2025 WSMO Speed Round

SMO Team

SR 1: How many permutations $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ satisfy

$$a_1 + a_2 = a_4 + a_5?$$

SR 2: Charlie has five consecutive integers and chooses one of them. The sum of her four numbers that she did not choose is 2025. What number did she choose?

SR 3: There exists a unique positive integer n such that $n^3 + 6n^2 + 11n + 21$ is a perfect cube. Find the value of n.

SR 4: Suppose that three fair six-sided dice are tossed so that the three numbers that they land on are the roots of monic cubic $P(x) = x^3 + ax^2 + bx + c$. What is expected value of $200 + 4a + 3b + 4c$?

SR 5: If a, b are real numbers such that

$$\frac{a}{b} + \frac{b}{a} = \frac{a}{b^2} + \frac{b}{a^2} = 10,$$

then $ab = \frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

SR 6: Suppose $\mathcal{R}_1$ and $\mathcal{R}_2$ are rectangles such that every vertex of $\mathcal{R}_1$ lies on a different side of $\mathcal{R}_2$. The side lengths of $\mathcal{R}_1$ are in a $3 : 4$ ratio, and the side lengths of $\mathcal{R}_2$ are in a $8 : 9$ ratio. Then, the ratio between the perimeters of $\mathcal{R}_1$ and $\mathcal{R}_2$ is $m : n$, where m and n are relatively prime positive integers. Find $m + n$.

SR 7: Let $ABCDEF$ be an equiangular hexagon with $AB = DE = 4$, $BC = EF = 7$, and $CD = FA$. Point P lies in the interior of the hexagon such that $[APB] = 12\sqrt{3}$, $[BPC] = 14\sqrt{3}$, and $[CPD] = 9\sqrt{3}$. Find the perimeter of the hexagon.

SR 8: Alice rolls a number n on a fair six-sided die. Bob then rolls n fair twenty-sided dice, getting $a_1, a_2, \ldots a_n$. Let a be the maximum integer such that 2^a divides into $\prod_{i=1}^{n} a_i!$. The expected value of a can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 9: Let $\mathcal{S}$ denote the set of all permutations of the first 6 positive integers. A permutation $(a_1, a_2, \ldots, a_6)$ is chosen uniformly at random from the elements of $\mathcal{S}$. Suppose that there are r distinct values among the remainders when $a_1 + 1, a_2 + 2, \ldots, a_6 + 6$ are each divided by 6. The expected value of r can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

SR 10: Complex numbers a, b, c satisfy the following system of equations:

$$\begin{aligned}(a+b)^2 &= ab+39,\\ (b+c)^2 &= bc+61,\\ ab+bc+ca &= 38.\end{aligned}$$

Find the sum of all possible values of $(a+b+c)^2$.