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A FLOW-BASED PROGRAMMING ENVIRONMENT FOR KALEIDOSCOPE PATTERNS

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Abstract: *In this research, we explore kaleidoscope patterns with a flow-based programming environment. Kaleidoscope patterns are generated by mirror reflections in lines or circles. Flow-based programming is one of the programming paradigms. It enables us to overlook structures of constructions by mirrors. Also, it is easy to extend functions of kaleidoscope patterns editor by making original scripts or nodes. In order to render an image of a kaleidoscope, it may take a long time to compute kaleidoscope patterns according to an arrangement of mirrors. We also developed an algorithm to render patterns generated by reflections in real time. The algorithm is called Iterated Inversion System (IIS.) Thanks to the algorithm, it makes it easy to design and render*

kaleidoscope patterns generated by mirror reflections. We can find many varieties of patterns with the software based on flow-based programming. This research is in the interdisciplinary area and contains mathematics, arts, and technology.

1 INTRODUCTION

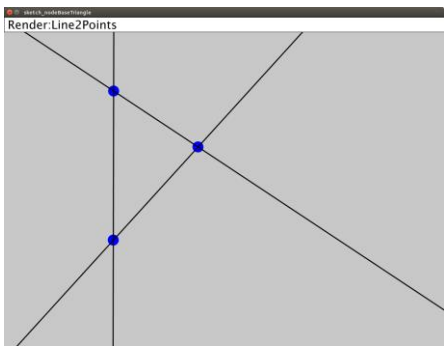

Figure 1

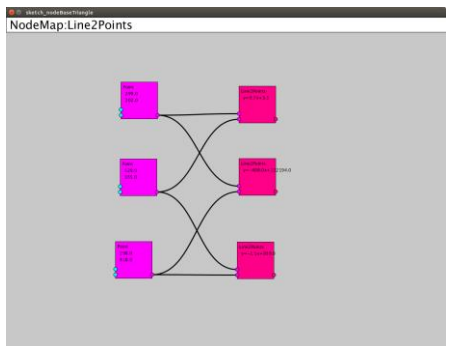

Figure 2

Interactive geometry software is software that creates geometrical constructions, and we can manipulate geometrical objects by hand keeping relationships between the geometrical objects. For example, see Figure 1. There are three points, and they form three lines. When we move one of the points, the lines are also moved according to the positions of points. Famous interactive geometry software is *GeoGebra*, *Cabri Geometry*, or *Cinderella*.

Flow-based programming is one of the programming paradigms. See Figure 2. All processes and data are represented as nodes. We also call the figure *graph* because the figure looks like a plane graph with nodes and edges. In Figure 2, there are three point nodes, and they are connected to the three line2point nodes by edges. The data is sent left to right through the edges. Finally, the geometrical objects are rendered in canvas as in Figure 1. There are three lines and three crossing points.

There are many programming environments based on flow-based programming. For example, *vvvv*, *TouchDesigner*, or *Pure Data*. In addition, 3D modeling software like *blender* often has a shader node graph editor to construct shaders easily. In this way, flow-based programming is used various fields of creative coding such as audio and visual.

However, there is few mathematical software adopting flow-based programming. So, we are developing interactive geometry software called *Flower* generating kaleidoscope patterns based on flow-based programming.

Drawing kaleidoscope patterns with computers often takes a long time because the number of reflected images of kaleidoscope increases exponential order. So, we use an algorithm to draw symmetry patterns including kaleidoscope patterns fast. The algorithm is called *Iterated Inversion System (IIS)* (Nakamura, 2016, pp.367-370).

Application range of the algorithm is wide, and we have found two application of IIS (Nakamura, 2017, pp.159-166) (Nakamura, 2018, pp.171-178). In this research, we apply IIS to interactive geometry software with flow-based programming.

2 IMPLEMENTATION

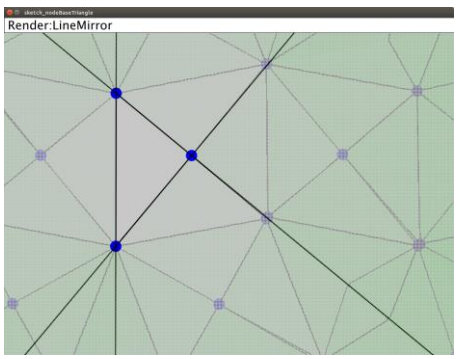

Figure 3

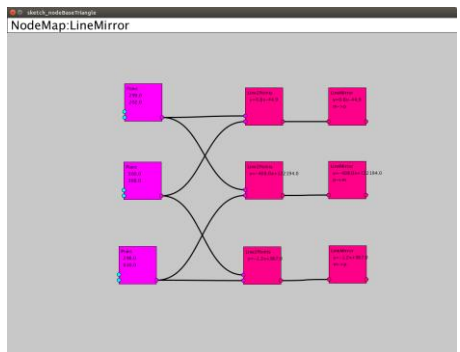

Figure 4

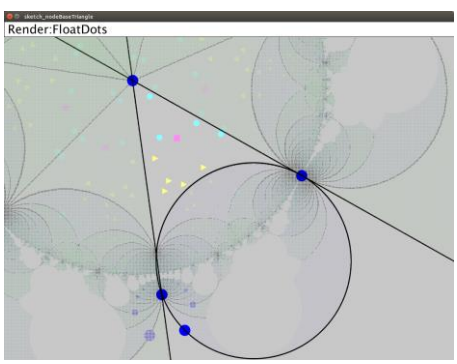

Figure 5

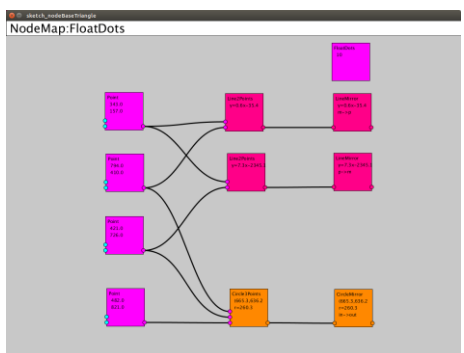

Figure 6

See Figure 3. This is the same construction as Figure 1, but the lines turn into mirrors of half-planes. The half-planes reflect inside of itself. The inner parts of the half-planes are colored by light green. This is one of the simple kaleidoscope patterns. If we move one of the three points in Figure 3, the triangle shaped by three mirrors is also moved. Figure 4 shows the above condition programmed by flow-based programming. There are also three point nodes and three line nodes, and line nodes are connected to line mirror nodes. In the same way as for interactive geometry software, we can move geometrical components keeping relationships between the components.

See Figure 5. Here, one of the mirrors is a circle. We adopt a circle as a mirror. A mirror reflection by a circle is an inversion in a circle. A mirror of the circle generates complicated patterns. Figure 5 includes some beads besides. They are also reflected by mirrors. Figure 6 shows the graph to render a picture of Figure 5. We can find there are

four control points, three mirrors (two half-planes and one circle,) and ten beads from Figure 6.

In order to render kaleidoscope patterns especially reflections through circles, it often takes a long time because reflected images increase exponentially. So, we use the algorithm called Iterated Inversion System (IIS.) The algorithm is simple. For each point in the plane, if the point is in the mirror, we apply a reflection of the mirror. We continue iterating the reflections until the point transformed outside of all of the mirrors. Finally, we determine color according to the number of reflections or refer the pixel value of the iterated point. We can parallelize the algorithm easily. So, the figures are rendered in real time when we use parallel processing. For more details of the algorithm, read (Nakamura, 2017, pp.160-163).

3 SUMMARY

In this paper, we showed a flow-based programming environment for kaleidoscope patterns. There are some advantages of the constructions by flow-based programming. Firstly, we can understand easily the procedure of constructions. We look out over the whole of the graph of the constructions, and we can understand relationships between mathematical components. Secondly, Flow-based programming is flexible. We can insert processes wherever we like. For example, we can use time series data to give motion to mathematical components such as points or circles later. Thirdly, flow-based programming has high extensibility. The users of the software can make their own node or script, and add or extend functions of the software. Software with flow-based has prominent features.

Thanks to the combination of IIS and flow-based software, we can continue drawing any figures in real time. Not only we create geometric construction, but also it makes it easy to search for artistical expressions.

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