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A Dynamic Programming Algorithm-based Clustering Model and Its Application to Interval Type-2 Fuzzy Large-Scale Group Decision Making Problem

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***Abstract*—This paper focuses on employing the dynamic programming algorithm to solve the large-scale group decision making problems, where the preference information takes the form of linguistic variables. Specifically, considering the linguistic variables cannot be directly computed, the interval type-2 fuzzy sets are employed to encode them. Then, new distance model and similarity model are respectively developed to measure the relationships between the interval type-2 fuzzy sets. After that, a dynamic programming algorithm-based clustering model is proposed to cluster the decision makers from the overall perspective. Moreover, by taking both the cluster center and the group size into consideration, a new model is introduced to determine the weights of clusters and decision makers, respectively. Finally, a centroid-based ranking method is developed to compare and rank the alternatives, and two illustrative experiments are provided to illustrate the effectiveness of the proposed method. Comparisons and discussions are also conducted to verify its superiority.**

***Index Terms*—Dynamic programming algorithm-based clustering model, Interval type-2 fuzzy sets, Centroid-based ranking method, Large-scale group decision making.**

I. Introduction

With the rapid development of science and technology, more and more decision makers are involved in the decision making process [1]. To fuse the individual preference information, group decision making (GDM) technology has been increasingly adopted and studied [2-5]. However, the traditional GDM mainly focuses on a small group of decision makers, which may bias the decision result [6]. Therefore, the large-scale group decision making (LSGDM) technology has received increasing attraction [7-10]. The LSGDM mainly consists of the following features: (1) There is usually involved in a huge number of decision makers (normally more than 20 [11-13]). (2) As the popularization of social media, the decision makers can participate in the decision making process at any place [14]. (3) Decision makers have mutual communication [15].

Due to the complexity and uncertainty of reality, it is difficult for the decision makers to express their opinions by crisp numbers [16-18]. Hence, various fuzzy sets and corresponding optimization models have been proposed to describe the real situations [19-24]. Word is one of the basic expression modes and can intuitively express the opinions of decision makers. Therefore, the LSGDM problems within the context of linguistic variables are emerging as one of the most important topics [14, 25-27]. Wang *et al*. [14] employed the linguistic variables to express preference information and then converted them into cloud models. To manage the comparative linguistic expressions (CLEs) information involved in the LSGDM problems, Zhang *et al*. [27] transformed the CLEs into the linguistic distribution assessments and designed a linguistic distribution-based optimization approach to solve the LSGDM problems. Liu *et al*. [25] developed a large group dependence assessment model to solve the LSGDM problems under the interval 2-tuple linguistic variables environment. To deal with the vague and uncertain features in the complex micro-grid planning problems, Ren *et al*. [26] proposed a social network analysis-based LSGDM method with hesitant fuzzy linguistic term sets.

In spite of the extensive researches dedicated to the linguistic LSGDM problems have been conducted, there is still one outstanding problem. That is, the meaning of the same linguistic variable is usually different among individuals. For example, several decision makers express their opinions as “good”, but the term “good” could have different semantic implications for

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each of them. However, most of the existing studies in LSGDM fail to reflect this difference. Different from these studies, the type-2 fuzzy set (T2FS), which was initially defined by Zadeh [28], can effectively describe this difference. To be more specific, the membership degree of the T2FS is denoted by fuzzy set instead of crisp number, which is more effective to express the difference and the fuzziness contained in linguistic variable [29, 30]. However, the heavy calculation burden of the T2FS hinders its applications in many practical problems. For that, Mendel *et al*. [29] defined the interval type-2 fuzzy set (IT2FS) by setting the secondary membership degree to 1. Since the advent of the IT2FS, it has been used in various fields, i.e., pattern recognition [31, 32], fuzzy logic system [33, 34] and optimization algorithm [35, 36]. Of note, the IT2FS has the advantages of the T2FS in describing linguistic variable and its computation is much simpler than that of the T2FS [30]. Therefore, using the IT2FS to encode the linguistic variable involved in the decision problems has received increasing attention [37-41]. In this case, we try to extend the application of the IT2FS for LSGDM problems under the linguistic environment.

Reasonable clustering analysis can simplify the decision making process [42], hence diverse clustering models have been proposed [13, 14, 43-45]. Based on the traditional hierarchical clustering algorithm, Wang *et al*. [14] defined an improved clustering approach to cluster the decision makers. Ding *et al*. [45] presented a sparse representation-based intuitionistic fuzzy clustering approach to investigate the intra-relations among the decision makers. In [44], by taking the decision makers' opinion reliability into account, an alternative ranking-based clustering method was introduced to improve the efficiency of the consensus model. To improve the accuracy and efficiency of the decision making process, Zhong and Xu presented a new clustering method by integrating the correlation and consensus [13]. Although, the existing clustering methods have made significant contributions for solving the LSGDM problems, there is still one key issue that has not been well addressed, that is, most of these methods are mainly based on the pairwise comparison, which is very complex and time consuming. Therefore, it is meaningful and necessary to develop a new clustering approach to cluster the decision makers.

In the LSGDM problems, the weights of clusters and decision makers have a vital impact on the final decision result(s). For that, various weight determination models have been introduced to determine the weights of clusters and decision makers [14, 44, 46, 47]. For example, based on the decision risks of clusters, Xu *et al*. [47] proposed a new weight scheme to calculate the weights of clusters. And, they assumed that the decision makers in the same cluster have equal importance. By taking the cluster size and the variance into consideration, Wang *et al*. [14] proposed a hybrid model to determine the weights of clusters. They also believe that the decision makers in the same group share the same importance. Xu *et al*. [46] developed a dynamic weight determination model to explore the weight vector of the clusters. In [44], the weights of clusters and decision makers were obtained by the following two principles: a) the decision makers in the same group should enjoy equal importance; b) the more decision makers in a cluster, the larger the weight should be assigned to the cluster. Despite the various contributions have been made, the above methods fail to reflect the differences between the decision makers. In practice, the importance of the decision makers usually varies with decision making problems, even if they are in the same cluster. In addition, the clusters' centers have an important impact on the weights of clusters, and the higher the degree of centralization is, the larger the weight will be [15, 23].

Even though the existing studies have made significant contributions to solving the LSGDM problems, there are still some limitations.

(1) The clustering process plays a vital role in solving the LSGDM problems, which is the main difference between the traditional GDM and LSGDM. However, nearly all the existing clustering methods divided the decision makers based on the similarity between two adjacent decision makers instead from the overall perspective, which may result in time consuming and reduce the decision efficiency.

(2) Most of the existing LSGDM methods assumed that the decision makers are independent or have the same degree of relevance. However, there is usually some relevance among the decision makers, and the relevance degrees are generally different for different decision makers. Ignoring those relations or differences may lead to an inaccurate and unreasonable result.

(3) The weights of clusters are critical for the LSGDM problems. Most of the existing studies determine the clusters' weights only rely on the number of decision makers in the clusters. However, this may lead to an unreasonable result when the clusters have the same number of decision makers but with different cohesions [23]. Moreover, owing to the diverse professional knowledges of the decision makers, the importance of the decision makers is often different. Nevertheless, almost all the existing models assumed that the decision makers in the same cluster share the equal importance, which may cause inaccuracy in the final result(s).

To address the three gaps, we first develop a new distance measure and a similarity measure for the IT2FSs. Based on the proposed similarity measure, the relevance graph of decision makers is constructed. Then, a new clustering method based on the dynamic programming algorithm is proposed to cluster the decision makers. After the clustering process, a new weight determination model is introduced to determine the weights of clusters and decision makers. Finally, a centroid-based ranking method is developed to address the LSGDM problems. To verify the effectiveness of the proposed clustering model, we apply it to a case study concerning renewable energy resources (RERs) evaluation. Additionally, the feasibility of the full method is demonstrated by a case study about the construction of subway station. Meanwhile, the comparisons with other methods [6, 14, 15] are also made to show the superiority of our method. The main contributions of this paper are summarized as follows:

(1) By counting the similar preference information between two decision makers, the similarity between them is provided. Based on the obtained similarities, some new

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rules are developed to explore the relevance degrees between the decision makers. So, the differences between the decision makers can be effectively reflected and the relevance degrees can be characterized in a more accurate manner.

(2) A dynamic programming algorithm-based interval type-2 fuzzy clustering method is developed to detect the maximum relevance path. The decision makers distributed on the maximum relevance path will be clustered into the same group. Compared to the existing clustering models, the proposed model can cluster decision makers from the overall perspective instead of only ensuring the maximum similarity between two adjacent decision makers. In this way, we can not only obtain a satisfactory clustering result, but also reduce the number of iterations.

(3) A new model is introduced to determine the weights of clusters. The new model can consider both the size and the centralization degree of cluster, and thus improves the weight determination model that only considers the cluster size. Moreover, a new weight determination rule is designed for the decision makers to ensure that the decision maker who has the more important influence on others will be assigned a bigger weight, rather than simply assuming that the decision makers in the same cluster have equal importance.

The rest of this paper is organized as follows: Section II introduces some basic concepts of T2FSs, computing with word of IT2FSs and Nie-Tan operator. Section III develops a dynamic programming algorithm-based interval type-2 fuzzy clustering method. In Section IV, a centroid-based ranking method is proposed to address the LSGDM problems within the context of linguistic variables. Section V demonstrates the superiority of the proposed method by comparative studies and related discussions. The conclusions are presented in Section VI.

II. Preliminaries

In this section, some basic concepts of T2FSs, computing with word of IT2FSs and Nie-Tan operator are introduced, which will be used in the subsequent sections.

A. Type-2 Fuzzy sets

The T2FS is initially proposed by Zadeh [28], which can better depict the fuzziness by setting a secondary membership. However, due to the heavy calculation, there are few researches on the T2FS. For that, Mendel *et al.* [29, 48] proposed the IT2FS on the basis of the T2FS. Of note, the calculation of IT2FS is much easier than that of T2FS. Hence, the IT2FS has been widely used in various fields, such as high-tech risk evaluation [40], medical diagnosis [49], wastewater treatment technology evaluation [41], and so forth.

Definition 1 [29]: A T2FS $\tilde{A}$ can be expressed by a type-2 membership function (MF) $\mu_{\tilde{A}}(x,u)$, shown as: $\tilde{A}=\left\{\left((x,u),\mu_{\tilde{A}}(x,u)\right)\mid\forall x\in X,\forall u\in J_x\subseteq[0,1]\right\}$, where X is the domain of $\tilde{A}$ and μ is the MF of x.

The T2FS can be also denoted as: $\tilde{A}=\int_{x\in X}\int_{\mu\in J_x}\mu_{\tilde{A}}(x,u)/(x,u)$, where J_x is an interval in $[0,1]$ and $\mu_{\tilde{A}}(x,u)$ denotes the secondary membership function at x.

Definition 2 [29, 50, 51]: For a T2FS $\tilde{A}$, if all $\mu_{\tilde{A}}(x,u)=1$, then $\tilde{A}$ is called an IT2FS, which can be expressed as: $\tilde{A}=\int_{x\in X}\int_{\mu\in J_x}1/(x,u)$, $J_x\in[0,1]$. For the discrete case, $\int$ is replaced by $\sum$.

Moreover, due to the secondary memberships of the IT2FS are all uniformly weighted for each primary membership of *x*. The IT2FS can also be represented by the footprint of uncertainty (FOU): $FOU(A)=\left\{(x,u):u\in\left[\underline{\mu}_{\tilde{A}}(x,u),\overline{\mu}_{\tilde{A}}(x,u)\right]\right\}$, where $\underline{\mu}_{\tilde{A}}(x,u)$ and $\overline{\mu}_{\tilde{A}}(x,u)$ respectively denote the lower and upper membership function of $\tilde{A}$.

B. Computing With Word of IT2FSs

To encode word by the IT2FS, Wu *et al.* [52] proposed an enhanced interval approach. Suppose there are N survey interval data $\left\{[a_k,b_k]\right\}_{k=1}^{N}$ for one word. The enhanced interval approach mainly involves the following steps.

● **Bad data processing:** After eliminating the interval survey data that do not meet the following conditions, the N interval survey data will reduce to n interval survey data.

$$0\le a_k\le b_k\le 10 \tag{1}$$

$$b_k-a_k<10 \tag{2}$$

● **Outlier processing:** Box and Whisker tests are performed on a_k, b_k and $L_k=b_k-a_k$, respectively. The intervals which do not satisfy the following conditions will be rejected:

$$a_k\in\left[Q_l(0.25)-1.5IQR_l,Q_l(0.75)+1.5IQR_r\right] \tag{3}$$

$$b_k\in\left[Q_r(0.25)-1.5IQR_r,Q_r(0.75)+1.5IQR_r\right] \tag{4}$$

$$L_k\in\left[Q_L(0.25)-1.5IQR_L,Q_L(0.75)+1.5IQR_L\right] \tag{5}$$

After this step, there will be n' interval data left.

● **Tolerance limit processing:** Only the remaining interval data satisfy the following conditions will be received:

$$a_k\in\left[m_l-k\sigma_l,m_l+k\sigma_l\right] \tag{6}$$

$$b_k\in\left[m_r-k\sigma_r,m_r+k\sigma_r\right] \tag{7}$$

$$L_k\in\left[m_L-k\sigma_L,m_L+k\sigma_L\right] \tag{8}$$

● **Reasonable-interval processing:** The interval data which do not meet the following will be eliminated:

$$2m_l-\xi^*\le a_k<\xi^*<b_k\le 2m_r-\xi^* \tag{9}$$

where

$$\xi^*=\frac{\left(m_r\sigma_l^2-m_l\sigma_r^2\right)^2\pm\sigma_l\sigma_r\sqrt{\left(m_l-m_r\right)^2+2\left(\sigma_l^2-\sigma_r^2\right)\ln\frac{\sigma_l}{\sigma_r}}}{\sigma_l^2-\sigma_r^2} \tag{10}$$

● **Construct the IT2FS:** The remaining interval data can be mapped into the embedded T1FSs by equating means and standard deviations of interval data to that of the embedded T1FSs, which is defined as follows:

$$m_{MF}=\frac{\int_{a_{MF}}^{b_{MF}} x\cdot u_{MF}(x)dx}{\int_{a_{MF}}^{b_{MF}} u_{MF}(x)dx}=\frac{a_k+b_k}{2} \tag{11}$$

$$\frac{\int_{a_{MF}}^{b_{MF}} (x-m_{MF})^2\cdot u_{MF}(x)dx}{\int_{a_{MF}}^{b_{MF}} u_{MF}(x)dx}=\frac{b_k-a_k}{\sqrt{12}} \tag{12}$$

Then, the IT2FS can be constructed from the embedded T1FSs. The detailed explanations for the approach are provided in [52].

Remark 1. Generally, word means different things to different people. For example, score the word "good" from 0.1 to 1, for some people [0.6, 0.8] means "good", but others may think [0.7, 0.9] is "good". Most of the existing fuzzy sets encode the word into one data regardless of the individual difference and collective opinions. In this case, those fuzzy sets may encode the "good" on the basis of [0.6, 0.8] or [0.7, 0.9], which is difficult to reflect the two opinions simultaneously. In contrast, the IT2FS can well reflect the two opinions by taking both [0.6, 0.8] and [0.7, 0.9] into consideration. The detailed explanations of the approach are provided in [52].

C. Nie-Tan Operator

As a fuzzy set, the IT2FS cannot be directly compared, hence it is necessary to defuzzify it before comparison. Centroid is considered as the most popular way to defuzzify IT2FS [48]. Initially, Mendel *et al*. [48, 53] proposed the Karnik-Mendel (KM) algorithm to compute the centroid of the IT2FS. Considering the heavy calculation of KM algorithm, Nie *et al*. [54] subsequently developed a new type-reduction method, named Nie-Tan operator. Moreover, Li *et al*. [55] proved that the results of Nie-Tan operator are as accurate as the KM algorithm by four random sampling methods.

Definition 3 [54, 55]: The MFs of the embedded sets of an IT2FS satisfy $\underline{\mu}(x)\le\mu(x)\le\overline{\mu}(x)$. If the centroid of an embedded set is equivalent to the center of gravity (COG) of IT2FS, the embedded set is called representative embedded set. Then, the COG of IT2FS $\tilde{A}$ is denoted as:

$$C(\tilde{A})=\frac{\int_{x_{\min}}^{x_{\max}} \mu^*(x)\cdot xdx}{\int_{x_{\min}}^{x_{\max}} \mu^*(x)dx} \tag{13}$$

where $\mu^*(x)$ is the MF of the representative embedded set, and $\mu^*(x)=\frac{1}{2}\cdot\left(\underline{\mu}(x)+\overline{\mu}(x)\right)$.

III. Dynamic Programming Algorithm-Based Clustering Model

For the LSGDM problems, conducting reasonable clustering analysis can improve the decision efficiency and accuracy. This section first defines a new interval type-2 fuzzy similarity model to measure the relevance between the decision makers. Then, a dynamic programming algorithm-based interval type-2 fuzzy clustering method is proposed to cluster the decision makers.

A. The Description of The Large-Scale Group Decision Making Problems

Due to the complexity of environment and the subjectivity of human thinking, the decision makers tend to use linguistic variables to express their opinions. Generally, the same linguistic variable has different meanings for different decision makers. The IT2FS can effectively reflect this difference and has been widely used to encode word.

To solve a complex decision problem, we invite p decision makers $D=\{d_1,d_2,\cdots,d_p\}$ to evaluate m alternatives $Z=\{Z_1,Z_2,\cdots,Z_m\}$ with respect to n attributes $C=\{C_1,C_2,\cdots,C_n\}$. And, $\lambda=\{\lambda_1,\lambda_2,\cdots,\lambda_n\}$ is the weight vector of the n attributes, where $0\le\lambda_j\le 1$, $\sum_{j=1}^{n}\lambda_j=1$. Moreover, $\omega=\{\omega_1,\omega_2,\cdots,\omega_p\}$ denotes the weight vector of the p decision makers, which satisfies $0\le\omega_j\le 1$, $\sum_{j=1}^{p}\omega_j=1$. To improve the efficiency of decision making, we classify the p decision makers into L clusters based on the dynamic programming algorithm. The weight vector of the L clusters can be denoted as $w=\{w_1,w_2,\cdots,w_L\}$, where $0\le w_j\le 1$, $\sum_{j=1}^{L}w_j=1$.

It is assumed that the evaluation information of the decision makers is expressed by linguistic variables. And, S_{ij}^k is the linguistic variable corresponding to the evaluation information given by the decision maker d_k for alternative Z_i under the attribute C_j . To make decision with word, we encode the linguistic variable into the IT2FS. Specifically, r_{ij}^k is the IT2FS corresponding to linguistic variable S_{ij}^k , where $r_{ij}^k=\left(\left(\overline{a}_{1ij}^k,\overline{a}_{2ij}^k,\overline{a}_{3ij}^k,\overline{a}_{4ij}^k,\overline{h}(r_{ij}^k)\right),\left(\underline{a}_{1ij}^k,\underline{a}_{2ij}^k,\underline{a}_{3ij}^k,\underline{a}_{4ij}^k,\underline{h}(r_{ij}^k)\right)\right)$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

B. Fuzzy Distance Measure and Similarity Measure of IT2FSs

Definition 4: Let $\tilde{A}=(\tilde{A}_1,\tilde{A}_2,\cdots,\tilde{A}_n)$ and $\tilde{B}=(\tilde{B}_1,\tilde{B}_2,\cdots,\tilde{B}_n)$ be any two sets of IT2FSs, then the generalized centroid-based distance measure between $\tilde{A}$ and $\tilde{B}$ is defined as:

$$D(\tilde{A},\tilde{B})=\left[\sum_{i=1}^{n}\left|C(\tilde{A}_i)-C(\tilde{B}_i)\right|^{\lambda}\right]^{\frac{1}{\lambda}} \tag{14}$$

If $\lambda=1$, then the generalized centroid-based distance measure reduces to the Hamming distance measure.

If $\lambda=2$, then the generalized centroid-based distance measure reduces to the Euclidean distance measure.

Specially, if $n=1$, the *Eq.*(14) denotes the distance between two interval type-2 fuzzy numbers.

It can be easily proved that the proposed distance model satisfies the following distance axioms:

(1) $d\left(\tilde{A},\tilde{B}\right)\geq 0$

(2) $d\left(\tilde{A},\tilde{B}\right)=d\left(\tilde{B},\tilde{A}\right)$

(3) $d\left(\tilde{A},\tilde{C}\right)\leq d\left(\tilde{A},\tilde{B}\right)+d\left(\tilde{B},\tilde{C}\right)$

For the interval type-2 fuzzy large-scale group decision making (IT2-FLSGDM) problems, the distance between $\tilde{r}_{ij}^{\alpha}$ and $\tilde{r}_{ij}^{\beta}$ can be calculated by:

$$D\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)=\left|C\left(\tilde{r}_{ij}^{\alpha}\right)-C\left(\tilde{r}_{ij}^{\beta}\right)\right| \tag{15}$$

where $\tilde{r}_{ij}^{\alpha}$ denotes the evaluation value provided by the decision maker d_{α} of the alternative Z_i under the attribute C_j; similarly, $\tilde{r}_{ij}^{\beta}$ denotes the evaluation value provided by the decision maker d_{β}.

Based on the *Eq.*(15), the similarity between $\tilde{r}_{ij}^{\alpha}$ and $\tilde{r}_{ij}^{\beta}$ can be calculated as:

$$S\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)=\frac{1}{1+\zeta D\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)} \tag{16}$$

Let $\delta\in[0,1]$ be the threshold for the similarity between $\tilde{r}_{ij}^{\alpha}$ and $\tilde{r}_{ij}^{\beta}$. If $S\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)\geq\delta$, we consider that the opinion between d_{α} and d_{β} for the alternative Z_i under the attribute C_j is similar or consistent. And, ζ is the tune parameter, where $\zeta>0$.

The main contribution of the tune parameter ζ is to increase the flexibility of the similarity. For instance, if the decision makers deem that two IT2FSs are similar, but the distance between them is far larger than 1, then a small ζ is needed to adjust the similarity. Conversely, if two IT2FSs are deemed dissimilar, but the distance between them is very minor, then the value of ζ should be large. So far, there is no effective way to set ζ accurately. In this paper, we believe that the value of ζ is the average of values provided by all decision makers.

Remark 2. The parameter δ is the similarity threshold between the preference information. The larger value of δ means a higher similarity requirement between preference information. Setting an appropriate δ can not only save the running time, but also improve the accuracy of decision making. In practice, decision makers can set appropriate value of δ according to the specific problem.

Then, the overall similarity between the decision makers d_{α} and d_{β} can be calculated by:

$$S\left(d_{\alpha},d_{\beta}\right)=\frac{NSAP\left(S\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)\geq\delta\right)}{N} \tag{17}$$

where $NSAP\left(S\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)\geq\delta\right)$ is the number of similarity degrees that holds $S\left(\tilde{r}_{ij}^{\alpha},\tilde{r}_{ij}^{\beta}\right)\geq\delta$ and N is the total number of evaluation values of d_{α} or d_{β}.

C. A New Clustering Model Based on Dynamic Programming Algorithm

Dynamic programming algorithm was initially proposed by Bellman and Dreyfus [56], which is an effective tool to solve the multi-stage decision-making (MSDM) problems. According to the Bellman optimality principle, the optimal value of the MSDM problems is obtained by adopting the following performance index function:

$$f_k\left(s_k\right)=\underset{\{u_k,\ldots,u_n\}}{opt}\;V_{k,n}\left(s_k,u_k,\ldots,s_{n+1}\right) \tag{18}$$

where "*opt*" is the abbreviation, which can be *min* or *max*; s_k and u_k are the state variable and decision variable of stage k.

In this paper, the dynamic programming algorithm is applied to detect the maximum relevance path among the decision makers.

Let η be the threshold of similarity between the decision makers, if $S\left(d_{\alpha},d_{\beta}\right)\geq\eta$, we consider that there is relevance between d_{α} and d_{β}, and the relevant degree is $\xi_{\alpha\beta}$. Otherwise, we consider that they are irrelevant.

The relevance degree between d_{α} and d_{β} can be calculated by:

$$\xi_{\alpha\beta}=\begin{cases}S\left(d_{\alpha},d_{\beta}\right), & if\ S\left(d_{\alpha},d_{\beta}\right)\geq\eta\\ 0, & otherwise\end{cases} \tag{19}$$

Based on *Eq.*(19), we can draw the relevance graph of decision makers. After that, a dynamic programming algorithm-based clustering model is defined to detect the maximum relevance path among the decision makers. And, the decision makers distributed on the maximum relevance path have the largest overall relevance and should be clustered into the same group.

Remark 3. The value of η means the minimum similarity requirement between the decision makers. An inappropriate η may lead to more clusters and heavier calculation. Hence, selecting suitable η can improve the decision efficiency and accuracy. Generally, the value of η is equal to that of δ, but for some special problems, the values of the two parameters may be different. In this paper, we deem η is equal to δ.

Supposed d_1 is the starting point of the graph, and d_2 is the decision maker with $\xi_{12}=\max\limits_{1<\beta\leq t}\left\{\xi_{1\beta}\right\}$ and $\xi_{1\beta}\geq\eta$; similarly, d_3 is the decision maker with $\xi_{13}=sub\max\limits_{1<\beta\leq t}\left\{\xi_{1\beta}\right\}$ and $\xi_{1\beta}\geq\eta$. This is an iterative algorithm, which will be terminated when all decision makers have been labelled. The relevance graph of all decision makers is shown in Fig.1 (presented in the Appendix).

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According to Fig.1 and the relevance degrees, these decision makers are divided into different stages. And, the state variables of each stage can be summarized as: $s_0=\{d_1\}$, $s_1=\{d_2,d_3,d_4\}$, $s_2=\{d_5,d_6,d_7,d_8,d_9\}$, $s_3=\{d_{10},d_{11},d_{12},d_{13},d_{14},d_{15}\}$, $s_4=\{d_{16},d_{17},d_{18},d_{19}\}$, $s_5=\{d_{20},d_{21},d_{22},d_{23},d_{24},d_{25}\}$, $s_6=\{d_{26},d_{27},d_{28}\}$ and $s_7=\{d_t\}$. Inspired by the idea of dynamic programming algorithm, the maximum relevance path from s_k to s_t can be obtained by the following performance index function:

$$f_k(s_k)=\max_{1\le\alpha\le t}\{\xi_{k\alpha}\times f_\alpha(s_\alpha)\} \tag{20}$$

where $\xi_{k\alpha}$ is the potential relevance degree between s_k and s_α, $\xi_{k\alpha}\ge\eta$ and $\alpha>k$; $f_\alpha(s_\alpha)$ denotes the path with the maximum relevance from s_α to s_t.

The maximum relevance path from $s_1=\{d_1\}$ to $s_t=\{d_t\}$ can be obtained via *Eq*.(20). Then, we divide the decision makers distributed on the maximum relevance path into the same group, named as Q_1. Of note, if the maximum relevance from $s_1=\{d_1\}$ to $s_t=\{d_t\}$ is less than the threshold η, we start this process from the previous stage. To divide the remaining decision makers, we continue the iteration algorithm until all decision makers have been clustered.

The proposed clustering model is a convergent algorithm, which is mainly reflected by the following two aspects: (1) After each iteration, the number of the remaining decision makers decreases. (2) The path obtained by the clustering model is the maximum relevant path. Now, we will explain the two aspects in detail. For aspect (1), let $N_t(d)$ be the number of the remaining decision makers after the *t*-th iteration. Since, some decision makers will be eliminated and clustered into a certain group after each iteration, it is obvious that $N_t(d)\ge N_{t+1}(d)$. As for aspect (2), we assume that $d_1\to d_i\to\cdots\to d_j\to d_t$ is the optimal solution obtained by the proposed clustering model and the maximum relevance degree is $f_1(s_1)$. From *Eq*. (20), it is easy to observe that $f_1(s_1)$ is the maximum relevance path among all potential paths from d_1 to d_t.

The process of clustering decision makers is summarized in Fig.2.

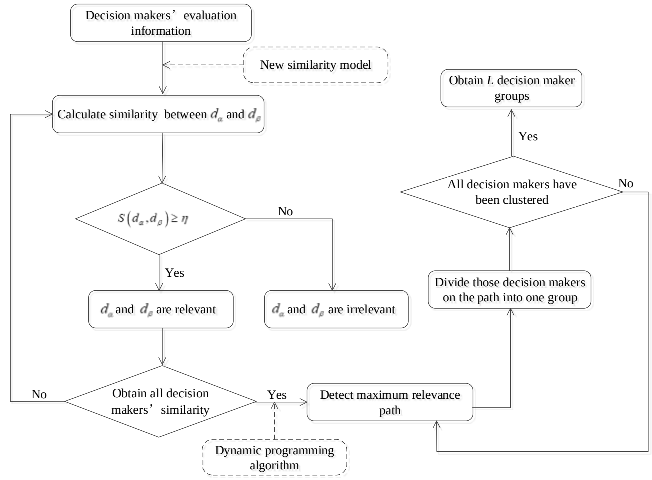

Fig.2. The process of clustering decision makers

Remark 4. The dynamic programming algorithm-based clustering model is an iteration algorithm. In each iteration process, we optimize the relevance paths in reverse order and divide the decision makers distributed on the path with maximum relevance into the same group. The proposed model clusters the decision makers from the overall perspective, which can reduce iteration times and bring a more precise result.

Example 1 Supposed that there are 5 decision makers and the similarities between them are summarized in Table I. To illustrate the effectiveness of the proposed clustering model, we cluster the 5 decision makers by the proposed clustering model. The details are shown as follows:

TABLE I
THE SIMILARITIES BETWEEN THE 5 DECISION MAKERS

	d_1	d_2	d_3	d_4	d_5
d_1	**1**	0.6	0.25	0.7	0.3
d_2	0.6	**1**	0.9	0.5	0.46
d_3	0.25	0.9	**1**	0.38	0.4
d_4	0.7	0.5	0.38	**1**	0.5
d_5	0.3	0.46	0.4	0.5	**1**

In this case, we assume that if the similarity between two decision makers is no less than 0.5, there is a direct path between them. By applying the dynamic programming algorithm, the problem is divided into 3 stages. To be more specific, there is one reachable state in the first stage, which can be expressed as $s_1=\{d_1\}$; similarly, the reachable states in the second and third stages can be respectively expressed as $s_2=\{d_2,d_4\}$ and $s_3=\{d_3,d_5\}$. Since the third stage contains two reachable states, we set a virtual final stage $s_t=\{d_t\}$ and the relevance between d_3 and d_t is equal to that between d_5 and d_t, both of which are 1. We gradually explore the maximum relevance path from s_1 to s_t by the backstepping algorithm. Specifically, when $k=3$, there are two reachable states and the values of the relevance path is $f_3(s_3)=1$. Then, according to *Eq*.(20), the optimal value in the second stage ($k=2$) is $f_2(s_2)=\max\{0.9\times1,0.5\times1\}=0.9$. Finally, when $k=1$, the

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optimal value is $f_1(s_1)=0.54$ and the corresponding maximum relevance path is $d_t \to d_3 \to d_2 \to d_1$. And, the clustering result obtained by the proposed clustering model is $Q_1=\{d_1,d_2,d_3\}$ and $Q_2=\{d_4,d_5\}$. From the obtained result and Table I, we can observe that although d_4 shares the maximum similarity to d_1, he/she is not chosen. The reason is that if we choose $d_1 \to d_4$, then $d_4 \to d_5 \to d_t$ will be subsequently chosen. However, the similarity between d_4 and d_5 is only 0.5, which is much less than that between d_2 and d_3. That is to say, if we choose d_4, the overall relevance of the cluster will be reduced. Therefore, our approach can cluster the decision makers from the overall perspective instead of only ensuring the maximum similarity between two adjacent decision makers.

IV. A Centroid-Based Ranking Method for THE IT2-FLSGDM Problems

In this section, we first utilize the dynamic programming algorithm to detect the center of cluster. By integrating the center with the group size, a new weight determination model is proposed to determine the weights of clusters and decision makers. Then, a centroid-based ranking method is developed to address the LSGDM problems with IT2FSs information.

A. A New Weight Determination Model

Based on the dynamic programming algorithm-based clustering model, the decision makers can be classified into L groups, named as $Q_1,Q_2,\cdots,Q_L$. The number of the decision makers in each group is $q_1,q_2,\cdots,q_L$. By integrating the cluster center with the group size, we develop a new weight determination model to determine the weights of clusters and decision makers, respectively.

Based on the degree of relevance between the decision makers, we can further obtain the consistency coefficient between the decision makers. Taking d_k and d_α in cluster Q_1 as an example, the consistency coefficient between them can be denoted as $l^1_{k\alpha}$.

$$l^1_{k\alpha}=\begin{cases}\xi^1_{k\alpha}, & if\ S(d_k,d_\alpha)\geq\eta\\ \max\{\xi^1_{k\beta}\cdot\xi^1_{\beta\alpha}\}, & if\ S(d_k,d_\alpha)<\eta, S(d_k,d_\beta)\geq\eta, S(d_\beta,d_\alpha)\geq\eta\\ 0, & otherwise\end{cases} \tag{21}$$

Let $MR(l^1_1)=\min\{\max\{l^1_{12}\},\max\{l^1_{13}\},\cdots,\max\{l^1_{1q_1}\}\}$ be the minimum consistency coefficient of decision maker d_1 in Q_1; similarly, we can obtain the minimum consistency coefficient of other decision makers in Q_1, denoted as $\{MR(l^1_2),MR(l^1_3),\cdots,MR(l^1_p)\}$. Then, let $CMR(l^1_\beta)=\max\{MR(l^1_1),MR(l^1_2),\cdots,MR(l^1_p)\}$ be the central consistency coefficient of Q_1, and l^1_β is the center of Q_1, $1\leq\beta\leq p$; similarly, we can obtain the central consistency coefficients and the centers of other groups.

Based on the central consistency coefficient, we can calculate the concentration degree of Q_1 as:

$$CD(Q_1)=\frac{CMR(l^1_\beta)}{\sum_{s=1}^{L}CMR\{l^s_\nu\}},\ 1\leq s\leq L \tag{22}$$

where l^s_ν is the center of Q_s, $l_\nu\in Q_s$; $MR\{l^s_\nu\}$ is the central consistency coefficient of Q_s.

By taking the group size into consideration, the scale proportion of Q_1 can be calculated by:

$$SP(Q_1)=\frac{q_1}{\sum_{s=1}^{L}q_s},1\leq s\leq L \tag{23}$$

Then, we can obtain the weight of Q_1 based on *Eq.*(22) and *Eq.*(23):

$$w_1=\theta\cdot\frac{CMR(l^1_\beta)}{\sum_{s=1}^{L}CMR\{l^s_\nu\}}+(1-\theta)\cdot\frac{q_1}{\sum_{s=1}^{L}q_s} \tag{24}$$

Similarly, the weights of other groups are denoted as $\{w_2,w_3,\cdots,w_L\}$.

Besides, the weight of the decision maker d_1 can be calculated by the following equation:

$$\omega_1=\frac{\sum_{\alpha\in Q_1}l^1_{1\alpha}}{\sum_{k\in Q_1}\sum_{\alpha\in Q_1}l^1_{k\alpha}} \tag{25}$$

The weights of other decision makers can also be determined by *Eq.*(25) and are denoted as $\{\omega_2,\cdots,\omega_p\}$.

B. A New Ranking Method Based on The Centroids of The IT2FSs

Once the weights of clusters and decision makers are determined, the interval type-2 fuzzy weighted average (IT2FWA) operator [57] is employed to obtained the overall preference information for each alternative. Due to the overall preference information is characterized by IT2FSs. Thus, an appropriate ranking method is needed to compare them. Considering the superiority of the centroid in type-reduction, this paper proposes a centroid-based ranking method to compare IT2FSs. Assume that there are two IT2FSs $\tilde{A}$ and $\tilde{B}$, we can obtain the centroids of $\tilde{A}$ and $\tilde{B}$ by the Nie-Tan operator [55]. Then, the ranking order of $\tilde{A}$ and $\tilde{B}$ can be defined as follows:

(1) If $C(\tilde{A})>C(\tilde{B})$, then $\tilde{A}$ is superior to $\tilde{B}$, denoted by $\tilde{A}\succ\tilde{B}$.

(2) If $C(\tilde{A})=C(\tilde{B})$, then $\tilde{A}$ is indifferent to $\tilde{B}$, denoted by $\tilde{A}\approx\tilde{B}$.

(3) If $C(\tilde{A}) < C(\tilde{B})$, then $\tilde{A}$ is inferior to $\tilde{B}$, denoted by $\tilde{A} \prec \tilde{B}$.

Remark 5 The centroid-based ranking method ranks the IT2FSs by the value and the membership degree of each point in IT2FSs, which can avoid information loss and bring a more accurate and reliable result. Besides, Nie-Tan operator has a simple closed-form representation and it is an accurate type-reduction method for IT2FSs. The results of Nie-Tan operator are more accurate than that of the KM method when UMF and LMF are both asymmetric [55].

A flowchart of the centroid-based ranking method for the IT2-FLSGDM problems is shown in Fig.3. And, the detailed procedures are summarized in Algorithm 1 (presented in the Appendix).

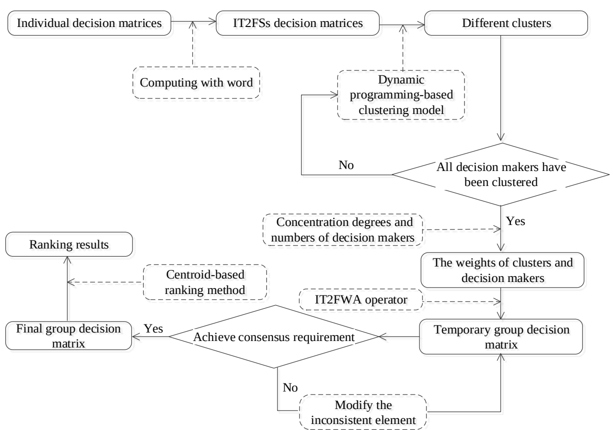

Fig.3. The process for the IT2-FLSGDM problems

Remark 6. The consensus threshold γ is the minimum requirement for the overall consensus degree. If the overall consensus degree fails to meet the threshold γ, there are some obvious differences between the clusters and the overall group. To achieve a higher consensus level, we need to modify the element which does not satisfy the consensus threshold γ.

V. Numerical Example and Comparison With Other Approaches

In this section, the proposed method is implemented on two illustrative experiments from different perspectives. In Section V-A, an illustrative experiment is carried out to intuitively illustrate the features of clustering and to check the effectiveness of the dynamic programming algorithm-based clustering model. In Section V-B, we demonstrate the feasibility of the full method by an illustrative example. Comparisons and discussions are also conducted to demonstrate the superiority of the proposed method. For reproducibility purposes, the pseudocode of the proposed method is provided in Appendix. The detail experiments are elaborated in the following sections.

A. An Illustrative Experiment with Clustering Model

Energy is one of the most important materials for human survival, and is the driving force for their social and economic development. During the last few years, China emerged growing energy crisis in terms of the rapid expansion of civilization. As an effective tool to alleviate energy crisis, the renewable energy sources (RESs) have attracted increasing attraction. Selecting appropriate RESs usually involves various factors and stakeholders with diverse expertise.

To alleviate the energy crisis, a city in China is committed to developing renewable energy for power generation. After some preliminary discussions, four RESs (hydro power, solar power, wind power, biomass power) are selected for further evaluation. Each of them is subjected to various attributes, which are briefly introduced in Table II (presented in the Appendix). Without loss of generality, we deem that these attributes share equal importance. To make a scientific decision, the government invite 20 decision makers (denoted as $d_1, d_2, \ldots, d_{20}$) to evaluate the four RESs. These decision makers are form different fields, including professors of renewable energy investments, experts of project origination, local resident representatives, experts of environmental protection, and so on.

For the sake of convenience, the four RESs are denoted as Z_1, Z_2, Z_3 and Z_4, respectively. The 20 decision makers express their opinions by numbers or linguistic terms, and the linguistic terms are denoted as {*ED*=extremely dissatisfied, *VD*=very dissatisfied, *D*= dissatisfied, *M*= medium, *S*= satisfied, *VS*= very satisfied, *ES*= extremely satisfied}. Due to the space limitations, only the preference information of d_1 is provided in Table III (presented in the Appendix).

To improve the efficiency and accuracy of decision making, it is essential to cluster the 20 decision makers into different groups. The relevance between the 20 decision makers are summarized in Table IV (presented in the Appendix). Additionally, some related parameters are set as follows: (1) the parameter δ and η are deemed to be equal and $\delta=\eta=0.5$. (2) the tune parameter ζ is set to be 0.5. (3) the consensus threshold γ is set as $\gamma=0.5$.

We implement the proposed clustering model on this experiment and the results are: $Q_1=\{d_1,d_8\}$, $Q_2=\{d_2,d_{13},d_{14},d_{20}\}$, $Q_3=\{d_3,d_6,d_{10},d_{11},d_{12},d_{19}\}$, $Q_4=\{d_4,d_5,d_9,d_{15},d_{16},d_{18}\}$ and $Q_5=\{d_7,d_{17}\}$. The obtained clusters for the 20 decision makers are shown in Fig.4.

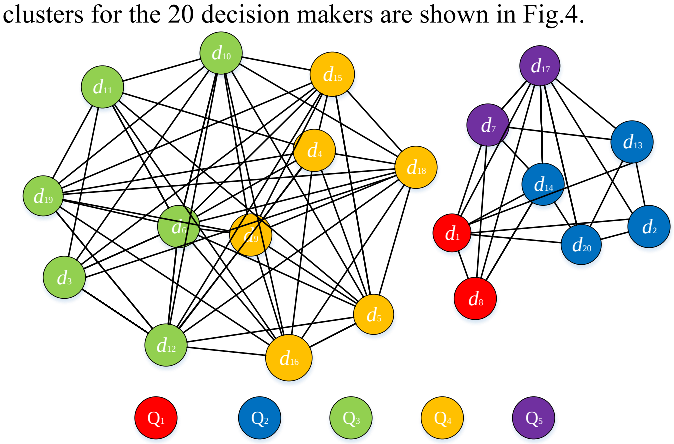

Fig.4. Clusters for the 20 decision makers

From Fig.4, we can observe that the proposed clustering model can effectively divide the 20 decision makers into 5 clusters from the overall perspective. Take the cluster Q_4 as an example, the maximum relevance path is

$d_{15} \to d_{18} \to d_5 \to d_9 \to d_{16} \to d_4$ and the overall relevance degree is 1. Especially, the decision maker d_{18} has the same relevance degree to d_5 and d_{12}, but the relevance degrees of their follow-up paths are different, and the path with the larger follow-up relevance will be selected. Therefore, our clustering model can cluster the decision makers form the overall perspective instead of just considering the relevance degree between two decision makers.

B. An Illustrative Example with Full Method

In this section, we will implement the full proposed method in a LSGDM problem to show its feasibility. Suppose a city plans to build a new metro station to alleviate the traffic congestion. There are three available locations, which are denoted as Z_1, Z_2, Z_3 ; the main factors affecting the site selection of metro station include: C_1 safety, C_2 economy, C_3 traffic condition. The weights of the three factors are $\lambda_1 = 0.3$, $\lambda_2 = 0.25$, $\lambda_3 = 0.45$, respectively. 20 decision makers are invited to evaluate the available locations with respect to the three factors. To improve the efficiency of the decision making, we need to classify the 20 decision makers into different clusters, and the weight of the *j*-th cluster can be denoted as w_j. Moreover, the weight vector of the 20 decision makers is$\{ \omega_1, \omega_2 \cdots \omega_{20} \}$. The preference information given by the decision makers is linguistic terms. The original evaluation information is summarized in Table V (presented in the Appendix).

To select the best location, we adopt the proposed method by the following steps:

Step1: Encode the linguistic variables into IT2FSs via *Eqs*.(1)-(12), and the corresponding relationships between the linguistic variables and the IT2FSs are summarized in Table VI.

TABLE VI
THE CORRESPONDING RELATIONSHIPS BETWEEN LINGUISTIC VARIABLES AND IT2FSS

Linguistic variables	IT2FSs
ED	[(0.2,1,1.5,2.3,1), (0.8,1.3,1.8,0.65)]
VD	[(0.8,1.8,2.5,3.9,1), (1.8,2.2,2.6,0.4)]
D	[(1,2.5,5,7.8,1), (2.8,3.5,4.2,0.36)]
M	[(3.6,4.5,5.5,6.9,1), (4.8,5,5.2,0.28)]
S	[(3.4,5.5,7.6,9.6,1), (5.8,6.5,7.2,0.4)]
VS	[(6.6,7.8,8.9,9.8,1), (7.7,8.2,8.7,0.58)]
ES	[(7.4,9.4,9.9,10,1), (9,9.9,10,1)]

Step 2: Divide all decision makers into different clusters by the dynamic programming algorithm-based clustering model, and the results are shown in Table VII.

TABLE VII
THE CLUSTERING RESULTS

Cluster	q_l	d_k
Q_1	4	d_1, d_2, d_{13}, d_{20}
Q_2	4	d_4, d_5, d_{10}, d_{16}
Q_3	5	d_3, d_6, d_{12}, d_{15}, d_{18}
Q_4	2	d_7, d_8
Q_5	3	d_9, d_{11}, d_{19}
Q_6	2	d_{14}, d_{17}

Step 3: Let $\theta = 0.5$, then we can obtain the weights of clusters and decision makers via *Eqs.*(21)-(25). The obtained results are summarized in Table VIII.

TABLE VIII
THE WEIGHTS OF CLUSTERS AND DECISION MAKERS

Cluster	w_j	ω_l
Q_1	0.2	$\omega_1 = 0.19$, $\omega_2 = 0.25$, $\omega_{13} = 0.27$, $\omega_{20} = 0.28$
Q_2	0.22	$\omega_4 = 0.26$, $\omega_5 = 0.27$, $\omega_{10} = 0.33$, $\omega_{16} = 0.27$
Q_3	0.33	$\omega_3 = 0.15$, $\omega_6 = 0.22$, $\omega_{12} = 0.22$, $\omega_{15} = 0.21$, $\omega_{18} = 0.20$
Q_4	0.06	$\omega_7 = 0.5$, $\omega_8 = 0.5$
Q_5	0.13	$\omega_9 = 0.33$, $\omega_{11} = 0.33$, $\omega_{19} = 0.35$
Q_6	0.06	$\omega_{14} = 0.5$, $\omega_{17} = 0.5$

Step 4: Construct the temporary group decision information for each cluster by *Eq*.(26). Due to the space limitations, only the temporary group information of Q_1 is provided as follows.

$$\tilde{r}_{11}^{1} = \left[(5.28, 7.3, 8.53, 9.67, 1), (7.4, 7.97, 8.37, 0.4) \right]$$

$$\tilde{r}_{12}^{1} = \left[(0.48, 1.37, 1.96, 3.04, 1), (1.26, 1.71, 2.17, 0.4) \right]$$

$$\tilde{r}_{13}^{1} = \left[(5.97, 7.99, 8.99, 9.78, 1), (7.92, 8.61, 8.91, 0.28) \right]$$

$$\tilde{r}_{21}^{1} = \left[(2.03, 3.21, 3.88, 5.04, 1), (3.14, 3.63, 3.98, 0.4) \right]$$

$$\tilde{r}_{22}^{1} = \left[(4.83, 6.64, 7.57, 8.59, 1), (6.71, 7.18, 7.5, 0.4) \right]$$

$$\tilde{r}_{23}^{1} = \left[(0.58, 1.71, 3.15, 4.89, 1), (1.74, 2.34, 2.93, 0.36) \right]$$

$$\tilde{r}_{31}^{1} = \left[(5.97, 7.99, 8.99, 9.78, 1), (7.92, 8.61, 8.91, 0.4) \right]$$

$$\tilde{r}_{32}^{1} = \left[(0.85, 2.22, 4.35, 6.78, 1), (2.43, 3.09, 3.75, 0.28) \right]$$

$$\tilde{r}_{33}^{1} = \left[(4.2, 5.95, 7.35, 8.86, 1), (6.25, 6.64, 7.07, 0.28) \right]$$

Step 5: The temporary group decision information for all clusters can be obtained by *Eq*.(27), and the results are shown as follows:

$$\tilde{r}_{11}^{t} = \left[(2.1, 3.48, 4.68, 6.19, 1), (3.56, 4.07, 4.55, 0.28) \right]$$

$$\tilde{r}_{12}^{t} = \left[(2.68, 4.15, 5.23, 6.52, 1), (4.22, 4.7, 5.15, 0.36) \right]$$

$$\tilde{r}_{13}^{t} = \left[(2.39, 3.84, 5.25, 6.96, 1), (3.94, 4.51, 4.98, 0.28) \right]$$

$$\tilde{r}_{21}^{t} = \left[(2.42, 3.84, 4.91, 6.23, 1), (3.89, 4.38, 4.85, 0.36) \right]$$

$$\tilde{r}_{22}^{t} = \left[(2.9, 4.32, 5.54, 7.11, 1), (4.49, 4.91, 5.32, 0.28) \right]$$

$$\tilde{r}_{23}^{t} = \left[(1.91, 3.21, 4.76, 6.62, 1), (3.37, 3.9, 4.4, 0.28) \right]$$

$$\tilde{r}_{31}^{t} = \left[(2.74, 4.07, 4.94, 6.15, 1), (4.05, 4.57, 4.93, 0.28) \right]$$

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$\tilde{r}^t_{32} = \left[\left(4, 5.75, 7.08, 8.47, 1\right), \left(5.88, 6.42, 6.85, 0.28\right)\right]$

$\tilde{r}^t_{33} = \left[\left(2.07, 3.37, 4.38, 5.79, 1\right), \left(3.43, 3.89, 4.32, 0.28\right)\right]$

Step 6: Let $\gamma = 0.5$, we can obtain the consensus degrees of the six clusters as: $S\left(G^t_1, G^t\right)$=0.11, $S\left(G^t_2, G^t\right)$=0.78, $S\left(G^t_3, G^t\right)$=0.67, $S\left(G^t_4, G^t\right)$=0.67, $S\left(G^t_5, G^t\right)$=0.22, $S\left(G^t_6, G^t\right)$=0.33. Then the overall consensus degree can be calculated by *Eq*.(29), and $S\left(G^t\right)$=0.54. Because of $S\left(G^t\right)$>0.5, the clusters achieve the essential consensus.

Step 7: Calculate the overall evaluation value for each alternative via *Eq*.(30). And, the results are shown as follows:

$R_1 = \left[\left(2.38, 3.81, 5.07, 6.62, 1\right), \left(3.9, 4.43, 4.9, 0.28\right)\right]$;

$R_2 = \left[\left(2.31, 3.67, 5, 6.63, 1\right), \left(3.81, 4.3, 4.77, 0.28\right)\right]$;

$R_3 = \left[\left(2.75, 4.17, 5.23, 6.57, 1\right), \left(4.23, 4.73, 5.14, 0.28\right)\right]$.

Step 8: Based on *Eq*.(13), we can obtain the centroids of the overall evaluation values: $C\left(R_1\right) = 4.52$, $C\left(R_2\right) = 4.42$, $C\left(R_3\right) = 4.73$. Hence, the ranking order of the three locations is $Z_3 \succ Z_1 \succ Z_2$, which means the best alternative is Z_3.

C. Comparison With Other Approaches

To verify the superiority of our method, we compare the proposed method with other existing methods. For a fair comparison, we also use the existing methods to solve the LSGDM problem provided in subsection V-B, and the original evaluation information is consistent with ours. The detailed pseudocodes or algorithm steps can be found in [6, 14, 15]. The comparison results are shown in Table IX.

TABLE IX
COMPARISON RESULTS WITH OTHER METHODS

Methods	Ranking values	Ranking orders
Wang's method [14]	$\hat{s}\left(R_1\right) = 3.651$, $\hat{s}\left(R_2\right) = 3.664$, $\hat{s}\left(R_3\right) = 4.516$	$Z_3 \succ Z_2 \succ Z_1$
Wu's method [15]	$cor\left(Z_1\right) = 0.88$, $cor\left(Z_2\right) = 0$, $cor\left(Z_3\right) = 0.72$	$Z_1 \succ Z_3 \succ Z_2$
He's method [6]	$O\left(Z_1\right) = 4.5$, $O\left(Z_2\right) = 4.54$, $O\left(Z_3\right) = 4.81$	$Z_3 \succ Z_2 \succ Z_1$
The proposed method	$C\left(R_1\right) = 5.16$, $C\left(R_2\right) = 5.14$, $C\left(R_3\right) = 5.36$	$Z_3 \succ Z_1 \succ Z_2$

The different aspects of the four methods are outlined in Table X (presented in the Appendix) to clearly illustrate the differences between our method and other existing methods.

In the following, we conduct a more detailed analysis to illustrate the superiority, validity, and novelty of the proposed method.

(1) Wang *et al*. [14] firstly converted the linguistic variables into the cloud models and proposed a new similarity measure to estimate the similarity between two clouds. The corresponding relationships between the linguistic variables and the cloud models are summarized in Table XI. Then, they developed a new cluster model based on the similarity measure to divide the decision makers into different clusters. Finally, the ranking order of the alternatives can be obtained by the estimated score of the cloud model. For the case concerned building the metro station construction, the ranking order of the alternatives is $Z_3 \succ Z_2 \succ Z_1$.

TABLE XI
THE CORRESPONDING RELATIONSHIPS BETWEEN LINGUISTIC VARIABLES AND CLOUD MODELS

Linguistic variables	**Cloud models**
ED	(0, 3.75, 0.25)
VD	(4.5, 3.208, 0.214)
D	(7.7, 2.292, 0.153)
M	(10, 1.917, 0.128)
S	(12.3, 2.292, 0.153)
VS	(15.5, 3.208, 0.214)
ES	(20, 3.75, 0.25)

Obviously, there are some differences between Wang's method [14] and our method. The main reasons for these differences are summarized as follows:

- To effectively depict the uncertainty, Wang *et al*. [14] applied the cloud models to handle linguistic variables. The core of converting linguistic variable into cloud model is to determine the parameter θ_i by the linguistic scale function. And, the essence of linguistic scale function is to apply the linguistic symbolic model to deal with the linguistic variable. However, the linguistic symbolic model cannot clearly describe the fuzziness and randomness of qualitative concepts [60, 61].
- In Wang's method [14], the decision makers were clustered based on the similarity between two clouds; however, this may lead to local optimal solution. The reason is that only the similarity between two adjacent decision makers is considered, which may result in high similarity between two decision makers, but the overall similarity is not the highest. Different from Wang's method, our method can consider the relevance among all decision makers, and bring more reasonable clustering results.

(2) By applying Wu's method [15], the linguistic variables are also encoded into IT2FSs. For the reason of fairness, we assume that the correspondence between the linguistic variables and the IT2FSs is the same as ours. In [15], Wu *et al*. clustered the decision makers by the Louvain method and calculated the group preference information by the IT2FWA operator. They ranked the alternatives by the TOPSIS method. And, the obtained ranking order is $Z_1 \succ Z_3 \succ Z_2$. Obviously, the result is different from our method, the main reasons are summarized as follows:

- The clustering results obtained by Wu's method are $Q_1 = \left\{e_1, e_7, e_8, e_{14}, e_{17}\right\}$, $Q_2 = \left\{e_2, e_{13}, e_{20}\right\}$,

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$Q_3 = \{e_3, e_6, e_{10}, e_{11}, e_{12}, e_{15}, e_{16}, e_{19}\}$ and $Q_4 = \{e_4, e_5, e_9, e_{18}\}$, and the corresponding weight vector is {0.154, 0.462, 0.308, 0.077}. Obviously, it is unreasonable that the cluster Q_2 with the least decision makers has the maximum weight. This is because that Wu's method ignores the influence of group size. In comparison, our method can not only consider the concentration degree but the group size of cluster, which is more reasonable and reliable.

- Wu *et al*. [15] employed the Louvain method to cluster the decision makers. The Louvain method can effectively classify the decision makers while the relevance between the decision makers is same. However, the relevance degree between different decision makers is usually different. Thus, Wu's clustering model may be not universal to all situations. Moreover, in [15], the TOPSIS method was applied to rank the alternatives. Generally, the TOPSIS needs to employ the distance model to pre-process IT2FSs, which may lead to information loss and cause inaccuracy in the final results. In comparison, the proposed method can effectively depict the relevance degree among decision makers and produce an accurate and reliable result.

(3) He *et al*. [6] proposed a shadowed set-based decision method to solve the LSGDM problems within the context of linguistic variables. They first encode the linguistic variables by the shadowed sets, and the corresponding relationships between the linguistic variables and the shadowed sets are summarized in Table XII. Then, they developed a shadowed set-based clustering model to cluster the decision makers. After the clustering process, the TODIM method was employed to compare and rank the alternatives. The ranking order of the alternatives obtained by He's method is $Z_3 \succ Z_2 \succ Z_1$, which is different from our method. The main reasons for this difference are summarized as follows:

TABLE XII
THE CORRESPONDING RELATIONSHIPS BETWEEN THE LINGUISTIC VARIABLES AND THE SHADOWED SETS

Linguistic variables	**Shadowed sets**
ED	(0.23, 0.3, 1.01, 1.28)
VD	(1.01, 1.39, 2.32, 2.79)
D	(2.35, 3.32, 3.73, 4.87)
M	(4.02, 4.85, 5.4, 6.37)
S	(5.75, 6.51, 7.2, 8.12)
VS	(7.47, 7.63, 8.61, 8.89)
ES	(8.72, 9.31, 9.59, 9.89)

- Inspired by the limitation of Wu's clustering method [15], He *et al*. developed an improved clustering method, which can cluster the decision makers with different degrees of relevance. However, the clustering method proposed in [6] still clusters the decision makers from the local perspective. Besides, He *et al*. did not consider the consensus adjustment process, which may reduce the reliability of the results. Moreover, in [6], the decision makers in the same cluster are set to share equal importance. However, due to the differences in the expertise and social reputation of the decision makers, the importance of the decision makers usually varies with decision problems, even if they are in the same cluster. Therefore, simply thinking that the decision makers in the same cluster share the equal importance may bring an inaccurate result. In conclusion, compared to He's method, the result obtained by our method is more reasonable and reliable.

Consensus is an important issue in LSGDM, which means a unanimous agreement among all participants concerning all alternatives. It is often unrealistic to reach a complete agreement for all decision makers owing to their different attitudes, motivations, and perceptions. In practice, when the consensus degree meets a certain threshold, it is acceptable. Generally, the higher the consensus level, the more reliable the decision result. Therefore, we take the consensus degree as a standard to intuitively illustrate the superiority of our method. The results of different methods are summarized in Table XIII.

TABLE XIII
INITIAL CONSENSUS DEGREE OF THE FOUR METHODS

Methods	The number of clusters	Initial consensus degree
Wang's method [14]	4	0.46
Wu's method [15]	4	0.48
He's method [6]	3	0.48
Our method	6	0.54

From Table XIII, we can observe that our method shares the highest consensus level among the four methods. This is because that our method can cluster decision makers form the overall perspective instead of only ensuring a maximum relevance between two adjacent decision makers. So, compared to the other methods, our method can enhance the consensus level and produce a more reliable result.

D. Discussions

In this subsection, some discussions are introduced to better understand the meaning and setting of some related parameters. Specifically, we will implement the proposed method at different parameter levels to explain the effects of different parameters on the clusters and decision results.

(1) This paper deems that if the degree of similarity is larger than η=0.5, there is relevance among decision makers. However, for different problems, the threshold η is usually different. And, different thresholds may bring different number of clusters, which will affect the efficiency and accuracy of the decision making. Moreover, for the same threshold, the different values of tune parameter ζ will produce different relevance degrees among the decision makers, and result in different clustering results. To explore the relationships between the number of clusters and the values of parameters η and ζ, we implement the proposed method at different parameter levels ($\eta = 0, 0.1, \cdots, 1$; $\zeta = 0, 0.1, \cdots, 1$). The results are shown in Fig.5 (presented in the Appendix).

For Fig.5, we can observe that both η and ζ have an effect on the clustering results. Especially, when $\zeta = 0$ or 0.1, the number of clusters is always 1, regardless of the value of η . The reason is that the relevance degrees among the decision makers are the same and equal to 1 at $\zeta = 0$ or 0.1 . Moreover, for the same ζ , the number of clusters will increase with the increase of η . This is because a larger η means more independent decision makers and fewer relevance paths among them. Similarly, for the same η , the larger the ζ is, the more clusters will be. In conclusion, the clustering results are manipulated by parameters η and ζ . In practice, different clustering results may lead to different iterations and running time. Thus, setting an appropriate threshold can improve the efficiency of decision making.

(2) Form the previous analysis, we know that different η and ζ may bring different clustering results, and the different clusters may lead to the different decision results. To reflect the influences of different parameters on the decision results, we assess the obtained decision results at different parameter levels ($\eta = 0, 0.1, \cdots, 1$; $\zeta = 0, 0.1, \cdots, 1$). In Fig.6 to 8 (presented in the Appendix), we respectively show the changes of alternatives Z_1 , Z_2 and Z_3 at different parameter levels.

From Fig.6 to 8, we can observe that the variations of parameters η and ζ will affect the decision results. To be more specific, no matter which of η and ζ changes, it may lead to different clustering results, and produce different overall values of alternatives. Moreover, their influences on the overall values are irregular, which means that the increasing of η or ζ may not lead to a larger or a smaller overall value of alternative and vice versa. This is because the clustering result has impact on the overall value, but there is no linear relationship between them. Therefore, for different decision problems, we should set different thresholds according to the real situations.

(3) The threshold γ is the minimum requirement for consensus. If the decision makers fail to meet the consensus requirement, they need to adjust their preference information, and may bring different decision results. To explore the influence of the consensus threshold γ on the final ranking order, we assess the obtained ranking order of the alternatives with different values of γ . The value of γ varied from 0 to 1 in steps of 0.1. And, the corresponding results are summarized in Table XIV and Fig.9 (presented in the Appendix).

From Table XIV and Fig.9, we can observe that the overall values and ranking orders of alternatives will change when $\gamma > 0.5$. This means that if $\gamma > 0.5$, the decision makers need to adjust their preference information to meet the consensus requirement. Moreover, the larger the threshold γ is, the more modifications and iterations will be. Meanwhile, the increase of the iterations will inevitably decline the efficiency of decision making. On the other hand, if the threshold is too low, it will result in conflict and reduce the quality of the decision result. Therefore, to improve the efficiency and quality of decision making, we should set different thresholds for different problems.

(4) The main contribution of the tune parameter θ is to increase the flexibility of the clustering weights. To reflect the influence of θ on the produced results, we conduct an analysis on parameter θ . The value of θ varied from 0.1 to 1 in a step of 0.1. The obtained decision results under different values of θ are summarized in Table XV and Fig.10 (presented in the Appendix).

The results show that the overall ranking values of the alternatives will change with different values of parameter θ , but the ranking order of the alternatives is constant. The reason is that although the tune parameter will affect the clustering weights, it will not change the clustering results and the preference information of the decision makers. This means that our method can maintain the stability of ranking under a certain parameter perturbation when the clustering results and preference information are unchanged. According to the discussions, this paper demonstrates that the proposed approach yields reasonable results and presents suitable outcomes to support the decision makers.

VI. Conclusions

Since large-scale decision makers are involved in the decision making process, we developed a dynamic programming algorithm-based clustering model to manage them. The proposed clustering model significantly differs from the previous clustering models and can cluster the decision makers from the overall perspective. Considering the weight vector of the clusters is vital for the decision making, a new weight determination model integrating the network center and the group size was developed to determine the weights of clusters. The new weight determination model can consider both the size and the relevance of clusters, and produce more reasonable weights. Further, a centroid-based ranking method was introduced to directly rank the IT2FSs, which can avoid information loss and bring more accurate and reliable result. As the real decision situation becomes increasingly complex, it is believed that this method will be widely used in the decision making field. The effectiveness of the proposed method was demonstrated by two illustrative experiments, and comprehensive comparisons with other approaches were also made to illustrate the superiority of our method.

Meanwhile, there are several promising and solid future research directions:

(1) Decision support system (DSS) can not only reduce the computational burden, but also promote the application of our method in other real-life problems. Therefore, developing a DSS might be one future direction to extend our proposed method.

(2) In practice, due to the limited knowledge of human and the complexity of reality, the decision makers may fail to provide complete preference information. Thus, integrating some new technologies to deduce the incomplete information might be another promising future direction to improve our method.

(3) Running time is an important indicator for the LSGDM problems. The different research backgrounds and various knowledges of the decision makers may result in a low-level consensus, and affect the efficiency of the proposed method. Thus, how to efficiently obtain a high-level consensus on the basis of ensuring that the

opinions of all decision makers are reliable is still a worthy topic.

(4) The complex nature of the uncertainty encountered in the real world indicates that the generalized T2FS is needed in the real-world applications [19]. Hence, in the future, the proposed method can be extended to the case of T2FS environment.

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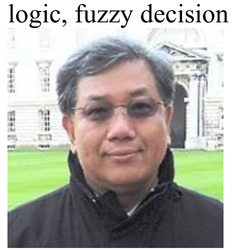

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