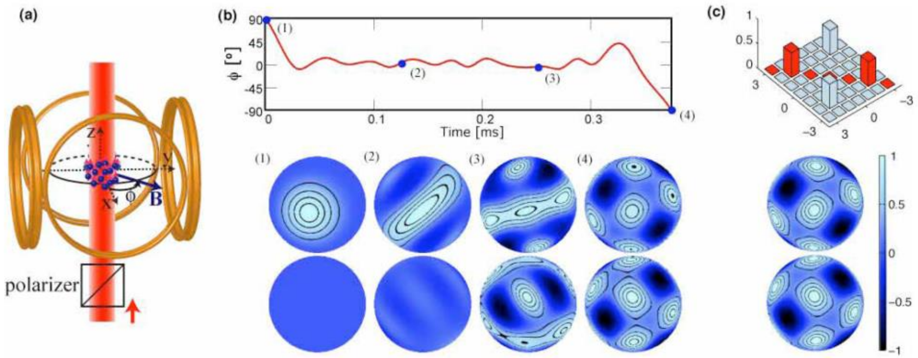
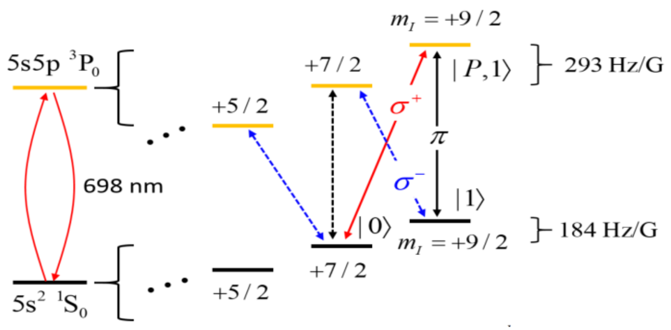
Quantum Control of Nuclear Spin for Quantum Logic with Qudits





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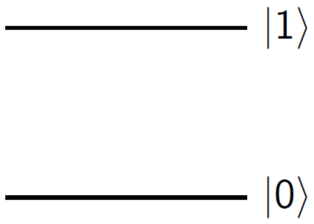
APS March Meeting 2021

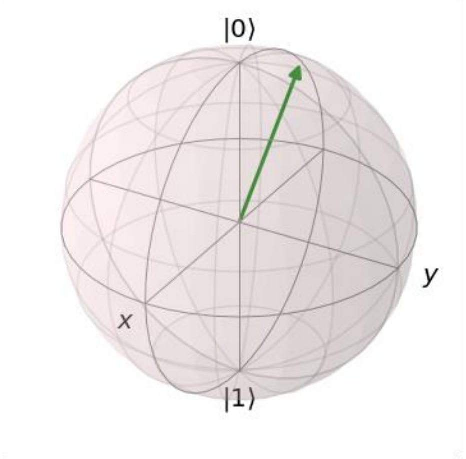
Center for Quantum Information and Control

Quantum Computing Beyond Qubits

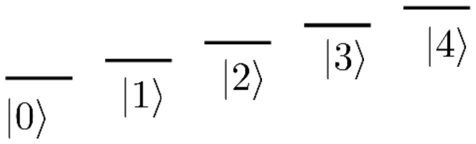
Qubits (d=2):



$$|\psi\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix}$$



Qudits (d=5):



$$|\psi\rangle = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

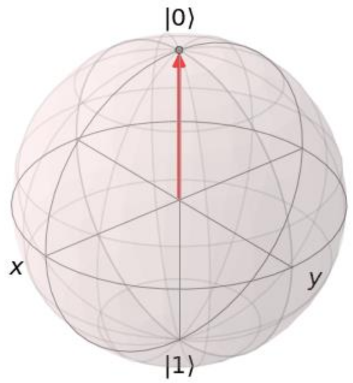
A Complex Visualization

- ❑ Universal Gate set for Quantum computing:
  - ❖ For Qubits: SU(2) + one entangling gate such as CNOT
  - ❖ For Qudits: SU(d) + one entangling gate.

Why Quantum Control?...

- ❑ Advantages of Quantum computing with qudits[1] (d>2)
  - ❖ Has the potential for storing more information in fewer systems
  - ❖ Improved thresholds for fault tolerance

- ❑ Implementation of Universal gates (challenge of Qudits) :
  - ❖ Qubits :SU(2) is easy!



Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, arXiv preprint arXiv:2008.00959 (2020).

Why Quantum Control?...

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- Implementation of Universal gates (challenge of Qudits) :
  - Qubits :SU(2) is easy!
  - For d>2, it is not straightforward to implement one qudit gates.
  - How do we do it?

Solution: Implementing SU(d) gates using Quantum control!

- Another challenge with qudits is to do with measurements..

Y. Wang, Z. Hu, B. C. Sanders, and S. Kais, arXiv preprint arXiv:2008.00959 (2020).

Encoding a Qudit in Nuclear Spin

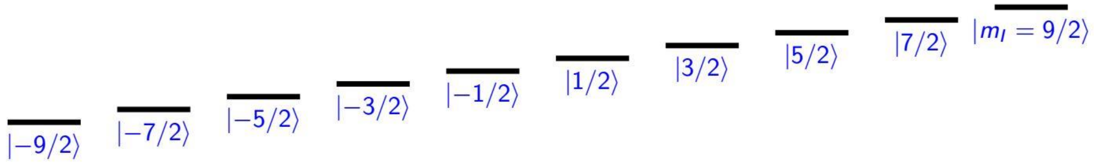
Strontium: ^{87}Sr[1]

- Alkaline Earth Elements attracted significant attraction of late as optical clocks and potential platform for quantum information

- The ground space of ^{87}Sr, $J = 0$ and $F = I + J = I$

F: Total angular momentum
I: Nuclear Spin
J: electron angular momentum

- $I = 9/2$ gives d=10 (qudecimal)



[1] *Boyd, M. M. (2007). High precision spectroscopy of strontium in an optical lattice: Towards a new standard for frequency and time, Citeseer.* ***68****.*
[2] *Martin, M. J. (2013). "Quantum metrology and many-body physics: pushing the frontier of the optical lattice clock." JILA Ph. D Thesis.*

Hamiltonian: Quasi-static magnetic fields and light shift

- The Qudit system control involves the magneto-optical field Hamiltonian[1],

$$\boxed{H = -\mu.\boldsymbol{B} - \frac{1}{4}E_i(t)^* E_j(t)\alpha_{ij}}$$

- Using a monochromatic light with polarization along z

$$\boxed{H(t) = \Omega\left[\cos(C(t)\pi)I_x + \sin(C(t)\pi)I_y\right] + \beta I_z^2.}$$

- Designing a Unitary map: Generate SU(d) gates
  - Optimize $\boxed{\mathcal{F}[\boldsymbol{c}(t)] = \frac{\left|\mathrm{tr}\left(U_{target}^{\dagger}U[\boldsymbol{c}(t)]\right)\right|^2}{d^2}}$ the trace overlap.
  - The total number of free parameters in a unitary operator is $\boxed{d^2\text{-}1}$

[1] *Merkel, S. (2009). "Quantum control of d-dimensional quantum systems with application to alkali atomic spins."* arXiv preprint arXiv:0906.4790

Controllability of Quantum System[1]:

- “Unitary Controllability” : create unitary map in finite time

- Control Hamiltonian $$H(t) = H_0 + \sum_{k=1}^{K} c_k(t) H_k,$$

  - $c_k(t)$ control waveforms that can be manipulated
  - $\{H_0, H_1, H_2, ..., H_K\}$ are Hermitian, which leads to unitary dynamics for the group su(d)
  - Finding the optimal control waveforms : maximize $\mathcal{F}[\boldsymbol{c}(t)]$ and involves looking for control wave forms that satisfies, $\nabla_c \mathcal{F}[\boldsymbol{c}(t)] = 0$ using gradient based optimization like GRAPE.

- Controlling the phase $H(t) = \Omega\left[\cos(C(t))I_x + \sin(C(t))I_y\right] + \beta I_z^2$ is controllable[1]

[1] *Merkel, S. (2009). "Quantum control of d-dimensional quantum systems with application to alkali atomic spins."* *arXiv preprint arXiv:0906.4790.*

Quantum Control: Unitary maps

Generating Unitary Maps:

- Quantity to optimize $\mathcal{F}_{gate}[\boldsymbol{c}(t)] = \frac{\left|\mathrm{tr}\left(U^{\dagger}_{target}U[\boldsymbol{c}(t)]\right)\right|^2}{d^2}$ the trace overlap

- X-gate for Qubit:

$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle \qquad X|k\rangle = |(k+1) \bmod 2\rangle$$

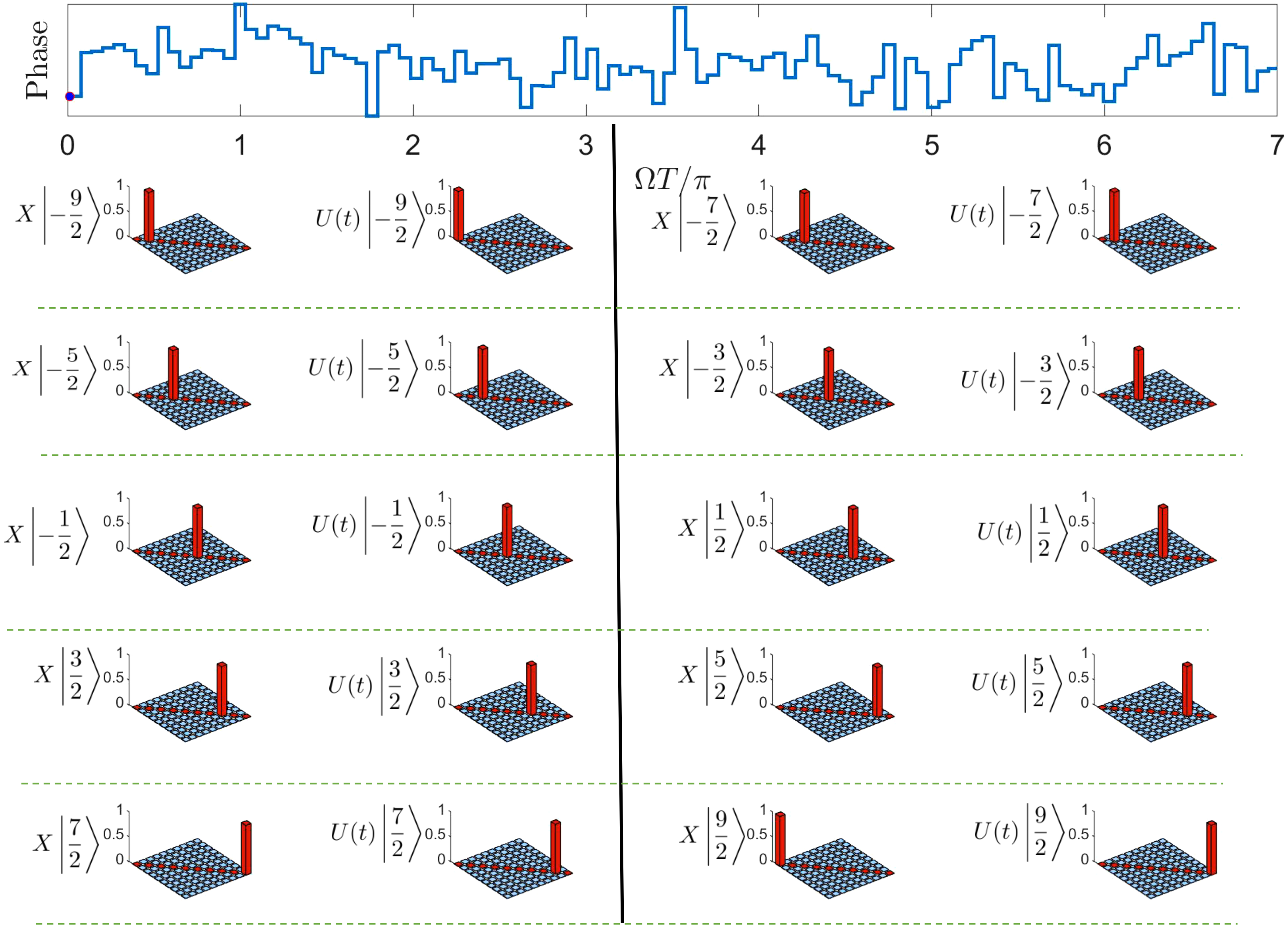
Generalized X gate for Qudit

$$X|k\rangle = |(k+1) \bmod \mathrm{d}\rangle$$

- Z-gate for Qudit : $Z|k\rangle = \omega^k |k\rangle\,;\;\; \omega = \exp(2\pi i/d)$

- Discrete generalization of the Weyl-Heisenberg group: $\omega^l X^p Z^m$ where $l, p, m \in \mathbb{Z}_d$

X-gate Preparation

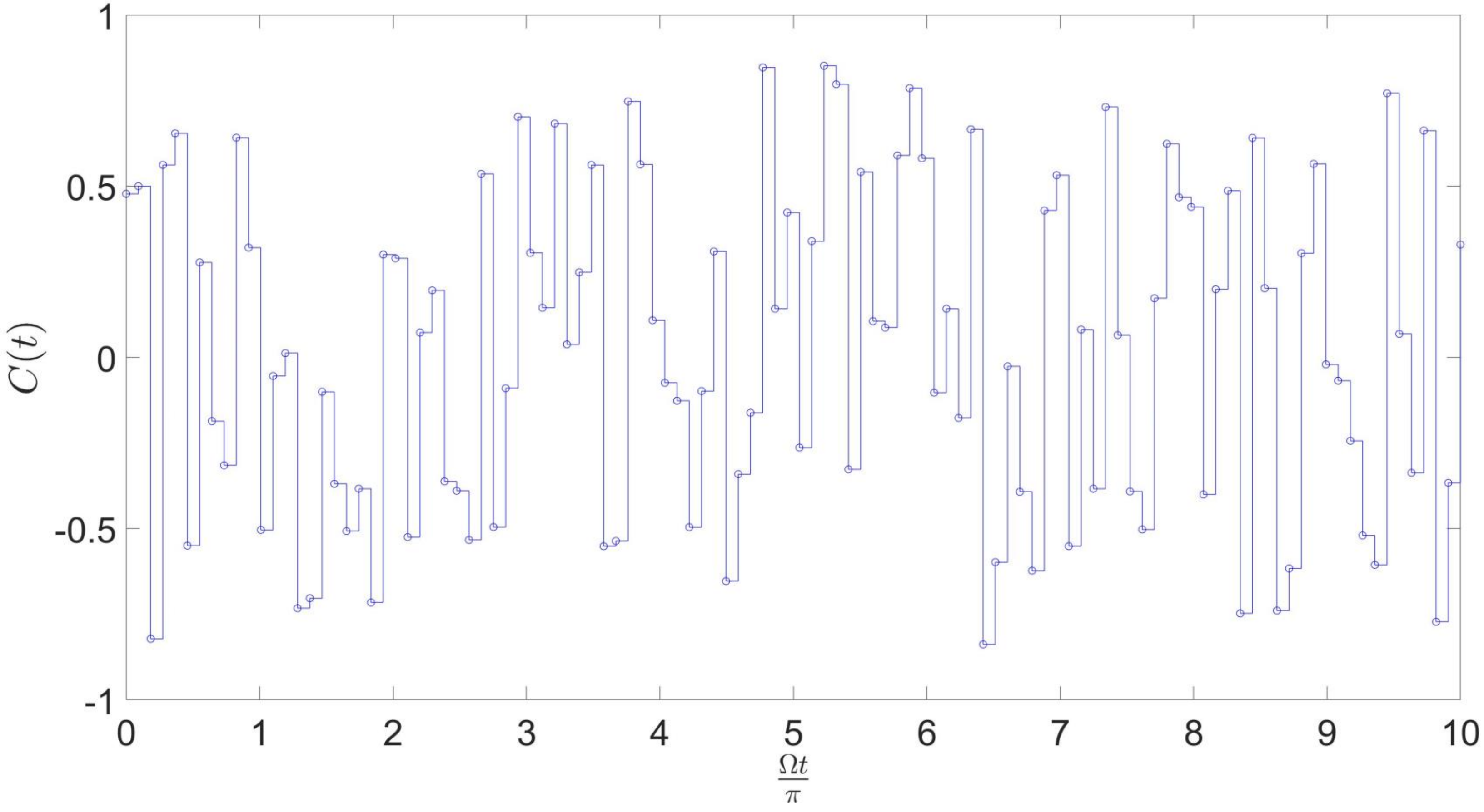


N: Total Number of controls
N=110
T: Total time for the Simulation
T=7π
I=9/2
d=10
Trace overlap =0.9999

Quantum control: Generating SU(10)

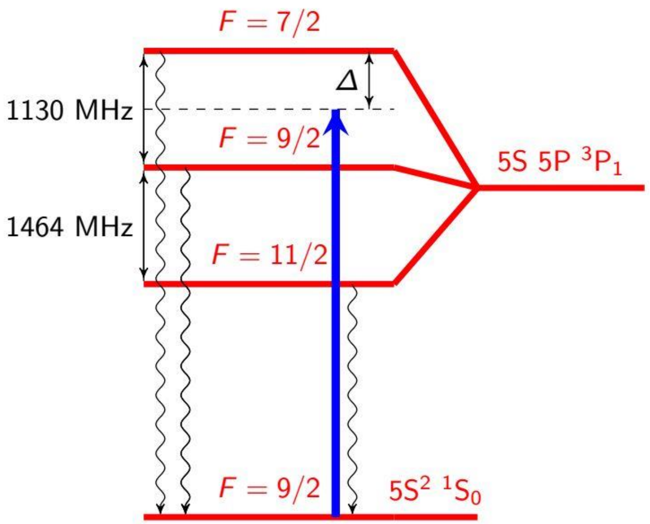
- Designing a Haar Random unitary gate: We choose a random unitary gate in this d=10 and see how the quantum control works

N: Total Number of controls
N=110
T: Total time for the Simulation
T=10π
I=9/2
d=10
Trace overlap =0.9999



Decoherence: Modelling Light Shift Interaction

$$\frac{\Gamma}{2\pi} = 7.5 \text{ KHz}$$



$$H_{\mathrm{LS}} = -\frac{1}{4}E_i(t)^* E_j(t)\alpha_{ij} = \beta I_z^2$$

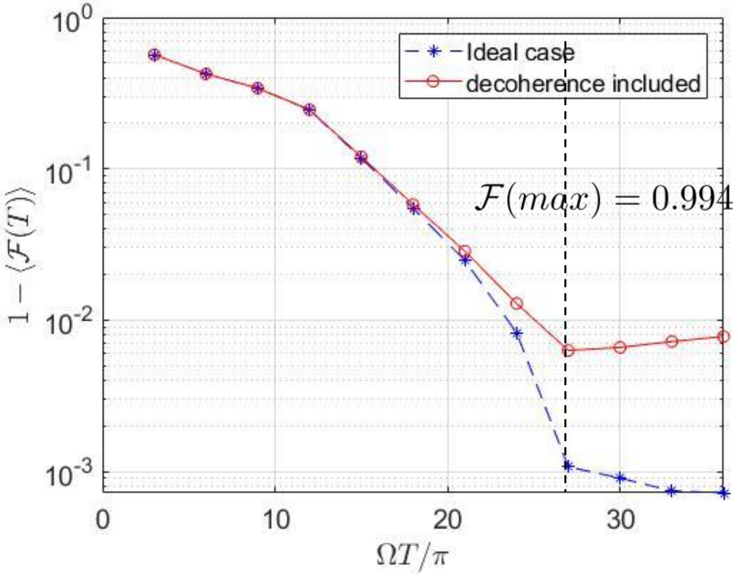
- However, this process induces decoherence[1]
- The decoherence process involve the spontaneous emission of population from the excited state.

[1] *I. H. Deutsch and P. S. Jessen, Optics Communications 283, 681 (2010).*

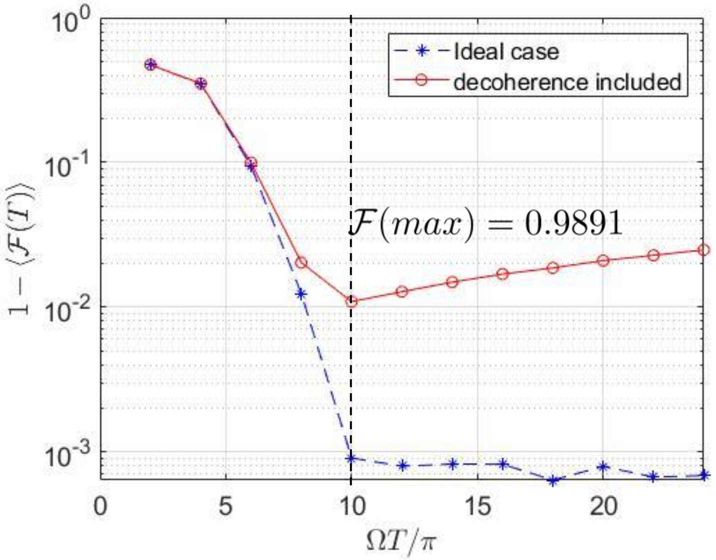
Decoherence: Unitary Mapping

$$H(t) = \Omega\left[\cos(C(t))I_x + \sin(C(t))I_y\right] + \beta I_z^2.$$

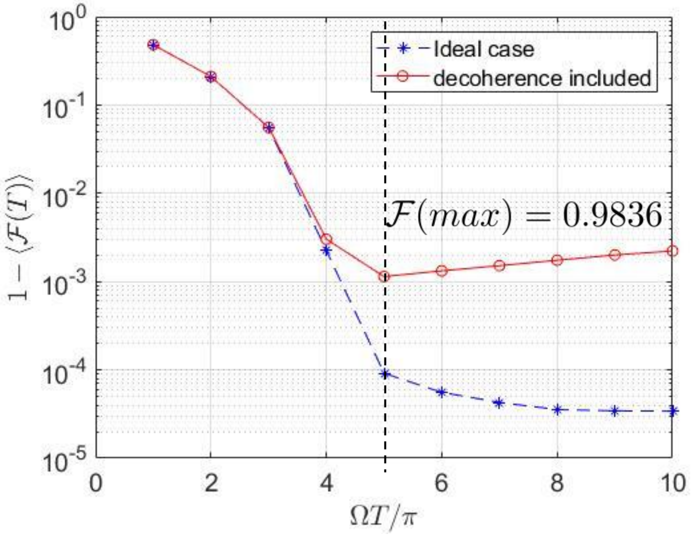
$$\mathcal{F} = \frac{1}{3\mathcal{N}}\sum_{i=0}^{3}\mathrm{Tr}\{U_{tar}\rho_i U_{tar}^{\dagger}\mathcal{J}(T)\rho_i\}$$ [1]



β/Ω=0.1



β/Ω=0.5



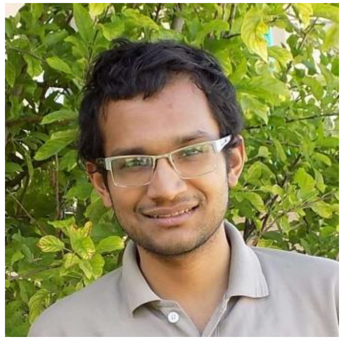
β/Ω=1

- Unitary mapping: fidelity>0.993 for d=10 in presence of decoherence
- Equivalent to fidelity ≈ 0.999 ($\mathcal{F}^{\frac{\log(2)}{\log(10)}}$) for qubit system

[1] C. P. Koch, Journal of Physics: Condensed Matter 28, 213001 (2016).

Summary:

- Fundamental Qudit gates in SU(d) is obtained using Quantum control protocols.
- A fidelity larger 0.99 in d=10, for unitary mapping in presence of decoherence.
- The effect of inhomogeneities is studied in detail.



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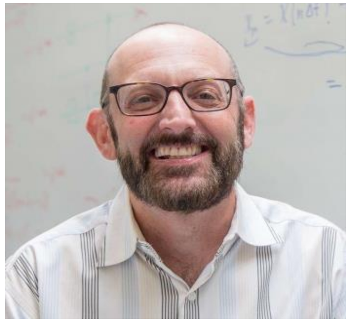

Ivan Deutsch(UNM)

Thanks for the attention!

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Studying inhomogeneities: Unitary Mapping

$$H(t) = \Omega\left[\cos(C(t))I_x + \sin(C(t))I_y\right] + \beta I_z^2.$$

$$\langle \mathcal{F}(T)\rangle = \int d\epsilon p(\{\epsilon\})F(T,\epsilon)$$

Assume the case in which the parameter is uncertain about the expected value with some width.