2024 SSMO Accuracy Round

SMO Team

AR 1: Let a time of day be three-full if exactly three of its digits are 3s when displayed on a 12-hour clock in the $hh : mm : ss$ format. How many seconds of the day are three-full?

AR 2: Equilateral triangle N is inscribed within circle O. A smaller equilateral triangle P is inscribed within N such that the vertices of P lie on the midpoints of N. The ratio of the areas between O and P can be expressed as $\frac{a\pi\sqrt{b}}{c}$, for relatively prime positive integers a, c and squarefree b. Find $a + b + c$.

AR 3: Three distinct random integers a, b, and c are selected so that $1 \leq a, b, c \leq 10$. Let the probability that $5|a! + b! + c!$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Find $m + n$.

AR 4: Right triangle ABC has a right angle at C and hypotenuse 1. Let points D and E lie on AC such that $\angle BDC = \angle BEC = 45°$. A, D, C, and E are colinear in that order. Given that $EA = 13DA$, the area of $\triangle ABC$ can be expressed as $\frac{m}{n}$ for relatively prime m and n. Find $m + n$.

AR 5: Let $ABCD$ be a convex quadrilateral such that $\angle ABC = 120°$ and $\angle ADC = 60°$. If $AB = BC = CD = 5$, the area of $ABCD$ can be expressed as $a + b\sqrt{c}$, where a, b, and c are positive integers and c is squarefree. Find $a + b + c$.

AR 6: Three six-sided dice are rolled. Then, the product of the three numbers on the top faces in calculated. If the probability of getting the product that is not a perfect square can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers, what is $m + n$?

AR 7: Find the value of $a + b + c + d$ given

$$\begin{aligned}
&a^2 + d^2 = b^2 + c^2 = 361, \\
&ac + bd = 247, \text{ and} \\
&ab + cd = 570.
\end{aligned}$$

AR 8: $ABCD$ is a convex cyclic quadrilateral with $AB = 2, BC = 5, CD = 10$, and $AD = 11$. Let W, Y, X, and Z be the midpoints of sides AB, BC, CD, and DA respectively. If $|(WX^2 - YZ^2)|$ can be expressed as $\frac{m}{n}$, for relatively prime positive integers m and n, find $m + n$.

AR 9: In the game of S - set, there are 3^{12} unique cards, each card containing a 12 traits of variants A, B, and C. A full house in the game of Sset is defined to be a hand of cards in

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which for each trait, all cards in the hand either share the same variant or have all different variants. The number of hands that can be considered a full house in the game of set can be expressed as $a^b + c^d + e^f$, where a, b, c, d, e, and f are positive integers and $a + c + e$ is minimized. Find $a + b + c + d + e + f$.

AR 10: Bobby is spinning a rigged wheel with three sections labeled A, B, and C. Two integers $0 \leq x, y \leq 50$ are chosen randomly and independently, such that there is a $x\%$ chance the wheel lands on A and a $y\%$ chance the wheel lands on B. Given that the wheel lands on C the first time, the probability that it will land on C if Bobby spins it again can be expressed as $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m + n$.