^3, 2T^3]$ does Kate have a winning strategy?

RR 4 Part 3: Let $T = TNYWR$. A particle moves in the coordinate plane such that at any time t , its position is

$$\left(\sum_{a=1}^{T-1} \cos(at), \sum_{a=1}^{T-1} \sin(at) \right).$$

Over the time interval $t \in (0, k]$, the particle lies on at least one coordinate axes T times. If the minimal value of k can be written as $\frac{m\pi}{n}$ for relatively prime positive integers m and n , find $m + n$.