

Suppose, X_1, \dots, X_d are i.i.d. samples from the distribution $\text{Uniform}(\{1, 2, \dots, t+2\} \setminus \{i\})$. Also, assume the function f is defined such that $f(i) = D_i(t)$. Now, we shall replace the sum in the above equation by an expectation over the i.i.d. random variables X_1, \dots, X_d .

$$\begin{aligned}
1 + P'_i(t) &\leq 1 + \frac{(t+1)^d}{2^d(t+1)^d} \mathbb{E} \left[\left(\prod_{j=1}^d f(X_j) \right) \times \frac{1}{t+2-d} \right] \\
&= 1 + \frac{1}{2^d(t+2-d)} (\mathbb{E} [f(X_1)])^d \\
&= 1 + \frac{1}{2^d(t+2-d)} \left(\frac{2(t+1) - D_i(t)}{t+1} \right)^d \\
&\leq 1 + \frac{1}{t+2-d} = \frac{t+3-d}{t+2-d}.
\end{aligned}$$