Computational Neurodynamics

Solutions for Exercise Sheet 2 (Unassessed)
Neural populations and Braitenberg vehicles

Solutions to written questions

Question 4.

a) The LIF model is comprised of three equations:

$$\tau \frac{dv}{dt} = v_r - v + RI$$

$$\text{if } v \geq \vartheta \text{ and } t - t_{spike} > \alpha \text{ then } \begin{cases} v \leftarrow v_r \\ t_{spike} \leftarrow t \end{cases}$$

The first equation describes the temporal evolution of the membrane potential in the absence of a spike. The second equation describes the behaviour of the neuron as the potential reaches a threshold θ, at which point the neuron is considered to spike and the voltage is reset to its resting value v_r. This equation also describes the refractory period of the neuron, whereby the neuron is prevented from spiking more than once in a time interval of duration α.

b) Given the simplicity of the ODE of the LIF neuron, we can analytically calculate its time course $v(t)$. We do this by first separating the ODE into v and t:

$$\frac{1}{\tau} dt = \frac{dv}{v_r - v + RI}$$

We now perform a definite integral on both sides of the equation. On the RHS, we integrate from the resting potential (where the neuron gets reset after a spike) $v = v_r$ to the firing threshold $v = \theta$. On the LHS, we integrate from $t = 0$ to the critical time t_c when the neuron reaches the threshold.

$$\int_0^{t_c} \frac{dt}{\tau} = \int_{v_r}^{\theta} \frac{dv}{v_r - v + RI}$$

$$\frac{t_c}{\tau} = \ln(RI) - \ln(v_r - \theta + RI)$$

Solving for t_c in this equation will show what time the neuron reaches the threshold. Plugging the parameters on the RHS yields $\ln(RI) - \ln(v_r - \theta + RI) = 1.4$, and solving for t_c shows that $t_c = 7\text{ms}$.

Finally, if the neuron spikes once every 7ms, we conclude that its firing rate will be $\frac{1}{7\ \text{ms}} = 0.142\ \text{kHz} = 142\text{Hz}$.

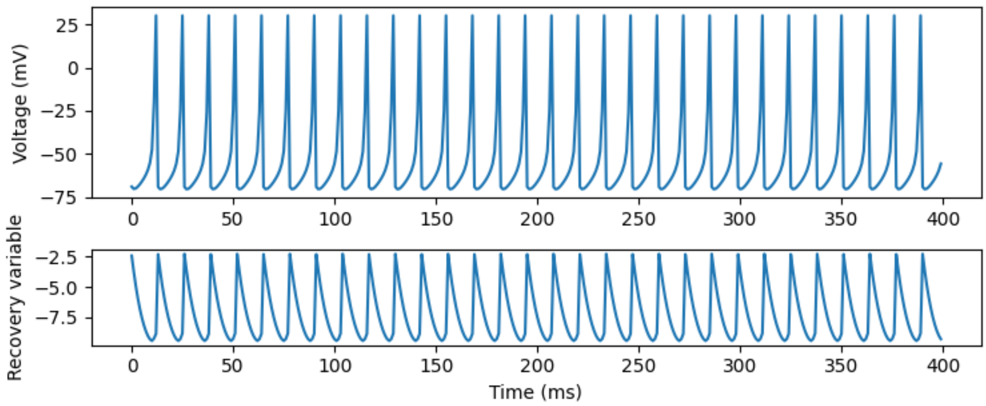
Question 5.

a) The missing element is the spike update (or reset) rule, that is run whenever the neuron records a spike:

$$\text{if } v \geq 30 \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d \end{cases}$$

This is necessary to reproduce spiking dynamics appropriately – without it, *v* would diverge to infinity and the recovery variable would not be able to reproduce the refractory period of real neurons.

b) If we were to increase *a*, the recovery variable *u* would decay much faster, allowing the voltage to increase more quickly following a spike, and thereby effectively shortening the neuron's refractory period. This would result in a neuron with a higher firing rate. A neuron with increased *a* would behave as follows:



Question 6.

a) The parameter *d* represents the after-spike jump of the recovery variable *u*. Increasing *d* would result in a larger jump, therefore a longer refractory period, and less likelihood that the neuron will fire twice in a short period of time.

b) In this scenario the recovery variable u does not play any role, so the model simplifies to a Quadratic Integrate and Fire (QIF) model.

By solving the second-degree equation $0.04v^2 + 5v + 140 = 0$, we can rewrite the voltage ODE as

$$\frac{dv}{dt} = 0.04(v + 42.3)(v + 82.7)$$

This motivates the definition of two values: a resting potential $v_r = -82.7$mV and a critical potential $v_c = -42.3$mV. The neuron will show different behaviours depending on the relationship between $v(0)$, v_r, and v_c:

- If $v(0) < v_r$, then $dv/dt > 0$ and the voltage will monotonically increase until a stable equilibrium is achieved at $v = v_r$.
- If $v_r < v(0) < v_c$, then $dv/dt < 0$ and the voltage will monotonically decrease until a stable equilibrium is achieved at $v = v_r$.
- If $v(0) > v_c$, then $dv/dt > 0$ and the voltage will diverge to infinity.