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Investigating the asymptotic properties of import elasticity estimates

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ABSTRACT

Feenstra (1994) is widely implemented in international trade to estimate elasticities of substitution. Through a Monte Carlo experiment, simulated estimates suggest substantial biases due to weak instruments. However, the derivation of the elasticity of substitution drastically mitigates these biases.

1. Introduction

Feenstra (1994) develops a method to quantify product variety changes into equivalent price changes which has allowed subsequent work to estimate the gains from new imported varieties in a new trade theory model context. In a global economy characterized by a representative consumer with CES preferences, he derives an exact price index taking into account changing variety sets over time. His derivation demonstrates the importance of consistently estimating a demand side elasticity of substitution across varieties, and he proposes an instrumental variables technique that yields asymptotically consistent and efficient estimates of underlying supply and demand elasticities of substitution.

The relatively small estimated F-statistics produced using Feenstra (1994)'s original data (see Table A.1) suggest that the classic instrumental variables problem addressed by these estimates seems to suffer from weak instruments. Staiger and Stock (1997) shows the detrimental effect of weak instruments on estimates of parameters using two-stage least squares (2SLS). I establish the presence of weak instruments in simulated data based on actual trade data, and that the instruments appear to drive significant small sample biases in the estimator. Furthermore, this paper attests that the presence of ME exacerbates slow convergence and small sample biases of the estimator.

The qualitative results of Monte Carlo experiments are threefold: First, the estimates of the parameters used to calculate the elasticity of substitution suffer from significant small sample biases, and converge relatively slowly. Second, the calculations of the elasticity of substitution drastically mitigate the biases of the constructed parameters while calculations of the inverse supply elasticity do not. Third, the use of limited information maximum likelihood (LIML) decreases small sample biases.

The paper proceeds as follows. The subsequent section discusses the theory and assumptions behind the estimating equation. Section 3 details the data generating process I use and how it compares to actual data. Section 4.1 discusses my initial simulation results, which are expanded upon in 4.2 to allow for ME. I then implement a modified LIML technique in Section 5. Section 6 concludes.

2. Supply, demand, and estimation

Feenstra (1994) specifies demand from a representative consumer with CES preferences,

$$S_{it} \equiv \frac{p_{it}x_{it}}{\sum_{i\in I_t} p_{it}x_{it}} = \frac{b_{it}^{\sigma-1}p_{it}^{1-\sigma}}{\sum_{i\in I_t} b_{it}^{\sigma-1}p_{it}^{1-\sigma}} = b_{it}^{\sigma-1}p_{it}^{1-\sigma}c(p_t, I_t, b_t)^{\sigma-1}$$

Market shares and prices of imports are denoted by s_{it} and p_{it}, respectively. The elasticity of substitution is $\sigma > 1$. The set of varieties available at time t is denoted by $I_t \subset \{1,....,N\}$, and x_{it} are the aggregate quantities of each of these varieties i consumed in period t. Consumers scale the value of consumption by a taste parameter b_{it}, which is allowed

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to be random. Demand is coupled with the supply equation from monopolistically competitive exporters,

$$p_{it} = \left(\frac{\sigma}{\sigma-1}\right) exp(v_{it}) x_{it}^{\omega}.$$

The inverse supply elasticity is given by $\omega \geq 0$, and v_{it} embodies a random technology factor assumed independent of b_{it}.

In order to eliminate unobservables, Feenstra (1994) takes first and referenced differences of the market outcomes (denoted by Δ and superscript k). This results in the system of equations,

$$\begin{aligned} \Delta^k lnp_{it} &\equiv \Delta lnp_{it} - \Delta lnp_{kt} = -\rho\epsilon_{it}^k + \delta_{it}^k \\ \Delta^k lns_{it} &\equiv \Delta lns_{it} - \Delta lns_{kt} = -(\sigma-1)\Delta^k lnp_{it} + (\sigma-1)\epsilon_{it}^k \\ &= (\sigma-1)(1-\rho)\epsilon_{it}^k + (\sigma-1)\delta_{it}^k. \end{aligned} \quad (1)$$

For notational convenience, let $\rho \equiv \frac{\omega(\sigma-1)}{1-\omega\sigma} \in \left[0, \frac{\sigma-1}{\sigma}\right)$. Fluctuations of differenced prices and shares are driven solely by the demand and supply errors for each good-time pair. Intuitively, elasticity scaled taste and technology shocks dictate movements in market outcomes within variants.

The error terms are assumed to vary independently across time and product space. Multiplying the equations for the differenced errors scaled by $\frac{1}{(1-\rho)}$ yields the tractable estimating equation,

$$\begin{aligned} &Y_{it} = \theta_1 X_{1it} + \theta_2 X_{2it} + u_{it}, \text{ where} \\ &u_{it} = \frac{\epsilon_{it}^k \delta_{it}^k}{(1-\rho)}, Y_{it} \equiv \left(\Delta^k lnp_{it}\right)^2, X_{1it} \equiv \left(\Delta^k lns_{it}\right)^2, \text{ and} \\ &X_{2it} \equiv \left(\Delta^k lns_{it}\right)\left(\Delta^k lnp_{it}\right). \end{aligned} \quad (2)$$

Taking advantage of the panel nature of the data, Feenstra (1994) demonstrates an IV regression with product indicators as the instruments produces consistent estimates of θ_1 and θ_2.[1]

The estimation procedure notably corresponds to Hansen (1982)'s method of moments where the moment condition, $E[u_{it}] = 0$, is approximated by choosing the parameters which minimize the weighted sum of squares residuals. Consistency relies on $T \to \infty$, and the assurance that the vectors $\overline{X}_{1i}$ and $\overline{X}_{2i}$ are not asymptotically proportional. Using the consistent estimates of θ_1 and θ_2 one can calculate,

$$\hat{\rho} = \begin{cases} \dfrac{1}{2} + \left(\dfrac{1}{4} - \dfrac{1}{4 + \frac{\hat{\theta}_2}{\hat{\theta}_1}}\right)^{\frac{1}{2}} & \text{for } \theta_1, \theta_2 > 0 \\ \dfrac{1}{2} + \left(\dfrac{1}{4} - \dfrac{1}{4 + \frac{\hat{\theta}_2}{\hat{\theta}_1}}\right)^{\frac{1}{2}} & \text{for } \theta_1 > 0 \text{ and } \theta_2 < 0, \end{cases} \quad (3)$$

resulting in, $\hat{\sigma} = 1 + \left(\frac{2\hat{\rho}-1}{1-\hat{\rho}}\right)\frac{1}{\hat{\theta}_2}$,

which consistently estimate the underlying parameters σ and ρ.[2] To correct for assumed heteroskedasticity, the estimated residuals are used to weight Eq. (2) by $\frac{1}{\hat{s}_i}$, where $\hat{s}_i^2 = \sum_t \frac{\hat{u}_{it}^2}{T_i}$.

[1] The IV estimation is equivalent to estimating $\overline{Y}_i = \theta_1 \overline{X}_{1i} + \theta_2 \overline{X}_{2i} + \overline{u}_i$ with WLS, which Feenstra (1994) demonstrates is consistently estimated with the proper weighting scheme.

[2] Broda and Weinstein (2006) expand the estimator to handle scenarios where Eq. (3) does not yield economically feasible estimates with $\hat{\theta}_1$ and $\hat{\theta}_2$. I implement their constrained grid search technique over the set of economically feasible values of σ and ρ to minimize the GMM objective function implied by the IV estimation is proposed.

2.1. Controlling for measurement error

In practice the econometrician observes unit values, rather than actual prices in trade data, which are expected to contain ME. The standard estimates are then no longer consistent. To allow for simple ME, suppose differenced log unit values are given by,

$$\begin{aligned} \Delta^k lnUV_{it} &= \Delta^k lnp_{it} + \mu_{it}^k \quad \text{which by Equation 1,} \\ &= -\rho\epsilon_{it}^k + \delta_{it}^k + \mu_{it}^k \quad \text{where } \mu_{it}^u \text{ satisfies,} \end{aligned} \quad (4)$$

Assuming μ_{it} independent of the error terms, one can rewrite Eq. (2) as,

$$\begin{aligned} &\left(\Delta^k lnUV_{it}\right)^2 = 2\sigma_\mu^2 + \theta_1\left(\Delta^k lns_{it}\right)^2 + \theta_2\left(\Delta^k lns_{it}\right)\left(\Delta^k lnUV_{it}\right) + v_{it}, \quad \text{where} \\ &v_{it} = u_{it} + \left[\left(\mu_{it}^k\right)^2 - 2\sigma_\mu^2\right] + 2\left(\Delta^k lns_{it}\right)\left(\mu_{it}^k\right) - \theta_2\left(\Delta^k lns_{it}\right)\left(\mu_{it}^k\right). \end{aligned}$$

With the same regularity conditions as before, simply including a constant in the estimation procedure will yield consistent and efficient estimates of the constructed parameters.

3. Data generating process

I first choose values of the parameters for the underlying elasticities, σ and ω,[3] then I generate data that satisfy Eqs. (1) and (2). I draw heteroskedastic variances for ϵ_{it} and δ_{it} from a Uniform distribution such that[4]

$$\begin{aligned} &\epsilon_{it} \sim N\left[0, \sigma_{\epsilon,i}^2\right] \quad \text{where,} \quad \sigma_{\epsilon,i}^2 \in (0, 9), \\ &\delta_{it} \sim N\left[0, \sigma_{\delta,i}^2\right] \quad \text{where,} \quad \sigma_{\delta,i}^2 \in (0, 9). \end{aligned}$$

Finally, I randomly eliminate observations to generate an unbalanced panel mimicking actual data.

These variances seem arbitrary (and extreme) at first glance, but my choice of parameter values make for a relatively clean comparison between my generated data and Feenstra (1994)'s original data for Steel Bars and Typewriters. Fig. A.1 compares Kernel Density estimates of a random draw of my generated data with their similar counterpart in the actual data[5]. Simulated data do not match the dispersion of actual data, but the shape of the distributions are comparable.

4. Simulation results

4.1. Base case — no measurement error

Fig. A.2 is produced from averaging estimates over 100 Monte Carlos for various levels of T and N. It highlights the importance of T in

[3] I arbitrarily fix $\sigma = 3$ and $\omega = .5$, which imply $\rho = .4$. These values appear to be inconsequential for generating my results (I present evidence in Section 5), thus I choose values similar to previous estimates using actual data.

[4] Specifically, I make draws of variances from a Uniform$\in$[0,3] distribution. I have explored many combinations of variance intervals for ε_{it} and δ_{it} with little to no difference, except it extreme cases.

[5] For visual ease, I truncate the kernel density estimates at $Y = 8$ and $X_2 = 6$. It is extremely rare, less than 1% of all observations, that the data lie beyond my levels of truncation.

the convergence of $\hat{\sigma}$.[6] The number of varieties observed by the econometrician, N, does not appear to impact the asymptotics of my estimates for small or large values. Thus, I fix $N=55$ for the remainder of my analysis.[7]

The bias of the average estimated coefficients from 100 Monte Carlo simulations across various levels of T are presented in Fig. A.3. The results suggest small sample biases upwards of 60% for the constructed coefficient $\hat{\theta}_1$. These estimated biases are doubled by $\hat{\theta}_2$, and the rate of convergence to the actual values for both coefficients is extremely slow. The remarkable feature of this estimation are the opposing results from the calculations of $\hat{\sigma}$. The largest bias is observed for the smallest sample when $T=5$, but this is a meager 4%. Additionally, $\hat{\sigma}$ seems to converge almost instantaneously to its true value, reaching a bias of less than 1% by $T=15$. Conversely, the structural calculations of $\hat{\omega}$ do not appear to mitigate the estimated biases.

4.2. The effect of measurement error

To introduce the problem of ME generally faced by the practitioner, I draw $\mu_{it} \sim N[0, 0.15]$. These random draws of ME are used to produce data using Eq. (4). The underlying data are identical to each Monte Carlo replication from Section 4.1. Fig. A.4 shows that the added inefficiency caused by the inclusion of unobserved ME considerably slows the asymptotic convergence and increases small sample biases of each estimated variable. The ability of $\hat{\sigma}$ calculations to relieve the biases of the simulated coefficients is lessened for the two smallest samples. With ME, $\hat{\sigma}$ reaches a maximum bias of 17%, which is 4 times larger than initial estimates. However, the convergence of $\hat{\sigma}$ is still rapid, and approaches a zero bias by $T=25$.[8] The qualitative results for $\hat{\omega}$ from Section 4.1 still hold, however the biases at each time interval are magnified with ME.

5. Limited information maximum likelihood

Fig. A.5 investigates possible gains from changing the estimation strategy. I estimate the problem using Fuller (1977)'s modified LIML. Beginning with data generated free of ME in Panel (a), LIML appears to vastly improve the estimates. Estimates of σ slightly improve, and the small sample biases, along with the slow rates of convergence, all but disappear for estimates of ω. Comparing Panels (a) and (b) of Fig. A.5 demonstrates that the introduction of ME muffles the beneficial effects of LIML for estimates of both σ and ω.

Finally, modified LIML estimates using Feenstra (1994)'s original data are reported at the bottom of Table A.1. As suggested by my simulations, the $\hat{\theta}'$ change considerably. The significant changes in $\hat{\omega}$ are also expected, but the magnitude of the differences in $\hat{\sigma}$ across techniques comes as somewhat of a surprise. The modified LIML suggests that the data for each product possess a larger degree of ME than initial estimates using 2SLS indicate, and the gains of modified LIML may be magnified from better recognition of the ME.

6. Conclusion

Using Monte Carlo simulations I have thoroughly investigated the hurdles faced by practitioners applying Feenstra (1994)'s method for estimating structural elasticities underlying common international trade models. Small sample biases are a significant feature of estimates of the constructed parameters, and likely arise from weak instruments. Yet, and most importantly, the biases are practically eliminated by the structural form of σ, but not ω. Furthermore, the pragmatic existence of MEin prices adversely affects the estimates.

The adoption of the *Harmonized System* frequently restricts trade samples to 1990 and onward. This implies modern studies are forced into having small sample sizes likely possessing weak instruments. In which case, the gains from estimating elasticities across traded goods using modified LIML rather the standard method may be considerable.

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Appendix A. Figures and tables

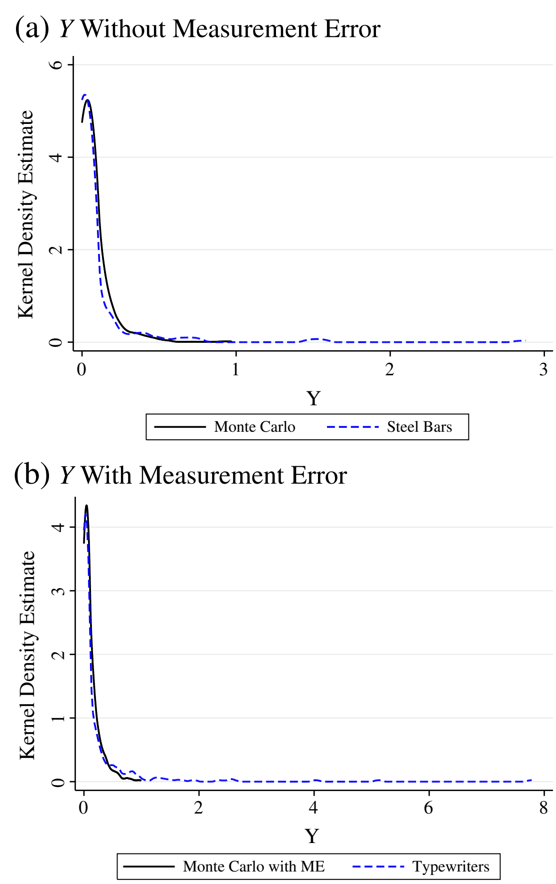

Fig. A.1. Kernel density estimates comparing Monte Carlo data with actual data.

[6] I have produced analogous figures for $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\rho}$ and $\hat{\omega}$, yielding the same results.

[7] See Blonigen and Soderbery (2009) and Broda and Weinstein (2006), for some recent evidence of the number of varieties within goods supporting my choice of N.

[8] These biases are not unsettling economically. Broda and Weinstein (2006) estimate consumer gains from new varieties, which hinge upon estimates of σ, to be approximately 2.59% of GDP. Assuming their estimates of σ were biased upward by the 10% suggested by my Monte Carlo, with their sample size and ME a rough calculation yields estimated consumer gains of approximately 2.88% of GDP using their average estimates.

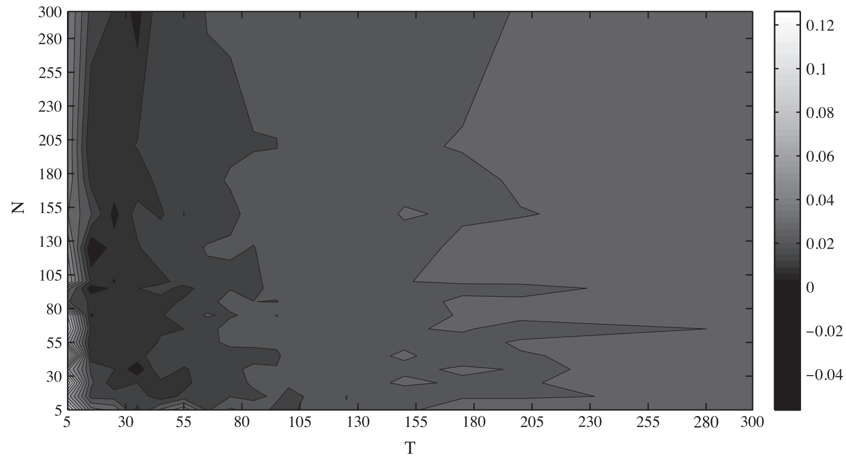

Fig. A.2. Contour plot examining $\frac{\hat{\sigma}-\sigma}{\sigma}$ without measurement error.

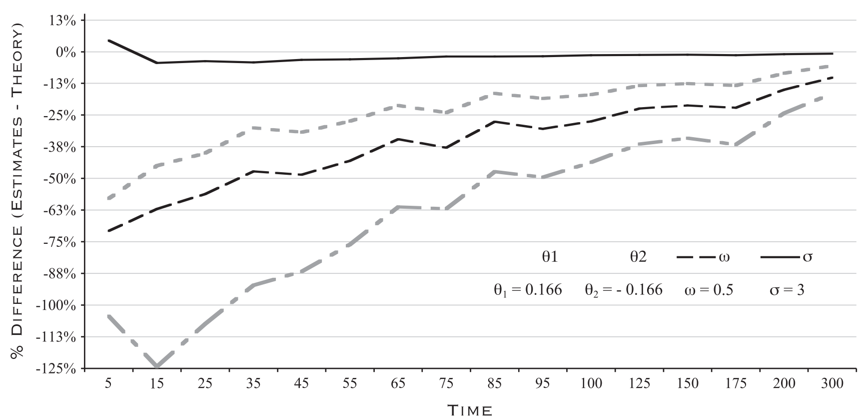

Fig. A.3. Asymptotic properties without measurement error.

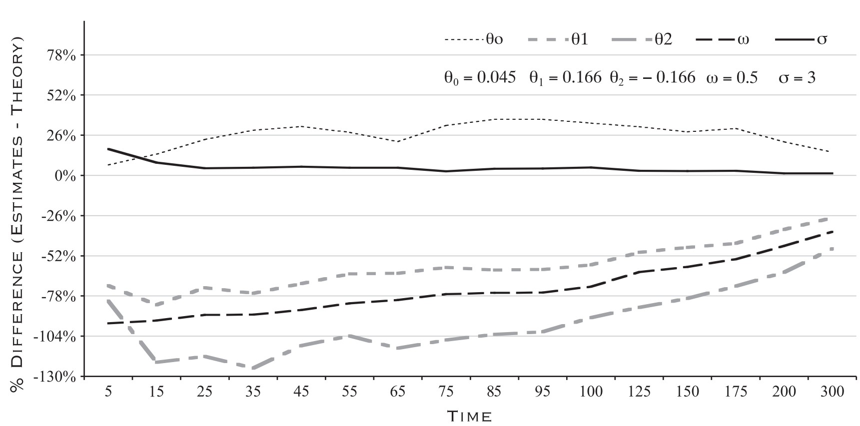

Fig. A.4. The asymptotic effect of measurement error.

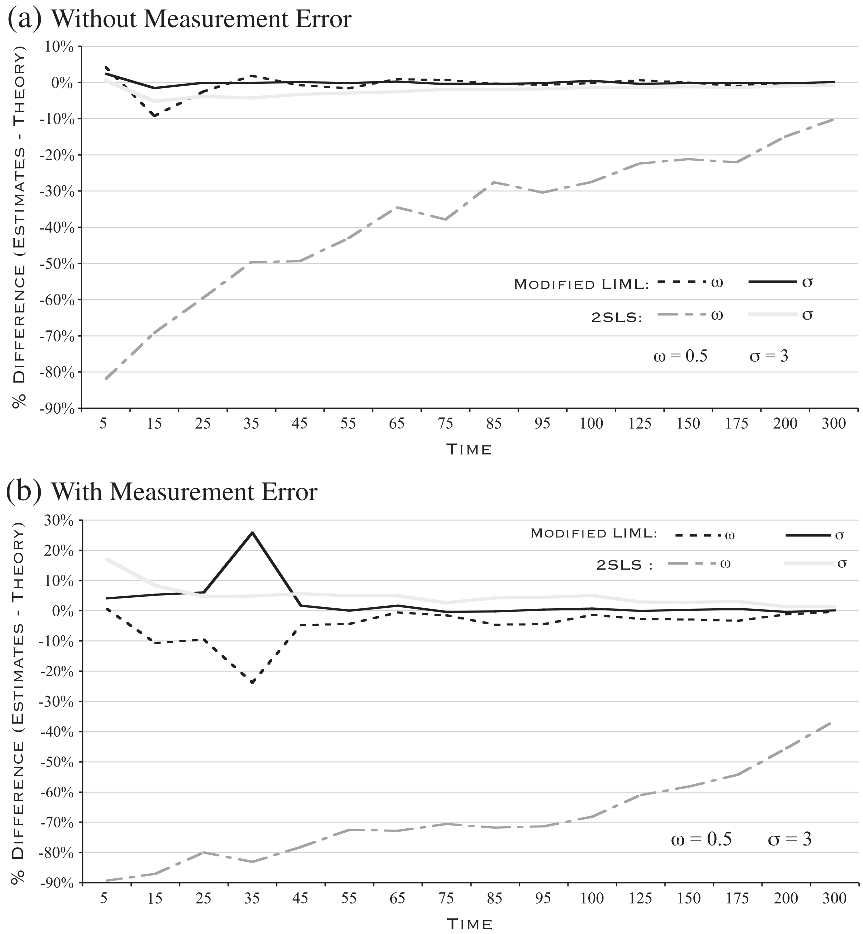

Fig. A.5. Comparing the asymptotic properties of 2SLS and modified LIML.

Table A.1
Comparing the estimates of (Feenstra, 1994) with Similar Monte Carlo data.

Estimate	Knit shirts	Steel bars	Steel sheets	TV receivers	Typewriters	Monte Carlo[a]	Monte Carlo[b]
2SLS							
$\hat{\theta}_0$	0.0436**	−0.0003	0.0079	0.0576	0.0410*	–	0.0656
$\hat{\theta}_1$	0.0684***	0.0509***	−0.0015	0.1446***	0.1739***	0.0948	0.0249
$\hat{\theta}_2$	0.1235**	−0.2537**	−0.3166***	0.9313***	−0.1712*	−0.3568	−0.4231
$\hat{\sigma}$	5.829	3.592	4.206	8.378	2.956	2.884	3.148
$\hat{\omega}$	0.4938	0.1521	−0.0047	−15.9587	0.5154	0.2190	0.1002
$\hat{\rho}$	0.6148	0.2550	−0.0154	0.8873	0.3995	0.2401	0.0921
T	22	22	22	16	22	25	25
N	62	16	30	16	26	50	50
Obs.	651	220	353	133	312	691	691
F statistics used to identify weak instruments							
F_{X1}	3.81	4.73	2.17	5.19	3.49	10.40	10.59
F_{X2}	1.13	1.31	1.59	2.42	1.47	6.15	4.98
$F_{Cragg\text{–}Donald}$	1.00	1.09	1.23	1.98	0.81	2.42	1.49
$F_{Stock\text{–}Yogo\ CV}$[3]	1.89	3.47	2.51	3.47	2.68	1.97	1.97
Modified LIML							
$\hat{\theta}_0$	0.1290	−0.0775	−0.0334	0.1989	−0.2065	–	0.0403
$\hat{\theta}_1$	0.1234	0.1548	−0.0014	0.1828	0.5681	0.1637	0.1695
$\hat{\theta}_2$	−0.6669	−0.6918	−1.4329	1.0108	−1.2598	−0.1751	−0.1756
$\hat{\sigma}$	2.223	2.150	1.698	7.387	1.620	2.995	3.077
$\hat{\omega}$	0.1776	0.2166	−0.0010	−6.9748	0.5441	0.4671	0.3769
$\hat{\rho}$	0.1557	0.1699	−0.0007	0.8817	0.1794	0.3884	0.3624

Significance levels, $^{*}p<0.05$, $^{**}p<0.01$, $^{***}p<0.001$, are calculated using Standard Errors reported in Feenstra (1994). Standard errors are not calculated for simulated or modified LIML estimates, thus significance stars are not included. Monte Carlos are parameterized such that $\theta_1=0.1667$, $\theta_2=-0.1667$, $\sigma=3$, $\omega=0.5\rho=0.4$, and in the presence of ME, $\theta_0=0.045$. [3]$F_{Stock-Yogo\ CV}$ are 5% maximal Fuller relative bias critical values, and thus provide stringent significance levels for the existence weak instruments when compared to $F_{Cragg\text{–}Donald}$.

[a] Values are averaged over 100 Monte Carlo estimates with data free of ME.

[b] Values are averaged over 100 Monte Carlo estimates with data containing measurement error.

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