A Multi-Objective Model for Bank ATM Networks*

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Abstract: In this paper we present an application of the core solution concepts for multi-objective games to a bank ATM network model. In these games, the worth of a coalition is given by a subset of vectors of the k-dimensional space rather than by a scalar. The paper investigates how an ATM network model based on multi-objective cooperative game theory could be used as an alternative way of setting interchange fees paid by the customer's bank to the one that owns the ATM. © 2004 Wiley Periodicals, Inc. Naval Research Logistics 52: 165–177, 2005.

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1. INTRODUCTION

Nowadays, Automated Teller Machines (ATMs) are the main way that banks dispense cash to their customers. Although each financial institution has its own ATMs, the banks have joined into networks so that their customers have access to more machines and the service can continue even if there is a failure of their individual ATM system.

Most ATM networks use interchange fees to adjust the imbalance in usage of the network by customers of the different banks. Thus bank i will pay a fee of f_{ij} to bank j each time a customer of bank i uses an ATM of bank j. Traditionally, these interchange fees have been established by bilateral agreements but, as was shown by Gow and Thomas [5], n-person cooperative game theory can be applied to investigate the interchange fee structure for ATM networks.

The model for bank ATM networks proposed is based on a multi-objective game involving two criteria: the costs involved in cash withdrawals by the customers of the banks joined in a network, and the quality of the service the network offers. In such a model, one is interested in the overall utility that each coalition can guarantee by itself, and has to be decided which allocation of the total cost is fair, in the sense that the allocation is compatible with the coalition structure. The model also provides a proposal about the number of ATMs that each of the banks should operate.

In the classical analysis of games, when dealing with several criteria, several authors have suggested converting vector-valued games into weighted scalar games (Blackwell [2], Bergstresser and Yu [1], Hwang and Lin [7] and Jörnsten, Lind, and Tind [8]). In Blackwell [2], Bergstresser and Yu [1], and Hwang and Lin [7] weights represent a distribution of probability on the criteria and consequently, the weighted payoff represents the expected payoff. In Jörnsten, Lind, and Tind [8], there is another interpretation of weights as tradeoffs between the criteria.

Recently cooperative games with several criteria have been considered from a multicriteria perspective. An extensive analysis for the class of vector-valued transferable utility (TU) games is found in Hinojosa [6], and Fernández, Hinojosa and Puerto [4] where the classical individual and collective rationality principles are extended using two dif-

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ferent orderings in the payoff space. The first ordering reflects a very compromising attitude in negotiation with coalitions admitting payoffs that are not worse in all the components than any payoffs that they can ensure by themselves. The second is a more restrictive ordering under which the only payoffs accepted by a coalition are those that give more in all the components than all the payoffs that they can guarantee by themselves.

In the most general setting, the multicriteria nature of the problem requires the worth of a coalition to be represented as a set of vectors. These games have been studied in Jörnsten, Lind, and Tind [8] and van der Nouweland, Aarts, and Borm [11]. The most analyzed application in the literature is the multicriteria production game (see Fernández et al. [3] and Nishizaki and Sakawa [9]). We define the class of set-valued games to be multiobjective games.

The objective of this paper is to propose a multi-objective TU game to model an ATM network situation. The analysis of the problem in the multiple criteria framework will produce more realistic solutions in which the different criteria considered are taken into account simultaneously.

The paper is organized as follows. The general theoretical results of multi-objective games and the core definitions are given in Section 2. The ATM network is modelled as a game in Section 3. The data for a specific network with the resultant fair allocations are discussed in Section 4, and the final section draws some conclusions.

2. CORE CONCEPTS FOR MULTI-OBJECTIVE GAMES

In this section we review the core concepts for multi-objective games. These concepts are used in Section 3 for the analysis of the ATM network model.

A multi-objective game is a pair (N, V), where $N = \{1, 2, \dots, n\}$ is the set of players and V is a function which assigns to each coalition $S \subseteq N$ the *characteristic set* of coalition S, V_S. V_S is a nonempty comprehensive and compact subset of $\mathbb{R}^k$, such that $V_\emptyset = \mathbf{0}$. Here a set $A \subset \mathbb{R}^k$ is called comprehensive if $\mathbf{b} \in A$ and $\mathbf{a} \in \mathbb{R}^k$ is such that $0 \le a_j \le b_j$, $\forall\, j = 1, 2, \dots, k$ implies $\mathbf{a} \in A$.

The characteristic function in these games are set-to-set maps instead of the usual set-to-point maps. Vectors in V_S represent the vector-valued utilities that the members of coalition S can guarantee by themselves. Among the points of the characteristic set V_S, we will be interested in those that are nondominated. If we denote by $E(V_S)$ the nondominated vectors of V_S,

$$E(V_S) = \{\mathbf{a} \in V_S, \text{ such that } \nexists\ \mathbf{b} \in V_S, \mathbf{a} \le \mathbf{b}, \mathbf{a} \ne \mathbf{b}\}.$$

It is easy to verify that one-objective games (the case $k = 1$) can be identified with scalar cooperative games, (N, v), where v is a real valued characteristic function. Vector valued games can be seen as multi-objective games with $|V_S| = 1$, $\forall\ S \subseteq N$. Similarly, multi-objective games for which the maxima of the characteristic sets are unique, that is, $|E(V_S)| = 1$, $\forall\ S \subseteq N$, can be treated as vector-valued games.

In general, if a multi-objective game is played and all the players in N decide to cooperate, then the question that arises is how an attainable nondominated commodity bundle $\mathbf{z} \in E(V_N)$ should be divided among the various players. It is worth noting that this is the same situation which appears in scalar cooperative games, where the worth of $v(N) \in \mathbb{R}$ has to be allocated among the players.

The natural extension of the idea of allocation used in scalar games to vector-valued games consists of using a payoff matrix (an element of $\mathbb{R}^{k\times n}$) whose rows are allocations of the criteria. Since the payoffs are vectors, the allocations in these games are matrices with k rows (criteria) and n columns (players):

$$X = \begin{pmatrix} x_1^1 & x_1^2 & \cdots & x_1^n \\ x_2^1 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_k^1 & x_k^2 & \cdots & x_k^n \end{pmatrix}.$$

The ith column, X^i, in matrix X represents the payoffs of ith player for each criteria; therefore, $X^i = (x_1^i, x_2^i, \dots, x_k^i)^t$ are the payoffs for player i. The jth row, X_j, in matrix X is an allocation among the players of the total amount obtained in each criteria; $X_j = (x_j^1, x_j^2, \dots, x_j^n)$ are the payoffs corresponding to criteria j for each player. The sum $X^S = \sum_{i\in S} X^i \in \mathbb{R}^k$ is the overall payoff obtained by coalition S. We will define the set of allocations of the game as the set of all $k \times n$ matrix such that $X^N \in E(V_N)$, and we will denote this set by $I^*(N, V)$.

Among all the allocations of the multi-objective game, we are interested in those allocations which cannot be beaten with respect to any coalitions. In scalar games, to beat an allocation with respect to a coalition means to find another allocation which gives more worth to all the members of that coalition (equivalently, which gives no less worth). Nevertheless, in multi-objective games to get more worth is not equivalent to not getting less worth, and there are two ways of analyzing the situation which correspond to two different solution concepts for these games. In the first one, we do not admit less worth, componentwise, than all the utility vectors that we already can guarantee by ourselves. In the second one, we accept compromise payoffs which are not in the set of vector that we can guarantee by ourself. Therefore, at least two different orderings are possible in the set of vector payoffs of a player or coalition in multi-objective games. One of them is a complete binary relation (although it is not transitive): a vector **a** is not worse than another **b** if **a** gets at least as much worth than **b** in at least one criterion. The second one is a partial order: a vector **a** is preferred to another **b** if **a** gets at least as much worth than **b** in all the criteria.

In the classical scalar theory allocations are acceptable to the players and to the coalitions provided they satisfy the individual and the collective rationality principles. The crucial point in the development of multi-objective games is the extension of these rationales from the scalar ordering to orderings in $\mathbb{R}^k$. This idea permits a new reformulation of the rationality principles considering two different relations, $\geq$ and $\not\leq$.

If we consider the relation $\geq$, allocation X verifies both the individual rationality principle and the collective rationality principle if $X^S \geq V_S$ for any $S \subset N$, where $X^S \geq V_S$ means $X^S \geq \mathbf{v}$, $\forall \mathbf{v} \in V_S$, that is, $X_j^S \geq v_j\ \forall\ j = 1, 2, \ldots, k$, $\forall\ \mathbf{v} \in V_S$. If we consider the relation $\not\leq$, then allocation X verifies both principles if $X^S \not\leq V_S$, which means $X^S \not\leq \mathbf{v}$, $\forall\ \mathbf{v} \in V_S$, that is, $X^S \notin (V_S - \mathbb{R}_+^k)\backslash E(V_S)$.

These rationality principles will lead us to two different core concepts for multi-objective games. In conventional games, an allocation is in the core of the game if it allocates to each player or coalition not less than the worth that the player or coalition can guarantee by himself. Notice that this concept depends on the meaning given to "not less." The weakest ordering, $\not\leq$, represents an attitude of compromise in the negotiation and leads us to the following definition of core, that we call the *nondominance core.*

DEFINITION 1: The *nondominance core* of the multi-objective game is the set of allocations, $X \in I^*(N, V)$, such that $X^S \not\leq V_S\ \forall\ S \subset N$, that is, $X^S \notin (V_S - \mathbb{R}_+^k)\backslash E(V_S)$. We will denote this set by $C(N, V; \not\leq)$.

It may happen that in some situations the preference structure assumed by the agents is stronger, and coalitions will only accept allocations if they get more than the worth given by the characteristic set. This assumption modifies the rationale of the decision process under the game and, therefore, the core concept will be modified accordingly. Proceeding similarly, we introduce now the concept of *preference core.*

DEFINITION 2: The *preference core* of the multi-objective game is the set of allocations, $X \in I^*(N, V)$, such that $X^S \geq V_S\ \forall\ S \subset N$. We will denote this set by $C(N, V; \geq)$.

In order to investigate when the preference core is nonempty, let us consider the maximal of each characteristic set. Because there is an upper boundary to V_S, there exists $\mathbf{v}^M(S)$, called the maximal of V_S, such that $\mathbf{v}^M(S) \geq \mathbf{v}$, $\forall\ \mathbf{v} \in V_S$, and if $\mathbf{w}$ is another k-dimensional vector verifying $\mathbf{w} \geq \mathbf{v}$, $\forall\ \mathbf{v} \in V_S$, then $\mathbf{v}^M(S) \leq \mathbf{w}$.

It is not difficult to see that a necessary and sufficient condition for nonemptiness of the preference core is that all the scalar games (N, v_j), $j = 1, 2, \ldots, k$, defined by $v_j(S) = v_j^M(S)$, $S \subset N$; $v_j(N) = z_j$, for some $\mathbf{z} \in E(V_N)$ are balanced. Recall that a cooperative game, (N, v), is balanced if for every nonnegative vector $(\alpha_S)_{S\subset N}$ satisfying $\sum_{S:I\in S\subset N} \alpha_S = 1$, we have $\sum_{S\subset N} \alpha_S v(S) \leq v(N)$.

It is worth noting that the *preference core* of a game can be empty; thus, in order to find suitable allocations, we may want to relax the requirements. On the other hand, if the *preference core* was too large, we may want to select a smaller set of best allocations. In practice, it may be interesting not only to look for nonnegative differences between X^S and V_S, but to establish certain bounds on these differences in order to obtain more adjusted solutions. This is the idea underlying the next solution concept, that is the extension to the multi-objective case of the notion of ε-core. Let $\mathbf{p} \in \mathbb{R}^k$.

DEFINITION 3: The **p**-*core* for the multi-objective game (N, V), $C(N, V; \mathbf{p}, \geq)$, is the set of allocations $X \in I^*(N, V)$, such that $X(S) \geq V_S - \mathbf{p}$, $\forall\ S \subset N$. We will denote this set by $C(N, V; \mathbf{p}, \geq)$.

Therefore, $C(N, V; \mathbf{p}, \geq) = \{X \in I^*(N, V), X^S \geq \mathbf{v} - \mathbf{p}, \forall\ \mathbf{v} \in V_S, \forall\ S \subset N\}$.

Vector **p** is interpreted as the admissible differences between the quantities allocated to the coalitions with respect to each of the objectives.

Notice that, for $\mathbf{p} \geq \mathbf{0}$, the concept is a relaxation of the notion of preference core, in that it allows these differences to be positive. For $\mathbf{p} \leq \mathbf{0}$ the concept becomes more restrictive than the preference core, requiring that the differences are below zero. The relationships between the generalized **p**-core and the preference core of the multi-objective game are stated below:

1. $C(N, V; \mathbf{0}, \geq) = C(N, V; \geq)$.
2. $C(N, V; \mathbf{p}, \geq) \subseteq C(N, V; \geq) \qquad \forall\ \mathbf{p} \leq \mathbf{0}$.
3. $C(N, V; \geq) \subseteq C(N, V; \mathbf{p} \geq) \qquad \forall\ \mathbf{p} \geq \mathbf{0}$.

With respect to the relationships between the **p**-core and the nondominance core it is easy to prove that in case **p** is nonpositive componentwise, that is, $\mathbf{p} \not\geq \mathbf{0}$, then $C(N, V; \mathbf{p}, \geq) \subseteq C(N, V; \not\leq)$. Moreover, even in case **p** is positive componentwise, if there does not exist any coalition $S \subset N$ such that $\mathbf{v}^M(S) - \mathbf{p} \in (V_S - \mathbb{R}_+^k)\backslash E(V_S)$, the above inclusion relation holds.

Decreasing the admissible levels, **p**, yields to solutions in the $C(N, V; \mathbf{p}, \geq)$, which are fairer, but the set can be empty. The objective is to find feasible allocations that reach as small maximum levels as possible. In order to establish the extension of the concept of least core for the multi-objective game, for each allocation we define the following vector.

DEFINITION 4: The maximum excess vector for allocation $X \in I^*(N, V)$, is the vector $p(X) \in \mathbb{R}^m$ whose components are

$$p_j(X) = \max_{\substack{S \in 2^N \\ \mathbf{v} \in (V_S - \mathbb{R}^k_+)}} (v_j - X_j^S), \qquad j = 1, 2, \dots, k.$$

The maximum excess vector is a vector-valued measure of the worst performance of an allocation in the set of all the coalitions. Indeed, the allocation is valued by the worst value attained with respect to each objective. Notice that the levels $p_j(X)$ can be also calculated as follows:

$$p_j(X) = \max_{S \in 2^N}(v_j^M(S) - X_j^S), \qquad j = 1, 2, \dots, k.$$

where $v_j^M(S)$ is the jth component of the maximal of V_S, $v^M(S)$. In order to formalize the idea of minimizing the worst performance, we consider those allocations that minimize the maximum excess vector, arriving at a natural extension of the concept of least core.

DEFINITION 5: An allocation $X \in I^*(N, V)$ is in the multi-objective least core of the game, $MLC(N, V)$, if $\nexists$ $Y \in I^*(N, V)$, such that $p(Y) \leq p(X)$, $p(Y) \neq p(X)$.

This extension of the least core generalizes the notion presented by Nisisaki and Sakawa [9], which consisted of minimizing the worst performance of an allocation measured by an scalar. In this sense, our definition keeps the multicriteria nature of the problem, allowing to treat the different objectives independently and not needing any intercriteria utility comparisons.

It can be proved that the problem of finding the allocations that minimize the excess vector, reduces to the problem of finding the allocations of each objective, such that the maximum differences with respect to the maximal value of each coalition is minimized. Thus the multi-objective least core can be obtained as the cartesian product of the least core of the k games of the maximals, (N, v_j), $j = 1, 2, \dots, k$, defined by $v_j(S) = v_j^M(S)$, $S \subset N$; $v_j(N) = z_j$, for $\mathbf{z} \in E(V_N)$.

In order to compute the $MLC(N, V)$, k linear programming problems have to be solved. For $j = 1, 2, \dots, k$, problem (LCP_j) is formulated as follows:

$$\begin{aligned} &\min \quad p_j \\ &s.t.: \quad v_j^M(S) - X_j^S \leq p_j \qquad \forall\, S \subset N, X_j^N = z_j, \end{aligned}$$

It follows that an allocation, X, is in $MLC(N, V)$ and $\mathbf{p}$ is the associated maximum excess vector if and only if $\forall$ $j = 1, \dots, k$, (X_j, p_j) is an optimal solution of problem (LCP_j).

3. ATM NETWORK MODEL

Consider an ATM network involving n financial organizations labeled $1, 2, \dots, n$. The aim is to develop a biobjective model with the costs involved in cash withdrawals by the customers of these organizations when they have a common network, and the quality of the service offered by the network measured by the number of ATMs. This is a conflicting situation in which increasing the quality of service offered by supplying more ATMs imply increasing the cost involved and, conversely, decreasing the costs involved in cash withdrawals imply decreasing the quality of service offered.

The notation for the model is as follows:

- S is any subset of $N = \{1, 2, \dots, n\}$,
- d_j is the fixed annual cost of operating one ATM by bank j,
- t_j is the total number of ATMs owned by bank j,
- c_{ij} is the variable cost of a transaction by a customer of bank i using an ATM of bank j,
- n_{ij} is the number of transaction each year by customer of bank i on ATMs owned by bank j,
- k is the average cost of cash withdrawals by means other than ATMs,
- m_j is the number of cash withdrawals by non-ATM means undertaken each year by bank j's customer.

The total cost of the existing service provision, C, is

$$C = \sum_{j \in N} R_j = \sum_{j \in N} \left(d_j t_j + m_j k + \sum_{i \in N} c_{ij} n_{ij} \right),$$

where R_j is the cost born by bank j in looking after its d_j machines, undertaking m_j non-ATM transactions and dealing with n_{ij} transactions from customers of bank i, $i = 1, 2, \dots, n$.

We consider the quality of the service offered to be proportional to the number of ATMs the coalition owns. Then the second objective is "number of ATMs in the coalition." The total utility in this criterion of the existing service, A, is

$$A = \sum_{i \in N} t_i.$$

In order to model this situation as a game, we are going to make assumptions about what would customer do if the network was smaller. If bank j leaves the network, the customers of the banks still in the network that used bank j's ATMs have the options of using the ATMs of the banks still in the network or choosing a non-ATM method of service provision. In this situation, the banks in the remaining network have the possibility of increasing the number of ATMs in order to get as many ATM transactions as possible from those customers. We assume in this case that coalition S could increase their ATMs by up to a fraction $\beta(S) = \sum_{i\in S}\sum_{j\notin S} n_{ij}/\sum_{i\in N}\sum_{j\notin S} n_{ij}$ of the number of ATMs that are now no longer in the coalition. $\beta(S)$ is the fraction of transactions on non-S ATMs which are performed by customers of S. However, such an increase will not always be possible or even desirable and so each coalition will decide a fraction λ, $0 \le \lambda \le 1$, of this quantity. The idea is that these extra ATMs try to reassure customers of S that the quality of the coalition has not deteriorated.

If we denote by $t_\lambda(S)$ the number of ATMs in coalition S when the banks in S decide to cooperate but do not cooperate with the banks outside S

$$t_\lambda(S) = \sum_{i\in S} t_i + \lambda \frac{\sum_{i\in S}\sum_{j\notin S} n_{ij}}{\sum_{i\in N}\sum_{j\notin S} n_{ij}} \sum_{j\notin S} t_j = \sum_{i\in S} t_i + \lambda\beta(S) \sum_{j\notin S} t_j,$$

where λ is the fraction of the total number of ATMs out of coalition S that coalition S is able to supply and that is a decision of the coalition (notice that, for $\lambda = 1$, the coalition is willing to supply enough ATMs to serve the transactions of its customers not now being served by the coalition).

Also if only coalition S forms, what happens to the ATM transactions where members of the banks in S used ATMs of banks that are not now part of S?. We assume a factor α_S will change to being ATM transactions on ATMs of the banks in S, while the rest will be satisfied by other methods. We consider this rate variable and proportional to the total number of transactions in the ATMs of coalition S:

$$\alpha_S = \frac{\sum_{i\in N}\sum_{j\in S} n_{ij}}{\sum_{i\in N}\sum_{j\in N} n_{ij}}.$$

The division of this fraction for bank i's customers between the ATMs of the banks still in the network is proportional to the usage of these ATMs by bank i's customers when there is a full network of all the banks. Thus, if we define:

$n_{ij}(S)$: the number of transaction each year by bank i customers using bank j's ATMs when the network consists of the banks in coalition S, where i and j are members of S,

$m_i(S)$: the number of non-ATM transactions each year by bank i's customers when the network consists of the coalition S, which includes i,

then

$$n_{ij}(S) = n_{ij} + \alpha_S \sum_{k\notin S} n_{ik} \frac{n_{ij}}{\sum_{r\in S} n_{ir}},$$

$$m_i(S) = m_i + (1-\alpha_S) \sum_{k\notin S} n_{ik}.$$

With these definitions, if we denote by $c_{\lambda(S)}$ the total cost of service provision for the customer of the banks in coalition S when the network only involves the banks in S,

$$c_\lambda(S) = \sum_{j\in S}\left(d_j t_j + m_j(S)k + \sum_{i\in S} n_{ij}(S) c_{ij}\right) + \lambda\beta(S)\left(\sum_{j\notin S} t_j\right)\frac{\sum_{j\in S} d_j}{|S|},$$

where we estimate the cost of the extra ATMs supplied by coalition S to be the average cost per ATM in the coalition.

In order to establish the characteristic sets for the coalitions, V_S, we have to think of the set of costs and number of ATMs that the whole coalition can simultaneously attain; therefore,

$$V_S = \left\{\begin{pmatrix} c_\lambda(S) \\ t_\lambda(S)\end{pmatrix} \in \mathbb{R}^2, \qquad 0 \le \lambda \le 1\right\} - \mathbb{R}^2_+.$$

To simplify the exposition, in what follows we will consider costs as negative magnitudes. Thus, V_S can be represented as

$$V_S = \left\{\begin{pmatrix} c \\ t\end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t\end{pmatrix} = \begin{pmatrix} -c_0(S) \\ t_0(S)\end{pmatrix} + \lambda\begin{pmatrix} -\Delta c(S) \\ \Delta t(S)\end{pmatrix}, \qquad 0 \le \lambda \le 1\right\} - \mathbb{R}^2_+,$$

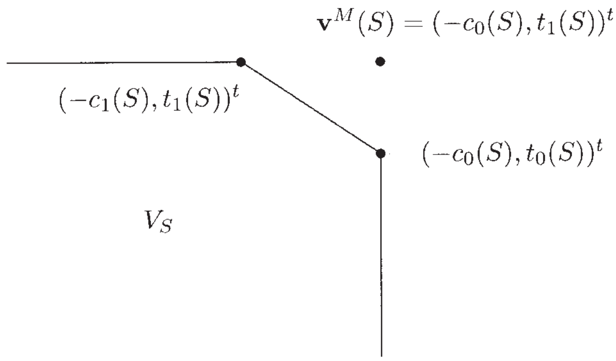


Figure 1. Shape of the characteristic set for coalition S.

where

$$c_0(S) = \sum_{j\in S}\left(d_j t_j + m_j(S)k + \sum_{i\in S} n_{ij}(S)c_{ij}\right),$$

$$t_0(S) = \sum_{i\in S} t_i, \qquad \beta(S)\sum_{j\notin S} t_j,$$

$$\Delta_c(S) = \beta(S)\left(\sum_{j\notin S} t_j\right)\frac{\sum_{j\in S} d_j}{|S|},$$

$$\Delta_t(S) = \beta(S) = \frac{\sum_{i\in S}\sum_{j\notin S} nij}{\sum_{i\in N}\sum_{j\notin S} n_{ij}}.$$

Notice that the minimum cost involved in cash withdrawals by the customers of coalition S is attained when $\lambda = 0$. Nevertheless, the maximum quality of the service offered by the network is obtained when $\lambda = 1$. The characteristic function of the game of the maximals is

$$\mathbf{v}^M(S) = (-c_0(S); t_1(S))^t, \qquad \forall\, S \subset N.$$

Figure 1 represents the shape of the characteristic set and the ideal point for coalition S.

4. EXAMPLE

In the following example we take the data in Gow and Thomas [5] from one of the UK four-bank networks:

- Variable transaction costs: We assume in the model that all banks have the same variable ATM transaction costs, but these are cheaper for their own customers than for those of other banks due to the additional technological cost of switching funds authorization transaction between computer installations of different institutions.

$$c_{ij} = \begin{cases} 0.15 & \text{per transaction if } i = j, \\ 0.25 & \text{per transaction if } i \neq j. \end{cases}$$

- Cost of servicing customers by means other than ATMs: We have taken the cost of a cash withdrawal by branch counter service, and we suppose it is the same for all the banks:

$$k_j = k = 0.75 \quad \text{per transaction} \qquad \forall\, j = 1, 2, 3, 4.$$

- Fixed annual cost of operating one ATM: The estimation of this cost is a fixed quantity for each bank:

$$d_j = d = 8{,}500 \quad \text{per year} \qquad \forall\, j = 1, 2, 3, 4.$$

- Number of cash withdrawals by means other than ATMs: As we only consider, initially, the transactions that use ATMs on the full network, we assume

$$m_j = 0 \qquad \forall\, j = 1, 2, 3, 4.$$

Thus

$$m_j(S) = (1 - \alpha_S)\sum_{k\notin S} n_{jk}.$$

- Total number of ATMs owned by bank j:

$$t_1 = 2{,}672, \qquad t_2 = 2{,}439, \qquad t_3 = 398, \qquad t_4 = 778.$$

- Annual ATM transaction volumes (see Table 1).

With these data, the total cost of the existing service provision, C, is

$$C = \sum_{j=1}^{4} R_j = \sum_{j=1}^{4}\left(d_j t_j + m_j k + \sum_{i=1}^{4} c_{ij} n_{ij}\right) = 134{,}864{,}500,$$

Table 1. Annual ATM transaction volumes.

Transaction Volumes (in millions)		ATM Bank 1	2	3	4	Total
User bank	1	157	30	0.7	3.2	190.9
	2	35	134	0.9	3.5	173.4
	3	2.2	1.6	32	7.5	43.3
	4	6.6	4.7	6.6	49	66.9
	Total	280.8	170.3	40.2	63.2	474.5

Table 2. Rates $\beta(S)$ and $\alpha(S)$ for each coalition.

S	{1}	{2}	{3}	{4}	{1, 2}	{1, 3}	{1, 4}
$\beta(S)$	0.12	0.13	0.03	0.04	0.08	0.18	0.20
$\alpha(S)$	0.42	0.36	0.08	0.13	0.78	0.51	0.56

S	{2, 3}	{2, 4}	{3, 4}	{1, 2, 3}	{1, 2, 4}	{1, 3, 4}	{2, 3, 4}
$\beta(S)$	0.18	0.20	0.04	0.22	0.20	0.21	0.22
$\alpha(S)$	0.44	0.44	0.22	0.87	0.92	0.64	0.58

and the actual costs are

$$R_1 = 56{,}717{,}000, \qquad R_2 = 50{,}061{,}500,$$
$$R_3 = 10{,}388{,}000, \qquad R_4 = 17{,}698{,}000.$$

On the other hand, the number of ATMs in the network is

$$A = \sum_{i=1}^{4} t_i = 6287.$$

The data about annual ATM transactions in the table above allow us to calculate the fraction of the total number of ATMs out of the coalition, $\beta(S)$, that coalition S would need to serve their customers.

The fractions, α_S, of ATM transactions, displaced when banks leave the network and replaced by ATM transactions on other machines, are taken to be proportional to the total number of transactions in the ATMs of coalition S.

Table 2 shows the rates $\beta(S)$ and $\alpha(S)$ for each coalition. With this information, we can define a biobjective game (N, V). The characteristic sets V_S and their corresponding maximals $\mathbf{v}^M(S)$ are as follows:

$$V_{\{1\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -63{,}079{,}472 \\ 2672 \end{pmatrix} + \lambda \begin{pmatrix} -3{,}805{,}854 \\ 448 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1\}) = (-63{,}079{,}472;\ 3120)^t,$$

$$V_{\{2\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -61{,}897{,}007 \\ 2439 \end{pmatrix} + \lambda \begin{pmatrix} -4{,}236{,}342 \\ 498 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{2\}) = (-61{,}897{,}007;\ 2937)^t,$$

$$V_{\{3\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -16{,}083{,}593 \\ 551 \end{pmatrix} + \lambda \begin{pmatrix} -1{,}302{,}414 \\ 153 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{3\}) = (-61{,}897{,}007;\ 2937)^t,$$

$$V_{\{4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -25{,}957{,}509 \\ 778 \end{pmatrix} + \lambda \begin{pmatrix} -2{,}037{,}915 \\ 240 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{4\}) = (-25{,}957{,}509;\ 1018)^t,$$

$$V_{\{1,2\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -105{,}793{,}909 \\ 5111 \end{pmatrix} + \lambda \begin{pmatrix} -802{,}387 \\ 94 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 2\}) = (-105{,}793{,}909;\ 5205)^t,$$

$$V_{\{1,3\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -74{,}041{,}637 \\ 3070 \end{pmatrix} + \lambda \begin{pmatrix} -4{,}953{,}629 \\ 583 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 3\}) = (-74{,}041{,}637;\ 3653)^t,$$

$$V_{\{1,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \middle/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -80{,}263{,}096 \\ 3450 \end{pmatrix} + \lambda \begin{pmatrix} -4{,}811{,}444 \\ 566 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 4\}) = (-80{,}263{,}096;\ 4016)^t,$$

Table 3. Scalar component games.

S	$v_1(S)$	S	$v_1(S)$	S	$v_1(S)$
{1}	−63,079,472	{2}	−61,897,007	{3}	−16,083,593
{4}	−25,957,509	{1, 2}	−105,793,909	{1, 3}	−74,041,637
{1, 4}	−80,263,096	{2, 3}	−72,991,755	{2, 4}	−79,274,145
{3, 4}	−35,066,656	{1, 2, 3}	−116,319,781	{1, 2, 4}	−123,596,098
{1, 3, 4}	−88,504,355	{2, 3, 4}	−87,032,353	N	**−134,864,500**

S	$v_2(S)$	S	$v_2(S)$	S	$v_2(S)$
{1}	2672	{2}	2439	{3}	398
{4}	778	{1, 2}	5111	{1, 3}	3070
{1, 4}	3450	{2, 3}	2837	{2, 4}	3217
{3, 4}	1176	{1, 2, 3}	5509	{1, 2, 4}	5889
{1, 3, 4}	3848	{2, 3, 4}	3615	N	**6287**

$$V_{\{2,3\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -72{,}991{,}755 \\ 2837 \end{pmatrix} + \lambda \begin{pmatrix} -5{,}354{,}034 \\ 630 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{2, 3\}) = (-72{,}991{,}755;\ 3467)^t,$$

$$V_{\{2,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -79{,}274{,}145 \\ 3217 \end{pmatrix} + \lambda \begin{pmatrix} -5{,}316{,}450 \\ 625 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{2, 4\}) = (-79{,}274{,}145;\ 3842)^t,$$

$$V_{\{3,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -35{,}066{,}656 \\ 1176 \end{pmatrix} + \lambda \begin{pmatrix} -1{,}767{,}709 \\ 208 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{3, 4\}) = (-35{,}066{,}656;\ 1384)^t,$$

$$V_{\{1,2,3\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -116{,}319{,}781 \\ 5509 \end{pmatrix} + \lambda \begin{pmatrix} -1{,}485{,}832 \\ 175 \end{pmatrix} \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 2, 3\}) = (-116{,}319{,}781;\ 5{,}684)^t,$$

$$V_{\{1,2,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -123{,}596{,}098 \\ 5889 \end{pmatrix} + \lambda \begin{pmatrix} -690{,}065 \\ 81 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 2, 4\}) = (-123{,}596{,}098;\ 5970)^t,$$

$$V_{\{1,3,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -88{,}504{,}355 \\ 3848 \end{pmatrix} + \lambda \begin{pmatrix} -4{,}418{,}987 \\ 520 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{1, 3, 4\}) = (-88{,}504{,}355;\ 4368)^t,$$

$$V_{\{2,3,4\}} = \left\{ \begin{pmatrix} c \\ t \end{pmatrix} \in \mathbb{R}^2 \Big/ \begin{pmatrix} c \\ t \end{pmatrix} = \begin{pmatrix} -87{,}032{,}353 \\ 3615 \end{pmatrix} + \lambda \begin{pmatrix} -4{,}954{,}112 \\ 583 \end{pmatrix}, \quad 0 \le \lambda \le 1 \right\} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(\{2, 3, 4\}) = (-87{,}032{,}353;\ 6287)^t,$$

$$V_N = \begin{pmatrix} -134{,}864{,}500 \\ 3615 \end{pmatrix} - \mathbb{R}^2_+,$$

$$\mathbf{v}^M(N) = (-134{,}864{,}500;\ 6287)^t.$$

In this application the characteristic set for the grand coalition, N, has a unique nondominated vector $\mathbf{z} = \mathbf{v}^M(N) = (-134{,}864{,}500;\ 6287)^t$ that represents the total cost and the total number of ATMs to be allocated. We are interested in finding fair allocations of vector $\mathbf{z} = (-134{,}864{,}500;\ 6287)^t$.

The preference core of the game is empty, because the scalar component game corresponding to the criteria that measure the quality of the service is not balanced (see the scalar component games in Table 3). Therefore, it is impossible to allocate the costs and the ATMs in a way that all the coalitions have less cost and more ATMs that they can guarantee. Nevertheless, there is a set of allocations for which no coalition can argue against, that is, the nondominance core is not empty. For instance, any allocation of the costs in the core of the corresponding

Table 4. Extreme points of the sets.

$MLC(N, V)$	Allocations of the Cost	Allocations of the ATMs
	$\begin{pmatrix} -53{,}437{,}699 \\ -48{,}111{,}323 \\ -13{,}019{,}580 \\ -20{,}295{,}897 \end{pmatrix}$	$\begin{pmatrix} 2516 \\ 2342 \\ 533 \\ 896 \end{pmatrix}; \begin{pmatrix} 2693 \\ 2519 \\ 356 \\ 719 \end{pmatrix}$
Extreme points		
	$\begin{pmatrix} -49{,}583{,}325 \\ -51{,}965{,}697 \\ -13{,}019{,}580 \\ -20{,}295{,}897 \end{pmatrix}$	$\begin{pmatrix} 2516 \\ 2333 \\ 533 \\ 905 \end{pmatrix}; \begin{pmatrix} 2693 \\ 2507 \\ 356 \\ 731 \end{pmatrix}; \begin{pmatrix} 2519 \\ 2333 \\ 530 \\ 905 \end{pmatrix}$

scalar component game together with any allocation of the ATMs would be an element of the nondominance core.

Among the allocations in the nondominance core, we look for those whose maximum deviations of the cost and the number of ATMs from its ideal values are minimized, i.e., allocations in the $MLC(N, V)$.

To this end, we consider the following two linear programming problems, labeled (P_j), $j = 1, 2$:

$$\left.\begin{array}{lll} \min & p_j & \\ s.t.: & v_j^M(S) - X_j^S \leq p_j & \forall\, S \subset N \\ & X_j^N = \mathbf{z}_j & \end{array}\right\} (P_j),$$

where $X_j = (x_j^1, x_j^2, x_j^3, x_j^4)$.

These problems have five variables and 15 constraints. Solving them with the software ADBASE (Steuer [10]), we obtain the associated maximum excess vector $\mathbf{p} = (-1{,}751{,}178;\ 604)$ and the set of allocations of $\mathbf{z} = (-134{,}864{,}500;\ 6287)$ in the $\mathbf{p}$-core. There is a segment of allocations of the total cost for which the coalition least happy with the allocation, pays 1,751,178 less than the minimum payment that it can guarantee by itself. The allocations of the ATMs form a polyhedron with five extreme where the coalition least happy has 604 less ATMs than the maximum quantity it could obtain. The extreme points of these sets are shown in Table 4. Notice that, as $p_1 < 0$, all these solutions are in the *nondominance core*.

We could consider any point in the $MLC(N, V)$ as a solution of the game, but hereafter in this paper we will concentrate on only one such point which corresponds to the nucleolus of each of the component games.

We can extract the nucleolus N_c and N_t, respectively, from the least core in the components of the game of the maximals, using any of the procedures in the literature (see, e.g., Nishizaki and Sakawa [9]).

The procedure used to obtain the nucleolus consists of replacing with equality the active constraints for any optimal solution in the corresponding (P_j) problem, and doing it repeatedly until the solution is unique. In each step at least one constraint becomes active, what means that at least one coalition has attained its minimum excess level. In Table 5, the maximum excess vectors and the coalitions corresponding to the active constraints in each step are showed when allocating the costs. Similarly when allocating the ATMs, the corresponding nucleolus N_t is obtained in three steps, which are summarized in Table 6. Notice that, in the cost component, the nucleolus is the allocation in the middle of the segment delimited by the two extreme points that define the least core.

Thinking of the nucleolus as a solution, Bank 1 and Bank 2 should decrease their number of ATM and assume smaller

Table 5. Algorithm steps when allocating costs.

	Cost		
Nucleolus	Step 1 (Least Core) $p_1 = 1{,}751{,}178$	Step 2 $p_1 = -3{,}678{,}365$	Step 3 (Nucleolus) $p_1 = -3{,}678$
Coalitions for which the level is attained	$\{3, 4\}$ $\{1, 2, 3\}$ $\{1, 2, 4\}$	$\{3\}$	$N_c = \begin{pmatrix} -51{,}510{,}513 \\ -50{,}038{,}510 \\ -13{,}019{,}580 \\ -20.295{,}897 \end{pmatrix}$

Table 6. Algorithm steps when allocating the ATMs.

	ATMs		
Nucleolus	Step 1 (Least Core) $p_2 = 604$	Step 2 $p_2 = 598$	Step 3 (Nucleolus) $p_2 = 515.5$
Coalitions for which the level is attained	$\{1, 3\}$	$\{1, 4\}$	$N_t = \begin{pmatrix} 2604.5 \\ 2424.5 \\ 444.5 \\ 813.5 \end{pmatrix}$
	$\{2, 4\}$	$\{2, 3\}$	

costs, particularly Bank 1. Bank 3 and Bank 4 should increase their number of ATMs and assume higher costs.

Our previous discussion outlines the way in which a multi-objective game theory approach can arrive at a fair cost to be paid by each bank in the network. This fair cost consists of the real costs incurred by the each bank, R_j, $j = 1, 2, 3, 4$, plus the side payments between the banks. So if the game theory model suggests that bank j, $j = 1, 2, 3, 4$, should really pay x_j, then bank j should pay an amount $x_j - R_j$ in side payments to the other banks to arrive at the solution suggested by the game model. One way to make these side payments is to establish transaction interchange fees, that is, each coalition member levies a fee on each transaction performed on its ATMs by the customers of the other banks in the coalition. In what follows we will discuss how to obtain transactions fees when from different decision criteria.

4.1. Interchange Fees

If f_{ij} is the fee charged by bank j for each transaction on its ATMs by customer of bank i (assuming that $fii = 0$), then a fair division of costs occurs when

$$x_j - R_j = \sum_{i \neq j} n_{ji} f_{ji} - \sum_{i \neq j} n_{ij} f_{ij}, \qquad j = 1, 2, 3, 4,$$

where $(x_1, x_2, x_3, x_4)^t = N_c$.

There are many different sets of interchange fees which give rise to the same division of total costs between the banks they are the solutions of a linear system with n equations (notice that only $n - 1$ are independent because $\sum_j x_j = \sum_j R_j = -z_1$) and $n(n - 1)$ unknowns. It is not possible to find a uniform interchange fee ($f_{ij} = f \; \forall \; i, j = 1, 2, 3, 4$) because with this condition the system is inconsistent. In order to obtain a set of interchange fees as uniform as possible, we propose a solution based on the minimization of the maximum average interchange fee (MinMax average fee) paid for each bank. Those fees are obtained solving the linear programming problem:

$$\min \quad v$$

$$s.t.: \quad \frac{\sum_{\substack{j=1 \\ j \neq i}}^{4} n_{ij} f_{ij}}{\sum_{\substack{j=1 \\ j \neq i}}^{4} n_{ij}} \leq v \qquad \forall \; i = 1, 2, 3, 4,$$

$$x_j - R_j = \sum_{\substack{i \\ i \neq j}} n_{ji} f_{ji} - \sum_{\substack{i \\ i \neq j}} n_{ij} f_{ij}, \qquad j = 1, 2, 3, 4,$$

$$f_{ij} \geq 0, \qquad i, j = 1, 2, 3, 4,$$

where $(x_1, x_2, x_3, x_4) = N_c$.

The solution of this problem is that the MinMax average fee to be paid in the network is 0.23, and there is a polyhedron of interchange fees sets in which this solution is attained. The constraint associated with Bank 3 is the only active one for all the interchange fees sets in the solution. Therefore, this bank cannot achieve an average fee smaller than 0.23. The other three banks, though, may be interested in reducing their own average fee, and so we want to find the solution to the above problem with this property. Replacing the inequality constraint for Bank 3 with an equality one, and solving the problem again, we obtain a MinMax average fee of 0.14. This is obtained by Bank 4, and so Bank 4 cannot get a smaller average fee. Repeating the procedure again, we obtain interchange fees sets in which Bank 3 pays an average fee of 0.23, Bank 4 pays an average fee of 0.14, and Banks 1 and 2 do not have to pay interchange fees. The interchange fees in this solution are $f_{31} = 1.19$ and $f_{41} = 0.39$.

If we impose on the interchange fees an additional condition of symmetry, that is, $f_{ij} = f_{ji} \; \forall \; i, j = 1, 2, 3, 4$, the resulting MinMax average fee is 0.34, and it is, again, Bank 3 which attains it. Following the same procedure as before, a unique solution is obtained in the next step with average interchange fee of 0.28 corresponding to $f_{13} = f_{31} = 1.75$, $f_{14} = f_{41} = 0.76$, $f_{24} = f_{42} = 0.02$.

Figure 2 represents the interchange fees sets and the average interchange fee in each case. The minimization of the maximum average fee yields solutions where one bank pays another but the other does not pay anything back for the same service. When we impose $f_{ij} = f_{ji} \; \forall \; i, j = 1, 2, 3, 4$, a very high fee results between Banks 1 and 3.

Instead of the symmetry condition, we will assume that bank i has to pay bank j a quantity per transaction at least equal to the cost c_{ij}, $f_{ij} \geq c_{ij}$. It is also reasonable to consider interchange fees sets with all the fees smaller than the average cost of cash withdrawals by means other than ATMs, k. Imposing these extra conditions, we obtain a higher average transaction fee for Bank 3, (0.41). Bank 4 cannot reduce its average fee from 0.34, and Banks 1 and 2 have the lowest average fees of 0.25. The interchange fees set is now

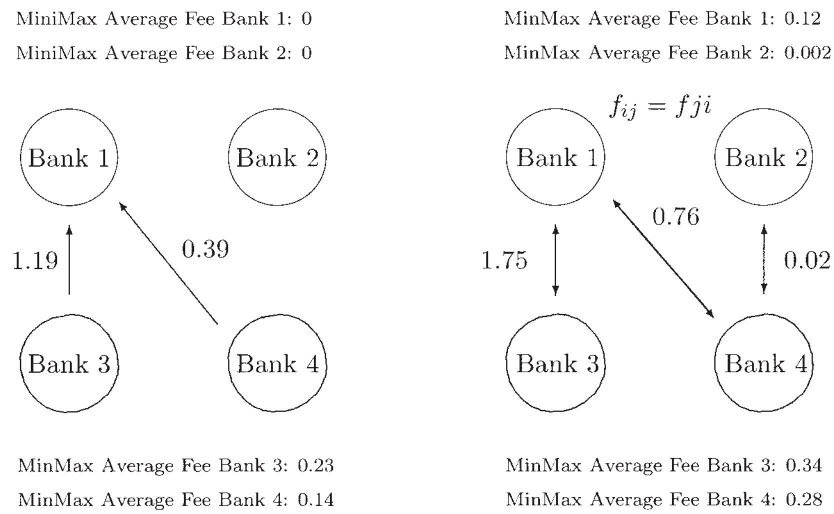


Figure 2. MiniMax average interchange fees.

$$(f_{12} = f_{21}) = f_{13} = f_{14} = f_{23} = (f_{24} = f_{42}) = (f_{34} = f_{43}) = 0.25,$$

$$f_{31} = f_{32} = 0.74, \qquad f_{41} = 0.5.$$

Notice that it is not possible to obtain an interchange fees set in which simultaneously $c_{ij} \leq f_{ij} \leq k$ and $f_{ij} = f_{ij}$ for all $i, j = 1, 2, 3, 4$.

Consider another criterium for obtaining balanced transaction fees. Any pair of banks should be interested on having interchange fee cash flows as small as possible. Thus we will try to choose the set of interchange fees which minimize the absolute difference in what a pair of banks pay each other. We want to minimize the maximum difference $|n_{ij}f_{ij} - n_{ji}f_{ji}|$ $i, j = 1, 2, 3, 4, i < j$.

To this end we now solve the linear programming problem:

$$\begin{aligned}
\min \quad & v \\
s.t.: \quad & n_{ij}f_{ij} - n_{ji}f_{ji} \leq v, \qquad i, j = 1, 2, 3, 4, \qquad i < j, \\
& n_{ji}f_{ji} - n_{ij}f_{ij} \leq v, \qquad i, j = 1, 2, 3, 4, \qquad i < j, \\
& x_j - R_j = \sum_{i \neq j} n_{ji}f_{ji} - \sum_{i \neq j} n_{ij}f_{ij}, \qquad j = 1, 2, 3, 4, \\
& f_{ij} \geq 0 \qquad \forall\, i, j = 1, 2, 3, 4,
\end{aligned}$$

where $(x_1, x_2, x_3, x_4) = N_c$.

The minimum difference obtained is 1,735,496. It corresponds to Bank 1 receiving this quantity from each of the others banks. Fixing the cash flow between Banks 2, 3, and 4, on one hand, and Bank 1, on the other hand, at this quantity, and solving again the problem to minimize the maximum cash flow among the remaining banks, we obtain that Bank 3 has to pay to Bank 2 and Bank 4 a quantity of 879,243 and that Bank 4 has to pay the same quantity to Bank 2. The interchange fees with which this cash flow levels can be attained are

$$f_{21} = 0.05, \qquad f_{31} = 0.79, \qquad f_{32} = 0.55,$$
$$f_{34} = 0.002, \qquad f_{41} = 0.26, \qquad f_{42} = 0.19.$$

This is an interesting solution because it also minimizes the maximum average. With these interchange fees, the averages fees for Banks 1, 3, and 4 are 0.23, 0.14, and 0, respectively. The corresponding averages only differ from those in the solution represented in Figure 2, when we do not impose $f_{ij} = f_{ji}$, $\forall\, i, j = 1, 2, 3, 4$, in the average fee of Bank 2. Bank 2 cannot obtain its MinMax average.

Finally, minimizing $\max_{i<j} |n_{ij}f_{ij} - n_{ji}f_{ji}|$, $i, j = 1, 2, 3, 4$, with the symmetry condition $f_{ij} = f_{ji}$, $\forall\, i, j = 1, 2, 3, 4$, the same cash flow levels as before can be attained (1,735,496 from Banks 2, 3, and 4 to Bank 1 and 879,243 from Bank 3 to Bank 2 and Bank 4 and from Bank 4 to Bank 2) with the following interchange fees set:

$$f_{12} = f_{21} = 0.35, \qquad f_{13} = f_{31} = 1.16, f_{14} = f_{41} = 0.51,$$
$$f_{23} = f_{32} = 1.26, \qquad f_{24} = f_{42} = 0.73, \qquad f_{34} = f_{43} = 0.02.$$

In this case, symmetry of the transaction fees make the average fees nearly uniform (Bank 1: 0.38, Bank 2: 0.4, Bank 3: 0.42, Bank 4: 0.39).

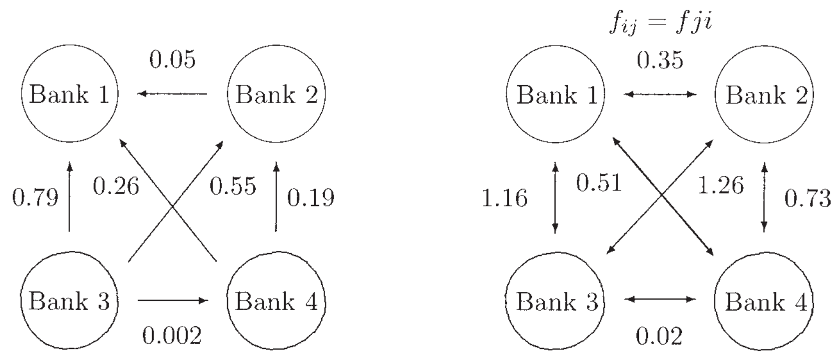


Figure 3. Interchange fees when we minimize $\max_{i<j} |n_{ij}f_{ij} - n_{ji}f_{ji}|$.

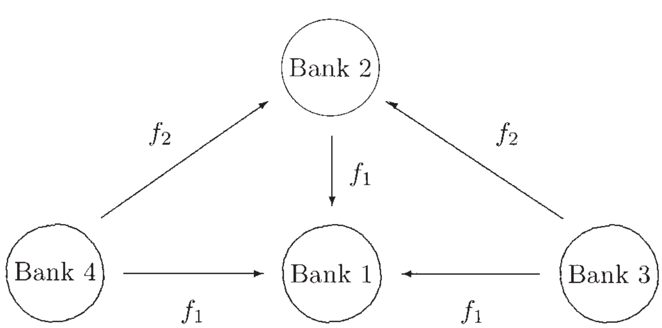
Figure 3 represents the interchange fees sets that minimize the flows, without and with the symmetry condition. The cash flows in both situations are the same and we represent it in Figure 4.

5. CONCLUSIONS

When dealing with interchange fees in an ATM network, cost is not the only variable to be considered in the model. The quality of the service is an important criterium that might be taken into account. Multi-objective game theory can be applied to investigate such interchange fee structure.

In this model it is assumed that the quality of the service depends, proportionally, on the number of ATM in the network and that the cost, for a subgroup of banks in the network, depends not only on the number of ATM still in the network and the cost to maintain them, but also on the new ATMs that the network introduces to replace those of the banks which have left the network.

Core solutions of the associated cooperative transferable utility, biobjective game have been established and have



$$f_1 = 1,735496$$

$$f_2 = 879,243$$

Figure 4. MiniMax interchange fees cash flows.

been proposed as a way to obtain fair allocations of the total cost of the existing service provision.

In the example considered, based on data from one of the UK four-banks network, we found that the preference core is empty and that the $MLC(N, V)$ is included in the non-dominance core and we concentrate on one of these solutions which is the nucleolus of the scalar component games corresponding to the vector-valued game of the maximals.

From this fair allocation of costs, we have obtained interchange fees from two different perspectives. On the one hand, we have minimized the average fee paid for each bank, and, on the other, we have looked for interchange fees that minimize the cash flows between any pair of banks. We have shown how, in our case, interchange fees that give minimum cash flows almost attain the MinMax average fees if we do not impose that every pair of banks pay the same for the same service. On imposing a symmetry condition, $f_{ij} = f_{ji}$, $\forall\ i < j$, the cash flows can be kept at their minimum levels, but the actual fees increase considerably. This forces up the average fees MiniMax average fee, but the fees themselves become very close to one other.

Looking at this multi-objective game has produced solutions, which are both different from the cost only game in Gow and Thomas [5] but are also understandable in terms of the context.

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