Untitled

Daniel Hsiao

November 12, 2018

Contents

1 Methodology **2**
- 1.1 Standard Approach . . . 2
- 1.2 With estimation error of w . . . 2
- 1.3 Negative weights . . . 3
- 1.4 Truncated weights . . . 3
- 1.5 Bias Correction . . . 4

2 Survey of Professional Forecasters (SPF) **5**

3 Empirical Results **7**
- 3.1 Proceure . . . 7
- 3.2 Ratio of Mean Squared Prediction Error . . . 8
- 3.3 Out-of-sample truncation selection . . . 8
- 3.4 Out-of-sample truncation result . . . 10
- 3.5 Bias weighting . . . 10

1 Methodology

1.1 Standard Approach

Consider the two variable case here for illustration purpose. We have two forecasts, y_1 and y_2, of the true variable y. We want to combine y_1 and y_2 with a weight w that we have $y_c = wy_1 + (1-w)y_2$. Assume they follow some distribution, e.g. $y_1 \sim D(0, \sigma_1)$, $y_2 \sim D(0, \sigma_2)$, and $corr(y_1, y_2) = \rho$. Then the variance of the combined forecast y_c is

$$Var(y_c) = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_1\sigma_2\rho, \tag{1}$$

and the optimal weight with minimal variance is

$$w^* = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}. \tag{2}$$

Equation 2 is the standard benchmark approach in the combination theory, where extensive research had been done on. **add research**. Equation 2 has a few empirical results that are against this approach. Two common alternatives are diagonal covariance matrix and equal weights.

Ignoring the correlation term ρ by setting $\rho = 0$, we get the inverse relation on the variance

$$w^* = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}. \tag{3}$$

This is a robust way to avoid the estimation of the covariance when the dimension goes up. The amount of parameter to estimate for the covariance with dimension n is $\frac{1}{2}n(n+1)$, which is quadratic in n. When the user only estimates the variances, the amount of parameter to estimate reduces to n, which greatly decreases the estimation error. **add citation**

Equal weights is another common approach that works better empirically. **add citation** The forecast combination is in this case just an arithmetic mean of all forecasts. The reason behind this is the fact that estimating weights increases or shifts the forecast errors due to additional estimation error in the estimation of w. We laborate on the estimation error of w more later.

add more on equal weight

1.2 With estimation error of w

We can also consider the weight as non-deterministic, but related with y, e.g., in a trivairate distribution with finite third and fourth moments. Under trivariate distribution, the variance of the weights influences the expected value and the variance of the combined forecast. The expected value and the variance of the combined forecast becomes

$$\begin{aligned} E(y_c) =& \mu + (cov(w, y_1 - y_2))^2 \\ var(y_c) =& E(w)^2\sigma_1^2 + (1-E(w))^2\sigma_2^2 + 2E(w)(1-E(w))\rho\sigma_1\sigma_2 \\ &+ E[(w-E(w))(y_1-y_2)(E(w)y_1 + (1-E(w))y_2 - \mu)] \\ &+ E[(w-E(w))^2(y_1-y_2)^2] - cov(w, y_1-y_2)^2. \end{aligned} \tag{4}$$

Equation **??** shows the general case of the forecast combination. If the covariance between w and $y_1 - y_2$ is not 0, the forecast is biased when combining, with bias $cov(w, y_1 - y_2)^2$. The variance also increases from euqation 1 with $E[(w-E(w))(y_1-y_2)(E(w)y_1 + (1-E(w))y_2 - \mu)] + E[(w-E(w))^2(y_1-y_2)^2] - cov(w, y_1-y_2)^2$.

can we prove that the change in var(yc) is positive? I tried to prove it but got stuck at $E[(w - E(w))(y_1 - y_2)(E(w)y_1 + (1 - E(w))y_2 - \mu)] + Var((w - Ew)(y_1 - y_2)) > 0$. This case the only requirements are that the individual forecast to be unbaised and that the weights sum up to 1.

Let $d = (d_1, d_2)'$ be the third moment between y_1, y_2, and $\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$ be the (co)variance matrix, we have

$$w^\dagger = w^*(1 + \frac{\sigma_{22}d_1 + \sigma_{11}d_2 - \sigma_{12}(d_1 + d_2)}{\sigma_{11}\sigma_{22} - 2\sigma_{12}}) - \frac{\sigma_{22}d_1 - \sigma 12 d_2}{\sigma_{11}\sigma_{22} - 2\sigma_{12}}. \tag{5}$$

The non-deterministic weight selection is a linear combination of the original weight. The non-deterministic weight does not change if the third moment is 0.

1.3 Negative weights

Looking back to equation 2, we examin the effect of high correlation term. Assume without loss of generality that $\sigma_1 = \sigma_2(1 + \delta)$, where $\delta > 0$, we rewrite the weight as

$$w = \frac{\sigma_2^2(-\rho\delta + (1 - \rho))}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}. \tag{6}$$

The numerator in w consist of σ_2^2 scaled with a weighted mean between $-\delta$ and 1 with weight ρ. When ρ is small, the weights is close to equation 3. When ρ is large, the negative difference in variance $-\delta$ takes over and results in negative weights. The level of negativity accounts for both the negtaive difference and correlation. The boundary case is

$$\rho = \frac{\sigma_2}{\sigma_1}, \tag{7}$$

which w decreases to 0 and $y_c = y_2$.

From equation 5, we look in the scaling parameter and the intercept adjustment vector. Under same condition where $\sigma_1 = \sigma_2(1 + \delta)$, we rewrite the determinant to

$$\sigma_{11}\sigma_{22} - 2\sigma_{12} = \sigma_2 2((1 + \delta)^2 - 2(1 + \delta)), \tag{8}$$

and the scaling factors

$$\begin{aligned} \frac{\sigma_{22}d_1 - \sigma 12 d_2}{\sigma_{11}\sigma_{22} - 2\sigma_{12}} &= \frac{d_1 - \rho(1 + \delta)d_2}{(1 + \delta)^2 - 2(1 + \delta)} \\ \frac{\sigma_{22}d_1 + \sigma_{11}d_2 - \sigma_{12}(d_1 + d_2)}{\sigma_{11}\sigma_{22} - 2\sigma_{12}} &= \frac{d_1(1 - \rho(1 + \delta)) + d_2((1 + \delta)^2 - \rho(1 + \delta)}{(1 + \delta)^2 - 2(1 + \delta))} \end{aligned} \tag{9}$$

and I'm lost in what I want to say

1.4 Truncated weights

To avoid the high correlated forecasts, we use truncation on the variable. The weight estimation is as follows

$$**insertlatexequationoftruncation** \tag{10}$$

where the sum of the weights are rescaled to 1.

Assume that there is no skewness in the joint distribution, e.g., w^* is unbiased estimator of the true w. The expected value of the weights $\tilde{w}$ is

$$
\begin{aligned}
E(\tilde{w}) &= E(I_{w>c,w<1-c}w + I_{w>1-c}) \\
&= \int_c^{1-c} wf(w)\,\mathrm{d}w + \int_c^{1-c} f(w)\,\mathrm{d}w.
\end{aligned} \tag{11}
$$

The bias is therefore

$$
E(\tilde{w}) - E(w^*) = \int_{-\infty}^{c} -wf(w)\,\mathrm{d}w + \int_{1-c}^{\infty}(1-w)f(w)\,\mathrm{d}w, \tag{12}
$$

In the first term, we have $w < c < 0$, which cancels out the negative sign and becomes positive. In the second term we have $w > 1-c > 1$, which gives a negative value in $1-w$. In general case where w can go above $1-c$ or below c and the skewness of w is not 0, the bias is non-zero. This increases MSE to the estimated y_c.

The variance is

$$
\begin{aligned}
Var(\tilde{w}) &= E(\tilde{w}^2) - E(\tilde{w})^2 \\
&= E[\tilde{w}^2],
\end{aligned} \tag{13}
$$

And the change in variance is

$$
\begin{aligned}
Var(\tilde{w}) - Var(w) = &\int_{-\infty}^{c} -w^2 f(w)\,\mathrm{d}w + \int_{1-c}^{\infty}(1-w^2)f(w)\,\mathrm{d}w \\
&-(\int_c^{1-c} wf(w)\,\mathrm{d}w + \int_{1-c}^{\infty} f(w)\,\mathrm{d}w)^2 + (\int_{-\infty}^{\infty} wf(w)\,\mathrm{d}w)^2.
\end{aligned} \tag{14}
$$

By Cauchy–Schwarz inequality, and take $g(w)$ as the weight generating function of $\tilde{w}$, we have

$$
\begin{aligned}
Var(\tilde{w}) - Var(w) \leq &\int_{-\infty}^{c} -w^2 f(w)\,\mathrm{d}w + \int_{1-c}^{\infty}(1-w^2)f(w)\,\mathrm{d}w \\
&-(\int_c^{\infty} g(w)f(w)\,\mathrm{d}w)^2 + \int_{-\infty}^{\infty} w^2 f^2(w)\,\mathrm{d}w.
\end{aligned} \tag{15}
$$

Since $|g(w)| \leq |w|$ for all w, thus $g^2(w) \leq w^2$ for all w. Equation 15 becomes

1.5 Bias Correction

Assume for the prediction error from forecast i, $\epsilon_i = y - y_i$, that we can decompose it into predictable term and unpredictable term:

$$
\epsilon_i = b_i + \xi_i. \tag{16}
$$

Then we can write the weight as a function that minimize the

2 Survey of Professional Forecasters (SPF)

To illustrate the empirical results, we use the data from ECB **(footnote link to data)** in this paper. The data, SPF, is a quarterly survey initiated by ECB, with the aim to obtain future estimates on inflation (HICP), RGDP and unemployment rate (UNEM) from the private sector. Every quarter, a gourp of professional forecasters from financial and non-financial instutition, such as economic research institutions, respond to the survey with the idea on the future economic. Starting 1999, SPF is the longest survey of macroeconomic expectation in the euro area. Until the date of this paper, there are 75 quarters of observation available, with 1999 Q4 as the first forecasted value, and 2018 Q2 as the last observed true macroeconomic indice.

The set up of the survey consist of multiple magnitudes of questions, ranging from different horizon to different distribution. The forecaststers are asked to provide their point forecast and the probability of a certain scenario to happen. The survey enables ECB to do quantitative assessment on the consensus of the market, like the distribution statistics and standard deviations. For this paper, we take the 2 most answered time periods, which is 1 year ahead and 2 year ahead as our data set for all HICP, RGDP, and UNEM.

To compare the forecasts with the actual macroeconomics, we obtain the true value from ECB data base **(footnote link to data)**. The data cannot be observed from the economic in 100% accuracy within the first time frame, and exihibits changes to the initial estimates after revision. We use the final estimate of the macroeconomics where possible. The use of final estimate is fine is due to the fact that the original forecast is not the real target to be forecasts.

Within the datasets, not all forecasters did a forecast every time period. To avoid singular outliers, we remove all forecasters with a total forecasted period of less than 24 quarter (6 years). The removal approach is inline with **(ref paper)**.

Following **(ref paper)**, we calculate the covariance by looking at the intersection between each forecasters.

$$\sigma_{i,j} = \frac{1}{|y_i \cap y_j|} \sum_{k \in \{y_i \cap y_j\}} (y_{i,k} - E(y_i)) * (y_{j,k} - E(y_j)) \tag{17}$$

When there are no intersection between 2 forecasters, we set the covariance value to 0. Additionally, we calculate the correlation by using the covariance divided by the standard deviation. Standard deviation is obtained from the square root of the diagonal.

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \tag{18}$$

The cleaned up gives us a preliminary view on the SPF data without the noises.

In figure 1 and table 2 we show the plots of the forecasts along side with the true value in the macroeconomics and the statistics of the covariance of the forecast error. To avoid too many lines on the figure by plotting all forecasts, we plot only the minimum, mean, and maximum from the forecasts. We see that there exist a high consistency across all forecasts, with two years ahead stronger than one year. The consistency in the forecast is lower in UNEM than the other two. Furthermore, many true values lies outside of the forecast range, with RGDP the worse of all three. More values outside of the forecast range suggest that restricting positive weights may be a strong limitation in the forecast combination.

We examine the amount of true value out side of forecast range by looking at the summary statistics. Let the ms be an indicator with 1 when the true value is outside of the forecast range, and 0 when the true value is inside. Table 1 shows the mean of the indicator. From the three macro topic, RGDP with 18% has the least chance within the forecast range, followed by HICP with 52%. UNEM with 62% has the highest chance to be in the forecast range. Changing from 1 year to 2 year generally does not influence the mean of the indicator a lot. From the results from figure 1 and table 1, we expect to have large effect using truncation in the forecast of RGDP, while HICP and UNEM does not have too strong effect. We also expect the two year ahead forecast will be better than the one year ahead.

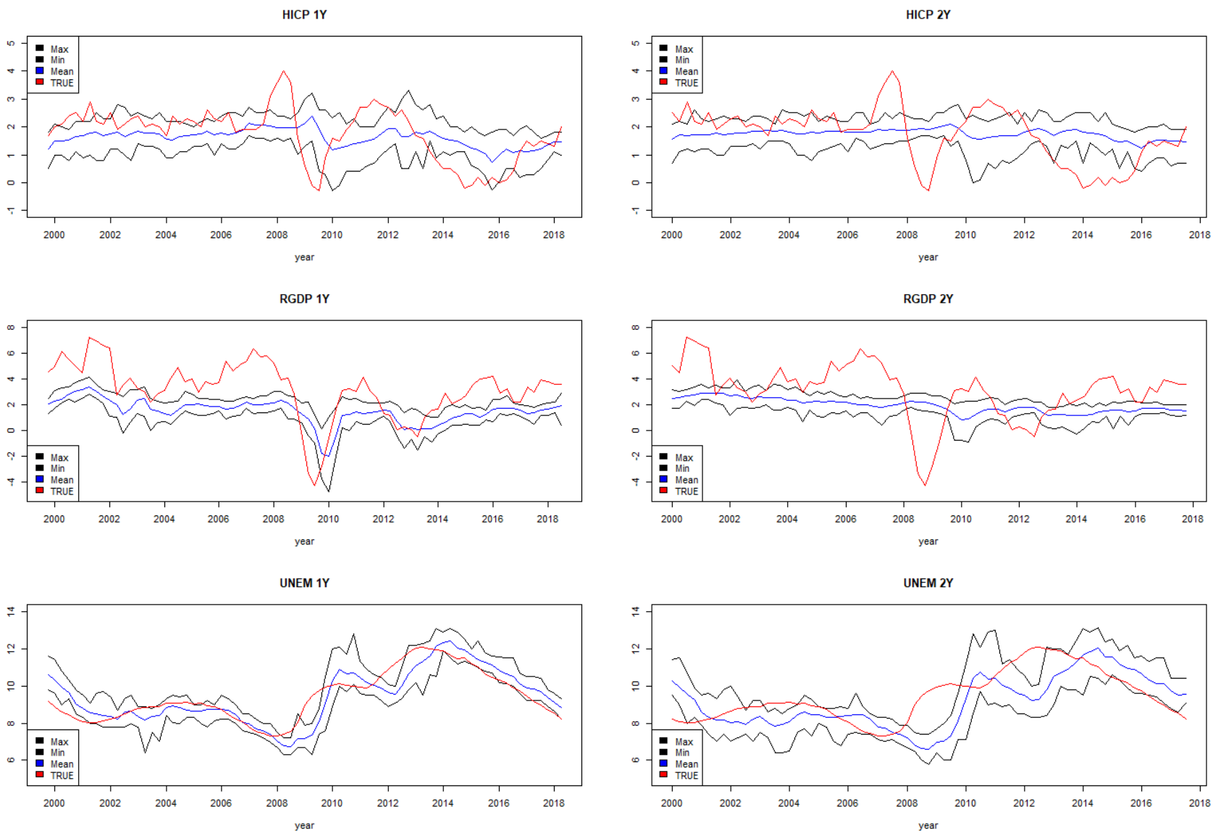

Figure 1: Survey of Professional Forecasters data illustration

Table 1: Mean of the model space indicator of the forecast. The indicators are split up into different forecast topics and different forecast horizons. From the three macro topic, RGDP has the least chance within the forecast range, followed by HICP. UNEM has the highest chance to be in the forecast range.

	Model Space Indicator					
Macro topic	HICP		RGDP		UNEM	
Horizon	1 year	2 year	1 year	2 year	1 year	2 year
Mean	0.45	0.51	0.83	0.82	0.37	0.39

The table 2 tell us on how the forecast error are correlated. The correlation are split up into different forecast topics and different forecast horizons. Diagonal element of the correlation matrix is not within the correlation when generating the summary statistics. For all of the series, the correlations are on average above 60%. In HICP and UNEM the correlation increases across all statistics when the forecast horizon increases, while RGDP remains the same. The lowest correlation to be found is -0.02, but this number is not too different than the minimum correlation in the other two topics.

Analysis on the simple weights given by equation 2 provides us a preliminary understanding on the variability. Table 3 shows the summary statistics of the weight. We see that for all macroeconomic topics, the mean and median variate around 0.02, which is close to the calculation from equal weights. Looking at the first and the third quantile, we see that half of the weights are between -0.6 and 0.6, which gives us an possible range of effectiveness of the truncation. The minimum and the maximum is extreme consider that 1 is 100%. With negativity up to -11, -21, and -14, and first and third quartile only up to -0.6 and 0.6, the weight indicates a strong thick tail behaviour. Looking further to the difference between 1 year and 2 year ahead, we see that in

Table 2: Summary statistics of the correlation of the forecast error. The correlation are split up into different forecast topics and different forecast horizons. For all of the series, the correlation are on average above 60%. In HICP and UNEM the correlation increases across all statistics when the forecast horizon increases, while RGDP remains the same.

	Correlation					
Macro topic	HICP		RGDP		UNEM	
Horizon	1 year	2 year	1 year	2 year	1 year	2 year
Minimum	0.03	0.02	0.11	0.13	-0.02	0.08
First Q	0.53	0.62	0.63	0.65	0.47	0.56
Mean	0.64	0.70	0.75	0.75	0.59	0.67
Median	0.66	0.72	0.81	0.80	0.62	0.70
Third Q	0.77	0.81	0.89	0.87	0.74	0.80
Maximum	0.96	0.97	0.98	0.98	0.94	0.95

two out of three macro topics, we see that the extreme values converge to 0, while RGDP becomes worse. On the other hand, the gap between first and the third quantile increases for all topics. This indicates the increase in the choice of negative weights in the general cases for 2 year horizon. The mean and median does not change much in relation to the forecsat horizon.

Table 3: Summary statistics of the weights from equation 2. The mean and median is close to the value from equal weights. However, extreme values exist in the weights, visible in the mimum and maximum. Based on the first and the third quantile, we see that half of the weights are between -0.6 and 0.6.

	Weights					
Macro topic	HICP		RGDP		UNEM	
Horizon	1 year	2 year	1 year	2 year	1 year	2 year
Minimum	-11.05	-6.21	-9.00	-21.79	-14.41	-5.64
First Q	-0.30	-0.59	-0.20	-0.63	-0.28	-0.40
Mean	0.04	0.04	0.01	-0.01	0.03	0.05
Median	0.05	0.03	0.04	0.01	0.04	0.03
Third Q	0.41	0.60	0.25	0.82	0.39	0.43
Maximum	12.94	7.56	8.74	16.34	11.26	7.90

We conclude in for the analysis of SPF that there exhibits characteristics that are not in the standard cases. By incorporating the possiblity of negative weights, we expect to see some improvement in the forecast errors.

3 Empirical Results

3.1 Proceure

We seek to evaluate the effect of truncating the weights with SPF data. The procedure to get the weights can be different between different researcher. Therefore we provide a through order of the steps we take in the weight estimation. We do the weights estimation for the truncation using the following steps:

Given each time to forecast, we take all the know observation before that forecast time period.

1. Find the nearest positive definite covariance matrix. This step is required for the covariance to be invertible. We employ the nearPD function from r package *Matrix*. The nearPD function first decompose the covariance into univariate variance and the correlation. The function then uses the algorithm by

higham on the correlation matrix to compute the nearest positive definite matrix. THe final results the the covariance matrix that is combined from the univariate variance and the correlation matrix.

2. Subset the covariance. Since there are more forecsaters in the estimation step than the amount of forecasters in the testing period, we take the sub matrix containing only the avaiable forecast on the testing period. That is, if 80 forecasters had made some forecasts before, but out of them, only 60 has a forecast this time, we discard the 20 extra forecasters.

3. Estimate the simple weight w^* from equation 2.

4. Truncate the weights with equation 10. The weight no has less values and does not sum to one. Due to the fact that our selection of the truncation parameter are negative, the summation is higher than one and we scale down the weights accordingly.

5. Combine the forecast using the weights.

3.2 Ratio of Mean Squared Prediction Error

We eavluate the performance of different weights by mean squared prediction error (MSPE). The trunacted MSPE is then compared with the MSPE from equal weight. The results of HICP, RGDP, and UNEM are given in table 4, 5, and 6 respectively. The MSPE of equal weight is not influenced by the changes in the truncation value. We see in table 4 that for truncation value

Table 4: Mean Squared Prediction Error (MSPE) of inflation with different truncated value. The MSPE of the truncated value is given in the ratio of tuncation to euqal weights. Larger than 1 means truncated is worse, where as smaller than 1 means truncated weight helps in reducing MSPE.

	HICP			
	1 Year Horizon		2 Year Horizon	
Threshold	Truncated Ratio	Equal MSPE	Truncated Ratio	Equal MSPE
$-\infty$	3.29	0.74	2.21	1.47
-10	2.82	0.74	2.21	1.47
-9	2.82	0.74	2.21	1.47
-8	2.82	0.74	2.21	1.47
-7	2.82	0.74	2.21	1.47
-6	2.82	0.74	2.08	1.47
-5	2.82	0.74	2.11	1.47
-4	2.82	0.74	2.15	1.47
-3	2.82	0.74	1.55	1.47
-2	0.81	0.74	0.99	1.47
-1	0.88	0.74	0.98	1.47
0	0.98	0.74	0.98	1.47

3.3 Out-of-sample truncation selection

In this section we show the forecast ability in weight truncation when the truncation parameter is selected out-of-sample (OOS). To perform the OOS selection of the truncation parameter, we follow the following step:

1. Estimate the normal weight in equation 2.

2. Calculate the mean squared error (MSE) with truncated weight. The truncation parameter ranges from 0 to -10 with a step of -0.1.

3. Select the truncation parameter with the lowest MSE. If there are multiple truncation points with the same MSE value, we take the largest truncation value within the same MSE.

Table 5: Mean Squared Prediction Error (MSPE) of economic growth with different truncated value. The MSPE of the truncated value is given in the ratio of tuncation to euqal weights. Larger than 1 means truncated is worse, where as smaller than 1 means truncated weight helps in reducing MSPE.

	RGDP			
	1 Year Horizon		2 Year Horizon	
Threshold	Truncated Ratio	Equal MSPE	Truncated Ratio	Equal MSPE
$-\infty$	1.44	3.54	6.07	3.36
-10	1.44	3.54	2.64	3.36
-9	1.44	3.54	1.03	3.36
-8	1.09	3.54	1.03	3.36
-7	1.1	3.54	1.02	3.36
-6	1.1	3.54	1.06	3.36
-5	1.1	3.54	1.07	3.36
-4	1.15	3.54	1.09	3.36
-3	0.88	3.54	1.08	3.36
-2	0.84	3.54	1.08	3.36
-1	0.86	3.54	1.14	3.36
0	0.97	3.54	1.01	3.36

Table 6: Mean Squared Prediction Error (MSPE) of nemployment with different truncated value. The MSPE of the truncated value is given in the ratio of tuncation to euqal weights. Larger than 1 means truncated is worse, where as smaller than 1 means truncated weight helps in reducing MSPE.

	UNEM			
	1 Year Horizon		2 Year Horizon	
Threshold	Truncated Ratio	Equal MSPE	Truncated Ratio	Equal MSPE
$-\infty$	1.16	0.33	4.35	0.81
-10	1.02	0.33	4.35	0.81
-9	1.02	0.33	4.35	0.81
-8	1.02	0.33	4.35	0.81
-7	1.02	0.33	4.35	0.81
-6	1.02	0.33	4.35	0.81
-5	1.02	0.33	2.32	0.81
-4	0.89	0.33	2.25	0.81
-3	0.93	0.33	1.95	0.81
-2	0.77	0.33	1.87	0.81
-1	0.73	0.33	1.42	0.81
0	0.93	0.33	0.88	0.81

4. Use the selected truncation parameter to forecast the combined prediction.

A remark on the selection of the largest truncation value. The reason to choose largest value is that the threshold is closer to the next changing point. Assume for exmaple, a weight with minimum of -1 is optimal, and any higher weight results in higher MSE. Then all truncation values between -10 and -1 gives the same minimal MSE. We therefore choose -1 to record. The choice does not influence the forecast, and is pure for the analysis later.

3.4 Out-of-sample truncation result

We evaluate the OOS forecast ability using the same way as the MSPE with in-sample truncation parameter selection. The MSPE ratio and the MSPE of the equal weight is given in table 7.

We see in table 7 that the truncation improves in all cases from the equal weights. The truncation works better in RGDP 1 year ahead forecast and UNEM 1 and 2 year ahead forecasts. If we compare the value to non truncated MSPE, we see that the truncation improves the forecasts of RGDP 2 year ahead for a large amount. On the other hand, there is relatively smaller improvement to UNEM 1 year and RGDP 1 year ahead. This results verify the idea that the truncation in general improves the upon equal weights even if the truncation uncertainty can increase the MSPE.

Table 7: Mean squared prediction error ratio when the selection of the truncation is also out-of-sample. On average the truncation improves the forecast in all cases.

	MSPE with OOS Truncation					
Macro topic	HICP		RGDP		UNEM	
Horizon	1 year	2 year	1 year	2 year	1 year	2 year
MSPE-ratio	0.97	0.96	0.93	0.96	0.92	0.88

To understand the oos truncation more, table 8 looks at the selection of the truncation. The results shows that on average, a truncation around -0.5 is selected, indicating a positive effect on the negative weights. HICP 2 year and RGDP 2 years select on average lower truncation, -0.8 and -1.0 respectively. UNEM selects -0.2 on average, highest among all. The minimum across all macro topics are at most -1, with RGDP 1 year, RGDP 2 year, and UNEM 1 year the lowest selected minimum. The first and the third quantile show that for the most cases, the truncation are not extreme, ranging between -0.5 to -0.2 in 1 year and -1 to 0 in 2 year horizon. We notice an increase in the variation when the horizon increases. This can be attibute to the uncertainty in the horizon adds uncertainty in the truncation parameter selection.

Table 8: Summary statistics of the truncation selected.

	Truncation					
Macro topic	HICP		RGDP		UNEM	
Horizon	1 year	2 year	1 year	2 year	1 year	2 year
Minimum	-1.0	-2.1	-3.3	-4.6	-3.0	-1.0
First Q	-0.6	-1.0	-0.5	-1.1	-0.4	-0.3
Mean	-0.5	-0.8	-0.5	-1.0	-0.4	-0.2
s Median	-0.4	-0.6	-0.2	-0.4	-0.2	-0.2
Third Q	-0.3	-0.3	-0.1	-0.2	-0.2	0.0
Maximum	0.0	0.0	0.0	0.0	0.0	0.0

3.5 Bias weighting

The bias weighting is an interesting case here. If the model in the bias estimation successfully estimates the predictable part and the unpredictable error, this can attirbute to a better weight selection with additional information gained. We estimate the bias by the following equation

$$\epsilon_i = b_i + \xi_i. \tag{19}$$