Combining the observations from the previous slide and using double expectation,

$$\mathbb{E}(N_k(t+1) - N_k(t) \,\big|\, \mathcal{F}_t) = \frac{(k-1)N_{k-1}(t)}{\sum_{j=1}^{t+1} D_j(t)} - \frac{kN_k(t)}{\sum_{j=1}^{t+1} D_j(t)} + \mathbb{1}_{\{k=1\}}$$

$$\implies \mathbb{E}(N_k(t+1)) - \mathbb{E}(N_k(t)) = \frac{k\left[\mathbb{E}(N_{k-1}(t)) - \mathbb{E}(N_k(t))\right]}{2t} - \frac{\mathbb{E}(N_{k-1}(t))}{2t} + \mathbb{1}_{\{k=1\}}.$$

We plan to further explore this recurrence relation in order to study the asymptotic behaviour of $N_k(t)$ and $P_k(t)$.