Numerical PDE's final Project

Peter Solfest Code available at `https://github.com/solter/python-FEM`

1 Problem 1

The equation $u_t + \left(\frac{u^2}{2}\right)_x = \varepsilon u_{xx}$ was solved on the interval $x \in [-1, 1]$, $\varepsilon = 0.01$ until $T = 0.1$.

With the boundary conditions $u(-1) = 1$ and $u(1) = 0$, and initial conditions $u(x, 0) = -.5x + .5$ and $u(x, 0) = 1 - x^2$.

$[-1, 1]$ was meshed into N equi-length intervals with $N = 40, 80$.

Both a standard FEM method and a streamline diffusion method were used to solve the system, using forward euler for the time integration, with 1 and 3 point quadrature schemes used for the nonlinear term.

The figures in appendix **??** display the solutions.

2 Problem 2

The equation $u_t + \left(\frac{u^2}{2}\right)_x = 0$ was solved on the interval $x \in [-1, 1]$, until $T = 0.05, 0.1$ and 0.2.

A periodic boundary condition was imposed, with the initial condition $u(x, 0) = 0.5(1 + \sin(\pi t))$.

$[-1, 1]$ was meshed into 160 equi-length intervals.

Finite volume methods were used to solve this. An ENO scheme (both 3^{rd} and 1^{st} order) were used for the interface value reconstructions. Both forward euler and a TVD RK3 solver were used for the time integration, and the numerical fluxes were reconstructed using Godunov and Global Lax-Friedrichs schemes.

The figures in appendix **??** display the solutions.

A Problem 1 Figures

Note that above each figure represents different times during the solution. The figures are labelled via the number of intervals (N), whether a standard or streamline method was used, and the initial condition.

Figure 1: $N = 40$ via standard with $u(x, 0) = -.5x + .5$

Figure 2: $N = 40$ via streamline with $u(x, 0) = -.5x + .5$

Figure 3: $N = 80$ via standard with $u(x, 0) = -.5x + .5$

Figure 4: $N = 80$ via standard with $u(x, 0) = -.5x + .5$

Figure 5: $N = 80$ via standard with $u(x, 0) = 1 - x^2$

Figure 6: $N = 80$ via standard with $u(x, 0) = 1 - x^2$

B Problem 2 Figures

Note that above each figure represents different times during the solution. Beneath each group of 4 figures the following code is used

- RO: The ENO reconstruction accuracy used
- OT: The order of accuracy for the time integration method (1 is forward euler, 3 is TVD RK3)
- flx: Whether a Godunov or GLF flux was used

Figure 7: RO = 1, OT = 1, Godunov

Figure 8: RO = 1, OT = 1, GLF

Figure 9: RO = 3, OT = 3, Godunov

Figure 10: RO = 3, OT = 3, GLF

Figure 11: RO = 3, OT = 1, Godunov

Figure 12: RO = 1, OT = 3, Godunov

Figure 13: RO = 1, OT = 1, Godunov

Figure 14: RO = 1, OT = 1, GLF