$$\frac{\partial U}{\partial t} + R(U) = L(U) + S$$

$$U = \begin{Bmatrix} \rho \\ \rho v_r \\ \rho v_\phi \\ \rho v_z \\ E \end{Bmatrix}, \quad R(U) = \frac{1}{r}\frac{\partial}{\partial r}\begin{Bmatrix} r\rho v_r \\ r(\rho v_r^2 + P_t) \\ r\rho v_r v_\phi \\ r\rho v_r v_z \\ r(E+P_t)v_r \end{Bmatrix} + \frac{1}{r}\frac{\partial}{\partial \phi}\begin{Bmatrix} \rho v_\phi \\ \rho v_\phi v_r \\ \rho v_\phi^2 + P_t \\ \rho v_\phi v_z \\ (E+P_t)v_\phi \end{Bmatrix}$$

$$+ \frac{\partial}{\partial z}\begin{Bmatrix} \rho v_z \\ \rho v_z v_r \\ \rho v_z v_\phi \\ \rho v_z^2 + P_t \\ (E+P_t)v_z \end{Bmatrix}$$

$$L(U) = \frac{1}{r}\frac{\partial}{\partial r}\begin{Bmatrix} 0 \\ r\tau_{rr} \\ r\tau_{\phi r} \\ r\tau_{zr} \\ r(v_r\tau_{rr} + v_\phi\tau_{\phi r} + v_z\tau_{zr}) \end{Bmatrix}$$

$$+ \frac{1}{r}\frac{\partial}{\partial \phi}\begin{Bmatrix} 0 \\ \tau_{r\phi} \\ \tau_{\phi\phi} \\ \tau_{z\phi} \\ v_r\tau_{r\phi} + v_\phi\tau_{\phi\phi} + v_z\tau_{z\phi} \end{Bmatrix}$$

$$+ \frac{\partial}{\partial z}\begin{Bmatrix} 0 \\ \tau_{rz} \\ \tau_{\phi z} \\ \tau_{zz} \\ v_r\tau_{rz} + v_\phi\tau_{\phi z} + v_z\tau_{zz} \end{Bmatrix}$$

$$S = \begin{Bmatrix} 0 \\ \frac{1}{r}\left(\rho v_\phi^2 + p_t\right) \\ -\frac{1}{r}\rho v_\phi v_r \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\tau_{\phi\phi}/r \\ \tau_{r\phi}/r \\ 0 \\ 0 \end{Bmatrix}$$

$$\nabla\cdot\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}\left(r v_r\right) + \frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\tau_{r\phi} = \tau_{\phi r} = \mu\left\{ r\frac{\partial}{\partial r}\left(v_\phi/r\right) + \frac{1}{r}\frac{\partial v_r}{\partial \phi}\right\}$$

$$\tau_{\phi z} = \tau_{z\phi} = \mu\left\{ \frac{\partial v_\phi}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \phi}\right\}$$

$$\tau_{zr} = \tau_{rz} = \mu\left\{ \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right\}$$

$$\tau_{rr} = 2\mu\left\{ \frac{\partial v_r}{\partial r} - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right\}$$

$$\tau_{\phi\phi} = 2\mu\left\{ \left(\frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r}\right) - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right\}$$

$$\tau_{zz} = 2\mu\left\{ \frac{\partial v_z}{\partial z} - \frac{1}{3}\left(\nabla\cdot\vec{v}\right)\right\}$$