Primary simulations with different schemes of "taboo"-ing

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Multiple simulations of "Depreferential attachment random graph through preferential attachment" (denoted by $(G_t^{(m,p)})_{t\geq 1}$) has been performed to obtain ideas about distribution of various statistics of the model through simulation. In this document I shall mainly focus on the statistic **degree of fixed vertex**. For $(G_t^{(m,p)})_{t\geq 1}$, the notation $D_i(t)$ stands for the degree of the i-th vertex at time t. For this notation to make any sesne, we must have $0 < i \leq t+2$.

For simplification, we have taken $m = 1$, i.e., when a new vertex comes, it comes with only $m = 1$ many half-edges. Also, three different methods have been taken for choosing the *taboo* vertices.

- **Hard-code:** In this method, the model has a parameter $p\,(0 < p < 1)$. At time point $t \geq 1$, we choose the $\lfloor (t+1)p \rfloor$ vertices in such a way that all the other vertices in $G_{t-1}^{(1,p)}$ had degree less than the chosen ones. The chosen vertices are made taboo.
- **Soft-code:** In this method too, the model has a parameter $p\,(0 < p < 1)$. At time point $t \geq 1$, we choose the $\lfloor (t+1)p \rfloor$ vertices in a preferential way, i.e., choosing a vertex v_j is proportional to its degree in $G_{t-1}^{(1,p)}$.
- **Soft-code (alternative):** This method does not have the parameter p. Instead, in this method, at time $t \geq 1$, one will choose only one vertex preferentially. That chosen vertex will be made taboo.

Based on these different taboo-ing schemes, multiple simulations have been performed by using three different values of the parameter p, and changing the target vertex i. For each of the cases, 500 simulations have been performed to compute mean, variance and standard deviation.

1 Hard-code

Consider the 'hard-code' taboo-ing scheme. In the following plots, we show the mean, variance and standard deviation respectively of $D_i(t)$ with $p \in \{0.01, 0.1, 0.25\}$ and $i \in \{10, 50, 200\}$. The size of the graph is kept fixed at 2000 in all of the cases.

Please note that, in the following plots, we used the following colour code

- Yellow: Mean of $D_i(t)$

- Red: Variance of $D_i(t)$
- Blue: Standard deviation of $D_i(t)$

1.1 p = 0.01

Figure 1: $\boldsymbol{p = 0.01, i = 10}$.

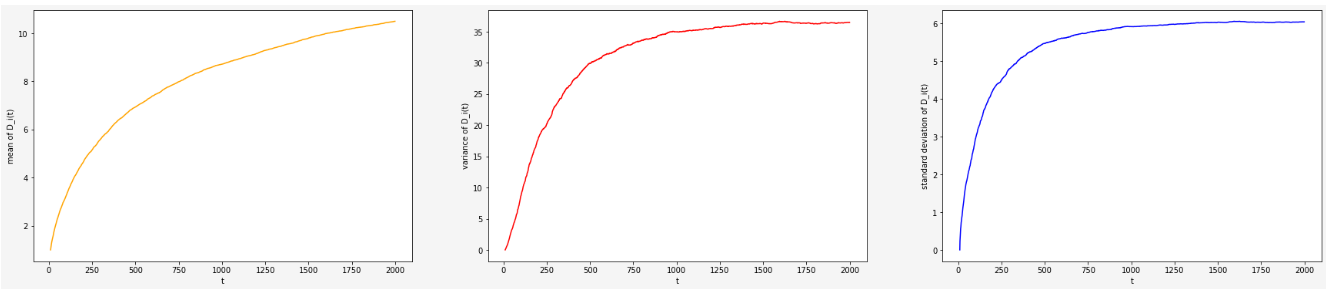


Figure 2: $\boldsymbol{p = 0.01, i = 50}$.

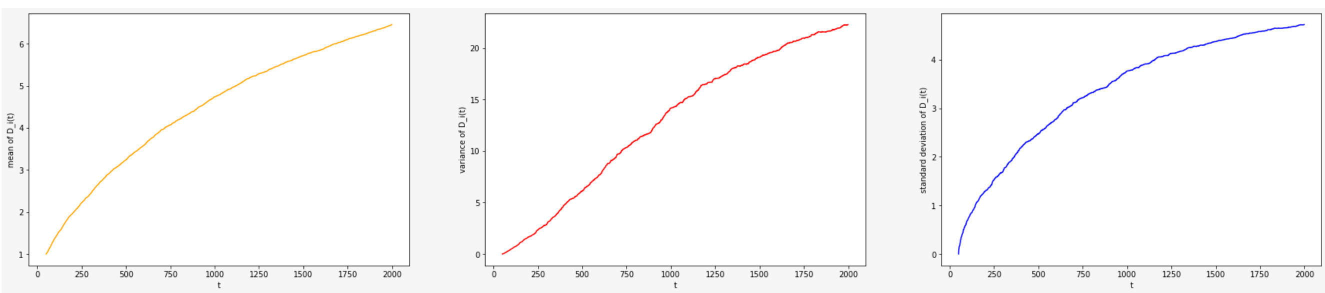
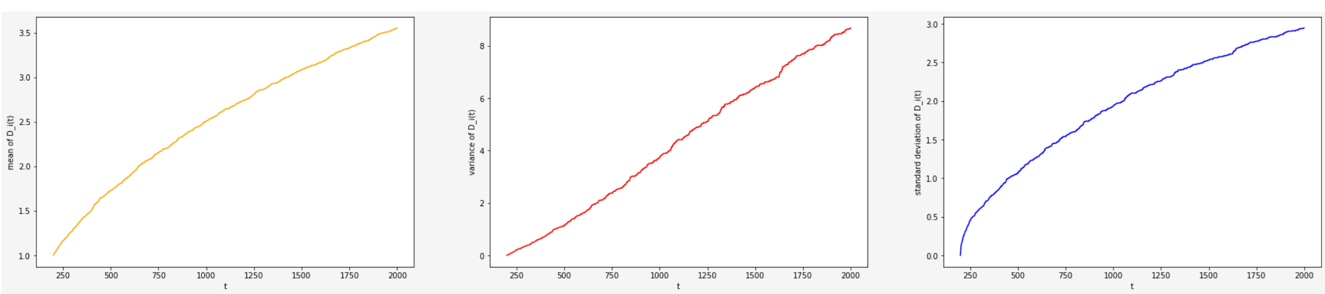


Figure 3: $\boldsymbol{p = 0.01, i = 200}$.



1.2 p=0.1

Figure 4: $\boldsymbol{p = 0.1, i = 10}$.

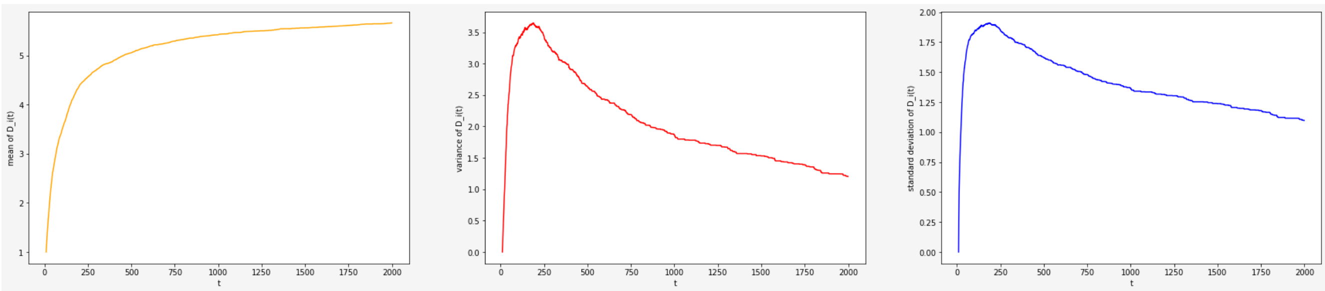


Figure 5: $\boldsymbol{p = 0.1, i = 50}$.

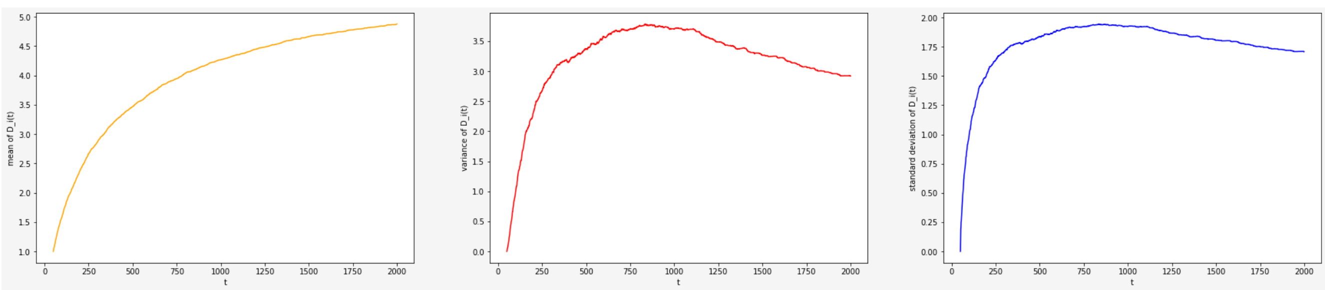
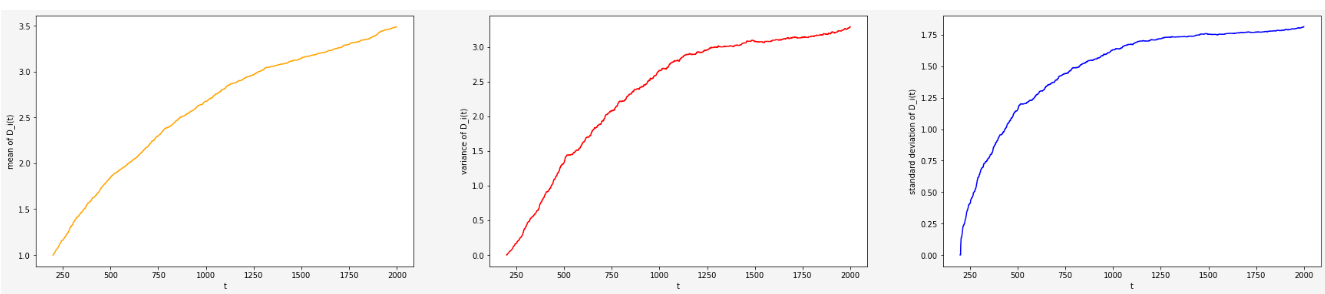


Figure 6: $\boldsymbol{p = 0.1, i = 200}$.



1.3 p=0.25

Figure 7: $\boldsymbol{p = 0.25, i = 10}$.

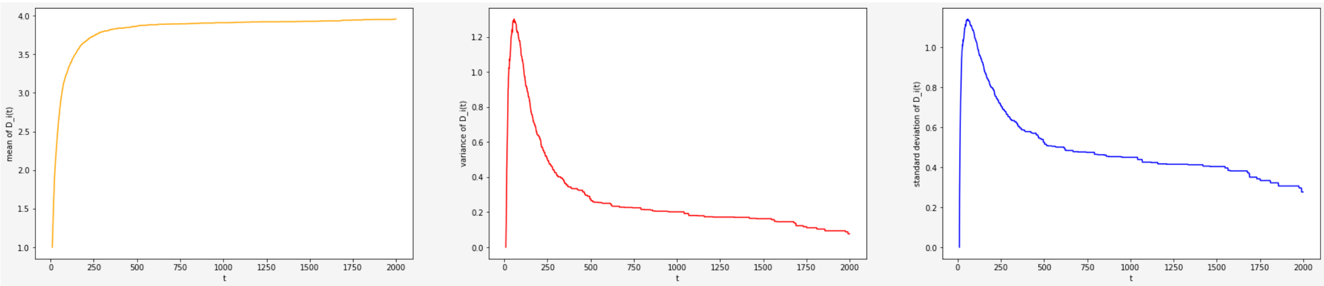


Figure 8: $\boldsymbol{p = 0.25, i = 50}$.

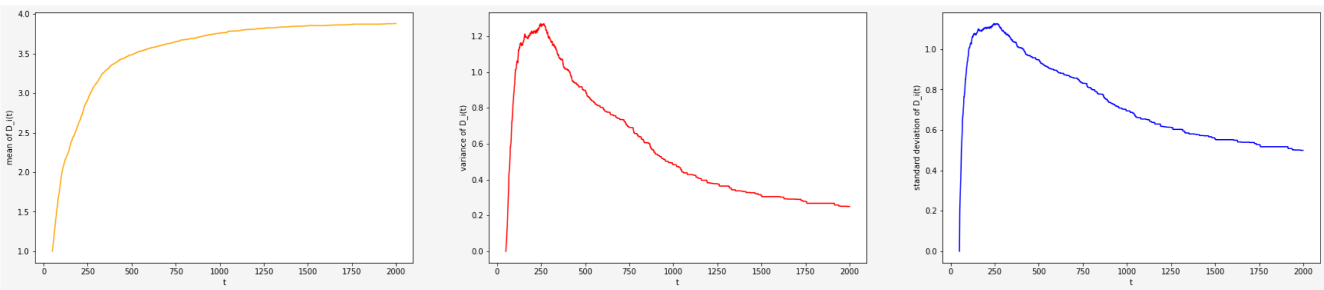
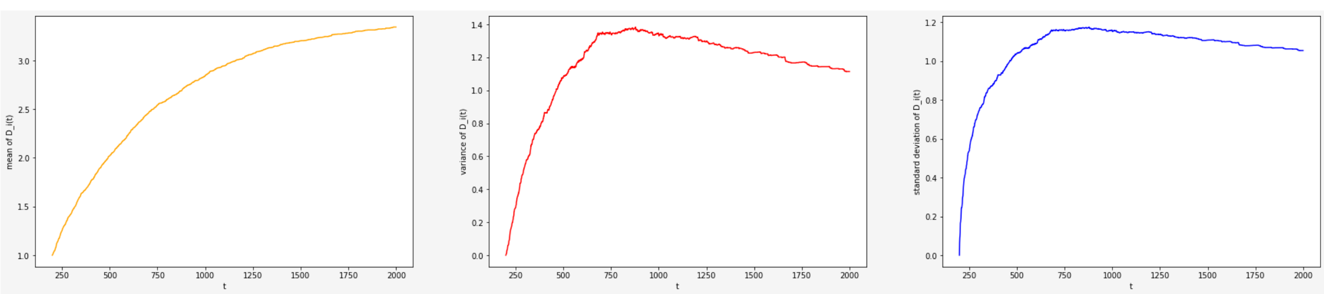


Figure 9: $\boldsymbol{p = 0.25, i = 200}$.



2 Soft-code

Now we adopt the 'soft-code' taboo-ing scheme. The other parameters p and i is varied as before.

2.1 p = 0.01

Figure 10: $\boldsymbol{p = 0.01, i = 10}$.

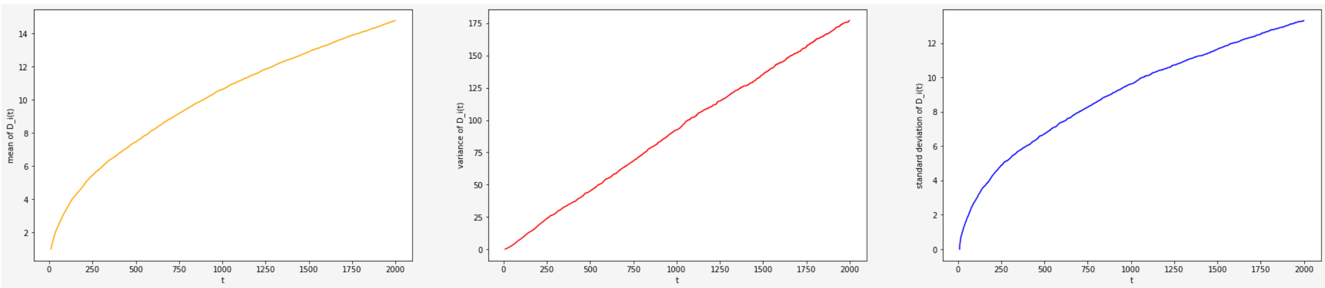


Figure 11: $\boldsymbol{p = 0.01, i = 50}$.

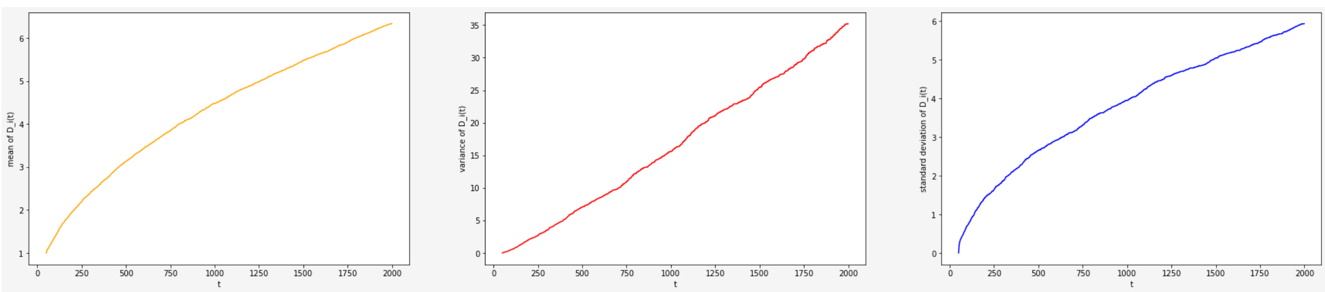
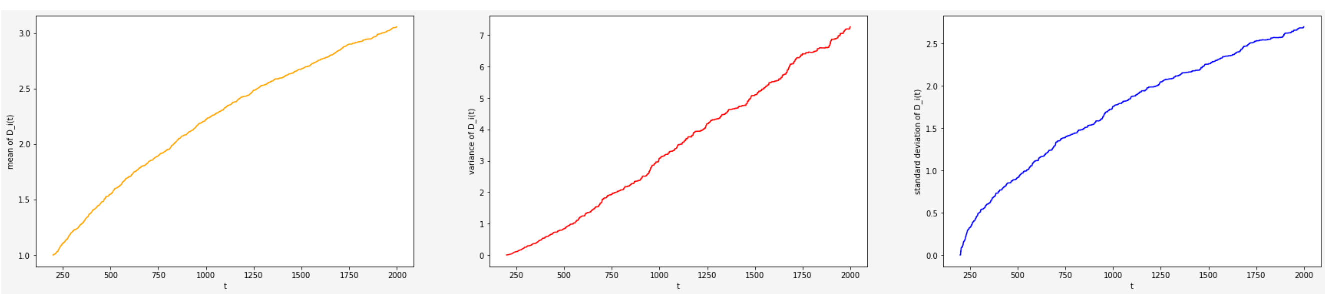


Figure 12: $\boldsymbol{p = 0.01, i = 200}$.



2.2 p=0.1

Figure 13: $\boldsymbol{p = 0.1, i = 10}$.

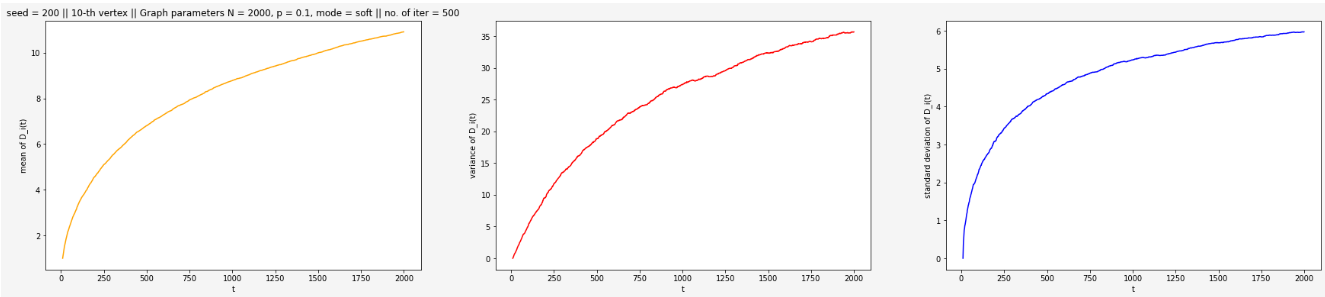


Figure 14: $\boldsymbol{p = 0.1, i = 50}$.

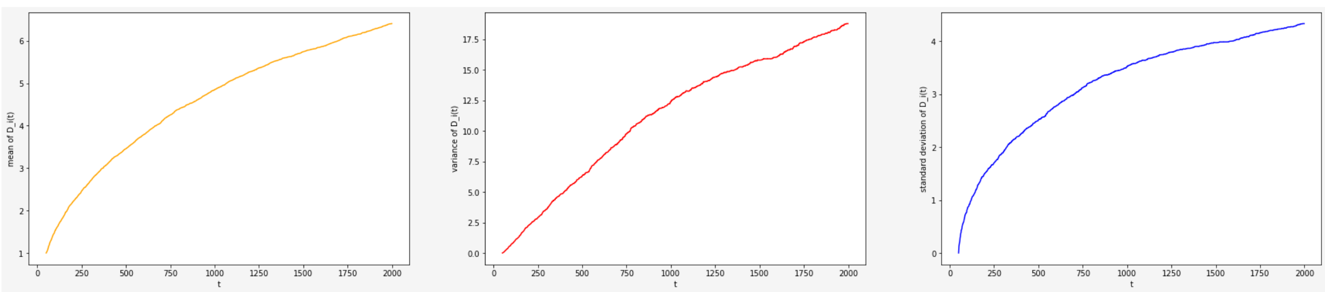
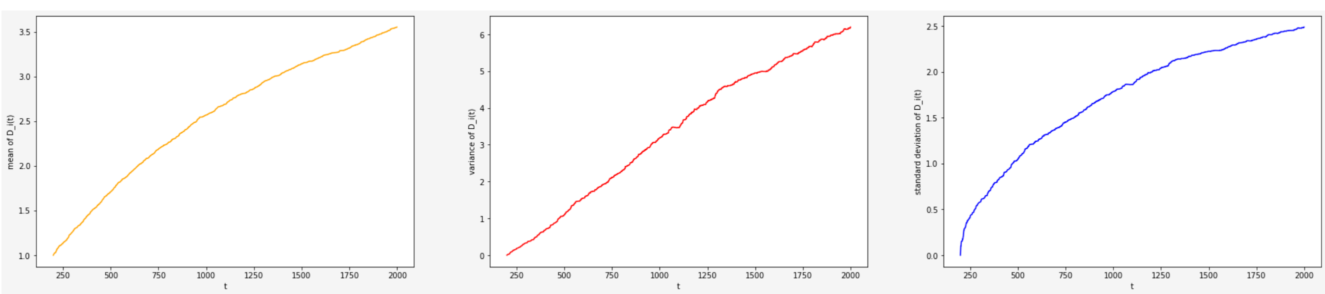


Figure 15: $\boldsymbol{p = 0.1, i = 200}$.



2.3 p=0.25

Figure 16: $\boldsymbol{p = 0.25, i = 10}$.

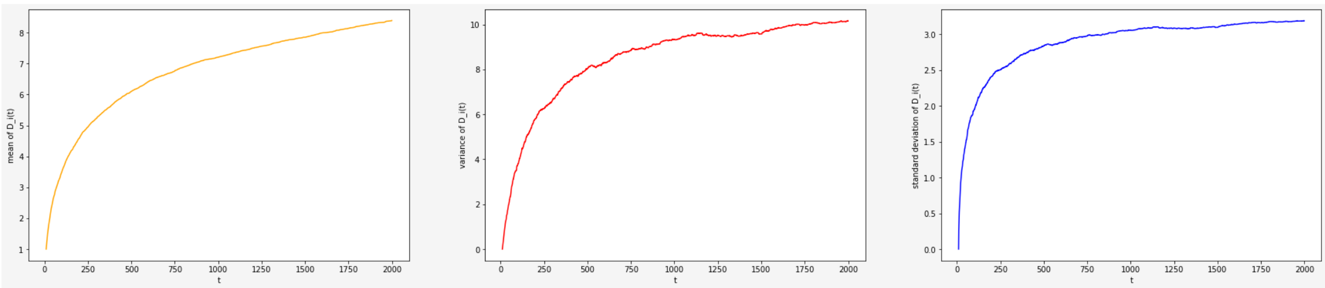


Figure 17: $\boldsymbol{p = 0.25, i = 50}$.

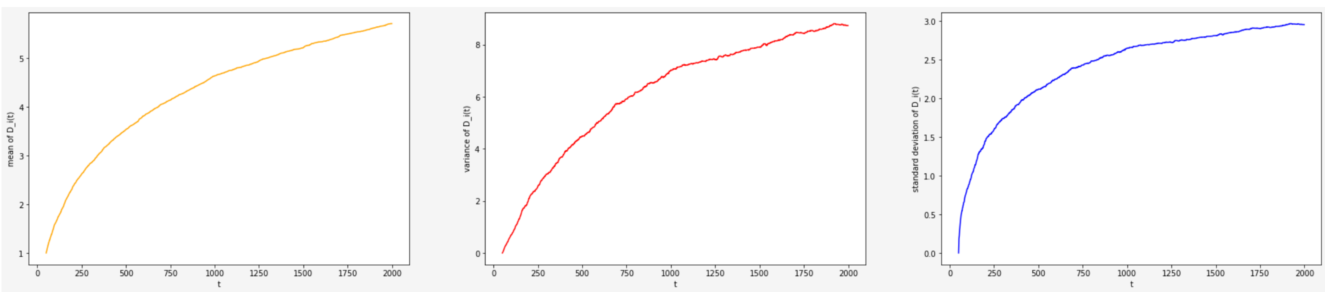
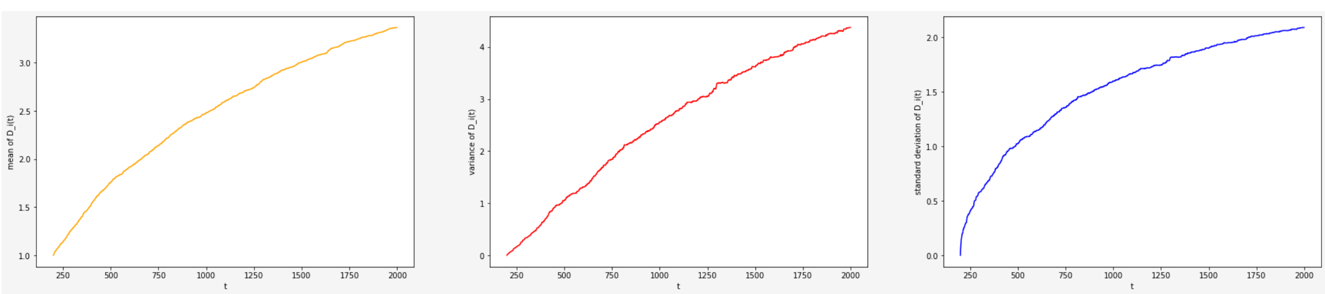


Figure 18: $\boldsymbol{p = 0.25, i = 200}$.



3 Soft-code (alternative)

For this last part, we adopt the 'soft-code (alternative)' taboo-ing scheme.This scheme has no parameter p. So, we just vary i in the set $\{10, 50, 200\}$.

Figure 19: $\boldsymbol{i = 10}$.

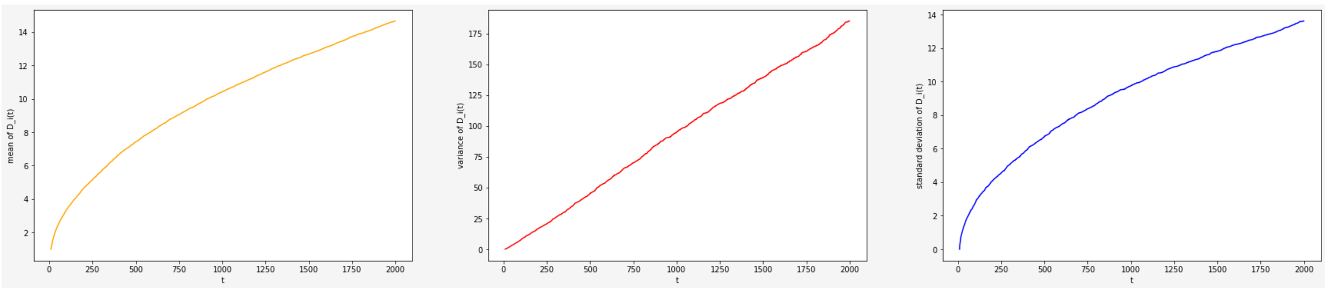


Figure 20: $\boldsymbol{i = 50}$.

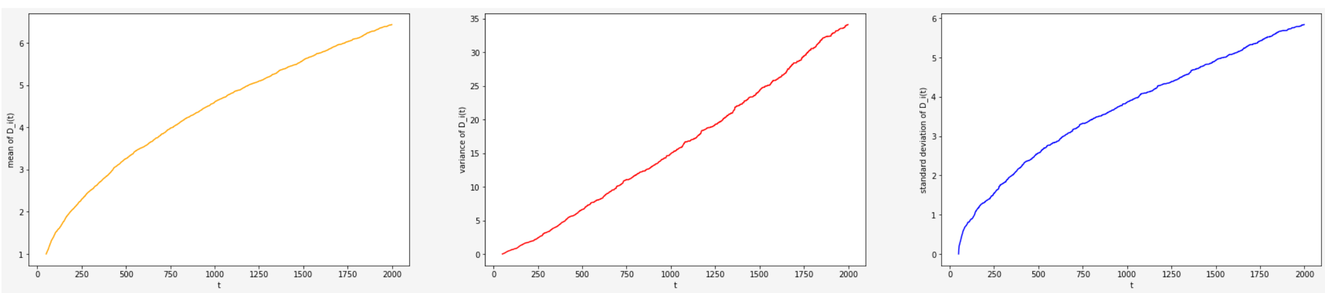


Figure 21: $\boldsymbol{i = 200}$.