

Instead of working with  $P_k(t)$ , which denotes the proportion of  $k$  degree vertices at time  $t$ , we define  $N_k(t)$  which would denote the total number of  $k$  degree vertices at time  $t$ . We define,

$$N_k(t) = \sum_{i=1}^{t+2} \mathbb{1}_{\{D_i(t)=k\}} = (t+2)P_k(t). \quad (1)$$

We start by finding the conditional expectation  $\mathbb{E}(N_k(t+1) \mid \mathcal{F}_t)$ .

$$\mathbb{E}(N_k(t+1) \mid \mathcal{F}_t) = N_k(t) + \mathbb{E}(N_k(t+1) - N_k(t) \mid \mathcal{F}_t). \quad (2)$$