

Now, conditional on \mathcal{F}_t , there are three ways $N_k(t+1) - N_k(t)$ can be non-zero. We list them as below.

- (I) The half-edge of the new vertex attaches to a particular $k - 1$ degree vertex. Then, $N_k(t)$ increases by 1 to $N_k(t+1)$. The probability of this event is $\frac{k-1}{\sum_{j=1}^{t+1} D_j(t)}$; and there are $N_{k-1}(t)$ many $k - 1$ degree vertices to choose from.
- (II) The half-edge of the new vertex attaches to a particular k degree vertex. Then, $N_k(t)$ decreases by 1 to $N_k(t+1)$. The probability of this event is $\frac{k}{\sum_{j=1}^{t+1} D_j(t)}$; and there are $N_k(t)$ many $k - 1$ degree vertices to choose from.
- (III) Another contribution to $k = 1$ arises from the newly arriving vertex itself. The newly arriving vertex at time $t + 1$ has degree exactly 1, since the half edge attached to it is not allowed to form a self-loop.