

Further, borrowing the notations for the i.i.d. random variables X_1, \dots, X_d as defined before

$$\begin{aligned}
1 + P'_i(t) &\geq 1 + \frac{(t+1)^d}{2^d(t+1)^d} \mathbb{E} \left[\left(\prod_{j=1}^d f(X_j) \right) \times \frac{1}{2t+1} \right] \\
&= 1 + \frac{1}{2^d(2t+1)} (\mathbb{E}[f(X_1)])^d \\
&= 1 + \frac{1}{2^d(2t+1)} \left(\frac{2(t+1) - D_i(t)}{t+1} \right)^d \\
&\geq 1 + \frac{1}{2^d(2t+1)} \\
&= \frac{2^d(2t+1) + 1}{2^d(2t+1)} = \frac{t + (1/2 + 2^{-d-1})}{t + 1/2}.
\end{aligned}$$