IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 8, 2019, accepted January 16, 2019, date of publication January 31, 2019, date of current version February 20, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2896286

An Improved Deng Entropy and Its Application in Pattern Recognition

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This work was supported in part by the China Scholarship Council, and in part by the startup fund from Northwest A&F University under Grant Z109021812.

ABSTRACT How to manage the uncertainty of the basic probability assignment accurately and efficiently is of significance and also an open issue. Plenty of functions have been established to cover the issue, especially Deng entropy recently. Deng entropy can deal with the more complex situation of the focal elements (propositions). However, Deng entropy has some limitations when the propositions are of the intersection. In this paper, a modified function is proposed by considering the scale of the frame of discernment and the influence of the intersection between statements on uncertainty. The proposed belief entropy provides a promising way to measure the uncertain information. Some numerical examples and an application in pattern recognition are used to show the efficiency and accuracy of the proposed belief entropy.

INDEX TERMS Entropy, Deng entropy, Shannnon entropy, Dempster-Shafer evidence theory, pattern recognition.

I. INTRODUCTION

The theory of evidence, which is also referred to as evidence theory or Dempster-Shafer theory (DST) has a wide range of applications in information fusion and decision-making. It has been used in uncertainty seasoning and is capable of taking all kinds of the information and data of the subjective world as the condition, then analyze and summarize the basic probability of the system, and thus make an accurate decision [1], [2].

Uncertainty measure (UM) can be represented as the quality of the information, which has been applied in feature selection [3], probability density estimation [4], machine learning [5], complex network [6], [7], quantum entanglement [8], complexity evaluation [9]. How to manage the uncertainty of the basic probability assignment (BPA) accurately and efficiently is of significance and also an open issue in DST. Plenty of functions have been developed for uncertainty modeling, they also took use of extended theories and hybrid methods to present for uncertainty measures. Some use the number of focal element, such as Dubois &Prade's weighted Hartley entropy [10], Klir&Ramer's discord measure [11], Klir&Parviz's strife measure [12], George & Pal's conflict measure [13]. Some use the belief function and plausibility function, such as Hohle's confusion measure [14], Yager's dissonance measure [15], distance-based measure [16]–[18], interval-value based measure [19], [20]. Especially, Deng entropy [21] which is proposed by Prof. Deng recently is an efficient function to manage the uncertain information and it is an extension of Shannon entropy. Deng entropy considered the more complex situation of the focal element (proposition). After investigation of Deng entropy carefully, we found that Deng entropy didn't consider the influence of the intersection between statements on uncertainty.

In this paper, we consider and analyze the influence of the intersection between statements on uncertainty in BPA in the frame of Deng entropy, so that we find out a much more efficient way to solve the problems. We propose the new belief entropy by combining the fixed frame of discernment and Deng entropy's idea, which can make up for the previous shortcomings and could be wider applications in the future. Thus, it is very feasible to define an uncertainty event. Some numerical examples and an application in pattern recognition are used to illustrate the effectiveness of the proposed entropy.

The paper is organized as follows. The preliminaries briefly introduce some concepts about Dempster-Shafer evidence theory, Shannon entropy, Deng theory and some

The associate editor coordinating the review of this manuscript and approving it for publication was Alberto Cano.

uncertainty measures in Dempster-Shafer framework in Section 2. In Section 3, the new belief entropy is proposed and some examples will be given to test the capacity. In Section 4, an application is given with the proposed Entropy. The conclusions and ongoing or future work are given in the Section 6.

II. PRELIMINARIES

In this section, some preliminaries are briefly introduced, including Dempster-Shafer evidence theory, Shannon entropy, Deng entropy and several typical uncertainty measures based on Dempster-Shafer framework.

A. DEMPSTER-SHAFER THEORY OF EVIDENCE

The Dempster-Shafer theory (DST) of evidence, which was first proposed by Dempster [22] and then developed by Shafer [23], is regarded as a generalization of the Bayesian theory of probability. Due to its ability to handle uncertainty or imprecision embedded in the evidence, the DST has increasingly been applied in fault diagnosis [24]–[26], risk analysis [27]–[29], reliability analysis [30]–[32], conflict information fusion [33]–[35], dependent information fusion [36]–[39], medical decision making [40]–[44], temporal information fusion [45], multi-modal information integration [46], complex electromechanical systems [47]–[49], etc.

The introduction of DST are briefly summarized as following: (1)

1) "Frame of discernment" [23]:
Let $\Theta = \{H_1, H_2, \ldots, H_N\}$ be a finite set of n elements, and $P(\Theta)$ denote the power set composed of 2^N elements of Θ.

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \ldots, \Theta\} \quad (1)$$

2) "Basic probability assignment (BPA)" [23]:
The BPA function is defined as a mapping of the power set $P(\Theta)$ to a number between 0 and 1.

$$m : P(\Theta) \rightarrow [0, 1] \quad (2)$$

and which satisfies the following conditions:

$$m(\emptyset) = 0, \quad \sum_{A \subseteq P(\Theta)} m(A) = 1 \quad (3)$$

The mass $m(A)$ represents how strongly the evidence supports A.

3) "Belief and plausibility functions" [23]:
The belief function Bel is defined as

$$Bel : P(\Theta) \rightarrow [0, 1] \quad and\ Bel(A) = \sum_{B \subseteq A} m(B) \quad (4)$$

and the plausibility function Pl is defined as

$$\begin{aligned} Pl &: P(\Theta) \rightarrow [0, 1] \\ Pl(A) &= 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \end{aligned} \quad (5)$$

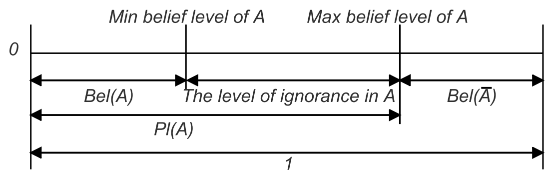

FIGURE 1. The relation between Bel and Pl.

Bel(A) and *Pl(A)* are the lower limit and the upper limit, respectively, of the belief level of hypothesis A which is illustrated in Figure 1. Both imprecision and uncertainty can be represented by them.

4) "Dempster's combination rule":
Two bodies of evidence X and Y regarding Θ can be used to calculate the belief level for some new hypothesis C as follows:
The measure of conflict K is given as

$$K = \sum_{X \cap Y = \emptyset, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y) \quad (6)$$

and the mass function after combination is

$$\begin{aligned} m(C) &= m_i(X) \oplus m_{i'}(Y) \\ &= \begin{cases} 0, \quad if\ X \cap Y = \emptyset, \\ \dfrac{\sum_{X \cap Y = C, \forall X, Y \subseteq \Theta} m_i(X) \times m_{i'}(Y)}{1 - K}, \\ \quad if\ X \cap Y \neq \emptyset. \end{cases} \end{aligned} \quad (7)$$

B. DISCOUNTING OF BPA

A discounting coefficient $\alpha \in [0, 1]$ represents the weight (reliability) of the evidence, then the discounted evidence m^α can be defined as follows [23]:

$$m^\alpha(\Theta) = \alpha m(\Theta) + (1 - \alpha) \quad (8)$$

$$m^\alpha(A) = \alpha m(A) \quad \forall A \subset \Theta \quad and\ A \neq \Theta \quad (9)$$

C. PIGNISTIC PROBABILITY TRANSFORMATION (PPT)

Beliefs manifest themselves at two levels: the credal level (from credibility) where belief is entertained, and the pignistic level where beliefs are used to make decisions. The term "pignistic" was proposed by Smets [50] and originates from the word pignus, meaning 'bet' in Latin. Pignistic probability is used for decision making and uses Principle of Insufficient Reason to derive from BPA. It represents a point estimate in a belief interval and can be determined as

$$bet(A_i) = \sum_{A_i \subseteq A_k} \frac{m(A_k)}{|A_k|} \quad (10)$$

where A_k is the focal element.

D. SHANNON ENTROPY

Shannon entropy is an uncertain measure of information volume in a system which plays an important roles in information theory [51]. The Shannon entropy is H is denoted by:

$$H = -\sum_{i=1}^{N} p_i \log p_i \tag{11}$$

where N is the number of basic states in a system, and p_i is the probability of state i appears satisfying $\sum_{i=1}^{N} p_i = 1$.

There still some limitations in Shannon entropy for DST, thus, the concept of entropy in the framework of Dempster-Shafer evidence theory is an open issue. Plenty of researchers have extended many measured functions in the framework of it.

E. UNCERTAINTY MEASURES BASED ON DEMPSTER-SHAFER FRAMEWORK

Assume that X is FOD, A and B are focal elements of the mass function, and $|A|$ denotes the cardinality of A. Then, definitions of some uncertain measures in DST framework are briefly introduced as follows.

1) HOHLE'S CONFUSION MEASURE

Hohle's confusion measure is one of earlier confusion measures for D-S theory was due to Hohle [14].

$$C_H(m) = -\sum_{A\subseteq X} m(A) \log_2 Bel(A) \tag{12}$$

2) YAGER'S DISSONANCE MEASURE

Dissonance measure of BPA was defined by Yager [15] as follow:

$$E_Y(m) = -\sum_{A\subseteq X} m(A) \log_2 Pl(A) \tag{13}$$

3) DUBOIS & PRADE'S WEIGHTED HARTLEY ENTROPY

Dubois & Prade's weighted Hartley entropy is shown as follow [10]:

$$E_{DP}(m) = m(A) \log_2 |A| \tag{14}$$

4) KLIR & RAMER'S DISCORD MEASURE

Another discord measure of BPA was defined by Klir and Ramer [11], as follow:

$$D_{KR}(m) = -\sum_{A\subseteq X} m(A) \log_2 \sum_{B\subseteq X} m(B) \frac{|A\cap B|}{|A|} \tag{15}$$

5) KLIR & PARVIZ'S STRIFE MEASURE

Klir & Parviz's strife measure is denoted as follow [12]:

$$S_{KP}(m) = -\sum_{A\subseteq X} m(A) \log_2 \sum_{B\subseteq X} m(B) \frac{|A\cap B|}{|B|} \tag{16}$$

6) GEORGE & PAL'S CONFLICT MEASURE

The total conflict measure prospered by George & Pal, denoted as H_{GP}, is defined as follow [13]:

$$H_{GP}(m) = \sum_{A\subseteq X} m(A) \sum_{B\subseteq X} m(B) \left(1 - \frac{|A\cap B|}{|A\cup B|}\right) \tag{17}$$

F. DENG ENTROPY

Deng entropy is a generalization of Shannon entropy in Dempster-Shafer framework [21].

$$E_d(m) = -\sum_{A\subseteq X} m(A) \log_2 \frac{m(A)}{2^{|A|}-1} \tag{18}$$

where A is a proposition in mass function m,$|A|$ denotes the cardinality of proposition A, and X is the FOD.As shown in the above definition, Deng entropy, specially, if the belief is only assigned to single elements, Deng entropy can be degenerated to the Shannon entropy.

$$E_d = -\sum_{i} m(\theta_i) \log \frac{m(\theta_i)}{2^{|\theta_i|}-1} = -\sum_{i} m(\theta_i) \log m(\theta_i)$$

Uncertainty plays a significant role in some fields since it is the foundation and prerequisite to quantitatively study the questions. There is no doubt that Deng entropy provides a promising way to measure uncertain degree and to handle more uncertain information. For more details about Deng entropy, please refer to [21]. Deng entropy has obtained lots of concerns from the theory perspective recently [52]–[54], which has been applied in the ordered propositions fusion [55], multi-sensor data fusion [56]. Related work of Deng entropy is also investigated in evidential reasoning [57], pedestrian detection [58], recognizing fatigue driving [59], distributed object recognition [60], combination rule of D-S theory [61].

Based on the frame of Deng entropy, other two uncertainty measure are presented as follows.

1) Pan *et al.*s' ENTROPY

In DST, the probability interval [Bel(A), Pl(A)] can be obtained more information based on the basic probability assigned to each focal elements. Here formula use the probability interval to extend method of measuring uncertain as follow [62]:

$$H_{Bel}(m) = -\sum_{A\subseteq 2^{\theta}} \frac{Bel(A)+Pl(A)}{2} \log \frac{Bel(A)+Pl(A)}{2(2^{|A|}-1)} \tag{19}$$

2) Zhou *et al.*s' ENTROPY

Another belief entropy in the framework of Dempster-Shafer is given by Zhou as follow, which considers the scale of FOD, i.e. $|X|$ [63]:

$$E_{Md}(m) = -\sum_{A\subseteq X} m(A) \log_2 \left(\frac{m(A)}{2^{|A|}-1} e^{\frac{|A|-1}{|X|}}\right) \tag{20}$$

In [52], some new properties of Deng entropy has been discussed. In this paper, we focus some limitations of Deng entropy to deal with the BPA with intersection of focal elements. Then we propose a new function of belief entropy in the frame of Deng entropy. We first give the proposed belief entropy and then apply two examples to illustrate the effectiveness of the proposed belief entropy.

III. PROPOSED BELIEF ENTROPY

Suppose there is a BPA denoted by m, the proposed belief entropy is denoted as follow,

$$E(m) = -\sum_{A\subseteq X} m(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}e^{\sum\limits_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right) \quad (21)$$

where $|A|$ denotes the cardinality of proposition A, and $|A\cap B|$ is the cardinality of the intersection of A and B.

In addition, it is easy to verify that the proposed entropy is degenerated into Deng entropy when the focal elements are not in intersection. What is more, the proposed entropy is degenerated into Shannon entropy when the belief is only assigned to single elements.

In the next section, we use two examples to present the effectiveness of the proposed entropy.

A. COUNTER-EXAMPLE 1

Example 1: Now let us consider this example, there is a target identification, assume that two reliable sensors report the detection results on their own. Firstly, assume the FOD in this example is $X = \{a, b, c, d\}$. The results are presented by BOEs listed as follows:

$$m_1 \colon m_1(\{a, b\}) = 0.4,\ m_1(\{c, d\}) = 0.6$$
$$m_2 \colon m_2(\{a, c\}) = 0.4,\ m_2(\{b, c\}) = 0.6$$

Intuitively, the uncertainty of m_1 is larger than that of m_2 for the focal elements of m_2 are of intersection. Next, we use the Deng entropy [21] and Zhou's method [63] to investigate the uncertainty of m_1 and the uncertainty of m_2.

Solve the question using Deng entropy, the uncertainty measures are calculated as follows:

$$E_d(m_1) = -\sum_{A\subseteq X} m_1(A)\log_2\frac{m_1(A)}{2^{|A|}-1} = 0.4\log_2\frac{0.6}{2^2-1} = 2.5559$$

$$E_d(m_2) = -\sum_{A\subseteq X} m_2(A)\log_2\frac{m_2(A)}{2^{|A|}-1} = 0.4\log_2\frac{0.6}{2^2-1} = 2.5559$$

Solve the question with Zhou's belief entropy, the uncertainty measures are calculated as follows:

$$\begin{aligned} E_{Md}(m_1) &= -\sum_{A\subseteq X} m_1(A)\log_2\left(\frac{m_1(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right) \\ &= -0.4log_2\left(\frac{0.4}{2^2-1}e^{\frac{2-1}{4}}\right) - 0.6log_2\left(\frac{0.6}{2^2-1}e^{\frac{2-1}{4}}\right) \\ &= 2.3155 \end{aligned}$$

$$\begin{aligned} E_{Md}(m_2) &= -\sum_{A\subseteq X} m_2(A)\log_2\left(\frac{m_2(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right) \\ &= -0.4log_2\left(\frac{0.4}{2^2-1}e^{\frac{2-1}{4}}\right) - 0.6log_2\left(\frac{0.6}{2^2-1}e^{\frac{2-1}{4}}\right) \\ &= 2.3155 \end{aligned}$$

As shown to us, the results given by Deng entropy are the same. However, although the BOEs have the similar mass value assignment, but the number of the target isn't same, the FOD of the first one has four candidate targets as a, b, c and d, but the FOD of the second one only has three targets as a, b and c. On the basis of the logical thinking, it is expected that their results should be different, the first one have more uncertainty than the second one because of the larger information volume.

Zhou *et al.*s' method consider this problem, and his belief entropy addresses the issue to a certain degree. However, there is still some room for the improvement, because his measure doesn't pay attention to the framework of discernment given firstly. In the example, recall the belief entropy, the length of the frame is 4 in the first one, but the length of the framework is 3 in the second one, it fails to consider the influence of the intersection between statements on uncertainty if the scale of FOD is given. We hope that a much more precise measure to refine the result should be taken,so based on the fixed framework of discernment $\theta = \{a, b, c, d\}$ and Deng entropy, we get a new belief entropy.

Then, we use the proposed belief entropy to investigate Example 1, the new belief entropy for these two BOEs is calculated using Eq.(21) as follows:

$$\begin{aligned} E(m_1) &= -\sum_{A\subseteq X} m_1(A)\log_2\left(\frac{m_1(A)}{2^{|A|}-1}e^{\sum\limits_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right) \\ &= -0.4\log_2\left(\frac{0.4}{2^2-1}e^{0}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{0}\right) \\ &= 2.5559 \end{aligned}$$

$$\begin{aligned} E(m_2) &= -\sum_{A\subseteq X} m_2(A)\log_2\left(\frac{m_2(A)}{2^{|A|}-1}e^{\sum\limits_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right) \\ &= -0.4\log_2\left(\frac{0.4}{2^2-1}e^{\frac{1}{15}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{1}{15}}\right) \\ &= 2.4597 \end{aligned}$$

From the results, the effectiveness of the proposed entropy can be obtained comparing Deng's entropy [21] and Zhou' entropy [63].

The comparison results of different uncertainty measures are obtained shown in the Table 1.

It can be concluded that most of the entropies have the same shortage that couldn't measure the differences of uncertain degree between two BOEs, their measures cannot give a much more accurate and reliable result except KR [11], KP [12], GP [13]. However, they can't deal with the more complex situation of the focal elements (propositions), i.e. the simultaneity of the propositions. Hence, our focus lands in the frame of Deng entropy. In this paper, the proposed

TABLE 1. Uncertainties using different entropy measures.

Entropy	m_1	m_2
Hohle [14]	0.4422	0.4422
Yager [15]	0.0644	0.0644
DP [10]	1.0000	1.0000
KR [11]	0.9710	0.3990
KP [12]	0.9710	0.3990
GP [13]	0.4800	0.3200
Deng [21]	2.5559	2.5559
Zhou [63]	2.3155	2.3155
Pan [62]	2.3155	2.3155
Proposed	2.5559	2.4597

belief entropy can not only make use of more available information to measure the different uncertain degree effectively, but also consider the influence of the intersection between statements on uncertainty. Comparing the existing work, the proposed belief entropy is more reasonable comparing the previous uncertainty measures.

B. COUNTER-EXAMPLE 2

In order to compare the capacity of the proposed belief entropy, recall the other example in as follows.

Example 2: Think about another target identification, assume that three reliable sensors report the detection results on their own. Comparing with example 1, the number of the element in this one isn't same. The results are presented by BOEs listed as follows:

$$m_1 : m_1(\{a,b\}) = 0.2, m_1(\{c,d\}) = 0.6, m_1(\{e,f\}) = 0.2$$
$$m_2 : m_2(\{a,b\}) = 0.2, m_2(\{b,c\}) = 0.6, m_2(\{c,f\}) = 0.2$$
$$m_3 : m_3(\{a,b\}) = 0.2, m_3(\{b,c\}) = 0.6, m_3(\{e,f\}) = 0.2$$

Intuitively, the uncertainties of m_1, m_2, and m_3 are not the same. In addition, the uncertainty of m_3 should be largest, and the uncertainty of m_2 should be smallest.

Solve the question using Deng entropy, the uncertainty measures are calculated as follows:

$$\begin{aligned}
&E_d(m_1)\\
&= -\sum_{A\subseteq X} m_1(A)\log_2\frac{m_1(A)}{2^{|A|}-1}\\
&= -0.2\log_2\frac{0.2}{2^2-1} - 0.6\log_2\frac{0.6}{2^2-1} - 0.2\log_2\frac{0.2}{2^2-1}\\
&= 2.9559\\
&E_d(m_2)\\
&= -\sum_{A\subseteq X} m_2(A)\log_2\frac{m_2(A)}{2^{|A|}-1}\\
&= -0.2\log_2\frac{0.2}{2^2-1} - 0.6\log_2\frac{0.6}{2^2-1} - 0.2\log_2\frac{0.2}{2^2-1}\\
&= 2.9559\\
&E_d(m_3)\\
&= -\sum_{A\subseteq X} m_3(A)\log_2\frac{m_3(A)}{2^{|A|}-1}\\
&= -0.2\log_2\frac{0.2}{2^2-1} - 0.6\log_2\frac{0.6}{2^2-1} - 0.2\log_2\frac{0.2}{2^2-1}\\
&= 2.9559
\end{aligned}$$

From the results above, Deng entropy cannot make a difference between m_1 and m_2.

Solve the question with Zhou *et al.*s' entropy, the uncertainty measures are calculated as follows:

$$\begin{aligned}
E_{Md}(m_1) &= -\sum_{A\subseteq X} m_1(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{1}{6}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right)\\
&= 2.4750\\
E_{Md}(m_2) &= -\sum_{A\subseteq X} m_2(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{1}{6}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right)\\
&= 2.4750\\
E_{Md}(m_3) &= -\sum_{A\subseteq X} m_3(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}e^{\frac{|A|-1}{|X|}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{1}{6}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{6}}\right)\\
&= 2.4750
\end{aligned}$$

From the results above, Zhou *et al.*s' entropy cannot make a difference between m_1 and m_2 as the same as Deng entropy.

Using the proposed belief entropy, recall Example 2, the new belief entropy for these two BOEs is calculated as follows:

$$\begin{aligned}
E(m_1) &= -\sum_{A\subseteq X} m_1(A)\log_2\left(\frac{m_1(A)}{2^{|A|}-1}e^{\sum\limits_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{0}{63}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{0}{63}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{0}{63}}\right)\\
&= 2.9559\\
E(m_2) &= -\sum_{A\subseteq X} m_2(A)\log_2\left(\frac{m_2(A)}{2^{|A|}-1}e^{\sum\limits_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{63}}\right) - 0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{0}{63}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{63}}\right)\\
&= 2.9193
\end{aligned}$$

$$
\begin{aligned}
E(m_3) &= -\sum_{A\subseteq X} m_3(A)\log_2\left(\frac{m_3(A)}{2^{|A|}-1}e^{\sum_{\substack{B\subseteq X\\ B\neq A}}\frac{|A\cap B|}{2^{|X|}-1}}\right)\\
&= -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{1}{63}}\right)-0.6\log_2\left(\frac{0.6}{2^2-1}e^{\frac{0}{63}}\right)\\
&\quad -0.2\log_2\left(\frac{0.2}{2^2-1}e^{\frac{0}{63}}\right)\\
&= 2.9376
\end{aligned}
$$

From results in the Example 2, we can conclude that our proposed method improve the performance of Deng entropy, and Zhou *et al.* s' method.

TABLE 2. Uncertainties using different entropy measures.

Entropy	m_1	m_2	m_3
Hohle [14]	0.4644	0.4644	0.4644
Yager [15]	0.0644	0.0644	0.0644
DP [10]	1.0000	1.0000	1.0000
KR [11]	1.3710	0.5932	0.9731
KP [12]	1.3710	0.5932	0.9731
GP [13]	0.5600	0.4000	0.4800
Deng [21]	2.9559	2.9559	2.9559
Zhou [63]	2.4750	2.4750	2.4750
Pan [62]	0.5416	0.8959	0.5416
Proposed	2.9559	2.9193	2.9376

The comparison results of different uncertainty measures are given in the Table 2. It is very obvious that the proposed method overcome the shortcomings of the previous work.

IV. APPLICATION

In this section, an application in pattern recognition using Iris dataset [64] is investigated using the proposed entropy. The role of the proposed entropy is used to measure the quality of the evidence information, we assume that the larger entropy, the lower quality of the evidence information, the smaller entropy, the higher quality of the evidence information. Then the higher quality of the evidence information, we give them the lager weights in the process of combination, the lower quality of the evidence information, we give them the smaller weights in the process of combination.

In order to realize patter recognition, the method of generating BPA should firstly be considered. Lots of methods have been discussed to deal with this issue [65]. In this paper, we applied the simplest model (interval model to obtain BPA) by Kang *et al.* [66] to discuss the role of the proposed entropy.

A. PATTERN RECOGNITION CONSIDERING THE PROPOSED BELIEF ENTROPY

Firstly, we review the similarity of the interval numbers of generating BPA [66].

1) SIMILARITY OF INTERVAL NUMBERS

$A=[a_1,a_2]$ and $B=[b_1,b_2]$ are two interval numbers then their similarity $S(A,B)$ is defined as follows:

$$S(A,B)=\frac{1}{1+\alpha D(A,B)} \tag{22}$$

Of which $\alpha > 0$ is coefficient of support, $D(A,B)$ is the distance of interval number A and interval number B, the function of D refers to [67].

Secondly, we review the flow of obtaining BPA by interval model from [66].

2) METHOD OF GENERATING BPA

The main thought of generating BPA using interval number is conclude: First, use the collective samples contribute model of interval number. Then, obtain the distance of testing sample and model of interval number. Finally, get the reciprocal of this distance to generate degree of similarity which can be normalized in order to get BPA. The steps of generating BPA are listed as follows: (1)

1) Construct the model of interval number from the max-min value of collective samples.
2) Calculate the distance between unidentified sample property value and interval number.
3) Calculate the degree of similarity between unidentified sample property value and interval number using Eq. (22).
4) Normalize the similarity and generate the BPA.

After obtaining the BPA, we using the proposed belief to modify the BPA.

B. IMPROVED BPA USING THE PROPOSED ENTROPY

In this part, a discounting BPA is given based on results of BPA calculated by Kang's method, the process is listed as follows: (1)

1) Calculate the proposed belief entropy value of each unidentified sample property value using Eq. (21).
2) Use the Proposed value as significance of each unidentified sample property value in order to calculate the weight of each property by using the function $\omega_i(x)=\frac{e^{-x_i}}{\sum e^{-x_i}}$.
3) Select the maximum of these entropy value as target. Get the new discounted weight by making each Entropy value divided by the maximum value.
4) Allocate the new discounted weight to previous BPA using Eq.(8) and Eq. (9).

C. SIMULATION EXPERIMENT

The overall idea of the experimental design is: we use the Iris dataset as a verification database in order to introduce the application of the new generation method in classification recognition rate. The experiment is depicted as follows:

1) Select 120 samples from Iris Data Set randomly, of which select 40 samples for each kind of Iris. And then use sample of max-min value to generate model of interval number, as is shown in Table 3.
2) Each kind of Iris is still has 10 samples in remaining 30 samples which is regarded as unknown test sample (Suppose the selected sample data is [6.3, 2.7, 4.9, 1.8, Iris virsicolor]).

TABLE 3. The sample statistical model of interval numbers.

Item	SL	SW	PL	PW
m(Se)	[4.4,5.8]	[2.3,4.4]	[1.0,1.9]	[0.1,0.6]
m(Ve)	[4.9,7.0]	[2.0,3.4]	[3.0,5.1]	[1.0,1.7]
m(Vi)	[4.9,7.9]	[2.2,3.8]	[4.5,6.9]	[1.4,2.5]
m(Se,Ve)	[4.9,5.8]	[2.3,3.4]	[1.0,1.0]	[1.0,1.0]
m(Se,Vi)	[4.9,5.8]	[2.3,3.8]	[1.0,1.0]	[1.0,1.0]
m(Ve,Vi)	[4.9,7.0]	[2.2,3.4]	[4.5,5.1]	[1.4,1.7]
m(Se,Ve,Vi)	[4.9,5.8]	[2.3,3.4]	[1.0,1.0]	[1.0,1.0]

TABLE 4. BPAs based on Kang's method and final fusion result.

Item	SL	SW	PL	PW	Combined BPA
m(Se)	0.1071	0.0863	0.0625	0.1003	0.0053
m(Ve)	0.1745	0.1556	0.1837	0.2398	0.4852
m(Vi)	0.1465	0.1252	0.1817	0.3016	0.4463
m(Se,Ve)	0.1325	0.1705	0.0000	0.0000	0.0000
m(Se,Vi)	0.1325	0.1242	0.0000	0.0000	0.0000
m(Ve,Vi)	0.1745	0.1677	0.5715	0.3578	0.0632
m(Se,Ve,Vi)	0.1325	0.1705	0.0000	0.0000	0.0000
Proposed Entropy	2.8038	2.8706	1.5506	1.6133	

TABLE 5. The modified BPAs based on the proposed belief entropy and final fusion result.

Item	SL	SW	PL	PW	Combined BPA
m(Se)	0.0306	0.0231	0.0625	0.0942	0.0115
m(Ve)	0.0498	0.0416	0.1837	0.2252	0.3606
m(Vi)	0.0418	0.0334	0.1817	0.2833	0.3940
m(Se,Ve)	0.0378	0.0456	0.0002	0.0002	0.0000
m(Se,Vi)	0.0378	0.0332	0.0002	0.0002	0.0000
m(Ve,Vi)	0.0498	0.0448	0.5715	0.3361	0.2339
m(Se,Ve,Vi)	0.7523	0.7784	0.0002	0.0610	0.0000

3) Get the value of BPA by calculating the degree of similarity and proposed entropy. Then the each group of BPA is shown in Table 4.
4) Because there are four properties, we can generate four pieces of evidence shown in Table 4, then we calculate the belief entropy of each BPA using Eq. (21). Then we use the process in Section IV-B to discount the previous BPAs. The discounted BPAs are shown in Table 5
5) Then obtain the fusion value by using the DS rule of composition using Eq. (7).
6) The type of unknown sample is determined by Combined BPA. The maximum probability value of PPT using Eq. (10) for the combined BPA of property is final result.

Form the final result of Table 4 and Table 5, the proposed method can make a right recognition, i.e. the selected sample is 'Iris versicolor', but Kang' s method cannot make the right decision.

In consideration of scale of samples, we tested all 150 samples as the result. In order to understand the effect of this method in recognition of Iris Data Set, we make coefficient of support $\alpha = 5$ as Kang's method [66], under the condition of the sample statistical model of interval numbers in Table 3. After testing statistics of all 150 samples, we get result that global recognition rate is 96.67% and the recognition rate of Setosa, Versicolor, Virginica is 100%, 96%, 94% respectively, but the result of previous method by Kang is that global recognition rate is 95.33% and the recognition rate of Setosa, Versicolor, Virginica is 100%, 98%, 90% respectively. The comparing result is shown in Table 6. It is clearly shown that the recognition rate is improved using discounting method forced by the proposed belief entropy. At the same time, this new method also exert the excellent advantage in other similar area.

TABLE 6. The final fusion result (recognition rate).

Item	Setosa	Versicolor	Virginica	global
Kang's method	100%	96%	90%	95.33%
Proposed method	100%	96%	94%	96.67%

V. CONCLUSION

In this paper, a modified function is proposed by considering the scale of the frame of discernment (FOD) and the influence of the intersection between statements on uncertainty. Some numerical examples and an application in pattern recognition are used to show the efficiency and accuracy of the proposed belief entropy. Results show that the proposed belief entropy overcome the shortcomings of the previous work, the improved recognition rate enhanced effectiveness of the propose belief entropy. Further study of this work will be focused on the application prospect of the proposed measures and it provides a promising way to measure the uncertain degree in decision making, fault diagnosis pattern recognition, risk analysis and so on.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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