

Primary simulations with different schemes of “taboo”-ing

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Multiple simulations of “Depreferential attachment random graph through preferential attachment” (denoted by $(G_t^{(m,p)})_{t \geq 1}$) has been performed to obtain ideas about distribution of various statistics of the model through simulation. In this document I shall mainly focus on the statistic **maximal degree**. For $(G_t^{(m,p)})_{t \geq 1}$, the maximal degree of a vertex is defined as below.

$$M_t = \max_{i \in [t+2]} D_i(t)$$

For simplification, we have taken $m = 1$, i.e., when a new vertex comes, it comes with only $m = 1$ many half-edges. Also, three different methods have been taken for choosing the *taboo* vertices.

- **Hard-code:** In this method, the model has a parameter p ($0 < p < 1$). At time point $t \geq 1$, we choose the $\lfloor (t+1)p \rfloor$ vertices in such a way that all the other vertices in $G_{t-1}^{(1,p)}$ had degree less than the chosen ones. The chosen vertices are made taboo.
- **Soft-code:** In this method too, the model has a parameter p ($0 < p < 1$). At time point $t \geq 1$, we choose the $\lfloor (t+1)p \rfloor$ vertices in a preferential way, i.e., choosing a vertex v_j is proportional to its degree in $G_{t-1}^{(1,p)}$.
- **Soft-code (alternative):** This method does not have the parameter p . Instead, in this method, at time $t \geq 1$, one will choose only one vertex preferentially. That chosen vertex will be made taboo.

Based on these methods, multiple simulations have been performed – (i) Once by changing the parameter p while keeping the final graph size fixed, and (ii) Once by keeping the parameter p fixed while changing the final graph size.

1 Hard-code

In the following plots, the empirical distributions of the maximal degree obtained from the simulations are displayed. First, we have varied the value of p by keeping the final graph size fixed. The final plot below shows the mean maximal degree as a function of varying p .

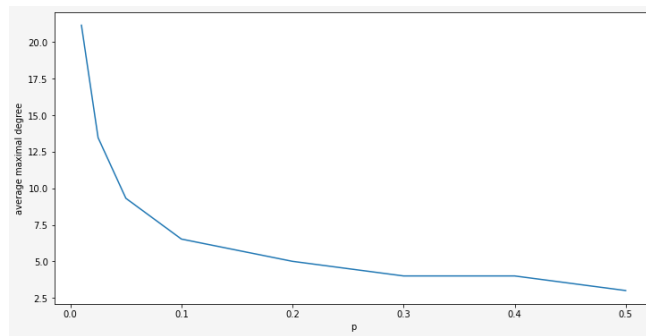
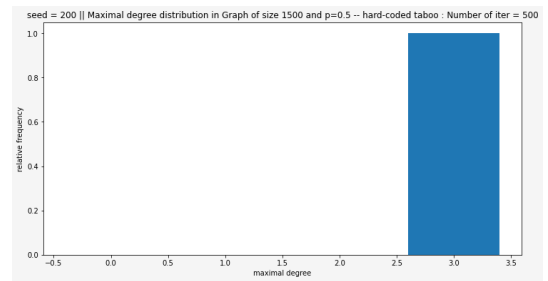
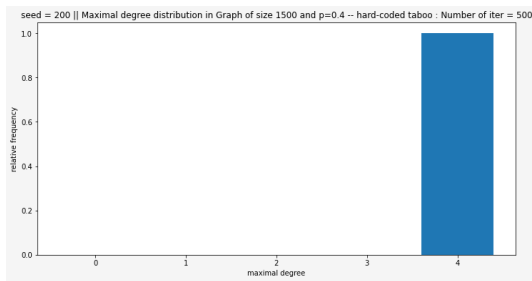
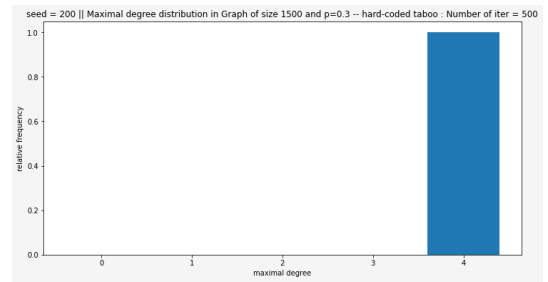
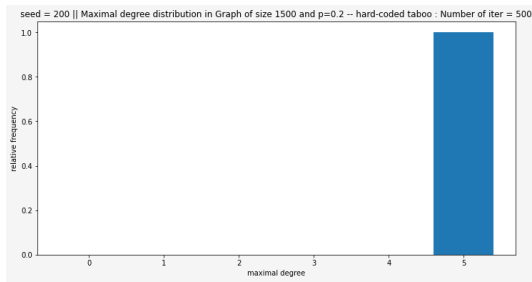
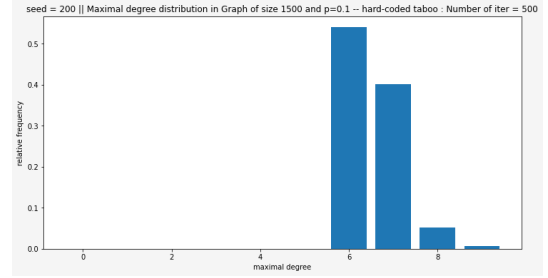
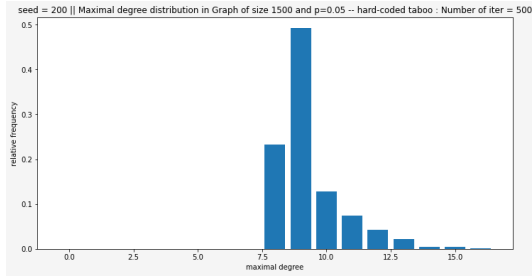
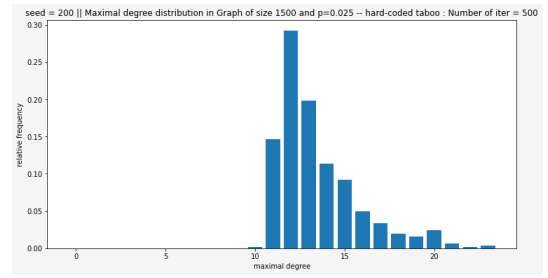
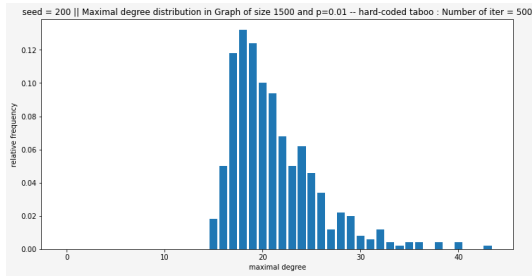
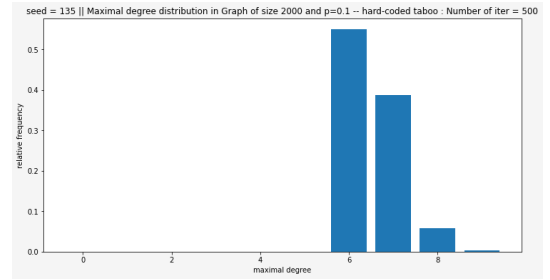
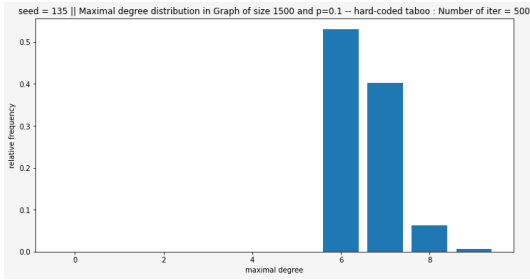
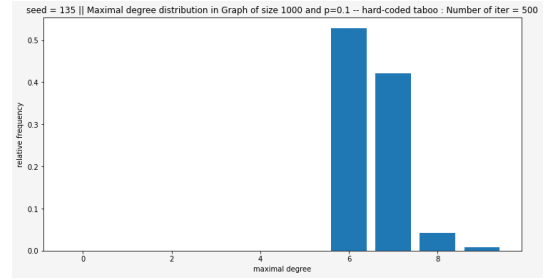
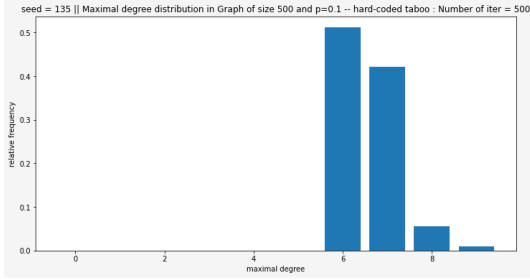


Figure: Maximal degree distributions as p is varied from 0.01 to 0.5 while keeping the graph size fixed at 2000

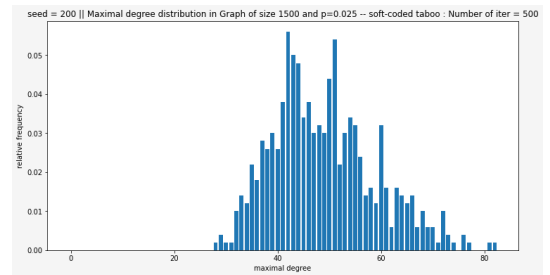
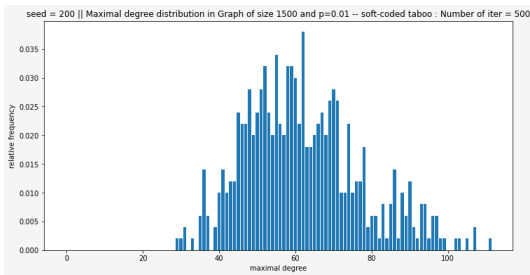
To our surprise, the distribution of maximal degree tend to be degenerate as p is increased from 0.01 to 0.5. For the next part, we have fixed $p = 0.1$ and varied the final graph size from 500 to 2000. Following are the obtained plots.



Apparently, the distribution of maximal degree does not depend much on the graph size when the parameter p is kept fixed.

2 Soft-code

In the following plots, the empirical distributions of the maximal degree obtained from the simulations are displayed. First, we have varied the value of p by keeping the final graph size fixed. The final plot below shows the mean maximal degree as a function of varying p .



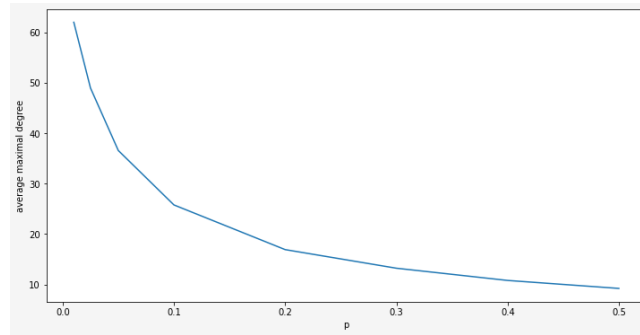
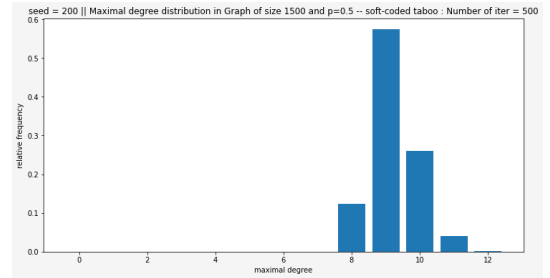
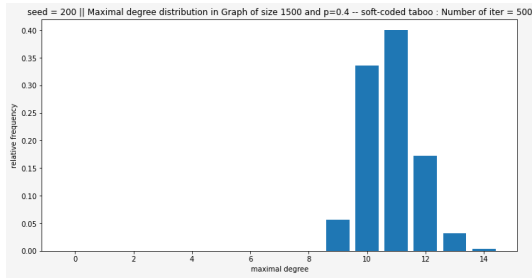
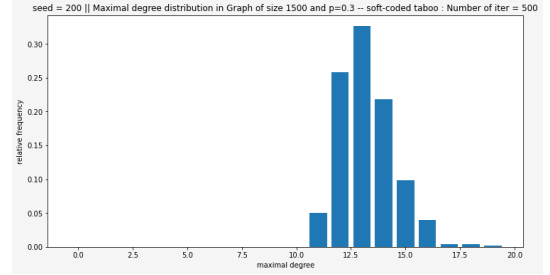
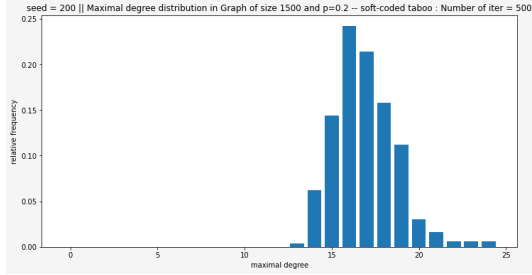
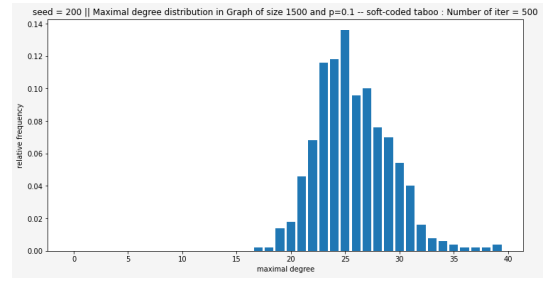
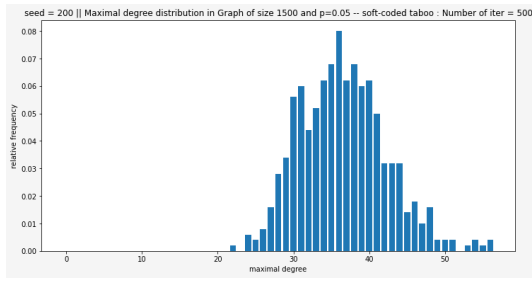


Figure: Maximal degree distributions as p is varied from 0.01 to 0.5 while keeping the graph size fixed at 2000

For the next part, we have fixed $p = 0.1$ and varied the final graph size from 250 to 2000. Following are the obtained plots. The final plot below shows the mean maximal degree as a function of varying n , the final graph size.

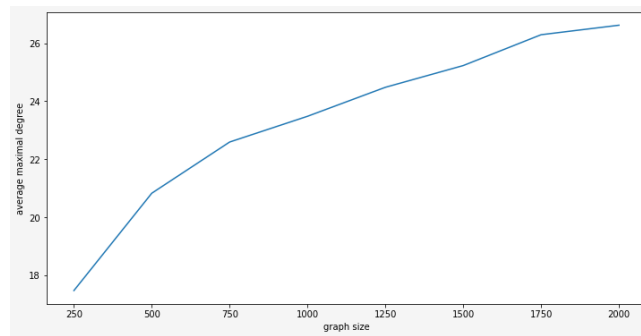
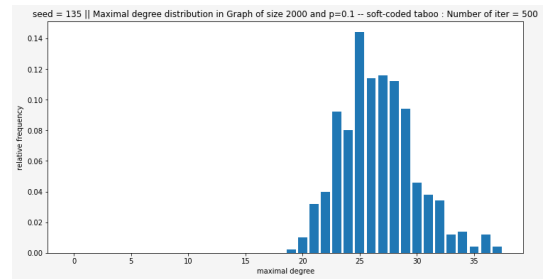
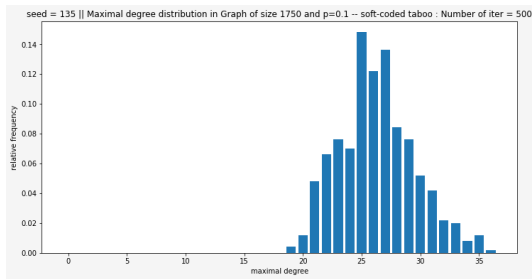
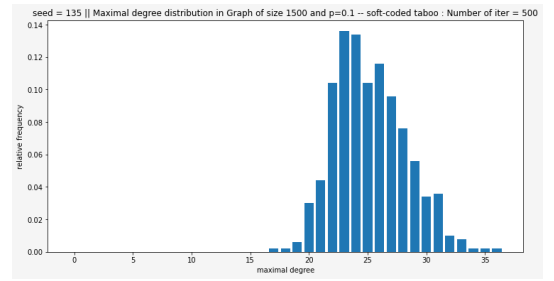
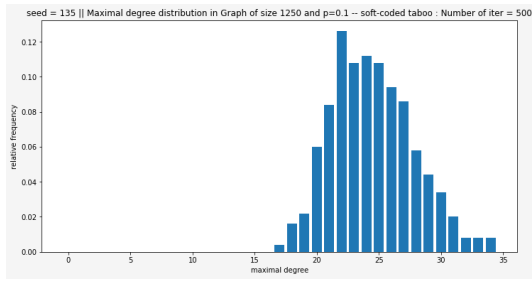
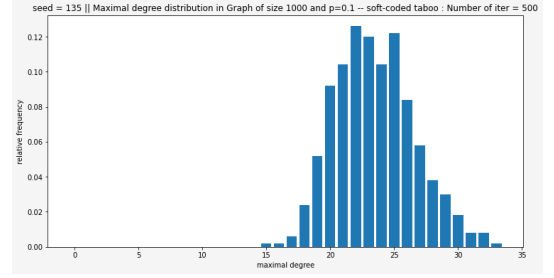
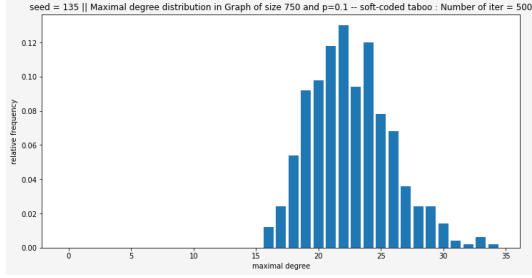
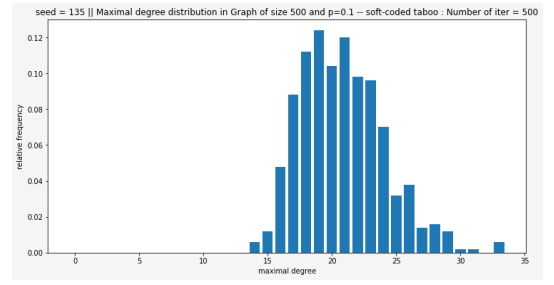
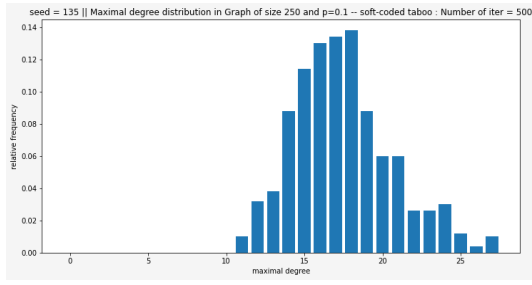
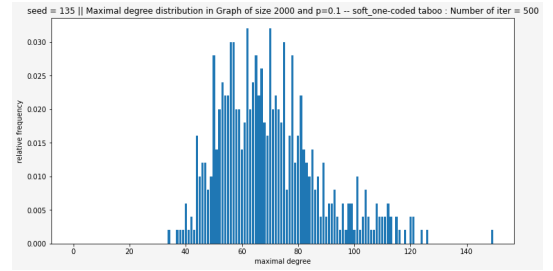
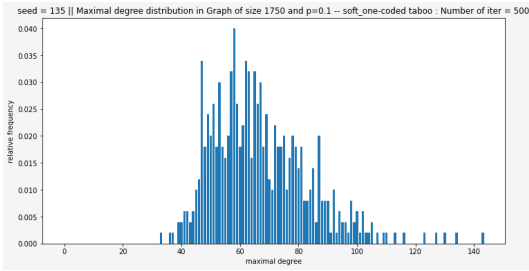
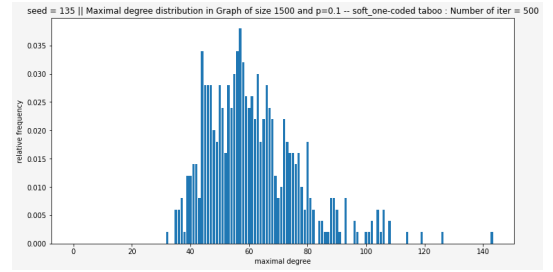
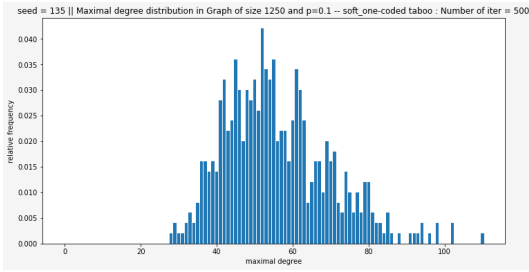
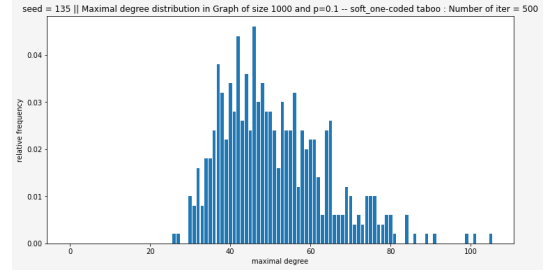
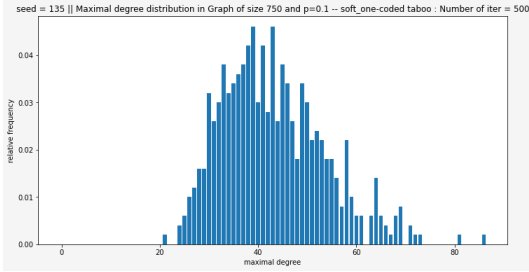
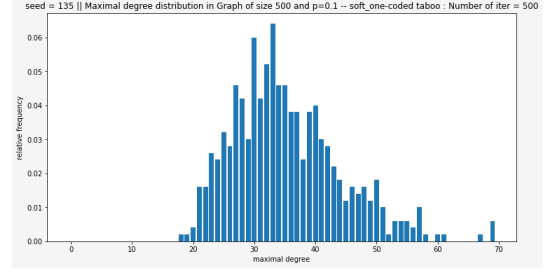
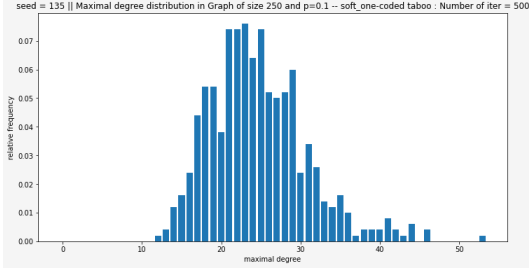


Figure: Maximal degree distributions as n is varied from 250 to 2000 while keeping p size fixed at $p = 0.1$

3 Soft-code (alternative)

In the following plots, the empirical distributions of the maximal degree obtained from the simulations are displayed. As the parameter p is not relevant here, we have just varied the final graph size, n . The final plot shows the change of mean maximal degree as function of the final graph size, n .



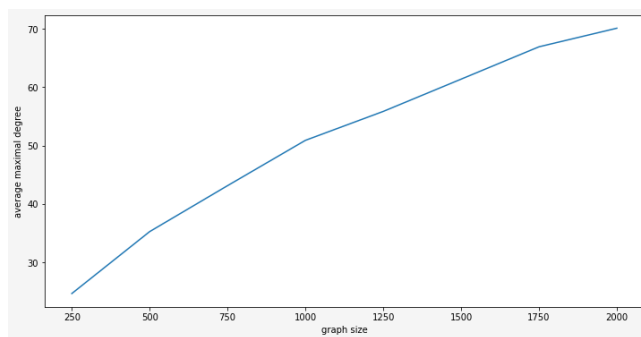


Figure: Maximal degree distributions as n is varied from 250 to 2000