$$\frac{\partial U}{\partial t} + R(U) = L(U) + S$$

$$U = \left\{ \begin{matrix} \rho \\ \rho v_r \\ \rho v_\theta \\ \rho v_\phi \\ E \end{matrix} \right\}$$

$$R(U) = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{ \begin{matrix} r^2 \rho v_r \\ r^2(\rho v_r^2 + p_t) \\ r^2 \rho v_r v_\theta \\ r^2 \rho v_r v_\phi \\ r^2 (E + p_t) v_r \end{matrix} \right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left\{ \begin{matrix} \sin\theta\, \rho v_\theta \\ \sin\theta\, \rho v_\theta v_r \\ \sin\theta (\rho v_\theta^2 + p_t) \\ \sin\theta\, \rho v_\theta v_\phi \\ \sin\theta (E + p_t) v_\theta \end{matrix} \right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left\{ \begin{matrix} \rho v_\phi \\ \rho v_\phi v_r \\ \rho v_\phi v_\theta \\ \rho v_\phi^2 + p_t \\ (E + p_t) v_\phi \end{matrix} \right\}$$

$$L(U) = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{ \begin{matrix} 0 \\ r^2 \tau_{rr} \\ r^2 \tau_{\theta r} \\ r^2 \tau_{\phi r} \\ r^2(v_r \tau_{rr} + v_\theta \tau_{\theta r} + v_\phi \tau_{\phi r}) \end{matrix} \right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left\{ \begin{matrix} 0 \\ \sin\theta\, \tau_{r\theta} \\ \sin\theta\, \tau_{\theta\theta} \\ \sin\theta\, \tau_{\theta\phi} \\ \sin\theta(v_r \tau_{r\theta} + v_\theta \tau_{\theta\theta} + v_\phi \tau_{\phi\theta}) \end{matrix} \right\} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left\{ \begin{matrix} 0 \\ \tau_{r\phi} \\ \tau_{\theta\phi} \\ \tau_{\phi\phi} \\ v_r \tau_{r\phi} + v_\theta \tau_{\theta\phi} + v_\phi \tau_{\phi\phi} \end{matrix} \right\}$$

$$
S = \begin{Bmatrix} 0 \\ \frac{1}{r}\left[\rho\left(v_\theta^2 + v_\phi^2\right) + 2p_t\right] \\ \frac{1}{r}\left[\left(\rho v_\phi^2 + p_t\right)\cot\theta - \rho v_r v_\theta\right] \\ -\frac{1}{r}\left(\rho v_\phi v_\theta \cot\theta + \rho v_\phi v_r\right) \\ 0 \end{Bmatrix}
+ \begin{Bmatrix} 0 \\ -\dfrac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r} \\ -\dfrac{\tau_{\phi\phi}\cot\theta - \tau_{r\theta}}{r} \\ \dfrac{\tau_{\theta\phi}\cot\theta + \tau_{r\phi}}{r} \\ 0 \end{Bmatrix}
$$

$$
\tau_{r\theta} = \tau_{\theta r} = \mu\left\{ r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} \right\}
$$

$$
\tau_{\theta\phi} = \tau_{\phi\theta} = \mu\left\{ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi} \right\}
$$

$$
\tau_{\phi r} = \tau_{r\phi} = \mu\left\{ \frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right) \right\}
$$

$$
\tau_{rr} = 2\mu\left[\frac{\partial v_r}{\partial r} - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right]
$$

$$
\tau_{\theta\theta} = 2\mu\left[\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{v_r}{r}\right) - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right]
$$

$$
\tau_{\phi\phi} = 2\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r} - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right]
$$

$$
\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}
$$