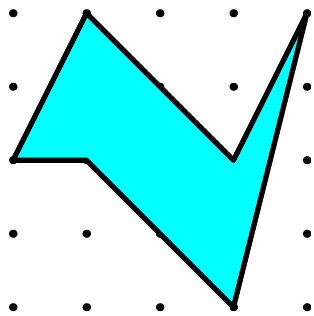
2023 WSMO Speed Round

SMO Team

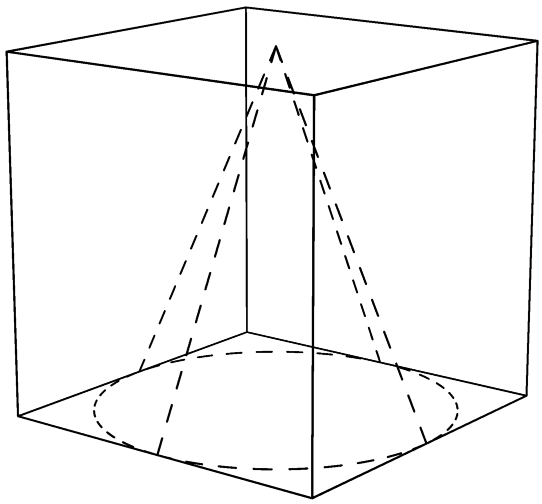
SR 1: Find the number of square units in the area of the shaded region.



SR 2: There are 4 tables and 5 chairs at each table. Each chair seats 2 people. There are 10 people who are seated randomly. Andre and Emily are 2 of them, and are a couple. If the probability that Andre and Emily are in the same chair is $\frac{m}{n}$, for relatively prime positive integers m and n, find $m+n$.

SR 3: There are 6 pairs of socks for each color of the rainbow (red, orange, yellow, green, blue, indigo, violet) in a sock drawer. How many socks must be drawn from the drawer to guarantee that a pair of red socks have been drawn?

SR 4: A right circular cone is inscribed in a right prism as shown. If the ratio of the volume of the cone to the volume of the prism is $\frac{m}{n}\pi$, for relatively prime positive integers m and n, find $m+n$.



SR 5: There exists a rational polynomial $f(x)$ such that for all x in the range $(0, 1)$, $f(x) = \sum_{n=1}^{\infty} nx^n$. If the maximum of $f(x)$ over $[6, 9]$ is $\frac{m}{n}$, for relatively prime positive integers m and n, find $m + n$.

SR 6: Let ABC be an equilateral triangle of side length 6. Points $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$ are chosen such that A_1, A_2, A_3 divide BC into four equal segments, B_1, B_2, B_3 divide AC into four equal segments, and C_1, C_2, C_3 divide AB into four equal segments. If i, j, k are chosen from the set $\{1, 2, 3\}$ independently and randomly, the expected area of $A_iB_jC_k$ is $\frac{a\sqrt{b}}{c}$, for squarefree b and relatively prime positive integers a and c. Find $a + b + c$.

SR 7: Let e, a, j be real numbers such that $e + a + j = 1$ and $e \geq -\frac{1}{3}$, $a \geq 1$ and $j \geq -\frac{5}{3}$. Find the maximum value of $\sqrt{3e + 1} + \sqrt{3a + 3} + \sqrt{3j + 5}$.

SR 8: In regular octagon $ABCDEFGH$ of sidelength 4, quadrilaterals $ACEG$ and $BDFH$ are drawn. Find the square of the area of the overlap of the two quadrilaterals.

SR 9: Suppose that b and c are the roots of the equation $x^2 - \log(16)x + \log(64)$. If $\sqrt{a + b} + \sqrt{a + c} = \sqrt{b + c}$, then $2^a = \frac{\sqrt{m}}{n}$, find $m + n$.

SR 10: Consider acute triangle ABC, H is the orthocenter. Extend AH to meet BC at D. The angle bisector of $\angle ABH$ meets the midpoint of AD, M. If $AB = 10, BH = 4$, then the area of ABC is $\frac{a\sqrt{b}}{c}$, for squarefree b and relatively prime positive integers a and c. Find $a + b + c$.