Then from the definition of $D_i^*(t)$, which converges almost surely as $t \to \infty$, we can write

$$
\begin{aligned}
D_i(t) &= D_i^*(t) \times \prod_{k=i-1}^{t-1} \{1 + P_i'(k)\} \\
&\le c_{i,d} D_i^*(t) \times \frac{\Gamma(t+3-d)}{\Gamma(t+2-d)},
\end{aligned}
$$

where $c_{i,d}$ is a finite constant. Using the identity

$$
\frac{\Gamma(t+a)}{\Gamma(t)} \to t^a(1 + O(1/t)), \quad \text{as } t \to \infty
$$

we see that $D_i(t)/t \le \zeta_i$ for large enough t where ζ_i is a random variable bounded in probability.