Combining all of the above observations, we obtain

$$\mathbb{E}(N_k(t+1) - N_k(t) \,\big|\, \mathcal{F}_t) = (k-1)N_{k-1}(t)P'_{i_0}(t) - kN_k(t)P'_{j_0}(t) + \mathbb{1}_{\{k=1\}}.$$

We assume $N_k(t)/t$ converges to $\mathbb{E}N_k(t)/t$ in probability and that

$$\mathbb{E}(N_k(t)) \approx tp_k \quad \text{for large } t.$$

Define by convention $N_0(t) = 0$ and then taking repeated expectations, we obtain the following recursion in p_k from the above equation.

$$p_k = \mathbb{1}_{\{k=1\}} + (k-1)tp_{k-1}P'_{i_0}(t) - ktp_kP'_{j_0}(t).$$