2023 SSMO Accuracy Round

SMO Team

AR 1: Mr. Sammy proposes a Hamburger Proclamation, which has 500 lines, divided into paragraphs of 5 lines each. It takes him 23 seconds to read each line. Additionally, he adds a 0.5 second pause between two lines in a paragraph, and a 2 second pause between paragraphs. If it takes him S minutes to read the whole Hamburger Proclamation, find $\lfloor 10S \rfloor$.

AR 2: Suppose that the average of all n-digit palindromes is denoted by P_n and the average of all n-digit numbers is denoted by N_n. Find $\left\lfloor \sum_{n=1}^{100} (P_n - N_n) \right\rfloor$.

AR 3: Suppose that a, b, c are real numbers such

$$\begin{aligned} a + b - c &= 4 \\ a^2 + b^2 + c^2 &= 14 \\ a^3 + b^3 - c^3 &= 34 \end{aligned}$$

Find the sum of all possible values of $a + b + c$.

AR 4: In square $ABCD$, point E is selected on diagonal AC. Let F be the intersection of the circumcircles of triangles ABE and CDE. Given that $AB = 10$ and $EF = 6$, find the maximum possible area of triangle BEC. (A circumcircle of some triangle $\triangle ABC$ is the circle containing A, B, and C)

AR 5: Define the *relationship* between two numbers a and b to be $\frac{\sigma(ab)}{\sigma(a)\sigma(b)}$ where $\sigma(x)$ is the number of divisors of x. Find the sum of integers $1 \le n \le 100$ which have a relationship of $\frac{3}{4}$ with 360.

AR 6: Let the roots of $P(x) = x^3 - 2023x^2 + 2023^{2023}$ be p, q, r. Find

$$\frac{p^2 + q^2}{p + q} + \frac{q^2 + r^2}{q + r} + \frac{r^2 + p^2}{r + p}$$

AR 7: Concentric circles ω and ω_1 are drawn, with radii 3 and 5, respectively. Chords AB and CD of ω_1 are both tangent to ω and intersect at P. If $PA = PC = 3$, then the sum of all possible distinct values of $[PAD]$ can be expressed as $\frac{m}{n}$, for relatively prime positive integers m and n. Find $m + n$.

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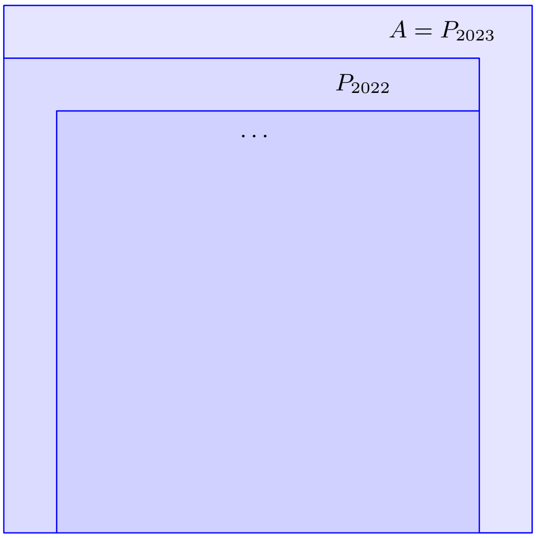
AR 8: There is a quadrilateral $ABCD$ inscribed in a circle ω with center O. In quadrilateral $ABCD$, diagonal AC is a diameter of the circle, $\angle BAC = 30°$, and $\angle DAC = 15°$. Let E be the base of the altitude from O onto side BA. Let F be the base of the altitude from E onto BO. Given that $EF = 3$, and that the product of the lengths of the diagonals of $ABCD$ is $a\sqrt{b}$, for some squarefree b, find $a + b$.

AR 9: Consider a 2023×2023 grid called $A = P_{2023}$. We take one of the four smaller 2022×2022 grids located in P_{2023} as P_{2022}. We repeat the process of taking smaller grids until we eventually converge at the unit square P_1.



Of the 4^{2022} distinct tuples of shrinking grids $(P_{2023}, P_{2022}, \dots P_1)$, let T be the number of these tuples such that their last element is the center square of the original grid A. Find the largest integer a such $2^a \mid T$.

AR 10: Let $\triangle ABC$ be a triangle such $AB = 13$, $BC = 14$, $CA = 15$. Let the incircle of $\triangle ABC$ touch BC at D, AC at E, and AB at F. Let ℓ_A be the line through the midpoints of AE and EF. Define ℓ_B and ℓ_C similarily. Let the area of the star created by the union of $\triangle ABC$ and the triangle bound by ℓ_A, ℓ_B, and ℓ_C be $\frac{p}{q}$ for relatively prime p and q. Find $p + q$.