Intro

Suppose p, q are nonnegative integers, and suppose A, B, C, D are respectively $p \times p$, $p \times q$, $q \times p$, and $q \times q$ matrices of complex numbers. Let

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

If D is invertible, then the Schur complement of the block D of the matrix M is the $p \times p$ matrix defined by

$$M/D := A - BD^{-1}C$$

And for A, respectively

$$M/A := D - CA^{-1}B$$

Properties

- In general, if A is invertible, then $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ CA^{-1} & I_q \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & M/A \end{bmatrix} \begin{bmatrix} I_p & 0 \\ A^{-1}B & I_q \end{bmatrix}$
- (Schur's formula) When A, respectively D, is invertible, the determinant of M is also clearly seen to be given by

  $\det(M) = \det(A)\det(M/A)$, respectively
  $\det(M) = \det(D)\det(M/D)$
- (Guttman rank additivity formula) If D is invertible, then the rank of M is given by

  $\mathrm{rank}(M) = \mathrm{rank}(D) + \mathrm{rank}(M/D)$, respectively
  $\mathrm{rank}(M) = \mathrm{rank}(A) + \mathrm{rank}(M/A)$