Then from the definition of $D_i^*(t)$, which converges almost surely as $t \to \infty$, we can write

$$\begin{aligned} D_i(t) &= D_i^*(t) \times \prod_{k=i-1}^{t-1} \{1 + P_i'(k)\} \\ &\geq b_{i,d} D_i^*(t) \times \frac{\Gamma(t + 1/2 + 2^{-d-1})}{\Gamma(t + 1/2)}, \end{aligned}$$

where $b_{i,d}$ is a finite constant. Again, we use the result

$$\frac{\Gamma(t+a)}{\Gamma(t)} \to t^a(1 + O(1/t)), \quad \text{as } t \to \infty$$

to obtain that $D_i(t)/t^{2^{-d-1}} \geq \zeta_i'$ for all large enough t where ζ_i' is a random variable bounded in probability.