CS685: Data Mining
Artificial Neural Networks

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Perceptron

- A perceptron is a simple binary *linear* classifier
- Input attributes $x_1, \ldots, x_n$ are weighted and summed
- A bias b is added as well
- Final class ($y = \pm 1$) is *sign* of the output
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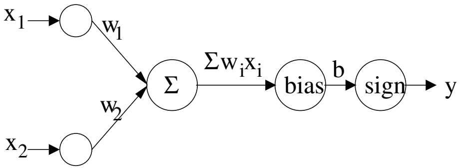
- The *sign* node is called the activation function
- It is a simple *linear* classifier
- Decision boundary is $w.x + b = \sum_{i=1}^{n} w_i x_i + b$
- Therefore, sign of $w.x + b$ predicts the class



Bias

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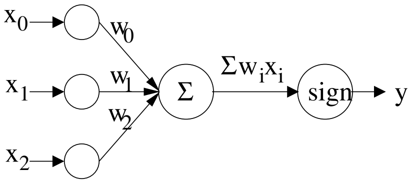
- Otherwise, hyperplane passes through origin
- Simple trick to model input uniformly: include 1 as x_0 of data
- Decision boundary becomes $w_0 x_0 + w.x = \sum_{i=0}^{n} w_i x_i$
- Weight on $x_0 = 1$ becomes the constant term, i.e., $w_0 = b$



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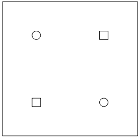

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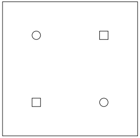

  - Combination of perceptrons can do it, though

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- Weights can be learned through *gradient descent* method as well

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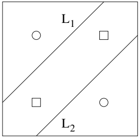

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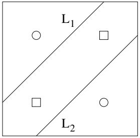
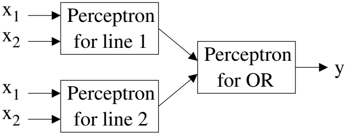


- Each hyperplane can be modeled by a perceptron and the outputs can be combined using OR



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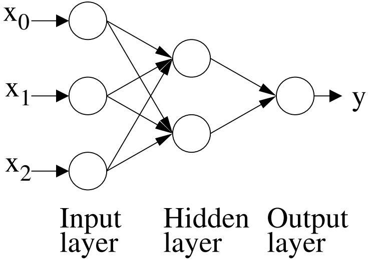

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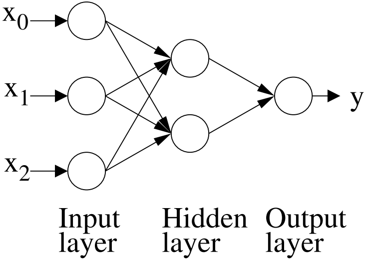

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- Learning through ANNs is also called connectionist learning

Connections

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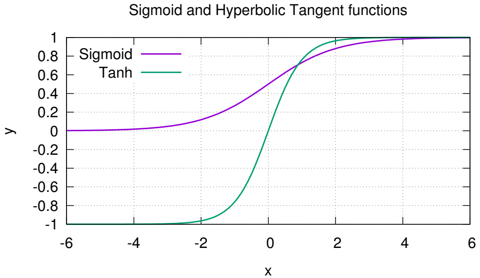
- Activation functions in the nodes are *non-linear*
  - Non-linear needed since combination of linear functions can only learn linear separators
- Activation function is sigmoid (logistic) or hyperbolic tangent

Sigmoid and Hyperbolic Tangent Functions

- If input of a node is x, then output y is

$$y = \sigma(x) = \frac{1}{1 + e^{-x}} \qquad y = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



- Approximates the step (or sign) function
- Continuous and differentiable
- Output constrained to $(0, 1)$ or $(-1, +1)$
- Scaled versions of each other
- Also called squashing functions

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- Weight from a node i to node j is w_{ij}
- Output from node i is O_i
- Input to node j is I_j

$$I_j = \sum_{\forall i} (w_{ij} \cdot O_i)$$

- Output from node j is O_j

$$O_j = \sigma(I_j)$$

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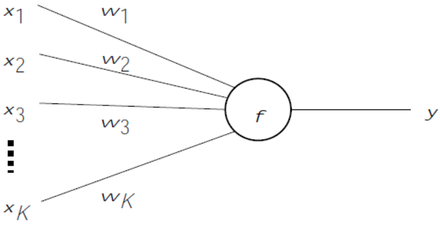

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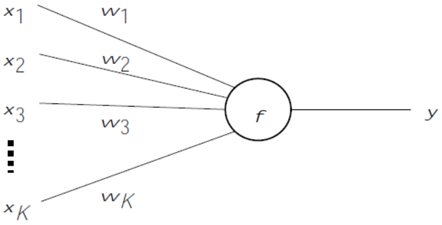

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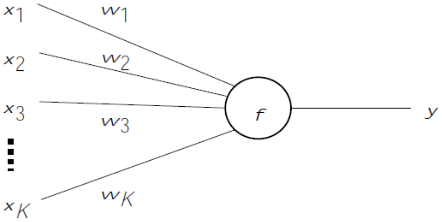

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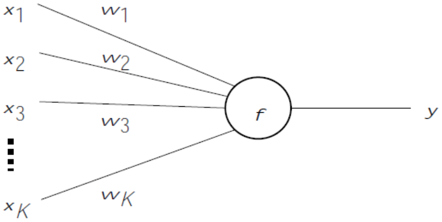

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- Properties of sigmoid function

$$\frac{d\sigma(u)}{du} = \sigma(u)\sigma(-u) = \sigma(u)(1 - \sigma(u))$$

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- Learning rate or *momentum* η controls the speed of training
- Can be continued till error is below a threshold

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- Notoriously non-explainable

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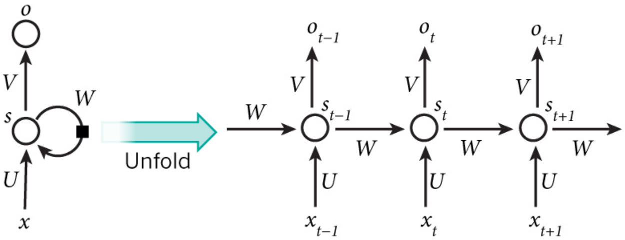

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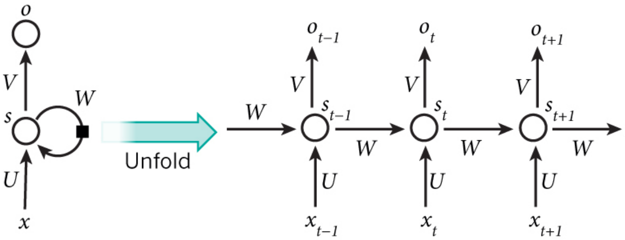

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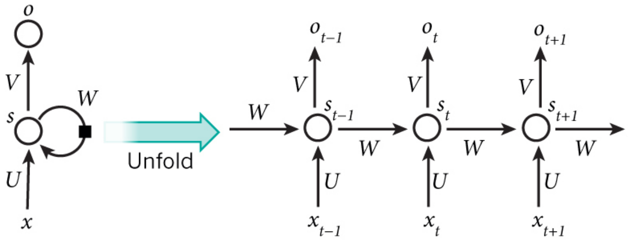
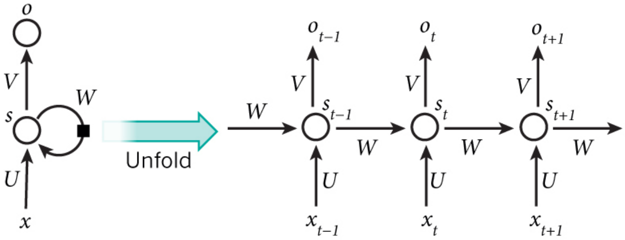


- Output depends on previous steps, i.e., it uses "memory"
- "Unrolling" or "unfolding" produces the layers
- If a 5-length *context* is needed, RNN is unfolded to 5 layers
- Same parameters U, V, W are shared across the layers
  - General deep networks are not constrained by this

Components of an RNN



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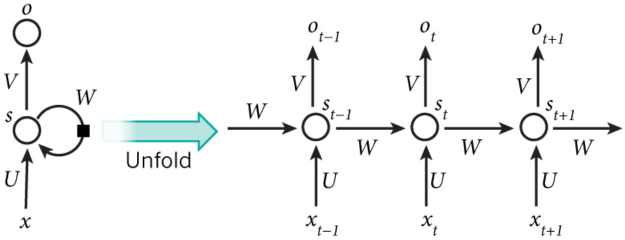

- x at each step is the "one-hot" vector (i.e., only 1 element is on)
- s_t is memory as it captures everything previous

$$s_t = f(U.x_t + W.s_{t-1} + b_s)$$

  - f is a non-linear function such as sigmoid or hyperbolic tangent
- o_t is the output at step t

$$o_t = g(V.s_t + b_o)$$

  - Generally, g is the *softmax* function to produce distributions

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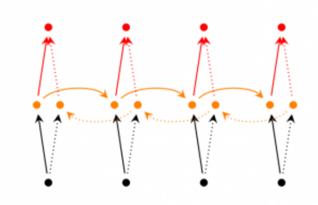
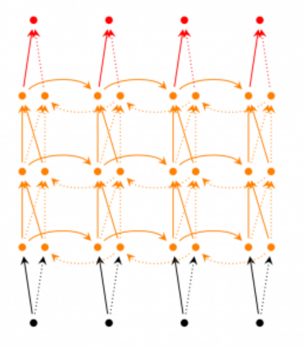
- Suffers from vanishing/exploding gradients problem for long chains

Types of RNNs

- Bi-directional RNNs use future as well as past to model present
- Deep bi-directional RNNs use multiple layers per time step





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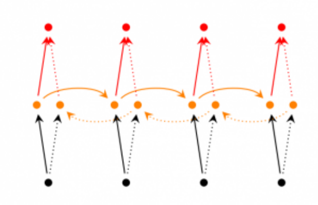

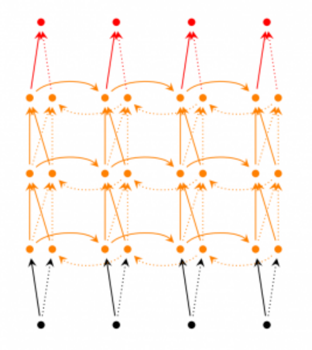

- Most famous is LSTM (long short-term memory) RNNs
  - Can use long-term memory or can ignore it
  - Instead of a simple non-linear function f at s_t, LSTM uses a complicated neural network structure

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  - Relaxed version is Leaky ReLU
- Includes *fully connected layers* (*dense layers*) as well