

Numerical PDE's final Project

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## 1 Problem 1

The equation  $u_t + \left(\frac{u^2}{2}\right)_x = \varepsilon u_{xx}$  was solved on the interval  $x \in [-1, 1]$ ,  $\varepsilon = 0.01$  until  $T = 0.1$ .

With the boundary conditions  $u(-1) = 1$  and  $u(1) = 0$ , and initial conditions  $u(x, 0) = -.5x + .5$  and  $u(x, 0) = 1 - x^2$ .

$[-1, 1]$  was meshed into  $N$  equi-length intervals with  $N = 40, 80$ .

Both a standard FEM method and a streamline diffusion method were used to solve the system, using forward euler for the time integration, with 1 and 3 point quadrature schemes used for the nonlinear term.

The figures in appendix A display the solutions.

## 2 Problem 2

The equation  $u_t + \left(\frac{u^2}{2}\right)_x = 0$  was solved on the interval  $x \in [-1, 1]$ , until  $T = 0.05, 0.1$  and  $0.2$ .

A periodic boundary condition was imposed, with the initial condition  $u(x, 0) = 0.5(1 + \sin(\pi t))$ .

$[-1, 1]$  was meshed into 160 equi-length intervals.

Finite volume methods were used to solve this. An ENO scheme (both 3<sup>rd</sup> and 1<sup>st</sup> order) were used for the interface value reconstructions. Both forward euler and a TVD RK3 solver were used for the time integration, and the numerical fluxes were reconstructed using Godunov and Global Lax-Friedrichs schemes.

The figures in appendix B display the solutions.

## A Problem 1 Figures

## B Problem 2 Figures

Note that above each figure is the time when it was extracted from. Beneath each group of 4 figures the following code is used

- RO: The ENO reconstruction accuracy used
- OT: The order of accuracy for the time integration method (1 is forward euler, 3 is TVD RK3)
- flx: Whether a Godunov or GLF flux was used

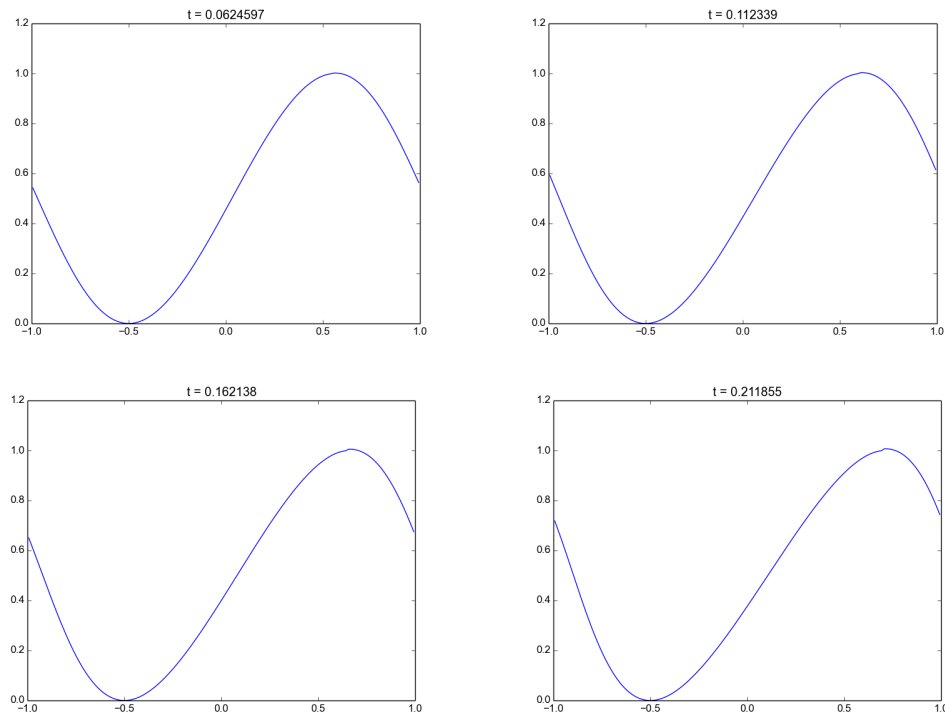
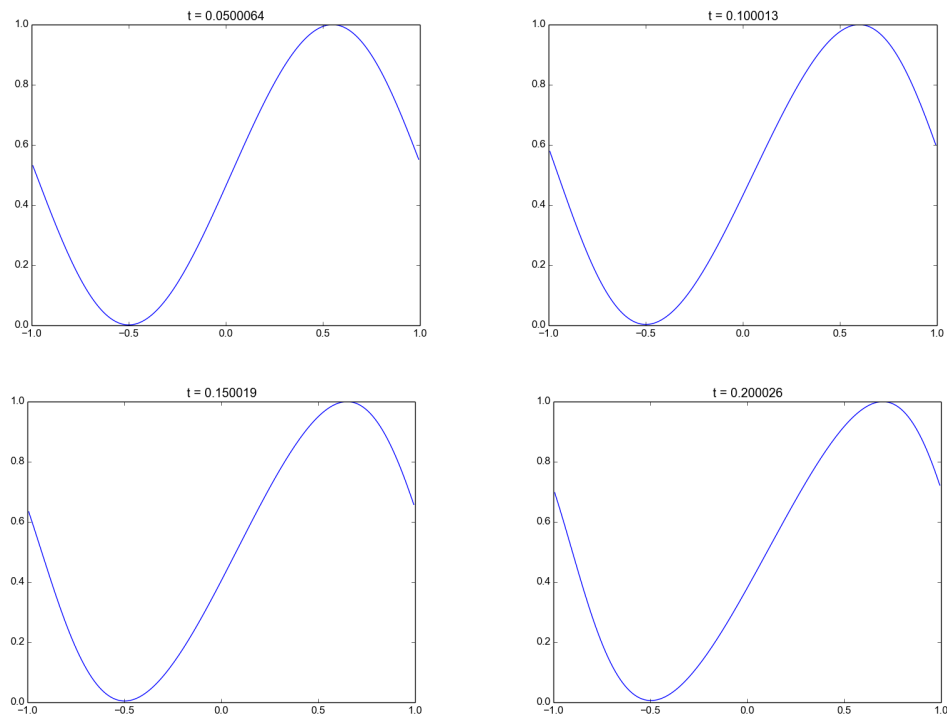
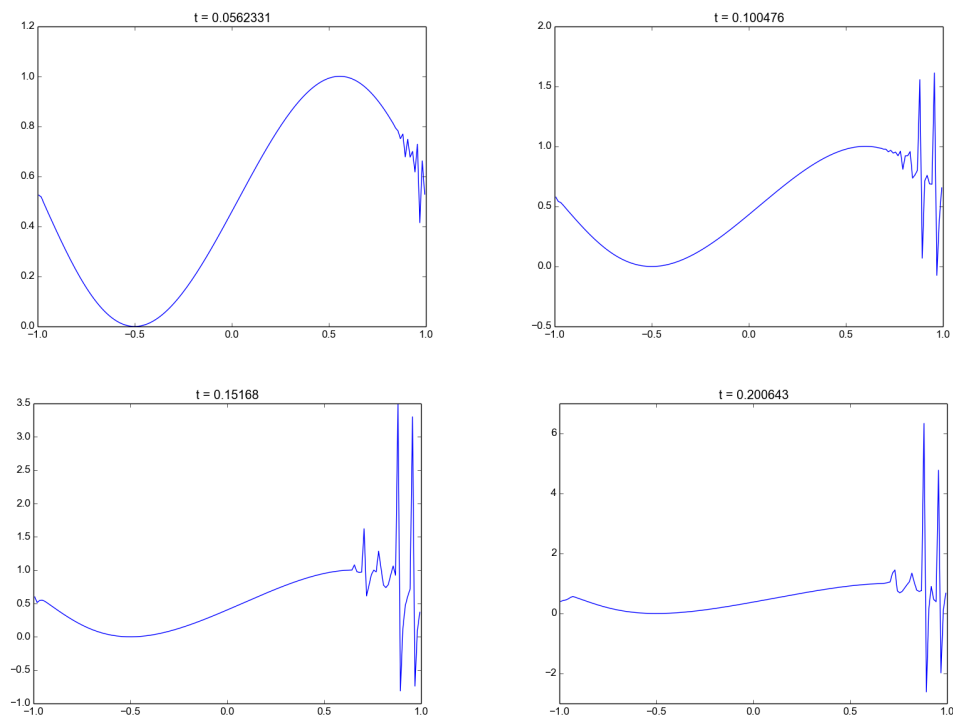
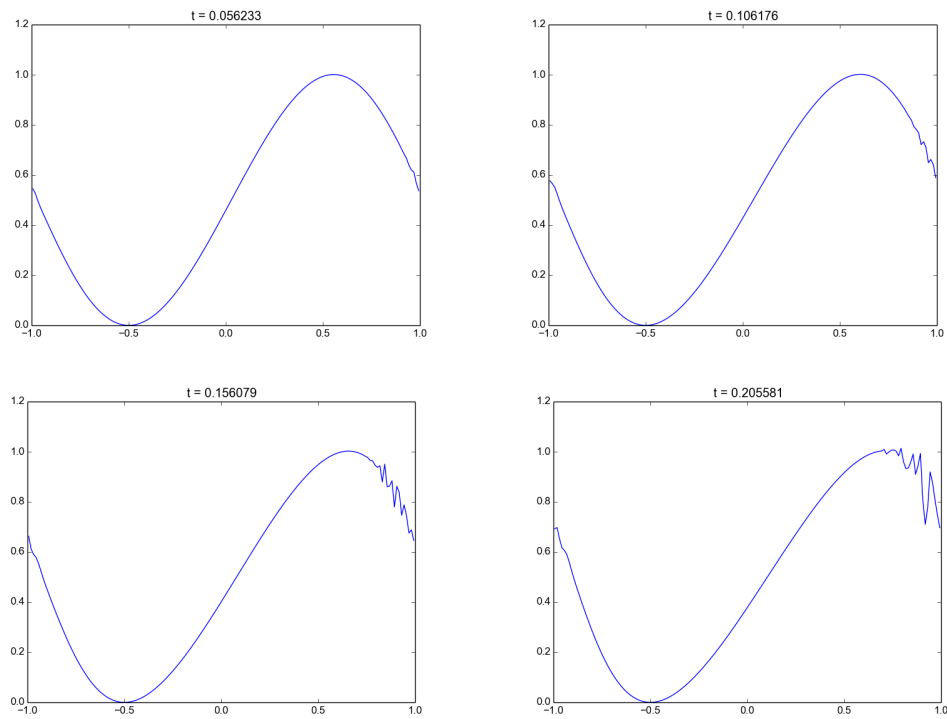
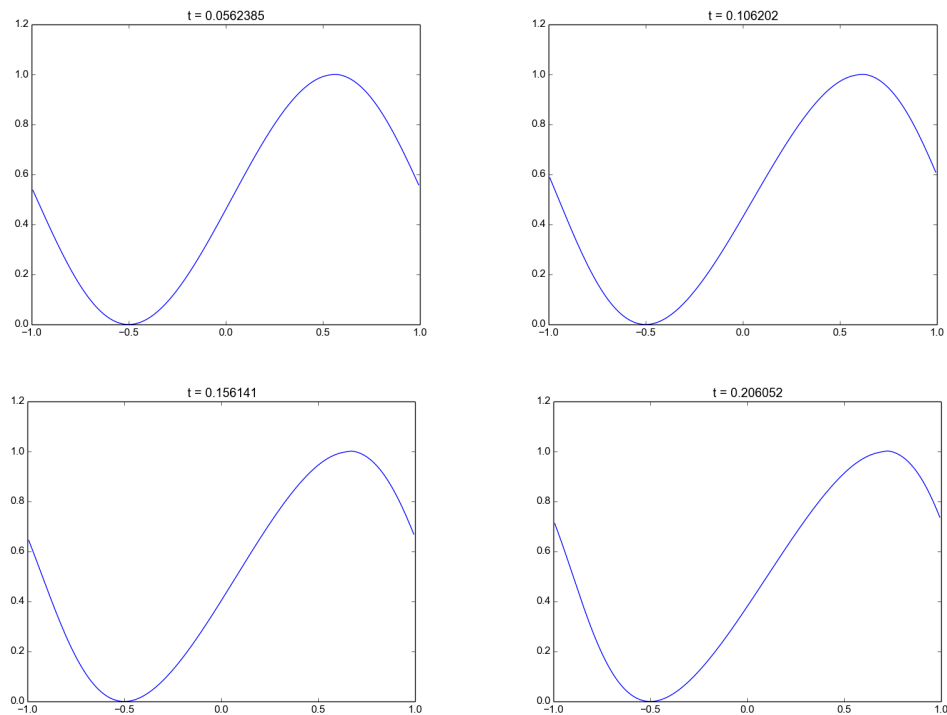
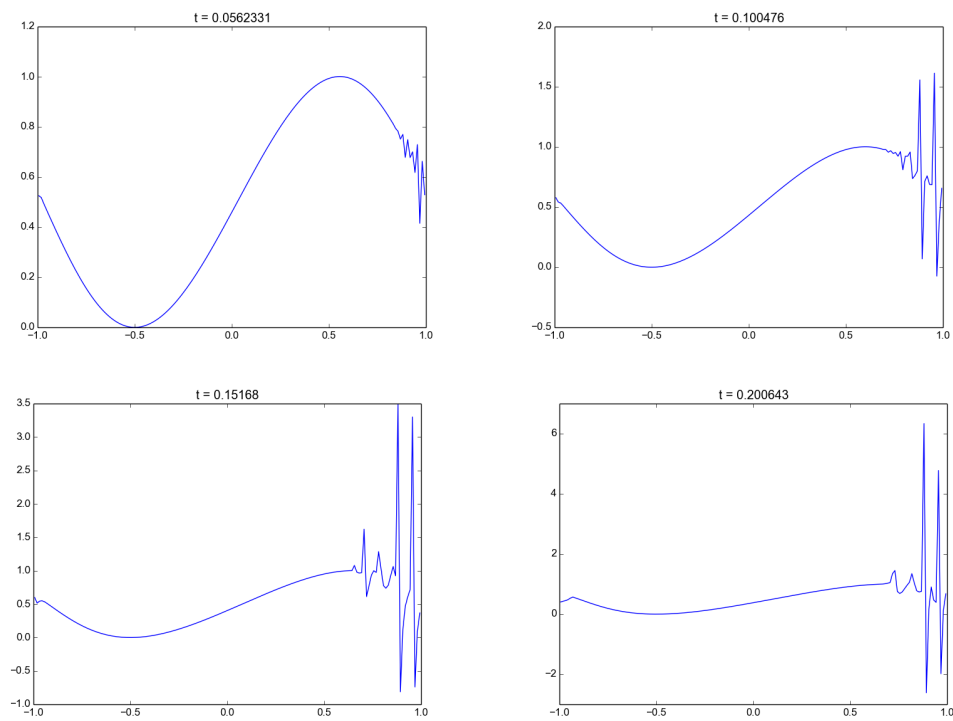
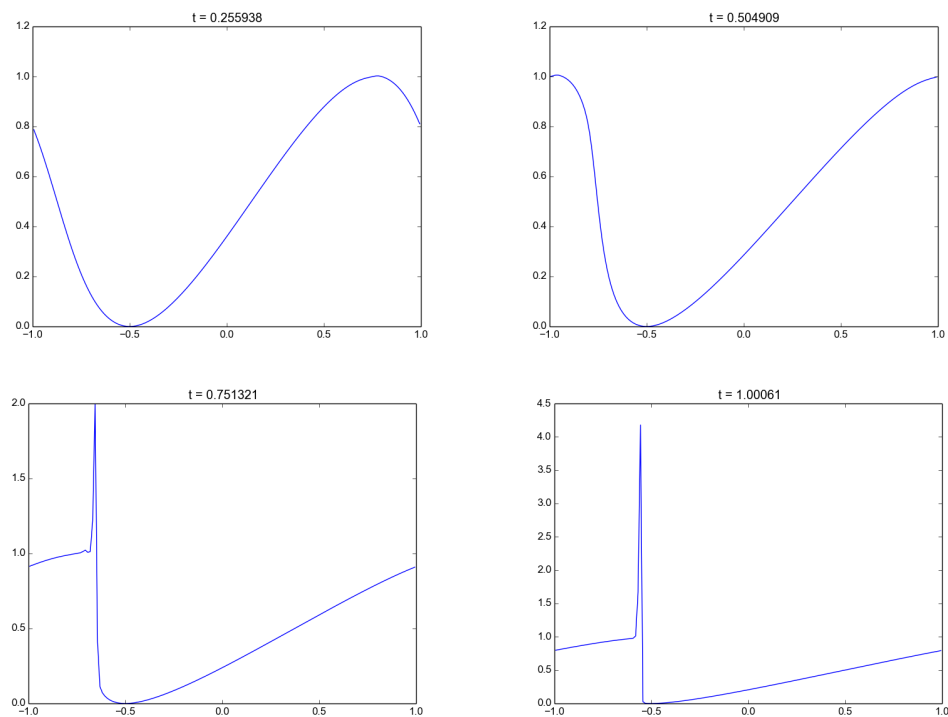
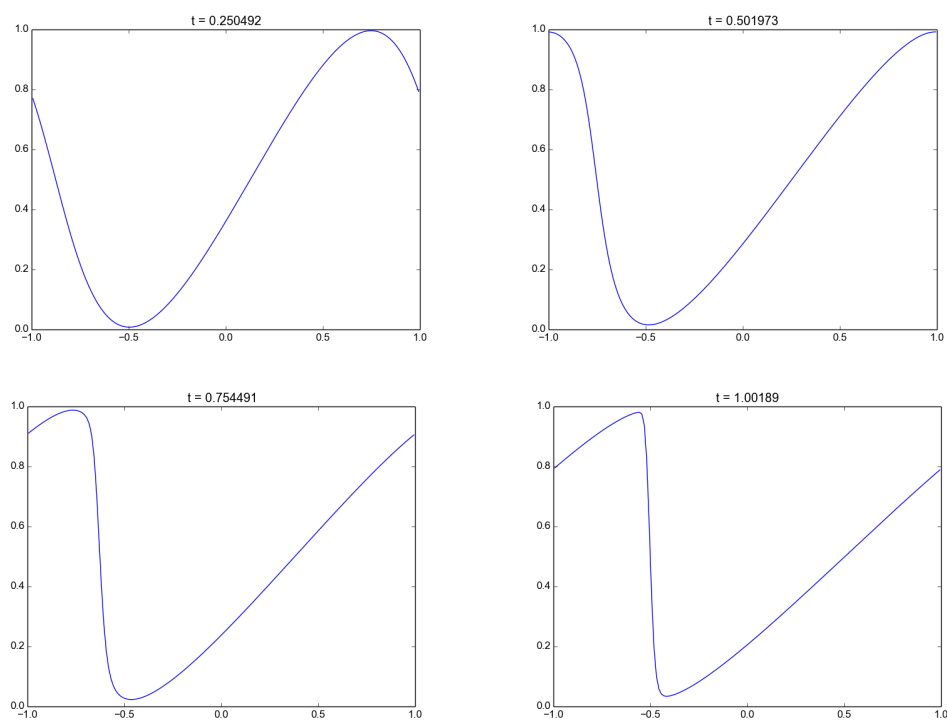


Figure 1: RO = 1, OT = 1, Godunov

Figure 2:  $RO = 1$ ,  $OT = 1$ , GLFFigure 3:  $RO = 3$ ,  $OT = 3$ , Godunov

Figure 4:  $RO = 3$ ,  $OT = 3$ , GLFFigure 5:  $RO = 3$ ,  $OT = 1$ , Godunov

Figure 6:  $RO = 1$ ,  $OT = 3$ , GodunovFigure 7:  $RO = 1$ ,  $OT = 1$ , Godunov

Figure 8:  $RO = 1$ ,  $OT = 1$ , GLF