2025 SSMO Accuracy Round

SMO Team

AR 1: An infinite sequence of real numbers $a_1, a_2, \ldots$ satisfies

$$a_n = a_1 + a_2 + \cdots + a_{n-1}$$

for all positive integers $n > 1$. Given that $a_{20} = 25$, find a_{25}.

AR 2: Let ABC be a triangle with circumcircle ω. The midpoint of AB is M, and the line CM intersects ω again at P. Given $\angle BMC = 120^\circ$, $\triangle BMC$ is isosceles, and $BC = 20$, the length of PM can be written as $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers such that a and c are relatively prime and b is square-free. Find $a + b + c$.

AR 3: Nonnegative real numbers x, y, and z satisfy

$$\frac{\sqrt{x} + 13}{y} = \frac{\sqrt{y} + 29}{z} = \frac{\sqrt{z} + 46}{x} = 2$$

and

$$\frac{\sqrt{x} + \sqrt{y} + \sqrt{z}}{x + y + z} = \frac{6}{25}.$$

Find the value of $x + y + z$.

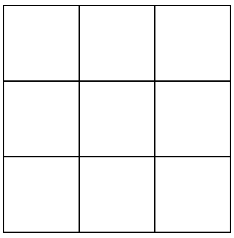
AR 4: Let a, b, and c be the roots of the polynomial $x^3 + 4x^2 - 3x - 4$. Suppose that

$$\left| \frac{a}{b + c} + \frac{b}{c + a} + \frac{c}{a + b} \right| = \frac{m}{n},$$

for relatively prime positive integers m and n. Find $m + n$.

AR 5: ABC is an isosceles triangle with base $BC = 6$ and $AB = AC$. Point M is the midpoint of BC such that $AM = 9$. Circle ω_1 is the circumcircle of ABC with radius R, and ω_2 is a circle passing through B and C with radius $2R$ and center on the opposite side of BC as A. Segment AM intersects ω_2 at point X and ω_1 at point Y, where X lies between A and Y. The length XY can be expressed as $m - \sqrt{n}$, where m and n are positive integers. Find $m + n$.

AR 6: Andy the ant starts at the square labeled 1. On each move Andy moves to any orthogonal square (a square with which his current square shares a side). What is the expected number of moves before Andy is in the square labeled 2?



AR 7: There is a unique ordered triple of positive reals (a, b, c) satisfying the system of equations

$$\begin{aligned} a^2 + 9 &= (b - 8\sqrt{3})^2 + 4 \\ b^2 + 4 &= (c - 8\sqrt{3})^2 + 49 \\ c^2 + 49 &= (a - 8\sqrt{3})^2 + 9. \end{aligned}$$

The value of $100a + 10b + c$ can be expressed as $m\sqrt{n}$, where m and n are positive integers such that n is square-free. Find $m + n$.

AR 8: We say that a permutation $(a_1, a_2, \ldots, a_{10})$ of the integers 1 through 10 inclusive is *peaked* if there do not exist three integers $1 \le i < j < k \le 10$ such that $a_i > a_j$ and $a_j < a_k$. Let $\mathcal{S}$ be the set of all peaked permutations. If $a_p = 9$ and $a_q = 4$, the expected value of $|p - q|$ over all permutations in $\mathcal{S}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is the value of $m + n$?

AR 9: For a positive integer n, let $r(n)$ denote the value of the binary number obtained by reading the binary representation of n from right to left. For example, $r(6) = r(110_2) = 011_2 = 3$. If k is the smallest positive integer such that the equation $n + r(n) = 2k$ has at least ten positive integer solutions n, find k.

AR 10: Let $ABCDE$ be a convex pentagon with $\angle BAC = \angle CAD = \angle DAE$ and $\angle ABC = \angle ACD = \angle ADE$. Let BD and CE meet at P. Given that $BC = 6$, $\sin \angle BAC = \frac{3}{5}$, and $\frac{AC}{AB} = 5$, the length of AP can be expressed as $\frac{m}{\sqrt{n}}$, where m and n are positive integers such that n is square-free. Find $m + n$.