

2023 SSMO Relay Round 5

SMO Team

RR 5 Part 1: Let S_n be the set of all rational numbers of the form $0.\overline{a_1a_2a_3\dots a_n}$, where n is an integer satisfying $n \geq 1$ and a_1, a_2, \dots, a_n are nonzero integers. If

$$n = 5 \left(\sum_{n=1}^{\infty} \left(\sum_{a \in S_n} \frac{a}{10^n} \right) \right),$$

find n .

RR 5 Part 2: Let $T = TNYWR$. Let $a_n = \text{lcm}\{1, 2, \dots, n\}$ for positive integers n . Compute

$$\sum_{i=1}^N a_i \pmod{720}.$$

RR 5 Part 3: Let $T = TNYWR$. Suppose that $x^2 + y^2 = N$. Find the remainder when the expected value of the square of the maximum value $ax + by$ is divided by 100, where a and b distinct members from the set $\{1, 2, \dots, N\}$.