Let us try to find some upper bound on the term $1+P_i'(t)$ for $t > d-2$. Observe that, for $t > d-2$,

$$
\begin{aligned}
1+P_i'(t) &= 1+\sum_{\substack{k_1,k_2,\ldots,\\ k_d\in[t+2]\setminus\{i\}}} \left[\left(\prod_{j=1}^{d}\frac{D_{k_j}(t)}{2(t+1)}\right)\times\frac{1}{2(t+1)-S_t(\{k_1,\ldots,k_d\})}\right] \\
&\leq 1+\sum_{\substack{k_1,k_2,\ldots,\\ k_d\in[t+2]\setminus\{i\}}} \left[\left(\prod_{j=1}^{d}\frac{D_{k_j}(t)}{2(t+1)}\right)\times\frac{1}{t+2-d}\right] \\
&= 1+\frac{1}{2^d(t+1)^d}\sum_{\substack{k_1,k_2,\ldots,\\ k_d\in[t+2]\setminus\{i\}}} \left[\left(\prod_{j=1}^{d}D_{k_j}(t)\right)\times\frac{1}{t+2-d}\right].
\end{aligned}
$$