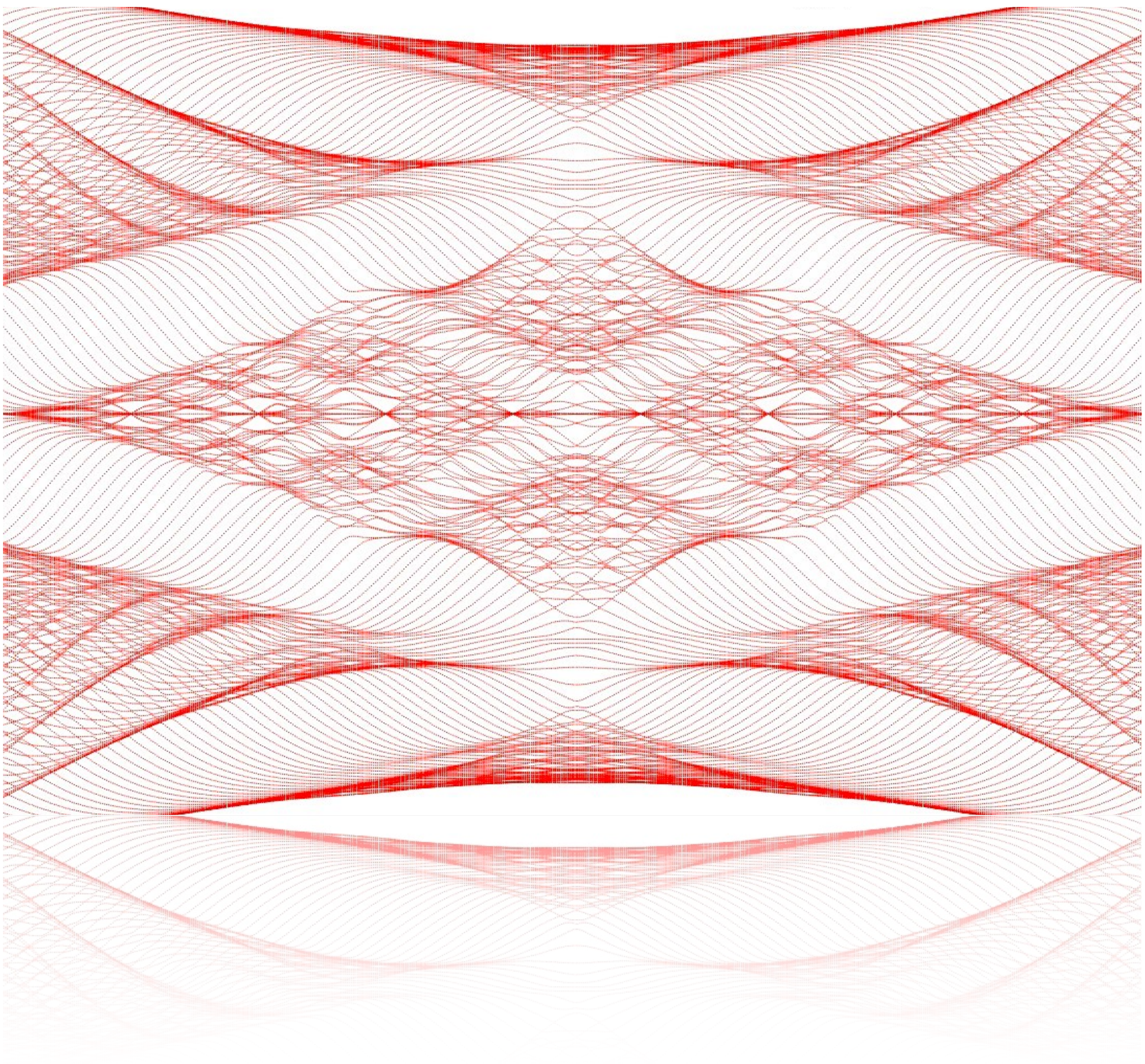
Lanczos transformation

For impurity problems

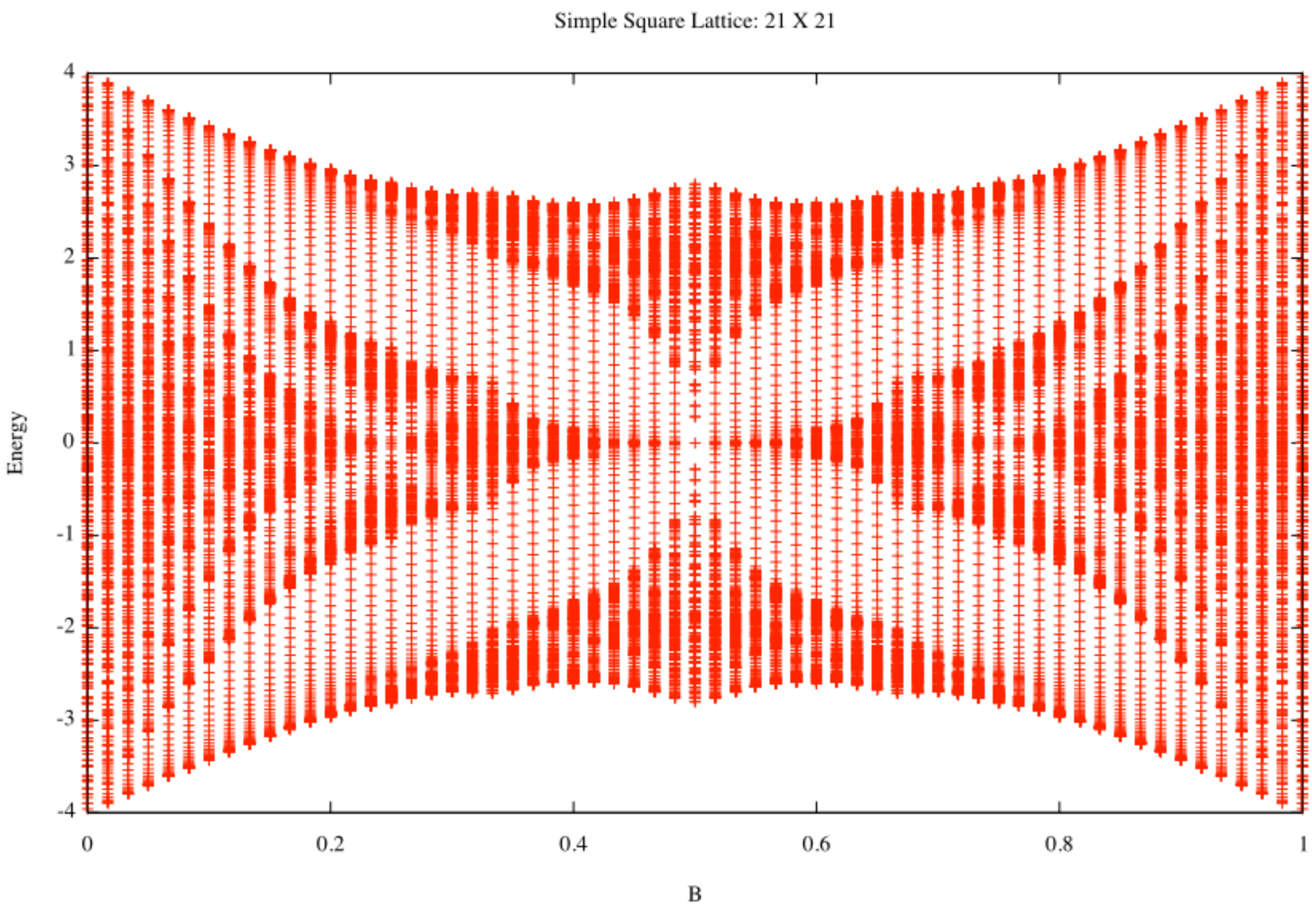
A short introduction



Hofstadter's Butterfly in a Square Lattice

$$\left\langle \psi(r) \middle| \psi(r \pm a\hat{y}) \right\rangle = t \qquad \left\langle \psi(r) \middle| \psi(r \pm a\hat{x}) \right\rangle = t \exp(\pm \frac{i}{\hbar} \int \vec{A} d\hat{x})$$

Let's say magnetic vector A is in the x direction; we get the energy levels as:



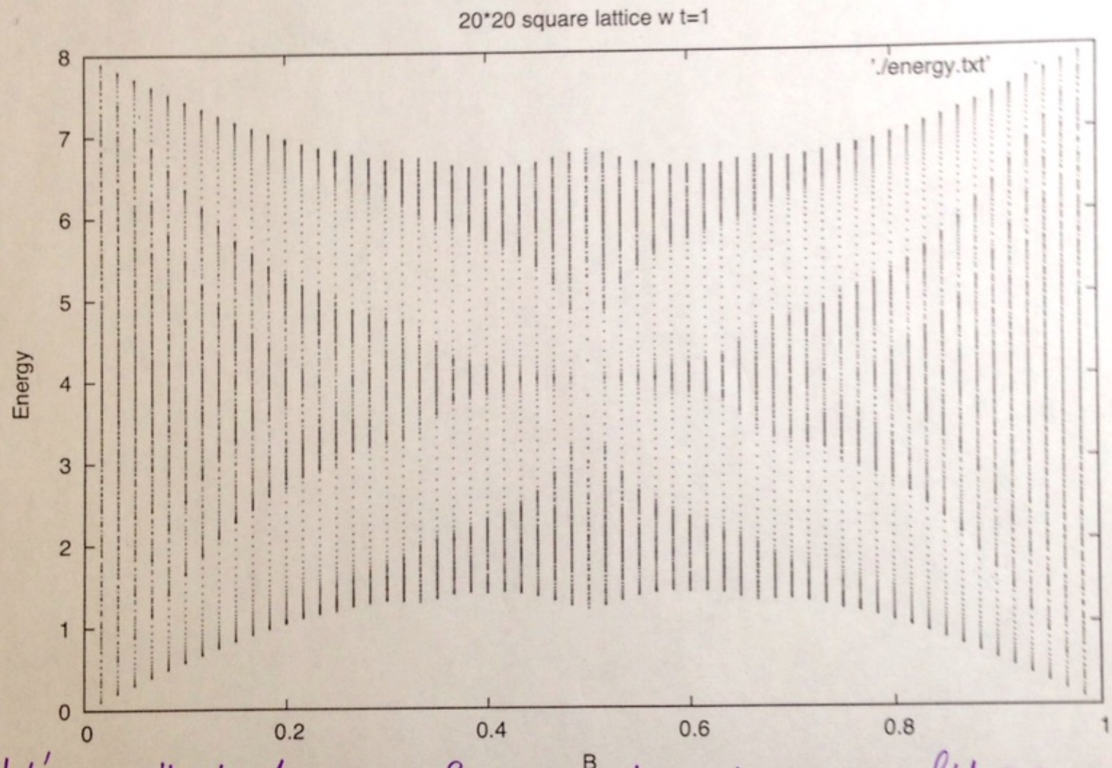
Hofstadter's Butterfly in square lattice:

$$\mathcal{H} = \sum_{nm} \left[t^x_{nm} |n+1,m\rangle\langle n,m| + t^y_{nm} |n,m+1\rangle\langle n,m| + H.C. \right] + \sum_{nm} \varepsilon_{nm} |n,m\rangle\langle n,m|$$

$\varepsilon_{nm} = 4t + V_{nm}$ (free electrons $\Rightarrow V=0$)

in the ansenbce of Magnetic field: $t^x_{nm} = t^y_{nm} = -t = -\frac{\hbar^2}{2m^* a^2}$

By having Magnetic field: $t^{x(y)}_{nm} = -t\, e^{-i\frac{e}{\hbar}\int A \cdot dl}$

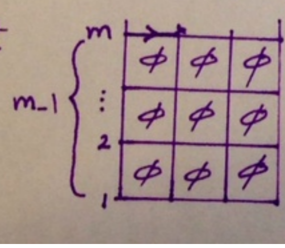


Let's use the Landau gauge for representing a homogeneoses field $B\hat{e}_z$:

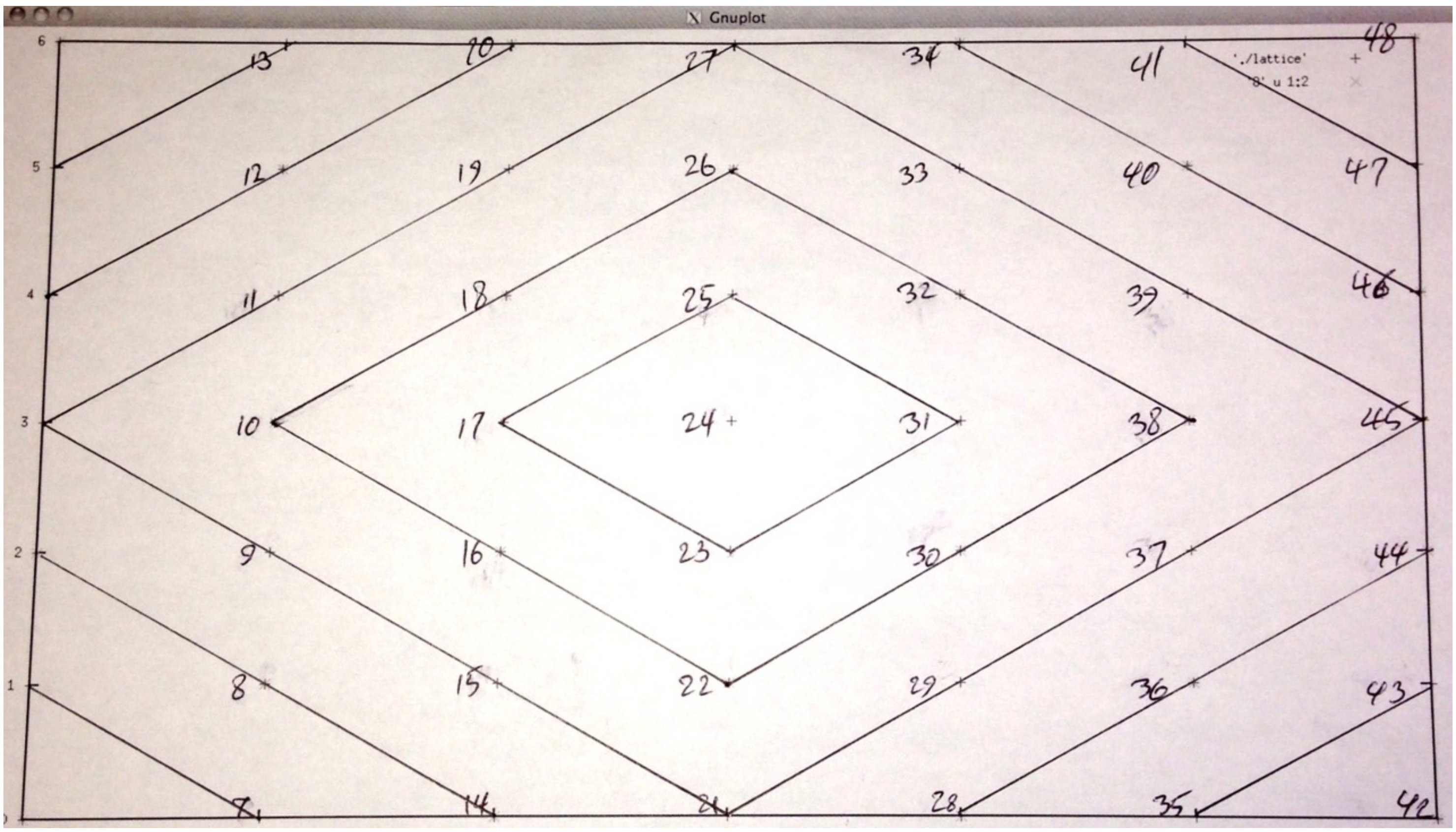
$\vec{A} = -By\,\hat{e}_x \qquad \phi_0 \equiv h/e \qquad \phi \equiv Ba^2$

$$\Rightarrow t^x = -t\, e^{-i\frac{2\pi}{h/e}\int A \cdot dl} = -t\, e^{-i\frac{2\pi}{\phi_0}(-Ba^2)(m-1)} = -t\, e^{i2\pi(m-1)\phi/\phi_0}$$

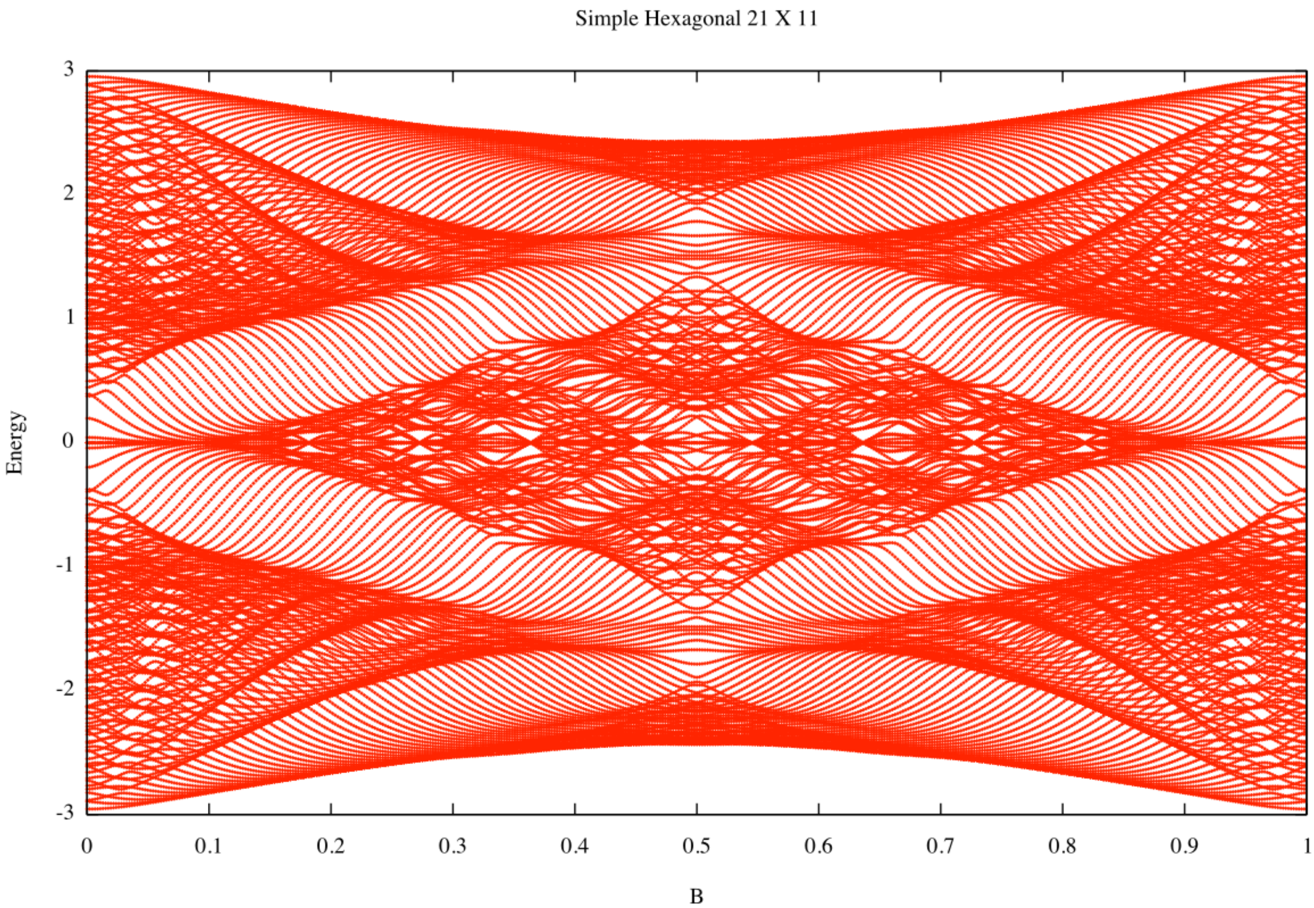
$t^y = -t$



Labeling method:

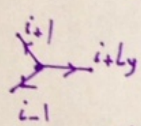


Hofstadter's Butterfly in a Hexagonal Lattice

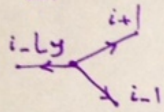


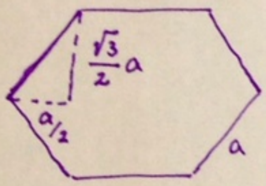
L_y : odd $L_x = \frac{L_y+1}{2}$

site odd



site even

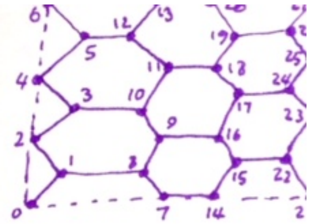




Area: $\frac{3\sqrt{3}}{2} a^2$

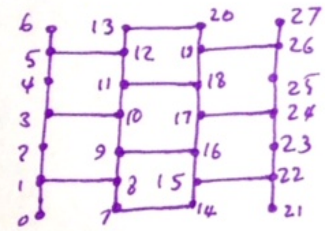
7 x 4

L_y x L_x

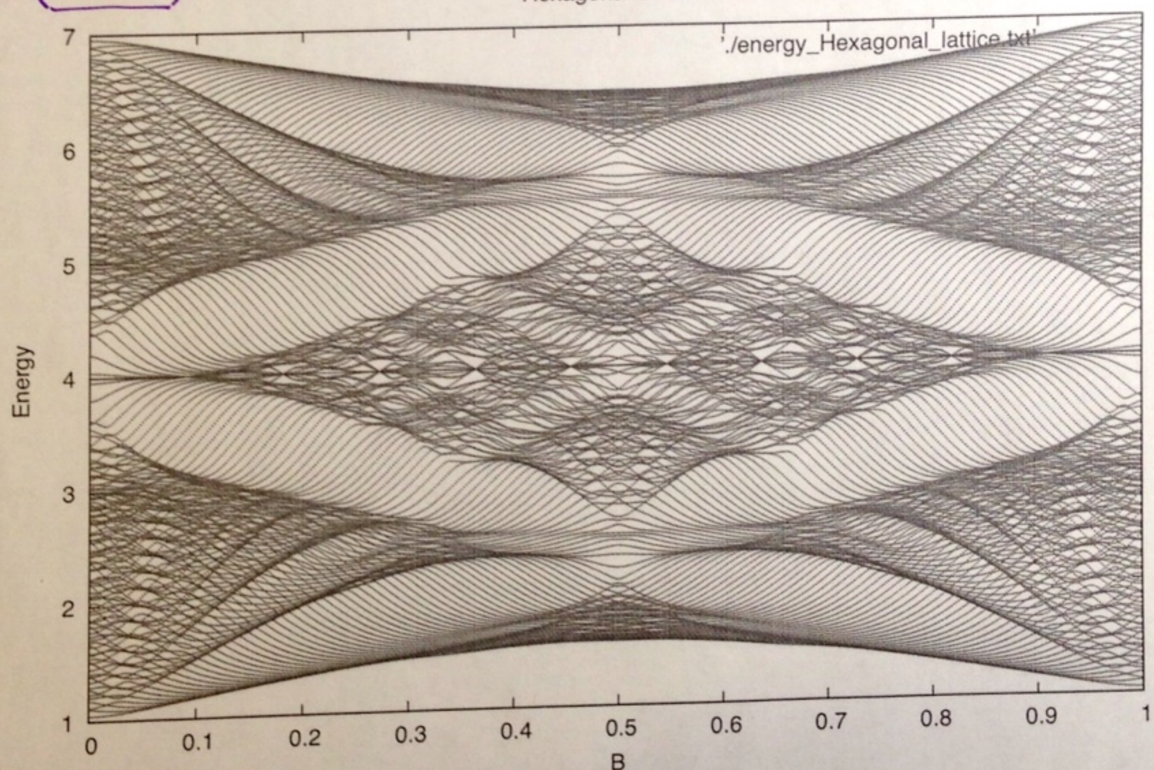


equivalent

(Not exactly)



Hexagonal 21 X 10



Labeling method: