Optimization Theory and Algorithm **Homework 2 - 07/03/2021**

Homework 2

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1 Homework

1. Poisson Regression: Suppose now that the b_i are counts and the $\mathbf{a}_i$ are some predictors in $\mathbb{R}^p$. Let λ_i be the expected number of events in sometime period, i.e.

$$P(b_i \mid \lambda_i) = \frac{e^{-\lambda_i}\lambda_i^{b_i}}{b_i!}$$

Based on the example of Generalized Linear Model, please derive the following optimization form to present the simple Poisson regression.

$$\min_{\mathbf{x}} f(\mathbf{x}) = \sum_{i=1}^{m} \exp\left(\langle \mathbf{a}_i, \mathbf{x}\rangle\right) - \langle \mathbf{b}, A\mathbf{x}\rangle.$$

Furthermore, please compute the gradient of $f(\mathbf{x})$.

2. Let

$$f(\mathbf{x}) = \sum_{i=1}^{m} \log\left(1 + \langle \mathbf{a}_i, \mathbf{x}\rangle\right) - \langle \mathbf{b}, A\mathbf{x}\rangle.$$

be the objective function of logistic regression, please compute the gradient and Hessian matrix of $f(\mathbf{x})$.

3. The textbook, Page 22, 1.4.

4. Please implement gradient descent method to solve the following problem

$$f(\mathbf{x}) = \frac{1}{2}\|A\mathbf{x} - \mathbf{b}\|_2^2, \tag{1}$$

where

$$A = \begin{bmatrix} 1, & 2, & 1 \\ 3, & 7, & 1 \\ 0, & 4, & 1 \end{bmatrix}$$

and $\mathbf{b} = (2, 12, 2)^\top$. Please specify the step for gradient descent for this problem.

2 Deadline

The deadline is March. 21. 2023.