On the nlogistic-sigmoid modelling for complex growth processes: in application to the COVID-19 pandemic

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Exponential growth: unrestricted (infinite) in 2-D. (x,y) [Shulman, 1998, Jeffery, 2016, Reed and Pearl, 1927]

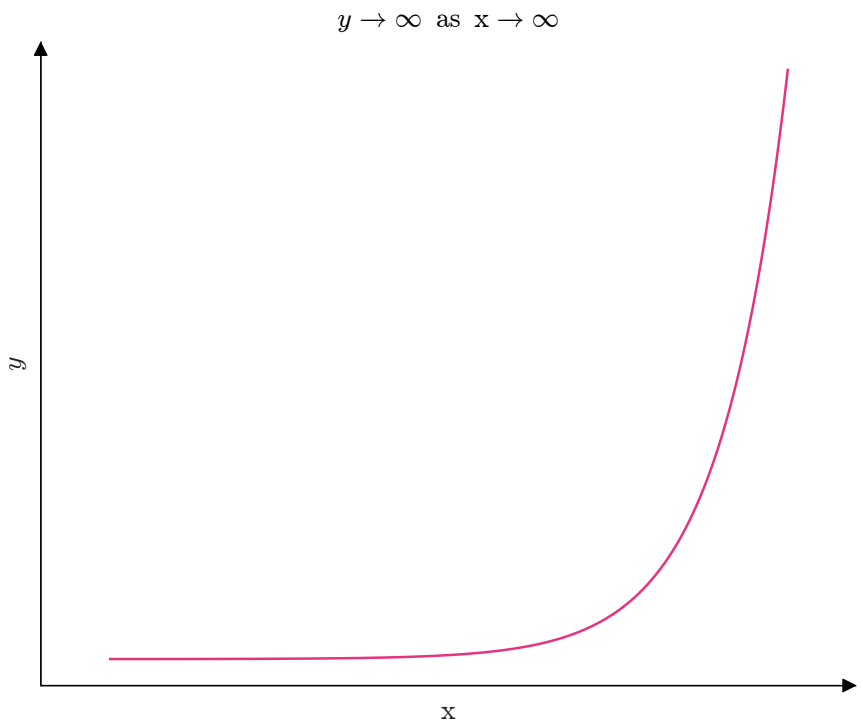


Figure 1: Exponential Growth

classic logistic growth

Logistic growth: restricted (finite) in 1-D (y), unrestricted (infinite) in 1-D (x). [Sze *et al.*, 2017, Taylor and Letham, 2018, Christopoulos, 2016, Xs *et al.*, 2012, Udell and Boyd, 2013, Lee *et al.*, 2020, Lipovetsky, 2010].

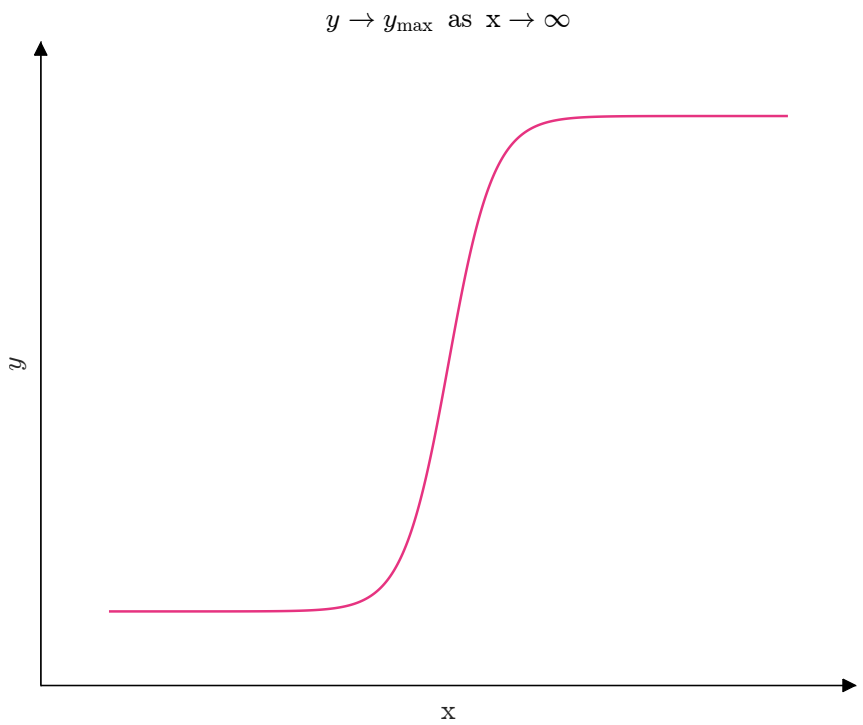


Figure 2: Logistic Growth

Limitations

This trend still continues, as can be seen in some recent works applying the logistic function. [Lee *et al.*, 2020, Wu *et al.*, 2020, Chowell *et al.*, 2019]

1. unrestricted on the 1-D (input-axis): input is taken as infinite.
2. can only deal with symmetric growth: enter over-parametrized models (Richard's models, hyperbolastic models). [Loibel *et al.*, 2006, Tabatabai *et al.*, 2005]
3. cumbersome analysis: for multiple sigmoids combined, parameter estimation is done separately. [Lipovetsky, 2010, Chowell *et al.*, 2019, Barker and Watt, 2020, Batista, 2020, Hsieh and Cheng, 2006]

Reality: multiple (n) Logistic growth: restricted (finite) in 2-D (x,y). The scientific basis for this is the constructal law origin of the logistic-sigmoid [Bejan and Lorente, 2011].

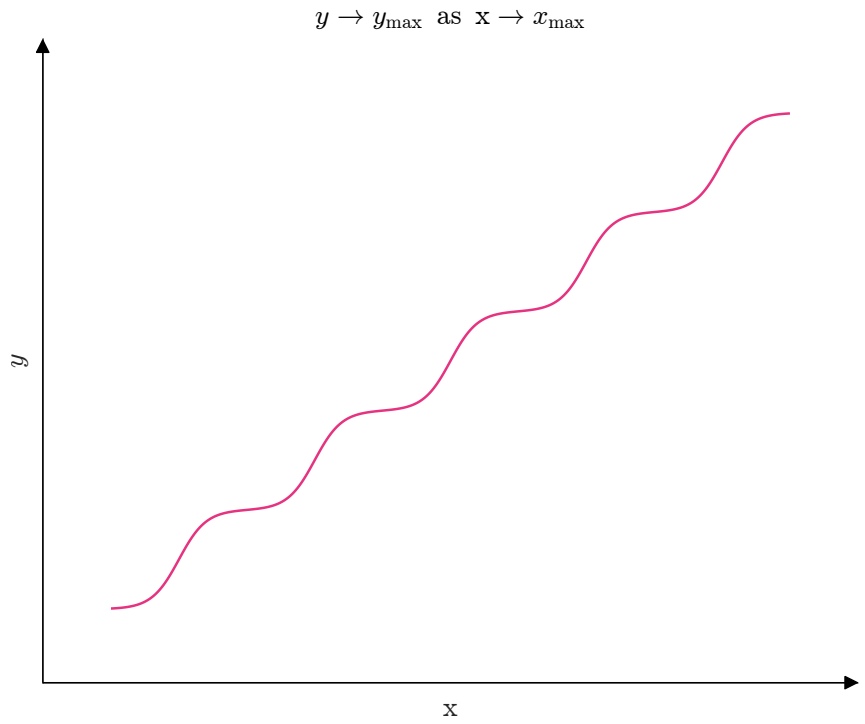


Figure 3: nLogistic Growth

This Work: nlogistic-sigmoid (NLSIG)

Benefits

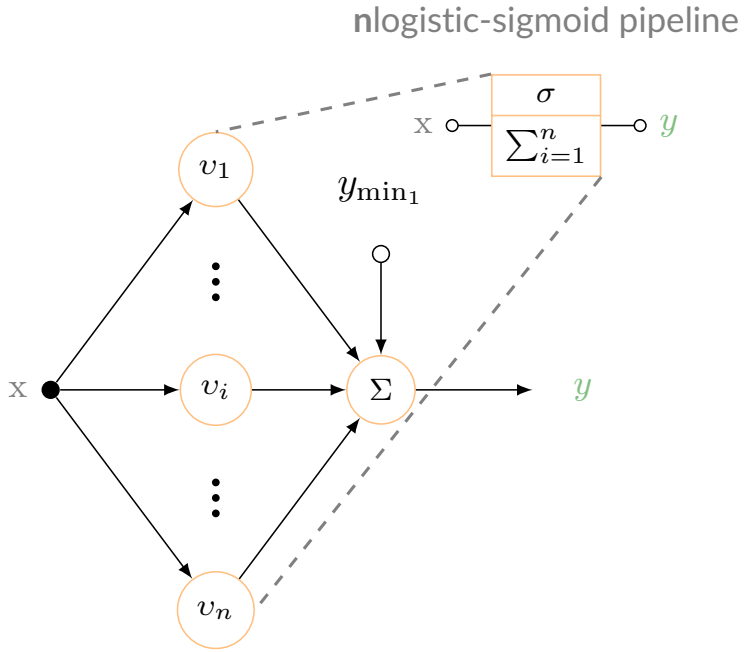
1. retains simplicity
2. unified definition
3. more efficient and improved modelling power

1. Presents a unified logistic function definition equivalent to a n-depth weighted–unweighted feed-forward neural network.
2. Introduces two characteristic metrics of the sigmoidal curve that can give more robust real-time projective measures on the geometric (exponential) state of a growth process.
3. Applies the SISO NLSIG pipeline to modelling the COVID-19 pandemic, which is a current real-world (natural) growth phenomena.

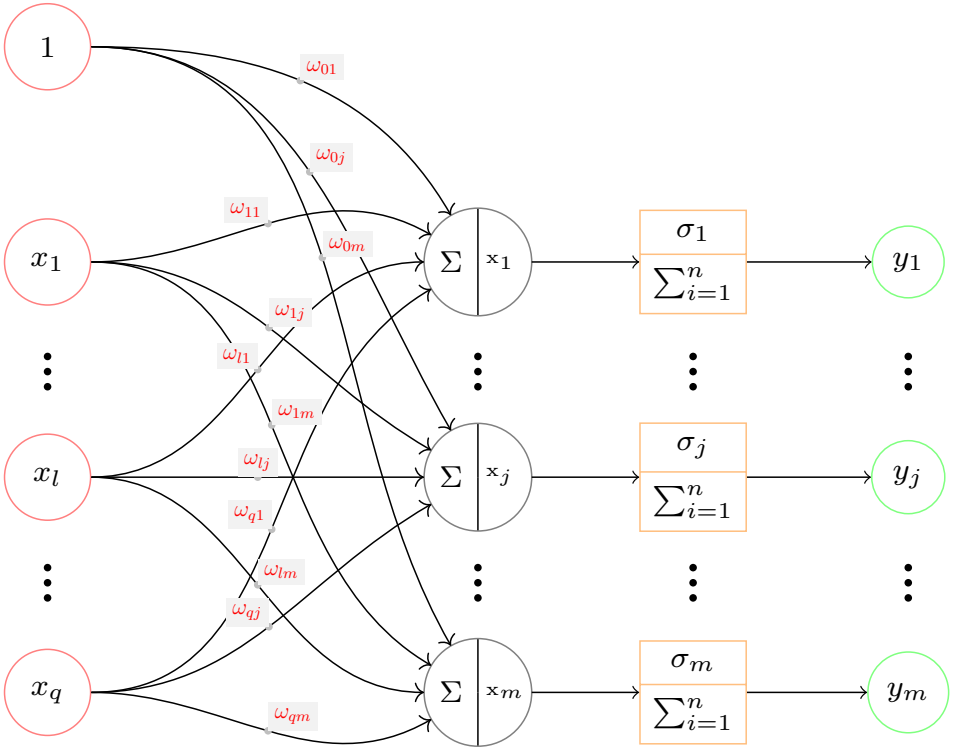
The nlogistic-sigmoid Pipeline

The nlogistic-sigmoid is essentially a neural network. [Udell and Boyd, 2013, Cybenko, 1989, Han and Moraga, 1995].

SISO Network



MIMO Network



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nlogistic-sigmoid function definition

Single-input (or Multiple-Input) Single-output Architecture

The single-output NLSIG denoted NLSIG$\pm$: $\mathbb{R}^q \mapsto \mathbb{R}^1$, where $n \in \mathbb{N}^+$ is defined as follows: Given an input $\mathrm{x} = w_0 + \sum_{l=1}^{q} w_l x_l$, such that $x \in \mathbb{R}^q$, $q \geq 1$ and $\mathrm{x} \in \mathbb{R}^1$, the odd output of the NLSIG is $y = \sigma(\mathrm{x}) \in \mathbb{R}^1$, the even output is $g = \sigma'(\mathrm{x}) \in \mathbb{R}^1$, with $2 \leq p \leq 7$ number of hyper-parameters for each of the i partitions, where:

$$y = y_{\min_1} + \sum_{i=1}^{n} \upsilon_i \tag{1}$$

$$g = \frac{dy}{d\mathrm{x}} = \sum_{i=1}^{n} \frac{d\upsilon_i}{d\mathrm{x}} = \pm \sum_{i=1}^{n} \alpha_i \ln(b_i) \upsilon_i \left(\frac{\upsilon_i}{\Delta y_i} - 1 \right) \tag{2}$$

nlogistic-sigmoid function definition ...

where the arbitrary sub-intervals for each i-th sub-sigmoid partitions are defined as:

$$\Delta \mathrm{x}_i = \mathrm{x}_{\mathrm{max}_i} - \mathrm{x}_{\mathrm{min}_i} \tag{3}$$

$$\Delta y_i = y_{\mathrm{max}_i} - y_{\mathrm{min}_i} \tag{4}$$

and the intermediate parameters are:

$$\upsilon_i = \frac{\Delta y_i}{D_i} \tag{5}$$

$$D_i = 1 + b_i^{\pm \alpha_{i\,i}(\mathrm{x} - \delta_i)} \tag{6}$$

$$\alpha_i = \frac{2\lambda_i}{\Delta x_i} \tag{7}$$

The first-order derivative with respect to the input variables x_l are:

$$g_l = \frac{dy}{dx_l} = w_l g \tag{8}$$

nlogistic-sigmoid function definition ...

Further, given a number of real data-points r_d, such that, $1 \leq d \leq \mathrm{D}$. The objective is to find y_d that minimizes in the least-squares sense, an error objective of the form: $E = \frac{1}{2}\sum_{d=1}^{D} e_d^2$, where $e_d = r_d - y_d$.

Partial Derivatives: Logistic Layer

At the core NLSIG input-output layer, the first-order partial derivatives of the output y to the hyper-parameters can be given as: let $t_i = \pm \ln(b_i) \upsilon_i \left(\frac{\partial y}{\partial y_{\max_i}} - 1 \right)$, then

nlogistic-sigmoid function definition ...

$$\frac{\partial y}{\partial y_{\max_i}} = \frac{v_i}{\Delta y_i} \tag{9}$$

$$\frac{\partial y}{\partial y_{\min_i}} = 1 - \frac{\partial y}{\partial y_{\max_i}} \tag{10}$$

$$\frac{\partial y}{\partial \delta_i} = -\alpha_i t_i \tag{11}$$

$$\frac{\partial y}{\partial \alpha_i} = (\mathrm{x} - \delta_i) t_i \tag{12}$$

$$\frac{\partial y}{\partial \lambda_i} = \frac{2}{\Delta \mathrm{x}_i} \frac{\partial y}{\partial \alpha_i} \tag{13}$$

nlogistic-sigmoid function definition ...

$$\frac{\partial y}{\partial \mathrm{x}_{\max_i}} = -\frac{\alpha_i}{\Delta \mathrm{x}_i}\frac{\partial y}{\partial \alpha_i} \tag{14}$$

$$\frac{\partial y}{\partial \mathrm{x}_{\min_i}} = \frac{\alpha_i}{\Delta \mathrm{x}_i}\frac{\partial y}{\partial \alpha_i} \tag{15}$$

$$\frac{\partial y}{\partial b_i} = \frac{\alpha_i}{b_i \ln(b_i)}\frac{\partial y}{\partial \alpha_i} \tag{16}$$

The jacobian and hessian matrix, where $1 \leq i \leq n$ are:

$$\mathbf{j}^i = \sum_{d=1}^{D} e_d \, \mathbf{j}_d^{i,y} \tag{17}$$

$$\mathbf{H}^i = \left(\mathbf{j}^i\right)^{\mathsf{T}} \mathbf{j}^i \in \mathbb{R}^{p \times p} \tag{18}$$

nlogistic-sigmoid function definition ...

where,

$$\mathbf{j}^{i,y} = \sum_{d=1}^{D} \mathbf{j}_d^{i,y} \tag{19}$$

$$\mathbf{j}_d^{i,y} = \Big[\frac{\partial y}{\partial b_i}, \frac{\partial y}{\partial \lambda_i}, \frac{\partial y}{\partial \mathrm{x}_{\mathrm{max}_i}}, \frac{\partial y}{\partial \mathrm{x}_{\mathrm{min}_i}}, \frac{\partial y}{\partial \delta_i}, \frac{\partial y}{\partial y_{\mathrm{max}_i}}, \frac{\partial y}{\partial y_{\mathrm{min}_i}} \Big] \in \mathbb{R}^{1\times p} \tag{20}$$

nlogistic-sigmoid function definition ...

Partial Derivatives: Weighted-Input Layer

At the weighted-input layer, the definitions for the partial-derivatives are:

$$\frac{\partial E}{\partial y} = e_j \tag{21}$$

$$\frac{\partial E}{\partial \mathrm{x}} = e\,\frac{dy}{d\mathrm{x}} \tag{22}$$

$$\frac{\partial y}{\partial \omega_l} = \begin{cases} \frac{dy}{d\mathrm{x}} & \text{if } l = 0 \\ x_l\,\frac{dy}{d\mathrm{x}} & \text{if } l > 0 \end{cases} \tag{23}$$

$$\frac{\partial E}{\partial \omega_l} = e\,\frac{\partial y}{\partial \omega_l} \tag{24}$$

nlogistic-sigmoid function definition ...

such that the jacobian matrix, hessian matrix, and gradient vector are respectively:

$$\mathbf{j}^w = \sum_{d=1}^{D} \mathbf{j}_d^w \tag{25}$$

$$\mathbf{H}^w = \left(\mathbf{j}^w\right)^\mathsf{T} \mathbf{j}^w \tag{26}$$

$$\mathbf{g}^w = \sum_{d=1}^{D} \left(\mathbf{j}_d^w\right)^\mathsf{T} e_d \tag{27}$$

where,

$$\mathbf{j}_d^w = \left[\frac{\partial E}{\partial \omega_0}, \ldots, \frac{\partial E}{\partial \omega_l}, \ldots \frac{\partial E}{\partial \omega_q}\right] \in \mathbb{R}^{1\times(q+1)} \tag{28}$$

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Y-variable to Inflection Ratio

The YIR is useful for indicating the state of the rate of incident increase or decrease over the curve's interval at a particular cumulative value. The theoretical idea is that at the y corresponding to peak inflection points, the YIR value is always 0.5 for the logistic curve.

$$\mathrm{YIR} = \frac{y - y_{\min_i}}{y_{\max_i} - y_{\min_i}} \tag{29}$$

such that for a growing or decaying process:

$$\begin{cases} \mathrm{YIR} < 0.5 & \text{if generally increasing} \\ \mathrm{YIR} \approx 0.5 & \text{if generally around a peak value} \\ \mathrm{YIR} > 0.5 & \text{if generally reducing} \end{cases} \tag{30}$$

X-variable to Inflection Ratio

The XIR is useful for indicating how far a certain x (e.g: time) is from the peak-inflection $\mathrm{x} = \delta_i$.

$$\mathrm{XIR} = b_i^{\alpha_i(\mathrm{x}-\delta_i)} \tag{31}$$

such that for a growing or decaying process:

$$\begin{cases} \mathrm{XIR} < 1 & \text{if pre-peak period} \\ \mathrm{XIR} \approx 1 & \text{if peak period} \\ \mathrm{XIR} > 1, \mathrm{XIR} \approx 0 & \text{if post-peak period} \end{cases} \tag{32}$$

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Modelling Workflow

Data Source

World Health Organization (`https://covid19.who.int`), Last Update Used: 30 November 2020

We developed a problem-based optimization workflow in MATLAB 2019b, as follows, for fitting the COVID-19 dataset to the nlogistic-sigmoid function.

Modelling Workflow ...

Procedure

1. find the possible locations of the inflection-points in the data, noting that the logistic-sigmoid curve at its core is characterized by three phases: pre-peak, peak and post-peak phases.
2. use the detected $3n$ inflection-points to find n and use as initial guess to the convex optimization solver to find an estimated solution to the inflection points and other three hyper-parameters.
3. applying the bootstrap method, find the 95% confidence intervals on the best-fit solution of the p hyper-parameters for each of i-partitions, obtained as a whole, by the solver.

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World Model

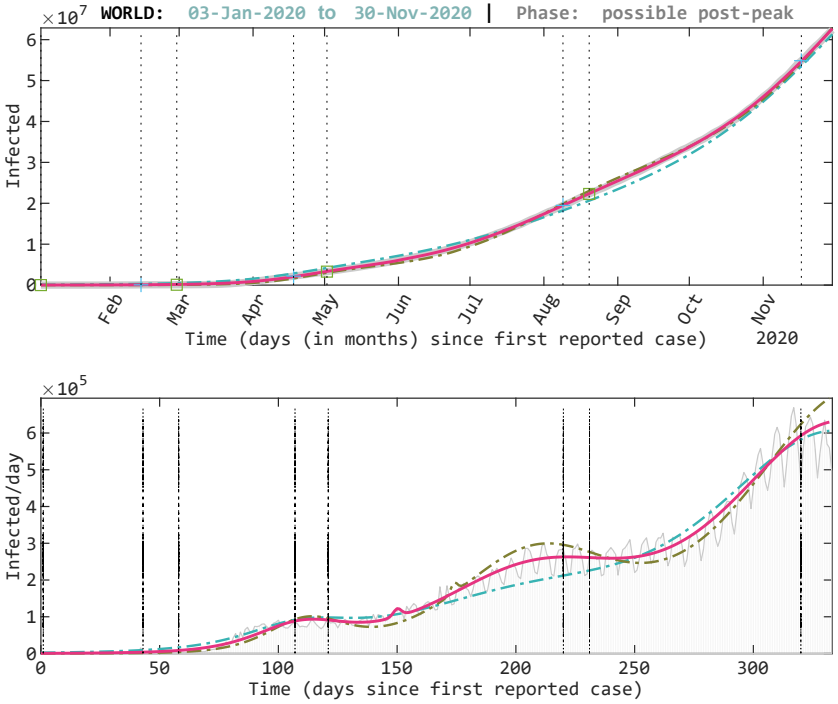
Growth process described by the nlogistic-sigmoid model (red line), actual COVID-19 data (gray lines and areas). Bootstrapped 95% upper and lower confidence Intervals (dashed brown and dashed blue).

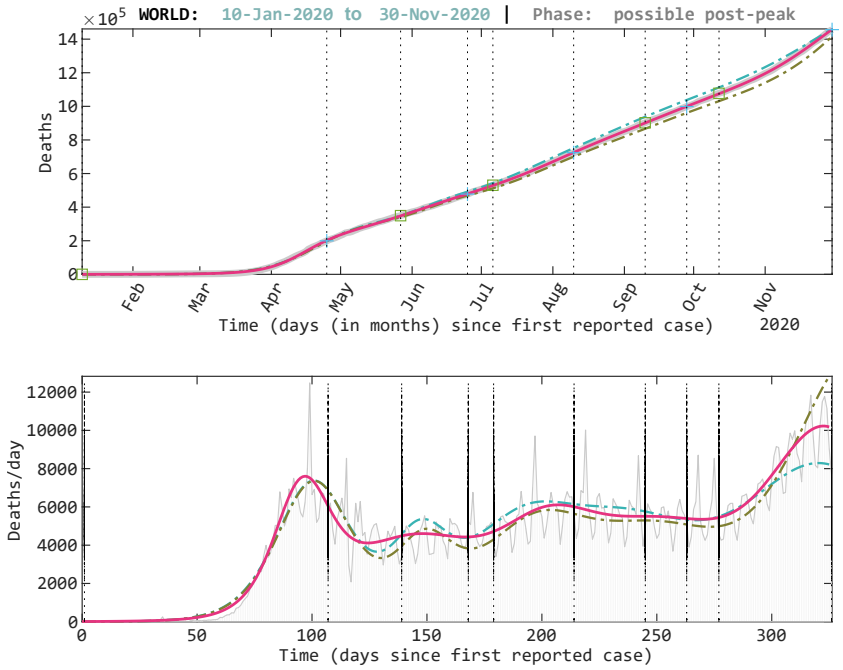
$$\text{YIR} = 0.535035[0.473952, 0.555294]$$

$$\text{XIR} = 1.09269[1.05804, 1.22407]$$

$$\text{YIR} = 0.549417[0.289723, 0.611299]$$

$$\text{XIR} = 0.973225[0.942817, 1.39597]$$





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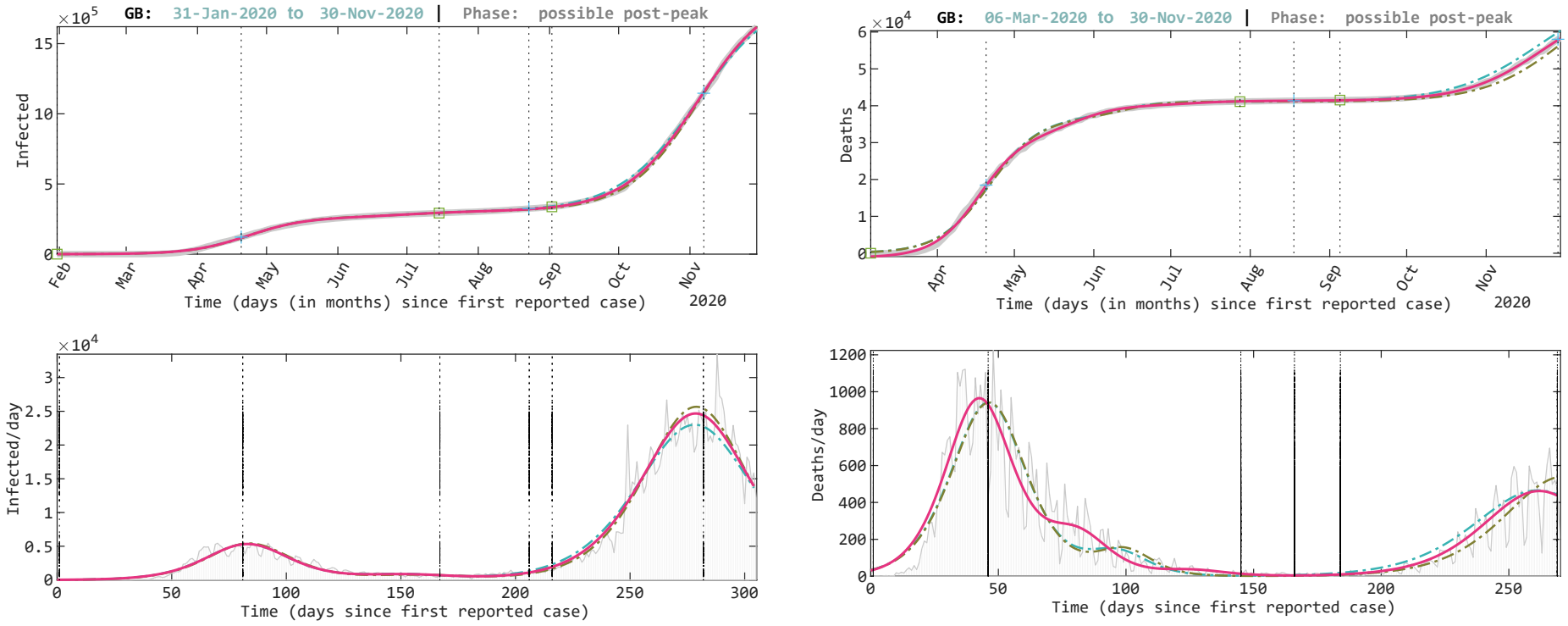
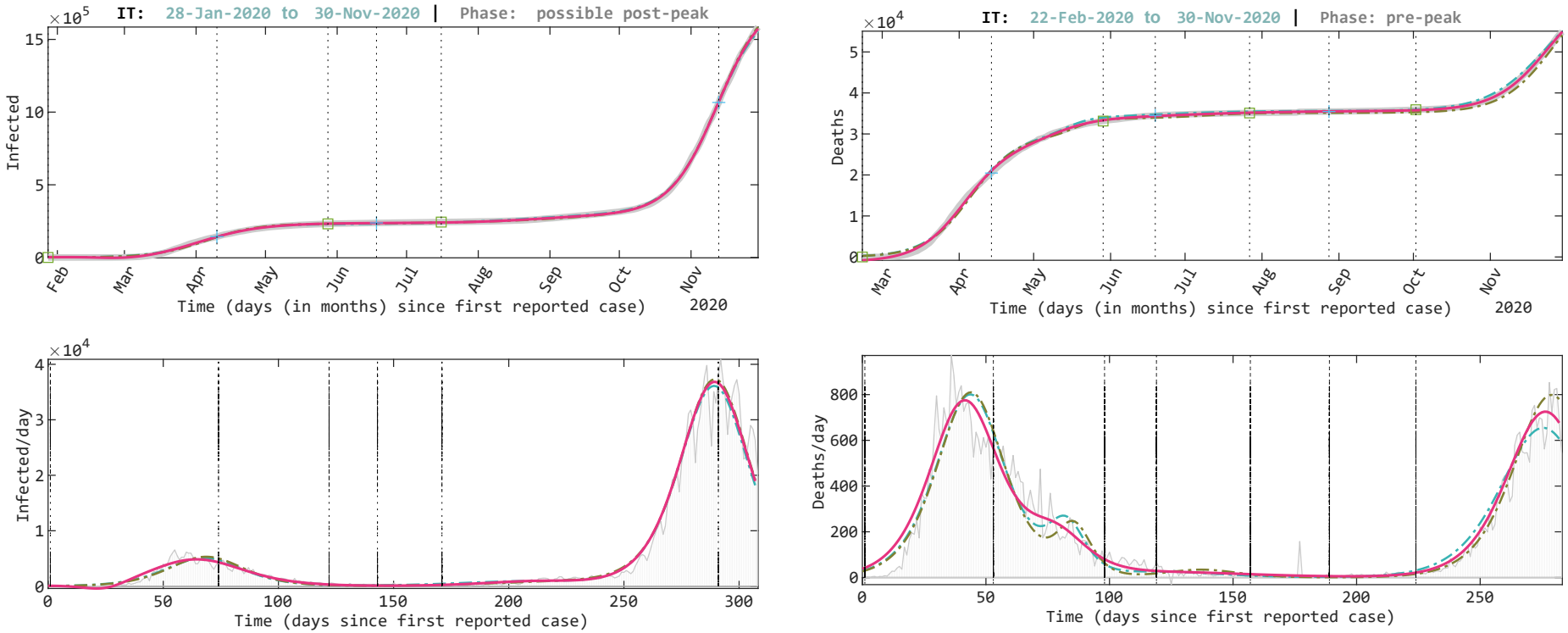
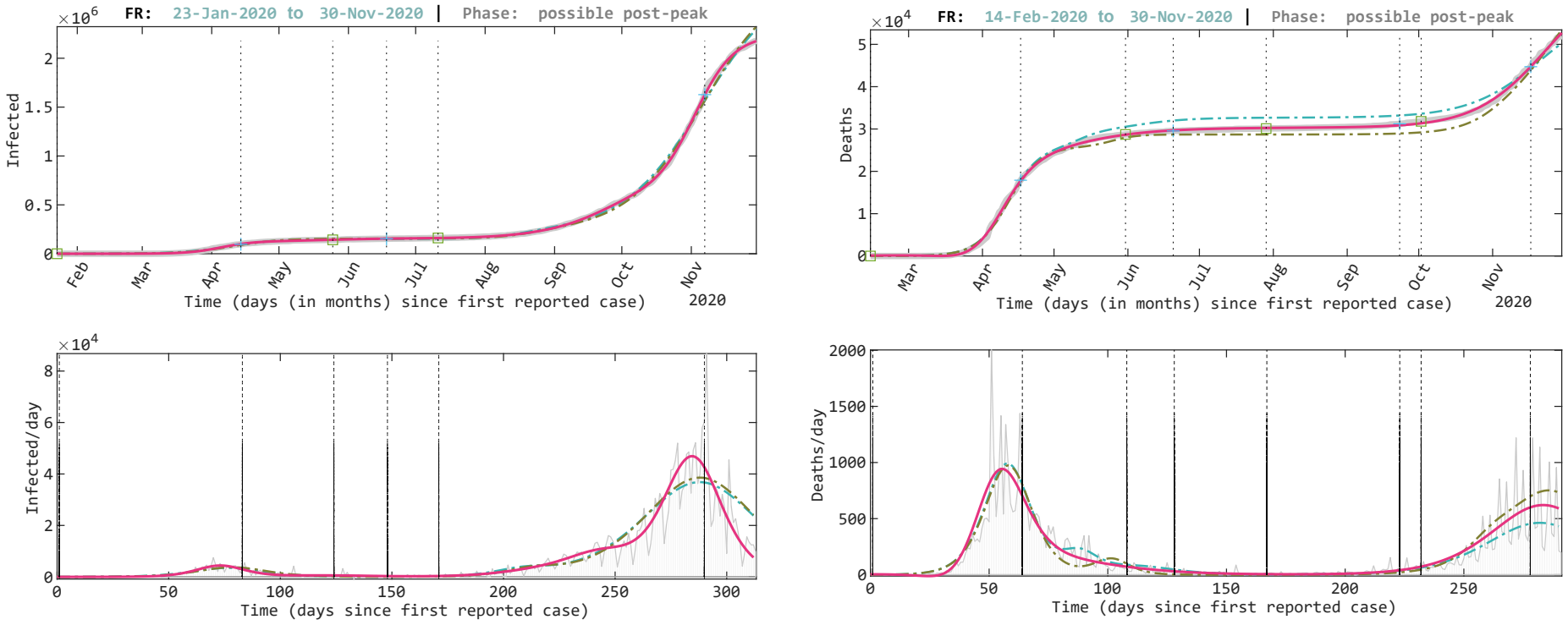
United Kingdom



Italy


IT: 28-Jan-2020 to 30-Nov-2020 | Phase: possible post-peak
×10^5
Infected
Feb Mar Apr May Jun Jul Aug Sep Oct Nov 2020
Time (days (in months) since first reported case)
IT: 22-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Deaths
Mar Apr May Jun Jul Aug Sep Oct Nov 2020
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

France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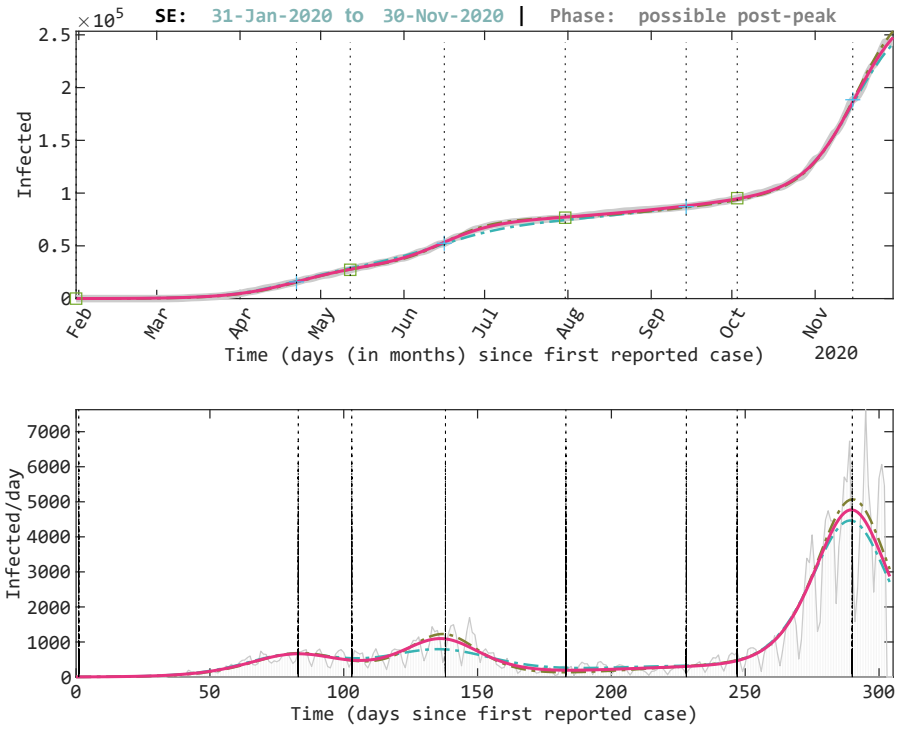
Infected
Time (days (in months) since first reported case)
FR: 14-Feb-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Sweden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×10^5
Infected
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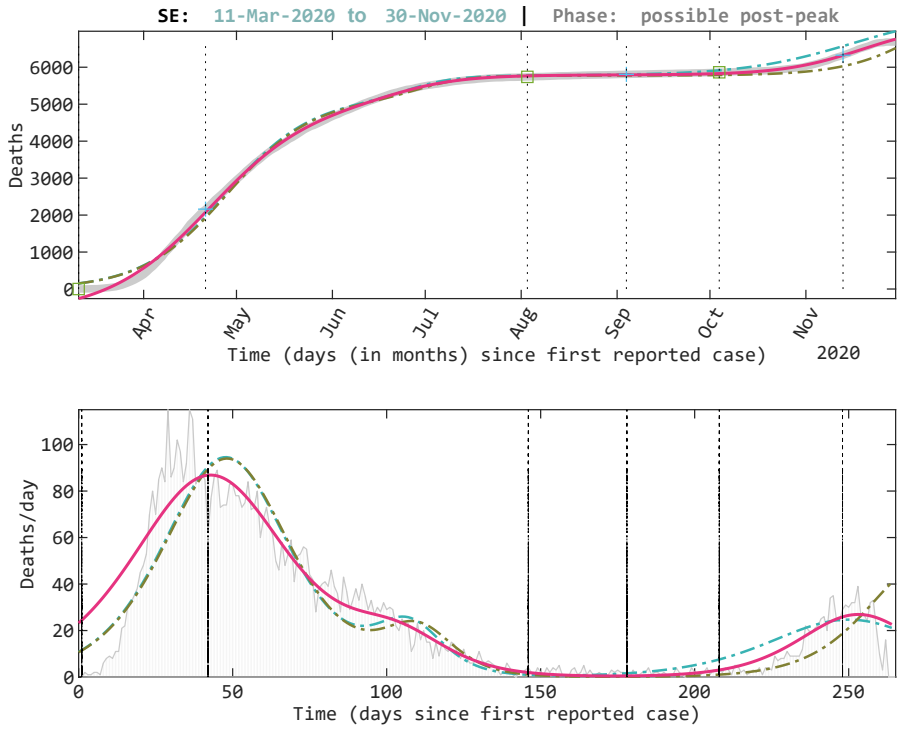
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)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Deaths
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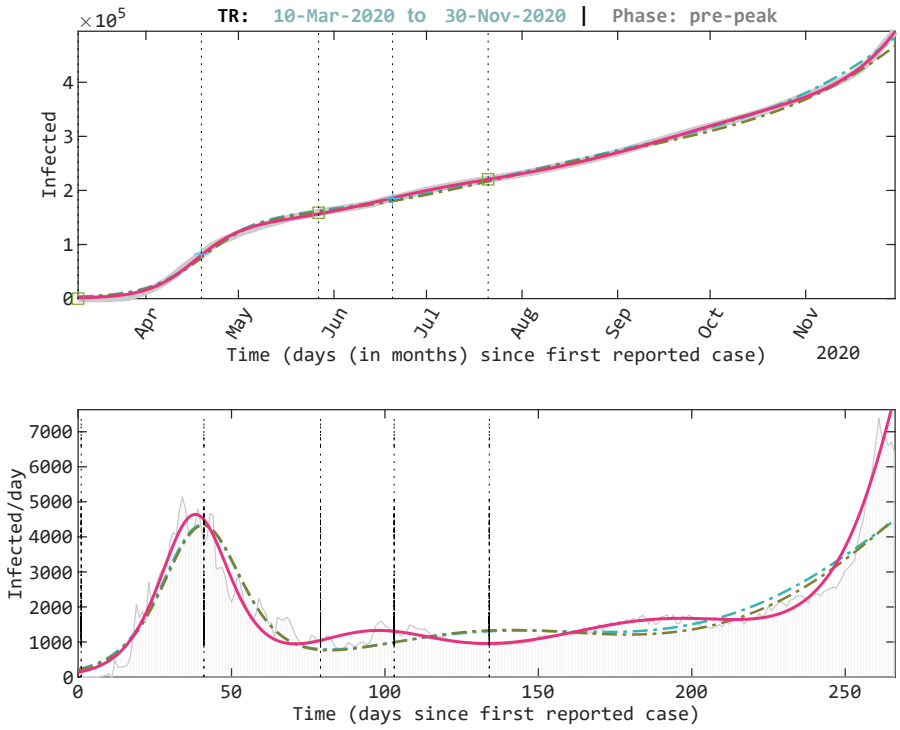
2020
Time (days (in months) since first reported case)
Deaths/day
Time (days since first reported case)

Turkey


TR: 10-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^5
Infected
Apr
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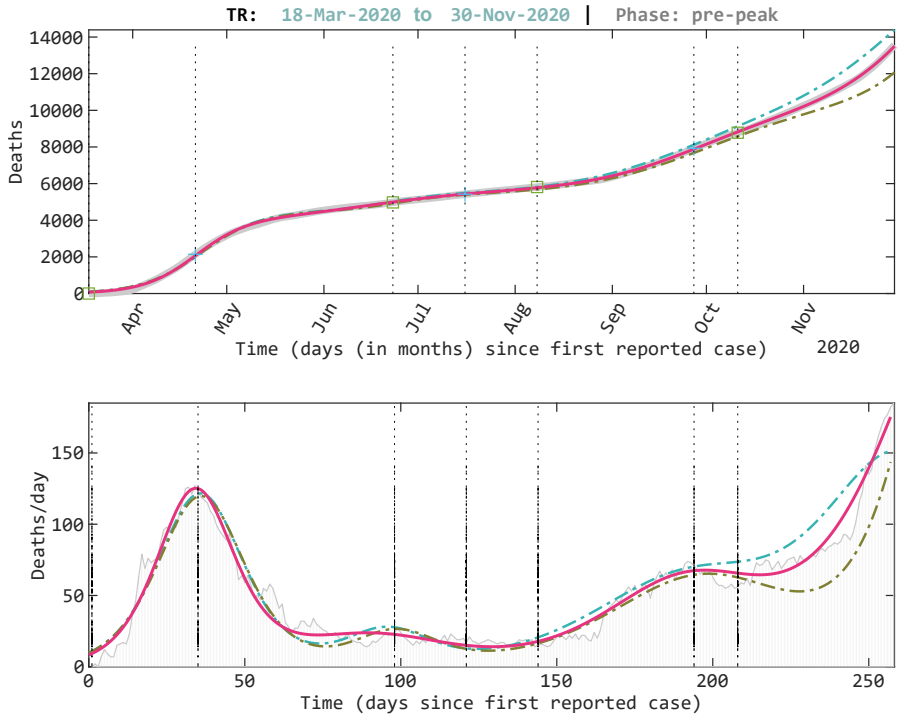
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)


TR: 18-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
Deaths
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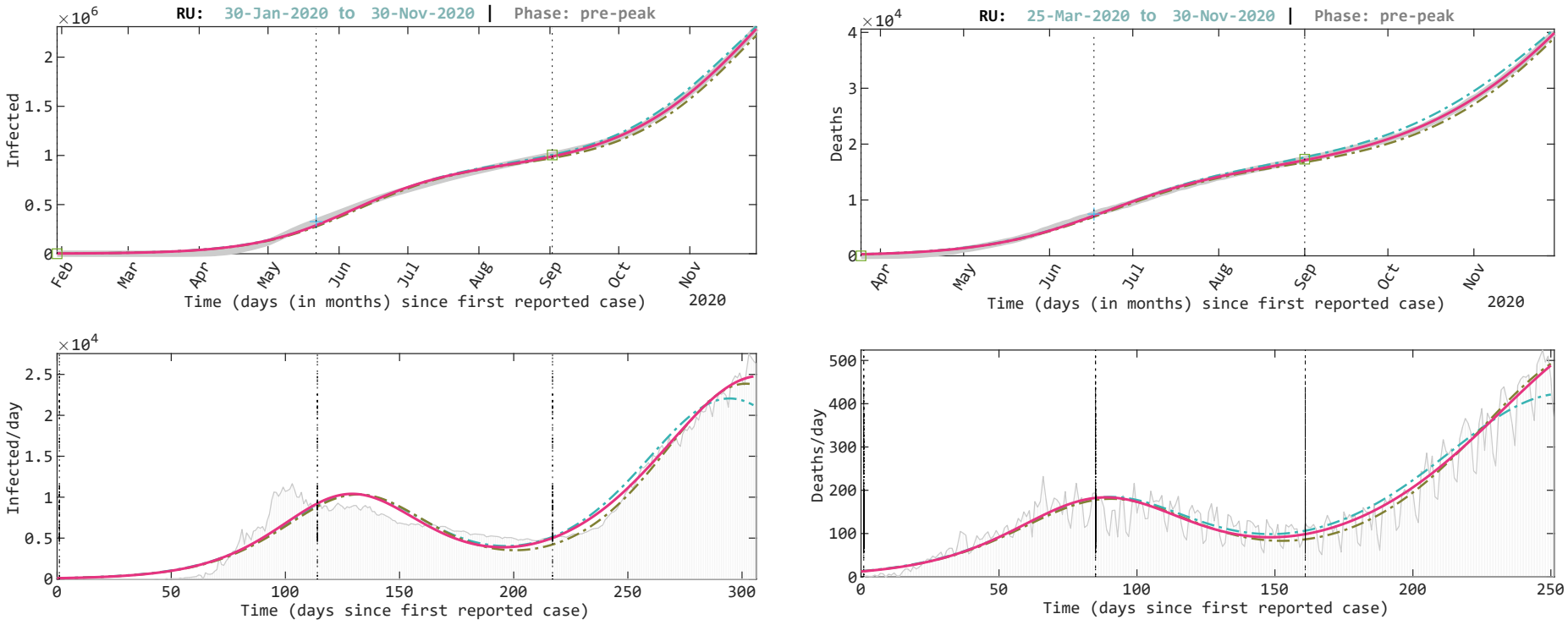
Time (days (in months) since first reported case)
Deaths/day
Time (days since first reported case)

Russia


RU: 30-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^6
Infected
Feb Mar Apr May Jun Jul Aug Sep Oct Nov
2020
Time (days (in months) since first reported case)
RU: 25-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Deaths
Apr May Jun Jul Aug Sep Oct Nov
2020
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

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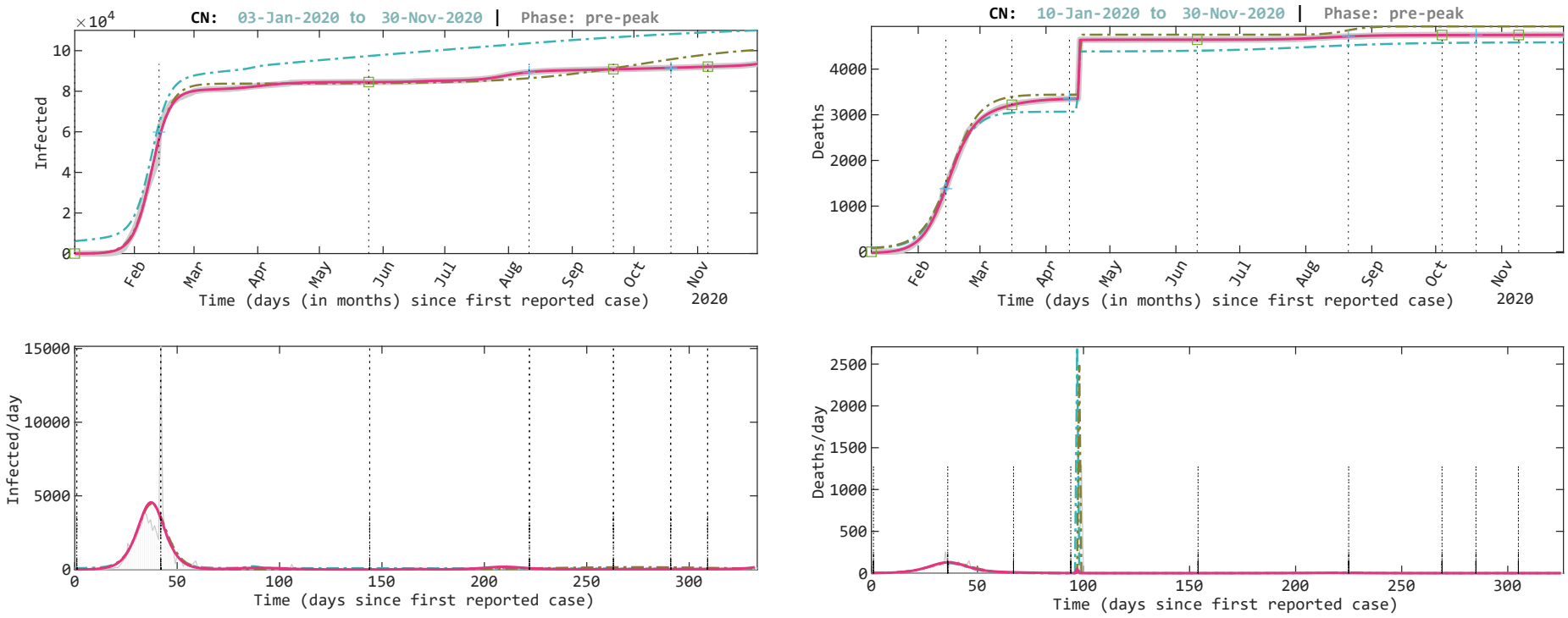
China


CN: 03-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
Time (days (in months) since first reported case)
Feb
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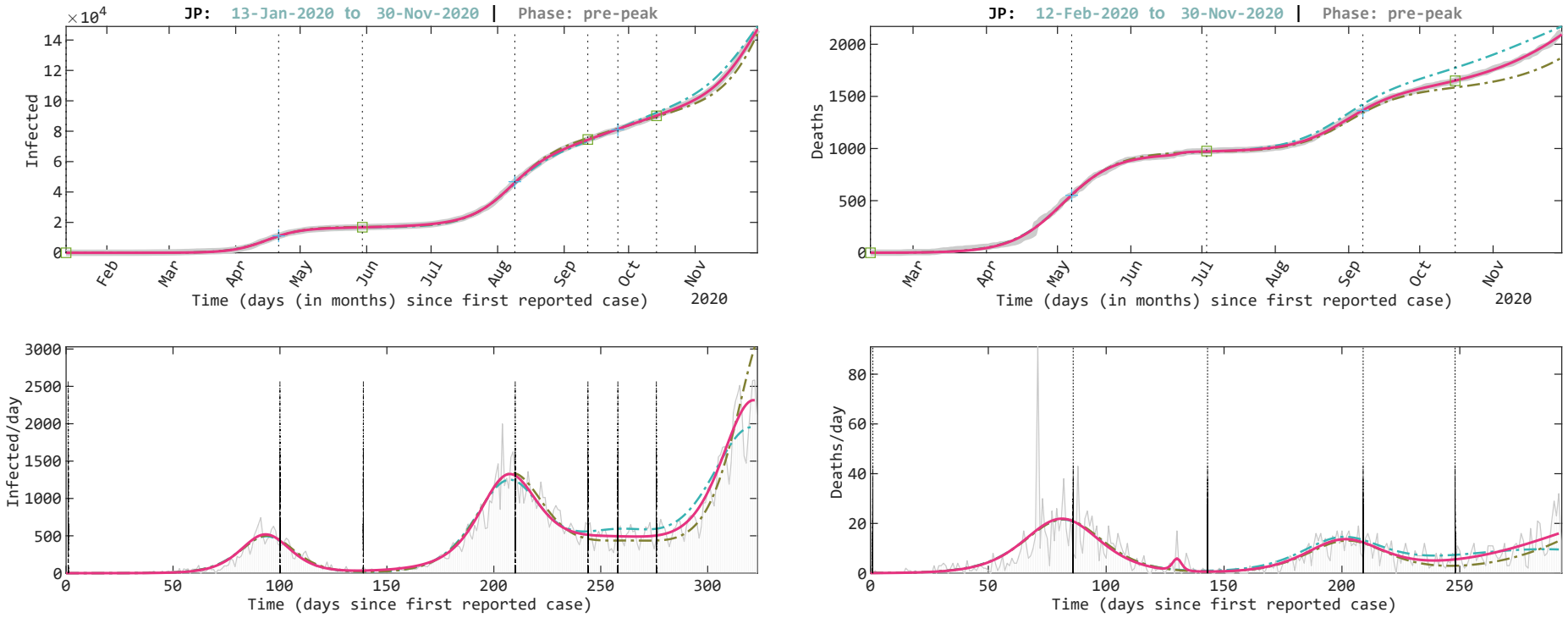
CN: 10-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
Deaths
Infected/day
Time (days since first reported case)
Deaths/day

Japan


JP: 13-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
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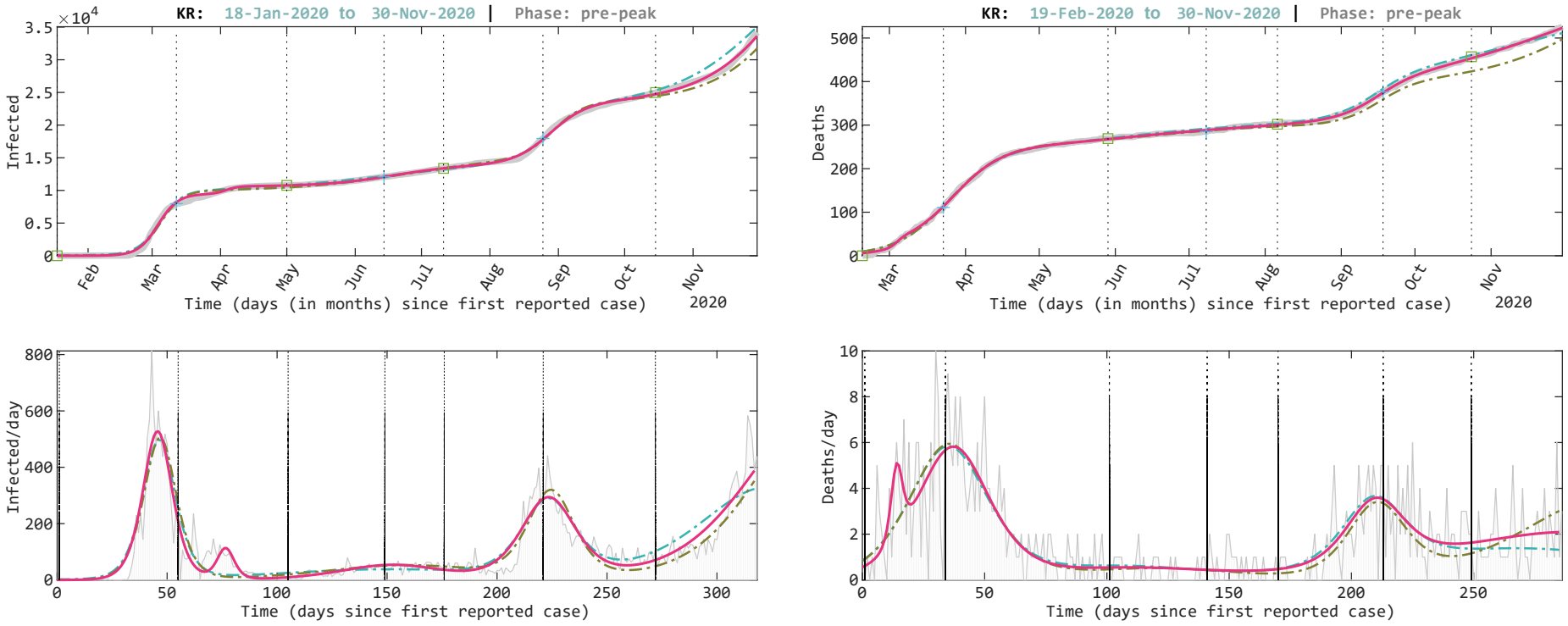
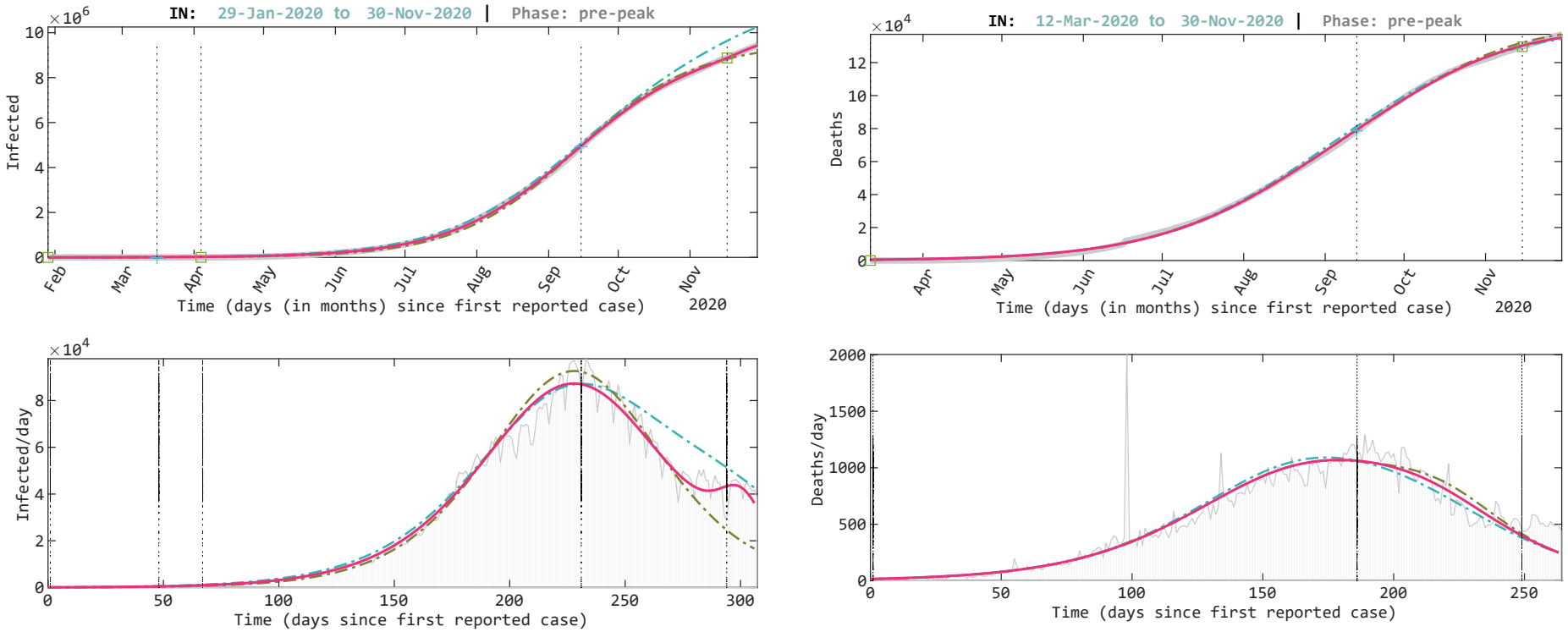
Time (days (in months) since first reported case)
JP: 12-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

South Korea


KR: 18-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
Feb Mar Apr May Jun Jul Aug Sep Oct Nov
2020
Time (days (in months) since first reported case)
KR: 19-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
Deaths
Mar Apr May Jun Jul Aug Sep Oct Nov
2020
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

India


IN: 29-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^6
Infected
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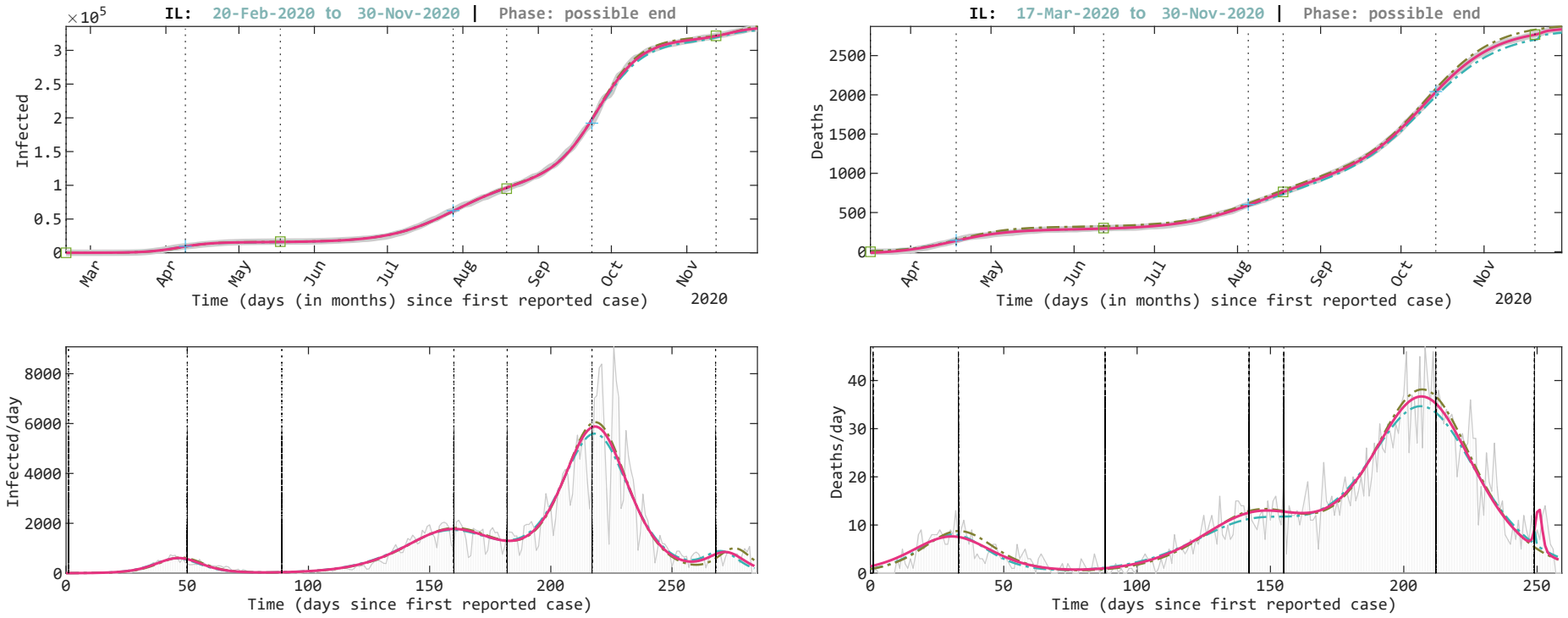
Time (days (in months) since first reported case)
IN: 12-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Deaths
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Israel


IL: 20-Feb-2020 to 30-Nov-2020 | Phase: possible end
×10^5
Infected
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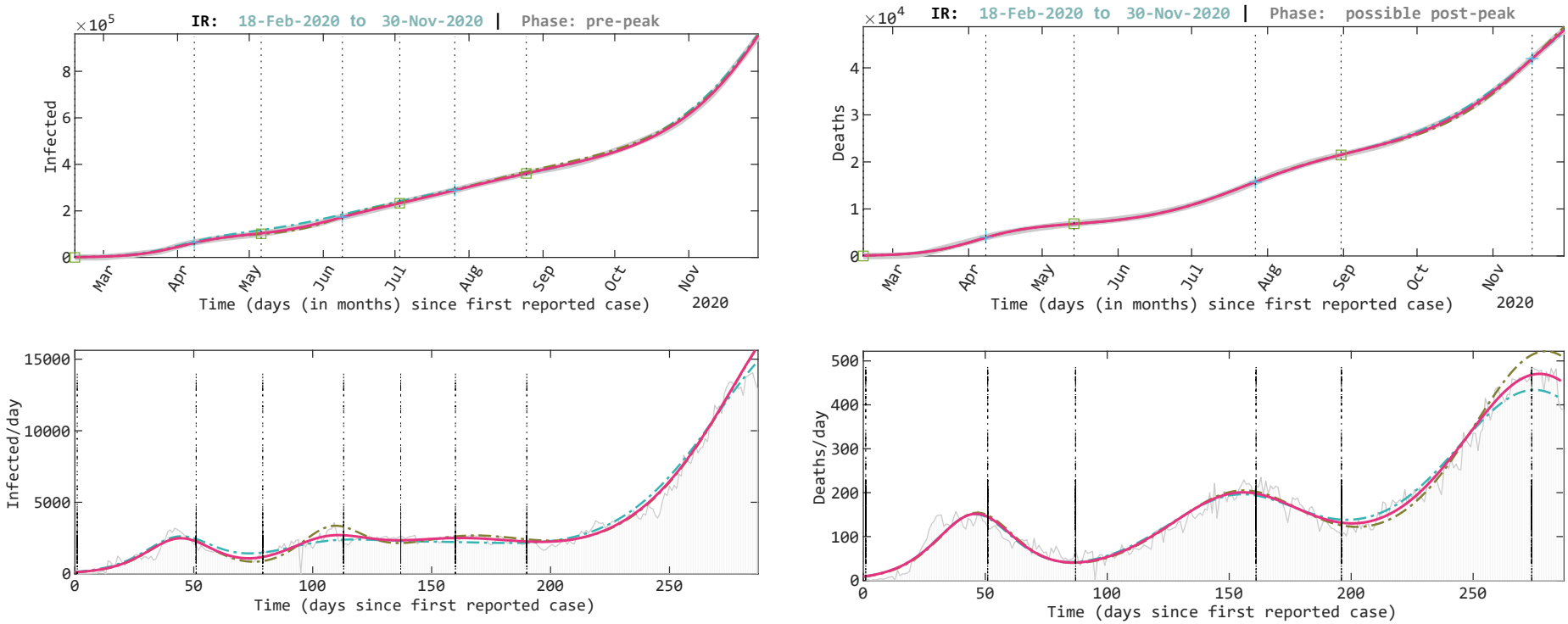
Time (days (in months) since first reported case)
IL: 17-Mar-2020 to 30-Nov-2020 | Phase: possible end
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Iran


IR: 18-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^5
Infected
Mar
Apr
May
Jun
Jul
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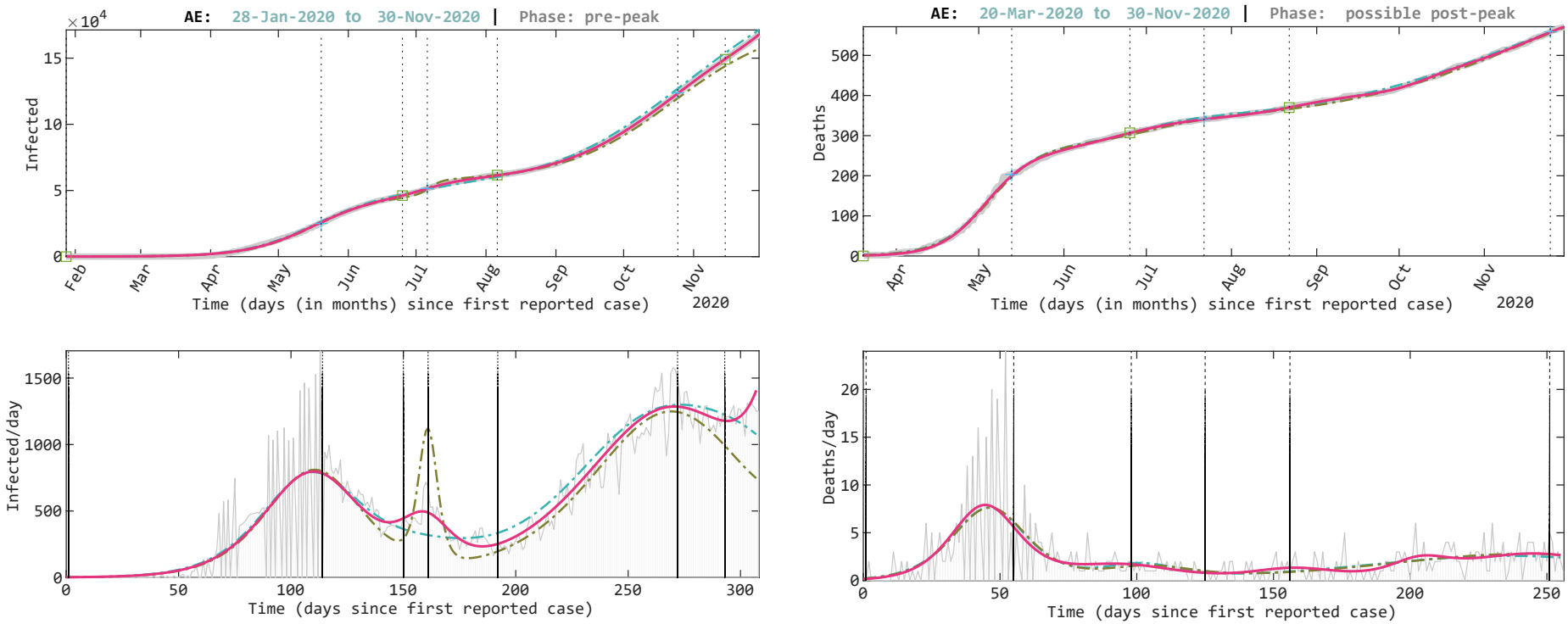
Sep
Oct
Nov
2020
Time (days (in months) since first reported case)
IR: 18-Feb-2020 to 30-Nov-2020 | Phase: possible post-peak
×10^4
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

United Arab Emirates



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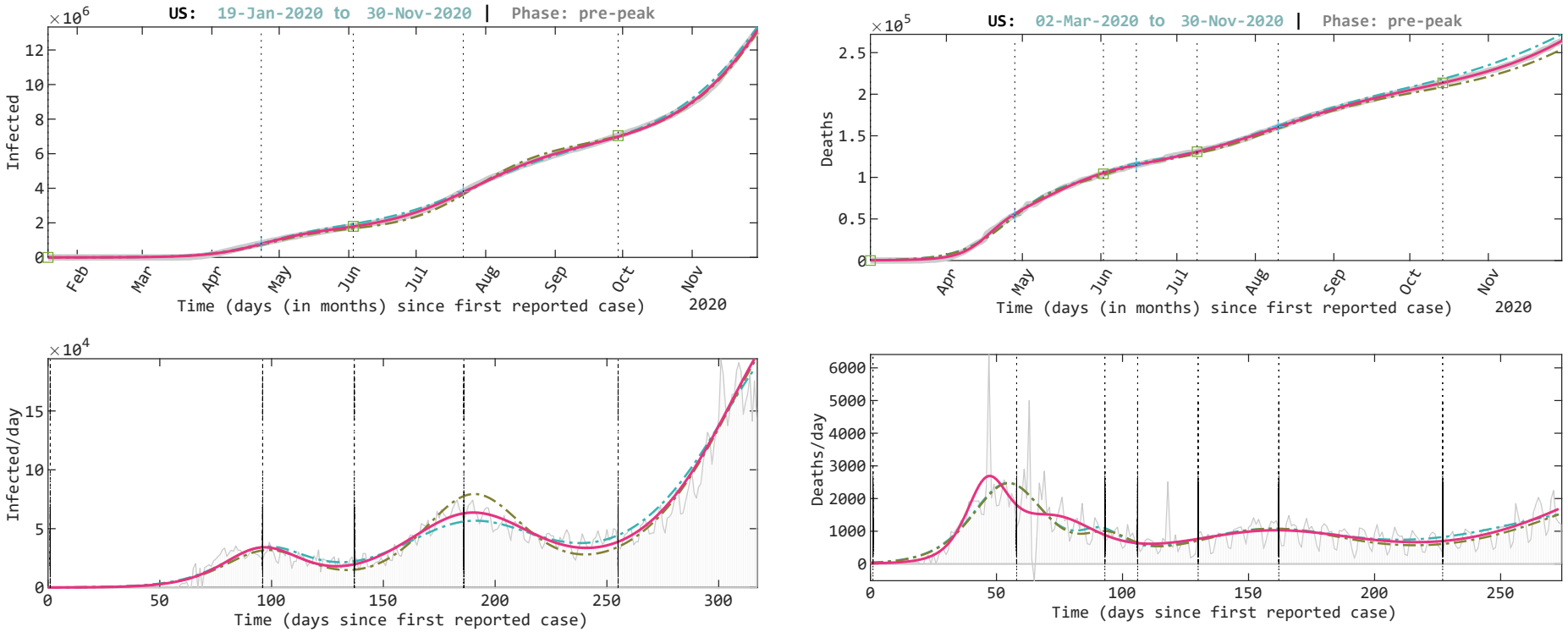
USA


US: 19-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^6
Infected
Feb
Mar
Apr
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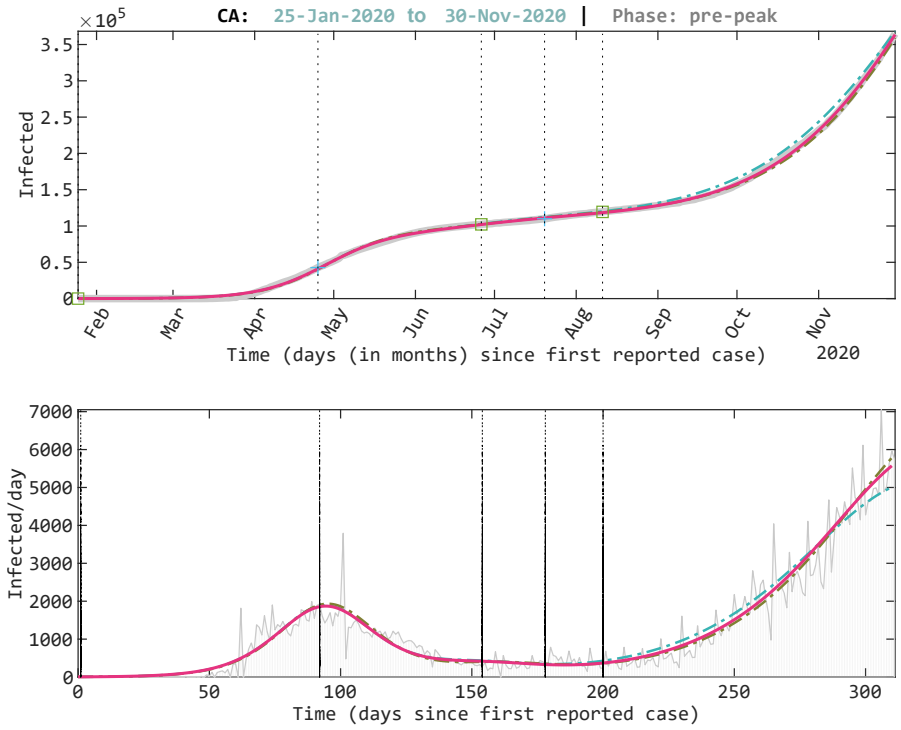
Nov
2020
Time (days (in months) since first reported case)
US: 02-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^5
Deaths
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Canada


CA: 25-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^5
Infected
Feb
Mar
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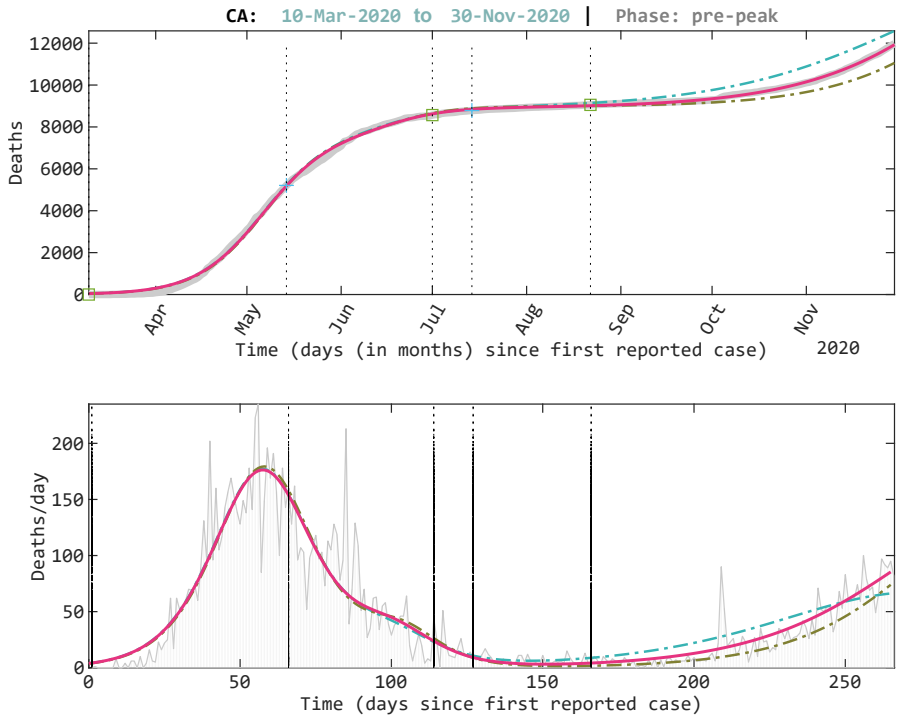
2020
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)


CA: 10-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
Deaths
Apr
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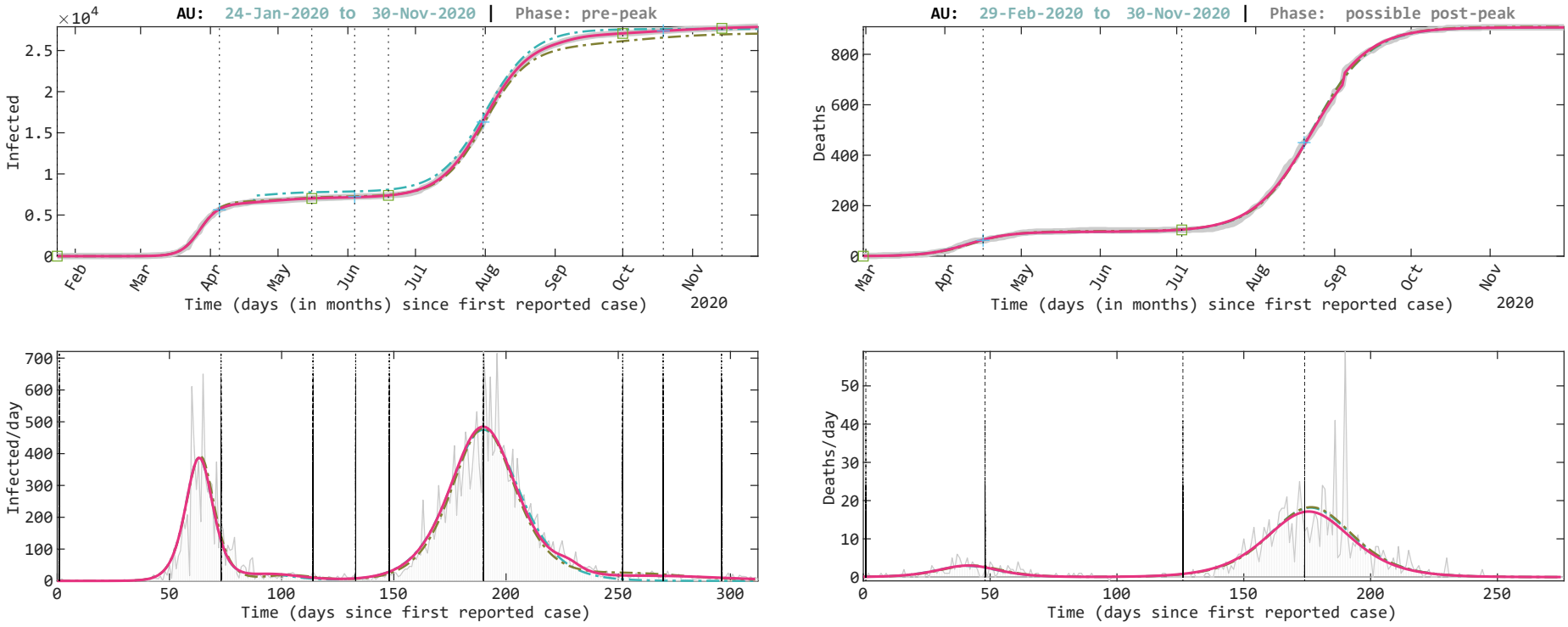
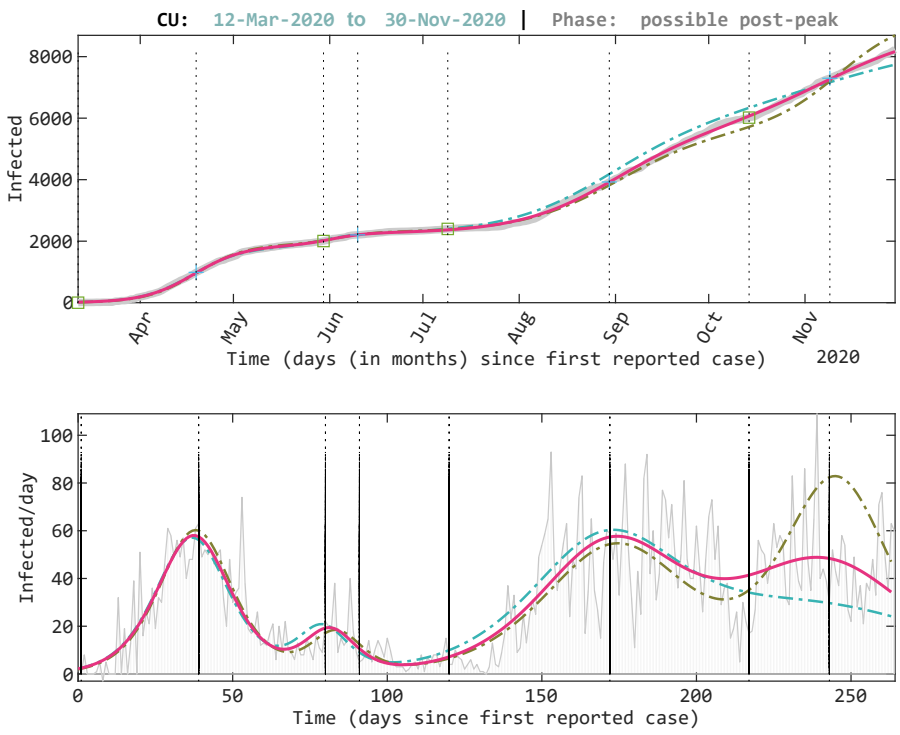
2020
Time (days (in months) since first reported case)
Deaths/day
Time (days since first reported case)

Australia


AU: 24-Jan-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
Time (days (in months) since first reported case)
2020
AU: 29-Feb-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
2020
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Cuba


CU: 12-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Infected
Time (days (in months) since first reported case)
Apr
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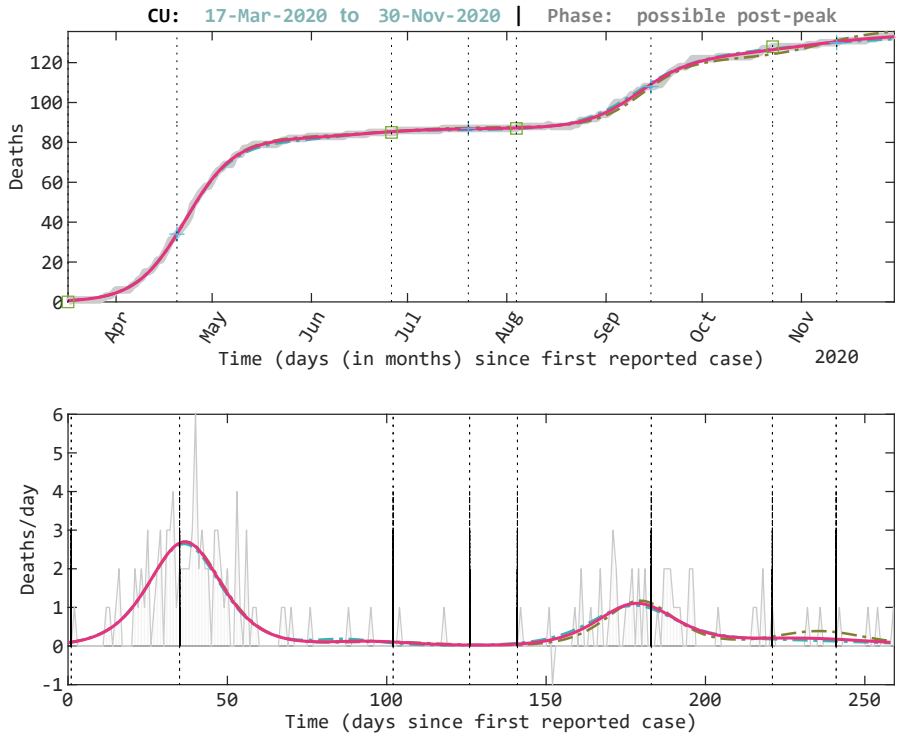
Nov
2020
Infected/day
Time (days since first reported case)


CU: 17-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Apr
May
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Sep
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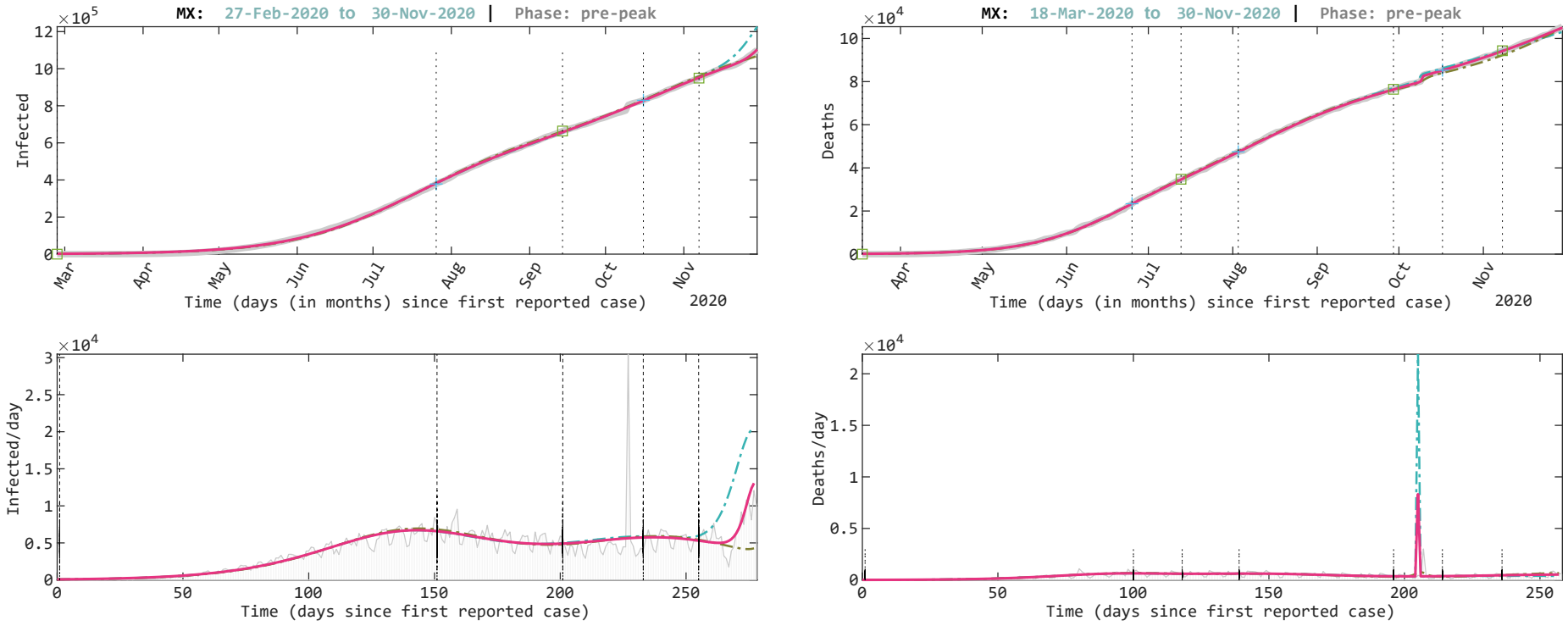
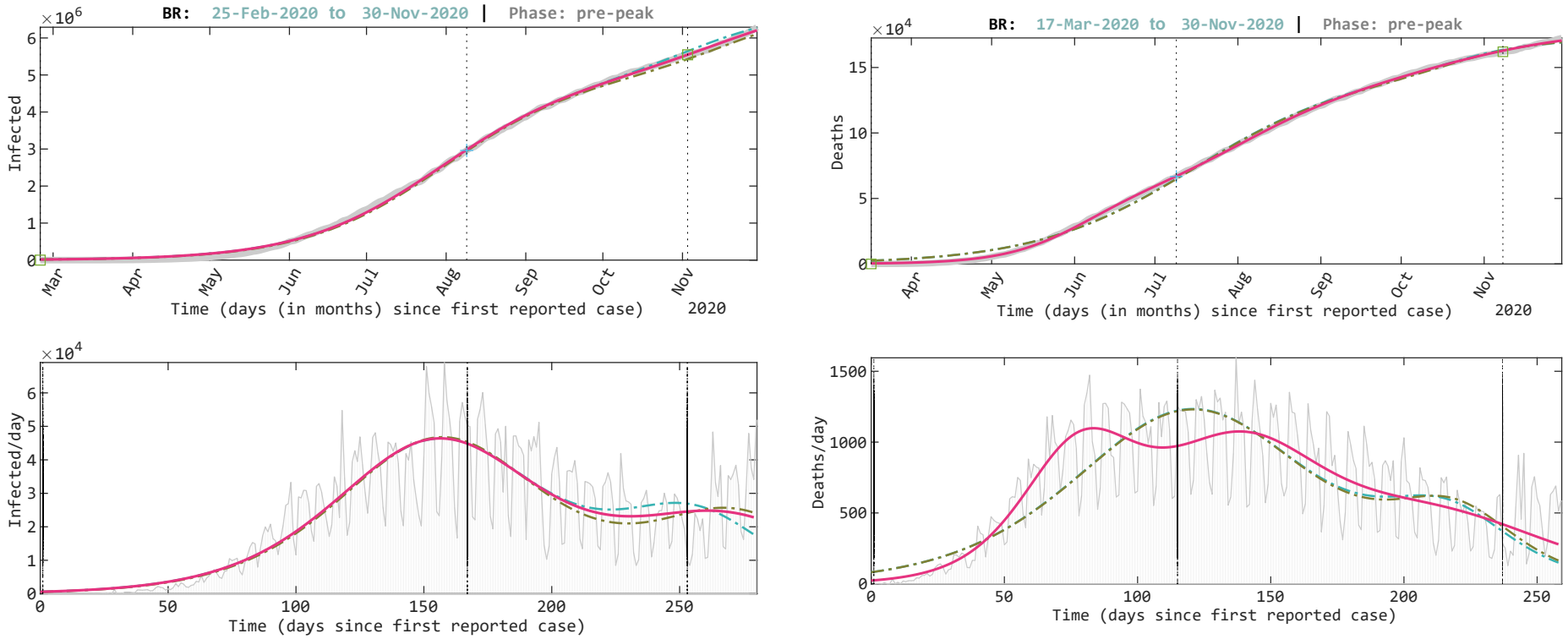
Nov
2020
Deaths/day
Time (days since first reported case)

Mexico


MX: 27-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^5
Infected
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
2020
Time (days (in months) since first reported case)
MX: 18-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Deaths
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
×10^4
Deaths/day
Time (days since first reported case)

Brazil


BR: 25-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^6
Infected
Mar
Apr
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Jun
Jul
Aug
Sep
Oct
Nov
2020
Time (days (in months) since first reported case)
BR: 17-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Deaths
Time (days (in months) since first reported case)
×10^4
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

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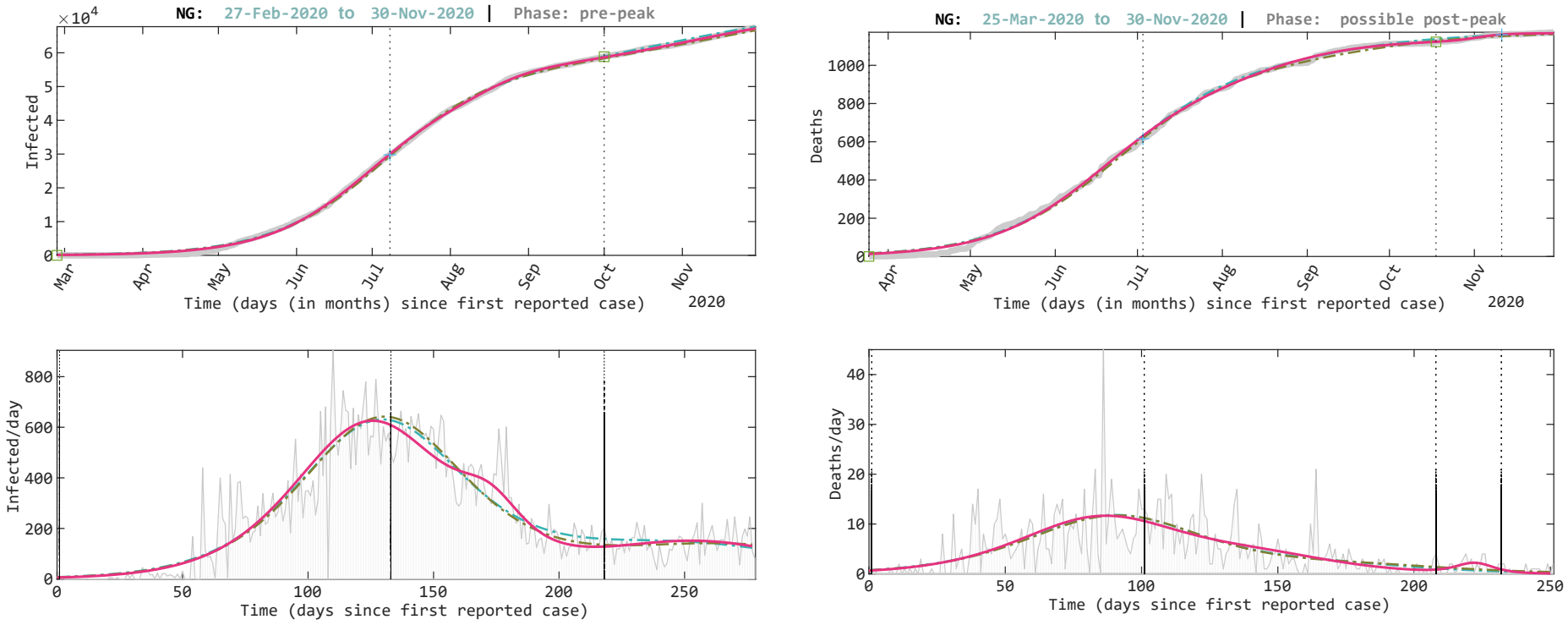
Nigeria


NG: 27-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
Mar
Apr
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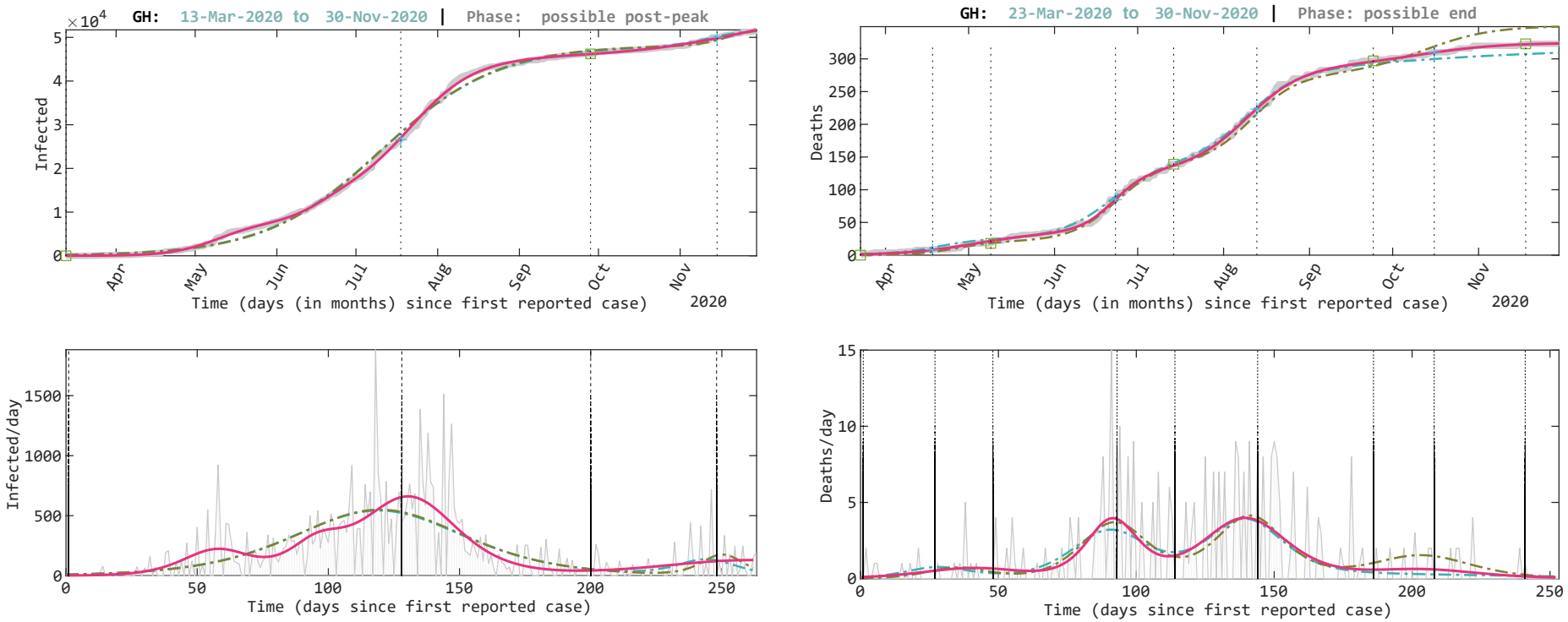
Nov
2020
Time (days (in months) since first reported case)
NG: 25-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Ghana


GH: 13-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
×10^4
Infected
Apr
May
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Aug
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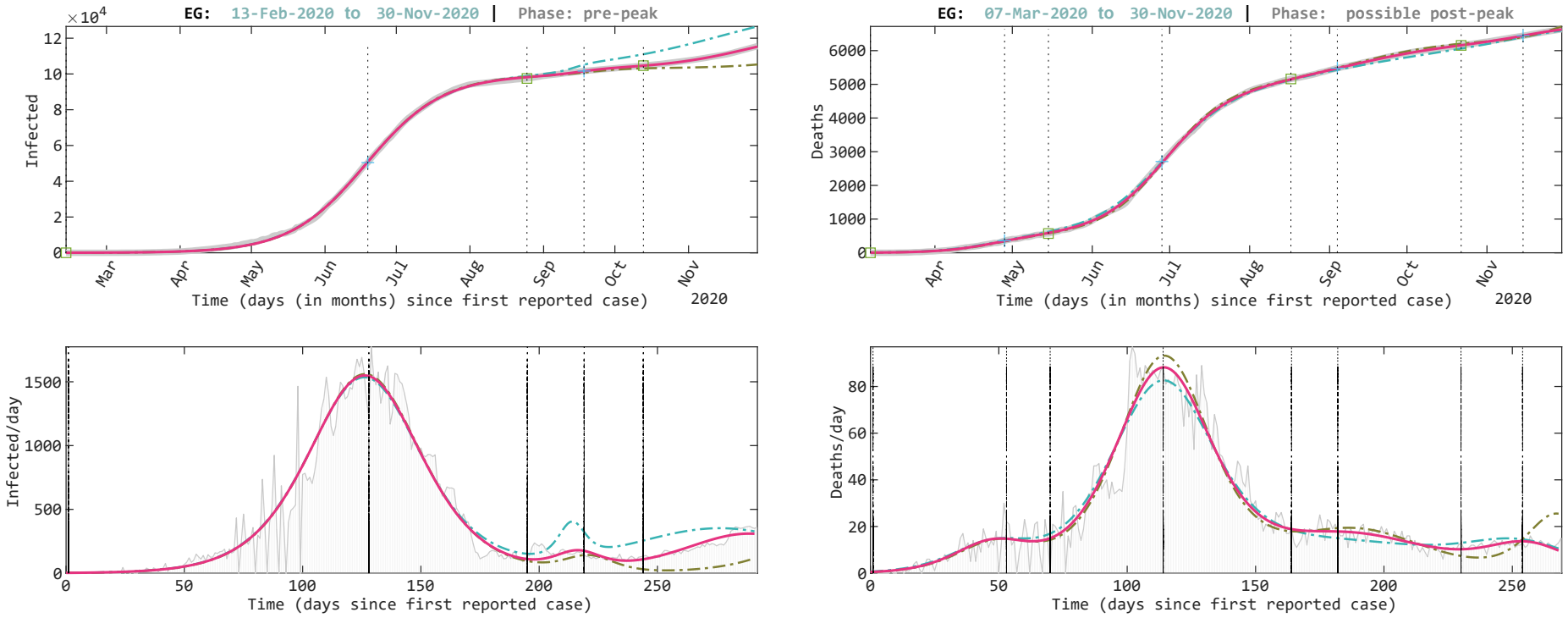
Oct
Nov
2020
Time (days (in months) since first reported case)
GH: 23-Mar-2020 to 30-Nov-2020 | Phase: possible end
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

Egypt


EG: 13-Feb-2020 to 30-Nov-2020 | Phase: pre-peak
×10^4
Infected
Mar
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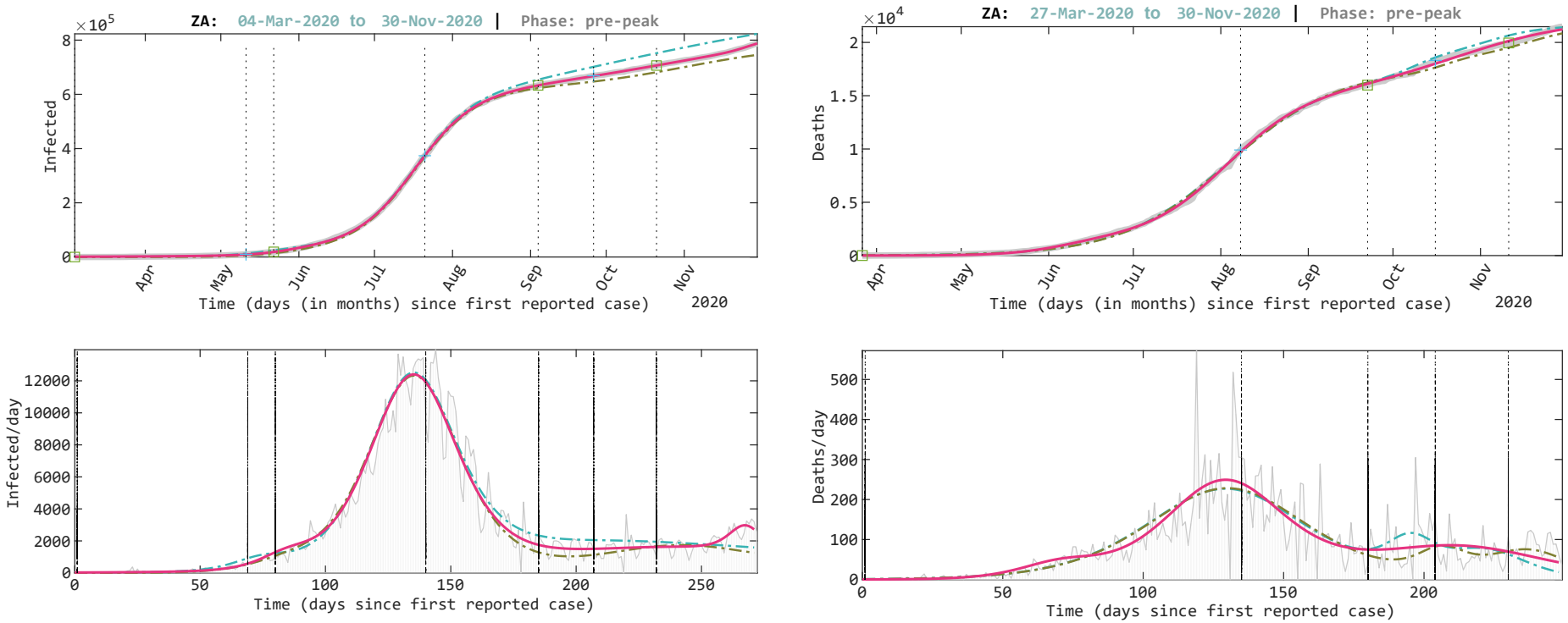
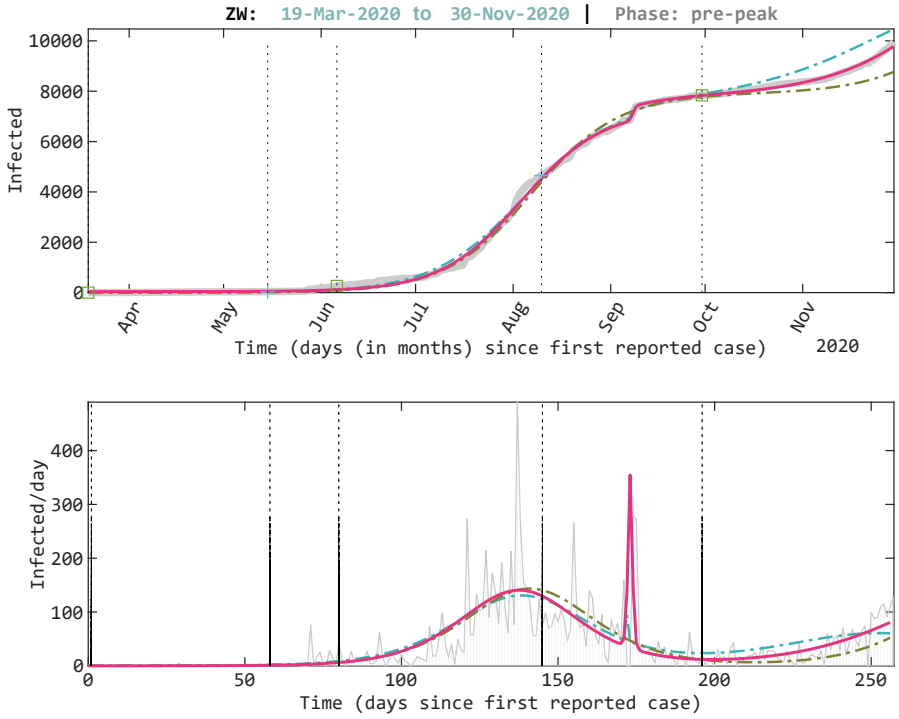
2020
Time (days (in months) since first reported case)
EG: 07-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

South Africa



Zimbabwe


ZW: 19-Mar-2020 to 30-Nov-2020 | Phase: pre-peak
Infected
Time (days (in months) since first reported case)
Apr
May
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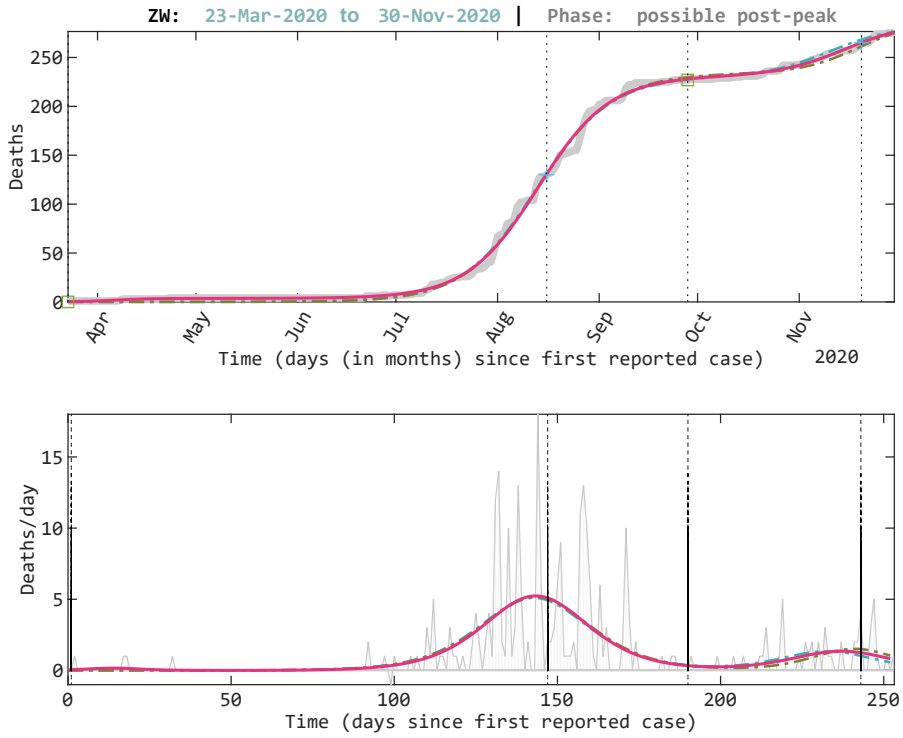
Nov
2020
Infected/day
Time (days since first reported case)


ZW: 23-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Apr
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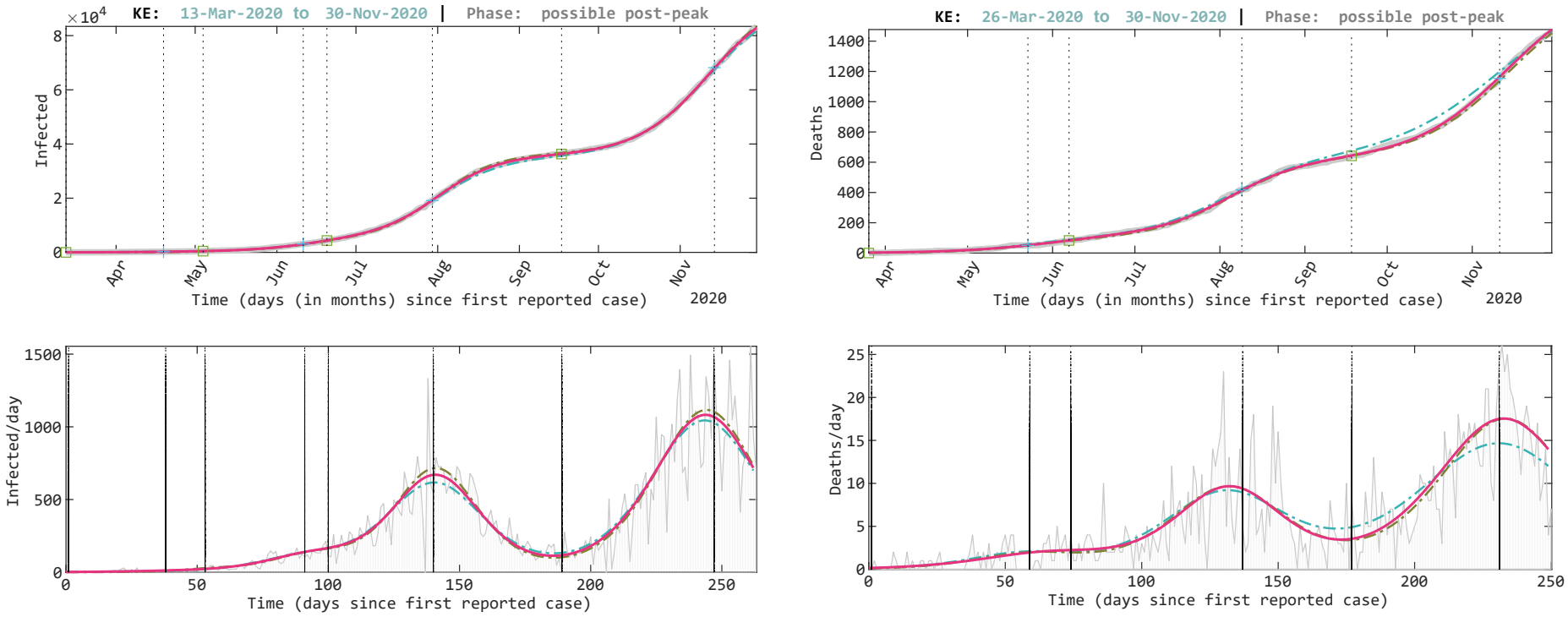
2020
Deaths/day
Time (days since first reported case)

Kenya


KE: 13-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
×10^4
Infected
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
2020
Time (days (in months) since first reported case)
KE: 26-Mar-2020 to 30-Nov-2020 | Phase: possible post-peak
Deaths
Time (days (in months) since first reported case)
Infected/day
Time (days since first reported case)
Deaths/day
Time (days since first reported case)

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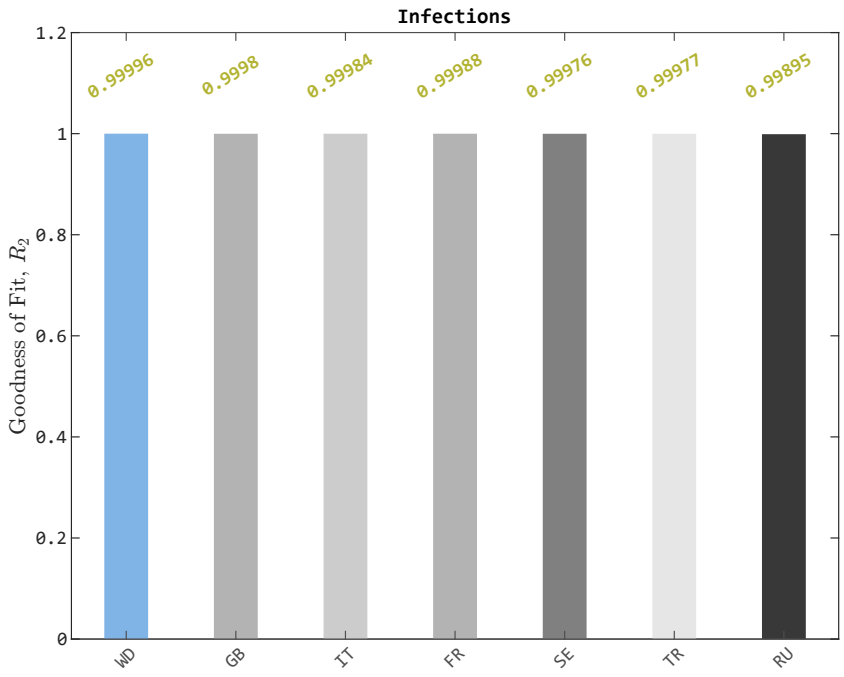
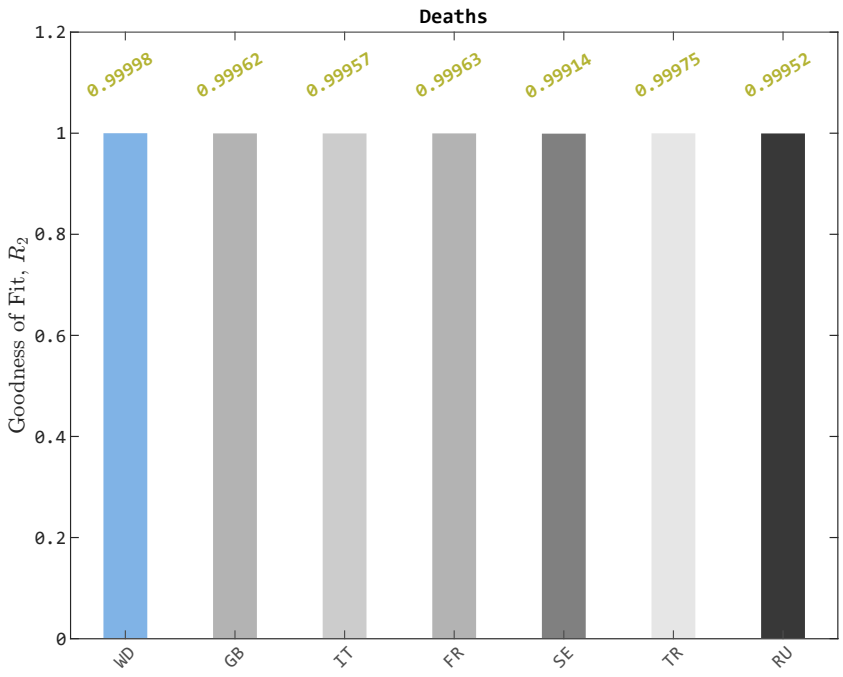
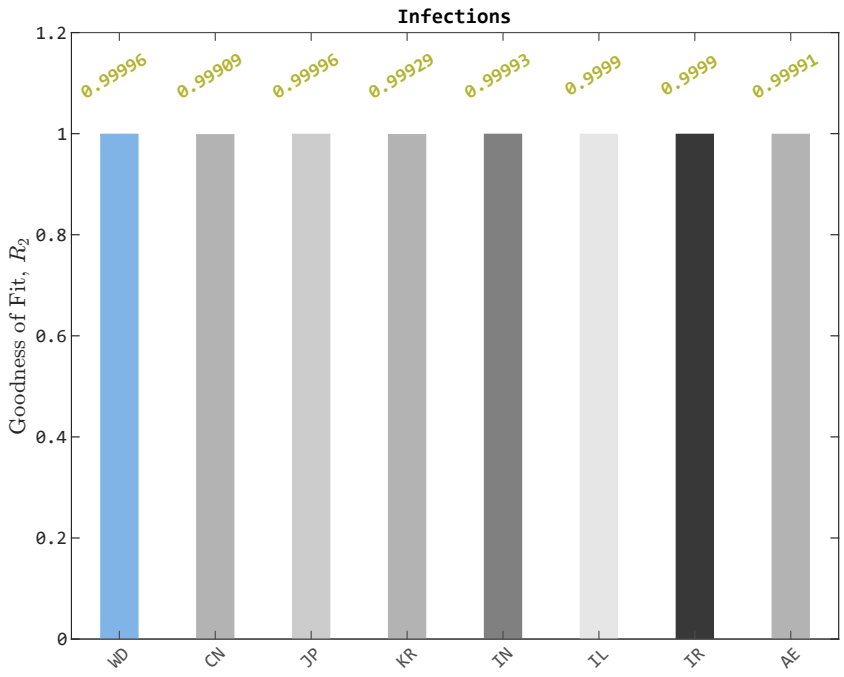
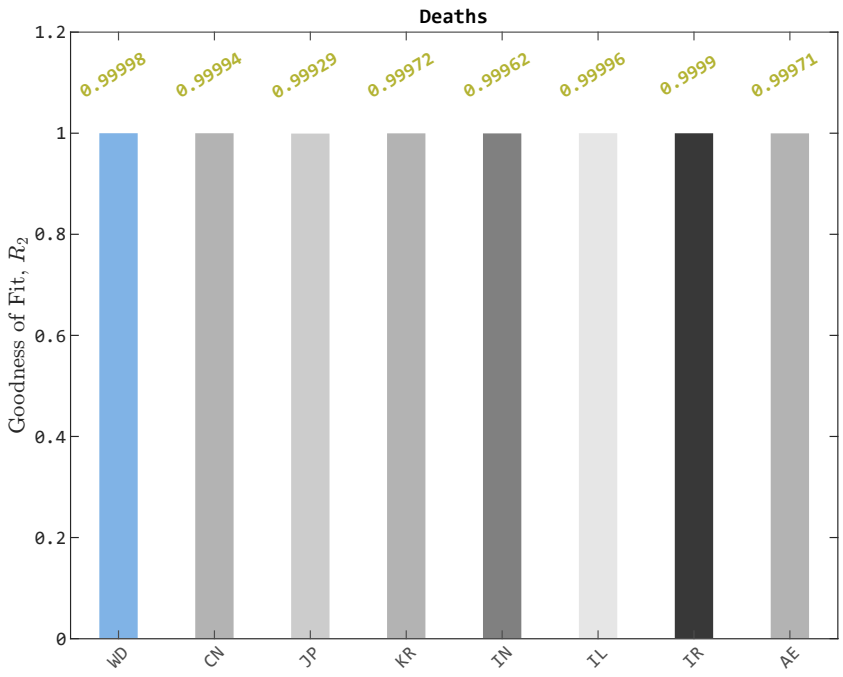
Europe: Goodness of Fit



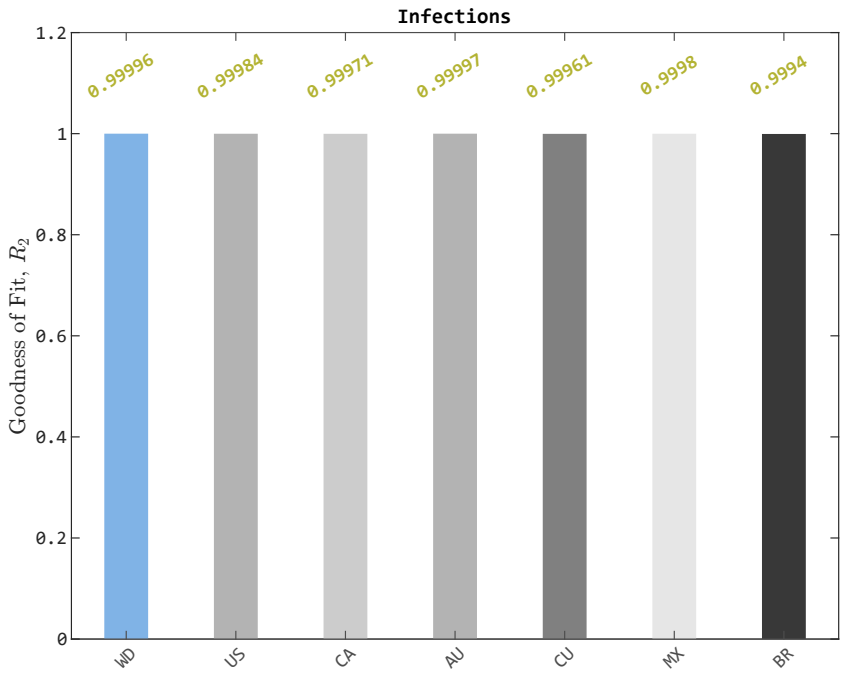


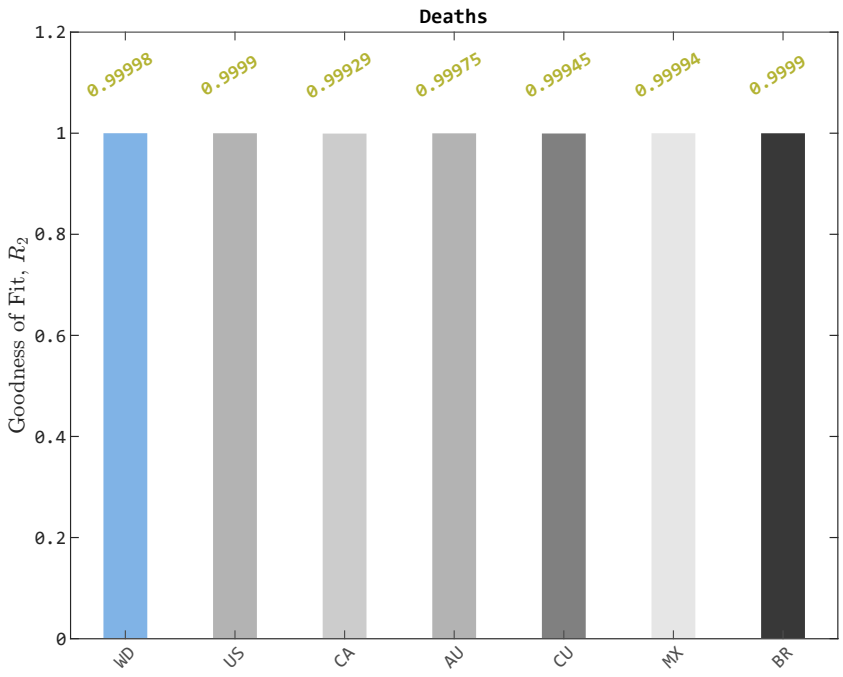
Asia: Goodness of Fit



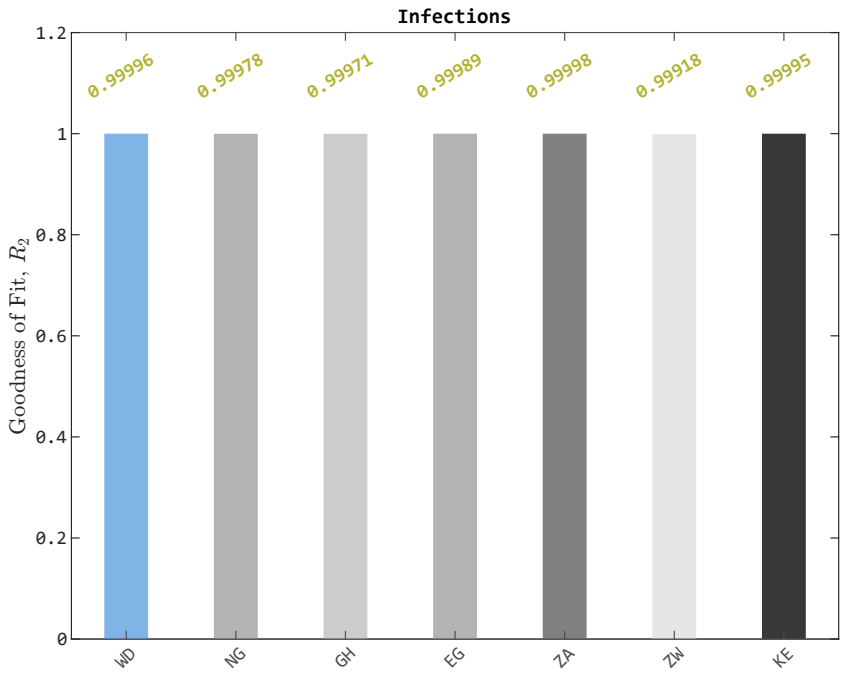


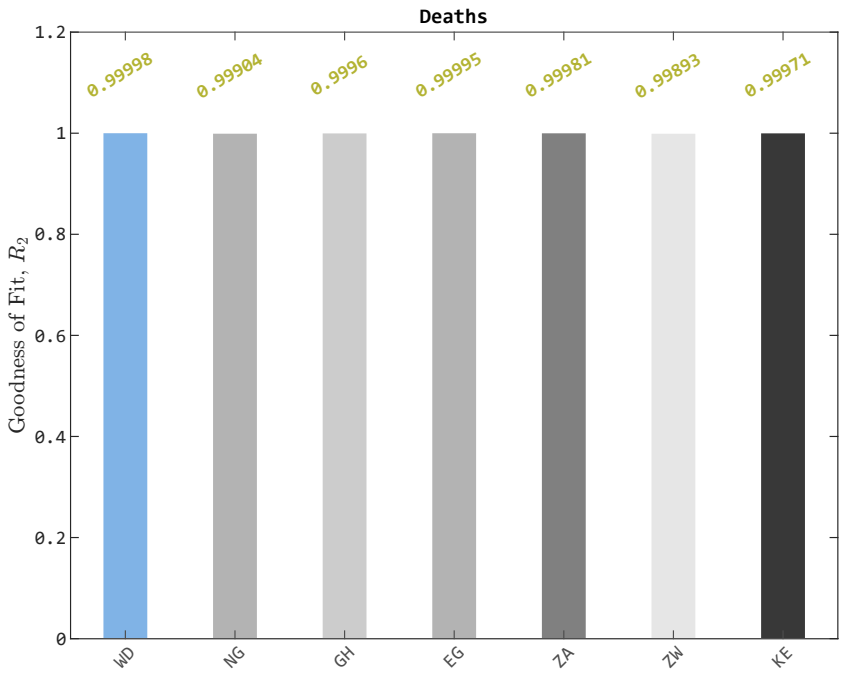
America and Australia: Goodness of Fit





Africa: Goodness of Fit





1 Introduction
- Background Story

2 nlogistic-sigmoid function
- SISO/MISO Pipeline

3 logistic curve metrics
- YIR and XIR

4 Application to the COVID-19 Pandemic
- Modelling: Optimization workflow
- Results: World Model and Metrics
- Europe
- Asia
- America and Australia
- Africa
- Results: Goodness of Fit

5 Conclusions

6 References

Conclusions

We have presented an extended definition for the logistic model, called the **n**logistic-sigmoid model that can be used to model the complex growth behaviour of real-world processes, such as the COVID-19 pandemic, with a high degree of accuracy. Our results reinforces that the COVID-19 pandemic growth follows the principle of logistic growth and in all applications provide statistically significant fit as high as an adjusted $R_2 \geq 99\%$ to the dataset describing the COVID-19 epidemic spread in each of the selected locales.

We strongly believe that the **n**logistic-sigmoid (NLSIG) can again increase the value of the logistic-sigmoid as an indispensable modelling tool in many areas of biomedical and epidemiological research and other scientific and engineering disciplines

Thank you!

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