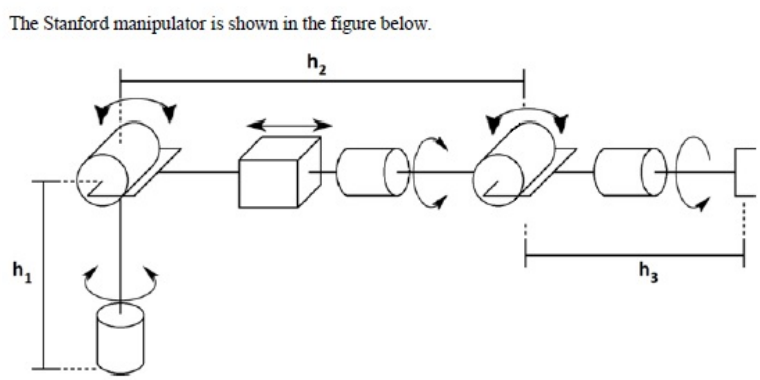
DoNRS Assignment 1

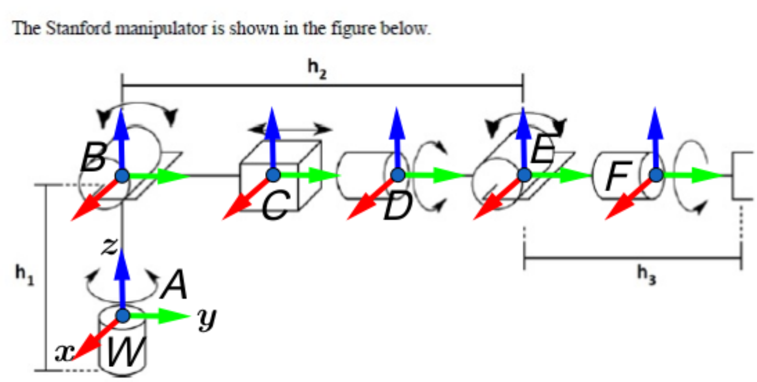
Milioshin Ilia

September 2024

1 Manipulator



2 Putting frames



World frame is corresponding with A-frame initially

3 Direct kinematics

All frames positions and orientations:

- $T_A = R_z(q_1)$
- $T_B = T_A T_z(h_1) R_x(q_2)$
- $T_C = T_B T_y(q_3 + h_2)$
- $T_D = T_C R_y(q_4)$
- $T_E = T_D R_x(q_5)$
- $T_F = T_E R_y(q_6) T_y(h_3)$

Joints D, E, and F form a spherical joint.

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_x(d) = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_y(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_z(d) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 Inverse kinematics

In this section we consider R_X as rotation part of T_X and r_X as translation part of T_X

From the structure of the Stanford manipulator we know $r_B = \begin{bmatrix} 0 & 0 & h_1 & 1 \end{bmatrix}^T$ and $r_E = T_F \begin{bmatrix} 0 & -h_3 & 0 & 1 \end{bmatrix}^T$. T_F is given because we solve the inverse kinematics problem.

Thus, $q_1 = \text{atan2}(y_E, x_E) - \pi/2$, $q_2 = \text{atan2}(z_E - h_1, \sqrt{x_E^2 + y_E^2})$, and $q_3 = \|r_E - r_B\| - h_2$

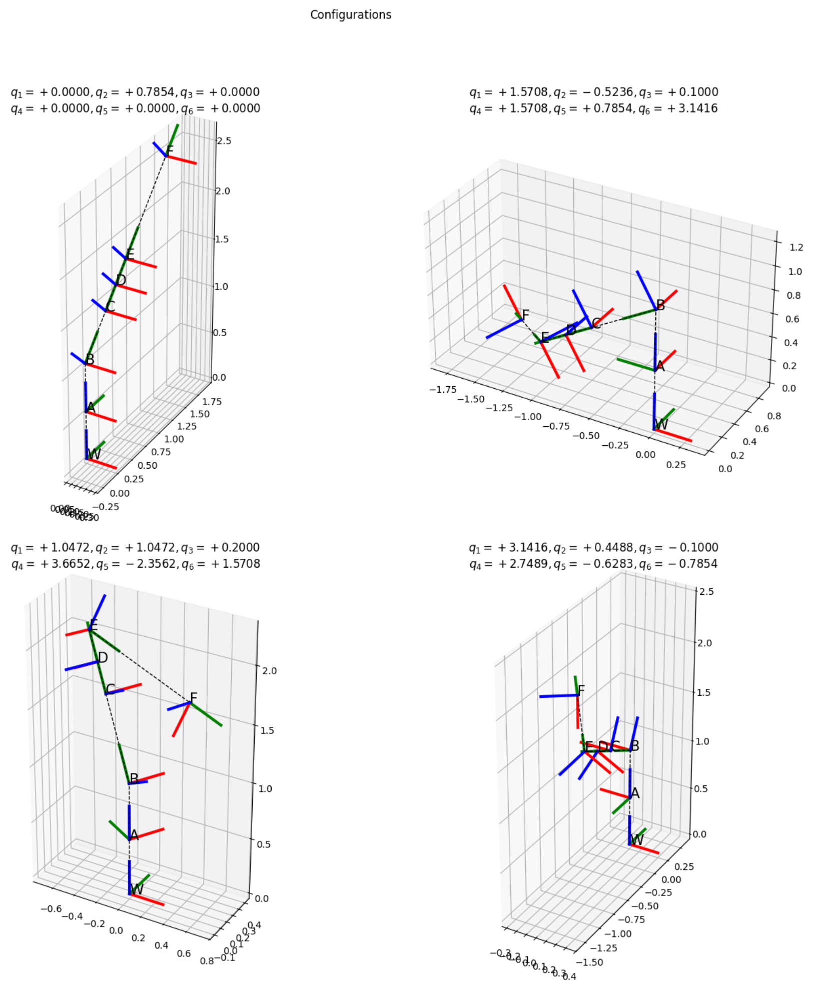
We can see that $\cos q_5 = \vec{j}_B \cdot \vec{j}_E$, therefore $q_5 = \pm \arccos \vec{j}_B \cdot \vec{j}_E$. In this system B, E, and F form a plane. Hence, $\vec{i}_E$ can take two values, that lead to two possible values of q_4, q_5 q_6. Let's find q_4, and q_6.

The right-hand rule defines the sign of the angle. The first $q_4 = \text{sign}[(\vec{i}_B \times \vec{i}_E) \cdot \vec{j}_B] \arccos(\vec{i}_B \cdot \vec{i}_E)$, the second option is $q_4 = \text{sign}[(-\vec{i}_B \times \vec{i}_E) \cdot \vec{j}_B] \arccos(-\vec{i}_B \cdot \vec{i}_E)$. Values of q_6 are $\text{sign}[(\vec{i}_E \times \vec{i}_F) \cdot \vec{j}_E] \arccos(\vec{i}_E \cdot \vec{i}_F)$, and $\text{sign}[(-\vec{i}_E \times \vec{i}_F) \cdot \vec{j}_E] \arccos(-\vec{i}_E \cdot \vec{i}_F)$ respectively. Here, $\vec{i}_B = (\vec{j}_B \times \vec{j}_E)/\|\vec{j}_B \times \vec{j}_E\|$, when $\vec{j}_B$, and $\vec{j}_E$ are co-linear we are free to choose any $\vec{i}_B$. For reference, see Fig. 1.

Now, we conduct an algorithm, on how the given T_F can find the two sets of generalized coordinates.

5 Some configurations

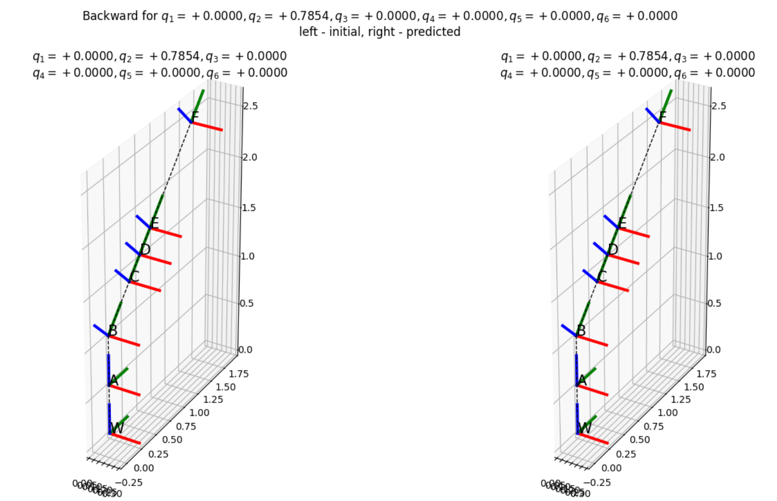
In this section the placements of frames a little bit changed in such a way that the origins of the frames did not correspond. In samples below, red stands for $\vec{i}$, green for $\vec{j}$, and blues for $\vec{k}$

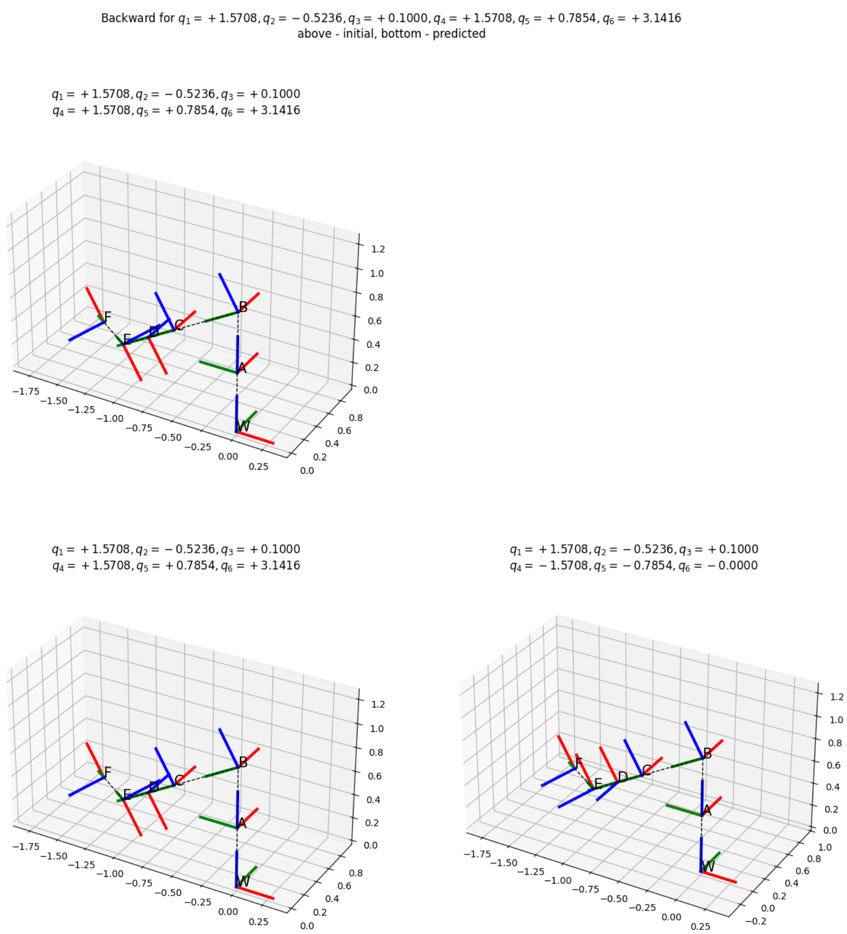


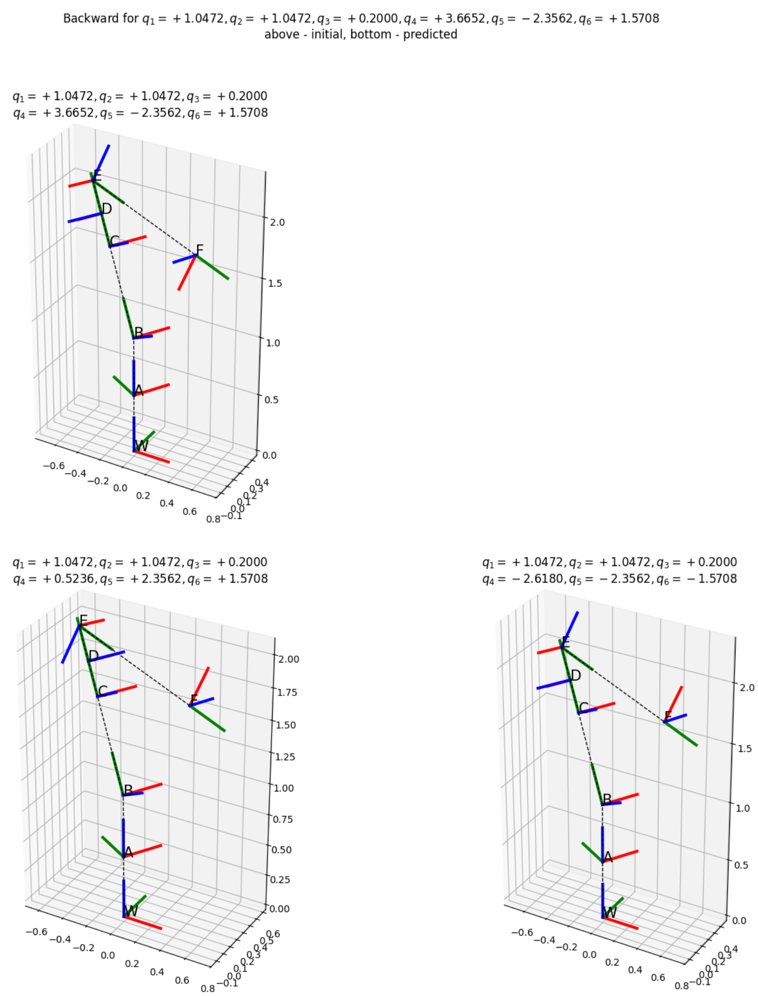
6 Inverse kinematics examples

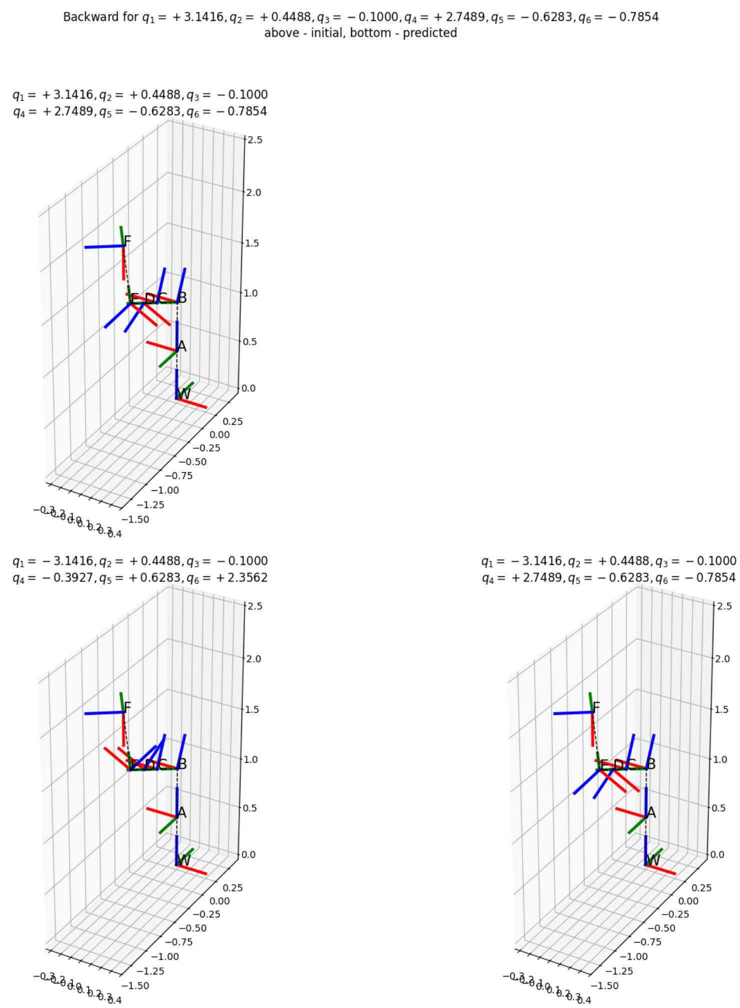
Now let's consider from the inverse kinematics solutions for the above configurations

7 Appendix


Backward for $q_1 = +0.0000, q_2 = +0.7854, q_3 = +0.0000, q_4 = +0.0000, q_5 = +0.0000, q_6 = +0.0000$
left - initial, right - predicted
$q_1 = +0.0000, q_2 = +0.7854, q_3 = +0.0000$
$q_4 = +0.0000, q_5 = +0.0000, q_6 = +0.0000$
$q_1 = +0.0000, q_2 = +0.7854, q_3 = +0.0000$
$q_4 = +0.0000, q_5 = +0.0000, q_6 = +0.0000$


Backward for $q_1 = +1.5708, q_2 = -0.5236, q_3 = +0.1000, q_4 = +1.5708, q_5 = +0.7854, q_6 = +3.1416$
above - initial, bottom - predicted
$q_1 = +1.5708, q_2 = -0.5236, q_3 = +0.1000$
$q_4 = +1.5708, q_5 = +0.7854, q_6 = +3.1416$
$q_1 = +1.5708, q_2 = -0.5236, q_3 = +0.1000$
$q_4 = +1.5708, q_5 = +0.7854, q_6 = +3.1416$
$q_1 = +1.5708, q_2 = -0.5236, q_3 = +0.1000$
$q_4 = -1.5708, q_5 = -0.7854, q_6 = -0.0000$


Backward for $q_1 = +1.0472, q_2 = +1.0472, q_3 = +0.2000, q_4 = +3.6652, q_5 = -2.3562, q_6 = +1.5708$
above - initial, bottom - predicted
$q_1 = +1.0472, q_2 = +1.0472, q_3 = +0.2000$
$q_4 = +3.6652, q_5 = -2.3562, q_6 = +1.5708$
$q_1 = +1.0472, q_2 = +1.0472, q_3 = +0.2000$
$q_4 = +0.5236, q_5 = +2.3562, q_6 = +1.5708$
$q_1 = +1.0472, q_2 = +1.0472, q_3 = +0.2000$
$q_4 = -2.6180, q_5 = -2.3562, q_6 = -1.5708$
E
D
C
F
B
A
W


Backward for $q_1 = +3.1416, q_2 = +0.4488, q_3 = -0.1000, q_4 = +2.7489, q_5 = -0.6283, q_6 = -0.7854$
above - initial, bottom - predicted
$q_1 = +3.1416, q_2 = +0.4488, q_3 = -0.1000$
$q_4 = +2.7489, q_5 = -0.6283, q_6 = -0.7854$
$q_1 = -3.1416, q_2 = +0.4488, q_3 = -0.1000$
$q_4 = -0.3927, q_5 = +0.6283, q_6 = +2.3562$
$q_1 = -3.1416, q_2 = +0.4488, q_3 = -0.1000$
$q_4 = +2.7489, q_5 = -0.6283, q_6 = -0.7854$
F
E
D
C
B
A
W

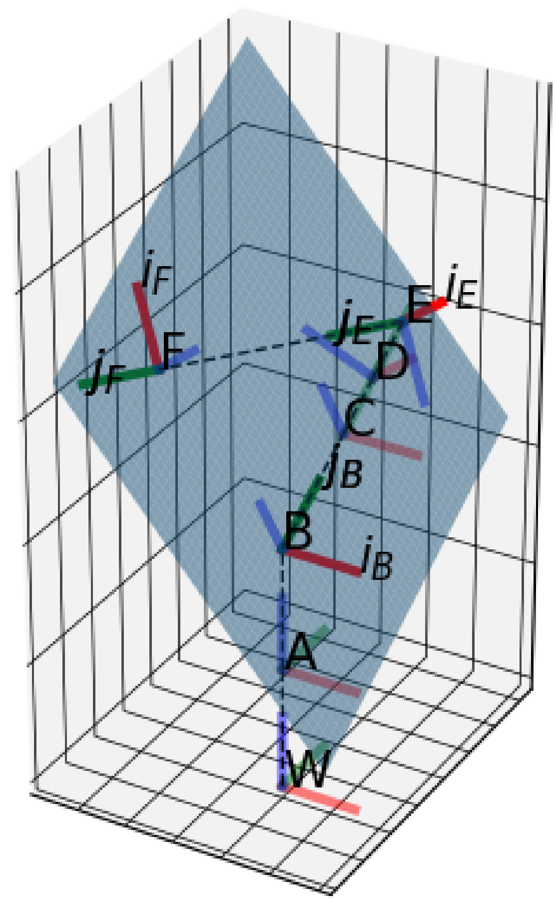


Figure 1: Explanation draft