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Estimating import supply and demand elasticities: Analysis and implications ☆

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ABSTRACT

Feenstra (1994) developed, and Broda and Weinstein (2006) refined, a structural estimator of import demand and supply elasticities. Working through the first principles of the methodology from Leamer (1981), this paper analyzes and improves the technique to provide a unified estimator of import supply and demand elasticities. The proposed LIML routine corrects small sample biases and constrained search inefficiencies. Previously used estimates are shown to overestimate the median elasticity of substitution by over 35%. Applied to US import data from 1993 to 2007, the biases of the standard estimates translate into an understatement of consumer gains from product variety by a factor of 6. To conclude, I investigate the implications of violations to the underlying assumptions of the model.

1. Introduction

Feenstra (1994)/Broda and Weinstein (2006) estimates (F/BW henceforth) of supply and demand elasticities have been heavily utilized in modern economic research. Studies using these estimates span international trade, open economy macroeconomics and labor economics.[1] Despite its wide use, I show the methodology possesses substantial biases that are rarely acknowledged.

This paper returns to the first principles of the technique, developed by Leamer (1981), to clarify the methodology. Leamer (1981)'s insights allow us to analyze deficiencies in the standard methodology of F/BW and motivate a "hybrid" estimator. The hybrid estimator proposed here combines limited information maximum likelihood (LIML) with a constrained nonlinear LIML routine. LIML addresses small sample bias while the nonlinear routine corrects grid search inefficiencies. Through Monte Carlo experiments and applications to actual data, I document the sources of bias in F/BW. In conjunction, I develop the intuition behind the improvements associated with the hybrid estimator. All methods of evaluation strongly support the hybrid estimator.

I show that the standard estimator is biased because it overweights outlier observations. The hybrid estimator better accounts for outlier observations, and significantly outperforms the standard method in Monte Carlo experiments. The estimators are then applied to import data. Correcting the biases of the standard estimates yields a 35% lower median demand elasticity for the universe of HS8 products imported by the US from 1993 to 2007. I demonstrate that bias in the standard estimates is responsible for understating consumer gains from product variety by a factor of 6 over the sample, and carries significant implications for a host of prominent studies.

I conclude by investigating the robustness of the methodology. First, I investigate violations to the assumed independence of errors through Monte Carlo experiments. When the supply and demand errors are positively correlated (e.g., endogenous quality), estimates of demand elasticities exhibit moderate bias of 10%–25% regardless of method. However, when errors are negatively correlated (e.g., hidden varieties), the hybrid estimator shows moderate bias while the standard method is significantly upward biased by 50%–125%. Estimates of the supply elasticity suffer regardless of method, but the hybrid estimator consistently outperforms the standard.

Next, I reestimate the model for various cuts of the data determined by income levels of each variety's country of origin in order to examine whether elasticities are feasibly identical across varieties. Hybrid

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[1] Trade topics range from studies of quality such as Khandelwal (2010), trade in intermediates Goldberg et al. (2010), and optimal tariffs Broda et al. (2008). Macroeconomic studies include Caballero et al. (2008). Labor topics include Iranzo and Peri (2009). This list hardly scratches the surface of the influential works utilizing Broda and Weinstein (2006)'s estimates (a simple count yields around 200 published articles using these estimates off-the-shelf).

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estimates of the demand elasticity are insensitive to the various specifications. The standard estimator, on the other hand, is extremely sensitive to the data used for estimation.[2] For the hybrid estimator, narrowing the sample to high income or OECD varieties leads to statistically different distributions only for the supply elasticity. Alleviating the bias of the standard estimator thus provides stronger support for the underlying assumptions of the model, but does suggest potential gains from a structural estimator that allows export supply elasticities to differ across varieties.

This paper proceeds as follows. Section 2 lays out the method of estimating import elasticities. Section 2.2 describes the first principles of the estimator. Section 3 presents Monte Carlo results supporting the hybrid estimator. Section 4 applies each estimator discussed to actual trade data. Section 5 investigates the robustness of the methodology across estimators, and Section 6 concludes.

2. Estimating import supply and demand

This section first describes the theoretical foundation of the Feenstra (1994)/Broda and Weinstein (2006) (F/BW) method to estimate import demand and export supply elasticities. Next, the econometric underpinnings of the estimator, which are drawn from Leamer (1981), are used to clarify the methodology. Finally, I lay out the steps that map Leamer (1981) to F/BW. Aligning F/BW with Leamer (1981) highlights sources of bias in the standard estimator. This section concludes by leveraging the intuition of the estimator to motivate an improved hybrid methodology.

2.1. Theory: the F/BW framework

We begin with the theoretical framework used by F/BW. The goal is to structurally estimate import demand and export supply elasticities in a common model of international trade. A representative consumer faces nested CES preferences over foreign and domestic goods and varieties. Denote the set of varieties v of good g available at time t by $I_{gt} \subset \{1,..., v,..., V\}$.[3] The aggregate quantity of each variety consumed in period t is x_{gvt}, and $\sigma_g > 1$ is the good specific constant elasticity of substitution. We also allow demand to contain a variety specific taste shock, denoted by b_{gvt}. Focusing on the variety nest for the imported good g, utility is given by,

$$X_{gt} = \left(\sum_{v \in I_{gt}} b_{gvt}^{\frac{1}{\sigma_g}} x_{gvt}^{\frac{\sigma_g - 1}{\sigma_g}} \right)^{\frac{\sigma_g}{\sigma_g - 1}}. \tag{1}$$

Demand for a given variety v of good g at time t is then $x_{gvt} = p_{gvt}^{-\sigma_g} b_{gvt} \left(\phi_{gt}(\mathbf{b}_{gt}) \right)^{\sigma_g - 1}$, where $\phi_{gt}(\mathbf{b}_{gt}) \equiv \left(\sum_{v \in I_{gt}} b_{gvt}\ p_{gvt}^{1-\sigma_g} \right)^{1/1-\sigma_g}$. Hence, the market share is,

$$s_{gvt} \equiv \frac{p_{gvt} x_{gvt}}{\sum_{v \in I_{gt}} p_{gvt} x_{gvt}} = \left(\frac{p_{gvt}}{\phi_{gt}(\mathbf{b}_{gt})} \right)^{1-\sigma_g} b_{gvt}. \tag{2}$$

Market share depends upon own price (p_{gvt}) relative to the price index ($\phi_{gt}(\mathbf{b}_{gt})$), and the variety specific random taste parameter (b_{gvt}).

[2] Kolmogorov–Smirnov tests reject the hypothesis that demand and supply elasticities are identically distributed across each cut of the data.

[3] Clarity regarding what is a variety and good is the key to understanding the estimator to follow. We rely on trade flows at the *HS*8 level of aggregation. Each *HS*8 will define a good. Then we employ the Armington (1969) assumption of national differentiation, so that varieties are defined by their country of origin. We can thus define the set {1,....,V} as the set of all potential exporters in the world.

Exporters are monopolistically competitive with upward sloping export supply of the form,

$$p_{gvt} = \left(\frac{\sigma_g}{\sigma_g - 1} \right) exp\left(\eta_{gvt} \right) \left(x_{gvt} \right)^{\omega_g}.$$

The inverse export supply elasticity for good g is given by $\omega_g \geq 0$, and η_{gvt} embodies a random technology factor. As with demand, we convert quantities supplied into market shares such that supply is written as,

$$p_{gvt} = \left(\sum_{v \in I_{gt}} exp\left(\frac{-\eta_{gvt}}{\omega_g} \right) p_{gvt}^{\frac{1+\omega_g}{\omega_g}} \right)^{\frac{\omega_g}{1+\omega_g}} exp\left(\frac{\eta_{gvt}}{1+\omega_g} \right) s_{gvt}^{\frac{\omega_g}{1+\omega_g}}.$$

Following Feenstra (1994), we wish to eliminate any time and good specific unobservables that would convolute the estimation of supply and demand elasticities. We eliminate good specific unobservables by first differencing prices and shares (denote first differences by Δ).[4] To eliminate time specific unobservables we difference again by a reference country k (denote reference differences by superscript k). This results in the system of equations,

$$\Delta^k lns_{gvt} \equiv \Delta lns_{gvt} - \Delta lns_{gkt} = -\left(\sigma_g - 1 \right) \Delta^k ln\left(p_{gvt} \right) + \varepsilon_{gvt}^k \tag{3}$$

$$\Delta^k lnp_{gvt} \equiv \Delta lnp_{gvt} - \Delta lnp_{gkt} = \left(\frac{\omega_g}{1+\omega_g} \right) \Delta^k ln\left(s_{gvt} \right) + \delta_{gvt}^k, \tag{4}$$

where $\varepsilon_{gvt}^k = \Delta^k ln(b_{gvt})$ and $\delta_{gvt}^k = \Delta^k \left(\frac{\eta_{gvt}}{1+\omega_g} \right)$ are unobservable demand and supply shocks, respectively. Eqs. (3) and (4) are the structural model's demand and supply curves.

As Feenstra (1994) shows, we can multiply ε_{gvt}^k and δ_{gvt}^k together in order to convert Eqs. (3) and (4) into one estimable equation. Define $\rho_g \equiv \frac{\omega_g(\sigma_g - 1)}{1+\omega_g \sigma_g} \in \left[0, \frac{\sigma_g - 1}{\sigma_g} \right)$, scale by $\frac{1}{(1-\rho_g)}$, and rearrange to produce Feenstra (1994)'s estimating equation,

$$\begin{aligned} &Y_{gvt} = \theta_{1g} X_{1gvt} + \theta_{2g} X_{2gvt} + u_{gvt}, \quad \text{where} \\ &Y_{gvt} \equiv \left(\Delta^k lnp_{gvt} \right)^2, X_{1gvt} \equiv \left(\Delta^k lns_{gvt} \right)^2, \\ &X_{2gvt} \equiv \left(\Delta^k lns_{gvt} \right) \left(\Delta^k lnp_{gvt} \right) \ \text{and} \ u_{gvt} = \frac{\varepsilon_{gvt}^k \delta_{gvt}^k}{\left(1 - \rho_g \right)}, \end{aligned} \tag{5}$$

where the coefficients, θ_{1g} and θ_{2g}, are nonlinear functions of σ_g and ρ_g such that,

$$\theta_{1g} \equiv \frac{\rho_g}{\left(\sigma_g - 1 \right)^2 \left(1 - \rho_g \right)} \text{ and } \theta_{2g} \equiv \frac{2\rho_g - 1}{\left(\sigma_g - 1 \right) \left(1 - \rho_g \right)}. \tag{6}$$

[4] This footnote makes three points. First, the underlying demand in logs and first differences for each variety can be written as,

$$\Delta ln\left(s_{gvt} \right) = \varphi_{gt} - \left(\sigma_g - 1 \right) \Delta ln\left(p_{gvt} \right) + \varepsilon_{gvt},$$

where $\varphi_{gt} \equiv (\sigma_g - 1)\Delta ln(\phi_{gt}(\mathbf{b}_{gt}))$ is a time-product specific random shock driven by the vector of random taste parameters $\mathbf{b}_{gt}$. The variety specific random shock, $\varepsilon_{gvt} = \Delta ln(b_{gvt})$, is driven by the random tastes of consumers across varieties. Second, the underlying supply in logs and first differences can be written as,

$$\Delta ln\left(p_{gvt} \right) = \psi_{gt} + \frac{\omega_g}{1+\omega_g} \Delta ln\left(s_{gvt} \right) + \delta_{gvt},$$

where $\psi_{gt} = \frac{\omega_g}{1+\omega_g} \Delta ln\left(\sum_{v \in I_{gt}} exp\left(-\eta_{gvt}/\omega_g \right) p_{gvt}^{1+\omega_g/\omega_g} \right)$ captures time-product specific shocks to production. The inverse supply elasticity for each product is $\omega_g \geq 0$. Random technology shocks to the production of each variety, η_{gvt}, manifest themselves through $\delta_{gvt} = \Delta(\eta_{gvt}/1 + \omega_g)$. Third, differencing by a reference variety in Eqs. (3) and (4) serves to eliminate the time-product shocks present in supply and demand (φ_{gt} and ψ_{gt}).

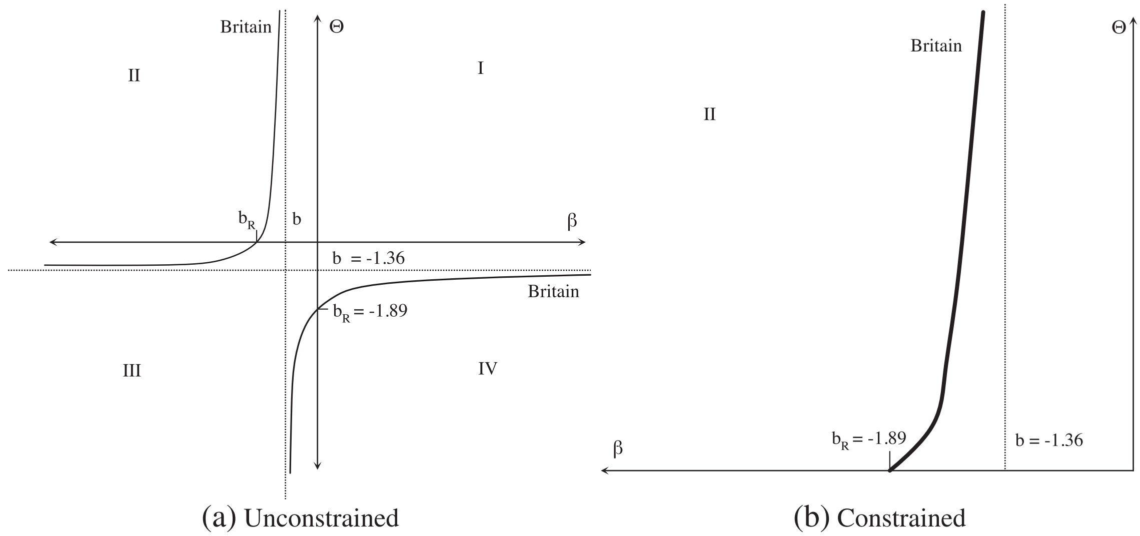


Fig. 1. Hyperbolae: British suits.

I next turn to estimation. Crucially, Feenstra (1994) assumes that ε_{gvt}^{k} and δ_{gvt}^{k} are independent.[5] Estimation will be employed separately good by good. Taking advantage of the panel nature of the data, Feenstra (1994) demonstrates that 2SLS estimation on Eq. (5), with variety indicators as instruments, produces consistent estimates of θ_{1g} and θ_{2g}. By Eq. (6) we can use the estimated coefficients to solve for the import demand and export supply elasticities. The estimation procedure corresponds to Hansen (1982)'s method of moments where the moment condition, $E[u_{gvt}]=0$, is approximated by the parameters that minimize the weighted sum of squares residuals.

While Feenstra (1994)'s estimator is consistent with the assurance of enough time periods, Soderbery (2010) demonstrates that it is biased for low T. This paper reaffirms that finding, and highlights significant biases when the estimator encounters data that tend toward infeasible elasticities. The following develops a unified estimator of import demand and export supply elasticities that addresses these biases. Acknowledging the first principles of the estimator, which are based on Leamer (1981)'s analysis of supply and demand, is the key to motivating the improved estimator. Providing a deep analysis of Feenstra (1994)'s adaptation of Leamer (1981) will also highlight within sample tools to evaluate the performance of our estimates.

2.2. Lifting the veil: Leamer (1981) revisited

In order to compare estimates to actual trade data and develop an improved estimator, it is valuable to acknowledge the structure of Feenstra (1994). The intuition for the preceding empirical strategy is drawn from Leamer (1981). Generically, let β denote the elasticity of demand and let Θ denote the elasticity of supply. Allow for variety specific fixed effects to enter demand (α_{gv}) and supply (γ_{gv}). Additionally, suppose there are unobserved components to both demand and supply, υ_{gvt} and μ_{gvt}. The relationship for a variety v of good g in Leamer (1981) is then,

$$\begin{aligned} \text{Demand}: &\quad \Delta ln\left(x_{gvt}\right) = \alpha_{gv} + \beta \Delta ln\left(p_{gvt}\right) + \upsilon_{gvt} \\ \text{Supply}: &\quad \Delta ln\left(x_{gvt}\right) = \gamma_{gv} + \Theta \Delta ln\left(p_{gvt}\right) + \mu_{gvt}, \end{aligned}$$

which is isomorphic to the preceding model. Leamer (1981) first solves the reduced form of these simultaneous equations. Assuming supply and demand errors are independent normal random variables, Leamer (1981) shows that the variance structure of the model can be used to define a hyperbola that characterizes the set of true supply and demand elasticities given observed data.

To show Leamer (1981) in action, I employ import data for the US from the *Center for International Data* aggregated to the *HS*8 level.[6] In general, Leamer (1981) and Feenstra (1994) can be applied to any industry g with well defined varieties v. With international trade data, goods are defined by specific *HS* codes, and varieties follow the Armington (1969) assumption of country of origin product differentiation. Consider the panel of US suit (*HS*61031960) imports from 1993 to 2007. Focus on the variety of suits imported from Britain. Regress the British suit imports on prices. The resulting OLS estimate in our data is $b \equiv \frac{cov(p,x)}{var(x)} = -1.36$. Since b is negative, it can be thought of as an *attenuated* estimate of β (i.e., the import demand elasticity).[7]

Leamer (1981) shows that while simple OLS estimates cannot identify a precise estimate of supply *or* demand, they can be used to create bounds on the set of potential true values. To construct these bounds, regress British suit prices on imports. Call this reverse OLS regression coefficient $b_r \equiv \frac{var(p)}{cov(p,x)} = -1.89$. Combining the "direct" estimate b with the "indirect" estimate b_r, Leamer (1981) demonstrates that the set of true elasticity estimates lie on the hyperbola,

$$(\beta - b)(\Theta - b) = \left(\frac{b}{b_r} - 1\right)(b_r * b). \tag{7}$$

Fig. 1 plots the hyperbola for British suits imported by the US in our data. For the case of British suits, the hyperbola is characterized by horizontal and vertical asymptotes at $b=-1.36$ and intersections on the Θ and β axes at $b_r=-1.89$.

Notice from Fig. 1(a) that the supply and demand elasticities can take on any value, but given one elasticity there is a unique solution for the other. Suppose for instance that we knew that the supply of British suits is perfectly inelastic ($\Theta=0$). Then plugging $\Theta=0$ into Eq. (7)

[5] Specifically, Feenstra (1994) requires three assumptions. The error term is zero in expectation, $E[u_{gvt}] = E[\varepsilon_{gvt}^{k}\delta_{gvt}^{k}] = 0$, the variances of the supply and demand shocks are not proportional (see Eq. (12) of Feenstra (1994, pp. 164)), and the assurance that $T \to \infty$. I will discuss violations of each of these assumptions subsequently, with extra attention paid to the correlation between error terms in Section 6.

[6] Remarkably, this discussion and subsequent estimation relies only on data recording import values and quantities, which the dataset tracks thoroughly. For a full description of the data, see Feenstra et al. (2002). The data are housed at http://cid.econ.ucdavis.edu/usix.html.

[7] If b were positive it would be an attenuated estimate of Θ (i.e., the export supply elasticity).

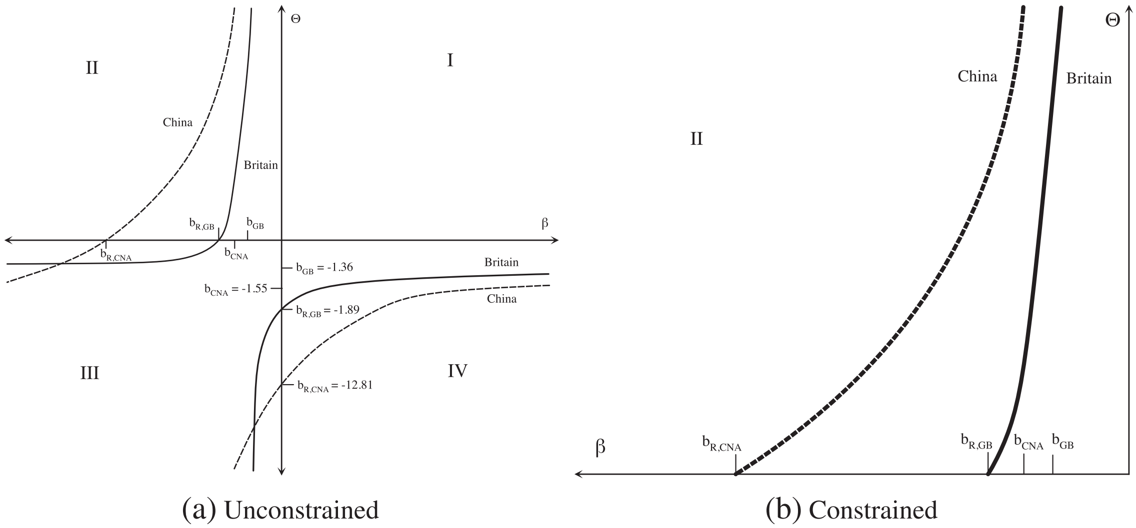


Fig. 2. Hyperbolae: British and Chinese suits.

yields $\beta = b_r = -1.89$. We can thus use Eq. (7)'s mapping of one elasticity into another to bound our estimates.

If we are willing to assume supply slopes upward and demand slopes downward, we can narrow the feasible range of estimates to the second quadrant. Fig. 1(b) imposes these feasibility constraints on the British hyperbola. Doing so demonstrates that our estimates are identifying a demand curve with an elasticity $\hat{\beta} \in [b_r, b] = [-1.89, -1.36]$. However, our estimates provide no useful information about the supply curve, which can take on any value $\hat{\theta} \in [0, \infty)$. This is precisely Leamer (1981)'s original point: Analysis of prices and quantities with a single variety can yield informative bounds for one, but not both, of the supply and demand elasticities.

The unique feature of trade data relative to the industry data Leamer (1981) presents is that trade data record detailed price and quantity outcomes for multiple varieties of the same good. Since these varieties generate data from the same underlying supply and demand equations, Feenstra (1994) asserts that the intersections of multiple hyperbolae can be used to pin down $\hat{\beta}$ and $\hat{\theta}$.

For the case of US suit imports, the data also record imports of a Chinese variety. The direct OLS estimate of imports on prices for the Chinese variety is $b = -1.55$ while the indirect estimate is $b_r = -12.81$. Fig. 2 overlays the hyperbolae coming from Chinese and British suit imports. While the intersections of the Chinese and British hyperbolae lie outside of the feasible region, their relationship to one another provides valuable information for estimating the demand and supply elasticities.

To provide more examples of how hyperbolae relate in the data, Fig. 3 plots the Leamer (1981) hyperbolae for four goods imported by the US. To narrow our focus, Fig. 3 constrains the hyperbolae to the feasible region where demand slopes down and supply slopes up (i.e., $\beta < 0$ and $\Theta > 0$). The most obvious difference between the goods are the number of hyperbolae they generate. Steel toe footwear contain the most imported varieties of the set presented, and thus the most hyperbolae. Noticeable differences between the hyperbolae across these goods emerge. Some goods present tightly concentrated hyperbolae (e.g., steel toe footwear), while others display a wider range of more disperse hyperbolae (e.g., motor vehicles). The combined intuition from Leamer (1981) and Feenstra (1994) is that if all of the hyperbolae for a particular good crossed at the same point we could precisely calculate the true supply and demand elasticities. It is clear from these figures that hyperbolae may not cross or may cross many times. Feenstra (1994)'s estimation framework proposes a solution to this feature of the data by estimating supply and demand elasticities that minimize the distance between the set of hyperbolae.

2.3. Feenstra (1994)'s transformation and estimation

In order to compare our estimates of the preceding trade model with the hyperbolae they are drawn from, it is necessary to transform Leamer (1981)'s framework. Without loss of generality, we can rewrite Leamer (1981)'s specification to match the functional form of Feenstra (1994). Replacing $\beta = 1 - \sigma$ and $\theta = \frac{1}{\omega}$ yields,

$$\begin{aligned} \text{Demand}: \quad & \Delta ln\left(x_{gvt}\right) = \alpha_{gv} + (1-\sigma)\Delta ln\left(p_{gvt}\right) + \upsilon_{gvt} \\ \text{Supply}: \quad & \Delta ln\left(x_{gvt}\right) = \gamma_{gv} + \frac{1}{\omega}\Delta ln\left(p_{gvt}\right) + \mu_{gvt}. \end{aligned}$$

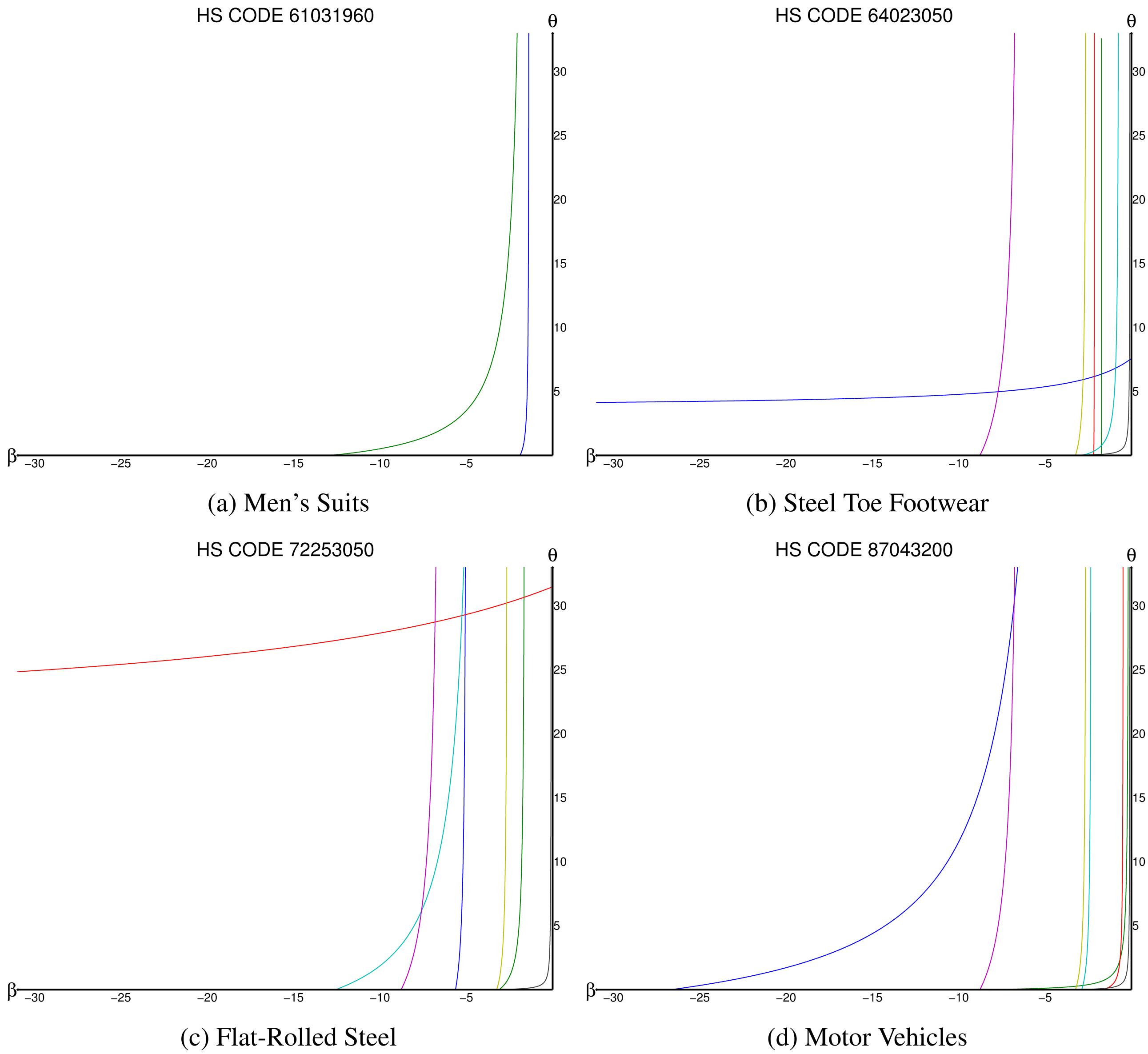
Notice these supply and demand equations are exactly Eqs. (3) and (4) less reference differencing. Applying Leamer (1981) will generate hyperbolae for each variety. As suggested before, Feenstra (1994)'s estimator minimizes the distance between the hyperbolae across different varieties. Practically, the first stage of Feenstra (1994)'s 2SLS procedure regresses X_{1gvt} and X_{2gvt} on country (i.e., variety) indicators. As noted by Feenstra (1994), the 2SLS estimation is equivalent to estimating $\overline{Y}_{gv} = \theta_{1g}\overline{X}_{1gv} + \theta_{2g}\overline{X}_{2gv} + \overline{u}_{gv}$, where an overbar represents the average within a variety over time, with WLS using the inverse of T as weights. The first stage of 2SLS thus maps each variety's hyperbola to a single observation defined by its variance of prices $(\overline{Y})$ and shares $(\overline{X}_1)$ along with its price and share covariance $(\overline{X}_2)$. The second stage then fits a line through these points by regressing $\overline{Y}$ on $\overline{X}_1$ and $\overline{X}_2$.

To further illustrate the relationship between Feenstra (1994)'s estimator and Leamer (1981)'s hyperbolae, consider dividing both sides of Eq. (5) averaged over time by the covariance of prices and shares $(\overline{X}_2)$. Doing so yields,

$$\frac{\overline{Y}_{gvt}}{\overline{X}_{2gvt}} = \theta_1\frac{\overline{X}_{1gvt}}{\overline{X}_{2gvt}} + \theta_2 + \overline{u}'_{gvt} \Rightarrow \frac{Var(p)}{Cov(p,s)} = \theta_1\frac{Var(s)}{Cov(p,s)} + \theta_2 + \overline{u}'_{gvt} \Rightarrow$$

$$b_r = \theta_1\frac{1}{b} + \theta_2 + \overline{u}'_{gvt}.$$

This transformation highlights how hyperbolae are qualitatively embedded in the estimator.



(a) Men's Suits

(b) Steel Toe Footwear

(c) Flat-Rolled Steel

(d) Motor Vehicles

Fig. 3. Constrained Leamer (1981) hyperbolae from representative imported products.

To illustrate the estimation procedure, Fig. 4 plots the Feenstra (1994) hyperbolae after they have been mapped into a single observation.[8] The data are weighted by the t-statistic produced in the first stage of the 2SLS procedure where larger dots correspond with larger t-statistics. This weighting provides intuition regarding which hyperbolae are most precisely estimated. The lines of best fit for each technique discussed in this paper applied to Eq. (5) are plotted through these transformed hyperbolae. Feenstra (1994) is denoted by 2SLS, while the method advocated here is denoted by LIML. Here we can see some of the difficulties of the estimation as outlier hyperbolae can substantially impact our estimates.

Feenstra (1994)'s 2SLS method gives equal weight to each country's hyperbola. Fig. 4(b) provides the starkest example of differences between 2SLS and LIML. There, for steel toe footwear, the equal weighting of 2SLS effectively discounts the two hyperbolae at the top of the picture. Following sections will argue explicitly that ignoring these hyperbolae, which are precisely estimated according to their first stage, leads to bias in 2SLS. Conversely, the LIML strategy advocated here reweights the sample hyperbolae by estimated residuals to better control for outlier hyperbolae.[9] The LIML fitted line is thus more attracted to the top hyperbolae in Fig. 4(b), which are precisely estimated, and less attracted to the imprecisely estimated hyperbolae furthest left. It is worth noting that increasing the number of hyperbolae may exacerbate rather than mitigate the biases. The following demonstrates that the precision of the hyperbolae, which is driven by the number of time periods in the sample, determines biases in 2SLS and gains from adopting LIML.

A parallel issue with the estimation is addressing the potential infeasibility of our estimates. If 2SLS produces $\hat{\theta}_1 < 0$, the resulting supply

[8] To ensure these data points represent the regression run on Eq. (5), I purge the $cov(X_1, X_2)$ from X_1 and the $cov(Y, X_2)$ from Y before plotting the data.

[9] Anderson and Rubin (1949) described the asymptotic properties of LIML. The LIML estimators used in this paper are Fuller (1977) adjusted with the adjustment parameter set equal to unity. Hausman et al. (2007) provide a thorough description of how the estimator is constructed and its relation to 2SLS (pages 3–5), and Anderson (2005) documents its origins. Notably, LIML is asymptotically equivalent to 2SLS, and the gains described by this paper are a consequence of small samples (in terms of T) inherent to trade data.

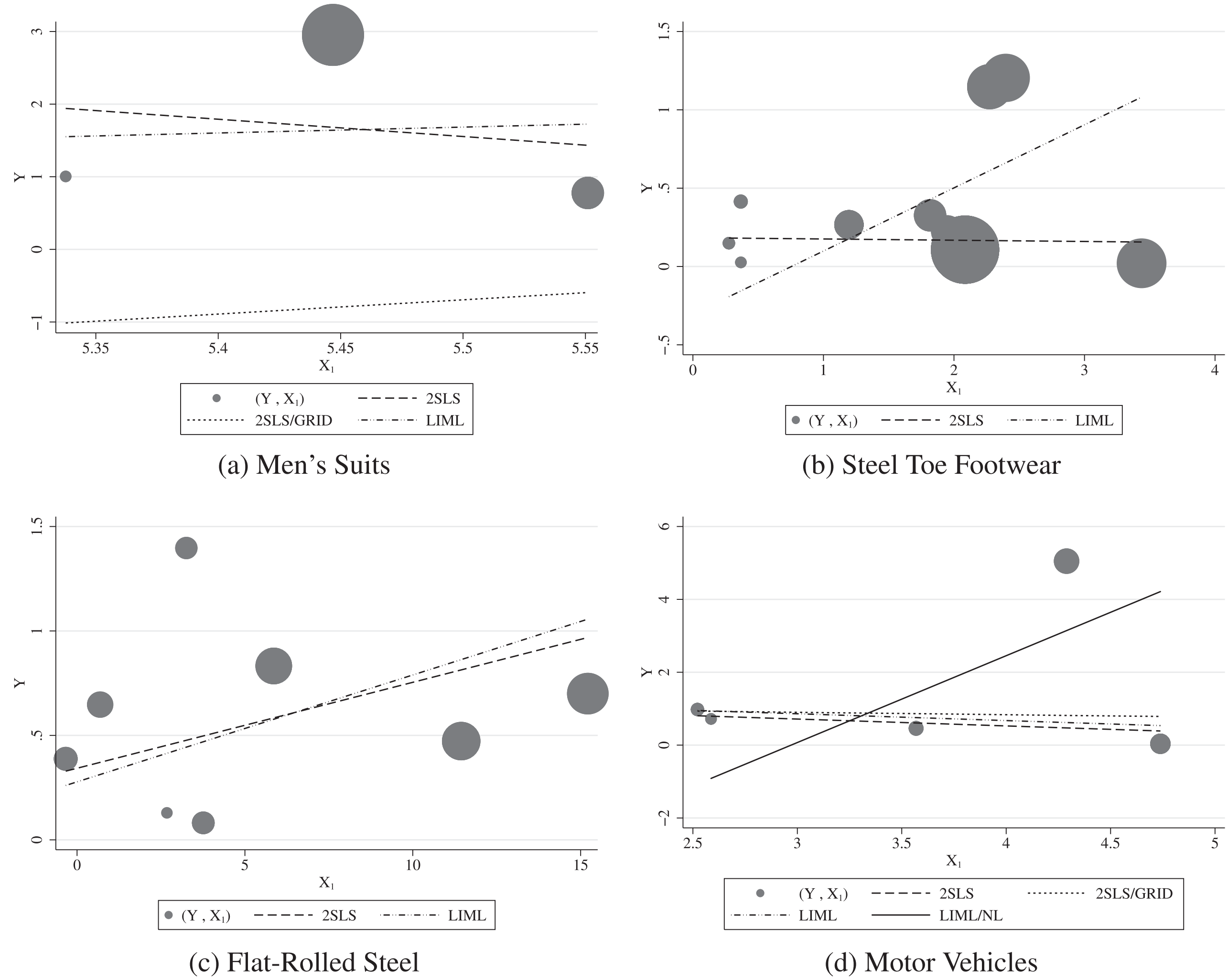


Fig. 4. Mapping of hyperbolae into data: estimating Eq. (5).
Notes: These figures plot Feenstra (1994) hyperbolae mapped into data. The data are weighted by their first stage t-statistics, where larger dots represent larger t-statistics. In some cases (e.g., Men's suits) there are more observations than hyperbolae since the data requirements for plotting hyperbolae are greater than Feenstra (1994)'s estimator.

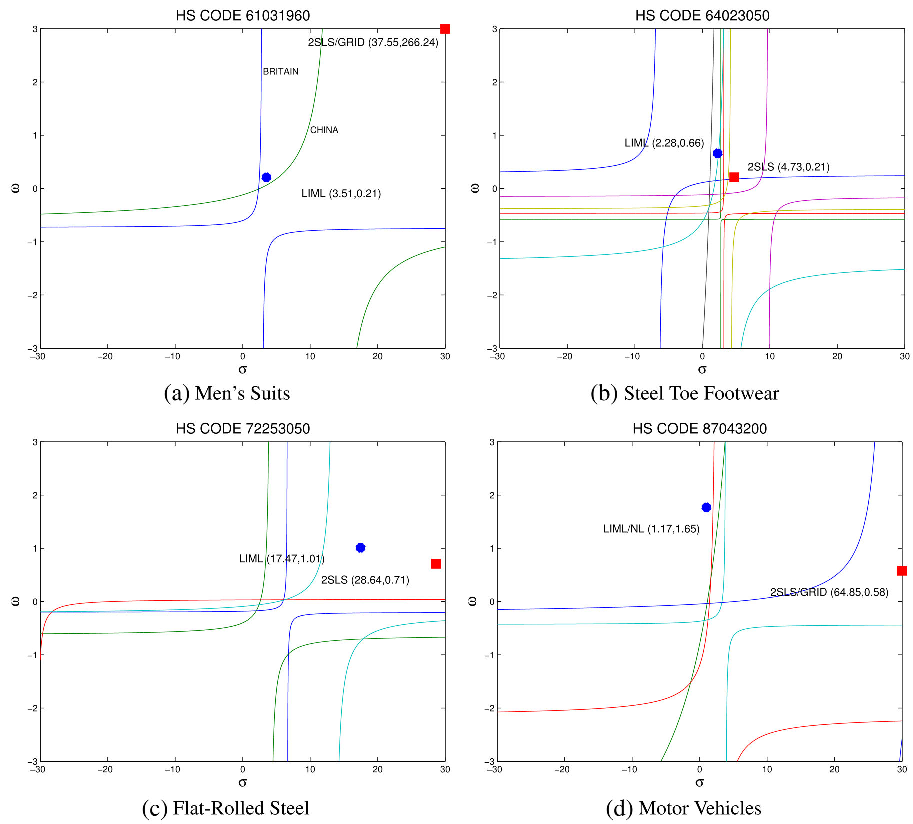
and demand elasticity estimates lie in the infeasible region.[10] Broda and Weinstein (2006) propose a constrained grid search in the second stage to replace infeasible estimates. Fig. 4(a) highlights this modification. The fitted 2SLS line is downward sloping, which leads to infeasible elasticities. There, Broda and Weinstein (2006)'s 2SLS/GRID is also plotted. The performance of F/BW's 2SLS and GRID method will be systematically analyzed in the following, but these raw data plots give insight into the main intuition: By equally weighting hyperbolae F/BW suffers from significant biases (i.e., its estimate lie far from all other plotted data points). The hybrid estimator developed here uses LIML to reweight outlier hyperbolae. If LIML results in infeasible estimates, it is then replaced with a nonlinear LIML routine constrained to the feasible regions. Fig. 4(d) shows the F/BW method labeled 2SLS and 2SLS/GRID along with the hybrid method labeled LIML and LIML/NL. While we cannot yet draw any concrete conclusions about the observed differences, Fig. 4 visually suggests that the hybrid method is less attracted to hyperbolae with imprecise first stages.

The following will use Monte Carlo simulations and estimates plotted against actual hyperbolae to demonstrate the superiority of the hybrid method. It is therefore worthwhile to introduce the Feenstra (1994) hyperbolae before they are mapped to data (i.e., transform hyperbolae plotted in (β, Θ) space to (σ, ω) space). Fig. 5 thus applies Feenstra (1994)'s transformation to the Leamer (1981) hyperbolae presented in Fig. 3. Additionally, Fig. 5 includes the actual estimated supply and demand elasticities from F/BW (2SLS and 2SLS/GRID) and the hybrid method (LIML and LIML/NL) alongside the actual hyperbolae.

Focus on flat-rolled steel in Fig. 5(c). The Feenstra (1994) 2SLS estimate is $\hat{\sigma} = 28.64$ and $\hat{\omega} = 0.71$, which is indicated by a square. The estimates from the methodology advocated in this paper are $\hat{\sigma} = 17.47$ and $\hat{\omega} = 1.01$, which I denote with a circle and label as LIML. These hyperbolae provide a useful context for evaluating our estimates next to actual data. The feasible region is where demand slopes down ($\sigma > 1$) and supply slopes up ($\omega \geq 0$). Fig. 6 plots the hyperbolae constrained to the feasible region along with various estimates from the model. At first glance, the estimators follow our notions of minimizing the distance between the set of hyperbolae. More scientific discussion will be presented in the following sections regarding the performance of our estimators in the context of these hyperbolae, but it appears that the hybrid estimates do better relate to the presented hyperbolae.

The remainder of the paper uses the preceding intuition to compare the performance of the standard 2SLS procedure with a hybrid LIML procedure. For the sake of continuity, the detailed discussion of implementation in practice is presented in Appendix A. I optimally normalize

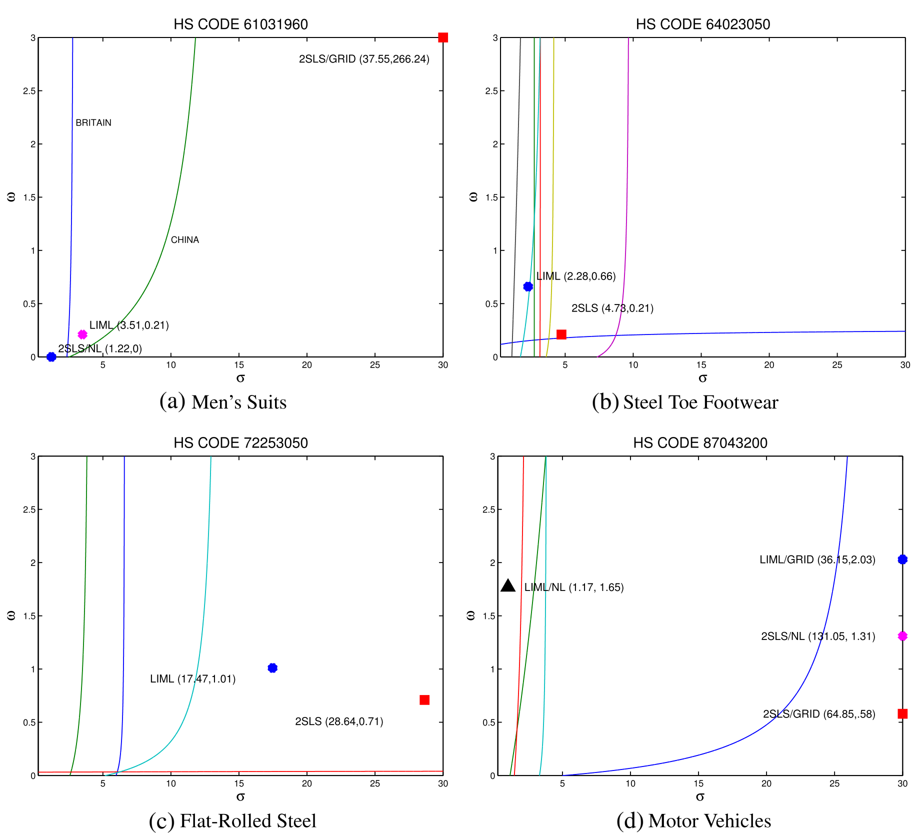
[10] Specifically, $\hat{\theta}_1<0\Rightarrow\hat{\rho}<0$ or $\hat{\rho}>1\Rightarrow\hat{\sigma}<1$ or $\hat{\omega}>0$, which violates our underlying assumptions.



(a) Men's Suits (b) Steel Toe Footwear

(c) Flat-Rolled Steel (d) Motor Vehicles

Fig. 5. Feenstra (1994) hyperbolae from representative imported products.



(a) Men's Suits (b) Steel Toe Footwear

(c) Flat-Rolled Steel (d) Motor Vehicles

Fig. 6. Constrained Feenstra (1994) hyperbolae from representative imported products.

Notes: These figures plot Feenstra (1994) hyperbolae constrained to the feasible region along with actual estimates of import demand and export supply elasticities. The F/BW estimate is denoted by ■ and recorded as $(\hat{\sigma}, \hat{\omega})$: 2SLS indicates Feenstra (1994) returned feasible estimates while 2SLS/GRID indicates Broda and Weinstein (2006)'s grid search was applied. The Hybrid estimate is denoted by ● and recorded as $(\hat{\sigma}, \hat{\omega})$: LIML indicates LIML returned feasible estimates while LIML/NL indicates the nonlinear LIML was applied.

how we take the methodology to the data in order to provide consistent comparisons of the standard F/BW estimator alongside alternative methodologies.

Going forward, Section 3 presents Monte Carlo analysis in order to compare the performance of the standard estimator to a hybrid estimator. Monte Carlo simulations suggest substantial numerical bias in the standard estimator. This bias is shown to be considerably reduced through the adoption of a hybrid LIML method. Section 4 applies each method to actual trade data. I utilize hyperbolae to provide intuition regarding how the hybrid method corrects the numerical biases of the standard method. Furthermore, the hyperbolae highlight that the hybrid method seems to fit actual data better than the standard method.

3. Monte Carlo analysis

In this section, simulated data are used to describe how estimated supply and demand elasticities respond to estimator choice. Each estimation is conducted in two steps. First we attempt to uncover supply and demand elasticities by estimating Eq. (5). Second, if the first step produces theoretically infeasible values (i.e., $\sigma \leq 1$, $\omega < 0$, or imaginary estimates) we reevaluate the GMM problem by constraining the parameter space to $\sigma > 1$ and $\omega \geq 0$. The goal of the Monte Carlo routine is to compare numerical biases produced from the standard method, 2SLS in the first step (i.e., Feenstra (1994)) followed by a constrained grid search if necessary (i.e., Broda and Weinstein, 2006), with the proposed hybrid method, LIML in the first step followed by constrained nonlinear LIML in the second. Additionally, I will consider LIML with a grid search to examine how the grid search fares compared to the constrained nonlinear LIML developed in this paper.

The data generating process closely follows Soderbery (2010), and is targeted at generating variances in prices and shares that are comparable to US import data at the *HS*8 level.[11] To match the notation from Soderbery (2010), define $\tilde{\varepsilon}_{vt} \equiv \frac{1}{\sigma-1}\varepsilon_{gvt}$ and $\tilde{\delta}_{vt} \equiv \frac{1+\omega}{1+\sigma\omega}\delta_{gvt}$. Differenced prices and shares can be written solely in terms of the differenced errors,

$$\Delta^k lnp_{vt} = \rho\ \tilde{\varepsilon}^k_{vt} + \tilde{\delta}^k_{vt}$$
$$\Delta^k lns_{vt} = (\sigma-1)(1-\rho)\tilde{\varepsilon}^k_{vt} - (\sigma-1)\tilde{\delta}^k_{vt}.$$

Draw the error terms from independent normal distributions with random variances coming from the uniform distribution such that,

$$\tilde{\varepsilon}_{vt} \sim N\left[0, \sigma^2_{\varepsilon,v}\right] \quad \text{where,} \quad \sigma^2_{\varepsilon,v} \sim U(0,9),$$
$$\tilde{\delta}_{vt} \sim N\left[0, \sigma^2_{\delta,v}\right] \quad \text{where,} \quad \sigma^2_{\delta,v} \sim U(0,9).$$

The randomly generated error terms create a balanced panel of prices and shares across varieties. In order to mimic actual data, I randomly drop observations for some varieties to generate an unbalanced panel. Finally, I impose measurement error to reflect the error associated with using unit values in place of actual prices.[12]

I generate 200 Monte Carlo data sets at varying levels of T with the maximum number of varieties fixed at 55.[13] I then apply the three estimators to each set of simulated data. Fig. 7(a) presents the results of the Monte Carlo estimates ignoring the constrained optimization. The vertical axis is the numerical bias of the estimated demand elasticity (defined as $\frac{\hat{\sigma}-\sigma}{\sigma}$) averaged across the Monte Carlo simulations. The small sample biases discussed in Soderbery (2010) are immediately evident — in the smallest sample, the numerical bias with 2SLS is around 30%. At every T interval LIML outperforms 2SLS, and substantially so for reasonable T.

Even though Feenstra (1994)'s 2SLS method is not a classic instrumental variables problem, we can still understand its biases in the context of weak instruments.[14] The rule of thumb proposed by Staiger and Stock (1997) to identify weak instruments in 2SLS is observing F-statistics from the first stage below ten. At the lowest interval where $T = 5$ the average F-statistic across the 200 generated data sets is 2.3. Increasing T by ten increases the average F-statistic by about two and cuts the simulated bias of 2SLS in half. Consequently, the simulations do not generate an average F-statistic exceeding ten until $T = 45$, at which point the numerical bias of 2SLS is around 5%.[15] This provides a convenient way to interpret the numerical bias in Feenstra (1994)'s 2SLS procedure as a consequence of weak instruments that are exaggerated in small samples. The deficiencies of 2SLS, and benefits of LIML, in the presence of weak instruments are well known.[16] The gains from LIML here are realized as the estimator places less weight on outlier (defined by their first stage statistical precision) hyperbolae than 2SLS.

Due to the perpetual reclassification of goods trade data present with small samples in terms of T. In the US we usually see panels of import data spanning 10–20 years — my sample includes 15 years of data. We have evidence of the performance of the original estimator in small samples, but further investigation of the constrained estimation is due. Notably, in both actual and simulated data, the constrained estimator is frequently triggered. The prevalence of an infeasible first step in simulated data is illustrated by Fig. 7(b). When $T = 15$, 2SLS triggers the constrained estimator 70% of the time, while LIML triggers the constrained estimator only 20% of the time. This frequency difference is presumably due to LIML's ability to better identify outlier hyperbolae that are pushing the estimator toward the infeasible region. Fig. 7(c) displays the bias of the second step estimates. The bias of the grid search is persistent and ranges from 15% to upwards of 40% for the lowest levels of T. The nonlinear estimator fares much better. The absolute bias is less than 5% in most cases and is always less than one third the size of the grid search.

Fig. 7(d) combines the first and second step estimates.[17] When $T = 15$ the simulated numerical bias of 2SLS is 20% compared to only 3% with the hybrid estimator. Additionally, numerical bias in the hybrid estimator converges to zero rapidly and outperforms the alternative estimators at each time interval.

To put our simulations in the context of hyperbolae, Fig. 8 displays the Feenstra (1994) mapping of hyperbolae to data for randomly generated data from a single Monte Carlo simulation at each time interval. These figures are the simulated counterparts to Fig. 4, and include the true regression line implied by the parameter assumptions of the

[11] Given the thousands of possible comparisons that can be made between each simulation and each good in the data, it is not tractable to present individual comparisons here. Figures comparing actual data to simulated are available upon request. Additionally, Soderbery (2010) describes the desirable performance of the simulations.

[12] Prices in the Monte Carlo are consequently replaced by,

$$\Delta^k lnUV_{vt} = \Delta^k lnp_{vt} + \mu^k_{vt} \quad \text{where} \quad \mu^k_{vt} \sim N[0, 0.15].$$

[13] Soderbery (2010) shows the estimator is insensitive to the number of varieties chosen and level of σ and ω. Thus, 55 varieties with $\sigma = 3$ and $\omega = 0.5$ are arbitrarily chosen to simulate actual data.

[14] The instruments here are country indicators. Their sole function is to convert hyperbolae into data. Ultimately, the 2SLS procedure averages price and quantity outcomes such that they are single observations representing price and quantity variances and covariances.

[15] Notably, Stock and Yogo (2005) generalize the rule of thumb across 2SLS and LIML for various numbers of instruments. With two endogenous regressors and 30 instruments, which is similar to the preceding Monte Carlo, the critical F-statistic to expect 2SLS bias under 10% is 63. For the same scenario using the Fuller adjusted LIML advocated here, the critical F-statistic is 2.28. This supports the numerical bias we see in the smallest sample when applying LIML and the rapid elimination of this bias as T increases.

[16] See, among others, Staiger and Stock (1997), Stock and Yogo (2005) and Hahn et al. (2004) for Monte Carlo analysis and thorough description of these biases.

[17] The magnitude of the biases are slightly different here when compared to Soderbery (2010). This is a direct consequence of the grid search. In Soderbery (2010) the grid search is done over $\sigma \in [1.05, 31.05]$ at equal intervals of 0.025, here the grid search is over $\sigma \in [1.05, 55.05]$ at intervals of 0.13 to more closely match Broda and Weinstein (2006).

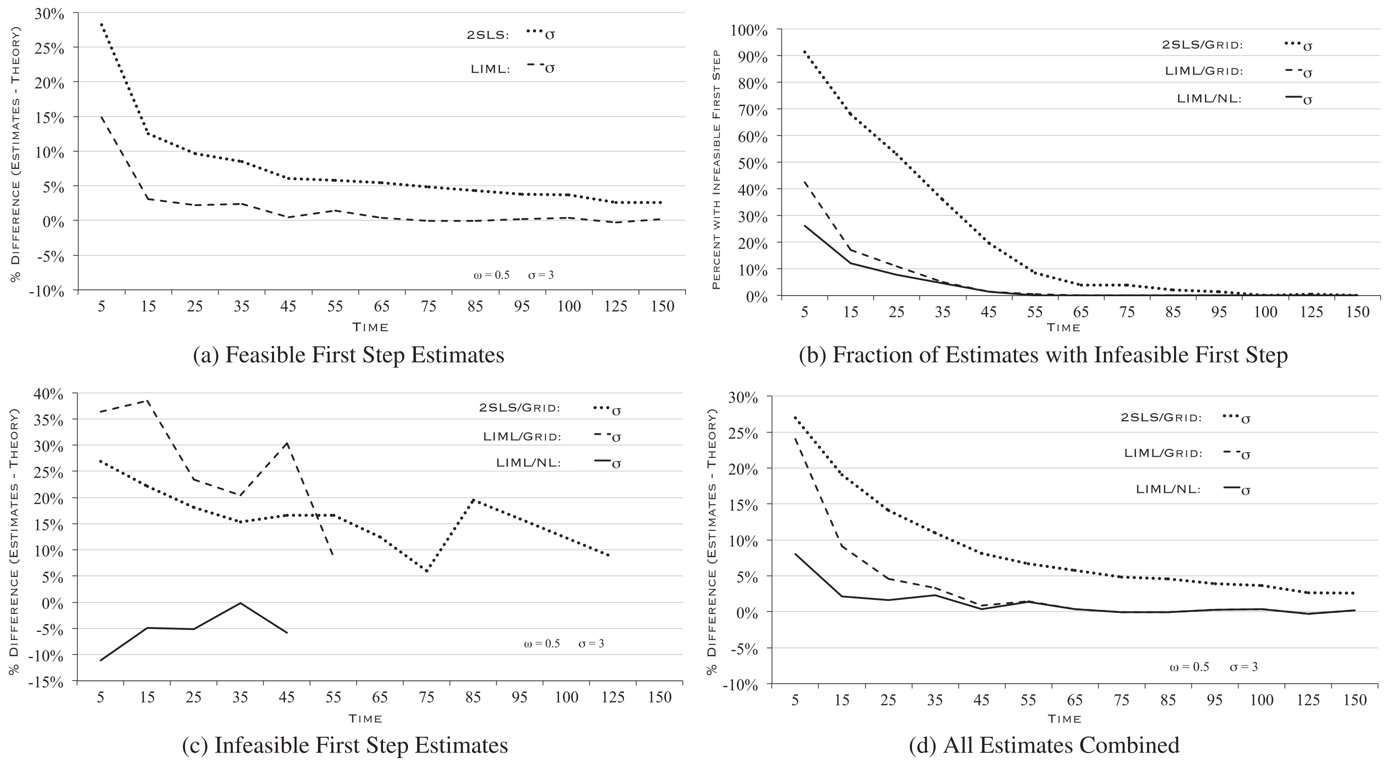


Fig. 7. Monte Carlo results.

simulated data along with fitted LIML and 2SLS lines. We can see the dispersion of the price and share variances and their covariance terms (i.e., the hyperbolae) is much greater in smaller samples. Fig. 8 also highlights the source of small sample bias with 2SLS. In small samples, the hyperbolae are more disperse and often far from the true line. The differences in weighting of hyperbolae between 2SLS and LIML are evident, as we can see LIML is less drawn toward outlier hyperbolae and consistently closer to the true line at every time interval.

4. Estimates of supply and demand

In this section, actual data are used to describe how estimated supply and demand elasticities respond to estimator choice. To reiterate, each estimation is conducted in two phases. First, we attempt to uncover supply and demand elasticities with Eq. (5). Second, if the first step produces theoretically infeasible estimates (i.e., $\sigma \leq 1$, $\omega < 0$, or imaginary values) we reevaluate the GMM problem by constraining the parameter space to $\sigma > 1$ and $\omega \geq 0$. The following will discuss deficiencies of each estimator at each step of estimation, offer intuition for any shortcomings, and highlight the improvements associated with the hybrid estimator.

Given the hundreds of thousands of elasticities estimated in this paper line by line comparisons are simply infeasible. The following will thus present summary statistics of our various estimates that characterize the distributional differences across estimators. Notably, the full set of elasticities and code applied in this analysis are available at my personal website.[18]

4.1. Feasible first step: 2SLS and LIML

I estimate supply and demand elasticities for the universe of *HS*8 goods imported by the US over the 15 year period from 1993 to 2007 using 2SLS and LIML in the first step.[19] I will start by comparing only the estimates where both procedures return feasible estimates of σ and ρ.

Fig. 5(b) and 5(c) provide examples from our data of goods where the estimator returns feasible values in the first step regardless of technique. Fig. 5(b) are the hyperbolae for all varieties of steel toe footwear imported by the US. The hyperbolae are relatively concentrated, save for one (the curves furthest right). Both 2SLS and LIML seem to approach the same mass of points where the majority of the hyperbolae intersect. Estimates are generated from a maximum of 15 years of data spanning 1993–2007. For the case of steel toe footwear, the 12 varieties exist for an average of 7.5 years. The 2SLS estimate of the demand elasticity is 4.73, which is more than double the LIML estimate of 2.28. This suggests that small sample biases play a significant role. This assertion is supported by Staiger and Stock (1997)'s rule of thumb. The first stage F-statistic for steel toe footwear is 4.55, which is well below the cutoff level of 10 proposed by Staiger and Stock (1997) to identify biased 2SLS due to weak instruments. Visually, Fig. 5(b) provides further support, as the LIML estimate lies more in the heart of the hyperbolae than 2SLS.

The hyperbolae for flat-rolled steel presented by Fig. 5(c) are considerably more disperse than those for footwear. This is partly due to the relatively small sample size. The 4 varieties exist for an average of 6.5 years. The resulting LIML estimate of the demand elasticity is significantly lower than that of 2SLS (17.47 versus 28.64), which again suggests bias in the 2SLS estimator. The assertion of weak instruments is supported by the first stage F-statistic, which is 1.65 for flat-rolled steel. The positioning of the estimates relative to the hyperbolae

[18] http://web.ics.purdue.edu/asoderbe/Site/Elasticities.html.

[19] To reiterate, I rely on import data from the *Center for International Data* aggregated to the *HS*8 level. For a full description of the data, see Feenstra et al. (2002). The data are housed at http://cid.econ.ucdavis.edu/usix.html.

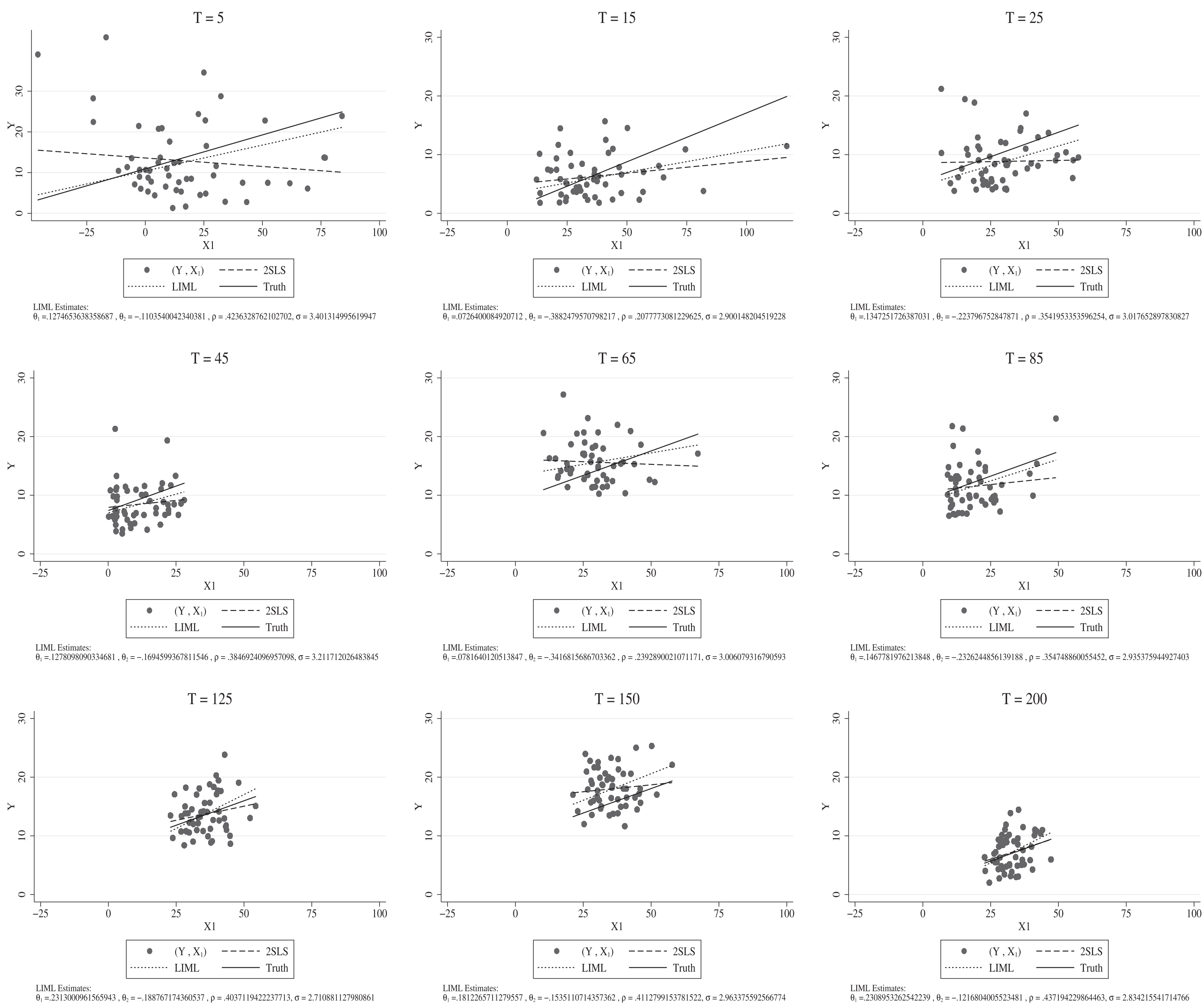


Fig. 8. Monte Carlo results: mapping hyperbolae into data.

indicates that the 2SLS estimator seems to overweight the outlier hyperbolae running across the middle of Fig. 5(c).

Table 1 presents the distribution of the σ and ρ estimates for all goods where the first stage of 2SLS and LIML is feasible.[20] The upward bias of σ_{2SLS} we saw in the Monte Carlo is reiterated by the distribution of estimates. The average 2SLS estimate is 10% greater than LIML, while the median is nearly 50% larger. Additionally, the estimated distributions of the demand elasticities are statistically significantly different from one another.[21]

4.2. Infeasible first step: constrained grid and nonlinear search

If the first step estimate yields $\hat{\theta}_1 < 0$, intractable estimates of supply and demand elasticities are produced. In this case, Broda and Weinstein (2006) propose a second step where we optimize the GMM problem from Eq. (5) by searching over feasible elasticities ($\sigma > 1$ and $\omega \geq 0$) only. We select a grid of potential values, calculate the GMM objective function from Eq. (5), and find the minimum. Fig. 9 presents a hypothetical objective function that would trigger such a search. The global optimum in this example occurs where $\hat{\theta}_1 < 0$. The grid search then constrains the elasticities, calculates the objective function at preselected points (marked with an *x*), and returns $\hat{\theta}_1 = 0$ as the minimization of GMM problem.

As a practical example, Fig. 5(d) are the hyperbolae for varieties of motor vehicles. There, regardless of the first step estimator, infeasible

Table 1
HS8 estimates: feasible first step.

Estimate	N	Percentile					Mean
		10th	25th	50th	75th	90th	
σ_{2SLS}	4577	1.843	2.289	3.129	4.879	8.251	5.026
σ_{LIML}	4577	1.304	1.538	2.131	3.536	7.110	4.526
ρ_{2SLS}	4577	0.116	0.236	0.406	0.579	0.742	0.418
ρ_{LIML}	4577	0.049	0.160	0.398	0.654	0.829	0.419

Notes: Estimates of σ are censored at 131.05 for exposition.

[20] For exposition, estimates are censored at 131.05. Footnote 21 in Broda and Weinstein (2006) describes the lack of significance of large σ for estimated variety gains, and provides the intuition as to why I do not lend much analysis to the right tail of the distribution.

[21] Kolmogorov–Smirnov tests confirm the entire distributions differ significantly.

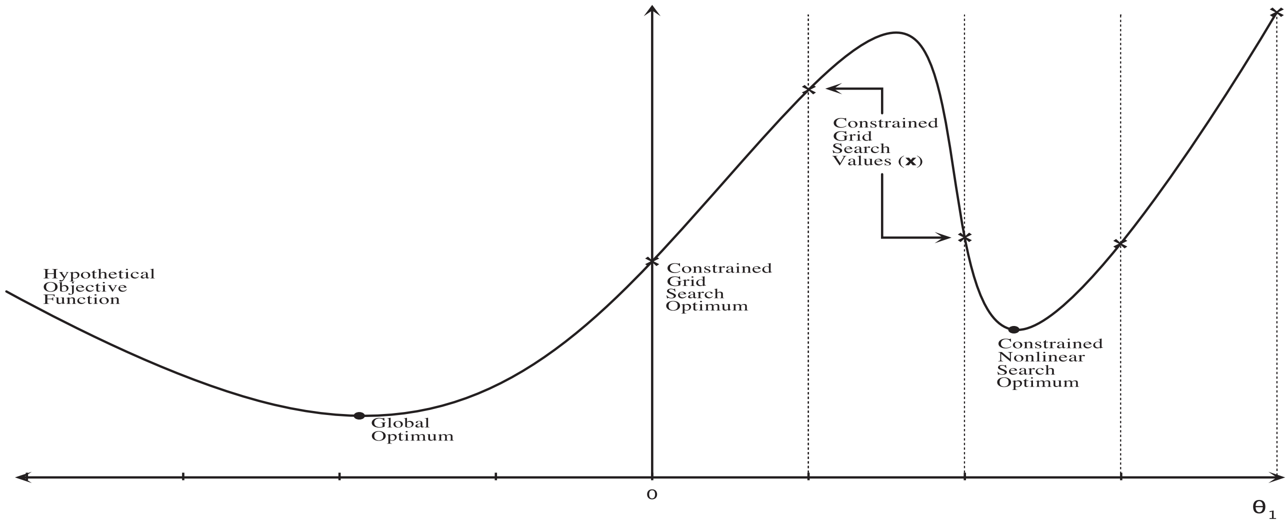


Fig. 9. Hypothetical GMM objective function.

estimates of supply and demand elasticities are produced. It is not surprising that the first step estimator is drawn toward an intractable estimate given the numerous intersections of hyperbolae in the infeasible region for motor vehicles (see also Fig. 5(b)). Fig. 6 imposes our feasibility constraints on hyperbolae just as the constrained search does. Before discussing the results from the different constrained optimization techniques some properties of the problem should be noted.

Actual grid search estimates suggest that the hypothetical situation in Fig. 9 is a prevalent one. The constrained grid search optimum occurs where $\hat{\theta}_1 \approx 0$. Fig. 10 displays the distributions of 2SLS and the grid searched estimates, $\hat{\rho}$ and $\hat{\theta}_1$. Over 90% of grid searched $\hat{\theta}_1$s are approximately zero. Recall that $\theta_1 \equiv \frac{\rho}{(\sigma-1)^2(1-\rho)}$ in the model. The form of θ_1 implies two ways to produce $\hat{\theta}_1 \approx 0$; either $\hat{\rho} = 0$ or $\hat{\rho} \rightarrow \frac{\sigma-1}{\sigma}$. This polarization is precisely what we see in the data where nearly 90% of the grid searches yield perfectly elastic ($\hat{\rho} = 0$) or inelastic ($\hat{\rho} \rightarrow \frac{\sigma-1}{\sigma}$) supply elasticities. The resulting grid search distributions of $\hat{\rho}$ and $\hat{\theta}_1$ are considerably different than those produced when the first step returned feasible estimates. These distributional differences can have perverse implications given the structure of the model.

Consider the case where $\rho = 0$. Eq. (5) becomes $Y_{gvt} = 0 * X_{1gvt} + \frac{-1}{\sigma-1} X_{2gvt} + \varepsilon^k_{gvt} \delta^k_{gvt}$, which fits well with the assertion that $\hat{\theta}_1 \approx 0$ minimizes the objective function. The problem reverts to choosing the optimal $\hat{\sigma}$ from a regression of the variance of prices on shares. Notably, this is in the same spirit as gravity type estimates of import demand, which implicitly assume horizontal supply to identify demand elasticities. However, if we instead assert $\rho \rightarrow \frac{\sigma-1}{\sigma}$, Eq. (5) becomes, $Y_{gvt} = \frac{1}{\sigma-1} * X_{1gvt} + \frac{\sigma-2}{\sigma-1} X_{2gvt} + \sigma \varepsilon^k_{gvt} \delta^k_{gvt}$. Then if $\hat{\theta}_1 \approx 0$ minimizes the objective function, the structure of the model essentially forces $\hat{\sigma}$ to be large. These patterns are precisely what we see from the 2SLS grid search. One could argue that the grid search is subject to Leamer (1981)'s critique, in that we are *at best* identifying one elasticity with precision but not the other. As was done by the gravity literature, when $\hat{\rho} = 0$ and $\hat{\theta}_1 \approx 0$, we can argue that $\hat{\sigma}$ is estimated with precision while the model provides no useful information about $\hat{\omega}$. However, when $\rho \rightarrow \frac{\sigma-1}{\sigma}$ and $\hat{\theta}_1 \approx 0$, both $\hat{\sigma}$ and $\hat{\omega}$ seem predetermined by the model, which calls into question the reliability of each.

An added issue with the grid search is illustrated by Fig. 9. Since the grid is coarse by design it has a possibility of missing the actual constrained optimum. The set of hyperbolae in Fig. 6(a) illustrates this point. The 2SLS/GRID estimate yields $\hat{\omega} \rightarrow \infty$, which is viable considering both hyperbolae contain large ω. However, the resulting estimate of σ is nowhere near the bounds of the hyperbolae suggesting the grid search is settling on a local minimum well away from the constrained optimum. A nonlinear search subverts the possibility of missing the optimum, and we can see the 2SLS/NL estimate gets much closer to where the constrained hyperbolae for suits in Fig. 7(a) intersect.

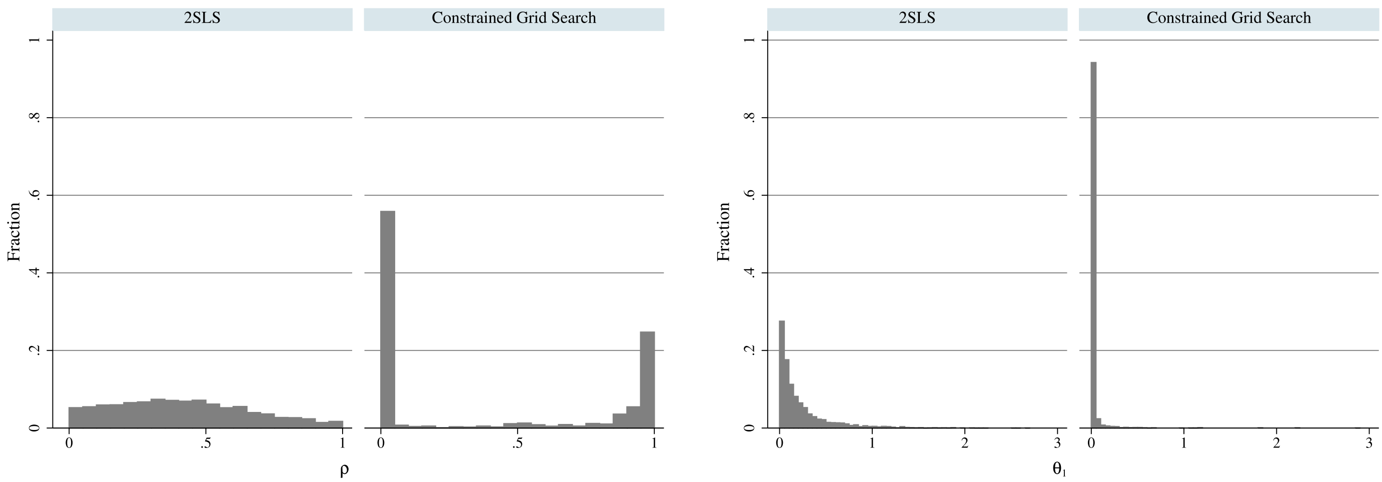
Replacing the grid search with nonlinear 2SLS was an improvement in the case of suits in Fig. 6(a), but to correct for small sample biases the nonlinear LIML procedure is called for. Consider the constrained hyperbolae for motor vehicles in Fig. 6(d). The 2SLS/GRID estimate seems to miss the mass of hyperbolae, as it tends toward $\hat{\rho} = 0$ (i.e., $\hat{\omega} \approx 0$). Replacing the grid search with a nonlinear search, labeled 2SLS/NL, moves $\hat{\omega}$ toward the mass of hyperbolae but provides no precision on the σ dimension. In this case, 2SLS gives too much weight to the outlier hyperbola which lies furthest to the right. Moving to LIML/GRID, the grid search is still based on 2SLS and lacks precision for $\hat{\sigma}$. The hybrid LIML/NL estimate is based wholly on LIML and gives considerably less weight to outlier hyperbolae. Most of the estimator's weight is placed on the hyperbola grouped furthest left, which leads to a more precise estimate of supply and demand elasticities with respect to the constrained hyperbolae.

Turning to the full set of estimates, Table 2 presents second stage estimates when both 2SLS and LIML are infeasible in the first step. The 2SLS/GRID, 2SLS/NL, and LIML/GRID all display this polarization of $\hat{\rho}$ discussed above. The implications of this polarization on our estimates

Table 2
HS8 estimates: infeasible first step.

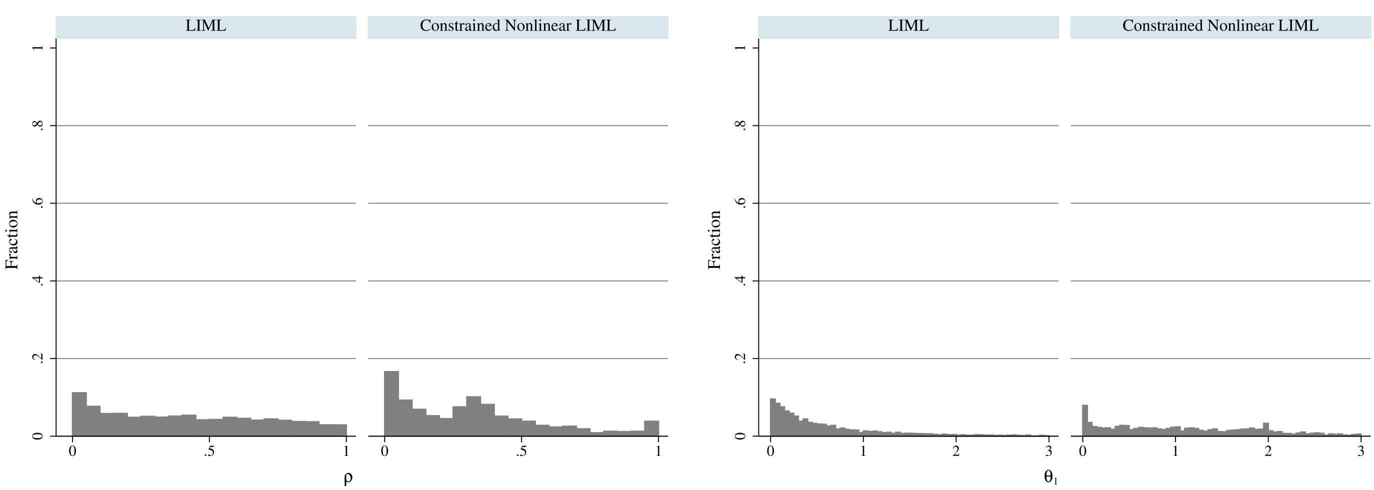
Estimate	N	Percentile					Mean
		10th	25th	50th	75th	90th	
$\sigma_{2SLS/GRID}$	1730	1.750	2.650	6.650	91.55	131.05	42.203
$\sigma_{2SLS/NL}$	2304	1.669	2.153	4.275	131.05	131.05	45.860
$\sigma_{LIML/GRID}$	1730	1.550	2.050	5.450	120.45	131.05	44.237
$\sigma_{LIML/NL}$	1583	1.077	1.270	1.525	1.988	4.384	6.791
$\rho_{2SLS/GRID}$	1730	0.000	0.000	0.000	0.959	0.992	0.362
$\rho_{2SLS/NL}$	2304	0.000	0.000	0.000	0.998	1.000	0.322
$\rho_{LIML/GRID}$	1730	0.000	0.000	0.000	0.959	0.992	0.370
$\rho_{LIML/NL}$	1583	0.024	0.104	0.309	0.464	0.713	0.332

Notes: Estimates of grid searched σ (denoted GRID) are censored at 131.05 by design, and estimates of nonlinear searched (denoted NL) σ are censored for exposition.



(a) Distribution of $\hat{\rho}$ (b) Distribution of $\hat{\theta}_1$

Fig. 10. Comparing first and second step estimates: standard estimator.



(a) Distribution of $\hat{\rho}$ (b) Distribution of $\hat{\theta}_1$

Fig. 11. Comparing first and second step estimates: hybrid estimator.

$\hat{\sigma}$ are reflected in the fact that the 2SLS estimates produce an average demand elasticity seven times the size of the hybrid method. The constrained 2SLS estimators additionally produce distributions of elasticities that are drastically different than those in Table 1 where the first step was feasible. Comparing Figs. 10 and 11, LIML/NL presents distributions of $\hat{\rho}$ and $\hat{\theta}_1$ that are more relatable to the unconstrained estimates. The result of the hybrid estimator correcting the unwarranted polarization is a distribution of $\hat{\sigma}$ that is considerably less concentrated in the right tail.[22]

Table 3
HS8 estimates: entire sample.

Estimate	N	Fraction $\hat{\theta}_1$<0	Percentile					Mean
			10th	25th	50th	75th	90th	
$\sigma_{2SLS/GRID}$	9468	0.377	1.834	2.375	3.688	8.664	87.15	19.288
$\sigma_{2SLS/NL}$	9363	0.370	1.741	2.238	3.391	7.018	131.05	20.521
$\sigma_{LIML/GRID}$	8624	0.390	1.350	1.650	2.530	6.955	118.85	19.941
$\sigma_{LIML/NL}$	7633	0.310	1.212	1.431	1.855	3.102	6.892	5.261
$\rho_{2SLS/GRID}$	9468	0.377	0.000	0.060	0.361	0.662	0.957	0.401
$\rho_{2SLS/NL}$	9363	0.370	0.000	0.022	0.327	0.628	0.997	0.382
$\rho_{LIML/GRID}$	8624	0.390	0.000	0.013	0.324	0.750	0.959	0.399
$\rho_{LIML/NL}$	7633	0.310	0.036	0.130	0.350	0.611	0.823	0.390

Notes: Estimates of grid searched σ (denoted GRID) are censored at 131.05 by design, and estimates of nonlinear searched (denoted NL) σ are censored for exposition.

4.3. Comparing F/BW to the hybrid estimator in the full sample

Table 3 presents estimates for the universe of HS8 goods across each of the estimators discussed. Focus on comparing the standard estimates (2SLS/GRID) from F/BW with the hybrid estimates (LIML/NL) proposed here. The average elasticity of substitution from the hybrid estimates is about one fourth that of the F/BW estimates, and the median is about half the size. The small sample

[22] One noticeable drawback of adopting LIML/NL is that the data requirements of the estimator result in about 10% less estimable elasticities.

Table 4
Examining whether elasticities are identical across varieties.

Estimate	Hybrid method: LIML/NL							Standard method: 2SLS/GRID						
	Goods	Frac	Percentile			Mean	K–S	Goods	Frac	Percentile			Mean	K–S
	N	$\hat{\theta}_1 < 0$	10th	50th	90th		PVal	N	$\hat{\theta}_1 < 0$	10th	50th	90th		PVal
σ_{ALL}	7633	0.31	1.21	1.86	6.89	5.26	1.00	9468	0.38	1.83	3.69	87.15	19.29	1.00
$\sigma_{Mid-High\ Inc.}$	8297	0.32	1.20	1.83	6.86	5.16	0.20	10320	0.38	1.79	3.60	86.85	19.14	0.04
$\sigma_{High\ Inc.}$	7601	0.32	1.21	1.84	6.87	5.90	0.90	9830	0.41	1.69	3.35	93.90	19.68	0.00
σ_{OECD}	7806	0.31	1.22	1.88	6.83	5.38	0.35	9949	0.40	1.72	3.45	93.05	19.73	0.00
$1/\omega_{ALL}$	7633	0.31	0.00	0.15	8.68	3.99	1.00	9468	0.38	0.01	3.82	100.00	26.73	1.00
$1/\omega_{Mid-High\ Inc}$	8297	0.32	0.00	0.19	9.25	4.21	0.69	10320	0.38	0.01	3.97	100.00	27.33	0.68
$1/\omega_{High\ Inc}$	7601	0.32	0.00	0.26	9.15	4.36	0.00	9830	0.41	0.01	3.04	100.00	26.61	0.00
$1/\omega_{OECD}$	7806	0.31	0.00	0.26	8.82	4.14	0.00	9949	0.40	0.01	3.14	100.00	26.52	0.00

Notes: Estimates of σ are censored at 131.05 and estimates of ω are censored at 100 for exposition. †K–S PVal is the probability value from a Kolmogorov–Smirnov test of distribution equality between the σ or ω in question and σ_{ALL} or ω_{ALL}.

improvements from LIML can be seen in the differences in the median elasticity, while the correction of the grid search inefficiencies is highlighted by a thinner right tail, and consequently a considerably lower average.

The hybrid estimator highlights a significant overestimation of elasticities of substitution in the literature. The impact of such an overstatement of σ has severe implications for our understanding of many channels driving gains from trade. One of the most direct implications of bias in $\hat{\sigma}$ is on calculations of consumer gains from product variety, which was the original impetus of Feenstra (1994). In Appendix B I apply the preceding estimates to study consumer gains from product variety in the US over the sample. The F/BW estimates of the elasticity of substitution are shown to understate variety gains by a factor of 6 relative to hybrid estimates. Given the profound implications of differences in our estimated elasticities a deeper examination of potential violations in the core assumptions behind the estimation technique is warranted.

5. Robustness of the methodology

Recall that the methodology hinges upon two key assumptions. First, supply and demand errors are uncorrelated. Second, supply and demand elasticities are identical across varieties (i.e., importing countries) within goods. The first assumption is fundamental not only to Feenstra (1994), but also to the analysis of Leamer (1981). I begin by investigating violations to the independence of errors through Monte Carlo and then evaluate whether actual data support the notion of identical elasticities across varieties.

Fig. 12 examines the effect of violations to the independence of errors through Monte Carlo analysis. Fixing $T = 100$, the figure plots the numerical bias in the simulated data for estimates of the demand elasticity at varying degrees of correlation between our error terms. Fig. 12 demonstrates that when the supply and demand errors are positively correlated (e.g., through endogenous quality) estimates of demand elasticities exhibit only moderate bias of 10%–25%, regardless of method. However, when errors are negatively correlated (e.g., hidden varieties) the hybrid estimator shows moderate bias while the standard method is significantly upward biased by 50%–125%.[23] Estimates of the supply elasticity suffer regardless of estimation technique, but the hybrid estimator consistently outperforms the standard. The patterns of the biases that result from violations to exogeneity seem to coincide with previous literature. Hallak and Schott (2011) discuss the role of quality on calculating price indexes with trade. Unobserved quality in their model implies a positive correlation between supply and demand error terms. This correlation could lead to downward biases in estimates of σ as consumers appear less responsive to price changes. Conversely, if there are hidden varieties we would expect the error terms to be negatively correlated. In which case, consumers would appear more responsive to price changes as they have a greater than observed set of varieties. These predicted patterns are supported by the Monte Carlo results.

It seems plausible that supply and demand elasticities could differ by income level or type of exporter. Table 4 investigates this possibility by examining subsets of the data. Consider using only varieties imported by the US from: 1) high or medium income, 2) high income, then 3) OECD countries to estimate the model.[24] The hybrid estimator supports the notion that the demand elasticity is constant across varieties as the Kolmogorov–Smirnov test cannot reject the hypothesis that the estimates from the various cuts of data yield a different distribution for $\hat{\sigma}$ using the entire data set. The standard estimator, on the other hand, is extremely sensitive to the data used for estimation. Kolmogorov–Smirnov tests reject the hypothesis that the distributions of demand and supply elasticities are identical across the various cuts of the data. These results support the notion that the hybrid estimator gives less weight to outlier hyperbolae than the standard estimator as it is less sensitive to the set of varieties used in estimation.

Notably, narrowing the sample to high income or OECD varieties leads to statistically different distributions of the supply elasticity even for the hybrid estimator. Given the nature of the estimates, there are potential gains from developing a structural estimator allowing export supply to differ across varieties. These gains would be most realized when revisiting studies that rely on precise estimates of export supply elasticities to analyze optimal trade policy (e.g., Broda et al. (2008)).[25]

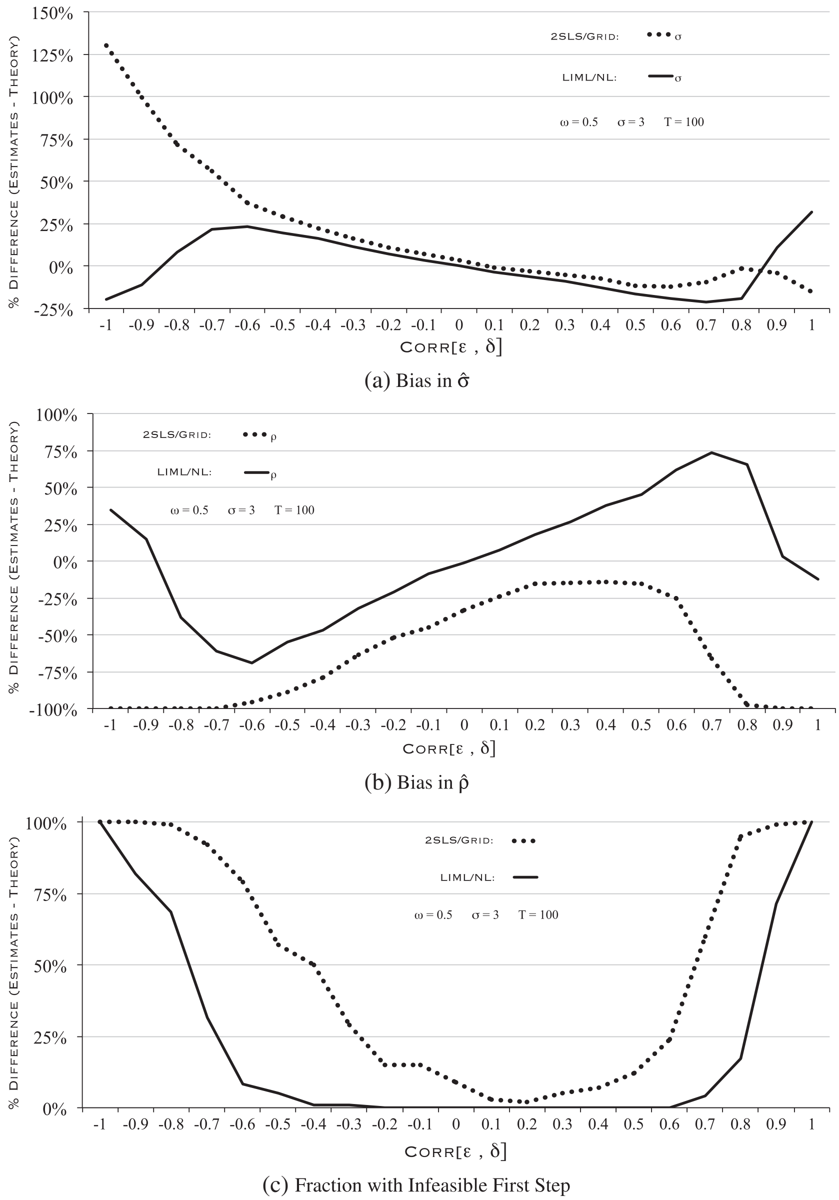
6. Conclusion

This paper proposes a hybrid estimator of supply and demand elasticities that incorporates recent advances into the standard Feenstra

[23] To remind the reader, hidden variety can result from the fundamental aggregation of trade data. See Feenstra (1994) for an extended discussion, and Blonigen and Soderbery (2010) for an application in the auto industry.

[24] I use the World Bank's definition over time of a country's income level and OECD status. The divisions of the data may lead to reselecting the reference variety. However, it is most often the case that the reference variety is supplied by a high income OECD country, and given the results, seems inconsequential.

[25] To be exact, Broda et al. (2008) use various measures of a country's market power as instruments to address potential measurement error in their supply elasticity estimates. Their instruments and analysis are based on relative levels of export supply elasticities across countries. The structural differences in the estimates of the supply elasticities presented here suggest that their IV method is warranted. Unfortunately, my estimates suggest a Pearson correlation of 0.117 between the standard and hybrid estimates across products, which call their core market power rankings into question.



(a) Bias in $\hat{\sigma}$

(b) Bias in $\hat{\rho}$

(c) Fraction with Infeasible First Step

Fig. 12. Monte Carlo results with correlated supply and demand errors.

(1994)/Broda and Weinstein (2006) method. The standard estimator suffers from small sample biases and erratic results due to constrained grid search deficiencies. The hybrid estimator performs favorably in Monte Carlo analysis and in relation to first principles.

The substantially lower estimated import demand elasticities produced by the hybrid estimator suggest prevailing estimates understate consumer gains from product variety by a factor of 6. Coupling the under estimation of the elasticity of substitution with the aggregation biases discussed by Blonigen and Soderbery (2010) implies a severe understating of consumer gains from product variety in previous studies. Additionally, the biases and instability of export supply elasticity estimates using the standard methodology necessitates reviewing outcomes such as those proposed by Broda et al. (2008) confirming the prevalence of importers setting tariffs in relation to export supply elasticities.

Appendix A. Implementation in practice

Implementing the estimator requires choosing from a myriad of available specifications. Starting with trade data, the practitioner must specify goods and varieties, decide upon a reference variety, control for measurement error as unit values take the place of transaction prices, correct for heteroskedasticity, and address small sample sizes. Adding to the difficulty, estimates where $\hat{\theta}_1<0$ cannot be used to uncover feasible values of $\hat{\sigma}$ and $\hat{\rho}$. The following discusses the tools available to practitioners for addressing these issues, and the methods chosen in this paper to provide consistent comparisons across estimators.

A.1. Good and varieties

The minimum requirement to estimate elasticities with this method is a panel consisting of three varieties and two overlapping time periods.[26] With the minimum satisfied, we turn to defining goods and varieties. Developed countries provide the luxury of disaggregate trade data down to an *HS*10 level, which is a viable way to define goods. Given the hundreds of thousand elasticities estimated in this paper, I opt to define a good as a unique *HS*8. As in Feenstra (1994) I

[26] While these conditions seem easily attainable given the scope of modern trade data, there still are a number of goods that do not satisfy the bare minimum (about 10% of all goods in 1993 to 2007 data).

follow the Armington (1969) National Differentiation assumption, and define varieties as product country pairs.[27]

A.2. Infeasible elasticities

Given the tractability assumptions behind supply and demand elasticities (i.e., $\omega \geq 0$ and $\sigma > 1$), if estimating Eq. (5) returns $\hat{\theta}_1 < 0$, the resulting $\hat{\omega}$ and $\hat{\sigma}$ are imaginary or infeasible.[28] In practice, imaginary elasticities are extremely common. Nearly 40% of all elasticities estimated directly from Eq. (5) produce infeasible elasticities here. In these instances, Broda and Weinstein (2006) propose a grid search over feasible σ and ρ to minimize the GMM objective function corresponding to Eq. (5).[29]

This paper will be the first to investigate the performance of the grid search in detail. I show that the grid search leads to polarized supply elasticity estimates: Over 90% of grid searched supply elasticities are either perfectly inelastic or elastic. The consequences for estimates of σ given this polarization can be perverse. Section 3 presents Monte Carlo evidence of substantial numerical bias in grid searched estimates. Section 4.2 puts this issue in the context of hyperbolae. Essentially, the grid search is triggered by outlier hyperbolae, and the polarization of estimates results from weak identification of the supply elasticity.

An added issue of the grid search is its potential to miss the constrained minimum of the objective function due to the coarseness of its evaluation. I show through Monte Carlo simulations that bias from the coarseness and polarization of the grid search is substantial, but can be improved by adopting nonlinear constrained LIML.[30]

A.3. Small sample bias

This paper uses numerical simulations to demonstrate that small sample biases are a real feature of the estimator.[31] These small sample biases are particularly striking when we realize common trade data have small T due to perpetual changes in goods classifications. Section 4.1 explains in greater detail the source of the small sample gains from LIML and how they translate into actual data. A key result of Soderbery (2010) is that small sample biases are a result of small T, and not impacted by the number of varieties, N, used in estimation. This result maintains in the Monte Carlo below.

Intuitively, small sample biases are a product of outlier hyperbola. The shape of individual hyperbola is fundamentally driven by T, but not the number of accompanying hyperbolae, N. While larger N gives us more hyperbolae to work with, their consistency is unaffected holding T constant. Consequently, numerical bias of the resulting estimates is also unaffected along the variety dimension. Intuitively, increasing the number of varieties requires increasing the number of instruments. With weak instruments (due to low T) this does little to reduce bias and in many cases may exacerbate it. Turning back to Feenstra (1994), consistency of the estimator is proven along the time dimension, and requires the existence of at least three hyperbolae that are not asymptotically identical. Holding T constant, consistency cannot be proven in Feenstra (1994).

A.4. Reference variety

With goods and varieties defined, we turn to the definition of a reference variety. The reference variety is key for the structural estimator, as it allows us to eliminate good specific unobservables. Feenstra (1994) states the reference variety should be a variety sold in every year, and in the event of multiple potential varieties, the "dominant supplier" should be selected. He strategically chooses a set of goods, such that Japan meets the criteria for the reference variety for each good. Mohler (2009) investigates the sensitivity of the estimation procedure to the choice of reference variety, and finds that the estimator does in fact appear to be more stable when the dominant supplier is chosen. Each estimation conducted in this paper endogenously chooses the reference variety to be the dominant supplier.[32] For accurate comparison, the reference variety is identical in each subsequent estimation.

A.5. Measurement error and heteroskedasticity

In the vast majority of international trade data, unit values take the place of transaction prices. The first step to control for measurement error is the use of import shares rather than quantities imported. As discussed by Kemp (1962), this adjustment should control for the error in quantities being correlated with unit values. It is also accepted that unit values themselves are measured with error. Suppose observed unit values contain additive random measurement error of the form $\Delta^k lnUV_{gvt} = \Delta^k lnp_{gvt} + \psi_{gvt}$. Feenstra (1994) proves that this type of measurement error can be controlled for by including a constant in the estimating equation such that,[33]

$$Y_{it} = \theta_0 X_{0it} + \theta_1 X_{1it} + \theta_2 X_{2it} + u_{it}, \tag{5}$$

where X_{0it} is a vector of ones. Broda and Weinstein (2006) expand Feenstra (1994)'s treatment of measurement error by allowing measurement error to depend on each variety's quantity sold and number of periods. They demonstrate that including a term inversely proportional to the variety's quantity sales can control for this more complex measurement error. Specifically, they include $X_{0it} = 1/T\sum_T (1/q_{gvt} + 1/q_{gvt-1})$.

A related issue is heteroskedasticity in the data. Feenstra (1994) weights Eq. (5) by the inverse of the estimated residuals to correct for heteroskedasticity. Assuming the average sample variance is inversely related to quantity sold, Broda and Weinstein (2006) weight the data by $T^{3/2}(1/q_{gvt} + 1/q_{gvt-1})^{-1/2}$ to correct for heteroskedasticity.

Mohler (2009)) provides a handful of specification tests, and advocates Feenstra (1994)'s original corrections.[34] This paper opts to control for measurement error with a constant, and correct for heteroskedasticity using the inverse of the estimated residuals, as in Feenstra (1994).[35]

Combining the preceding observations and studies, this paper proposes a hybrid estimator that applies Feenstra (1994)'s corrections

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[27] Although this is a common assumption, it is far from innocuous. Blonigen and Soderbery (2010) investigate the consequences of hidden varieties as a consequence of the Armington assumption on estimates of consumer gains from product variety. We uncover significant product churning hidden by this assumption that leads to significant understatement of consumer gains in the case of US automobiles.

[28] For example, Feenstra (1994) estimates the supply and demand elasticities for select products. The estimates of Eq. (5) for steel sheets are $\hat{\theta}_1 = -0.0015$ and $\hat{\theta}_2 = -0.317$. Transforming these estimates into demand and supply elasticities yields $\hat{\sigma} = 4.21$ and $\hat{\omega} = -0.005$. Since $\hat{\omega}$<0, the set of estimates are infeasible given the assumptions of the model.

[29] Broda and Weinstein (2006) evaluate the GMM objective function for each value of $\sigma \in [1.05, 131.5]$ and $\rho \in [0, \frac{\sigma-1}{\sigma})$ at 5% intervals apart. This paper opts for a slightly finer uniform grid. The GMM objective function is evaluated for $\sigma \in [1.05, 131.05]$ at fixed intervals 0.10 apart and $\rho \in [0, \frac{\sigma-1}{\sigma})$ at 50 equally spaced values.

[30] Additionally, by eliminating the need to evaluate the objective function at every point of the grid, nonlinear estimation drastically speeds up estimation time — applying the standard code to the universe of HS8 products requires two full days, while the nonlinear estimation is completed in half the time.

[31] Soderbery (2010) shows a bias in σ estimates near 20% in the smallest sample that do not fully dissipate until T exceeds 50. These results are reiterated in Section 3.

[32] Specifically, the reference variety k is the HS8-country pair supplying the US for the most time periods, and in the event of a multiple potential k, it is the variety with the highest sales. Refer to the Stata code implementing the hybrid estimator that accompanies this paper for details.

[33] This appendix has recently been included in Feenstra (2010, pp. 25–27).

[34] Anecdotally, I have also found Feenstra (1994)'s controls provide the most intuitive set of estimates.

[35] Monte Carlo simulations demonstrate that these controls perform well. Ultimately, the intuition of the estimator drawn form hyperbolae and Monte Carlo analysis suggests that ex-ante weighting schemes, including the intuition based weighting scheme in Broda and Weinstein (2006), will struggle to solve the underlying numerical biases of the estimator. Rather, a weighting scheme based on the precision of first-stage estimates, as implemented by LIML is key to addressing the bias of estimator.

for measurement error and heteroskedasticity to the data and uses LIML, along with constrained nonlinear LIML when the first step is infeasible, in order to correct for small sample biases and grid search inefficiencies. Following immediately, I contrast the estimators through Monte Carlo then apply the method to actual data.

Appendix B. Consumer gains from product variety

B.1. Theory

With estimated elasticities in hand, we can turn to the calculation of consumer gains from product variety over the sample. Working from the unit cost function, Feenstra (1994) defines the *exact* price index as,

$$\begin{aligned}\frac{\phi_{gt}^{M}\left(I_{gt},\mathbf{b}_{g}\right)}{\phi_{gt-1}^{M}\left(I_{gt-1},\mathbf{b}_{g}\right)} &= \pi_{g}^{M}\left(\mathbf{p}_{gt},\mathbf{p}_{gt-1},\mathbf{x}_{gt},\mathbf{x}_{gt-1},\mathbf{I}_{g}\right)\\ &= P_{g}\left(\mathbf{p}_{gt},\mathbf{p}_{gt-1},\mathbf{x}_{gt},\mathbf{x}_{gt-1},\mathbf{I}_{g}\right)\left(\frac{\lambda_{gt}}{\lambda_{gt-1}}\right)^{\frac{1}{\sigma_g-1}},\end{aligned}$$

which is the conventional Sato (1976) and Vartia (1976) price index[36] scaled by the λ ratio. The λ ratio captures gains (or losses) from a changing set of product varieties, and is defined,

$$\lambda_{gr} \equiv \frac{\sum_{v\in\mathbf{I}_g} p_{gvr}x_{gvr}}{\sum_{v\in\mathbf{I}_{gr}} p_{gvr}x_{gvr}} \qquad \text{for } r = t-1, t.$$

As varieties appear and disappear period by period, the λ ratio adjusts the price index to account for consumer valuation of these varieties. Intuitively, consumers gain (lose) from an excess value of new (disappearing) varieties. The *corrected* λ ratio is the λ ratio adjusted for consumer love of varieties of a given good (i.e., augmented by $\frac{1}{\sigma_g-1}$). Goods with higher σ are more homogeneous. Consequently, increasing or decreasing the set of varieties available to consumers has little effect on the exact price index, and hence welfare.

Aggregating over all goods in the economy yields the aggregate exact price index,

$$\Pi^{X} = \prod_{g\in G}\left(P_{g}^{X}\left(\mathbf{I}_{g}\right)\right)^{\mathbf{w}_{gt}}\left(\frac{\lambda_{gt}}{\lambda_{gt-1}}\right)^{\frac{\mathbf{w}_{gt}}{\sigma_g-1}}.$$

The weights, $\mathbf{w}_{gt}$, are ideal log-change at the goods level, and capture the relative importance of the good in total consumption.[37] Consumer gains from product variety in t are readily calculated as the inverse of $\Pi_{g\in G}\left(\lambda_{gt}\lambda_{gt-1}\right)^{\mathbf{w}_{gt}\sigma_g-1}$. Calculating end-point statistics is done so by simply taking the product year to year of the aggregate.

B.2. The impact of σ estimation on product variety gains

An end-point λ ratio less than one indicates net entry of varieties across each good over the sample. Table B.5 calculates end-point λ and welfare ratios associated with the universe of US imports from 1993 to 2007 across estimators. The λ ratio is around 0.958 for each methodology, suggesting net variety gains over the sample.[38]

The corrected λ ratio adjusts the variety gains by the substitutability of the new or disappearing varieties. This adjusted ratio is where differences in σ reflect in estimated variety gains. If elasticities are estimated with upward bias, the corrected λ ratio will be biased toward unity, suggesting a lower valuation of product variety. Table B.5 highlights the impact of σ on the λ ratio. As suggested by the preceding analysis of the estimator, correcting the λ ratio with the standard estimation of σ understates consumer valuation of product variety when compared with the hybrid estimates. The difference between the standard and the proposed hybrid estimates generate a disparity of nearly 10% in the corrected λ ratios.

The expansion in the set of available varieties indicated by the λ ratios translate into consumer gains. Calculating welfare over the sample is done by inverting the corrected λ ratio each period taken to the power of the US import share. The end-point ratio of welfare in Table B.5 is thus interpreted as the percent of GDP consumers would pay in 1993 to access the varieties available in 2007. Standard elasticities (2SLS/GRID) suggest consumers would forego 0.116% of their income in 1993 to access the expanded variety set in 2007.[39] Hybrid elasticities (LIML/NL) suggest consumers would pay significantly more for these varieties. Yielding a welfare gain estimate of 0.704%, the hybrid estimator reveals biases in the standard method understate consumer gains from imported varieties by a factor of 6 over the sample.[40]

[36] The conventional price index is $P_g^M\left(\mathbf{p}_{gt},\mathbf{p}_{gt-1},\mathbf{x}_{gt},\mathbf{x}_{gt-1},\mathbf{I}_g\right)\equiv\Pi_{v\in\mathbf{I}_g}\left(p_{gvt}p_{gvt-1}\right)^{\mathbf{w}_{gvt}\left(I_g\right)}$.

[37] Ideal log-change weights are defined as, $\mathbf{w}_{gt}\equiv\frac{\left(\frac{s_{gt}-s_{gt-1}}{\ln s_{gt}-\ln s_{gt-1}}\right)}{\sum_{v\in I_g}\left(\frac{s_{gt}-s_{gt-1}}{\ln s_{gt}-\ln s_{gt-1}}\right)}$.

[38] The slight difference between the λ ratio from 2SLS and LIML is due to the difference in the number of estimated elasticities. As mentioned previously, including the goods estimable with 2SLS but not LIML in the hybrid estimates only magnify the subsequent results. Specifically, welfare gains from the hybrid estimate rise from 0.704% to 1.031%.

[39] These estimates are in line with Broda and Weinstein (2006). From 1990 to 2001 using more disaggregate HS10 trade data, they estimate an end-point corrected λ ratio of 0.917 and corresponding welfare gains of 0.677%.

[40] The welfare implications of correcting estimated elasticities are not generalizable. The evidence in this paper is that the standard estimates of σ are biased upward at all points of the distribution. The impact of overestimating the elasticity of substitution on the corrected λ ratio, and hence welfare, depends upon the interaction of the bias with variety gains and losses. Since consumers are more responsive to variety losses as well as variety gains when σ is lower, the impact of correcting its biases is an empirical question. From 1993 to 2007, adopting the hybrid estimator demonstrates a severe downward bias in estimated consumer gains from product variety in the US.

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Table B.5
Consumer gains from product variety.

σ Estimation	End-point ratios		
	λ	Corrected λ	Welfare*
Broda and Weinstein (2006)†	N/A	0.917	0.677%
SLS/GRID	0.958	0.988	0.116%
SLS/NL	0.958	0.988	0.113%
LIML/GRID	0.959	0.961	0.370%
LIML/NL	0.967	0.928	0.704%

Notes: * Welfare gains each period are the inverse of the end point corrected λ ratio to the power of the US share of imports, as in Broda and Weinstein (2006). HS 84733010 is dropped from these calculations due to obvious misreporting. † Broda and Weinstein (2006) estimates for the period 1990 to 2001 at the HS10 goods level are reported.

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