BELLE2-NOTE-2016-0YY
August 6, 2016

Belle II Simulation

T. Ferber[1], D. Kim[2], M. Staric[3]

[1] *University of British Columbia, Vancouver*
[2] *Soongsil University, Seoul*
[3] *Jozef Stefan Institute, Ljubljana*

Abstract

An overview of the Belle II Simulation is presented. This document is a draft chapter of the B2TiP book.

1 Belle II Simulation

Section author(s): T. Ferber, D. Kim, M. Ritter, M. Staric, P. Urquijo

1.1 Introduction

This chapter describes the the simulation tools used in the studies presented in this report. This includes a brief review of the main event generators, the detector simulation and an overview of the expected beam backgrounds. Some analyses require very specific event generators whose description is given in the respective subsections. The reference cross sections for various physics processes are provided as well.

All simulations start with at least one event generator that simulates the primary physics process which is followed by a detailed detector simulation. Some studies include the effects of beam background which is simulated in specific background simulations and added to the physics event simulation.

1.2 Cross Sections

Fiducial cross sections for the most important physics processes are given in Table 1.1 at the default beam energy. In addition to the normalization values, a rough estimate for observable cross sections within acceptance and some typical generator level cuts are given by the indented values. The cut values (if any) for the non indented cross section values correspond to typical event generator cuts. The $\Upsilon(4S)$ cross section is calculated the on peak cross section assuming a collision energy at the $\Upsilon(4S)$ nominal mass and includes next–to–leading order initial state radiation corrections and a beam energy spread of 5 MeV [1].

1.3 Generators

Most studies in this report are based on three main event generators: `EvtGen 1.3` [?] is used to model the decays of B and D mesons into exclusive final states. `PYTHIA 8.2` [2] is used for inclusive decay final states and for the continuum production of light quark pairs. τ pair production is generated using KKMC 4.15 [3, 4] with the decays handled by TAUOLA [5]. In addition, the large cross section QED background processes $e^+e^- \to e^+e^-(\gamma)$ and $e^+e^- \to \gamma\gamma(\gamma)$ are simulated using BABAYAGA.NLO [6, 7, 8, 9, 10], and $e^+e^- \to e^+e^-e^+e^-$ and $e^+e^-\mu^+\mu^-$ are simulated using AAFH[1] [11, 12, 13]. Additional specialized event generators for certain analyses are described in the respective subsections.

All event generators use the same beam parameters such as the mean beam energies and the vertex position which are provided by a central data base. The effect of beam energy smearing is included in EvtGen and BABAYAGA.NLO only and modelled as single Gaussian for the HER and LER beams each, with a width of 5.13 MeV and 2.375 MeV respectively. The default vertex position is the detector center (0, 0, 0) and the

[1] This generator is sometimes also called BDK or DIAG36.

Table 1.1: Total production cross section from various physics processes from collisions at $\sqrt{s} = 10.573\,\mathrm{GeV}$.

Physics process	Cross section [nb]	Cuts	Reference
$\Upsilon(4S)$	1.05 ± 0.10	-	[1]
$u\bar{u}(\gamma)$	1.61	-	KKMC
$d\bar{d}(\gamma)$	0.40	-	KKMC
$s\bar{s}(\gamma)$	0.38	-	KKMC
$c\bar{c}(\gamma)$	1.30	-	KKMC
$e^+e^-(\gamma)$	300 ± 3 (MC stat.)	$10^\circ < \theta^*_{e's} < 170^\circ$, $E^*_{e's} > 0.15$ GeV	BABAYAGA.NLO
$e^+e^-(\gamma)$	74.4	e's (p >0.5GeV) in ECL	-
$\gamma\gamma(\gamma)$	4.99 ± 0.05 (MC stat.)	$10^\circ < \theta^*_{\gamma's} < 170^\circ$, $E^*_{\gamma's} > 0.15$ GeV	BABAYAGA.NLO
$\gamma\gamma(\gamma)$	3.30	γ's (p >0.5GeV) in ECL	-
$\mu^+\mu^-(\gamma)$	1.148	-	KKMC
$\mu^+\mu^-(\gamma)$	0.831	μ's (p >0.5GeV) in CDC	-
$\mu^+\mu^-\gamma(\gamma)$	0.242	μ's (p >0.5GeV) in CDC, ≥ 1 γ (E_γ >0.5GeV) in ECL	-
$\tau^+\tau^-(\gamma)$	0.919	-	KKMC
$\nu\bar{\nu}(\gamma)$	0.25×10^{-3}	-	KKMC
$e^+e^-e^+e^-$	39.7 ± 0.1 (MC stat.)	$W_{\ell\ell} > 0.5$GeV	AAFH
$e^+e^-\mu^+\mu^-$	18.9 ± 0.1 (MC stat.)	$W_{\ell\ell} > 0.5$GeV	AAFH

vertex smearing covariance matrix is calculated from the horizontal (x) and vertical (y) beam size at the IP and the bunch lengths (z) of the LER (σ_x=10.2 μm, σ_y=0.059 μm, σ_z=5mm) and HER (σ_x=7.75 μm, σ_y=0.059 μm, σ_z=6mm) and the angles of the beams with respect to the z–axis [14]. Normal distributed bunch densities are assumed for the calculation and the probability density functions for the two bunches are multiplied to get a resulting beam spot. Vertex position smearing is included for all generators.

`EvtGen` is an event generator originally developed for BaBar and CLEO, and also used within Belle. `EvtGen` accounts for cascade decays involving multiple vertices and spin configurations. Input data for each decay process is passed to the code as a complex amplitude. In cases where a number of complex amplitudes are invoked for the same process, these are added before the decay probabilities are calculated and consequently the interference terms, which are of significant importance in many B-physics studies, are included. `EvtGen` is controlled by means of a fairly complete decay table (`DECAY.DEC`), which lists all possible decay processes, their branching ratios and the model (amplitude) which is to be used to decay them. Belle II currently uses the simplified default `EvtGen` decay file for generic B events which lacks some improvements that were included in the Belle or BaBar decay files. Since `EvtGen` only handles exclusive final states, `PYTHIA 8.2` is used to produce final states not included in the decay file. Double counting is avoided by rejecting decays produced by `PYTHIA 8.2` that are already included in the decay file, and regenerating this event. `PHOTOS` is used to simulate final state radiation correction in decays [15]. `EvtGen` is also used to simulate $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ continuum events that are fragmented into final states using

PYTHIA 8.2. Unlike at Belle, the continuum light quark production currently does not include initial state radiation. In general, it is not straight forward to translate the Belle fragmentation settings to Belle II since the PYTHIA version has been changed from PYTHIA 6 to PYTHIA 8.2 and not all PYTHIA 6 parameters have PYTHIA 8.2 equivalents and vice versa. All currently used non–default PYTHIA 8.2 parameters are listed in Table 1.2. It is planned to have a full set of PYTHIA 8.2 tunes for Belle II based on Belle data before the start of Belle II data taking. It will need to be separately tuned for modelling inclusive decays in quark continuum samples, and in heavy meson decays.

KKMC is the default generator to simulate the two fermion final states $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$. The currently implemented version is based on the Belle implementation of KKMC4.19 including a modified interface for tau decays. KKMC generates multi–photon initial state radiation (ISR), final state radiation (FSR) and the interference of initial and final state radiation (IFI). These QED corrections are complete NLO for ISR, IFI and FSR and almost complete NNLO for ISR and FSR within the framework of exclusive coherent exponentiation (CEEX) based on Yennie–Frautschi–Suura exclusive exponentiation (YFS/EEX). τ decays are handled by TAUOLA-exp-11-10-2005, taking into account spin polarization effects and transverse spin correlations in τ decays as in KORALB. The hadronic currents for $\tau \rightarrow 4\pi$ are taken from CMD–2 ('binp'), all other from CLEO ('cleo'). Electroweak corrections within KKMC are implemented using the DIZET6.21 library of the ZFITTER project [16, 17]. The DIZET6.21 routine REPI for the calculation of the time–like real part of the electromagnetic coupling $\alpha_{QED}(s)$ has been replaced as described in [18] . The electroweak corrections are complete one–loop with some higher order extensions. The theoretical precision of the generator for lepton pairs is stated to be better than 0.5 % for both cross section and inclusive differential distributions within the detector acceptance for beam energies at and above the $\Upsilon(4S)$, including uncertainties due to vacuum polarization [18].

BABAYAGA.NLO is the default generator to simulate large angle (above about 5° in the CM frame) $e^+e^- \rightarrow e^+e^-(\gamma)$ (Bhabha) and $e^+e^- \rightarrow \gamma\gamma(\gamma)$ final states. BABAYAGA.NLO generates multi–photon initial state radiation (ISR), final state radiation (FSR) and the interference of initial and final state radiation (IFI) based on the matching of exact NLO corrections with a Parton Shower algorithm. Z exchange and $\gamma - Z$ interference are included at Born level. Narrow resonances and vacuum polarization corrections are included but no other electroweak corrections. The theoretical precision of the generator is stated to be about 0.1 % for both cross section and inclusive differential distributions within the detector acceptance.

The non–radiative four fermion final states $e^+e^- \rightarrow e^+e^-e^+e^-$ and $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$. AAFH includes all LO QED diagrams and their interference, but no higher order QED corrections, no weak corrections, and no Z–exchange. The leading order calculation is exact and includes final state mass kinematics. The leading order divergency of the process is controlled using a cut on the minimum invariant secondary fermion pair mass, with typically $W_{\ell\ell} = 0.5\,\mathrm{GeV}$.

1.4 Beam–induced background

Types and simulated samples

According to beam background simulation provided by the accelerator group the most important sources are radiative Bhabha's, Touschek scattering and beam–gas interactions. These backgrounds are simulated with a dedicated accelerator group software called SAD [19] which is not part of basf2.

SAD simulates the transportation of particles through the accelerator and if particle leaves the nominal beam trajectory and collides with the beam pipe or collimator in the Belle II

Table 1.2: `PYTHIA 8.2` parameters with changed values in Belle II.

Parameter name	Default	Belle II
StringFlav:etaSup	0.60	0.27
StringFragmentation:stopMass	1.0	0.3
StringZ:aLund	0.68	0.32
StringZ:bLund	0.98	0.62
StringZ:rFactC	1.32	1.0

experimental region, its position and momentum vector are saved to a file. The files normally correspond to one μs of running the accelerator at the nominal SuperKEKB luminosity. The data from SAD simulation are then passed to the Geant4 simulation within basf2 to produce background samples of a given type. The samples are saved in the standard basf2 root format. The events in these files correspond to the interaction of a single beam particle in the material of the interaction region and consist of simulated hits (SimHits) of all detector components. The equivalent accelerator running time and the background type are also saved within the files.

The two-photon QED background has been studied for the inner tracking detectors but is not included in the default background mixing yet. It is simulated within basf2 using the generator AAFH (see Section 1.3) followed by Geant4 simulation, and the output is saved in the same file format. Since the magnetic field inside the quadrupoles is not yet modelled properly in basf2, the simulation gives reliable results only for the vertex detectors (PXD and SVD), while for the other detector components the simulation tends to over–estimate this background. The output file therefore includes only PXD and SVD SimHits. Other backgrounds like synchrotron radiation and gammas from radiative Bhabha's are less intense and are currently not included in background mixing.

The background types are listed in Table 1.3. The rate of events is calculated from the number of events in the sample and the equivalent accelerator running time.

Background mixing

The simulated background samples are used to add background to the simulated events. Adding background to simulated events is done by adding SimHits; digitization is done after that. Possible pile–up of hits is therefore inherently included. The average number of background events of a given type to be added to a single simulated event is determined from the rate R of a particular background sample and the time window Δt in which the background is mixed

$$\bar{N} = sR\Delta t, \tag{1.1}$$

where s is an optional scaling factor. The number of background events added to a particular simulated event is then generated according to Poisson distribution with the mean $\bar{N}$. To simulate contributions from different bunches, the background events are shifted in time randomly within the time window. This means that all SimHits of a given background event are shifted by the same time and therefore the correlations between detector components are preserved. The discrete bunch nature is however neglected because of sufficiently small bunch spacing.

The size of the time window depends on the detector component. It ranges from 100 ns (TOP) to 26 μs (ECL). To reduce CPU time we chose the time window of $[-1.0,\ 0.8]$ μs, which fits the most detector components, except PXD and ECL; these two have time windows of $[-17.6,\ 8.5]$ μs

Table 1.3: Beam background types (12th background campaign).

type	source	rate [MHz]
radiative Bhabha	HER	1320
radiative Bhabha	LER	1294
radiative Bhabha (wide angle)	HER	40
radiative Bhabha (wide angle)	LER	85
Touschek scattering	HER	31
Touschek scattering	LER	83
beam-gas interactions	HER	1
beam-gas interactions	LER	156
two-photon QED	-	206

and $[-10.0,\ 10.0]$ μs, respectively. Additional background samples are used for mixing the background outside the default time window in these two cases.

Table 1.4 shows a comparison of the number of digitized hits (clusters for PXD and SVD) per event from beam–induced background with those from generic $B\overline{B}$ events.

Background Overlay

When experimental data become available we will use different method. Instead of using simulated beam background, the background overlay method will add background measured by random trigger. The background overlay is therefore done by adding the measured background event to the simulated one using digitized hits. Possible pile-up of hits must be taken into account with dedicated methods. These methods can model the pile-up only approximately since the measured background includes only the hits above the detection threshold.

A framework for background overlay has been designed to unify the method for all detector components. It consists of two basf2 modules and a base class for digitized hits (or clusters of hits). The first module, that must run in a single process mode, reads the data from background file (standard basf2 root file) and the second module, that can run in a multi-process mode, performs the overlay. Each class for digitized hits must implement two base class methods: the one that returns unique channel identifier of the hit and the one that implements the pile-up method, which is usually detector specific. The first method is used to identify channels where background hit has to be added to the existing simulated hit. If this happens, the second method is called. The return value then signals whether the pile-up criterion was fulfilled. If not, the background hit is added to the collection of simulated hits.

1.4.1 Detector Simulation

The simulation package of basf2 is based on the Geant4 software [20, 21]. With basf2 release 00–06–00 (December 2015), the version of Geant4 used for the simulation package has been upgraded from 9.6.2 to 10.1.2. There are two methods to supply the primary event to Geant4: One can use the particle gun class, which is part of the Geant4 package, or one can employ a specific generator software. For the latter case, the particles created by the generator package are sent to Geant4 for simulation, via the interface implemented in the basf2 simulation package. Most of the decay process of particles are described by the generator software. Short lived particles such as K_s are usually decayed by Geant4. Exchange bosons and initial particles such as e^- and e^+ is not passed to Geant4. During the simulation, Geant4 transports each primary particle step by step inside the detector and creates secondary particles. Digitization of hit information

Table 1.4: Number of digitized hits per event for beam-induced background (12th background campaign) and for generic $B\overline{B}$ events withount background. For PXD and SVD the clusters are counted instead of digits. Numbers in parenthesis are without two–photon QED background.

component	background	generic $B\overline{B}$
PXD	10000 (580)	23
SVD	284 (134)	108
CDC	654	810
TOP	150	205
ARICH	191	188
ECL	3470	510
BKLM	484	33
EKLM	142	34

in the sensitive area of the detectors is handled by separate basf2 modules, rather than using software objects incorporated into Geant4 [14]. The result from the Geant4 simulation is sent to *DataStore* to be used by other basf2 modules.

To simulate propagation of particles in the detector, physics processes on the interactions between the particles and the detector materials must be specified. These physics models can be either supplied by users or selected from the physics lists provided by the Geant4 group. The recommended physics list by the Geatnt4 group for the high energy physics experiments is `FTFP_BERT` [22], which has been the default physics list since basf2 release 00–04–00 (May 2014). The `FTFP` and `BERT` acronyms stand for hadronic shower models at different energies: The Fritiof quark–gluon string model at high energy and the Bertini intra–nuclear cascade model at low energy. The transition area between the two models depends on each particle type, typically from 4 to 5 GeV [22, Section 3],[23, 24, 25, 26]. `FTFP_BERT` contains all the standard electromagnetic processes provided by the Geant4 group [27].

The Geant4 software uses the concept of "range–cut" as the production threshold for secondary particles. Internally, Geant4 converts the distance of the range–cut to an energy threshold for each particle [28]. The basf2 simulation package is using the default number set by the Geant4 group, which is 0.7 mm. This parameter can be changed as a user option of basf2.

1.4.2 Magnetic field in basf2

Uncertainties in magnetic fields will affect the Belle II analysis in several ways. The magnetic field is an input to reconstruction of charged tracks. To obtain the optimal resolution of the charged track momentum, the magnetic field must be understood precisely. The reconstruction efficiencies of particles depend on the accuracy of magnetic field information. Differences between the magnetic field used for the detector simulation and the one used for collision data sets may results in systematic bias. Differences between the magnetic field used for the reconstruction and the real magnetic field may create a systematic bias as well.

Inside the Belle II detector, there are two sources of magnetic fields, the detector solenoid and the final focus system (QCS). The detector solenoid, which is comprised of an iron yoke and a superconducting solenoid, creates a uniform magnetic field of 1.5 T at the center of the detector [14]. The iron yoke is interlaced with the KLM detector. The QCS is an extension of the SuperKEKB collider, whose purpose is to focus the incoming e^+ and e^- beams at the collision point [29]. The main components of the QCS are eight superconducting quadrupole magnets. In addition, there are secondary superconducting magnets used for correction and compensation. On the surface, the magnetic fields generated by all the components of the QCS can be added linearly and used for simulation. However, due to the ferromagnetic yokes or shields around the main quadrupoles, non–linear characteristics are introduced in the magnetic field [29].

The Opera3D/TOSCA software [30] was used to produce precision models of the magnetic field [29]. The resulting 3D magnetic field map is incorporated into basf2 release version 00–07–00 (April 2016), and replaced the constant field of 1.5 T as the default map for simulation and reconstruction (see Fig. 1.1). Some analyses may still use the constant field map for the time being. The 3D magnetic field map is in the developing stage, in needs of detailed understanding. In–situ measurements of the Belle II magnetic field have been conducted (September 2015) to provide references for the model. More in–situ measurements and further analysis are planned to improve the precision of the field map up to 10^{-4}.

Bibliography

[1] BaBar, D. Boutigny et al., *The BABAR physics book: Physics at an asymmetric B factory*, in *Workshop on Physics at an Asymmetric B Factory (BaBar Collaboration Meeting) Pasadena, California, September 22-24, 1997*. 1998. `http://www-public.slac.stanford.edu/sciDoc/docMeta.aspx?slacPubNumber=SLAC-R-504`.

[2] *An Introduction to PYTHIA 8.2*, Comput. Phys. Commun. **191** (2015) 159–177, `arXiv:1410.3012 [hep-ph]`.

[3] S. Jadach, B. Ward, and Z. Was, *The Precision Monte Carlo event generator K K for two fermion final states in e+ e- collisions*, Comput.Phys.Commun. **130** (2000) 260–325, `arXiv:hep-ph/9912214 [hep-ph]`.

[4] S. Jadach, B. F. L. Ward, and Z. Was, *Coherent exclusive exponentiation for precision Monte Carlo calculations*, Phys. Rev. **D63** (2001) 113009, `arXiv:hep-ph/0006359 [hep-ph]`.

[5] N. Davidson, G. Nanava, T. Przedzinski, E. Richter-Was, and Z. Was, *Universal Interface of TAUOLA Technical and Physics Documentation*, Comput. Phys. Commun. **183** (2012) 821–843, `arXiv:1002.0543 [hep-ph]`.

[6] G. Balossini, C. Bignamini, C. M. C. Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Photon pair production at flavour factories with per mille accuracy*, Phys. Lett. **B663** (2008) 209–213, `arXiv:0801.3360 [hep-ph]`.

[7] G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *Matching perturbative and parton shower corrections to Bhabha process at flavour factories*, Nucl. Phys. **B758** (2006) 227–253, `arXiv:hep-ph/0607181 [hep-ph]`.

[8] C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, *The BABAYAGA event generator*, Nucl. Phys. Proc. Suppl. **131** (2004) 48–55, `arXiv:hep-ph/0312014 [hep-ph]`. [,48(2003)].

[9] C. M. Carloni Calame, *An Improved parton shower algorithm in QED*, Phys. Lett. **B520** (2001) 16–24, `arXiv:hep-ph/0103117 [hep-ph]`.

[10] C. M. Carloni Calame, C. Lunardini, G. Montagna, O. Nicrosini, and F. Piccinini, *Large angle Bhabha scattering and luminosity at flavor factories*, Nucl. Phys. **B584** (2000) 459–479, `arXiv:hep-ph/0003268 [hep-ph]`.

[11] F. A. Berends, P. H. Daverveldt, and R. Kleiss, *Complete Lowest Order Calculations for Four Lepton Final States in*

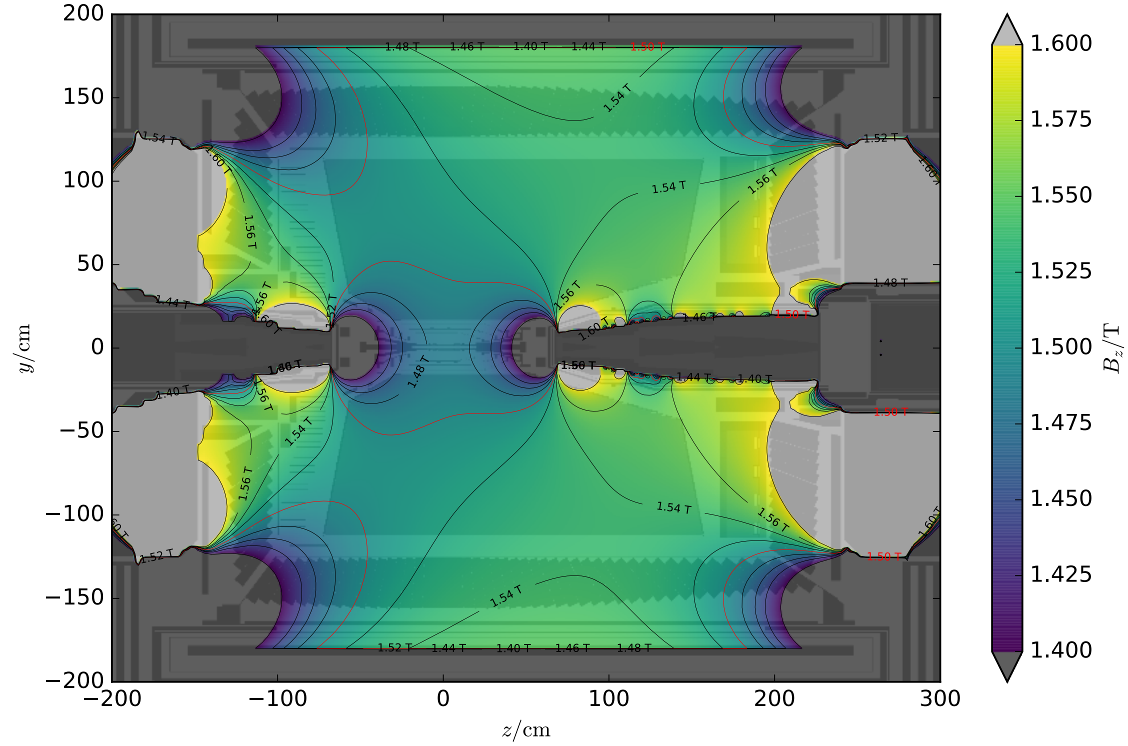

Figure 1.1: z component of the magnetic field map as used in basf2 release version 00–07–00.

electron-Positron Collisions, Nucl. Phys. **B253** (1985) 441.

[12] F. Berends, P. Daverveldt, and R. Kleiss, *Monte Carlo simulation of two-photon processes*, Computer Physics Communications **40** (1986) no. 2, 285 – 307. `http://www.sciencedirect.com/science/article/pii/0010465586901153`.

[13] F. A. Berends, P. H. Daverveldt, and R. Kleiss, *Radiative Corrections to the Process $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$*, Nucl. Phys. **B253** (1985) 421.

[14] T. Abe, Belle II Collaboration, *Belle II Technical Design Report*, `arXiv:1011.0352 [physics.ins-det]`.

[15] N. Davidson, T. Przedzinski, and Z. Was, *PHOTOS Interface in C++: Technical and Physics Documentation*, Comput. Phys. Commun. **199** (2016) 86–101, `arXiv:1011.0937 [hep-ph]`.

[16] D. Yu. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, and T. Riemann, *ZFITTER v.6.21: A Semianalytical program for fermion pair production in e+ e- annihilation*, Comput. Phys. Commun. **133** (2001) 229–395, `arXiv:hep-ph/9908433 [hep-ph]`.

[17] A. B. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. W. Grunewald, K. Monig, S. Riemann, and T. Riemann, *ZFITTER: A Semi-analytical program for fermion pair production in e+ e- annihilation, from*

version 6.21 to version 6.42, Comput. Phys. Commun. **174** (2006) 728–758, arXiv:hep-ph/0507146 [hep-ph].

[18] S. Banerjee, B. Pietrzyk, J. M. Roney, and Z. Was, *Tau and muon pair production cross-sections in electron-positron annihilations at $\sqrt{(s)}$ = 10.58-GeV*, Phys. Rev. **D77** (2008) 054012, arXiv:0706.3235 [hep-ph].

[19] *Strategic Accelerator Design (SAD)*, http://acc-physics.kek.jp/SAD. Accessed: 2016-05-06.

[20] S. Agostinelli et al., GEANT4, *GEANT4: A Simulation toolkit*, Nucl.Instrum.Meth. **A506** (2003) 250–303.

[21] J. Allison et al., *Geant4 developments and applications*, IEEE Trans. Nucl. Sci. **53** (2006) 270.

[22] Geant4 Hadronic Working Group, Electromagnetic Working Group, A. Dotti, *Simulation of Showers with Geant4*, in *Proceedings, International Conference on Calorimetry for the High Energy Frontier (CHEF 2013)*, pp. 247–253. 2013. http://inspirehep.net/record/1289762/files/CHEF2013_Andrea_Dotti.pdf.

[23] Geant4 Collaboration, *Physics reference manual Ver. 10.1*, Dec., 2014. http://geant4.org/.

[24] Geant4 hadronic Working Group, D. H. Wright, *Improving the Medium and Low Energy Physics Models in Geant4*, in *Proceedings, International Conference on Calorimetry for the High Energy Frontier (CHEF 2013)*, pp. 254–259. 2013. http://inspirehep.net/record/1289812/files/CHEF2013_Dennis_Wright.pdf.

[25] B. Andersson, G. Gustafson, and B. Nilsson-Almqvist, *A Model for Low p(t) Hadronic Reactions, with Generalizations to Hadron - Nucleus and Nucleus-Nucleus Collisions*, Nucl. Phys. **B281** (1987) 289–309.

[26] Geant4 Hadronic Working Group, V. V. Uzhinsky, *The Fritiof (FTF) Model in Geant4*, in *Proceedings, International Conference on Calorimetry for the High Energy Frontier (CHEF 2013)*, pp. 260–264. 2013. http://inspirehep.net/record/1289919/files/CHEF2013_Vladimir_Uzhinskiy.pdf.

[27] V. Ivanchenko et al., *Recent Improvements in Geant4 Electromagnetic Physics Models and Interfaces*, in *Progress in Nuclear Science and Technology*, vol. 2, pp. 898–903. 2011. http://www.aesj.or.jp/publication/pnst002/data/898-903.pdf.

[28] Geant4 Collaboration, *User's Guide for Application Developers Ver. 10.2*, Dec., 2015. http://geant4.org/.

[29] Y. Arimoto, N. Ohuchi, M. Tawada, K. Tsuchiya, H. Yamaoka, Z. Zong, B. Parker, and P. Wanderer, *Three Dimensional Field Analysis for Final Focus Magnet System at SuperKEKB*, in *Proceedings, 5th International Particle Accelerator Conference (IPAC 2014)*, p. WEPRI086. 2014. http://jacow.org/IPAC2014/papers/wepri086.pdf.

[30] Cobham Technical Services, Vector Fields Software, Oxford, England, *Opera-3D User Guide Ver. 15R3*, Oct., 2012. http://operafea.com/.