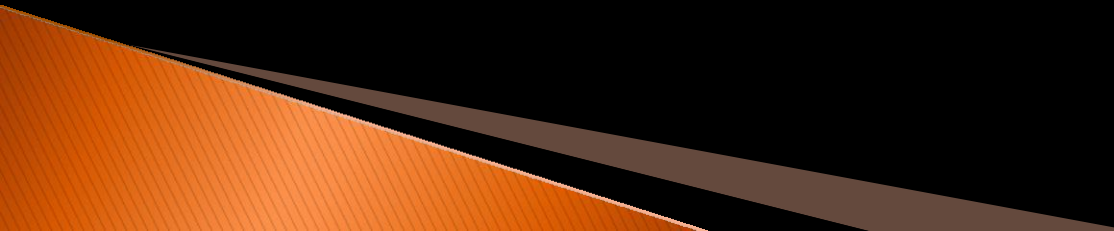


Sorting Algorithms

- Algorithms designed to arrange data in a specific order (ascending or descending)
- Key in tasks like searching, data organization, and optimization
- **Types of Sorting Algorithms:**
 1. **Comparison–Based Sorting:**
 - a) Compare elements to determine their order.
 - b) **Example:** Bubble Sort, Quick Sort, Merge Sort.
 2. **Non–Comparison–Based Sorting:**
 1. Use counting or distribution properties instead of direct comparisons.
 2. **Example:** Counting Sort, Radix Sort, Bucket Sort.


Bubble Sort

- Bubble Sort is a simple comparison-based sorting algorithm. It repeatedly steps through the list, compares adjacent items, and swaps them if they are in the wrong order.
 - Works by repeatedly "bubbling up" the largest element to the end.
 - Easy to understand and implement.
 - Best for Small or nearly sorted datasets.
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Bubble Sort algorithm

- **Input:**
 - A list of prices.
- **Output:**
 - The sorted list in ascending order.
- **Steps:**
 1. Start with the given list of prices.
 2. Outer loop: Repeat $n-1$ passes through the list (where n is the number of prices).
 - a) Assume the list is sorted at the start of each pass
 3. Inner loop: Compare each pair of adjacent prices in the unsorted portion of the list
 - a) If the left price is greater than the right price, swap them
 - b) If any swap occurs, mark the list as not sorted.
 4. If no swaps occur during a pass, the list is already sorted, so stop early
 5. Repeat until the entire list is sorted.
 6. Output the sorted list

Why Two Loops?

- **Multiple Passes:**
 - Bubble sort requires multiple passes over the list to sort it completely. The outer loop controls the number of passes.
 - **Comparison and Swapping:**
 - The inner loop performs the actual comparison and swapping of adjacent elements within each pass.
 - The outer loop ensures that each element gets its chance to be compared and potentially swapped.
 - The inner loop performs the pairwise comparisons and swaps within each pass.
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Example pass 1

- Price list = [99.99, 49.95, 299.49, 19.95]
- $n = 4$;
- Pass 1 ($i=0$):
 1. Comparison 1:
 - Compare 99.99 and 49.95 ($j=0$):
 - $99.99 > 49.95$, so swap.
 - List after swap: [49.95, 99.99, 299.49, 19.95]
 2. Comparison 2:
 - Compare 99.99 and 299.49 ($j=1$)
 - $99.99 < 299.49$, no swap.
 - List after swap: [49.95, 99.99, 299.49, 19.95]
 3. Comparison 3:
 - Compare 299.49 and 19.95 ($j=2$):
 - $299.49 > 19.95$, so swap.
 - List after swap: [49.95, 99.99, 19.95, 299.49]

Example pass 2

- List after pass1 = [49.95, 99.99, 19.95, 299.49]
- $n = 4$;
- Pass 2 ($i=1$):
 1. Comparison 1:
 - Compare 49.95 and 99.99 ($j=0$):
 - $49.95 < 99.99$, no swap.
 - List after swap: [49.95, 99.99, 19.95, 299.49]
 2. Comparison 2:
 - Compare 99.99 and 19.95 ($j=1$):
 - $99.99 > 19.95$, so swap.
 - List after swap: [49.95, 19.95, 99.99, 299.49]

Example pass 3

- List after pass 2 = [49.95, 19.95, 99.99, 299.49]
- $n = 4$;
- Pass 3 ($i=2$):
 1. Comparison 1:
 - Compare 49.95 and 19.95 ($j=0$):
 - $49.95 > 19.95$, so swap.
 - List after swap: [19.95, 49.95, 99.99, 299.49]
- Final sorted price list = [19.95, 49.95, 99.99, 299.49]

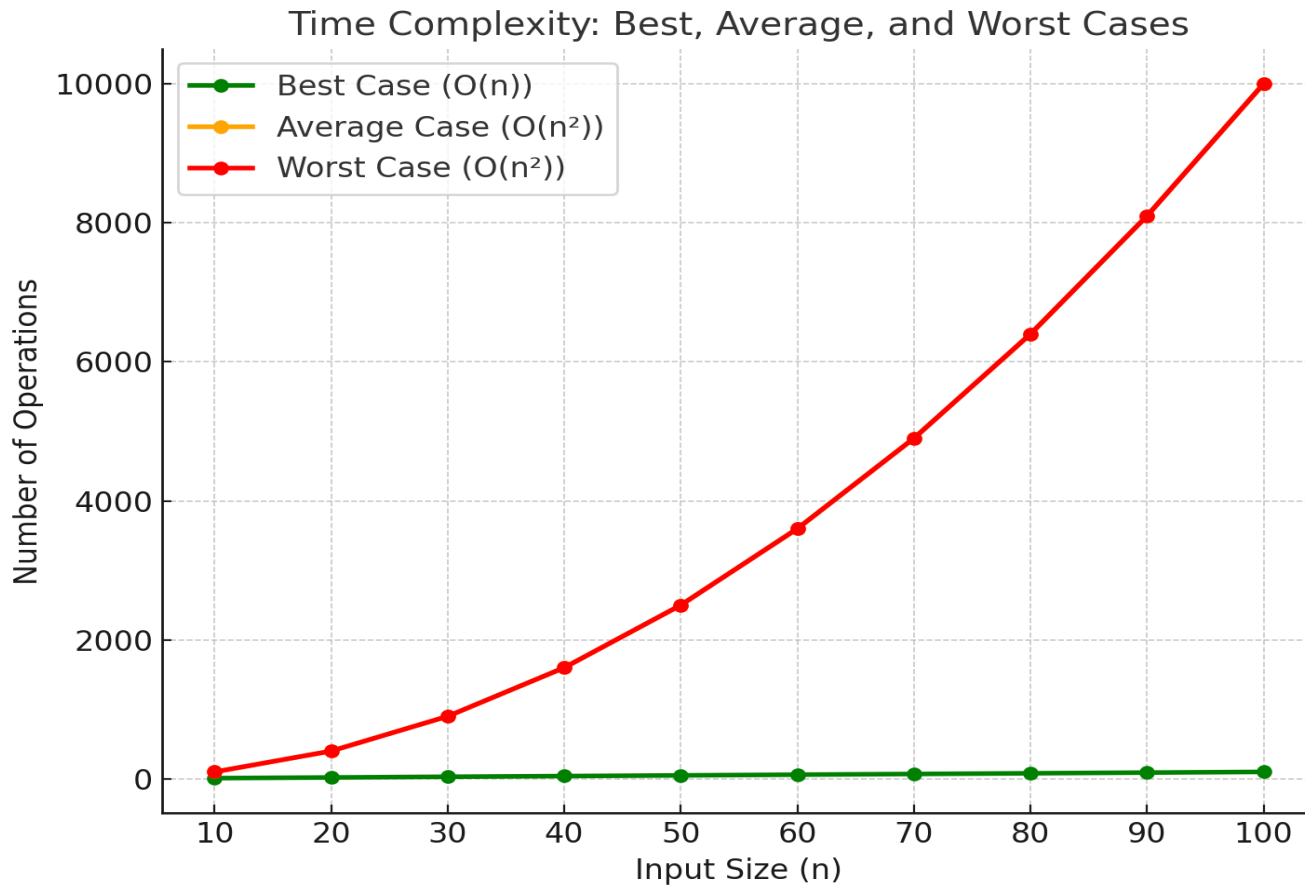
Time Complexity of Bubble Sort

- **Best Case $O(n)$:**
 - The list is already sorted
 - In this case, the outer loop only needs to run once, and the inner loop will terminate early without any swaps.
- **Average Case $O(n^2)$:**
 - The list is in a random order.
 - In most cases, the algorithm will require multiple passes through the array to sort it completely
- **Worst Case $O(n^2)$:**
 - The list is sorted in reverse order.
 - In this case, the maximum number of comparisons and swaps will be required.
- Due to its quadratic time complexity, bubble sort is generally not considered efficient for large datasets. It's better suited for small datasets or educational purposes.

Why $O(n^2)$?

- Outer Loop: Runs $n-1$ times.
 - For example: $n=4$, it runs 3 times.
- Inner Loop: Runs $n-i-1$ times for each outer loop iteration.
 - $i=0$: Inner loop runs 3 times, means $(n-1)$ times
 - $i=1$: Inner loop runs 2 times, means $(n-2)$ times
 - $i=2$: Inner loop runs 1 time, means $(n-3)$ times
- Total Comparisons:
 - $(n-1)+(n-2)+\dots+1$
- Using the Arithmetic Series Formula:
 - $$\frac{(\text{first term} + \text{last term}) \times \text{number of terms}}{2}$$
 - $$\frac{(n-1)+1 \times (n-1)}{2}$$
- $$\frac{n(n-1)}{2} = \frac{n^2 - n}{2} = \text{so the dominant term is } n^2 = O(n^2).$$

Quadratic time complexity



Space Complexity of Bubble Sort

- Space Complexity $O(1)$.
- The space complexity of bubble sort is $O(1)$ that means the amount of extra space required by the algorithm is constant, regardless of the input size.
- Bubble sort is an in-place sorting algorithm, which means it sorts the elements within the original array without requiring any additional data structures that grow with the input size.