Smart throttle

Assignment 0

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PART A

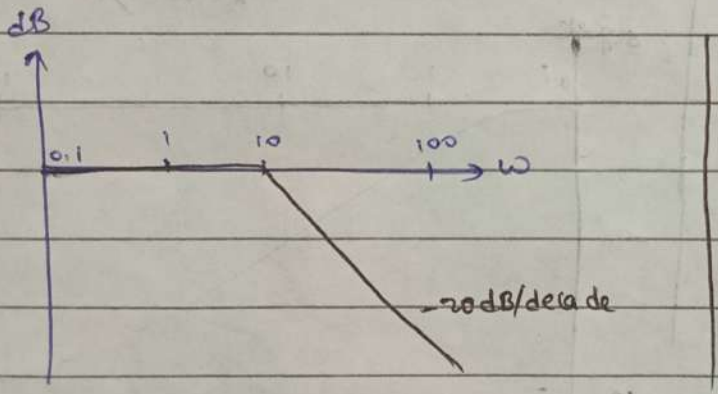
I. $G_1(s) = \frac{10}{s+10}$

Pole $s = -10$, $G_1(0) = \frac{10}{0+10} = 1$

α in dB $= 20\log_{10} 1 = 0$

In Y axis: $20\log_{10}\left|\frac{10}{s+10}\right| = 20\log_{10}\left|\frac{1}{1+\frac{s}{10}}\right|$

$\hookrightarrow \omega_c$



magnitude plot

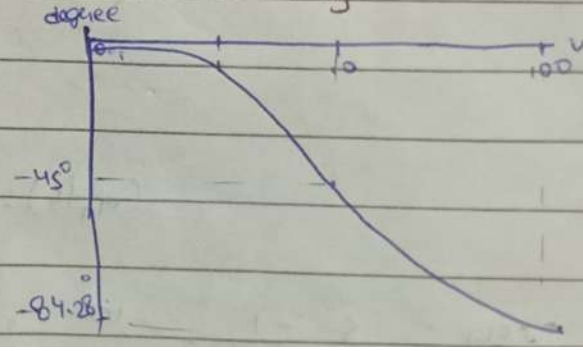
$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right) = -\tan^{-1}\left(\frac{\omega}{10}\right)$

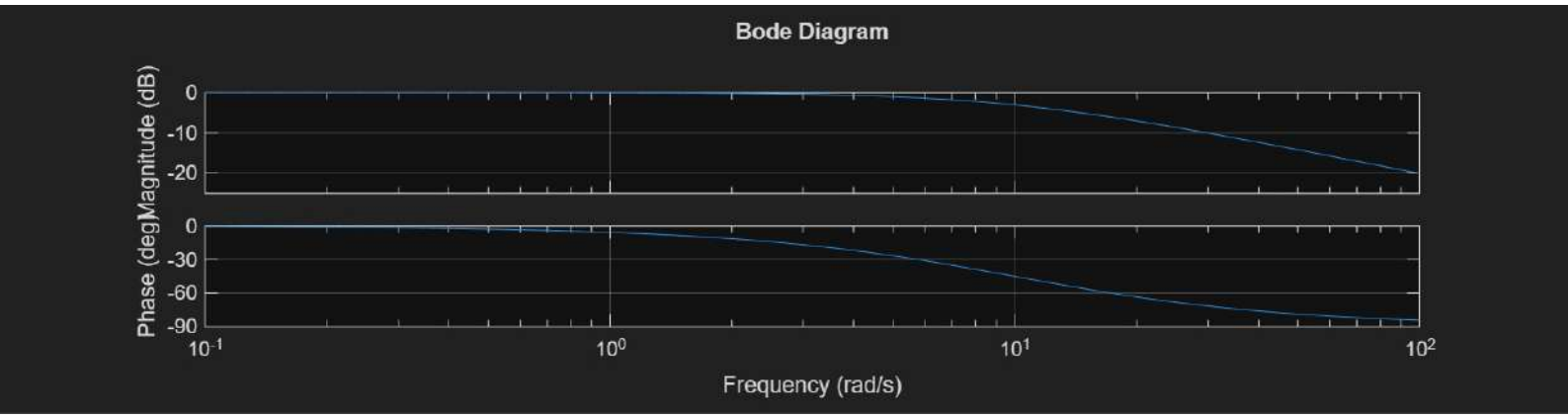
$\omega = 0.1 \Rightarrow \phi = -0.1°$

$\omega = 1 \Rightarrow \phi = -5.7°$

$\omega = 10 \Rightarrow \phi = -45°$

$\omega = 100 \Rightarrow \phi = -84.28$



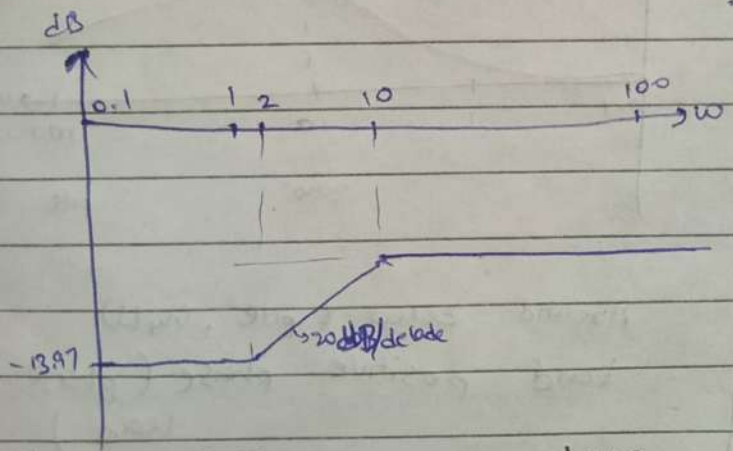

Bode Diagram
Magnitude (dB)
0
-10
-20
Phase (deg)
0
-30
-60
-90
10^-1
10^0
10^1
10^2
Frequency (rad/s)

2. $G_2(s) = \frac{s-2}{s+10}$

zero : $s = 2$, Pole: $s = -10$, $G_2(0) = \frac{-2}{10} = -\frac{1}{5}$

In Y-axis: $20\log_{10}\left|\frac{s-2}{s+10}\right| = 20\log_{10}\left|\frac{-2}{10}\right| + 20\log_{10}\left|\frac{s}{2}-1\right| - 20\log_{10}\left|\frac{s}{10}+1\right|$

$= -13.97 + 20\log_{10}\left|\frac{s}{2}-1\right| - 20\log_{10}\left|\frac{s}{10}+1\right|$



④ A RHP zero cause phase to decrease (-90° lag)

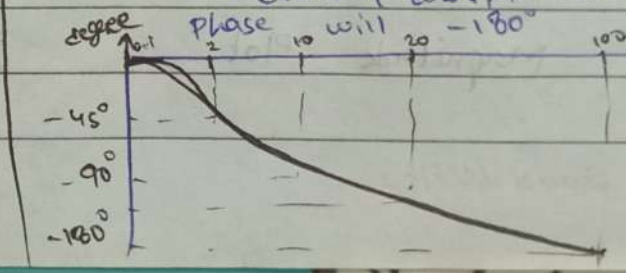
Phase

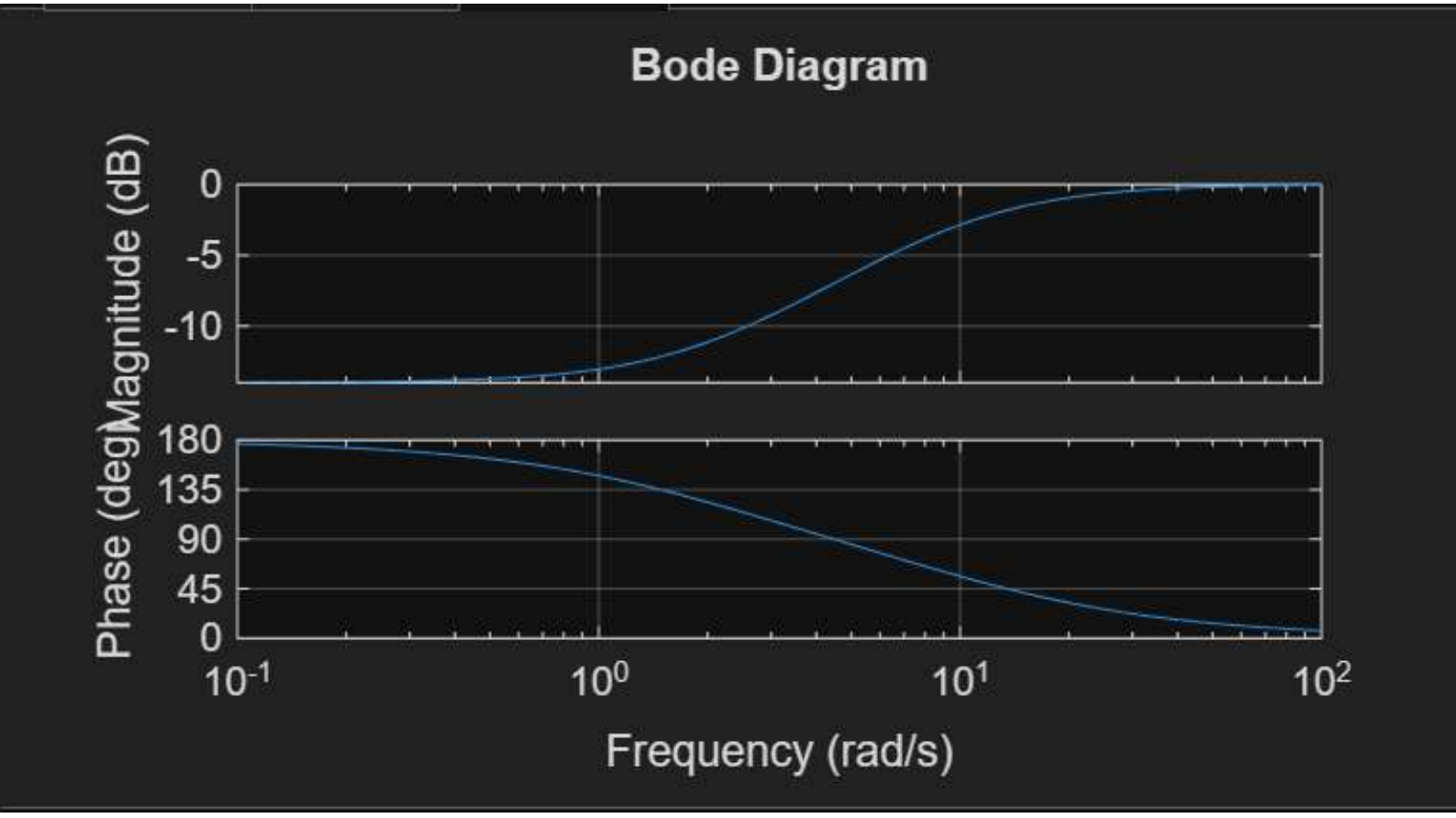
at $\omega = 2$, (zero of RHP) , phase will -45°

at $\omega = 10$, (pole of LHP) , phase will $-45^\circ + (-45^\circ)$

~~at $\omega = 20$, decade~~ , phase will $-90^\circ - 45^\circ = -135^\circ$

at $\omega = 100$, decade of zero & pole reach phase will -180°




Bode Diagram
Magnitude (dB)
0
-5
-10
Phase (deg)
180
135
90
45
0
10-1
100
101
102
Frequency (rad/s)

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3.

$$G_3(s) = \frac{100}{s^2 + 10s + 100}$$

pole $s_{1,2} = \frac{-10 \pm \sqrt{100 - 400}}{2} = \frac{-10 \pm 10\sqrt{3}j}{2} = -5 \pm 5\sqrt{3}j$

compare to standard form : $\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

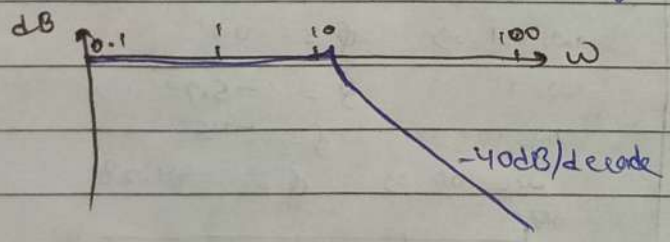
$\omega_n = \sqrt{100} = 10$; $2\zeta = 10 \Rightarrow \zeta = 0.5$

magnitude plot

DC gain = 1

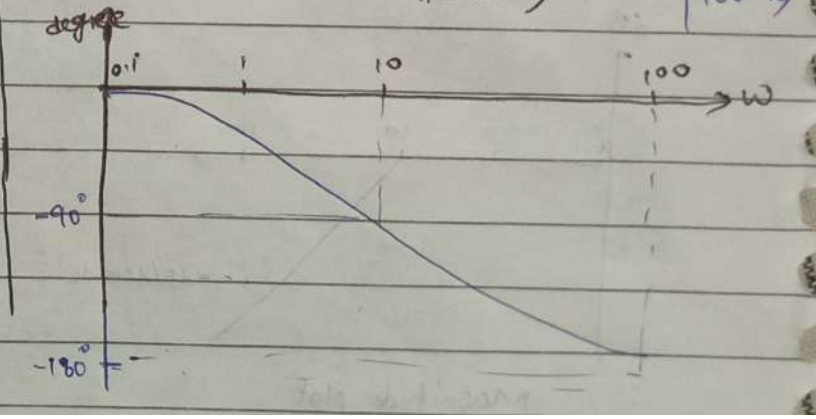
in y-axis: $20\log_{10} 1 = 0$

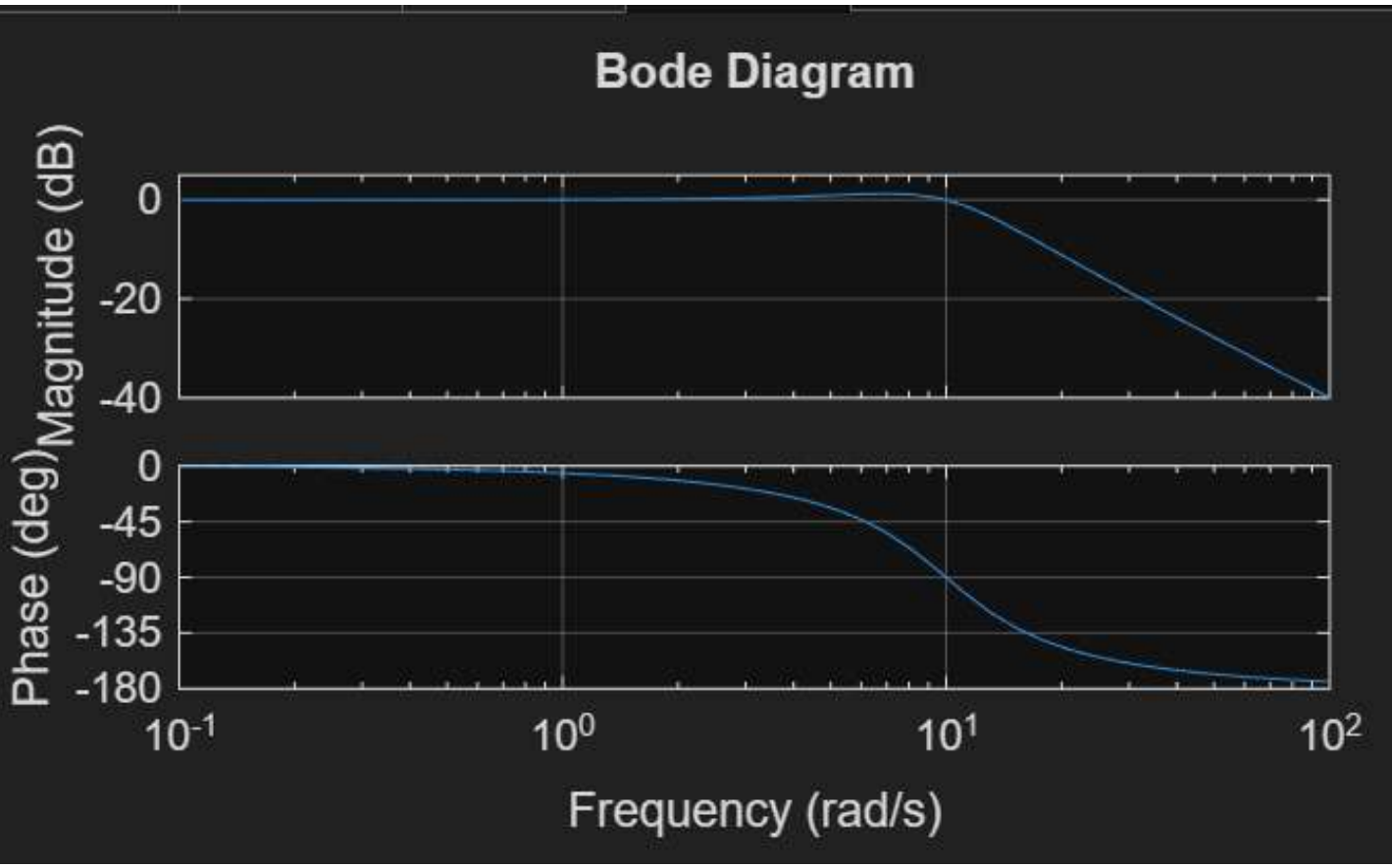
only bump at natural frequency



Phase plot

$\phi = -\tan^{-1}\left(\frac{2\zeta\omega\omega_n}{\omega_n^2 - \omega^2}\right) = -\tan^{-1}\left(\frac{10\omega}{100 - \omega^2}\right)$




Bode Diagram
Magnitude (dB)
0
-20
-40
Phase (deg)
0
-45
-90
-135
-180
10-1
100
101
102
Frequency (rad/s)

4.

$$G_4(s) = \frac{0.1s+1}{0.01s+1}$$

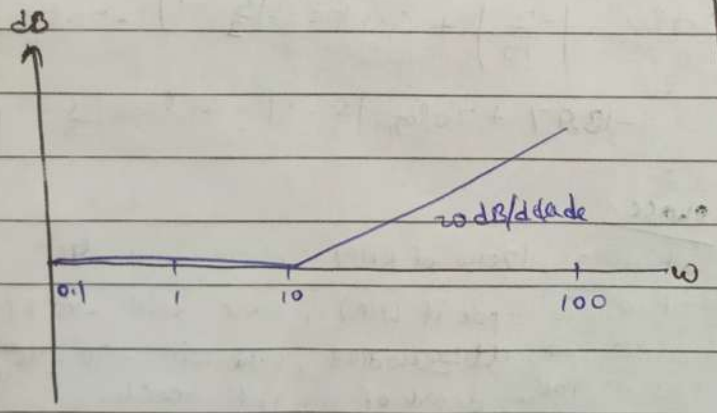
zero : $s = \frac{-1}{0.1} = -10$, pole : $s = \frac{-1}{0.01} = -100$

DC gain : 1

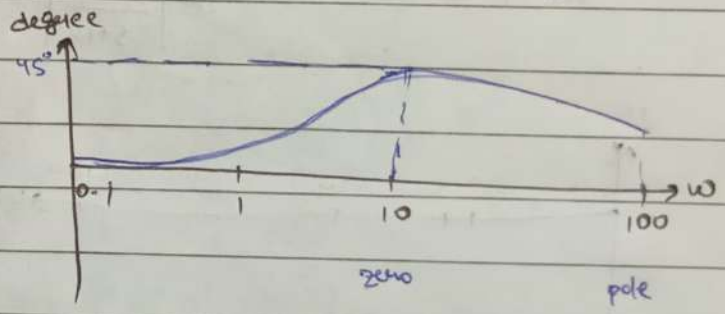
In Y-axis : $20\log_{10}|1| + 20\log_{10}|0.1s+1|$ $- 20\log_{10}|0.01s+1|$

at $\omega = 10$, (zero of LHP) phase lead $+45^\circ$

at $\omega = 100$ (pole of LHP) phase $\approx +90^\circ - 45^\circ \approx 45^\circ$

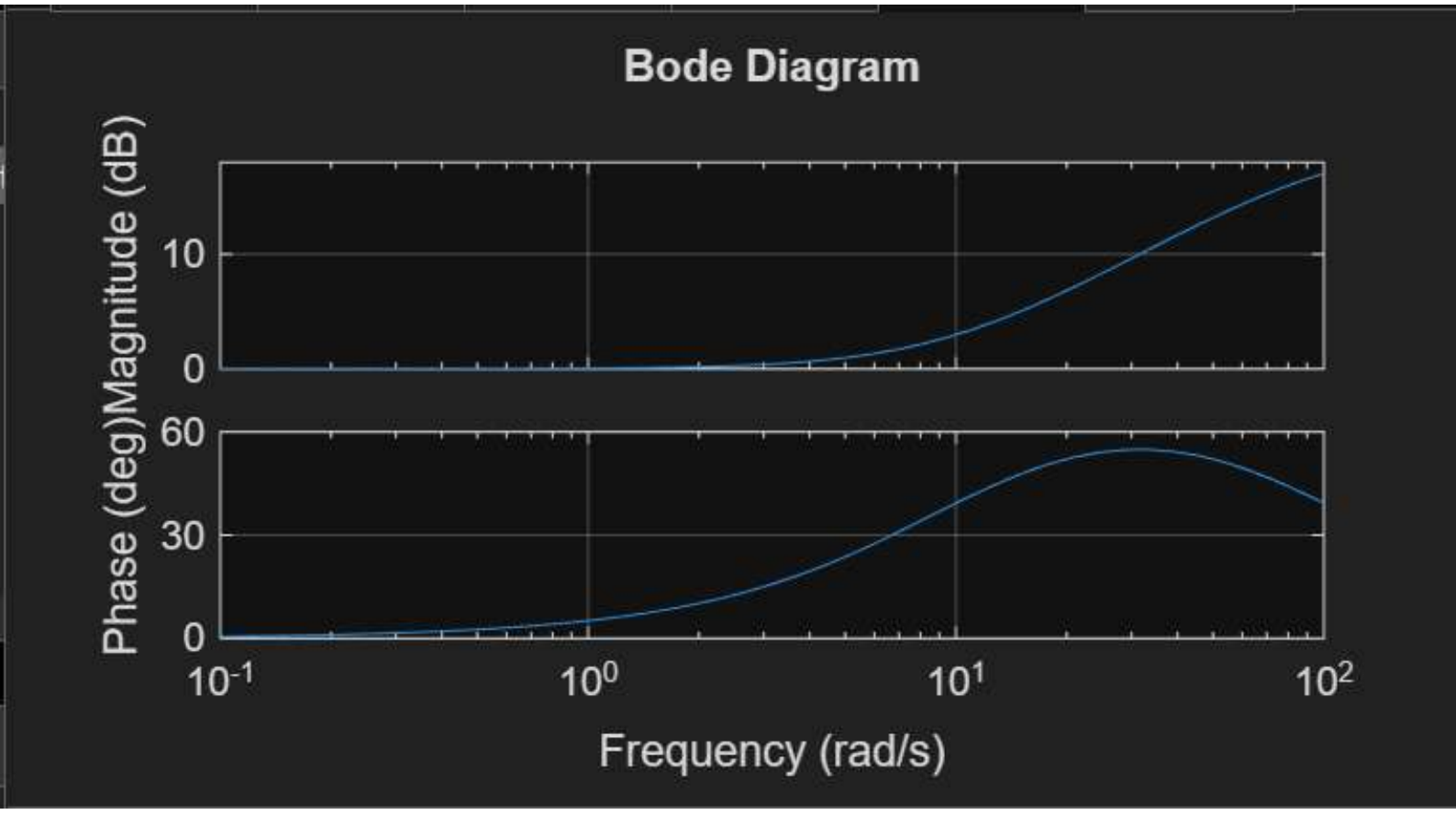


Magnitude Plot



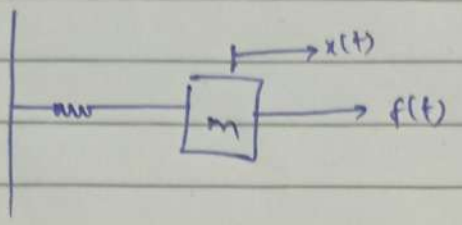
Around zero & pole, $G_4(s)$ tend positive phase (phase lead)

Good Write


Bode Diagram
Magnitude (dB)
10
0
Phase (deg)
60
30
0
10-1
100
101
102
Frequency (rad/s)

PART - B

Q2



B.1

$$\Sigma F = \frac{m d^2x}{dt^2}$$

$$f_{spring} + f_{damping} + F_{Ext} = \frac{m d^2x}{dt^2}$$

$$-(x + d\frac{dx}{dt} + f(t) = m\frac{d^2x}{dt^2}$$

$$\boxed{f(t) = m\frac{d^2x}{dt^2} + (x + d\frac{dx}{dt}}$$

2.

Zero initial condition : $x(0)=0$; $\dot{x}(0)=0$; $\ddot{x}(0)=0$

$$f(t) = m\frac{d^2x}{dt^2} + (x + d\frac{dx}{dt}$$

convert into laplace domain

$$F(s) = ms^2X(s) + cX(s) + dsX(s)$$

$$\boxed{F(s) = X(s)\left[ms^2 + c + ds\right]}$$

3.

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + c + ds}$$

B.2

For $m = 1\,kg$, $d = 4\,Ns/m$, $c = 16\,N/m$

1. $$\boxed{G(s) = \frac{1}{s^2 + 16 + 4s}}$$

2. poles $(s_{1,2}) = -2 \pm 2\sqrt{3}i$

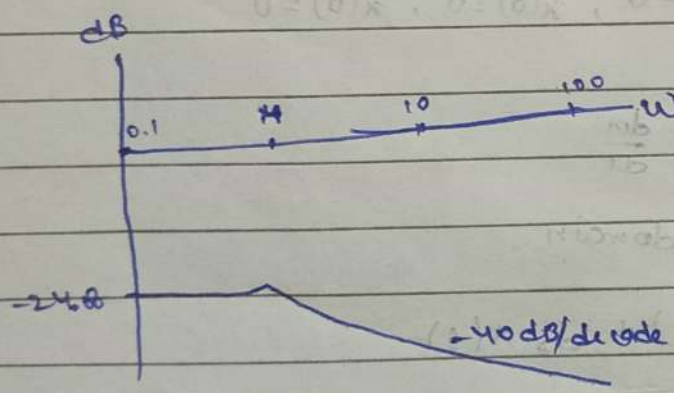
$$G(s) = \frac{1}{s^2+4s+16}$$

$$s_{1,2} = \frac{-4 \pm \sqrt{16-64}}{2} = \frac{-4 \pm 4\sqrt{3}i}{2} = -2 \pm 2\sqrt{3}i$$

3. Compare with standard form : $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

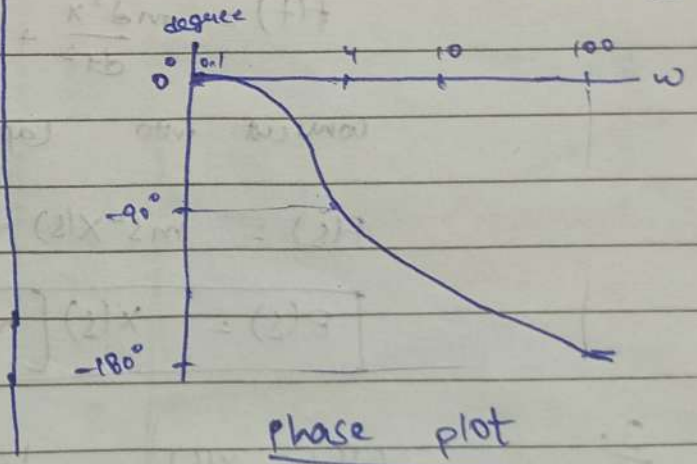
$\omega_n = 4$, $\zeta = 0.5$

DC gain $G(0) = \frac{1}{16}$ $\Rightarrow$ In dB $20\log_{10}\left(\frac{1}{16}\right) = -24.08$ dB

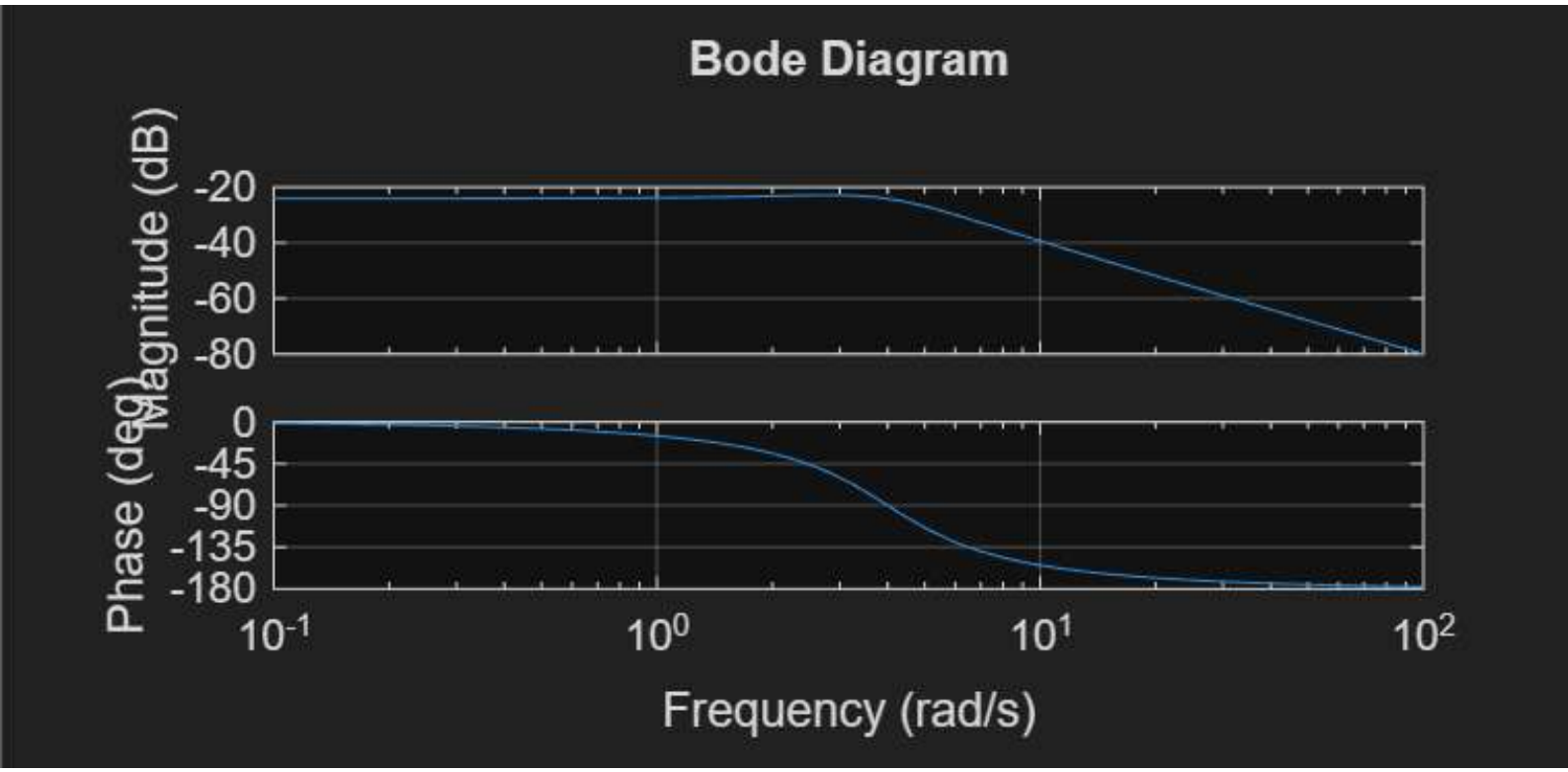


Magnitude plot

phase $\phi(\omega) = -\tan^{-1}\left(\frac{2\times0.5\times4\times\omega}{16-\omega^2}\right)$



Phase plot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