Desired end effector motion law:

$$\mathbf{r}(t),\ R(t) \tag{1}$$

Therefore, the desired velocity and angular velocity

$$\boldsymbol{v}(t),\ \boldsymbol{\omega}(t) \tag{2}$$

Let $J_p(\mathbf{q})$ and $J_r(\mathbf{q})$ describes relation between $\dot{\mathbf{q}}$ and end-effector velocity and angular velocity. Thus,

$$J(\mathbf{q})\dot{\mathbf{q}} = \begin{bmatrix} \boldsymbol{v}_e \\ \boldsymbol{\omega}_e \end{bmatrix} \tag{3}$$

where $J(\mathbf{q}) = \begin{bmatrix} J_p(\mathbf{q}) \\ J_r(\mathbf{q}) \end{bmatrix}$

Differentiate (2) and substituting desired motion law instead $\boldsymbol{v}_e$, $\boldsymbol{\omega}_e$ we get

$$J(\mathbf{q})\ddot{\mathbf{q}} = \begin{bmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} - \dot{J}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \tag{4}$$

For stability, we should take into account the displacement and velocity error.

$$\boldsymbol{e} = \begin{bmatrix} \boldsymbol{r}(t) - \boldsymbol{r}_e(t) \\ \text{skew to vector}(\log R_e^T(t)R(t)) \end{bmatrix} \tag{5}$$

where $\log(X)$ is matrix Y, such $e^Y = X$, X should be invertable; skew to vector of $\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$ is $\begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}$

$$\dot{\boldsymbol{e}} = \begin{bmatrix} \boldsymbol{v}(t) \\ \boldsymbol{\omega}(t) \end{bmatrix} - J(\mathbf{q})\dot{\mathbf{q}} \tag{6}$$

Combining (4), (5) and 6, we get $A = J(\mathbf{q})$ and $b = \begin{bmatrix} \dot{\boldsymbol{v}} & \dot{\boldsymbol{\omega}} \end{bmatrix}^T + K_p\boldsymbol{e} + K_d\dot{\boldsymbol{e}} - \dot{J}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}$. Now we can use the Udwadia-Kalaba approach:

$$M\ddot{\boldsymbol{q}} = Q + Q_c \tag{7}$$

$$Q_c = M^{1/2}(AM^{-1/2})^{+}(b - AM^{-1}Q) \tag{8}$$