

Since $m = 1$ the degree of a vertex can increase by at most one in unit time. So there exists some $t \geq 1$ such that $G_{t-1}^{(1,p)}$ has n vertices having the maximum degree $d = \lceil 1/p \rceil$ for some $n \in \mathbb{N}$.

Now we give a lower bound to the number of vertices present in the graph $G_{t-1}^{(1,p)}$. It has

- n many d degree vertices
- Each of the d degree vertices has n neighbours; so nd many neighbours
- While counting the neighbours, maximum $n - 1$ many may have been counted twice as neighbours can be shared

So, the graph has at least $n + nd - (n - 1)$ vertices.