

For $t + 2 > d$, i.e., for $t > d - 2$,

$$P_i(t) = \sum_{\substack{k_1, k_2, \dots, \\ k_d \in [t+2] \setminus \{i\}}} \left[\left(\prod_{j=1}^d \frac{D_{k_j}(t)}{2(t+1)} \right) \times \frac{D_i(t)}{2(t+1) - S_t(\{k_1, \dots, k_d\})} \right], \quad (1)$$

where $S_t(A)$ denotes the sum of the degrees at time t of the unique vertices indexed by the set $A \subset [t+2]$. On the other hand, for $0 \leq t \leq d-2$, there is no taboo-ing, and hence,

$$P_i(t) = D_i(t)/2(t+1).$$

Since $P_i(t) \geq 0$, $(D_i(t), \mathcal{F}_t)$ is a submartingale.