Brief Description of Dissertation topic

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The Albert-Barabasi model (1999) is an algorithm for generating random graphs through a preferential attachment mechanism. This model has been used to model several natural and human-made systems, including the world wide web, citation networks, and some social networks.

The important feature of the preferential attachment model proposed by Albert and Barabasi is that newly appearing edges are more likely to connect to vertices with large degrees, thus making these degrees even larger. This feature clarifies the nomenclature *preferential* attachment model. This phenomenon is often called the *rich gets richer* phenomenon.

Our aim is to suggest a random graph model where the *rich gets richer* phenomenon is restricted to some degree. The suggested model will be called **Depreferential attachment model through preferential attachment**. Below is a brief description of this model.

The model has two parameters p and m to start with. p is a real number in the interval $(0,1)$ and m is a positive integer. It starts with 2 vertices and an edge between them. At each discrete time point $t \geq 1$, a new vertex appears with m half-edges. The other end of each of these half-edges are to be attached to some of the already existing vertices. Let us denote the process by $(G_t^{(m,p)})_{t\geq 1}$. So, at time point $t=1$ the graph is $G_1^{(m,p)}$ and at time point $t=2$ the graph is $G_2^{(m,p)}$ etc. The algorithm to generate the specified random graph proceeds as below.

At time $t \geq 1$, there were total $(t+1)$ already existing vertices except the newly arrived vertex. Among them, we shall choose $\lfloor (t+1)p \rfloor$ many in such a way that the probability of choosing the vertex v_k is proportional to its degree in $G_{t-1}^{(m,p)}$. The chosen vertices are made taboo, i.e., any of the m half-edges of the newly arrived vertex would not attach to any of these taboo vertices. In the next step, each of the m half-edges are attached to non-taboo vertices in such a way that the probability of attaching an edge to v_j is proportional to its degree in $G_{t-1}^{(m,p)}$. This model differs from the Albert-Barabasi model in the way that there was no concept of creating taboo vertices in the latter.

We shall focus on the following statistics of the random graph $(G_t^{m,p})_{t\geq 1}$.

- **Degree of fixed vertex**: For $0 < i \leq t+2$, let $D_i(t)$ be the degree of the i-th vertex at time t. We shall investigate the nature of $D_i(t)$ as $t \to \infty$ fixing i.

- **Degree sequence**: The proportion of vertices with degree k at time t is denoted as

$$P_k(t) = \frac{1}{t} \sum_{i=1}^{t} \mathbb{1}_{\{D_i(t)=k\}}.$$

We shall investigate the asymptotic behaviour of $P_k(t)$ as $t \to \infty$.

- **Maximal degree**: The maximal degree of a vertex in $(G_t^{(m,p)})_{t\geq 1}$ is defined as

$$M_t = \max_{i \in [t+2]} D_i(t).$$

We want to study the behaviour of M_t as $t \to \infty$.