修士学位請求論文要旨

Iterated Inversion System: An Efficient Algorithm to Visualize Kleinian Groups Based on Inversions

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Kleinian group theory is one of the fields of mathematics studying Möbius transformation groups. Kleinian group theory is advanced by mathematicians in the nineteenth century. Felix Klein and his student, Robert Fricke studied Möbius transformation groups. Henri Poincare named such groups *Kleinian Groups*. Moreover, composing Geometry, Algebra, and Analysis, Poincare built foundation of Kleinian groups theory. Klein and Poincare were rivals to study Kleinian group theory.

Möbius transformation group is well suited to visualization and experiment; Actually, Klein and his students also leave beautiful visualized images of a Kleinian group without a computer. There are aspects that mathematicians advance research from visualization and experiment.

After a computer appeared, various visualization and calculation are performed by computer. Visualized images of Kleinian groups often have beautiful fractal shape. Thus, some people enjoy rendered images as arts. For example, *Fractalforums*[1] community gathers many fractal enthusiasts and discusses fractals. The fractals generated by Kleinian group theory also come up for discussion. In a sense, Kleinian group theory is an interdisciplinary area between mathematics and arts.

Mumford, Series, and Write wrote a book called *Indra's Pearls* The book is written for non-mathematician and contains explanation about Kleinian group theory, many beautiful visualized images, and methods of visualization. Thus, not only math enthusiast but also programmers enjoy the book.

However, the book deal with a small part of Kleinian group. Because of the properties of the group, we can not visualize all of the types of Kleinian groups in real time in spite of the growth of the performance of a personal computer.

Our goals are to visualize all of the Kleinian groups in real time by personal computer and help us understand the Kleinian group theory intuitively with software.

As the first step to the goals, we invent an algorithm called *Iterated Inversion System (IIS)*. IIS is an algorithm to render Kleinian group based on circle or sphere inversions. It visualizes not only two-dimensional Kleinian group but also three-dimensional Kleinian group. In this paper, we introduce the basic usage of the IIS algorithm and its applications.

[1] `https://fractalforums.org/`