We take resort to an appealing ansatz that states

$$\frac{S_t(\{k_1,\ldots,k_d\})}{2(t+1)} \stackrel{a.s.}{\to} 0, \quad \text{as } t \to \infty$$

where k_1, \ldots, k_d are the indices of the taboo vertices at time t. Assuming this, which still needs a rigorous proof, we get two tighter bounds.