

# 2023 SSMO Relay Round 5

SMO Team

**RR 5 Part 1:** Let  $S_n$  be the set of all rational numbers of the form  $0.\overline{a_1a_2a_3\dots a_n}$ , where  $n$  is an integer satisfying  $n \geq 1$  and  $a_1, a_2, \dots, a_n$  are nonzero integers. If

$$n = 5 \left( \sum_{n=1}^{\infty} \left( \sum_{a \in S_n} \frac{a}{10^n} \right) \right),$$

find  $n$ .

**RR 5 Part 2:** Let  $T = TNYWR$ . Let  $a_n = \text{lcm}\{1, 2, \dots, n\}$  for positive integers  $n$ . Compute

$$\sum_{i=1}^N a_i \pmod{720}.$$

**RR 5 Part 3:** Let  $T = TNYWR$ . Suppose that  $x^2 + y^2 = N$ . Find the remainder when the expected value of the square of the maximum value  $ax + by$  is divided by 100, where  $a$  and  $b$  distinct members from the set  $\{1, 2, \dots, N\}$ .

