

Quiz

- In a binary search, what is the maximum number of comparisons needed to find a target value in a sorted array of size 16?
 - A. 4
 - B. 8
 - C. 16
 - D. 32
- Hint: Think about how binary search divides the array into halves and the number of times this division happens.

Analyzing Merge Sort Time Complexity

- Base Case:

- `if (left < right) {`

- Time: $O(1)$.

- It's a single comparison that runs in constant time.

- Calculate Midpoint:

- `int mid = left + (right - left) / 2;`

- Time: $O(1)$.

- Arithmetic operations and assignment take constant time.

Analyzing Merge Sort Time Complexity

- Recursive Calls:

- `merge_sort(prices, left, mid);`
- `merge_sort(prices, mid + 1, right);`
- Time: This is where the recursion happens, dividing the array into smaller subarrays. Each recursive call breaks the array into two halves until size 1.
- Total time: $O(\log n)$ recursion depth.

- Merging:

- `merge(prices, left, mid, right);`
- Time: $O(n)$. Merging two halves involves comparing all elements in the subarrays, which takes linear time relative to the number of elements in the range `[left, right]`.

Analyzing Merge Sort Time Complexity

- Temporary Arrays (Copying):

```
vector<double> L(n1), R(n2);
```

```
for (int i = 0; i < n1; i++) L[i] = prices[left + i];
```

```
for (int j = 0; j < n2; j++) R[j] = prices[mid + 1 + j];
```

- Time: $O(n1 + n2) \approx O(n)$.
- Both loops iterate through the left and right subarrays to copy elements.

Analyzing Merge Sort Time Complexity

- Merge While Loop:

```
while (i < n1 && j < n2) {  
    if (L[i] <= R[j]) {  
        prices[k] = L[i];  
        i++;  
    } else {  
        prices[k] = R[j];  
        j++;  
    }  
    k++;  
}
```

- Time: $O(n)$.
- Each element in L and R is compared and copied into the main array.

Analyzing Merge Sort Time Complexity

- Remaining Elements in L or R:

```
while (i < n1) {  
    prices[k] = L[i];  
    i++;  
    k++;  
}
```

```
while (j < n2) {  
    prices[k] = R[j];  
    j++;  
    k++;  
}
```

- Time: $O(n)$.
- After one subarray is exhausted, the remaining elements in the other subarray are copied into prices.

Analyzing Merge Sort Time Complexity

- C++ Overall Analysis:
 - Lines like `if (left < right)` or calculating `mid` take constant time.
 - Recursion (`merge_sort`) divides the array in $O(\log n)$ steps.
 - Merging at each recursion level processes all n elements $\rightarrow O(n)$.
- Overall time complexity: $O(n \log n)$.

Merge sort Space Complexity analysis

- C++ Implementation:
 - Uses temporary arrays (L and R) for merging, requiring $O(n)$ extra space.
- Python Implementation:
 - Slicing (`prices[:mid]` and `prices[mid:]`) creates new subarrays, also requiring $O(n)$ space.
- Overall Space Complexity: $O(n)$.
- C++ is more memory-efficient and faster because it avoids creating new arrays during recursion (uses indices instead).
- Python's slicing adds an extra overhead, but the time complexity remains the same because the constant factors don't change the overall growth rate.