Conditional on $\mathcal{F}_t$, there are three ways $N_k(t+1) - N_k(t)$ can be non-zero.

(I) The half-edge of the new vertex attaches to a particular $k-1$ degree vertex. Then, $N_k(t)$ increases by 1 to $N_k(t+1)$. The probability of this event is

$$\sum_{\substack{1 \le i \le t+2: \\ D_i(t) = k-1}} \sum_{\substack{k_1, k_2, \ldots, \\ k_d \in [t+2] \setminus \{i\}}} \left\{ \prod_{j=1}^{d} \frac{D_{k_j}(t)}{2(t+1)} \right\} \frac{D_i(t)}{2(t+1) - S_t(\{k_1, \ldots, k_d\})}$$

$$= \sum_{\substack{k_1, k_2, \ldots, \\ k_d \in [t+2] \setminus \{i_0\}}} \left\{ \prod_{j=1}^{d} \frac{D_{k_j}(t)}{2(t+1)} \right\} \frac{(k-1) N_{k-1}(t)}{2(t+1) - S_t(\{k_1, \ldots, k_d\})} = (k-1) N_{k-1}(t) P_{i_0}(t),$$

where i_0 is a particular $k-1$ degree vertex at time t.