Feedback control over constrained robotic systems through the Udvadia-Kalaba approach

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Contents

Chapter 1

Introduction

The advancement of microelectronics and sensor technologies has catalyzed a widespread integration of robotic systems into various domains. Manipulators, delivery robots, and flying drones have become ubiquitous, presenting developers and researchers with novel challenges in terms of robustness, adaptiveness, and synchronization. Among these challenges, synchronization stands out as a critical issue, especially given the proliferation and increasing complexity of such systems. In real-world applications, synchronization of robotic systems is crucial across various industries. For instance, in manufacturing assembly lines, synchronized robots ensure smooth production flow and maximize throughput. Similarly, in warehouse logistics, coordinated robotic systems optimize order fulfillment times and enhance overall productivity. Furthermore, collaborative robotics scenarios in construction projects benefit from synchronized robotic systems for tasks like concrete pouring and steel beam placement, improving efficiency and minimizing delays. A common challenge in this realm involves effectively controlling robots constrained by a shared object. This paper proposes a straightforward methodology to address such problems through the Udvadia-Kalaba[1] approach.

This proposed methodology gains particular significance within the context of modern physics simulation, derivation, and optimization libraries. These tools offer substantial computational speed, enabling efficient problem-solving. In this article, we leverage MuJoCo[2], Pinocchio[3], and ProxSuite[4] for simulation, robotic dynamics computation, and optimization, respectively. Pinocchio, in particular, emerges as a key tool for computing the dynamics of robotic systems. However, its limitation to open-loop physics models poses challenges for controlling and synchronizing multiple robots.

Previous solutions have often involved constructing models with pre-existing constraints or employing the KKT (Karush-Kuhn-Tucker) approach. However, these methods suffer from computational complexity and issues with constraint prioritization. The proposed methodology combines the strengths of both approaches, integrating physical grounding from the former and simplicity of application from the latter. Notably, it allows for manual fine-tuning of constraint priorities and boasts superior computational speed through auto code generation.

The implementation of the proposed methodology demonstrates significant advancements in the control and synchronization of robotic systems constrained by shared objects. Through the integration of robust physics simulation, precise robotic dynamics computation, and efficient optimization techniques, the method achieves remarkable outcomes across various simulation experiments. By leveraging advanced simulation, computation, and optimization techniques, the method has improved levels of efficiency, accuracy, and adaptability in robotic operations, paving the way for further advancements in automation and robotics technology.

In subsequent chapters, we delve deeper into various aspects of the proposed method. Chapter 2 provides an exhaustive review of recent literature, highlighting the existing landscape of solutions and their limitations. Chapter 3 elucidates the methodology underlying the proposed approach, offering insights into its theoretical underpinnings. Implementation details and code snippets are presented in Chapter 4, demonstrating the practical application of the method. Chapter 5 undertakes a comparative analysis, pitting technique against established methods to gauge its efficacy. Finally, Chapter 6 summarizes findings, discusses implications, and outlines avenues for future research.

Chapter 2

Literature Review

The control of the interacting physical systems is a challenging task. There are few problems that should be solved to achieve the high efficiency and precision in the aforementioned problem. They are the following: the right mathematical model selection, the unified methodology for defining the interaction, and the well-defined control error. The first point is necessary to cover a wide range of physical systems. The second one is crucial to work with different mechanical connections. Finally, the last one is needed to stability and robustness analysis.

This literature review covers all these subproblems and explores them from the point of view of the work of Firdaus Udwadia and Robert Kalaba [1]. The section 2.1 considers a mathematical models that utilized in the recent studies. The 2.2 part reviews a rigorous methodology to define how physical systems can affect each other. The section 2.3 explores the previous research in control error defining and analysis on different manifolds.

The articles were selected to demonstrate a solid ground why some methods had been preferred in this study. Some of the papers were chosen to reference for theoretical background that was not included in this paper. Other articles had been reviewed for shown an existing techniques.

2.1 The mathematical model

This section contains review of existing mathematical models for physical systems. Moreover, this piece has a comparison of them in terms of numerical integration convenience and simplicity of defining the initial conditions. It is necessary to clarify that from now this study considers only systems of rigid body with only inner stiffness. The detailed explanation of such choice will be conduct later in the Chapter 3.

Firstly, lets consider the most popular model. The articles [1], [5] and [6] relies on it. Usually it is called the canonical manipulator equation. The model can be formulated as

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) = \boldsymbol{\tau} \tag{2.1}$$

where $M(\mathbf{q})$ is known as inertia matrix, $C(\mathbf{q}, \dot{\mathbf{q}})$ is Centrifugal-Coriolis matrix, $g(\mathbf{q})$ is a gradient of potential forces, and $\mathbf{q}$, $\boldsymbol{\tau}$ are generalized coordinates and torques respectively. This equation will be explored in details in the next Chapter 3.

Conversely, Udwadia [7] utilized a Hamiltonian view of the problem. The equations are following,

$$\begin{cases} \dot{\mathbf{q}} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}} \\ \dot{\mathbf{p}} = -\frac{\partial \mathcal{H}}{\partial \mathbf{q}} \end{cases} \tag{2.2}$$

where $\mathbf{q}$, $\mathbf{p}$, and $\mathcal{H}$ are generalized coordinates, generalized momentum, and

Hamiltonian respectively.

The aforementioned equations are proven to be equivalent. Thus, both models can be utilized for achieving the main goal. However, the crucial difference lies in the another plane.

In the vast majority of cases the both differential equations can be solved analytically. Therefore, it is necessary to use the numerical methods. The equation (2.1) has second time derivative. It forces to construct proper state variable for numerical integration. In the opposite the Hamiltonian equations (2.2) has only first derivative. Hence, the usage of it is simpler. However, the definition of initial conditions is harder. Nevertheless, the mentioned problem are already solved. So, choice of model fully lies on the specific of discussed task.

The most of reviewed papers relies on the canonical model (2.1). Therefore, this research will utilize this equation too.

2.2 The methodology to defining interaction

The second crucial point in this paper is a definition of interaction between systems. This piece considers how it can be achieved, and which methodology is better in the context of discussed question. The important remark of this section that only interaction via rigid bodies will be consider below.

The first discussed approach is analyzed in [8] and [9]. These articles propose to use a a force cone to define an contact between a physical system and solid surface. The actuation force can be formulated as

$$\boldsymbol{\lambda} = \sum_{i=1}^{N_d} \beta_i(\mathbf{n} + \mu\,\mathbf{d}_i),\ \beta_i \geqslant 0 \tag{2.3}$$

where $\mathbf{n}$ is a normal force, $\mathbf{d}_i$ is a tangent to contact vector, μ is the Coulomb friction coefficient, and N_d is a amount of used tangent vectors. This contact force later can be translated to joint space via respective jacobian - $J^T(\mathbf{q})$.

Using this approach it is possible to emulate an interaction between physical systems via rigid body. It can be achieved by defining the motion of connection body through force acting on it. However, this method cannot guaranty a stability. Moreover, constructing a feedback loop in this case is not trivial task.

The next straight forward solution is initially impose an interaction inside to mathematical model. In the discussed case it can be achieved by writing the $M(\mathbf{q})$, $C(\mathbf{q}, \dot{\mathbf{q}})$, and $g(\mathbf{q})$ in equation (2.1) as formulation of closed-loop system. Thus, the advantage of this method is unnecessary of stabilization, because it is guaranteed by a model itself. Futhermore, this formulation can be used with a great range of control techniques. However, [10] demonstrates a lower computation speed with comparison to open-loop algorithms [3]. Moreover, proposed closed-loop version requires a predefined description of a whole system. Thus, it cannot be used in the realtime in the dynamic environment.

Finally, the third approach is defining a right constraint. In this study the action of the rigid body on connected systems can be described by holonomic constraint. It can be formulated as

$$\varphi(\mathbf{q}, t) = 0 \tag{2.4}$$

Further this equation can be used in KKT (Karush-Kuhn-Tucker) technique to rigorous define system with imposed constraints. Moreover, using the equation (2.4) it is straight forward to construct a stabilization mechanism. As instance, the Baumgarte's approach can be used with KKT method to achieve this goal.

Nevertheless, this study does not rely on aforementioned technique (KKT), instead it utilized the Udwadia-Kalaba approach. Details are shown in the Chapter 3.

2.3 Control error

The last crucial subproblem is rigorous defining of control error for ensure stabilization. Especially, it is important in the context of chosen type of interaction. Therefore, this section reviews how to calculate the discrepancy between the current system state and desired one in the form of the equation (2.4).

The previous studies [11]–[15] concentrates on computing pose difference in space. The majority [12]–[15] of source explores problem through utilizing rotations matrices. On the other hand [11] uses a quaternions for achieving the goal. Let's compare these solutions.

The [11] states the advantage of quaternions on the rotations matrix. This research shows that proposed method achieves the *exponential asymptotic stability* against *almost global stability* of techniques from [12], [13]. Futhermore, the quaternion-based method in discussed study presents the superiority on the analogous ones. The authors proves that their technique avoids the unwinding phenomenon, i.e. error converges to zero by shortest path in $\mathbb{S}^3$.

The solution via utilizing the rotations matrices is presented in [12]–[15]. However, in aforementioned quaternion-based approach shows that basing on such structure methods has disadvantage. The naive error computation via matrices is calculation of dot product between base vectors of current and desired orientation. As mentioned above this approach can achieve only *almost global stability*. Nevertheless, studies [14], [15] relies on deep topology analysis of $SE(3)$ group. These research uses a Lie theory to construct a right discrepancy between attitudes. In the Chapter 3 the *exponential convergence* of method based on articles above is presented.

To conclude, in this study the rotation matrices are chosen to define a control error. As starting point of investigations articles [14], [15] are picked. The main reason of such choice is convenience of work in context of framework [3] that utilized for numerical experiments.

Chapter 3

Methodology

This chapter will present a detailed elucidation of the principles underpinning the proposed methodology. The initial section will offer a concise overview of the Udvadia-Kalaba's approach is accompanied by a comprehensive exposition of the physical rationales employed in this technique. Subsequently, these principles will be harnessed in the development of the intended methodology. Furthermore, this section will scrutinize the limitations and distinctive characteristics inherent in the proposed methodology.

3.1 Essentials of Udvadia-Kalaba approach

Let $\mathbf{q}$ be a vector of generalized coordinates of some physical system. Also let agree that $\dim \mathbf{q} = n_q$ and $\dim \dot{\mathbf{q}} = \dim \ddot{\mathbf{q}} = n_v$, where $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are generalized velocity and acceleration respectevly. Notably, that in the general case $n_q \neq n_v$ because $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ can be located in different spaces. Thus, the dynamics of the given system can be expressed:

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) = \boldsymbol{\tau} \tag{3.1}$$

where $M(\mathbf{q}) \succeq 0$ is known as general inertia matrix, $C(\mathbf{q}, \dot{\mathbf{q}})$ is Coriolis-Centrifugal matrix, and $g(\mathbf{q})$ is potential forces impact (usually gravitational impact). For further convenience, I would omit the parameters of matrix functions. In the case of Coriolis-Centrifugal and potential forces, it is common to combine them in one term (bias force) $\mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} - C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - g(\mathbf{q})$. Hence, I can rewrite 3.1 in the following manner

$$M\ddot{\mathbf{q}} = \mathbf{Q} \tag{3.2}$$

equation 3.2 is compact form of equation 3.1. It would be highly utilized in further investigations.

Let's consider the same physical system again with imposed constraints at this time. All holonomic constraints would be presented by $\varphi(\mathbf{q}, t) : \mathbb{S}_{n_q} \times \mathbb{R} \to \mathbb{R}^m$, and all non-holonomic by $\psi(\mathbf{q}, \dot{\mathbf{q}}, t) : \mathbb{S}_{n_q} \times \mathbb{R}^{n_v} \times \mathbb{R} \to \mathbb{R}^k$, where $\mathbb{S}_{n_q}$ generalized coordinates space in the most common case. All constraints are satisfied if and only if the above functions are equal to zero.

These constraints can be rewritten in the form

$$A(\mathbf{q}, \dot{\mathbf{q}}, t)\ddot{\mathbf{q}} - b(\mathbf{q}, \dot{\mathbf{q}}, t) \tag{3.3}$$

by differentiation of holonomic constraints twice and non-holonomic once. According to Udwadia-Kalaba the constrained force that would satisfy can be written

$$Q_c = M^{1/2}(AM^{-1/2})^{+}(b - AM^{-1}Q) \tag{3.4}$$

where $M^{\pm 1/2} = W\Lambda^{\pm 1/2}W^T$ (W, Λ gain by eigendecomposition), $[*]^{+}$ is the Moore-Penrose inverse. Under these forces, the solution of the forward dynamics takes the form

$$\ddot{\mathbf{q}} = M^{-1}Q + M^{-1/2}(AM^{-1/2})^{+}(b - AM^{-1}Q) \tag{3.5}$$

The equation 3.4 according to F. Udvadia and R. Kalaba [1] is the analytical solution to the minimization problem based on Gauss's principle of least constraint

$$\begin{aligned} \min_{\ddot{\mathbf{q}}} \quad & [\ddot{\mathbf{q}} - a]^T M [\ddot{\mathbf{q}} - a] \\ \text{s.t.} \quad & A\ddot{\mathbf{q}} - b = 0 \\ & a(\mathbf{q}, \dot{\mathbf{q}}, t) = M^{-1}Q \end{aligned} \tag{3.6}$$

While the solution proposed by F. Udvadia and R. Kalaba demonstrates precision, it is not without its drawbacks. Instability points and computationally intensive operations, such as the computation of matrix roots, present significant challenges. Moreover, a notable deficiency lies in the absence of prioritization within the methodology.

3.2 Brief Lie theory

The Lie group, a mathematical concept dating back to the 19th century, was first proposed by Sophus Lie, laying the foundation for continuous transformation groups. Although it was initially abstract, over time its influence has spread to various scientific and technological fields. Before proceeding further, it is necessary to consider the basics of Lie theory. The mathematical tools discussed in this section are crucial for the research discussed here.

Let $\mathcal{G}$ is smooth manifold that satisfies the group axioms. For any $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ from $\mathcal{G}$ the following statements are true:

$$\begin{aligned}
&\mathcal{X} \circ \mathcal{Y} \in \mathcal{G} && \text{(I)}\\
&\exists\, \mathcal{E} :\ \mathcal{E} \circ \mathcal{X} = \mathcal{X} \circ \mathcal{E} = \mathcal{X} && \text{(II)}\\
&\exists\, \mathcal{X}^{-1} :\ \mathcal{X}^{-1} \circ \mathcal{X} = \mathcal{X} \circ \mathcal{X}^{-1} = \mathcal{E} && \text{(III)}\\
&(\mathcal{X} \circ \mathcal{Y}) \circ \mathcal{Z} = \mathcal{X} \circ (\mathcal{Y} \circ \mathcal{Z}) && \text{(IV)}
\end{aligned} \tag{3.7}$$

For such group $T_{\mathcal{E}}\mathcal{G}$ is the Lia Algebra defined at $\mathcal{E}$ element. The geometric interpretation of algebra is tangent plane that touchs the smooth manifold (group). $\mathfrak{m} \equiv T_{\mathcal{E}}\mathcal{G}$ is always a vector space.

The next crucial defenition is a group action

$$f : \mathcal{G} \times \mathcal{V} \to \mathcal{V} \tag{3.8}$$

where $\mathcal{V}$ is some set. The operation defined above should satisfy the axioms ($v \in \mathcal{V}$; $\mathcal{X}, \mathcal{Y} \in \mathcal{G}$),

$$\begin{aligned}
&\mathcal{E} \cdot v = v && \text{(I)}\\
&(\mathcal{X} \circ \mathcal{Y}) \cdot v = \mathcal{X} \cdot (\mathcal{Y} \cdot v) && \text{(II)}
\end{aligned} \tag{3.9}$$

For instance, if the $\mathrm{SO}(n)$ rotations is considered as Lie group, than the transformation $R \cdot x \equiv Rx$ ($x \in \mathbb{R}^n$) is a group action.

The exponential map $\exp :\ \mathfrak{m} \to \mathcal{G}$ converts elements of Lie algebra to corresponding group. The log map do inverse operation. However, it is neccesary to remember that this map applicable only for "identity" algebra. However, there is a linear transformation between $T_{\mathcal{X}}\mathcal{G}$ and $T_{\mathcal{E}}\mathcal{G}$. It is called adjoin.

It is known that $\mathfrak{m}$ is isomorphic to the vector space $\mathbb{R}^m$. One can write it as $\mathfrak{m} \cong \mathbb{R}^m$. Due to convinience this isomorphism would be highly utilized. Moreover, in the bellow sections only algebras constructed on $\mathbb{R}^m$ are considered. Thus, a maping between sets can be defined in the following manner (hat-vee notation)

$$\begin{aligned} &[*]^{\wedge} : \mathbb{R}^m \to \mathfrak{m} \qquad \boldsymbol{\tau}^{\wedge} = \sum_{i=1}^{m} \tau_i E_i \\ &[*]^{\vee} : \mathfrak{m} \to \mathbb{R}^m \qquad \boldsymbol{\tau} = \sum_{i=1}^{m} \tau_i \mathbf{e}_i \end{aligned} \tag{3.10}$$

where $\mathbf{e}_i$ are a base of $\mathbb{R}^m$ and E_i are base vectors of $\mathfrak{m}$. Obviously, $\mathbf{e}_i^{\wedge} = E_i$. Using this mapping, the exponential / log map can be modified

$$\begin{aligned} \exp : &\qquad \mathcal{X} = \exp(\boldsymbol{\tau}^{\wedge}) \\ \log : &\qquad \boldsymbol{\tau} = \log(\mathcal{X})^{\vee} \end{aligned} \tag{3.11}$$

Or, in the most convinient form

$$\begin{aligned} \mathrm{Exp} : &\qquad \mathcal{X} = \exp(\boldsymbol{\tau}^{\wedge}) = \mathrm{Exp}(\boldsymbol{\tau}) \\ \mathrm{Log} : &\qquad \boldsymbol{\tau} = \log(\mathcal{X})^{\vee} = \mathrm{Log}(\mathcal{X}) \end{aligned} \tag{3.12}$$

Through this definitions it easy to introduce plus and minus operations

$$\begin{aligned} &\text{right-}\oplus : \mathcal{X} \oplus {}^{\mathcal{X}}\boldsymbol{\tau} \equiv \mathcal{X} \circ \mathrm{Exp}({}^{\mathcal{X}}\boldsymbol{\tau}) \in \mathcal{G} \\ &\text{right-}\ominus : \mathcal{Y} \ominus \mathcal{X} \equiv \mathrm{Log}(\mathcal{X}^{-1} \circ \mathcal{Y}) \in T_{\mathcal{X}}\mathcal{G} \end{aligned} \tag{3.13}$$

TODO: continue section

3.3 Defying constraints over multiple systems

This section introduces a common holonomic constraint, which serves as a foundation for further investigations. This constraint delineates a rigid body's behavior, applicable to multiple manipulators.

Let $M_1, M_2, \dots, M_p$ are p iinertia matrices for p independent systems. The i-th system has n_q^i generalized coordinates, n_v^i generalized velocity components and Q_i bias force. Thus, the common unconstrained dynamics is

$$\begin{bmatrix} M_1 & 0 & \dots & 0 \\ 0 & M_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & M_p \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_1 \\ \ddot{\mathbf{q}}_2 \\ \vdots \\ \ddot{\mathbf{q}}_p \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_p \end{bmatrix} \tag{3.14}$$

The equation 3.14 can be rewriten in the manner of 3.2. In such compact form it is convinient for futher analysis. Hense, let

$$M_s = \begin{bmatrix} M_1 & 0 & \dots & 0 \\ 0 & M_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & M_p \end{bmatrix} \tag{3.15}$$

$$q_s = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \dots & \mathbf{q}_p \end{bmatrix}^T \tag{3.16}$$

$$Q_s = \begin{bmatrix} Q_1 & Q_2 & \dots & Q_p \end{bmatrix}^T \tag{3.17}$$

It implies to the following dynamics equation that describes motion of p independent systems

$$M_s\ddot{\mathbf{q}}_s = \mathbf{Q}_s \tag{3.18}$$

Let R_{ij} and $\mathbf{p}_{ij}$ are rotation and position of j-th frame attached to some link of the i-th system respectevly. The rigid body connection between two frames now can be defined as

$$\begin{cases} R_{ij}R_d = R_{\alpha\beta} & (3.19) \\ (\mathbf{p}_{ij} - \mathbf{p}_{\alpha\beta})^T(\mathbf{p}_{ij} - \mathbf{p}_{\alpha\beta}) = l & (3.20) \end{cases}$$

where R_d is a fixed rotation matrix and l a distence between connection points inside a rigid body.

Substituting ${}^{\alpha\beta}_{ij}\mathbf{d} = \mathbf{p}_{ij} - \mathbf{p}_{\alpha\beta}$ and differencing respect to time twice the equation 3.20 transforms to

$${}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{\alpha\beta}_{ij}\mathbf{d} - l = 0 \Rightarrow {}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{\alpha\beta}_{ij}\dot{\mathbf{d}} = 0 \Rightarrow {}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{\alpha\beta}_{ij}\ddot{\mathbf{d}} + {}^{\alpha\beta}_{ij}\dot{\mathbf{d}}^T\,{}^{\alpha\beta}_{ij}\dot{\mathbf{d}} = 0 \tag{3.21}$$

The equation 3.21 can be expresed via generalized coordinates

$${}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{\alpha\beta}_{ij}\ddot{\mathbf{d}} + {}^{\alpha\beta}_{ij}\dot{\mathbf{d}}^T\,{}^{\alpha\beta}_{ij}\dot{\mathbf{d}} = 0 \Rightarrow {}^{\alpha\beta}_{ij}\mathbf{d}^T({}^{v}_{ij}J\ddot{\mathbf{q}}_s + {}^{v}_{ij}\dot{J}\dot{\mathbf{q}}_s) + \dot{\mathbf{q}}_s^T\,{}^{v}_{ij}J^T\,{}^{v}_{ij}J\dot{\mathbf{q}}_s = 0 \tag{3.22}$$

where ${}^{v}_{ij}J \equiv {}^{v}_{ij}J(\mathbf{q}_s)$ is the j-th frame velocity Jacobian of the i-th system. Now, it is possible to convert 3.20 constraint to 3.3 form

$$A_p(\mathbf{q}_s, \dot{\mathbf{q}}_s) = {}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{v}_{ij}J \tag{3.23}$$

$$b_p(\mathbf{q}_s, \dot{\mathbf{q}}_s) = -\,{}^{\alpha\beta}_{ij}\mathbf{d}^T\,{}^{v}_{ij}\dot{J}\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_s^T\,{}^{v}_{ij}J^T\,{}^{v}_{ij}J\dot{\mathbf{q}}_s \tag{3.24}$$

Now, let suppose that $\gamma_1, \gamma_2, \ldots, \gamma_w$ system connected by rigid body. Here γ_i is an index of the system, and $w \leq p$.

Chapter 4

Implementation

Chapter 5

Evaluation and Discussion

Chapter 6

Conclusion

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