Appendix

A. The Derivation of Q and G Matrix

For each robot r, the local subproblem in distributed explicit-based ADMM pose graph optimization is formulated as a quadratic program over its own trajectory variables $\mathcal{X}^r$ and the local copies $\mathcal{X}^{(r)}$ of the poses belonging to neighboring robots. The objective function is defined as:

$$\mathcal{L}^r = \frac{1}{2}\begin{bmatrix}\mathcal{X}^r\\ \mathcal{X}^{(r)}\end{bmatrix}^\top \begin{bmatrix}Q_{xx} & Q_{xx^{(r)}}\\ Q_{xx^{(r)}}^\top & Q_{x^{(r)}x^{(r)}}\end{bmatrix}\begin{bmatrix}\mathcal{X}^r\\ \mathcal{X}^{(r)}\end{bmatrix} + \begin{bmatrix}\mathcal{X}^r\\ \mathcal{X}^{(r)}\end{bmatrix}^\top \begin{bmatrix}G_x\\ G_{x^{(r)}}\end{bmatrix} \tag{18}$$

where Q_{xx} captures the second-order terms over the local pose variables $\mathcal{X}^r$, $Q_{x^{(r)}x^{(r)}}$ encodes the quadratic terms associated with the replicated variables $\mathcal{X}^{(r)}$, and $Q_{xx^{(r)}}$ represents the coupling between local and replicated variables. The linear term G_x aggregates the first-order contributions with respect to the local trajectory variables $\mathcal{X}^r$, primarily arising from intra-robot constraints. In contrast, $G_{x^{(r)}}$ collects the linear components associated with the replicated variables $\mathcal{X}^{(r)}$, including both inter-robot measurement residuals and consensus penalty terms. For each intra-robot edge $(i,j) \in \mathcal{E}^r_{\text{intra}}$, the connection Laplacian matrix can be constructed following the approach described in [1].

For each inter-robot edge $(i,j) \in \mathcal{E}^r_{\text{inter}}$, robot r retains the local pose $\mathcal{X}^r_i$ and a replicated copy $\mathcal{X}^{(r)}_{j|s}$ of the neighboring robot s's pose $\mathcal{X}^s_j$. The relative measurement $T^{rs}_{ij} \in \mathrm{SE}(d)$ encodes the transformation from pose i of robot r to pose j of robot s, and is associated with an information matrix $\Omega \in \mathbb{R}^{(d+1)\times(d+1)}$. The residual for this constraint is defined as

$$r^{(r)}_{ij} = \mathcal{X}^r_i T^{rs}_{ij} - \mathcal{X}^{(r)}_{j|s}, \tag{19}$$

whose squared Frobenius norm can be expressed as

$$\begin{aligned}\|r^{(r)}_{ij}\|^2_\Omega = &\operatorname{tr}\left(\mathcal{X}^r_i T^{rs}_{ij}\Omega T^{rs\top}_{ij}\mathcal{X}^{r\top}_i\right)\\ &+ \operatorname{tr}\left(\mathcal{X}^{(r)}_{j|s}\Omega\mathcal{X}^{(r)\top}_{j|s}\right) - 2\operatorname{tr}\left(\mathcal{X}^{(r)\top}_{j|s}\Omega\mathcal{X}^r_i T^{rs}_{ij}\right)\end{aligned} \tag{20}$$

Expanding the residual into quadratic and bilinear forms allows us to map each contribution to specific block entries of the augmented quadratic matrix Q_{aug} in the objective function $\frac{1}{2}X^\top Q_{\text{aug}}X$. The first term is purely quadratic in the local pose $\mathcal{X}^r_i$, contributing to the diagonal block of Q_{xx} as

$$Q_{xx}[i,i] \mathrel{+}= T^{rs}_{ij}\,\Omega\,T^{rs\top}_{ij}, \quad (i,j) \in \mathcal{E}^r_{\text{inter}}, \tag{21}$$

The second term is quadratic in the replicated variable $\mathcal{X}^{(r)}_{j|s}$, contributing to the diagonal block of $Q_{x^{(r)}x^{(r)}}$ as

$$Q_{x^{(r)}x^{(r)}}[j|s,\, j|s] \mathrel{+}= \Omega, \quad (i,j) \in \mathcal{E}^r_{\text{inter}}. \tag{22}$$

The third term is bilinear and couples the local and replicated poses, yielding the off-diagonal cross-terms

$$\begin{aligned}Q_{x,\,x^{(r)}}[i,\, j|s] &\mathrel{+}= T^{rs}_{ij}\,\Omega,\\ Q_{x^{(r)},\,x}[j|s,\, i] &\mathrel{+}= \Omega\,T^{rs\top}_{ij},\end{aligned} \tag{23}$$

which are symmetric counterparts in Q_{aug} and encode the interaction between the two variables.

In addition to inter-robot residuals, the ADMM framework enforces consensus not only between each replicated variable $\mathcal{X}^{(r)}_{j|s}$ and its corresponding consensus value $\mathcal{Z}^s_j$, but also between every local primary variable $\mathcal{X}^r_i$ that participates in inter-robot loop closures and the consensus value $\mathcal{Z}^r_i$. More generally, for the current robot r, any local primary pose $\{\mathcal{X}^r_i\}_{i\in\mathcal{V}^r_{\text{cross}}}$ and any replicated neighbor pose $\{\mathcal{X}^{(r)}_{j|s}\}_{j|s\in\mathcal{R}^r}$ are each coupled to their respective consensus values $\{\mathcal{Z}^r_i\}$ and $\{\mathcal{Z}^s_j\}$ via an augmented Lagrangian term with associated dual variables. The total consensus contributions can be written as

$$\begin{aligned}&\sum_{i\in\mathcal{V}^r_{\text{cross}}}\left(\frac{\rho}{2}\left\|\mathcal{X}^r_i - \mathcal{Z}^r_i\right\|^2_F + \langle\lambda^r_i,\, \mathcal{X}^r_i - \mathcal{Z}^r_i\rangle\right)\\ &+ \sum_{j|s\in\mathcal{R}^r}\left(\frac{\rho}{2}\left\|\mathcal{X}^{(r)}_{j|s} - \mathcal{Z}^s_j\right\|^2_F + \langle\lambda^{(r)}_{j|s},\, \mathcal{X}^{(r)}_{j|s} - \mathcal{Z}^s_j\rangle\right),\end{aligned} \tag{24}$$

where $\rho > 0$ is the penalty parameter. Expanding each term with respect to the relevant variable block and discarding constants yields, for a primary variable $\mathcal{X}^r_i$,

$$\begin{aligned}&\frac{\rho}{2}\operatorname{tr}\big((\mathcal{X}^r_i)^\top\mathcal{X}^r_i\big) - \rho\operatorname{tr}\big((\mathcal{Z}^r_i)^\top\mathcal{X}^r_i\big) + \operatorname{tr}\big((\lambda^r_i)^\top\mathcal{X}^r_i\big) \quad\Rightarrow\\ &\begin{cases}Q_{xx}[i,i] \mathrel{+}= \rho\, I_{d+1},\\ G_x[i] \mathrel{+}= -\rho\mathcal{Z}^r_i + \lambda^r_i,\end{cases}\end{aligned} \tag{25}$$

and for a replicated variable $\mathcal{X}^{(r)}_{j|s}$ is same. In this way, the consensus term adds ρI_{d+1} to the diagonal blocks of both Q_{xx} and $Q_{x^{(r)}x^{(r)}}$, and augments the corresponding linear term blocks G_x and $G_{x^{(r)}}$ by $-\rho\mathcal{Z} + \lambda$.