

CS685: DATA MINING

ASSOCIATION RULE MINING

Arnab Bhattacharya
arnabb@cse.iitk.ac.in

Computer Science and Engineering,
Indian Institute of Technology, Kanpur
<http://web.cse.iitk.ac.in/~cs685/>

1st semester, 2020-21
Mon 1030-1200 (online)

Association Rules

- Find which **itemsets** are associated

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- Called **association rule mining** or **itemset mining** or **basket mining**
- Extremely rare that this will happen always
- Not useful if such itemsets occur rarely

Parameters of Association Rules

- For both A and B to occur, $A \cup B$ must occur
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$$P(A, B) = \frac{|A \cup B|}{|T|} \geq s$$

- **Confidence:** If A occurs, B should occur in at least c (ratio of) transactions

$$P(B|A) = \frac{|A \cup B|}{|A|} \geq c$$

Example

Transaction Id	Itemsets
1	A, C, D
2	B, C, E
3	A, B, C, E
4	B, E

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Rule	Support	Confidence
$B \Rightarrow E$	0.75	1.00
$C \Rightarrow E$		

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$A \Rightarrow D$	0.25	0.50
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- **Strong rule**: An association rule whose confidence is more than or equal to the confidence threshold

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- **Strong rule**: An association rule whose confidence is more than or equal to the confidence threshold
- **Weak rule**: An association rule whose confidence is less than the confidence threshold

Finding Association Rules

- Mining association rules require two steps
 - Finding **frequent** itemsets
 - Generating **strong** association rules

Finding Association Rules

- Mining association rules require two steps
 - Finding **frequent** itemsets
 - Generating **strong** association rules
- The first step is more time-consuming

Brute-force Algorithm

- Generate a candidate itemset
- Test its support
- If frequent, accept
- Else, throw away

Brute-force Algorithm

- Generate a candidate itemset
- Test its support
- If frequent, accept
- Else, throw away
- Total number of possible itemsets is $2^n - 1$
- Checking each itemset requires scanning the entire transaction database
- Too impractical

Apriori Principle

- Candidate-generation-and-test paradigm
- **Apriori principle:** If an itemset is frequent, all its subsets must also be frequent
- Conversely, if an itemset X is infrequent, all its supersets are also infrequent
- This is an *anti-monotonic* property: if a set fails, its supersets fail as well

Apriori Algorithm

- Generates candidate itemsets in order of length
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- Candidate itemsets of length k is C_k
- Frequent itemsets of length $k - 1$ is F_{k-1}
- **Join step:** $C_k = F_{k-1} \bowtie F_{k-1}$
 - Join two candidates whose $k - 2$ items are common
 - Perform **subset checking**
- **Prune step:** $F_k = \{I \in C_k : |I| \geq s\}$
 - Retain only frequent itemsets

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- Requires k database scans for itemsets up to length k

Apriori Example

Transaction Id	Itemsets
0	1, 2, 5
1	2, 4
2	2, 3
3	1, 2, 4
4	1, 3
5	2, 3
6	1, 3
7	1, 2, 3, 5
8	1, 2, 3
9	6

Support threshold $s = 2$

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1



Apriori example (contd.)

Candidate set C_1

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1	6
2	7
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→

Frequent set F_1

Itemset	Frequency
1	6
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4	2
5	2

→

Apriori example (contd.)

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1	6
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5	2
6	1

→

Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2

→

Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

→

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

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Frequent set F_1

Itemset	Frequency
1	6
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Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

→

Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 5	2
2, 3	4
2, 4	2
2, 5	2

→

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

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Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2

→

Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

→

Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 5	2
2, 3	4
2, 4	2
2, 5	2

→

Candidate set C_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2
(1, 3, 5)	subset
(2, 3, 4)	subset
(2, 3, 5)	subset
(2, 4, 5)	subset

→

Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

→

Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2

→

Candidate set C_2

Itemset	Frequency
1, 2	4
1, 3	4
1, 4	1
1, 5	2
2, 3	4
2, 4	2
2, 5	2
3, 4	0
3, 5	1
4, 5	0

→

Frequent set F_2

Itemset	Frequency
1, 2	4
1, 3	4
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2, 3	4
2, 4	2
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Frequent set F_3

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1, 2, 3	2
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Apriori example (contd.)

Candidate set C_1

Itemset	Frequency
1	6
2	7
3	6
4	2
5	2
6	1

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Frequent set F_1

Itemset	Frequency
1	6
2	7
3	6
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5	2

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Itemset	Frequency
1, 2	4
1, 3	4
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Itemset	Frequency
1, 2	4
1, 3	4
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2, 3	4
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Candidate set C_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2
(1, 3, 5)	subset
(2, 3, 4)	subset
(2, 3, 5)	subset
(2, 4, 5)	subset

→

Frequent set F_3

Itemset	Frequency
1, 2, 3	2
1, 2, 5	2

Candidate set C_4

Itemset	Frequency
(1, 2, 3, 5)	subset

- *Transaction-wise partitioning*
 - Partition transactions into different sets
 - Find frequent and infrequent itemsets in each partition with support threshold s' (according to ratio of transactions in each partition)
 - For two equal partitions, $s' = s/2$
 - Report all itemsets that are frequent in all partitions
 - Prune all itemsets that are infrequent in all partitions

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- Find frequent and infrequent itemsets in each partition with support threshold s' (according to ratio of transactions in each partition)
 - For two equal partitions, $s' = s/2$
- Report all itemsets that are frequent in all partitions
- Prune all itemsets that are infrequent in all partitions

- *Item-wise partitioning*

- Partition items into different sets
- Find frequent itemsets in each partition
- Join only these frequent itemsets to form global candidates

FP-Growth

- Frequent pattern (FP)-growth
- Compact representation of entire transaction database as a tree
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- Items in *descending* order of support forms *flist* order
- Re-arranges items in every transaction in *flist* order

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- Nodes are items with corresponding count
- Each transaction is added as a path in the tree
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- Nodes of same item are linked using *node links*
- Two database scans

FP-Tree Example

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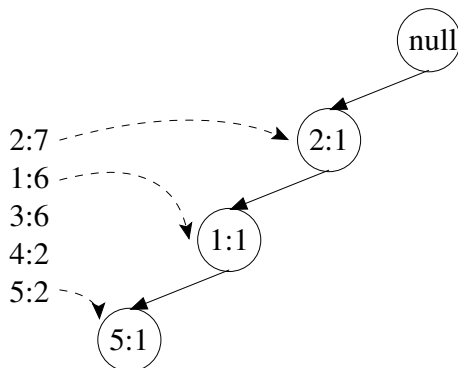
Support threshold $s = 2$

→

Flist order of items	
Item	Frequency
2	7
1	6
3	6
4	2
5	2

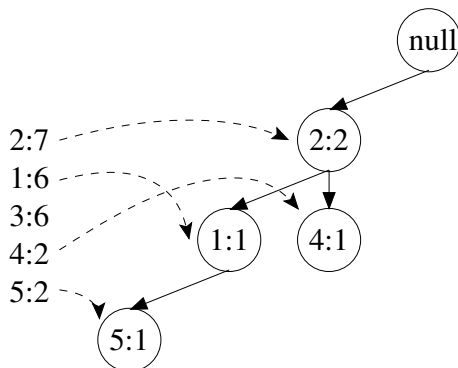
FP-Tree Construction

- Adding transaction 0: 2, 1, 5



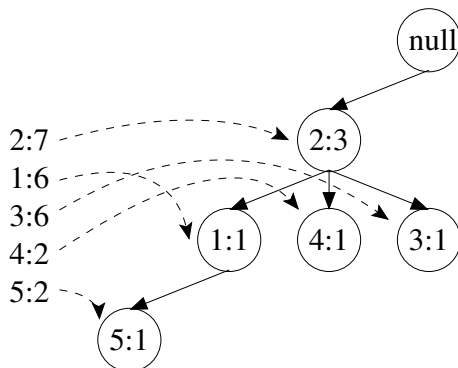
FP-Tree Construction (contd.)

- Adding transaction 1: 2, 4



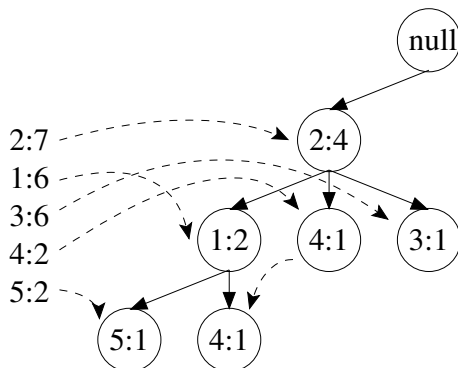
FP-Tree Construction (contd.)

- Adding transaction 2: 2, 3



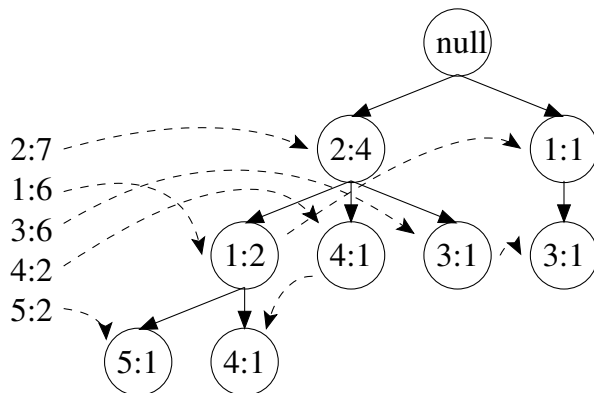
FP-Tree Construction (contd.)

- Adding transaction 3: 2, 1, 4



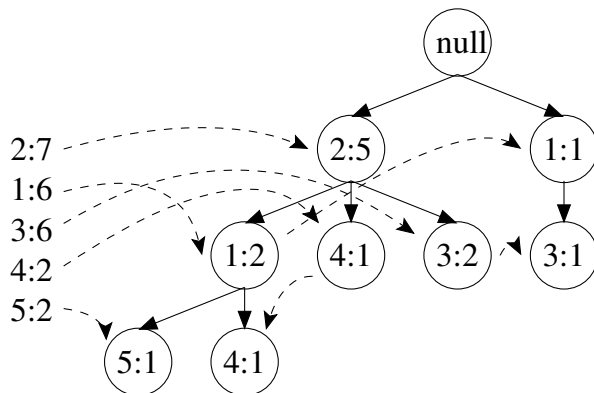
FP-Tree Construction (contd.)

- Adding transaction 4: 1, 3



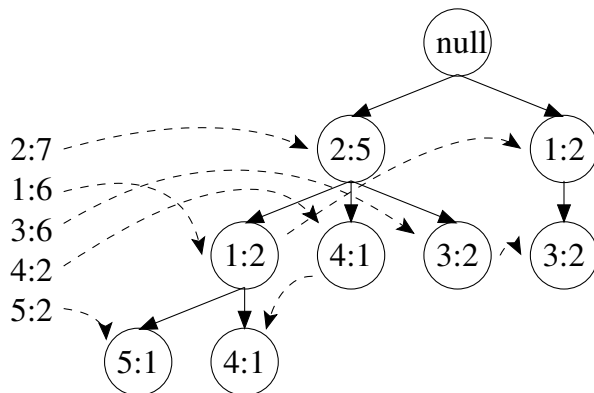
FP-Tree Construction (contd.)

- Adding transaction 5: 2, 3



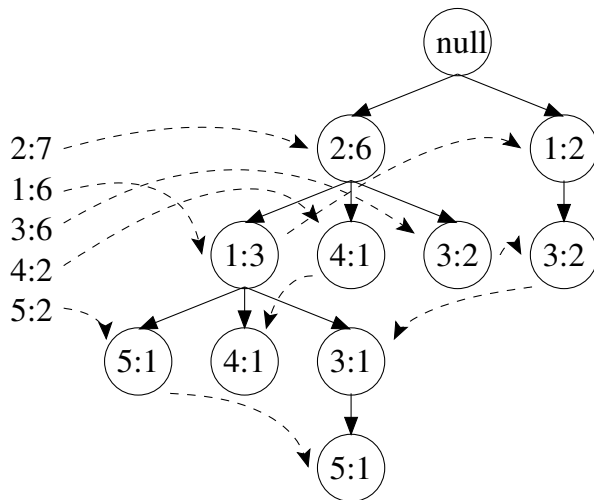
FP-Tree Construction (contd.)

- Adding transaction 6: 1, 3



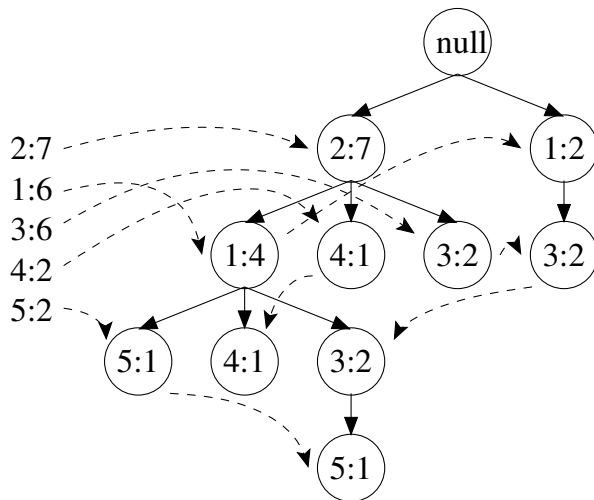
FP-Tree Construction (contd.)

- Adding transaction 7: 2, 1, 3, 5



FP-Tree Construction (contd.)

- Adding transaction 8: 2, 1, 3



FP-Tree Mining

- Starts with the item with the least support, say x
- *Projects* its paths from the base tree
- x is the suffix in all such paths

FP-Tree Mining

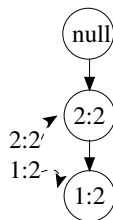
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- x is the suffix in all such paths
- A new FP-tree is built with only these paths (equivalently, transactions) with x removed
- This new FP-tree is *recursively* mined to find frequent patterns
- All such frequent patterns are appended with x and returned

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- Starts with the item with the least support, say x
- *Projects* its paths from the base tree
- x is the suffix in all such paths
- A new FP-tree is built with only these paths (equivalently, transactions) with x removed
- This new FP-tree is *recursively* mined to find frequent patterns
- All such frequent patterns are appended with x and returned
- The item with the next lowest count is continued with

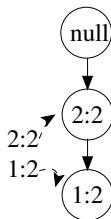
FP-Tree Mining Example

- For the least frequent item: 5
- Two prefix paths found by traversing node links are (2, 1): 1 and (2, 1, 3): 1
- This forms the **conditional pattern base**
- 3 is discarded as its support ($= 1$) is less than threshold
- From conditional pattern base, **conditional FP-tree** is then constructed



FP-Tree Mining Example

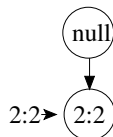
- For the least frequent item: 5
- Two prefix paths found by traversing node links are (2, 1): 1 and (2, 1, 3): 1
- This forms the **conditional pattern base**
- 3 is discarded as its support (= 1) is less than threshold
- From conditional pattern base, **conditional FP-tree** is then constructed



- Frequent patterns found are (1, 5): 2, (2, 1, 5): 2 and (2, 5): 2

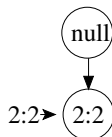
FP-Tree Mining Example (contd.)

- For the next least frequent item: 4
- Two prefix paths found by traversing node links are (2, 1): 1 and (2): 1
- This forms the **conditional pattern base**
- 1 is discarded as its support ($= 1$) is less than threshold
- From conditional pattern base, **conditional FP-tree** is then constructed



FP-Tree Mining Example (contd.)

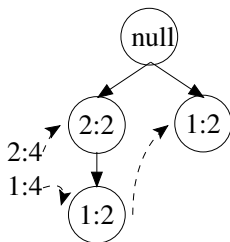
- For the next least frequent item: 4
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- Frequent patterns found are (2, 4): 2

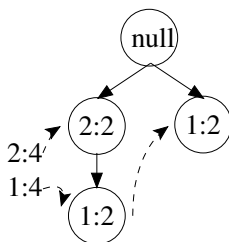
FP-Tree Mining Example (contd.)

- For the next least frequent item: 3
- Three prefix paths found by traversing node links are (2, 1): 2, (2): 2 and (1): 2
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FP-Tree Mining Example (contd.)

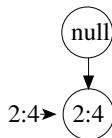
- For the next least frequent item: 3
- Three prefix paths found by traversing node links are (2, 1): 2, (2): 2 and (1): 2
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- Frequent patterns found are (1, 3): 4, (2, 1, 3): 2 and (2, 3): 4

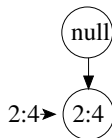
FP-Tree Mining Example (contd.)

- For the next least frequent item: 1
- One prefix path found by traversing node links is (2, 1): 4
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FP-Tree Mining Example (contd.)

- For the next least frequent item: 1
- One prefix path found by traversing node links is (2, 1): 4
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- Frequent patterns found are (2, 1): 4

FP-Tree Mining Example (contd.)

- For the most frequent item: 2
- Nothing needs to be done
- Assumption is that all 1-itemsets are already returned

Projected and Residual Databases

- Consider the item with the largest support, say x
- x partitions transactions into two parts
- Transactions containing x form the **projected database** P_x of x
- Transactions after removing x form the **residual database** R_x of x

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- Consider any (in-)frequent itemset I
 - If $x \in I$, then it will be (in-)frequent in P_x as well
 - If $x \notin I$, then it will be (in-)frequent in R_x as well
 - Frequency of I does not change in R_x

- **H-mine** is a partitioning-based algorithm
- It first sorts the items in *flist* order
- From each item, a pointer is linked to the first transaction that contain this item as the first in flist order
- All subsequent transactions of the same nature are chained
- Following the chain produces the projected database for that item
- The frequent itemsets are mined recursively then

Mining Closed and Maximally Frequent Itemsets

- Apriori algorithm works
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Mining Closed and Maximally Frequent Itemsets

- Apriori algorithm works
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- If any subset has same support, remove that subset
- Apriori may be run in reverse direction, starting with all items and then generating subsets as candidates
- A single support threshold across all itemset lengths may not be useful
- Chances of itemsets with larger length occurring are less
- **MLMS** model: **Multiple Length Minimum Support**
- Apriori works again
- If support at lesser length is smaller, e.g., $s_k < s_{k+1}$
 - All k -length subsets of frequent itemsets of length $k + 1$ are frequent
 - Conversely, if an itemset is pruned at length k , all its supersets of length $k + 1$ will be infrequent

Are Strong Association Rules Always Good?

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- Consider the rule $3 \implies 2$
- Support is 0.4 and confidence is 0.67
- However, support of 2 itself is 0.7
- When there is no influence, 2 occurs more frequently than when 3 is there
- The effect of 3 is thus *negative* on 2
- Just support and confidence thresholds are, therefore, not enough

Lift

- A correlation measure **lift**
- Lift measures how correlated the two itemsets are

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- If lift is > 1 , they are *positively* correlated
- Lift of the rule $3 \implies 2$ is $0.67/0.7 = 0.95$
- Thus, 3 and 2 are negatively correlated

Probabilistic Association Rule Mining

- Occurrence of an item in a transaction is not just presence or absence
- It is present with a probability $p \in [0, 1]$
- Applications
 - Medical: a patient may have cancer with 70% chance, hepatitis with 10% chance, etc.

Transaction id	Item A	Item B	Item C	Item D
0	0.9	0.8	0.0	0.2
1	0.7	0.7	1.0	0.3
2	0.2	0.5	0.9	0.5

- Support of 1-itemsets can be found by just adding the columns
- Support of larger itemsets can be found by adding the products of the corresponding probabilities
 - Support of (A) is $0.9 + 0.7 + 0.2 = 1.8$
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