



Full resolution photo on my Instagram [@feenafoto](https://www.instagram.com/feenafoto)

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# ECOR1043: Circuits

## Passive Filters

### Capacitor and Inductor Based Filters

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# First Order RC Filters (Lowpass & Highpass)

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## RC Low-pass Filter

- RC Low-pass filter

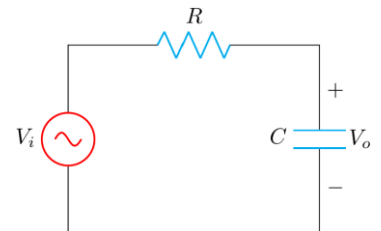
- The Transfer function H

$$V_o = \frac{Z_c}{R + Z_c} \times V_i$$

(using simple voltage divider)

$$H = \frac{V_o}{V_i} = \frac{Z_c}{R + Z_c}$$

where  $Z_c = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{-j}{2\pi f C}$



- Magnitude

$$H = \frac{X_c}{\sqrt{R^2 + X_c^2}}$$

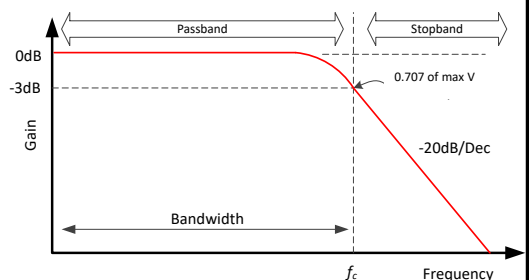
where  $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

- Cut-off frequency

$$f_c = \frac{1}{2\pi RC}$$

- Bandwidth

$$BW = 0 \text{ Hz to } f_c$$



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## RC Low-pass Filter

- Example 1a: Find the transfer function ( $H=V_o/V_i$ ) of the given RC LPF

$$H = \frac{V_o}{V_i} = \frac{Z_c}{R + Z_c}$$

1) Find  $Z_c$

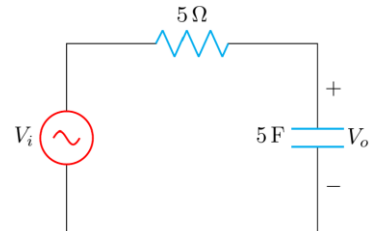
$$R = 5\Omega$$

$$Z_c = \frac{1}{j\omega C}$$

$$Z_c = \frac{1}{j\omega(5F)}$$

$$Z_c = \frac{1}{j5\omega}$$

$$Z_c = -\frac{j}{5\omega}$$



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## RC Low-pass Filter

- Example 1a: Find the transfer function ( $H=V_o/V_i$ ) of the given RC LPF (cont.)

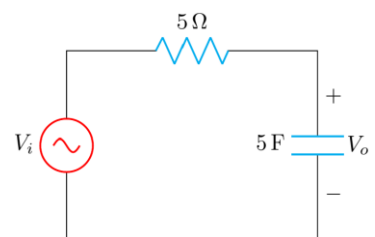
1) Find  $H = \frac{V_o}{V_i}$

$$Z_c = -\frac{j}{5\omega}$$

$$R = 5\Omega$$

$$H = \frac{V_o}{V_i} = \frac{Z_c}{R + Z_c}$$

$$H = \frac{V_o}{V_i} = \left( \frac{-\frac{j}{5\omega}}{5 - \frac{j}{5\omega}} \right)$$



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## RC Low-pass Filter

- Example 1b: Create the Bode plot for the **magnitude** of given transfer function. Use  $\omega = 0.2, 1, 10$  rad/s

3) Solve for the magnitude of  $\omega = 0.2$

$$H = \frac{V_o}{V_i} = \left( \frac{-\frac{j}{5\omega}}{5 - \frac{j}{5\omega}} \right)$$

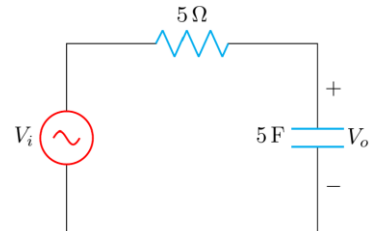
$$H = \frac{V_o}{V_i} = \frac{1/5\omega}{\sqrt{5^2 + \left(\frac{1}{5\omega}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1/1}{\sqrt{5^2 + \left(\frac{1}{1}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{26}} = 0.2$$

$$\text{Magnitude } H = \frac{X_c}{\sqrt{R^2 + X_c^2}}$$

Where  $X_c = \frac{1}{\omega C}$



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## RC Low-pass Filter

- Example 1b: Create the Bode plot for the magnitude of given transfer function. Use  $\omega = 0.2, 1, 10$  rad/s

3) Solve for the magnitude of  $\omega = 1$

$$H = \frac{V_o}{V_i} = \left( \frac{-\frac{j}{5\omega}}{5 - \frac{j}{5\omega}} \right)$$

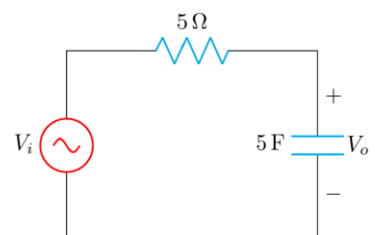
$$H = \frac{V_o}{V_i} = \frac{1/5\omega}{\sqrt{5^2 + \left(\frac{1}{5\omega}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1/5}{\sqrt{5^2 + \left(\frac{1}{5}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{0.2}{\sqrt{25.04}} = 0.04$$

$$\text{Magnitude } H = \frac{X_c}{\sqrt{R^2 + X_c^2}}$$

Where  $X_c = \frac{1}{\omega C}$



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## RC Low-pass Filter

- Example 1b: Create the Bode plot for the magnitude of given transfer function. Use  $\omega = 0.2, 1, 10$  rad/s

3) Solve for the magnitude of  $\omega = 10$

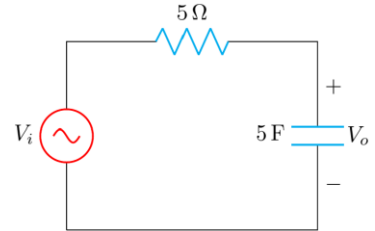
$$H = \frac{V_o}{V_i} = \left( \frac{-\frac{j}{5\omega}}{5 - \frac{j}{5\omega}} \right)$$

$$H = \frac{V_o}{V_i} = \frac{1/5\omega}{\sqrt{5^2 + \left(\frac{1}{5\omega}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1/50}{\sqrt{5^2 + \left(\frac{1}{50}\right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{0.02}{\sqrt{25.0004}} = 0.004$$

Magnitude  $H = \frac{X_c}{\sqrt{R^2 + X_c^2}}$   
Where  $X_c = \frac{1}{\omega C}$



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## RC Low-pass Filter

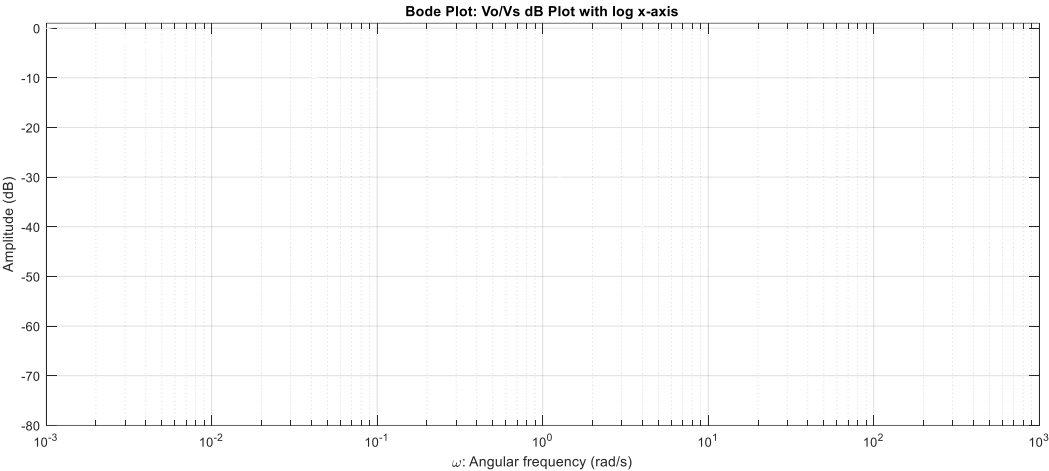
6) Convert values into dB

| Frequency (rad/s) | Amplitude [V/V] | Amplitude dB [ $20\log_{10}(V/V)$ ] |
|-------------------|-----------------|-------------------------------------|
| 0.2               | 0.2             | -14                                 |
| 1                 | 0.04            | -28                                 |
| 10                | 0.004           | -48                                 |

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# RC Low-pass Filter

| Frequency (rad/s) | Amplitude [V/V] | Amplitude dB [ 20log <sub>10</sub> (V/V)] |
|-------------------|-----------------|---|
| .001              | 1               | 0   |
| 0.2               | 0.2             | -14                                       |
| 1                 | 0.04            | -28                                       |
| 10                | 0.004           | -48                                       |

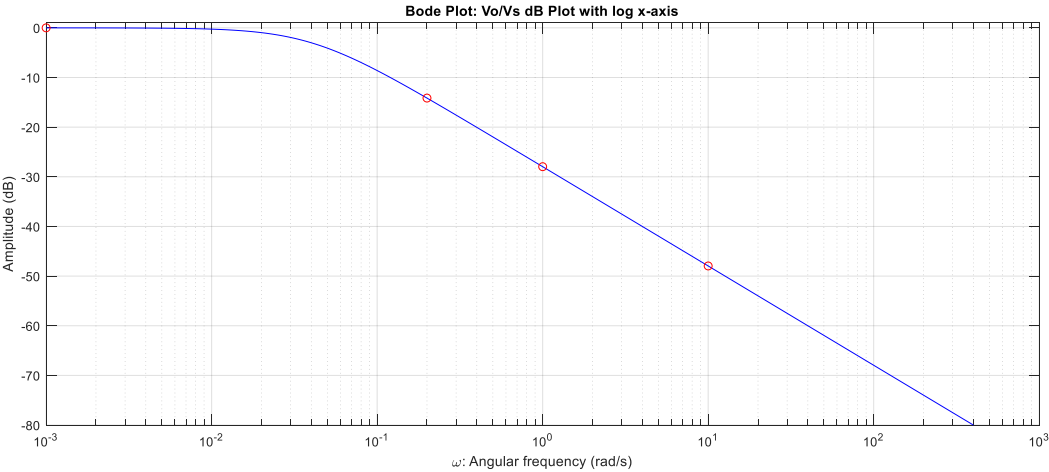


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# RC Low-pass Filter

| Frequency (rad/s) | Amplitude [V/V] | Amplitude dB [ 20log <sub>10</sub> (V/V)] |
|-------------------|-----------------|---|
| .001              | 1               | 0   |
| 0.2               | 0.2             | -14                                       |
| 1                 | 0.04            | -28                                       |
| 10                | 0.004           | -48                                       |



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## RC High-pass Filter

- RC High-pass filter

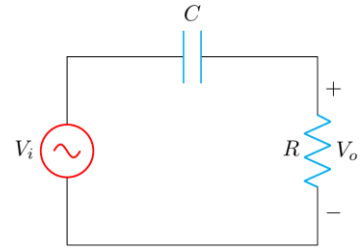
- The Transfer function

$$V_o = \frac{R}{R + Z_c} \times V_i$$

(using simple voltage divider)

$$H = \frac{V_o}{V_i} = \frac{R}{R + Z_c}$$

where  $Z_c = \frac{1}{j\omega C} = \frac{1}{j2\pi fC} = \frac{-j}{2\pi fC}$



- And magnitude

$$H = \frac{R}{\sqrt{R^2 + X_c^2}}$$

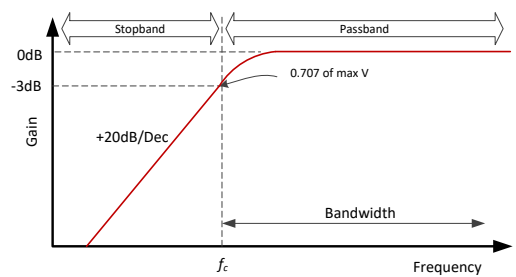
where  $X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

- Cut-off frequency

$$f_c = \frac{1}{2\pi RC}$$

- Bandwidth

$$BW = f_c \text{ to } \infty \text{ (ideally)}$$



## First Order RL Filters (Lowpass & Highpass)

## RL High-pass Filter

- RL High-pass filter

- Transfer function

$$V_o = \frac{Z_L}{R + Z_L} \times V_i \quad (\text{using simple voltage divider})$$

$$H = \frac{V_o}{V_i} = \frac{Z_L}{R + Z_L} \quad \text{where } Z_L = j\omega L = j2\pi fL$$

- Magnitude

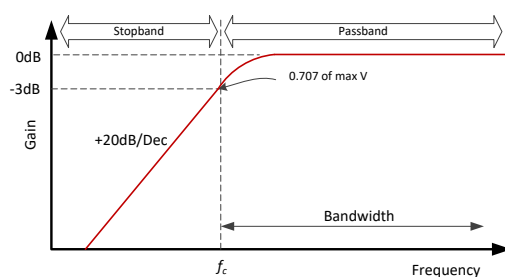
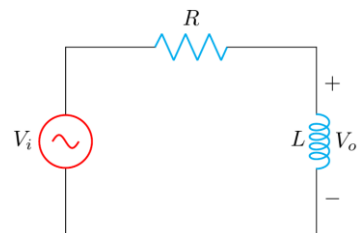
$$H = \frac{X_L}{\sqrt{R^2 + X_L^2}} \quad \text{where } X_L = \omega L = 2\pi fL$$

- Cut-off frequency

$$f_c = \frac{R}{2\pi L}$$

- Bandwidth

$$BW = f_c \text{ to } \infty \text{ (ideally)}$$



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## RL High-pass Filter

- Example 2a: Find the transfer function ( $V_o/V_i$ ) of given RL HPF circuit

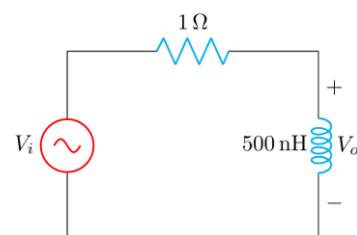
$$H = \frac{V_o}{V_i} = \frac{Z_L}{R + Z_L}$$

1) Find  $Z_L$ 

$$R = 1\Omega$$

$$Z_L = j\omega L$$

$$Z_L = j\omega(500 \text{ n}) \Omega$$



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## RL High-pass Filter

- Example 2a: Find the transfer function ( $V_o/V_i$ ) of given RL HPF circuit (cont.)

2) Find  $\frac{V_o}{V_i}$

$$Z_L = j\omega(500 \text{ n})$$

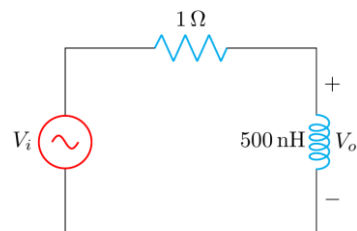
$$R = 1\Omega$$

$$H = \frac{V_o}{V_i} = \frac{Z_L}{R + Z_L}$$

$$H = \frac{V_o}{V_i} = \left( \frac{j\omega(500\text{n})}{1 + j\omega(500\text{n})} \right)$$

$$\frac{V_o}{V_i} = \left( \frac{j2\pi f(500\text{n})}{1 + j2\pi f(500\text{n})} \right)$$

What if I tell you to find its magnitude?



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## RL High-pass Filter

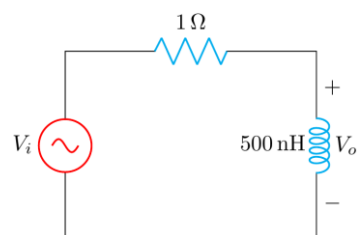
- Example 2b: Find  $f_c$  of the RL high-pass filter

3) Find  $f_c$

$$f_c = \frac{R}{2\pi L}$$

$$f_c = \frac{1\Omega}{2\pi(500 \times 10^{-9} \text{ H})}$$

$$f_c = 318.3 \text{ kHz}$$



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## RL High-pass Filter

- Example 2c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 100 \text{ Hz}$ ,  $f_c$ , and 1 MHz

4) Solve for the magnitude at  $f = 100 \text{ Hz}$

$$\frac{V_o}{V_i} = \left( \frac{j2\pi f(500n)}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{2\pi f(500n)}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

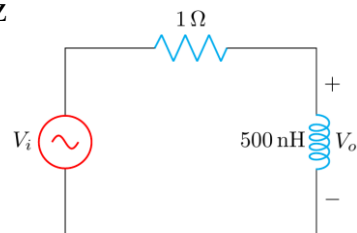
Magnitude  
where

$$H = \frac{V_o}{V_i} = \frac{Z_L}{\sqrt{R^2 + X_L^2}}$$

$$X_L = \omega L = 2\pi fL$$

$$H = \frac{V_o}{V_i} = \frac{2\pi 100(500n)}{\sqrt{1^2 + (2 \times \pi \times 100 \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = 0.000314$$



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## RL High-pass Filter

- Example 2c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 100 \text{ Hz}$ ,  $f_c$ , and 1 MHz

4) Solve for the magnitude at  $f_c = 318.3 \text{ kHz}$

$$\frac{V_o}{V_i} = \left( \frac{j2\pi f(500n)}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{2\pi f(500n)}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

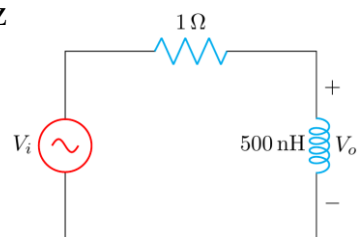
Magnitude  
where

$$H = \frac{V_o}{V_i} = \frac{Z_L}{\sqrt{R^2 + X_L^2}}$$

$$X_L = \omega L = 2\pi fL$$

$$H = \frac{V_o}{V_i} = \frac{2\pi \times 318.3k \times (500n)}{\sqrt{1^2 + (2 \times \pi \times 318.3k \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = 0.7071$$



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### RL High-pass Filter

- Example 2c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 100$  Hz,  $f_c$ , and **1 MHz**

4) Solve for the magnitude at  $f = 1$  MHz

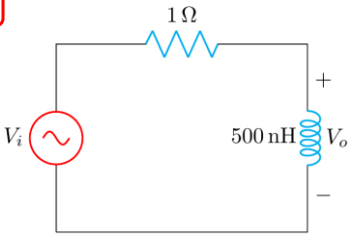
$$\frac{V_o}{V_i} = \left( \frac{j2\pi f(500n)}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{2\pi f(500n)}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

Magnitude where  $H = \frac{V_o}{V_i} = \frac{Z_L}{\sqrt{R^2 + X_L^2}}$   
 $X_L = \omega L = 2\pi fL$

$$H = \frac{V_o}{V_i} = \frac{2\pi \times 1M \times (500n)}{\sqrt{1^2 + (2 \times \pi \times 1M \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = .953$$



### RL High-pass Filter

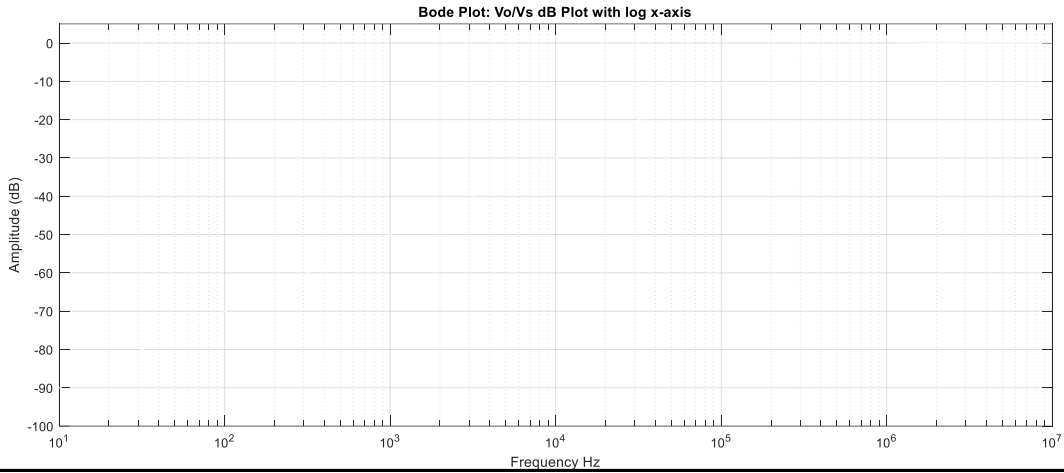
- Example 2c: Plot the Bode plot for the following frequencies

$$\frac{V_o}{V_i} = \left( \frac{(2\pi f)(500 \text{ nH})}{1 \Omega + j(2\pi f)(500 \text{ nH})} \right)$$

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 100            | 0.000314        | -70                                      |
| 318.3k         | 0.7071          | -3.01                                    |
| 1M             | 0.953           | -0.42                                    |

# RL High-pass Filter

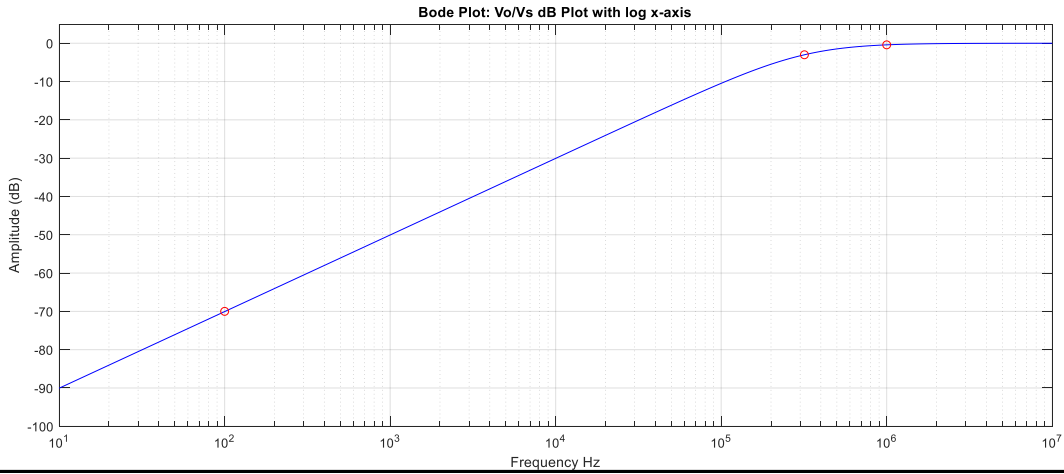
| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 100            | 0.000314        | -70                                      |
| 318.3k         | 0.7071          | -3.01                                    |
| 1M             | 0.953           | -0.42                                    |



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# RL High-pass Filter

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 100            | 0.000314        | -70                                      |
| 318.3k         | 0.7071          | -3.01                                    |
| 1M             | 0.953           | -0.42                                    |



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# RL Low-pass Filter

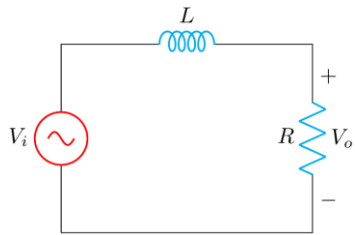
- RL Low-pass filter

- Transfer function

$$V_o = \frac{R}{R + Z_L} \times V_i$$

where  $Z_L = j\omega L = j2\pi fL$

$$H = \frac{V_o}{V_i} = \frac{R}{R + Z_L}$$



- Magnitude

$$H = \frac{R}{\sqrt{R^2 + X_L^2}}$$

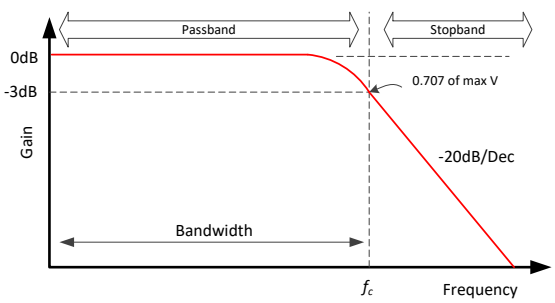
where  $X_L = \omega L = 2\pi fL$

- Cut-off frequency

$$f_c = \frac{R}{2\pi L}$$

- The bandwidth

$$BW = 0 \text{ Hz to } f_c$$



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## Alternative Low/High-pass Filters

| Filter Type | Typical Circuit | Transfer Function                           | Cut-off Frequency         | Bandwidth                     |
|-------------|-----------------|---|---------------------------|-------------------------------|
| RC LPF      |                 | $H = \frac{V_o}{V_i} = \frac{Z_c}{R + Z_c}$ | $f_c = \frac{1}{2\pi RC}$ | $BW = 0 \text{ Hz to } f_c$   |
| RC HPF      |                 | $H = \frac{V_o}{V_i} = \frac{R}{R + Z_c}$   | $f_c = \frac{1}{2\pi RC}$ | $BW = f_c \text{ to } \infty$ |
| RL LPF      |                 | $H = \frac{V_o}{V_i} = \frac{R}{R + Z_L}$   | $f_c = \frac{R}{2\pi L}$  | $BW = 0 \text{ Hz to } f_c$   |
| RL HPF      |                 | $H = \frac{V_o}{V_i} = \frac{Z_L}{R + Z_L}$ | $f_c = \frac{R}{2\pi L}$  | $BW = f_c \text{ to } \infty$ |

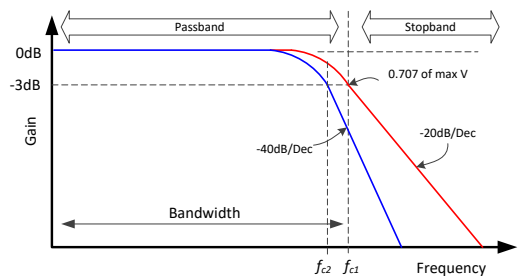
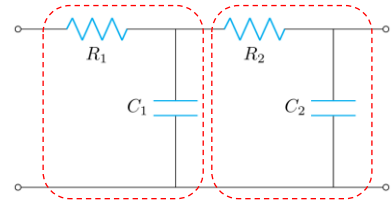
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## Higher Order RC Filters

- Higher order filters

- Sometimes the roll-off of -20dB/decade may not be enough to remove unwanted signals
- There we can use multiple stages of filtering to increase our roll-off rate
- First order filters are cascaded (arranged in series) to get higher order filters which increase the roll-off rate
- There is a limit to how many filters we can cascade (more in next lecture)



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## Practice Problems

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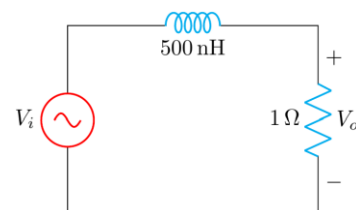
## RL Low-pass Filter

- Problem 1a: Find the transfer magnitude function ( $V_o/V_i$ ) of the circuit below

1) Find  $Z_L$

$$Z_L = j\omega L$$

$$Z_L = j\omega(500 \text{ nH})$$



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## RL Low-pass Filter

- Problem 1a: Find the transfer magnitude function ( $V_o/V_i$ ) of the circuit below

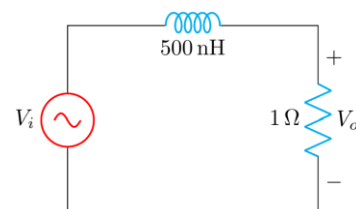
2) Find  $\frac{V_o}{V_i}$

$$Z_L = j\omega(500 \text{ n})$$

$$H = \frac{V_o}{V_i} = \left( \frac{R}{R + Z_L} \right)$$

$$H = \frac{V_o}{V_i} = \left( \frac{1}{1 + j\omega(500 \text{ n})} \right)$$

$$\frac{V_o}{V_i} = \left( \frac{1}{1 + j2\pi f(500 \text{ n})} \right)$$



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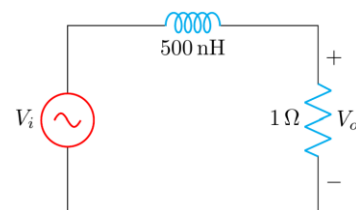
## RL Low-pass Filter

- Problem 1b: Find  $f_c$  of the RL low-pass filter

3) Find  $f_c$

$$f_c = \frac{R}{2\pi L}$$

$$f_c = \frac{R}{2\pi L} = \frac{1\Omega}{2\pi(500 \times 10^{-9} \text{ H})} = 318.3 \text{ kHz}$$



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## RL Low-pass Filter

- Problem 1c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 1 \text{ Hz}$ ,  $10 \text{ kHz}$ ,  $f_c$ , and  $500 \text{ kHz}$

4) Solve for the magnitude at  $f = 1 \text{ Hz}$

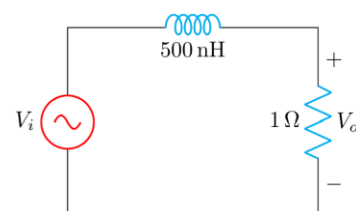
$$H = \frac{V_o}{V_i} = \left( \frac{1}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times 1 \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{1} = 1$$

Magnitude  $H = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2 + X_L^2}}$   
 where  $X_L = \omega L = 2\pi fL$



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## RL Low-pass Filter

- Problem 1c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 1$  Hz, 10 kHz,  $f_c$ , and 500 kHz

4) Solve for the magnitude at  $f = 10$  Hz

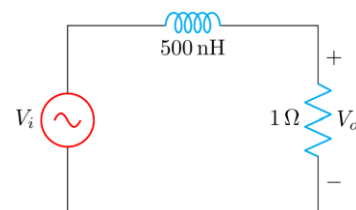
$$H = \frac{V_o}{V_i} = \left( \frac{1}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

Magnitude  $H = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2 + X_L^2}}$   
 where  $X_L = \omega L = 2\pi fL$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times 10k \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{1.0005} = 0.9995 = 1$$



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## RL Low-pass Filter

- Problem 1c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 1$  Hz, 10 kHz,  $f_c$ , and 500 kHz

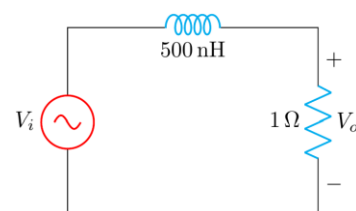
4) Solve for the magnitude at  $f = 318.3$  kHz

$$H = \frac{V_o}{V_i} = \left( \frac{1}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times 318.3k \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{1.414} = 0.70712$$



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## RL Low-pass Filter

- Problem 1c: Using the magnitude of transfer function ( $V_o/V_i$ ), create a bode plot using  $f = 1$  Hz, 10 kHz,  $f_c$ , and 500 kHz

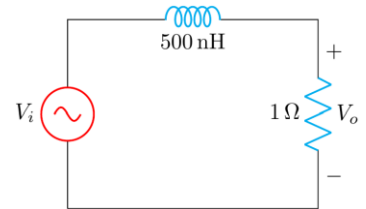
4) Solve for the magnitude at  $f = 500$  kHz

$$H = \frac{V_o}{V_i} = \left( \frac{1}{1 + j2\pi f(500n)} \right)$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times f \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{\sqrt{1^2 + (2 \times \pi \times 500k \times 500n)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{1}{1.862} = 0.537$$



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## RL Low-pass Filter

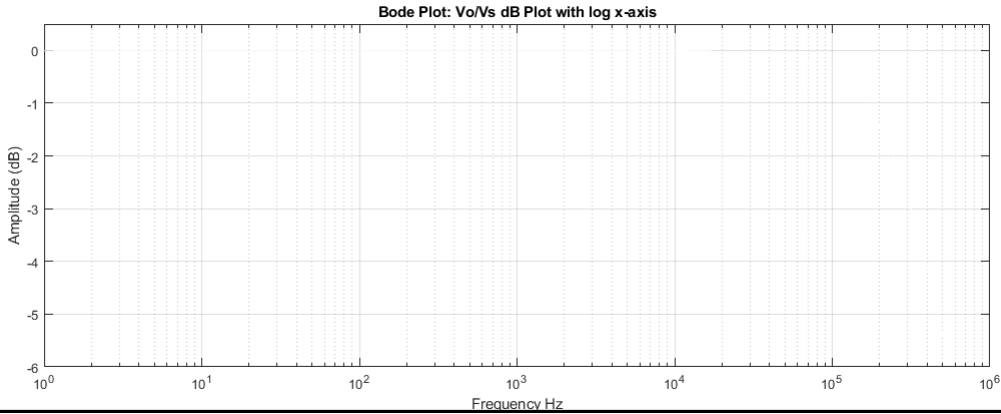
8) Convert values into dB

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [ $20\log_{10}(V/V)$ ] |
|----------------|-----------------|-------------------------------------|
| 1              | 1               | 0                                   |
| 10k            | 0.9995          | -0.004                              |
| 318.3k         | 0.70712         | -3.01                               |
| 500k           | 0.537           | -5.4                                |

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RL Low-pass Filter

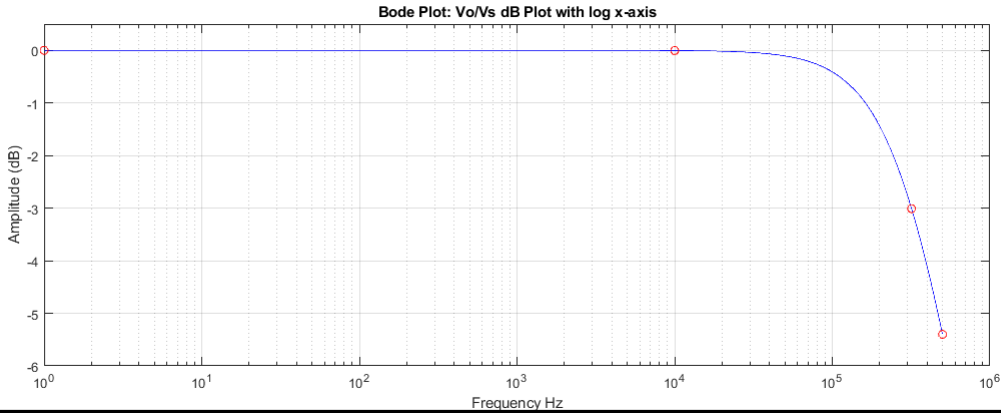
| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 1              | 1               | 0  |
| 10k            | 0.9995          | -0.004                                   |
| 318.3k         | 0.70712         | -3.01                                    |
| 500k           | 0.537           | -5.4                                     |



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RL Low-pass Filter

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 1              | 1               | 0  |
| 10k            | 0.9995          | -0.004                                   |
| 318.3k         | 0.70712         | -3.01                                    |
| 500k           | 0.537           | -5.4                                     |



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## RC High-pass Filter

- Problem 2a: Find the transfer function ( $H = V_o/V_i$ ) of circuit

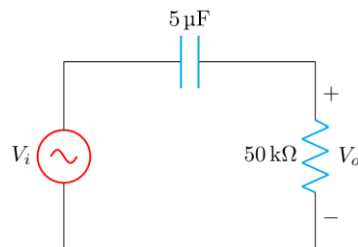
1) Find  $Z_c$

$$Z_c = \frac{1}{j\omega C}$$

$$Z_c = \frac{1}{j\omega(5\mu F)}$$

$$Z_c = -\frac{j}{\omega(5\mu)}$$

$$Z_c = -\frac{j}{2\pi f(5\mu)}$$



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## RC High-pass Filter

- Problem 2a: Find the transfer function ( $H = V_o/V_i$ ) of circuit

1) Find  $\frac{V_o}{V_i}$

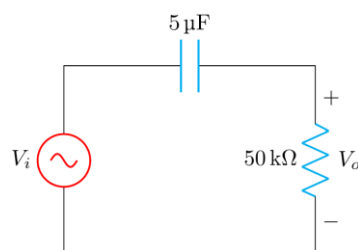
$$Z_c = -\frac{j}{(5\mu)2\pi f}$$

$$H = \frac{V_o}{V_i} = \left( \frac{R}{R + Z_c} \right)$$

Remember the voltage divider?

$$H = \frac{V_o}{V_i} = \left( \frac{50k}{50k + Z_c} \right)$$

$$H = \frac{V_o}{V_i} = \left( \frac{50k}{50k - \frac{j}{(5\mu)2\pi f}} \right)$$



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## RC High-pass Filter

- Problem 2b: Create the Bode plot for the magnitude of given transfer function using  $f = 0.01, f_c, 100 \text{ Hz}$

3) Solve for the magnitude of  $f = 0.01$

$$H = \frac{V_o}{V_i} = \left( \frac{50k}{50k - \frac{j}{(5\mu)2\pi f}} \right)$$

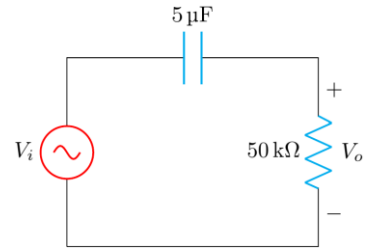
$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + \left( \frac{1}{(5\mu)2\pi f} \right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + \left( \frac{1}{(5\mu)2\pi \times .01} \right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + (3183098)^2}}$$

Magnitude  $H = \frac{R}{\sqrt{R^2 + X_c^2}}$  where  $X_c = \frac{1}{\omega C}$

$$H = \frac{V_o}{V_i} = 0.0157$$



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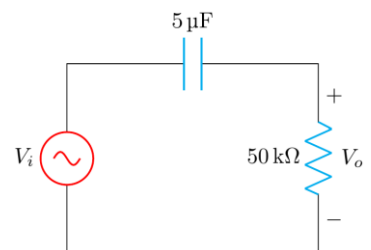
## RC High-pass Filter

- Problem 2b: Create the Bode plot for the magnitude of given transfer function using  $f = 0.01, f_c, 100 \text{ Hz}$

4) Find  $f_c$

$$f_c = \frac{1}{2\pi RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(50K \times 5 \times 10^{-6})} = 0.6366 \text{ Hz}$$



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## RC High-pass Filter

- Problem 2b: Create the Bode plot for the magnitude of given transfer function using  $f = 0.01, f_c, 100 \text{ Hz}$

3) Solve for the magnitude of  $f_c = 0.6366$

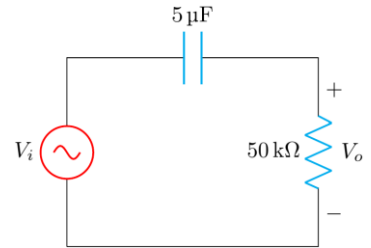
$$H = \frac{V_o}{V_i} = \left( \frac{50k}{50k - \frac{j}{(5\mu)2\pi f}} \right)$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + \left( \frac{1}{(5\mu)2\pi \times 0.6366} \right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + (50002)^2}}$$

$$H = \frac{V_o}{V_i} = 0.7071$$

Magnitude  $H = \frac{R}{\sqrt{R^2 + X_c^2}}$  where  $X_c = \frac{1}{\omega C}$



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## RC High-pass Filter

- Problem 2b: Create the Bode plot for the magnitude of given transfer function using  $f = 0.01, f_c, 100 \text{ Hz}$

3) Solve for the magnitude of  $f = 100$

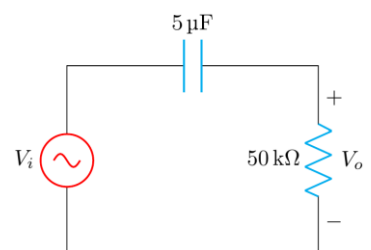
$$H = \frac{V_o}{V_i} = \left( \frac{50k}{50k - \frac{j}{(5\mu)2\pi f}} \right)$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + \left( \frac{1}{(5\mu)2\pi \times 100} \right)^2}}$$

$$H = \frac{V_o}{V_i} = \frac{50k}{\sqrt{(50k)^2 + (318.31)^2}}$$

$$H = \frac{V_o}{V_i} = 0.99998$$

Magnitude  $H = \frac{R}{\sqrt{R^2 + X_c^2}}$  where  $X_c = \frac{1}{\omega C}$



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# RC High-pass Filter

7) Convert values into dB

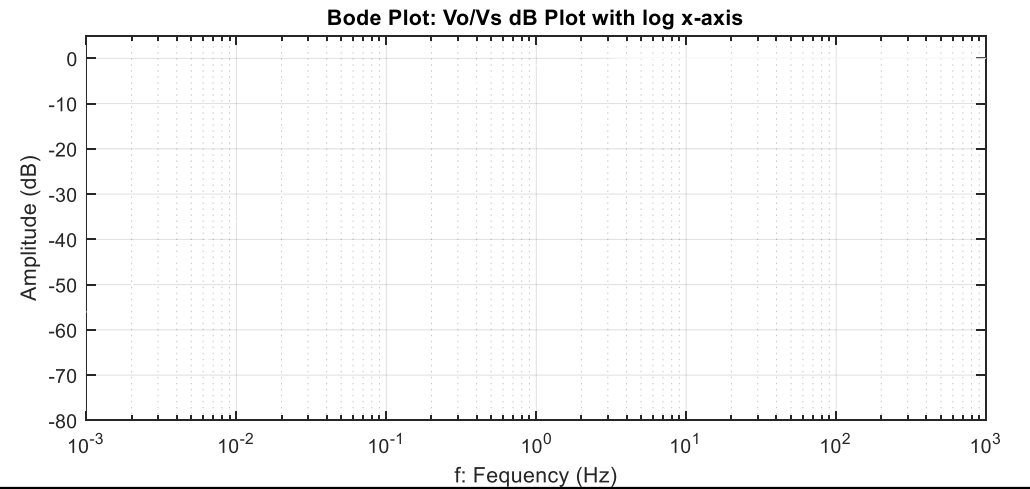
| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 0.01           | 0.0157          | -36.08                                   |
| 0.637          | 0.7071          | -3.01                                    |
| 100            | 0.99998         | -0.00017                                 |

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# RC High-pass Filter

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [20log <sub>10</sub> (V/V)] |
|----------------|-----------------|--|
| 0.01           | 0.0157          | -36.08                                   |
| 0.637          | 0.7071          | -3.01                                    |
| 100            | 0.99998         | -0.00017                                 |



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### RC High-pass Filter

| Frequency (Hz) | Amplitude [V/V] | Amplitude dB [ $20\log_{10}(V/V)$ ] |
|----------------|-----------------|-------------------------------------|
| 0.01           | 0.0157          | -36.08                              |
| 0.637          | 0.7071          | -3.01                               |
| 100            | 0.99998         | -0.00017                            |

