

## Intro

Suppose  $p, q$  are nonnegative integers, and suppose  $A, B, C, D$  are respectively  $p \times p$ ,  $p \times q$ ,  $q \times p$ , and  $q \times q$  matrices of complex numbers. Let

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

If  $D$  is invertible, then the Schur complement of the block  $D$  of the matrix  $M$  is the  $p \times p$  matrix defined by

$$M/D := A - BD^{-1}C$$

And for  $A$ , respectively

$$M/A := D - CA^{-1}B$$

## Properties

- In general, if  $A$  is invertible, then  $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ CA^{-1} & I_q \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & M/A \end{bmatrix} \begin{bmatrix} I_p & 0 \\ A^{-1}B & I_q \end{bmatrix}$
- (Schur's formula) When  $A$ , respectively  $D$ , is invertible, the determinant of  $M$  is also clearly seen to be given by
$$\det(M) = \det(A)\det(M/A), \text{ respectively}$$
$$\det(M) = \det(D)\det(M/D)$$
- (Guttman rank additivity formula) If  $D$  is invertible, then the rank of  $M$  is given by
$$\text{rank}(M) = \text{rank}(D) + \text{rank}(M/D), \text{ respectively}$$
$$\text{rank}(M) = \text{rank}(A) + \text{rank}(M/A)$$