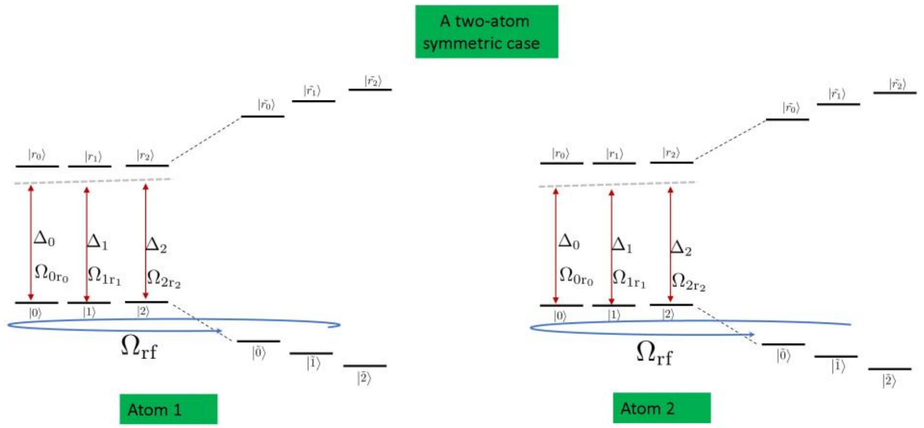

Qudit entanglers using quantum optimal control and Rydberg interaction on nuclear-spin

Sivaprasad Omanakuttan(UNM), Anupam Mitra(UNM),

Michael J. Martin (LANL), Ivan Deutsch (UNM)

DAMOP 2022 (May 30-June 3)

CQuIC Center for Quantum Information and Control

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NSF National Science Foundation

Los Alamos NATIONAL LABORATORY EST. 1943

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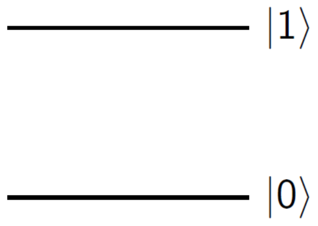
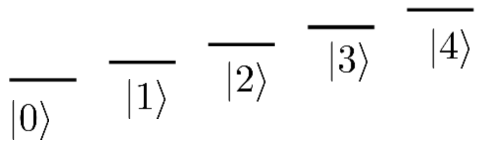
Quantum computing beyond qubits

Qubits
(d=2):



Qudits
(d>2)
(d=5):

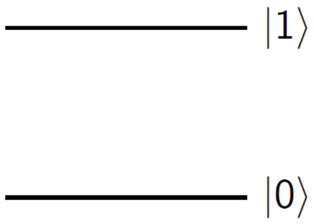


Universal gate set for quantum computing

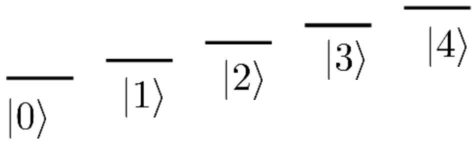
- Qubits : SU(2) + one entangling gate such as CNOT
- Qudits : SU(d) + one entangling gate

Quantum computing beyond qubits

Qubits
(d=2):



Qudits
(d>2)
(d=5):



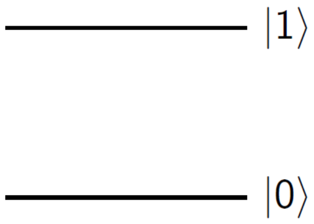
Universal gate set for qudits:

- Gates are harder to implement for qudits
  1. Constructive analytical approach [1][2]

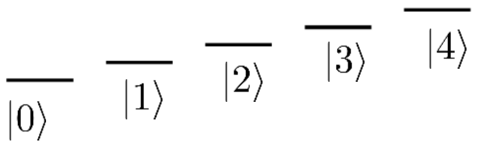
[1] M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, and T. Monz, arXiv:2109.06903 (2021)
[2] M. S. Blok, V. V. Ramasesh, T. Schuster, K. O'Brien, J. M. Kreikebaum, D. Dahlen, A. Morvan, B. Yoshida, N. Y. Yao, and I. Siddiqi, Phys. Rev. X 11, 021010 (2021)

Quantum computing beyond qubits

Qubits (d=2):



Qudits (d>2) (d=5):



Universal gate set for qudits:

- Gates are harder to implement for qudits
  1. Constructive analytical approach:
  2. Numerical approach: Quantum control

High fidelity entangling qudit gates using quantum control

[1] M. Ringbauer, M. Meth, L. Postler, R. Stricker, R. Blatt, P. Schindler, and T. Monz, arXiv:2109.06903 (2021)
[2] M. S. Blok, V. V. Ramasesh, T. Schuster, K. O'Brien, J. M. Kreikebaum, D. Dahlen, A. Morvan, B. Yoshida, N. Y. Yao, and I. Siddiqi, Phys. Rev. X 11, 021010 (2021)
[3] **SO**, A. Mitra, M. J. Martin, and I. H. Deutsch, Phys. Rev. A **104**, L060401 (2021).

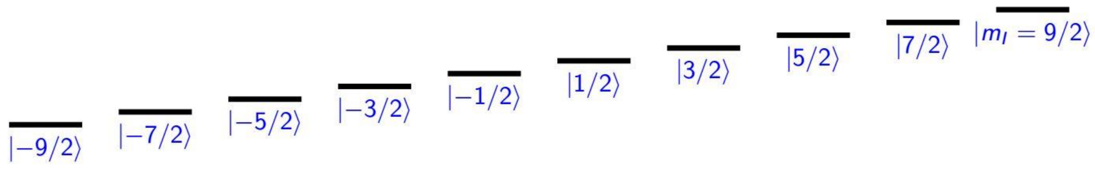
Nuclear Spin Qudit: Group-II Atoms

Strontium: ^{87}Sr[1][2]

- The ground space of ^{87}Sr, $J=0$ and $F=I+J=I$

F: Total angular momentum
I: Nuclear spin
J: Electron angular momentum

- $I=9/2$ gives d=10 (qudecimal) [$d=2F+1$]



[1] Boyd, M. M. (2007). High precision spectroscopy of strontium in an optical lattice: Towards a new standard for frequency and time, JILA Ph. D Thesis.
[2] Martin, M. J. (2013). "Quantum metrology and many-body physics: pushing the frontier of the optical lattice clock." JILA Ph. D Thesis.

Control Hamiltonian for quantum control

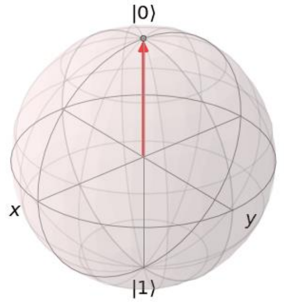
- We use the quantum control technique of GRAPE to generate qudit entanglers

- We have a controllable two qudit Hamiltonian,

$$\boxed{H(t) = H^{(1)}(t) + H^{(2)}(t) + H_{\text{Ent}}}$$

- The single atom Hamiltonian is given as,

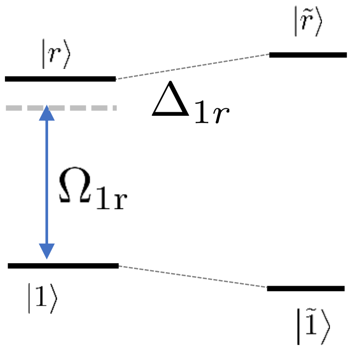
$$\boxed{H^{(\alpha)}(t) = -\boldsymbol{\mu}.\boldsymbol{B}_{\text{rf}}(t) = \Omega_{\text{rf}}\left[\cos\phi(t)F_x^{\alpha} + \sin\phi(t)F_y^{\alpha}\right] \equiv H_{\alpha}[\phi(t)]}$$



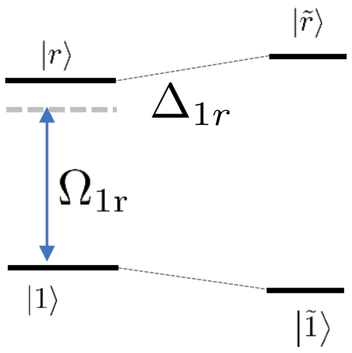
Larmor Precession

Rydberg blockade and Rydberg dressing[1][2]

$$H(t) = H_1[\phi(t)] + H_2[\phi(t)] + \boxed{H_{\text{Ent}}}$$



Atom 1

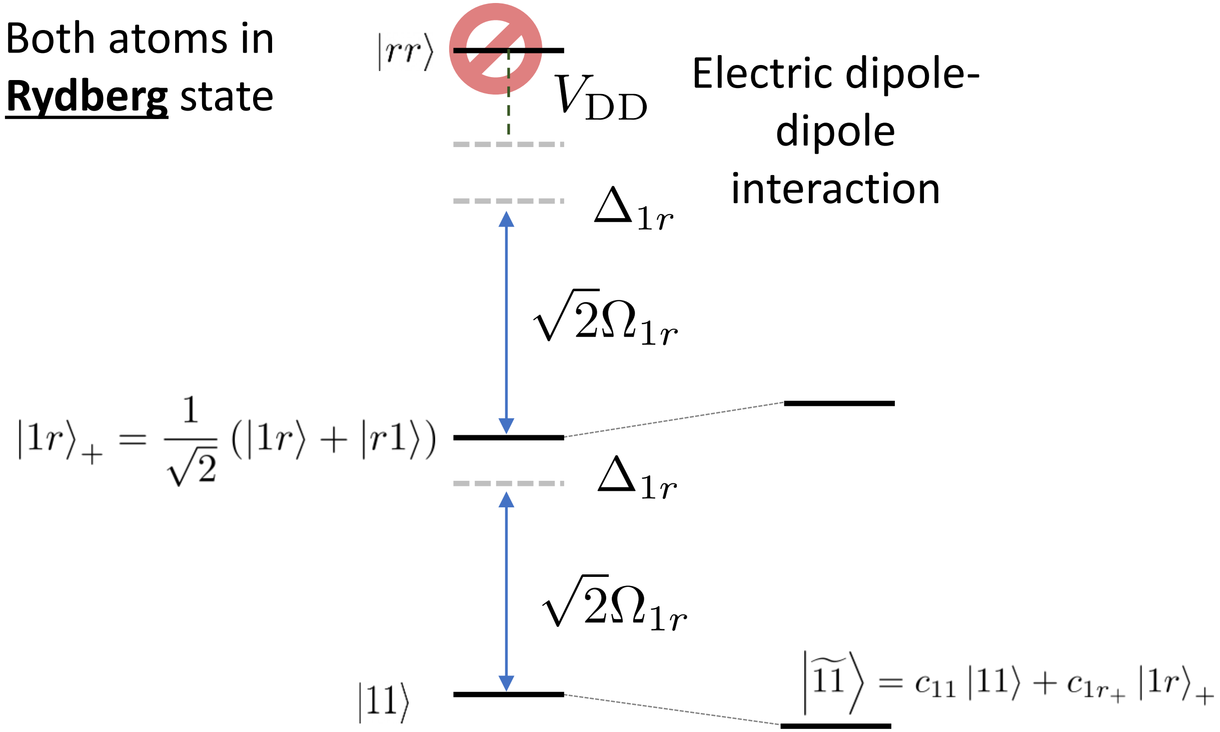


Atom 2

$|1\rangle$: **ground state**

$|r\rangle$: **highly excited state**

Rydberg blockade



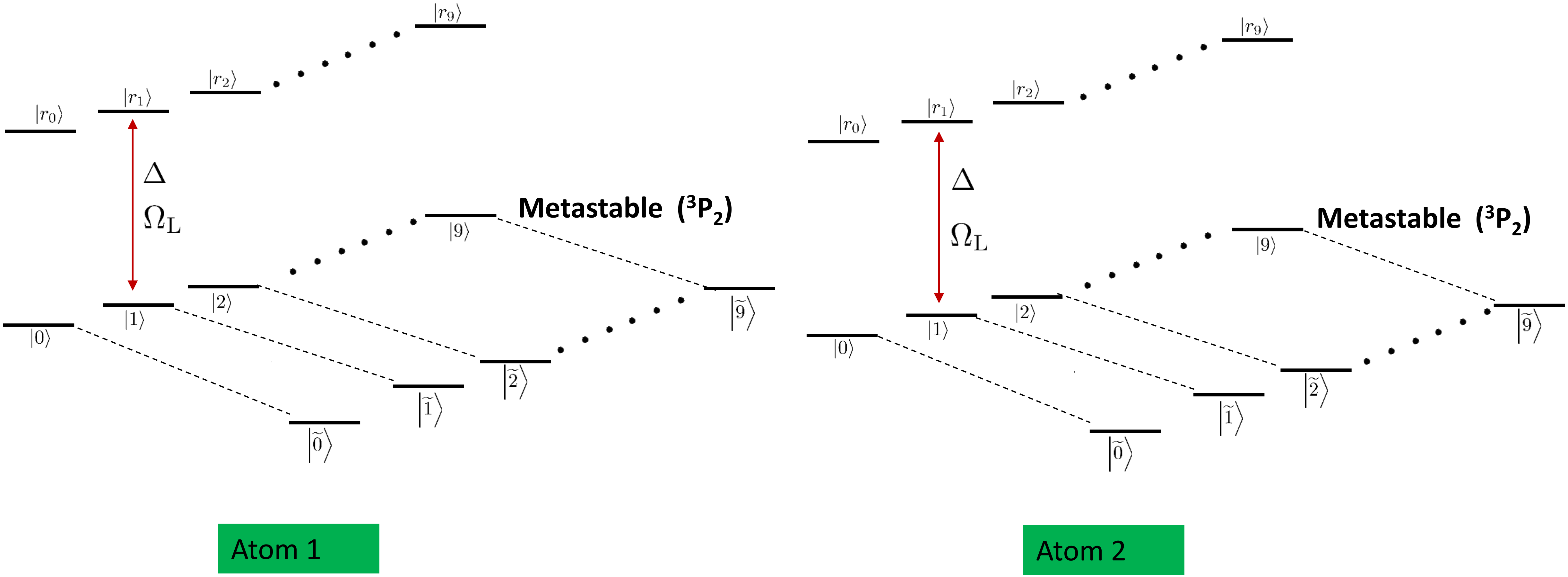
Rydberg blockade occurs when

$$|V_{\text{DD}}| \gg \Omega_{1r} \qquad |V_{\text{DD}}| \gg |\Delta_{1r}|$$

[1] A. Mitra, M. J. Martin, G. W. Biedermann, A. M. Marino, P. M. Poggi, and I. H. Deutsch, Phys. Rev. A 101, 030301 (R) (2020).
[2] J. E. Johnson and S. L. Rolston, Phys. Rev. A 82, 033412 (2010).

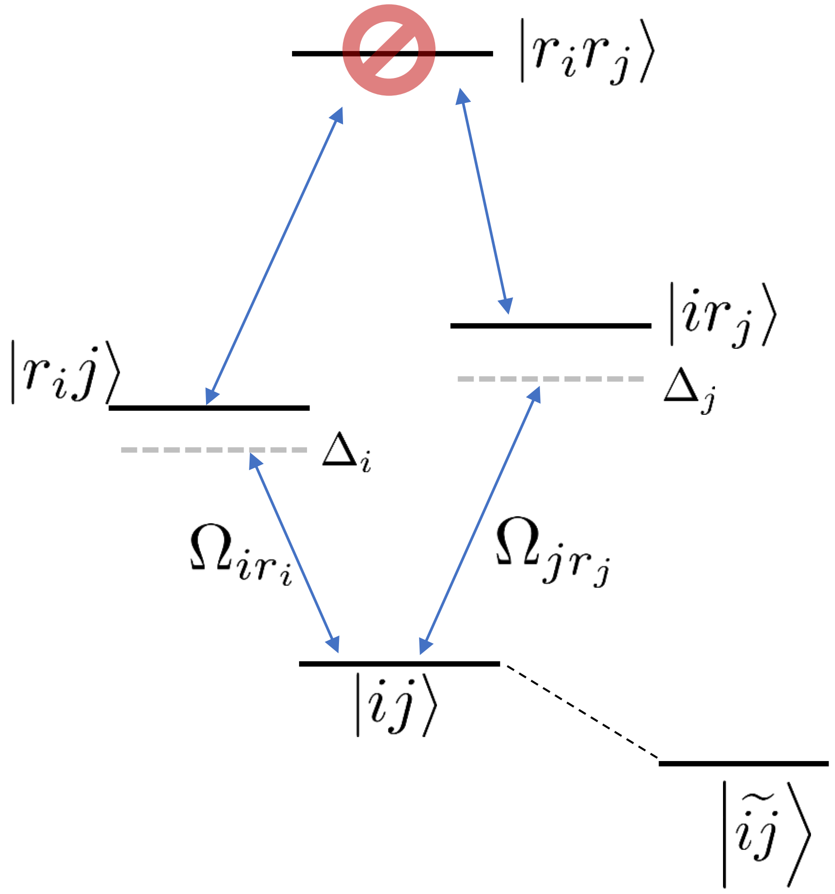
Control Hamiltonian for quantum control

$$H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\mathrm{Ent}}$$



Control Hamiltonian for quantum control

$$\boxed{H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\mathrm{Ent}}}$$

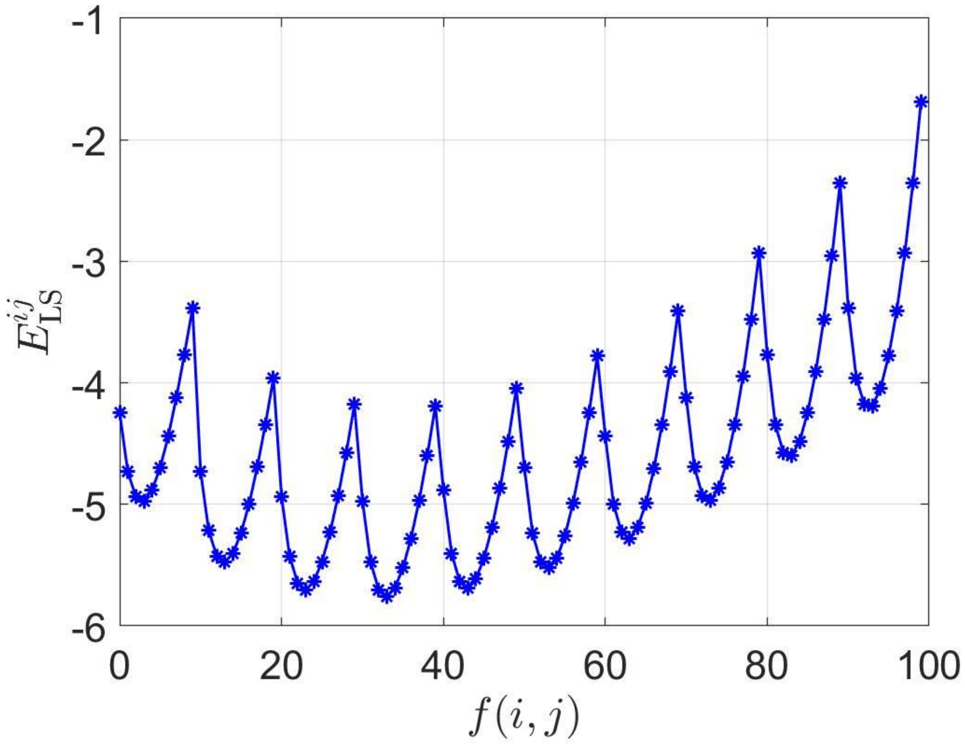


$$\left|\widetilde{ij}\right\rangle = C_{ij}\,|ij\rangle + C_{r_i j}\,|r_i j\rangle + C_{i r_j}\,|i r_j\rangle$$

$$\boxed{H_{\mathrm{Ent}} = \sum_{ij} E^{ij} \left|\widetilde{ij}\right\rangle \left\langle\widetilde{ij}\right|}$$

Understanding the entangling Hamiltonian

$$H_{\text{Ent}} = \sum_{ij} E^{ij} \left|\widetilde{ij}\right\rangle \left\langle\widetilde{ij}\right|$$



$$f(i,j) = 10i + j; 0 \leq i,j < 10$$

Tensor light shift kind of effect

$$H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\text{Ent}}$$

Quantum optimal control

- Partial isometry

$$\begin{pmatrix} \boxed{\begin{matrix} U_{11} & U_{12} & \cdots \\ U_{21} & U_{22} & \cdots \\ \vdots & \vdots & \ddots \\ U_{d1} & U_{d2} & \cdots \end{matrix}} & \begin{matrix} U_{1d} \\ U_{2d} \\ \vdots \\ U_{dd} \end{matrix} \end{pmatrix}$$

$$\xleftrightarrow{}$$
$$k$$

$$\text{Unitary Transformation: } \{|e_i\rangle\} \to \{|f_i\rangle\}$$

$$V_{\text{tar}} = \sum_{i=1}^{k} |f_i\rangle\langle e_i| + U_{\perp}$$

- Cost function: $\mathcal{F}[c(t)] = \frac{1}{k^2}\left|\sum_{i=1}^{k} \langle f_i| V[c(t)] |e_i\rangle\right|^2$

- Divide the total time T into N equal timesteps each of width Δt

- Finding the optimal control waveforms involves maximizing $\mathcal{F}[\boldsymbol{c}(t)]$

Quantum optimal control

- Partial isometry

$$\begin{pmatrix} U_{11} & U_{12} & \cdots & U_{1d} \\ U_{21} & U_{22} & \cdots & U_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ U_{d1} & U_{d2} & \cdots & U_{dd} \end{pmatrix}$$

k

$$\text{Unitary Transformation: } \{|e_i\rangle\} \to \{|f_i\rangle\}$$

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- Cost function: $\mathcal{F}[c(t)] = \frac{1}{k^2} \left| \sum_{i=1}^{k} \langle f_i| V[c(t)] |e_i\rangle \right|^2$

- Divide the total time T into N equal timesteps each of width Δt

- Number of free parameter: $N_{\text{min}}(k,d) = 2kd - k^2 - 1.$

Encoding a Qutrit entangler in Qudit

- We are interested in the CPhase which generalizes CZ gate

$$\text{CPhase}\,|j\rangle\,|k\rangle = \omega^{jk}\,|j\rangle\,|k\rangle\,; \quad \omega = \exp(2\pi i/d)$$

$$|0\rangle = \left|m_I = \frac{-9}{2}\right\rangle \qquad |1\rangle = \left|m_I = \frac{-7}{2}\right\rangle \qquad |2\rangle = \left|m_I = \frac{-5}{2}\right\rangle$$

$$|00\rangle \to |00\rangle \qquad |01\rangle \to |01\rangle \qquad |02\rangle \to |02\rangle \qquad |10\rangle \to |10\rangle \qquad |11\rangle \to e^{\frac{2\pi i}{3}}\,|11\rangle$$

$$|12\rangle \to e^{\frac{4\pi i}{3}}\,|12\rangle \qquad |20\rangle \to |20\rangle \qquad |21\rangle \to e^{\frac{4\pi i}{3}}\,|21\rangle \qquad |22\rangle \to e^{\frac{2\pi i}{3}}\,|22\rangle$$

$$\boxed{H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\text{Ent}}}$$

$$H_\alpha[\phi(t)] = -\boldsymbol{\mu}.\boldsymbol{B}_{\text{rf}}(t) = \Omega_{\text{rf}}\left[\cos\phi(t)F_x + \sin\phi(t)F_y\right]$$

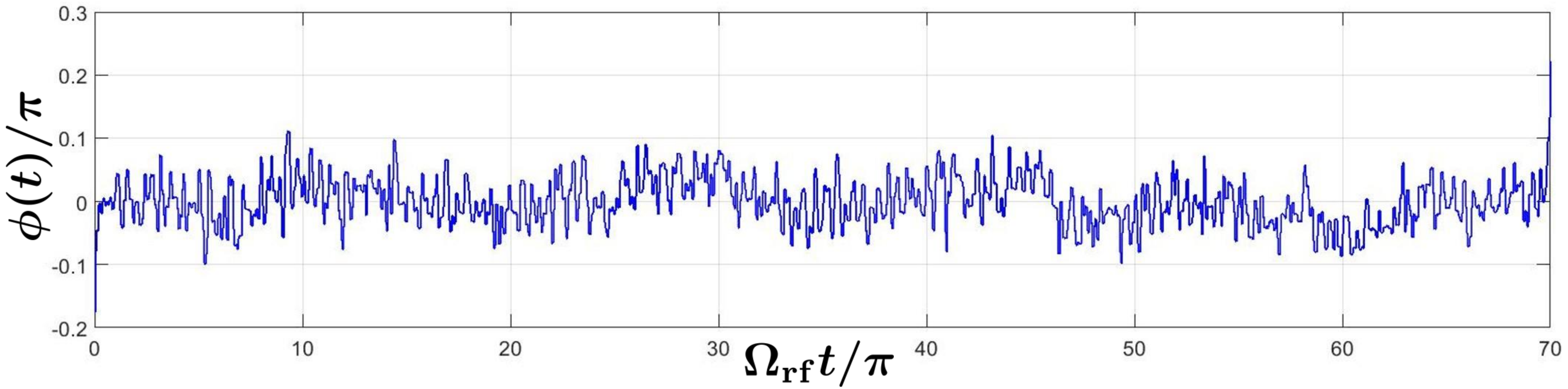
We control the system by changing the phase in a piecewise constant way with the cost function

$$\mathcal{F}[c(t)] = \frac{1}{k^2}\left|\sum_{i=1}^{k}\langle f_i|\,V[c(t)]\,|e_i\rangle\right|^2$$

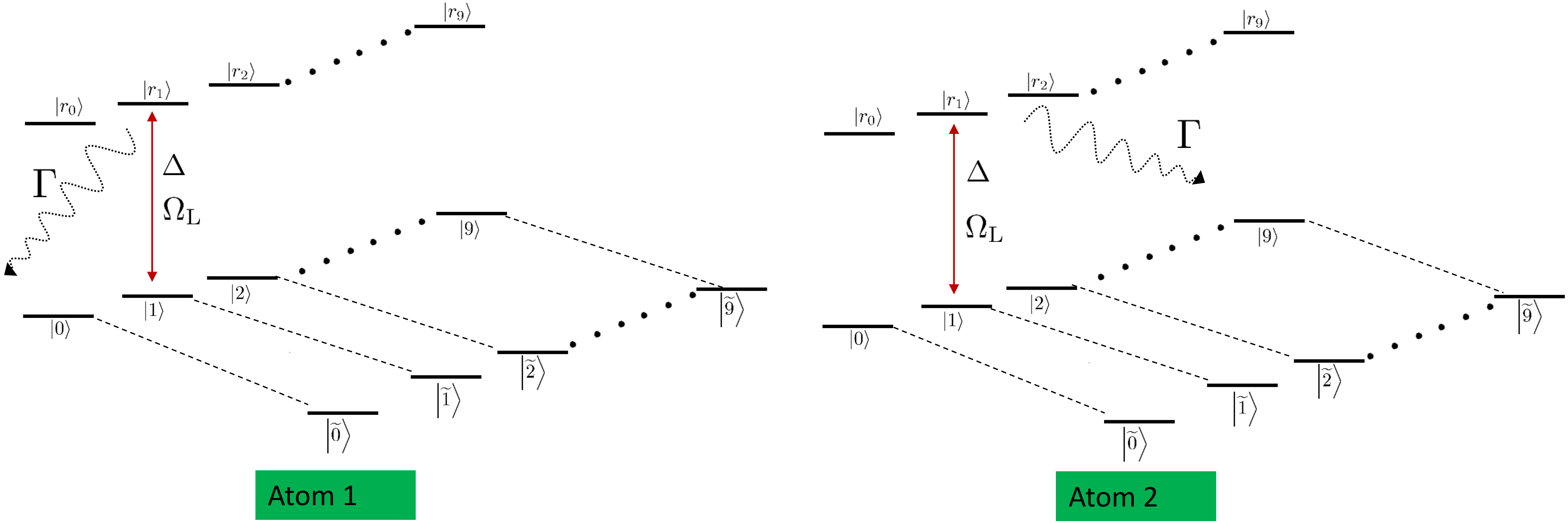
Encoding a Qutrit entangler in Qudit

$$H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\mathrm{Ent}}$$

N_{min}= 643 (minimum number of parameters)
N: Total Number of controls
N=1500
T: Total time for the Simulation
$\Omega_{rf}T=70\pi$
Fidelity=0.999



Decoherence: Effective Hamiltonian for qudits

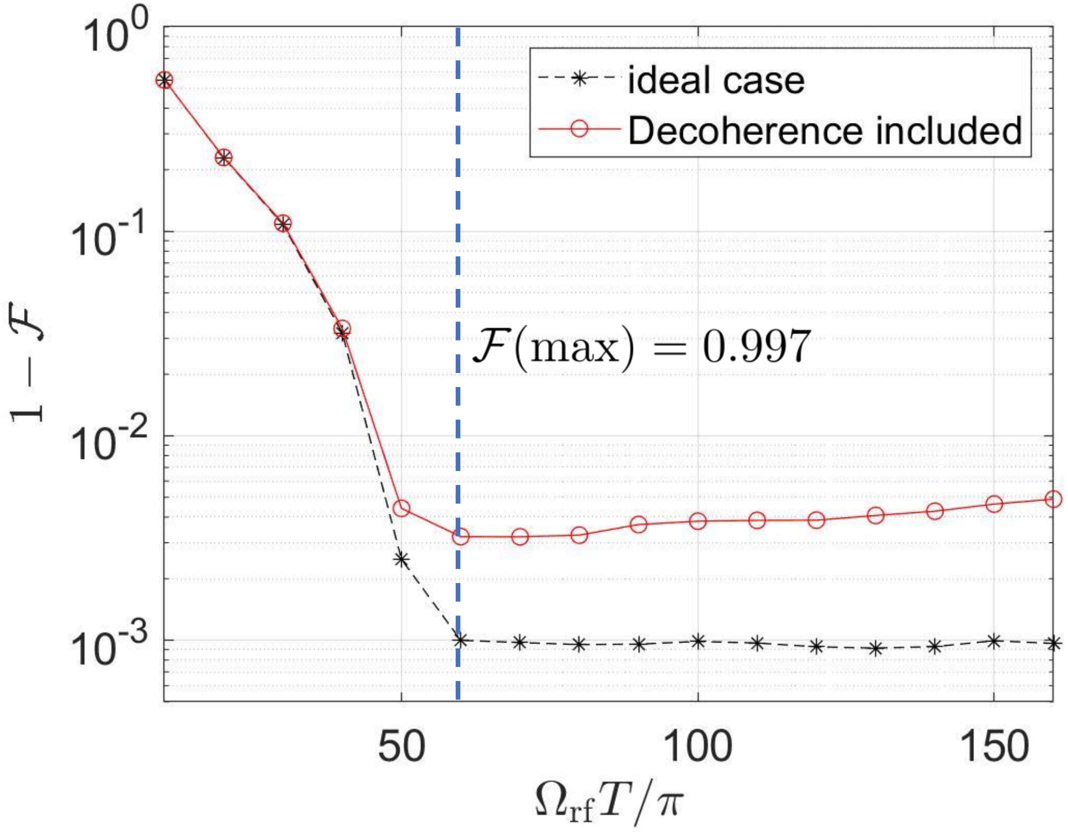


$$H_{\mathrm{Ent}}^{\mathrm{eff}} = \sum_{ij} \left(E^{ij} - i\gamma_{\mathrm{decay}}^{ij}/2 \right) \left| \widetilde{ij} \right\rangle \left\langle \widetilde{ij} \right|$$

Qutrit Cphase gate

$$\boxed{H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\mathrm{Ent}}}$$

$$\mathcal{F}[c(t)] = \frac{1}{k^2}\left|\sum_{i=1}^{k} \langle f_i| V_{\mathrm{eff}}[c(t)] |e_i\rangle\right|^2$$



Fidelity: Cphase gate

$$\boxed{H(t) = H_1[\phi(t)] + H_2[\phi(t)] + H_{\mathrm{Ent}}}$$

$$\mathcal{F}[c(t)] = \frac{1}{k^2}\left|\sum_{i=1}^{k} \langle f_i| V_{\mathrm{eff}}[c(t)]\, |e_i\rangle\right|^2$$

d	$\mathcal{F}(\max)$	$\Omega_{\mathrm{rf}} T_{\mathrm{opt}}$
2	0.998	24π
3	0.997	60π
5	0.9900	162π
7	0.9700	270π

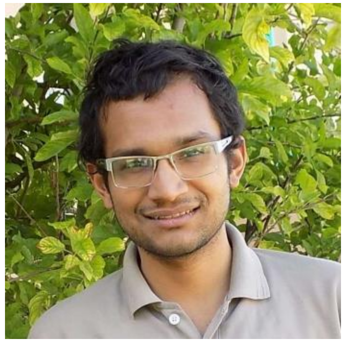
- This values can be further optimized
- A larger qudit stores much **more information** than a qubit system

Summary:

- A quantum control approach for creating CPhase gate for qudits with few μs.
- Decoherence induced by finite lifetime of the Rydberg states is studied in detail.

d	$\mathcal{F}(\max)$	$\Omega_{rf} T_{opt}$
2	0.998	24π
3	0.997	60π
5	0.9900	162π
7	0.9700	270π

- The preprint to appear in arXiv soon.



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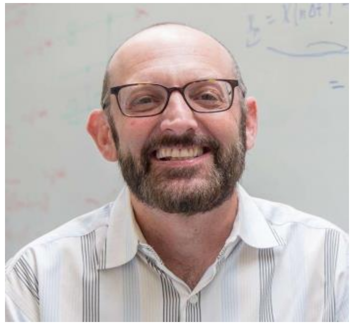

Ivan Deutsch(UNM)

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Thanks for the attention

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