Laplace's equation

Solveig Lunde

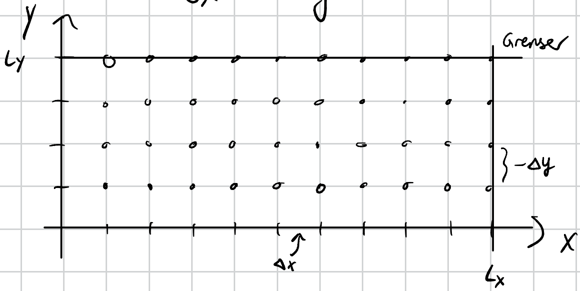
Laplace's ligning (numerisk løsning)

$$\Delta f = 0$$

$$\hookrightarrow \Delta = \nabla \cdot \nabla = \nabla^2 = \text{laplace operatoren}$$

I to romlige koordinater har vi

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

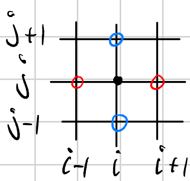


$$\Delta y = \Delta x$$
$$L_x = L_y = 1$$

$$x_i = (i-1)\Delta x, \quad i = 1, n_x$$

$$y_j = (j-1)\Delta y, \quad j = 1, n_y$$

$$f(x_i, y_j) = f_{i,j}$$



$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{(\Delta x)^2}$$

$$\left.\frac{\partial^2 f}{\partial y^2}\right|_{i,j} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{(\Delta y)^2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \qquad \Delta x = \Delta y$$

$$4f_{i,j} - f_{i+1,j} - f_{i-1,j} - f_{i,j+1} - f_{i,j-1} = 0$$

$$f_{i,j} = \frac{f_{i+1,j} + f_{i-1,j} + f_{i,j+1} + f_{i,j-1}}{4}$$