Report

Ilia Milioshin, RO-01

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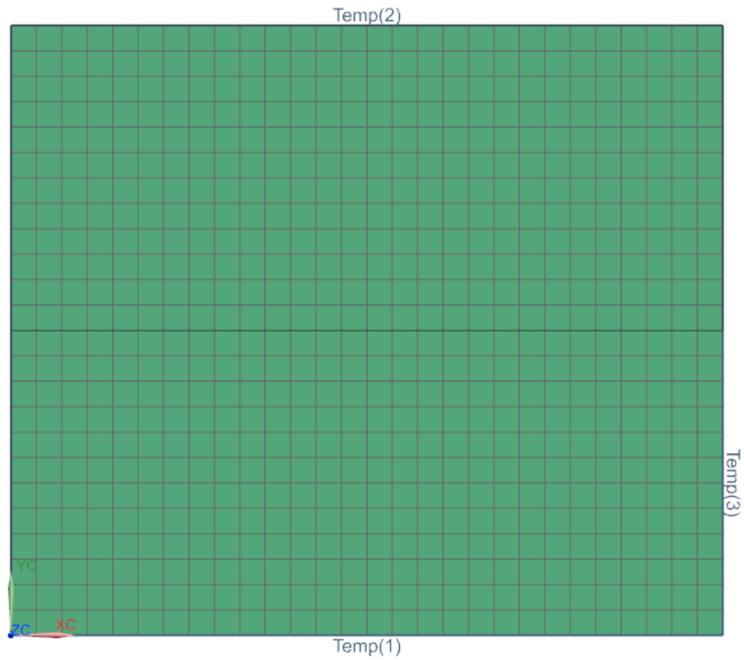
1 Intro



A plate, width - 7 cm, height - 6 cm

2 Approach

The heat equation:

$$\frac{\delta T}{\delta t} = a_x^2 \frac{\delta^2 T}{\delta x^2} + a_y^2 \frac{\delta^2 T}{\delta y^2} + a_z^2 \frac{\delta^2 T}{\delta z^2}$$

We consider only 2-d case, where $a_x = a_y = 1$:

$$\frac{\delta T}{\delta t} = \frac{\delta^2 T}{\delta x^2} + \frac{\delta^2 T}{\delta y^2}$$

Now let's take a look at approximation (implicit):

$$\frac{T_{i,j}^k - T_{i,j}^{k-1}}{\Delta t} = \frac{T_{i+1,j}^k - 2T_{i,j}^k + T_{i-1,j}^k}{\Delta x^2} + \frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{\Delta y^2}$$

Multiply both sides by $\Delta t \Delta x^2 \Delta y^2$:

$$AT_{i,j}^k + B(T_{i+1,j}^k + T_{i-1,j}^k) + C(T_{i,j+1}^k + T_{i,j-1}^k) = T_{i,j}^{k-1}\Delta x^2 \Delta y^2$$

Where, $A = \Delta x^2 \Delta y^2 + 2\Delta t(\Delta x^2 + \Delta y^2)$, $B = -\Delta t \Delta x^2$ and $C = -\Delta t \Delta y^2$

Let's define $T_{i,j}^k \equiv T_p^k$, where $p = i\hat{n} + j$, $\hat{n} = n - 2$ and $\hat{m} = m - 2$. Thus,

$$\begin{bmatrix} & & & & \cdots & & & & \\ \cdots & a_{p,p-n} & \cdots & a_{p,p-1} & a_{p,p} & a_{p,p+1} & \cdots & a_{p,p+n} & \cdots \\ & & & & \cdots & & & & \end{bmatrix} \begin{bmatrix} \vdots \\ T_{p-n}^k \\ \vdots \\ T_{p-1}^k \\ T_p^k \\ T_{p+1}^k \\ \vdots \\ T_{p+n}^k \\ \vdots \end{bmatrix} = \Delta x^2 \Delta y^2 \begin{bmatrix} \vdots \\ T_{p-n}^{k-1} \\ \vdots \\ T_{p-1}^{k-1} \\ T_p^{k-1} \\ T_{p+1}^{k-1} \\ \vdots \\ T_{p+n}^{k-1} \\ \vdots \end{bmatrix} + \mathbf{b}$$

Where:
$a_{p,p-n} = C$ if $i > 0$ else 0 and $b_p = -T_{-1,j}C$ if $i = 0$ else 0,
$a_{p,p+n} = C$ if $i < \hat{m} - 1$ else 0 and $b_p = -T_{\hat{m},j}C$ if $i = \hat{m} - 1$ else 0,
$a_{p,p-1} = B$ if $j > 0$ else 0 and $b_p = -T_{i,-1}B$ if $j = 0$ else 0,
$a_{p,p+1} = B$ if $j < \hat{n} - 1$ else 0 and $b_p = -T_{i,\hat{n}}B$ if $j = \hat{n} - 1$ else 0,
$a_{p,p} = A$

Therefore, we can rewrite the above equation in the following manner:

$$\mathbb{T}\mathbf{T}^k = \Delta x^2 \Delta y^2 \mathbf{T}^{k-1} + \mathbf{b} \Rightarrow$$

$$\mathbf{T}^k = \mathbb{T}^{-1}(\Delta x^2 \Delta y^2 \mathbf{T}^{k-1} + \mathbf{b})$$

Now we can simulate it.

3 Results

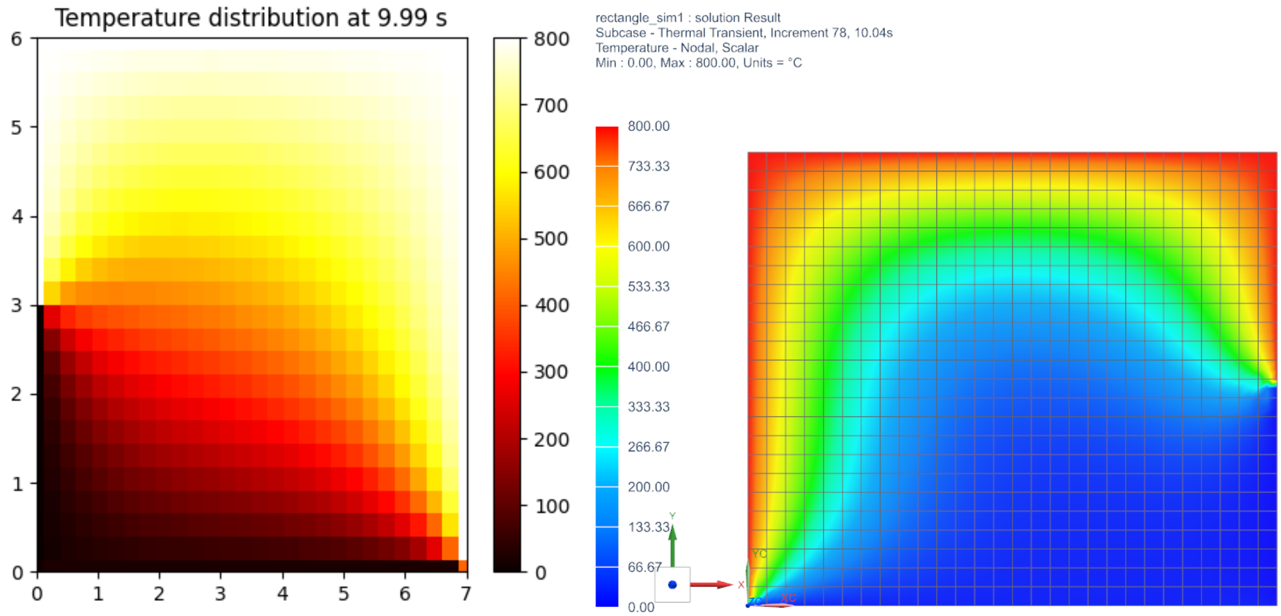


Figure 1: Python simulation (left) VS Nx simulation (right)

Note: All evaluations can be found in the following GitHub repository.