FoRC Final Report

Milioshin Ilia

December 2024

1 Dynamics

Let the dynamics of the system be:

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) + D\dot{\mathbf{q}} + F_c(\dot{\mathbf{q}}) = \boldsymbol{\tau}$$

where $M(\mathbf{q})$, $C(\mathbf{q}, \dot{\mathbf{q}})$, and $g(\mathbf{q})$ is an uncertain mass matrix, Coriolis, and gravity, respectively. $D\dot{\mathbf{q}}$, and $F_c(\dot{\mathbf{q}})$ are unknown viscous and Coulomb friction.

Below we will omit the parameters of those quantities, and use $\delta(\mathbf{q}, \dot{\mathbf{q}})$ instead of $D\dot{\mathbf{q}} + F_c(\dot{\mathbf{q}})$

2 Control signal

$$\boldsymbol{\tau} = \hat{M}\mathbf{v} + \hat{C}\dot{\mathbf{q}} + \hat{g}$$

where v is control input, and $\hat{[.]}$ denotes best estimates of uncertain quantities.

3 Sliding surface

$$\begin{aligned}
& M\ddot{\mathbf{q}} + C\dot{\mathbf{q}} + g + \delta = \hat{M}\mathbf{v} + \hat{C}\dot{\mathbf{q}} + \hat{g} && \Rightarrow \\
\Rightarrow\quad & M\ddot{\mathbf{q}} = \tilde{C}\dot{\mathbf{q}} + \tilde{g} - \delta + \hat{M}\mathbf{v} && \Rightarrow \\
\Rightarrow\quad & \ddot{\mathbf{q}} = M^{-1}(\tilde{C}\dot{\mathbf{q}} + \tilde{g} - \delta) + M^{-1}\hat{M}\mathbf{v} && \Rightarrow \\
\Rightarrow\quad & \ddot{\mathbf{q}} = f + B\mathbf{v}
\end{aligned}$$

where $f = M^{-1}(\tilde{C}\dot{\mathbf{q}} + \tilde{g} - \delta)$, $B = M^{-1}\hat{M}$, and $\tilde{[.]}$ is difference between actual and calculated quantities

Let the sliding variable be $\mathbf{s} = \dot{\tilde{\mathbf{q}}} + \Lambda\tilde{\mathbf{q}}$, here $\tilde{\mathbf{q}} = \mathbf{q}_d - \mathbf{q}$. Using $\mathbf{v}_n = \ddot{\mathbf{q}}_d + \Lambda\tilde{\mathbf{q}}$, we obtain

$$\frac{1}{2}\frac{d}{dt}\|\mathbf{s}\|^2 = \mathbf{s}^T\dot{\mathbf{s}} = \mathbf{s}^T[\ddot{\tilde{\mathbf{q}}} + \Lambda\dot{\tilde{\mathbf{q}}}] = \mathbf{s}^T[\mathbf{v}_n - \ddot{\mathbf{q}}] = \mathbf{s}^T[\mathbf{v}_n - f - B(\mathbf{v}_n + \mathbf{v}_s)] =$$
$$= \mathbf{s}^T[((I - B)\mathbf{v}_n - f) - B\mathbf{v}_s] = \mathbf{s}^T[\mathbf{w} - B\mathbf{v}_s] \leq \|\mathbf{s}\|\|\mathbf{w}\| - \mathbf{s}^T B\mathbf{v}_s$$

If we try $\mathbf{v}_s = k\frac{\mathbf{s}}{\|\mathbf{s}\|}$, we get $\frac{1}{2}\frac{d}{dt}\|\mathbf{s}\|^2 \leq \|\mathbf{s}\|\|\mathbf{w}\| - \frac{k}{\|\mathbf{s}\|}\mathbf{s}^T B\mathbf{s}$. Due to M and $\hat{M}$ are bound, they have bounded singular values. Therefore, we can find some σ greater than all possible singular values of B. Hence,

$$\frac{1}{2}\frac{d}{dt}\|\mathbf{s}\|^2 \leq \|\mathbf{s}\|\|\mathbf{w}\| - \frac{k}{\|\mathbf{s}\|}\mathbf{s}^T B\mathbf{s} \leq \|\mathbf{s}\|\|\mathbf{w}\| - \frac{k\sigma}{\|\mathbf{s}\|}\|\mathbf{s}\|^2 =\leq \|\mathbf{s}\|(\|\mathbf{w}\| - k\sigma) < -\eta\|\mathbf{s}\|$$

Therefore, $k\sigma > \eta + \|\mathbf{w}\|$ is condition of sliding mode.

Finally, we formulate a control loop in the following way:

$$\begin{cases} \boldsymbol{\tau} = \hat{M}\mathbf{v} + \hat{C}\dot{\mathbf{q}} + \hat{g} \\ \mathbf{v} = \mathbf{v}_n + \mathbf{v}_s \\ \mathbf{v}_n = \ddot{\mathbf{q}}_d + \Lambda\tilde{\mathbf{q}} \\ \mathbf{v}_s = k\frac{\mathbf{s}}{\|\mathbf{s}\|} \end{cases}$$

For better performance and to reduce chattering, it is preferable to use

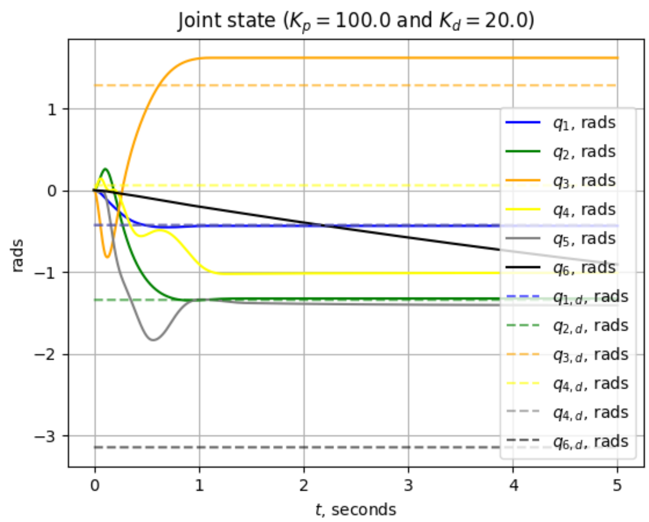
$$\mathbf{v}_s = \begin{cases} k\frac{\mathbf{s}}{\|\mathbf{s}\|}, \ \|\mathbf{s}\| \geq \epsilon \\ k\frac{\mathbf{s}}{\epsilon}, \ \|\mathbf{s}\| < \epsilon \end{cases}$$

4 Simulation results

In the simulation, the manipulator has an additional 4 kg attached to the end effector, and the viscous and Coulomb friction is turned on.

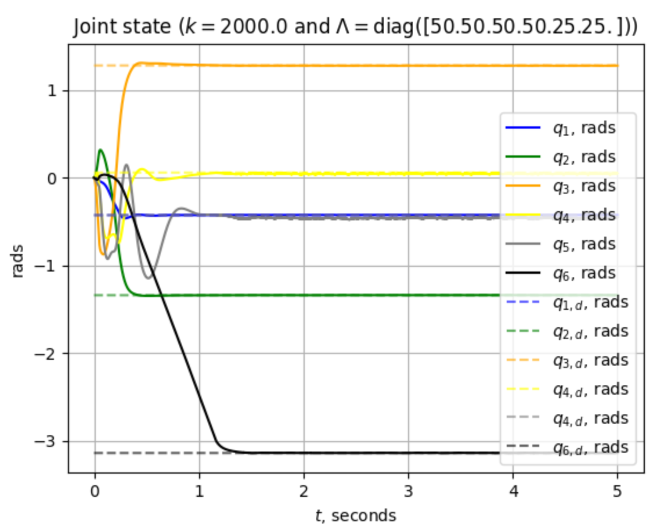
The desired joint position is $\begin{bmatrix} -0.4264 & -1.3405 & 1.2812 & 0.0592 & -0.4264 & -3.1415 \end{bmatrix}$

4.1 Usual PD controller

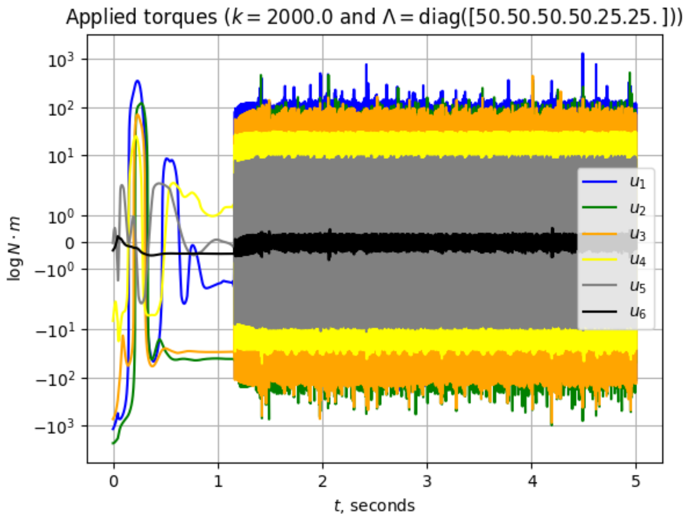


As you see the desired joint position cannot be reached by the PD controller within the given circumstances

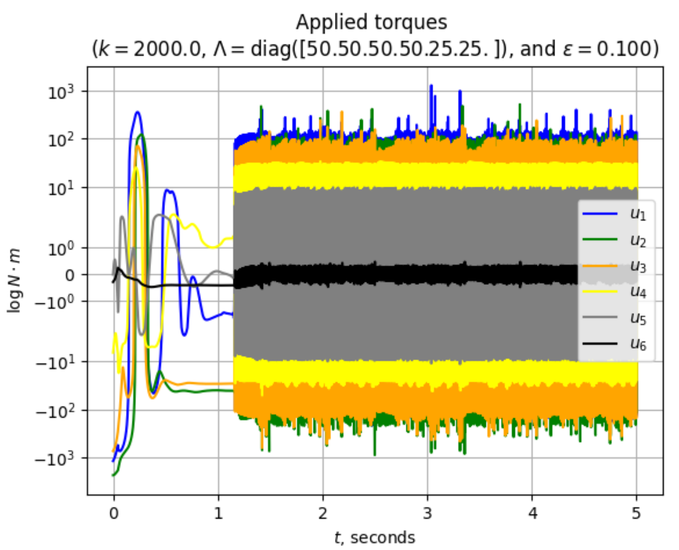
4.2 Robust controller, no saturation

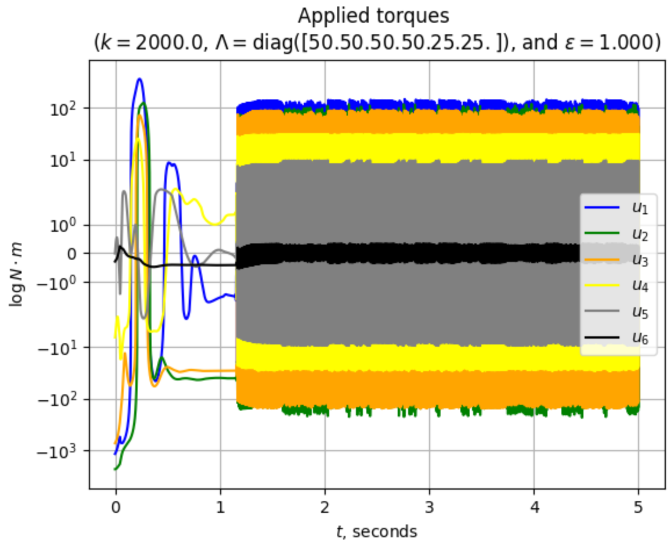


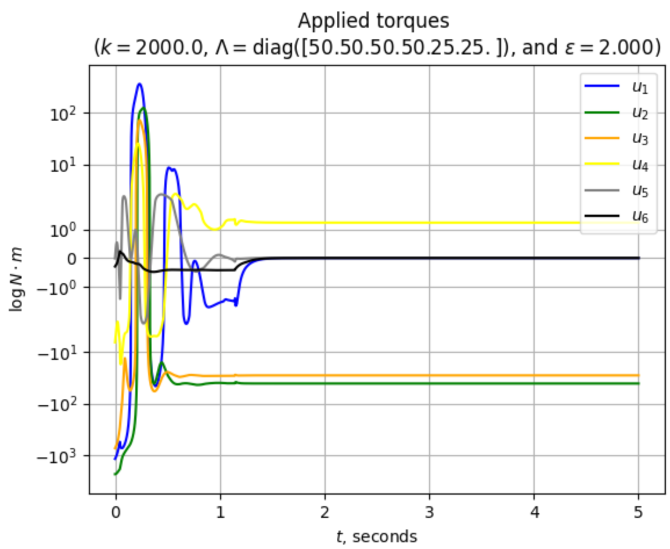
However, the chattering effect appears.



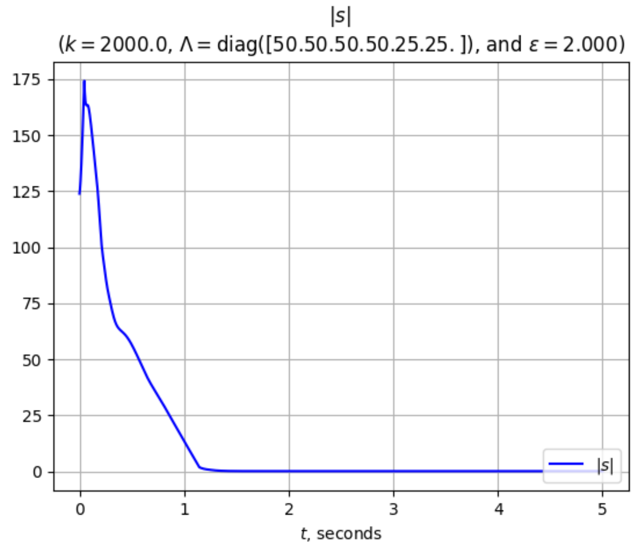
4.3 Robust controller, with different saturation







The best performance is achieved with $\epsilon = 2$, with this value we have nice (no chattering) convergence.



$|s|$
($k = 2000.0$, $\Lambda = \mathrm{diag}([50.50.50.50.25.25.\ ])$, and $\varepsilon = 2.000$)
175
150
125
100
75
50
25
0
0
1
2
3
4
5
t, seconds
$|s|$