CS685: Data Mining
Density-Based Clustering Methods

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Mon 1030-1200 (online)

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- Density-Based Spatial Clustering of Applications with Noise (DBSCAN)
- Ordering Points to Identify the Clustering Structure (OPTICS)
- Density-based Clustering (DENCLUE)

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- Uses notions of density-reachability and density-connectivity

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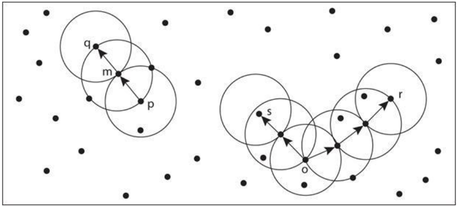

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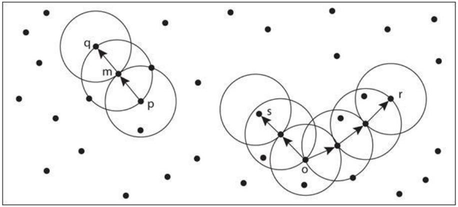

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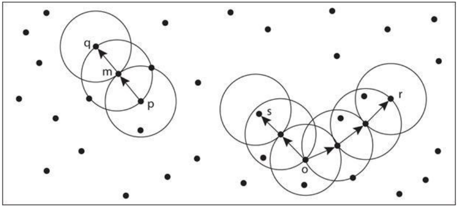

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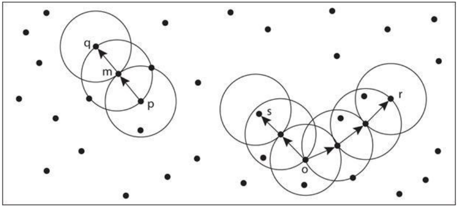

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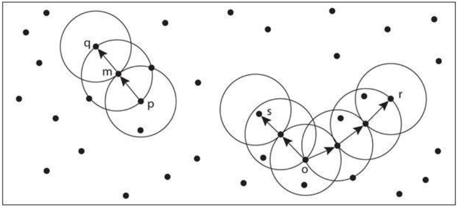

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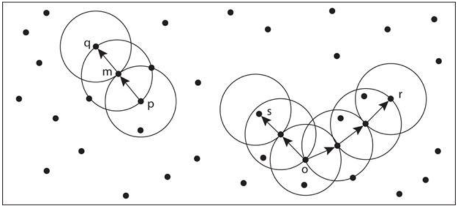

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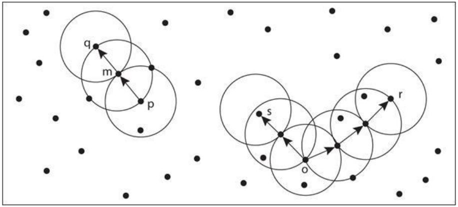

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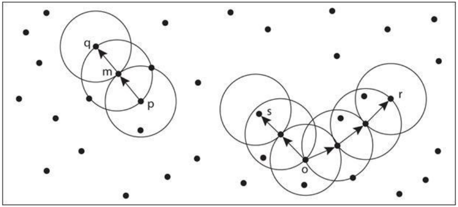

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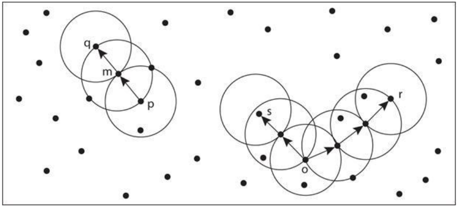

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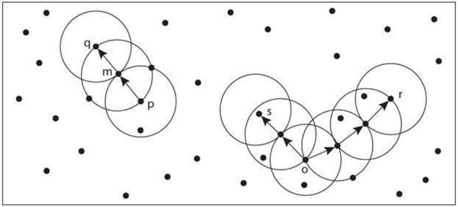

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- A density-based cluster is the closure of density-connected points
- A set of points C is a cluster if
  - For any two points $p, q \in C$, p and q are density-connected
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- Assumes clusters have similar densities
- Depends heavily on the parameters ϵ and τ

OPTICS

- Ordering Points To Identify the Clustering Structure (OPTICS)
- More a data ordering algorithm than a clustering method
- Tries to overcome the difficulty of choosing ϵ in DBSCAN
- Produces a cluster ordering of the data
- A linear order that represents the density-based clustering structure of the data

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- Uses notions of core distance and reachability distance

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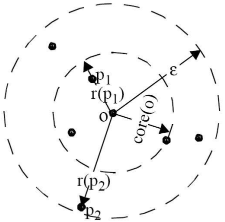
- The reachability distance of point p to point q is the *minimum* radius that makes q *directly density-reachable* from p
- p must become a core point
- It is, therefore, the *maximum* of *core distance* of p and distance of p to q
  - If p is not a core point with respect to ϵ, it is considered *undefined*



Reachability Plot

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- For each point in the database, it plots its reachability distance to the *nearest* core point
- Bumps mark the boundaries of clusters
- Valleys denote the clusters
  - Deeper the valley, denser the cluster

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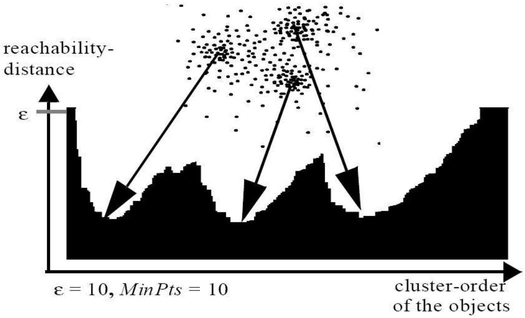

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- Still uses parameters: τ, ϵ

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- Local peaks denote cluster centres
- Points are attracted towards the nearest peak

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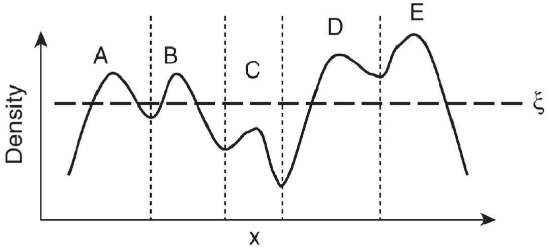
- Cluster corresponding to C is discarded
- Clusters A and B remain separated
- Clusters D and E get merged



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- Points are attracted towards local density attractors using *hill climbing*
- Uses the gradient of the Gaussian kernel

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- Generalization of several clustering methods
- Can be very slow
- Density estimated only at actual data points
- Influence function constrained to a range
- May use grids where each point influences its own cell and the neighboring cells only