

# ALGORITHM

- Input: A sorted array of book titles, 'book\_list' of n elements, and a target book title, 'target\_book'.
  - Output: The index of target\_book in book\_list, or -1 if target\_book is not found.
1. Set low = 0
  2. Set high = n - 1
  3. Set loc = -1
  4. While low <= high do:
    - a) Set mid = (low + high) / 2
    - b) If book\_list[mid] == target\_book:
      - I. Set loc = mid
      - II. Break
    - c) Else If book\_list[mid] < target\_book:
      - I. Set low = mid + 1
    - d) Else:
      - I. Set high = mid - 1

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5. If  $\text{loc} \geq 0$ :
  - a) Print "Book is found at location 'loc' and searching is successful."
6. Else:
  - a) Print " 'loc' Book is not found and searching is unsuccessful."

# Time Complexity Analysis

## Binary Search

- **Best Case:**  $O(1)$  – Target is the middle element in the first step
- **Average Case:**  $O(\log n)$  – As the search is halved each time, it takes  $\log_2(n)$  steps on average.
- **Worst Case:**  $O(\log n)$  – Target is not found, or it's in the last possible half.
- Binary search is faster than linear search  $O(n)$ , as binary search minimizes comparisons.

# Why is Time Complexity $O(\log n)$ ?

- Logarithmic Reduction: Each iteration reduces the search space by half.
- Formula Derivation: For an array size of  $n$ , after each split:
  - $n/2 \dots \approx$  reaches 1
  - Formula:  $n/2^k$  where  $k$  is the number of steps or divisions we make.
  - the process continues until  $n/2^k = 1$
  - we multiply both sides by  $2^k = n = 2^k$
  - To solve for  $k$ , we use the base-2 logarithm on both sides:  $\log_2(n) = \log_2(2^k)$
  - $\log_2(n) = k$
  - So  $k = \log_2(n)$

# Example 1

- $n = [2, 5, 7, 12, 15, 18, 21, 24, 30, 35]$
- Target element = 15
- Step 1:
  - Low = 0, high = 9
  - $\text{Mid} = (\text{low} + \text{high}) // 2 = 4$
  - Element at index 4 is 15 (target found)
- **Result:** Found 15 in just 1 step (best case)

# Example 2

- $n = [2, 5, 7, 12, 15, 18, 21, 24, 30, 35]$
- Target element = 21
- **Step 1:**
  - Low = 0, high = 9
  - $\text{Mid} = (\text{low} + \text{high}) // 2 = 4$
  - Element at index 4 is 15 (less than 21)
  - Set low = 5
- **Step 2 low = 5, high = 9:**
  - $\text{Mid} = (\text{low} + \text{high}) // 2 = 7$
  - Element at index 7 is 24 (greater than 21)
  - Set high = 6

## Example 2 continue

- $n = [2, 5, 7, 12, 15, 18, 21, 24, 30, 35]$
- Target element = 21
- **Step 3 low = 5, high = 6:**
  - $\text{Mid} = (\text{low} + \text{high}) // 2 = 5$
  - Element at index 5 is 18 (less than 21)
  - Set low = 6
- **Step 4: low = 6, high = 6**
  - $\text{Mid} = (\text{low} + \text{high}) // 2 = 6$
  - Element at index 6 is 21 (target found)
- **Result: Found 21 in 4 steps (worst case).**

# $O(\log n)$

- $n = [2, 5, 7, 12, 15, 18, 21, 24, 30, 35]$
- $n = 10$
- The number of steps in binary search is roughly  $\log_2(10) \approx 3.32$  so we round up to 4
- This tells us that on average and in the worst cases, we need at most 4 steps



# Space Complexity of Binary Search

- Iterative Binary Search:  $O(1)$  space, as it only needs a few variables for indices.
- Recursive Binary Search:  $O(\log n)$  space, as it requires stack space for each recursive call.
- The iterative approach is more space-efficient, while recursion provides a simpler code structure.