Full resolution photo on my Instagram @feenafoto

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ECOR1043: Circuits

Resistive Circuits

Single Node-pair Circuits

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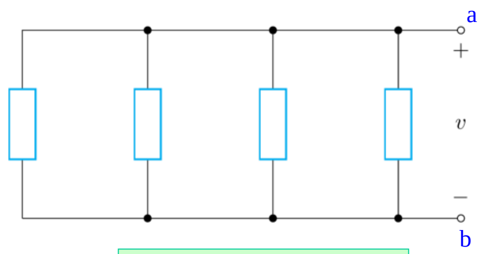
Single Node Pair Circuit

- These circuits are characterized by all the elements having the same voltage across them i.e., the elements are in parallel



Single node pair circuit

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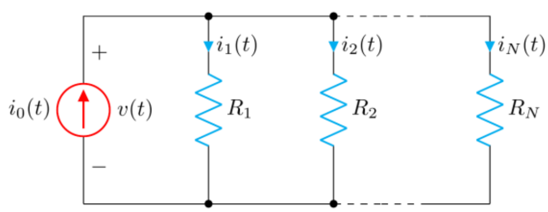
Equivalent Resistances: Parallel

- Equivalent Resistance
  - Simplifies our analysis by combining parallel resistors
  - Resistors are in parallel if they share the same two nodes (same voltage)
  - The equation to determine parallel resistance is given as:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$



- Special cases
  - If there are only two resistors in parallel, their sum would be

$$R_p = \frac{R_1 \times R_2}{R_1 + R_2}$$

  - If there are N parallel resistors of **equal value R**, their sum would be

$$R_p = \frac{R}{N}$$

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Current Divider: General Equation

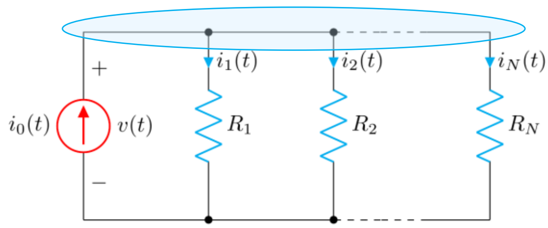
- In a single node circuit with multiple resistors and a current source, we can find the current through any resistor using Current Divider
- According to Current Divider, current $i_k(t)$ through any resistor R_k is given as:

$$\boxed{i_k(t) = \frac{R_p}{R_k} i_0(t)} \longrightarrow A$$

- Where

$$R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

$i_0(t)$: Current source

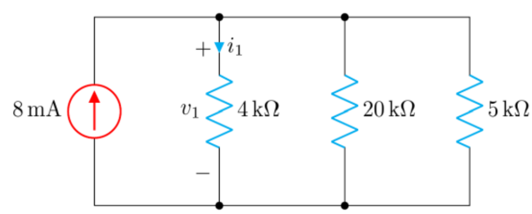


- In other words, in single node circuits, current through any resistor is inversely proportional to its value

`Derivation of eq. A is given in Appendix-A at the end`

Single Node Pair Circuit

- Ex. 1: Find R_p, i_1, v_1, and power associated with the source



`Finding` R_p

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

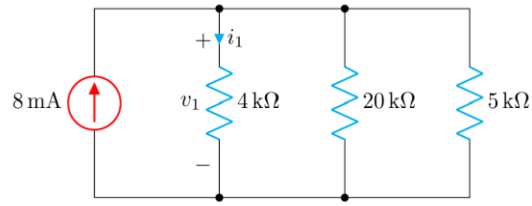
$$\frac{1}{R_p} = \frac{1}{4k\Omega} + \frac{1}{20k\Omega} + \frac{1}{5k\Omega}$$

$$\frac{1}{R_p} = \frac{5+1+4}{20k\Omega}$$

$$R_p = 2k$$

Single Node Pair Circuit

- Ex. 1: Find R_p, i_1, v_1, and power P_s associated with 8mA source



`Finding` I_1 `using general current divider`

$$i_k(t) = \frac{R_p}{R_k} i_o(t)$$

$R_p = 2k$

$$i_1 = \frac{2k\Omega}{4k\Omega} \times (8mA) = 4mA$$

`Finding` v_1`, using Ohm's law`

$V = IR$

$v_1 = i_1 \times 4k\Omega$

$v_1 = 4mA \times 4k\Omega = 16V$

`Finding` P_s

$P_s = -IV$

$P_s = -8mA \times 16V = -128\ mW$

So power is supplied by current source

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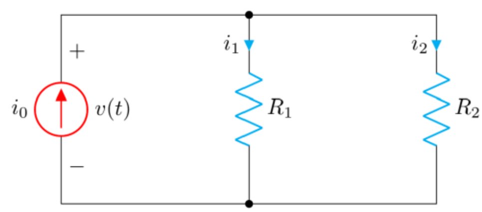
Current Divider: Special Case

- For the special case of two resistors R_1 and R_2, the current divider becomes:



$$i_1 = \frac{R_2}{R_1 + R_2} \times i_0$$

$$i_2 = \frac{R_1}{R_1 + R_2} \times i_0$$

- Use this when possible
- Reduce the parallel circuit to this if possible and convenient

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Current Divider: Example

- Ex. 2: Find i_1 and i_2

Finding i_1

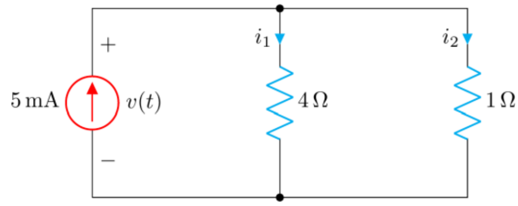
$$i_1 = \frac{R_2}{R_1 + R_2} \times i_0$$

$$i_1 = \frac{1\,\Omega}{1\Omega + 4\Omega} \times 5\,mA = 1mA$$

Finding i_2

$$i_2 = \frac{R_1}{R_1 + R_2} \times i_0$$

$$i_2 = \frac{4\,\Omega}{1\Omega + 4\Omega} \times 5\,mA = 4mA$$



We can check our answers using KCL:

$$i(t) = i_1(t) + i_2(t)$$
$$5\,mA = 1\,mA + 4\,mA$$
$$5\,mA = 5\,mA$$

LHS = RHS

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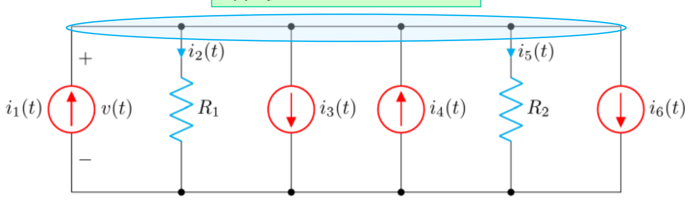
Equivalent Current Sources

– Similar to our equivalent voltage sources in a single loop circuits, we can make equivalent current sources.

Apply KCL to this node



– Assume currents entering the node are +ve and applying KCL

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

– Collect current source terms, using the sign of the net value to determine the direction of the equivalent source.

$$[i_1(t) - i_3(t) + i_4(t) - i_6(t)] - i_2(t) - i_5(t) = 0$$

$$[i_1(t) - i_3(t) + i_4(t) - i_6(t)] = i_2(t) + i_5(t)$$

$$i_0(t) = i_2(t) + i_5(t)$$

where: $i_0(t) = [i_1(t) - i_3(t) + i_4(t) - i_6(t)]$

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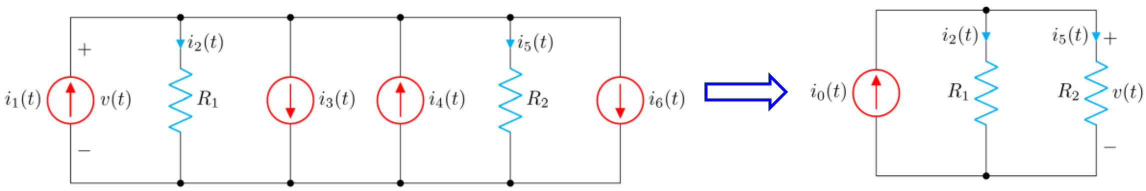
Equivalent Current Sources



$i_0(t) = [i_1(t) - i_3(t) + i_4(t) - i_6(t)]$

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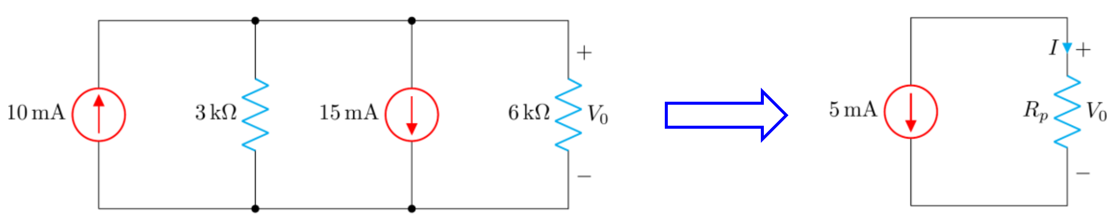
Multiple-source/Resistor Networks

- Ex 3. Find V_o and power associated with the current sources
  - Hint: Start by finding the equivalent current source and resistance



$R_{eq} = R_p = \dfrac{1}{\dfrac{1}{3k\Omega} + \dfrac{1}{6k\Omega}} = 2k\Omega$

```
Here, it is convenient to use the
special case of two resistors in
parallel, we get same value
```

$R_p = \dfrac{R_1 \times R_2}{R_1 + R_2} = \dfrac{3k \times 6k}{3k + 6k} = \dfrac{18k^2}{9k} = 2k\Omega$

Finding V_0

$V = IR$

$V_0 = I \times R_p$

$V_0 = -5mA \times 2k\Omega$

$V_0 = -10V$

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Multiple-source/Resistor Networks

- Ex 3 (cont.): Find V_0 and power associated with the sources
  - Now that V_0 is known, return to original circuit to determine individual powers associated with each current source

$P_{10mA} = -VI$

$V_0 = -10V$

$P_{10mA} = -(V_0)(10mA)$

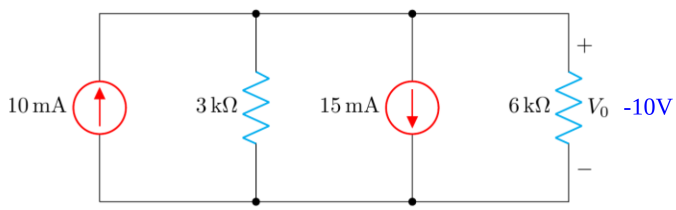
$P_{10mA} = -(-10)(10mA)$

$P_{10mA} = 100\ mW$

$P_{15mA} = VI$

$P_{15mA} = V_0\ (15mA)$

$P_{15mA} = -150mW$



So power supplied by 15mA current source is 150 mW
and power *absorbed* by 10mA current source is 100 mW

Find the power absorbed by resistors which should total 50 mW

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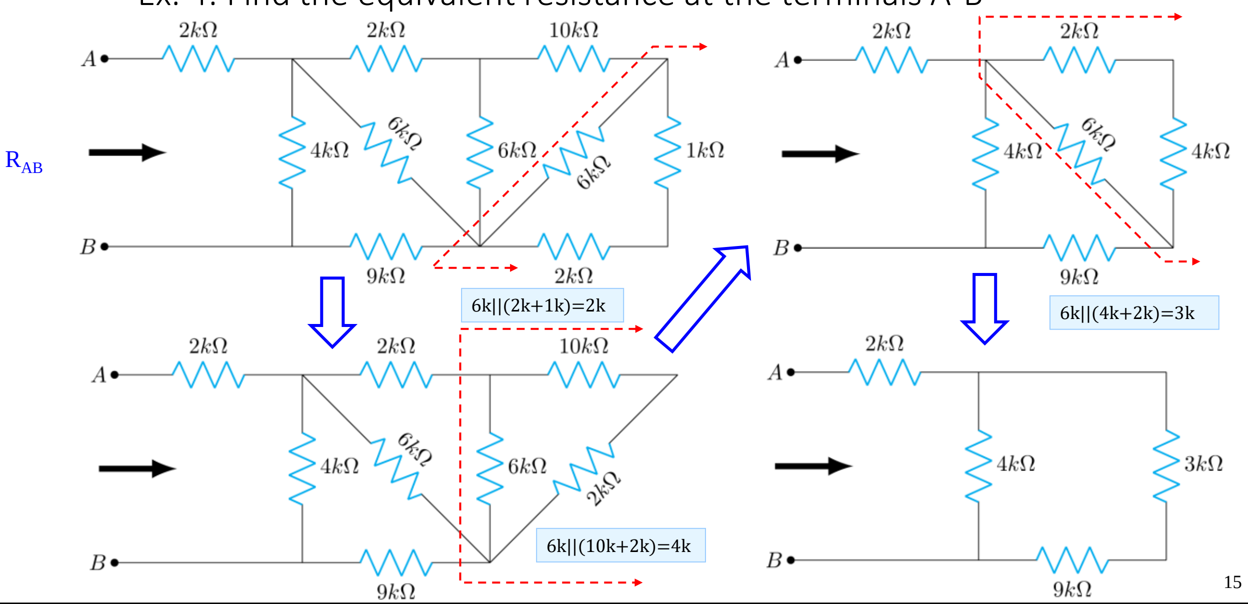
Series & Parallel Resistor Combinations

- Ex. 4: Find the equivalent resistance at the terminals A-B



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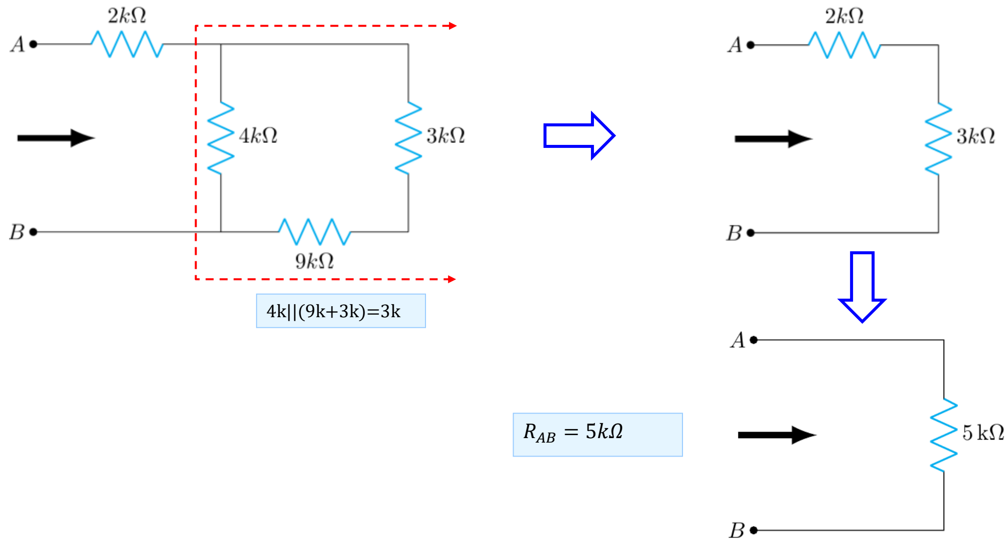
Series Parallel Resistor Combinations

- Ex. 4: Find the equivalent resistance at the terminals A-B



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Practice Problems

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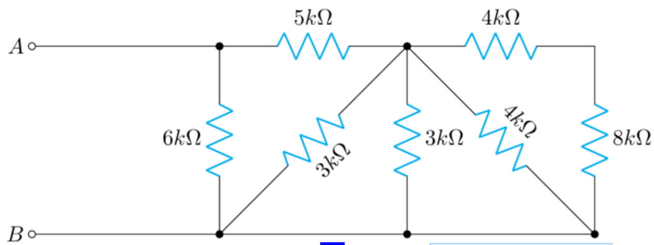
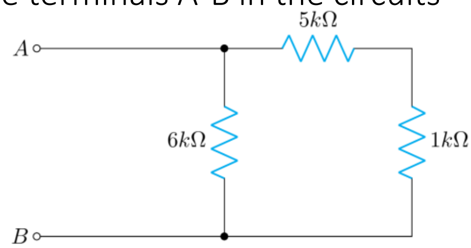
Series Parallel Resistor Combinations

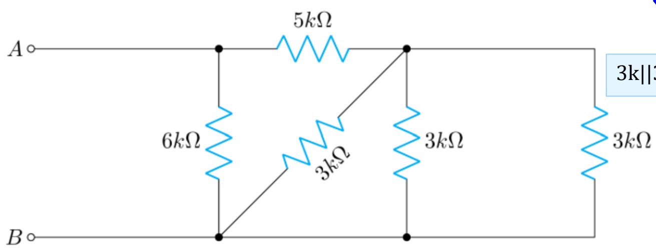
- Pro. 1: Find the equivalent resistance at the terminals A-B in the circuits



4k||(4k+8k)=3k



R_{AB} =6k||(5k+1k)=3kΩ



3k||3k||3k=1k

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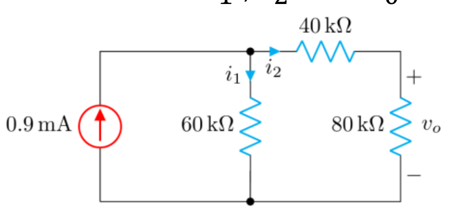
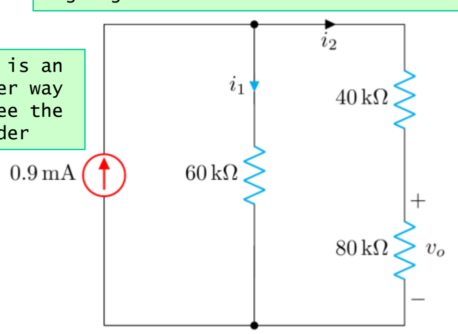
Single Node Pair Circuit

- Pro. 2: Find i_1 , i_2 and v_0



When in doubt… redraw the circuit to Highlight electrical connections!!

This is an easier way to see the divider



Applying current divider, we get

$$i_1 = \frac{40k + 80k}{(40k + 80k) + 60k} \times 0.9 \times 10^{-3}$$
$$= 0.6mA$$

$$i_2 = \frac{60k}{60k + (40k + 80k)} \times 0.9 \times 10^{-3}$$
$$= 0.3mA$$

For voltage, just use Ohm's law

$$v_o = 80k \times i_2 = 24V$$

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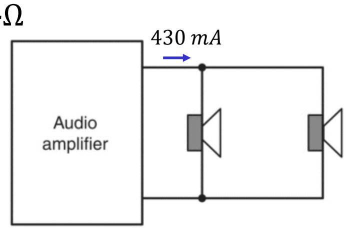
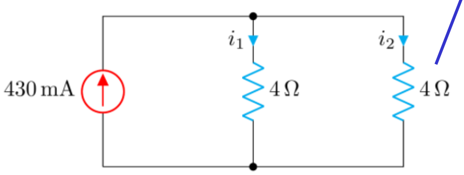
Single Node Pair Circuit

- Pro. 3:
  - Find the power absorbed by the car audio speakers which have resistance of 4Ω



The circuit for the above system is given below:



Since the resistors are equal, so current will be equally distributed (you don't need to use voltage divider, but if you did, you will find the same answer)

$$i_1 = i_2 = \frac{430}{2} = 215mA$$

Therefore, power per speaker

$$P = i^2R$$

$$P = i^2R$$
$$= (215 \times 10^{-3})^2 \times 4$$
$$= 184.9\ mW$$

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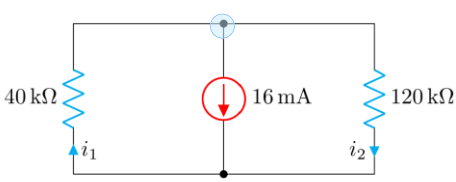
Single Node Pair Circuit

- Pro. 4:
  - Find the currents i_1 and i_2 and the power absorbed by the 40-kΩ resistor



Using current divider for i_1

$$i_1 = \frac{120}{120+40}(16)$$

$$i_1 = 12\ mA$$

Power absorbed by 40kΩ resistor

$$P_{40k} = (12mA)^2 \times 40k\Omega = 5.76W$$

There are more than one options to compute i_2. You can use current divider or apply KCL at top node

Using current divider for i_2

$$i_2 = -\frac{40}{120+40}(16) = -4mA$$

Using KCL at the top node

$$i_2 + 16m - i_1 = 0$$

$$i_2 + 16m - 12m = 0$$

$$i_2 = -4mA$$

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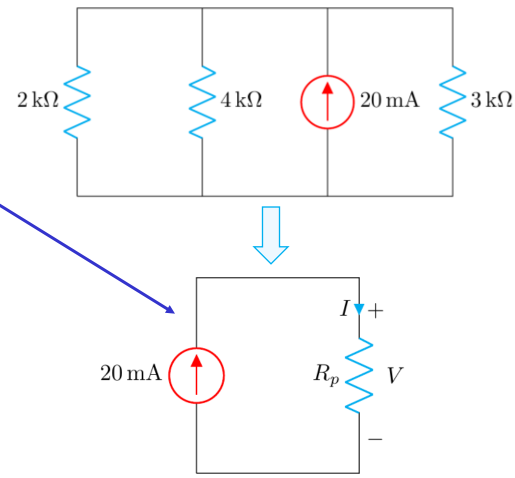
Single Node Pair Circuit

- Pro. 5: Determine power delivered by source

To find out power delivered by current source we have to use

$P = VI$

We know I which is 20mA, so we have to find voltage V across the current source (which will be across all the elements as they all are parallel)
So to find voltage across all the elements, we reduce the circuit to its equivalent circuit



$$\frac{1}{R_p} = \frac{1}{2k} + \frac{1}{4k} + \frac{1}{3k}$$
$$= \frac{6+3+4}{12k}$$
$$R_p = \frac{12}{13}k$$

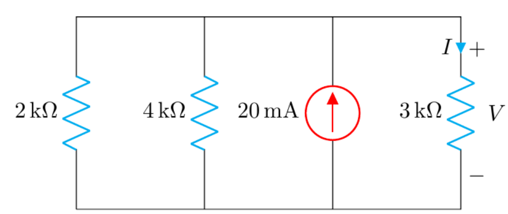
Now find V

$$V = IR$$
$$V = 20m \times \frac{12}{13}k$$
$$V = 18.46\,V$$

Single Node Pair Circuit

- Pro. 5 (cont): Determine power delivered by source

$V = 18.46\,V$ is appearing across all the elements
Since current of the current source is entering the negative terminal, we use



$P = -VI$

Therefore

$$P = -VI$$
$$P = -(18.46 \times 20m)$$
$$P = -369.2\ mW$$

Therefore, power delivered by the source is 369.2mW

Single Node Pair Circuit

- Pro. 5 (cont): Determine power delivered by source

Alternate solution:

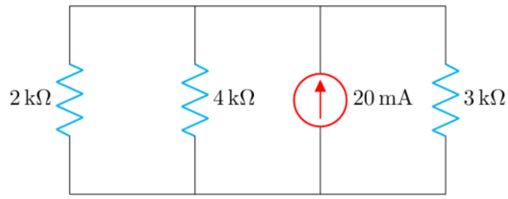
Instead of finding the power delivered directly, we can find the power absorbed by the circuit, which will be equal to power delivered by the source.

$P = \boldsymbol{I^2} \times R_p$

$P = (20mA)^2 \times \boldsymbol{R_p}$

where

$$\frac{1}{R_p} = \frac{1}{2k} + \frac{1}{4k} + \frac{1}{3k}$$
$$= \frac{6+3+4}{12k}$$
$$R_p = \frac{12}{13}k$$



$$P == (20 \times \boldsymbol{10^{-3}})^2 \times \frac{12}{13} \times 10^3$$
$$P = \frac{4.800}{13} W$$
$$P = 369.2\ mW$$

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Appendix-A

Derivation of General Case of Current Divider

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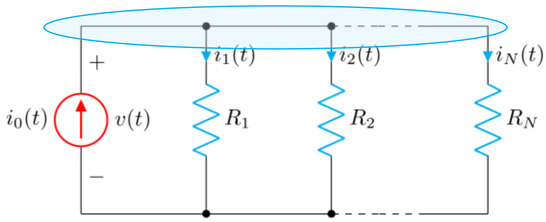
Current Divider: General Equation

– Similar to the voltage divider we can find an equation for a current divider:



– Start by applying KCL at the top node of the circuit:

$$i_o(t) = i_1(t) + i_2(t) + \ \dots + i_{N(t)} \longrightarrow \text{A}$$

– Reminder: From Ohm's Law we can rearrange to solve for current:

$$V = IR$$

$$I = \frac{V}{R}$$

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Current Divider: General Equation

– Substitute rearranged Ohm's Law into equation (A) for each current

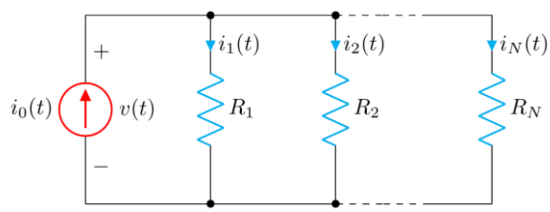
$$i_o(t) = \frac{v(t)}{R_1} + \frac{v(t)}{R_2} + \ \dots + \frac{v(t)}{R_N}$$

– Solving for i_1(t): factor out v(t) and solve for it:

$$i_o(t) = \left(\frac{1}{R_1} + \frac{1}{R_2} + \ \dots + \frac{1}{R_N}\right) v(t)$$

$$v(t) = \frac{i_0(t)}{\frac{1}{R_1} + \frac{1}{R_2} + \ \dots + \frac{1}{R_N}}$$

$$v(t) = i_0(t) \times \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ \dots + \frac{1}{R_N}}\right)$$



– Notice we can substitute R_p in:

$$v(t) = i_0(t) R_p \longrightarrow \text{B}$$

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Current Divider: General Equation

- For any specific current through a resistor we can specify it through Ohm's Law:

$$\boxed{v(t) = i_1(t)R_1} \longrightarrow \text{C}$$

- Now we equate equations (B) and (C) and solve for $i_1(t)$

$$\boxed{i_1(t)R_1 = i_0(t)R_p}$$

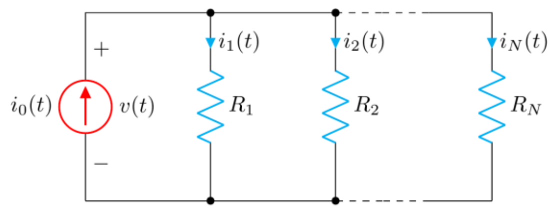
$$\boxed{i_1(t) = \frac{i_0(t)R_p}{R_1}}$$

- The same algebra can be done for any given resistor, so the general equation is:

$$\boxed{i_k(t) = \frac{i_0(t)R_p}{R_k}}$$

- Rearranging the terms

$$\boxed{i_k(t) = \frac{R_p}{R_k} i_0(t)}$$



$$\boxed{R_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}}$$

White Board