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1

ECOR1043: Circuits

Multi-Node Analysis

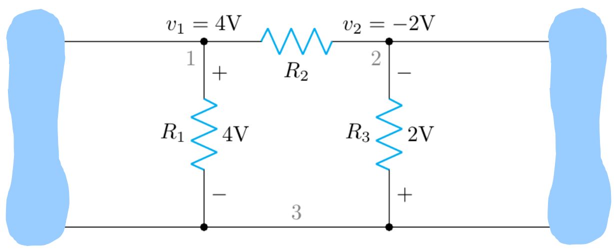
Develop systematic techniques to determine all the voltages in a circuit

3

Nodal Analysis

- A Multi-Node circuit is a circuit with more nodes than the basic node pair circuit we discussed
  - Due to the added nodes, analysis becomes more complex
  - Therefore, we introduce a new type of analysis

- Multi-Node Analysis: Nodal Analysis
  - This method uses the "Nodal" equations of Kirchhoff's Current Law as well as Ohm's Law to find the voltages around the circuit.

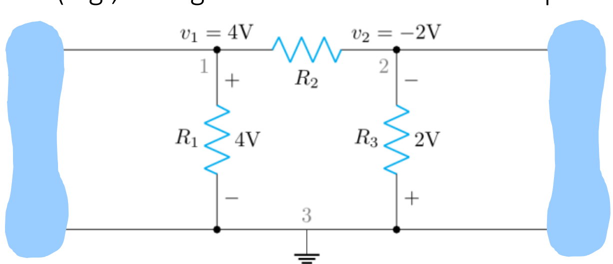


4

4

Nodal Analysis

- Before we start: Set a reference node (ground)
  - Defining the reference node is vital: The statement V_1=4V is meaningless by itself without a reference
  - Therefore, to give the measurement meaning we assume the reference is ground
  - Any node voltage (i.e., v_1, and v_2) is measured in reference to ground unless otherwise defined (e.g., voltage difference between two points)



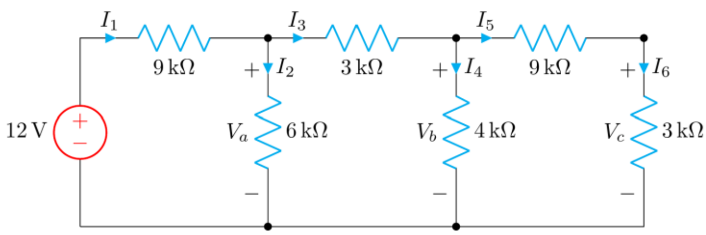
What if we want to solve for
$v_{12} =$ ____? *or* $v_{21} =$ ____?

5

5

Nodal Analysis

- Steps for Nodal Analysis:
  1) Identify all nodes and select a reference node
  2) Identify all known node voltages
  3) At each node with an unknown voltage, write a KCL equation
  4) Replace currents in terms of node voltages
  5) Solve for unknown voltages as needed
  6) Use voltages to solve for any desired values



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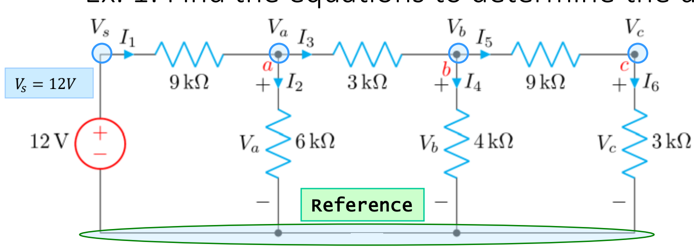
6

Nodal Analysis

- Ex. 1: Find the equations to determine the unknown node voltages



Steps:

1) Identify all nodes and select a reference node

2) Identify all known node voltages

3) At each node with an unknown voltage, write a KCL equation (For this example, assume currents entering the nodes are negative)

$@\,node\ a: \sum_{k=1}^{n} i_k = 0$

$-I_1 + I_2 + I_3 = 0$

$@\,node\ b: \sum_{k=1}^{n} i_k = 0$

$-I_3 + I_4 + I_5 = 0$

$@\,node\ c: \sum_{k=1}^{n} i_k = 0$

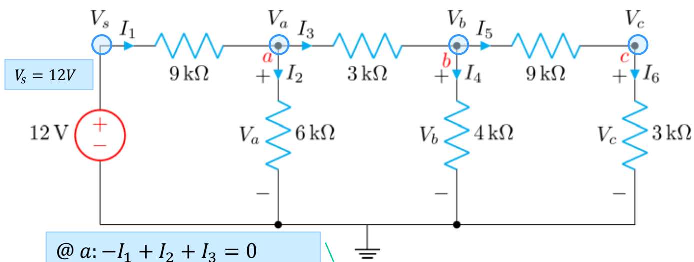
$-I_5 + I_6 = 0$

7

7

Nodal Analysis

- Ex. 1: Find the equations to determine the unknown node voltages



Steps:

1) Identify all nodes and select a reference node

2) Identify all known node voltages

3) At each node with an unknown voltage, write a KCL equation (For this example, assume currents entering the nodes are negative)

4) Replace currents in terms of node voltages (I=V/R)

$@\,a: -I_1 + I_2 + I_3 = 0$

$$-\frac{V_s - V_a}{9k} + \frac{V_a}{6k} + \frac{V_a - V_b}{3k} = 0$$

$@\,b: -I_3 + I_4 + I_5 = 0$

$$-\frac{V_a - V_b}{3k} + \frac{V_b}{4k} + \frac{V_b - V_c}{9k} = 0$$

$@\,c: -I_5 + I_6 = 0$

$$-\frac{V_b - V_c}{9k} + \frac{V_c}{3k} = 0$$

Shortcut: Skip writing these equations...

Practice writing these directly

These are the three equations to solve for three unknowns V_a, V_b, V_c

Nodal Analysis

- Ex. 2: Find the equations to determine unknown voltages V_1 and V_2

Steps:
1) Identify all nodes and select a reference node
2) Identify all known node voltages
3) Write a KCL equation for rest of the nodes

$@\,Node\ 1$: Sum of currents=0

Assuming:
- All the currents are leaving the node
- Currents leaving the node are positive

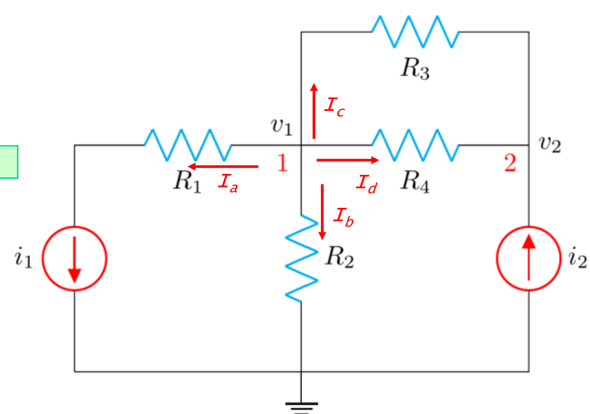
$I_a + I_b + I_c + I_d = 0$

Note: $I_a = i_1$

$i_1 + I_b + I_c + I_d = 0$

4) Replace currents in terms of node voltages

$$i_1 + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$



Nodal Analysis

- Ex. 2 (cont.): Find the equations to determine voltages V_1 and V_2

@ Node 2: Sum of currents=0

Assuming:
- All the currents are entering the node
- Currents entering the node are positive

$I_g + I_e + I_f = 0$

Note: $I_e = I_c$, $I_f = I_d$, and $I_g = i_2$

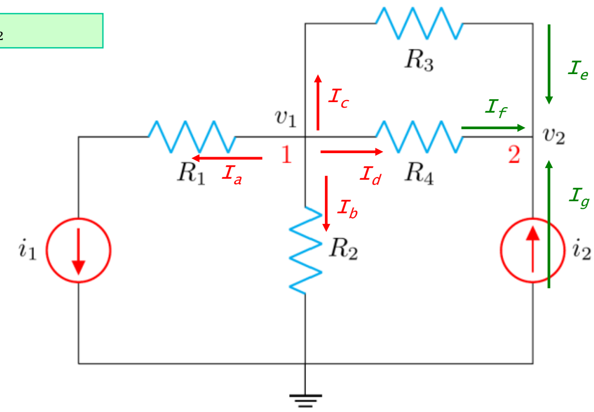
4) Replace currents in terms of node voltages

$$i_2 + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

Now we have two eq which can be solved for two unknowns (i.e. v_1 & v_2)

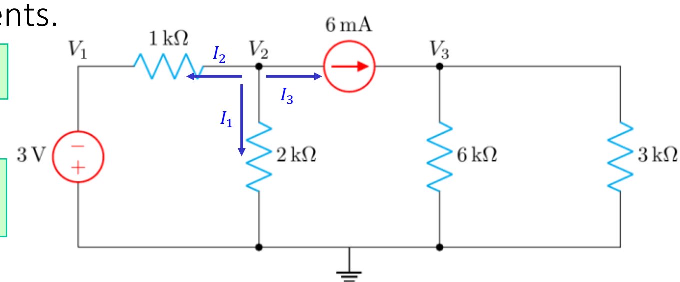
$$i_1 + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$

$$i_2 + \frac{v_1 - v_2}{R_3} + \frac{v_1 - v_2}{R_4} = 0$$



Nodal Analysis

- Ex. 3: Find all the branch currents.



One node is connected to the reference via a voltage source, so voltage is known at V_1

$V_1 = -3\,V$

Assuming:
- All currents are leaving node 2
- Currents leaving the nodes are positive

Apply KCL at node 2

$I_1 + I_2 + I_3 = 0$

$I_3 = 6mA$

Replace currents in terms of node voltages and known values and solve for V_2

$$\frac{V_2}{2k\Omega} + \frac{V_2 - V_1}{1k\Omega} + 6mA = 0$$

$V_1 = -3\,V$

$$\frac{V_2}{2k\Omega} + \frac{V_2 - (-3)}{1k\Omega} + 6mA = 0$$

$$\frac{V_2}{2k\Omega} + \frac{V_2 + 3}{1k\Omega} + 6mA = 0$$

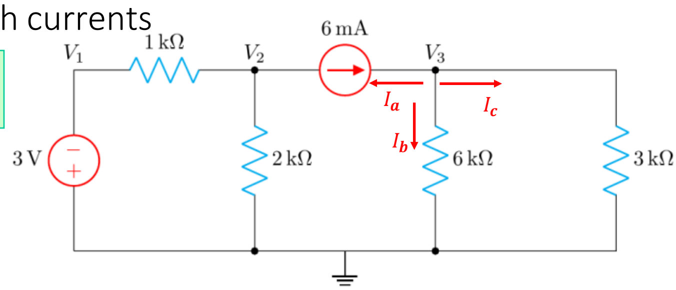
$$V_2 + 2V_2 + (2 \times 3) + 12 = 0$$

$$3V_2 + 18 = 0$$

$$V_2 = -6\,V$$

Nodal Analysis

- Ex. 3 (cont.): Find all the branch currents



Assuming:
- All currents are leaving node 3
- Currents leaving the nodes are positive

Apply KCL at node 3

$I_a + I_b + I_c = 0$

$I_a = -6mA$

$-6mA + I_b + I_c = 0$

Replace currents in terms of node voltages and known values and solve for V_3

$$-6mA + \frac{V_3}{6k} + \frac{V_3}{3k} = 0$$

$$-36 + V_3 + 2V_3 = 0$$

$$3V_3 = 36$$

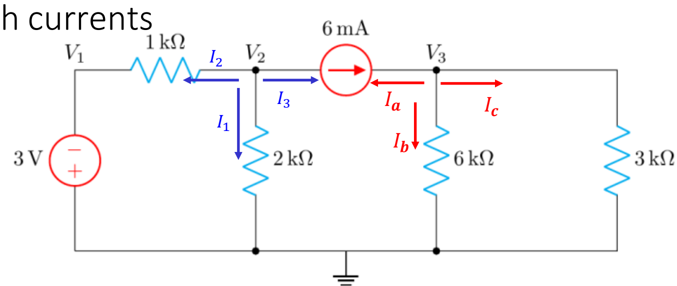
$$V_3 = \frac{36}{3} = 12V$$

12

12

Nodal Analysis

- Ex. 3 (cont.): Find all the branch currents



We now have:
$V_1 = -3V$
$V_2 = -6V$
$V_3 = 12V$

Use Ohm's Law to find I_1, I_2, I_b and I_c:

V=IR

I_1

$$I_1 = \frac{V_2}{2k\Omega}$$

$$I_1 = -\frac{6V}{2k\Omega} = -3\ mA$$

I_2

$$I_2 = \frac{V_2 - V_1}{1k\Omega}$$

$$I_2 = \frac{-6-(-3)}{1k\Omega} = -3\ mA$$

I_b

$$I_b = \frac{V_3}{6k\Omega}$$

$$I_b = \frac{12V}{6k\Omega} = 2\ mA$$

I_c

$$I_c = \frac{V_3}{3k\Omega}$$

$$I_c = \frac{12V}{3k\Omega} = 4\ mA$$

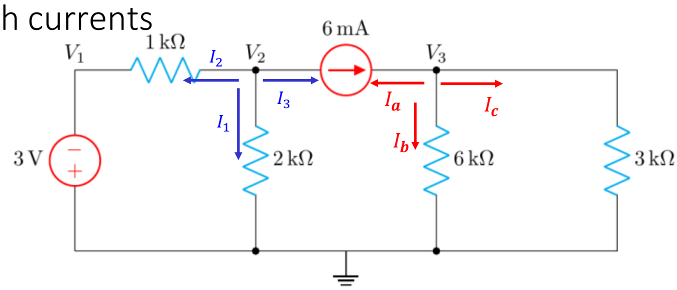
Do these values make sense?
How can you check?

13

13

Nodal Analysis

- Ex. 3 (cont.): Find all the branch currents
  - Alternate solution for I_b and I_c:



To find I_b and I_c apply current divider:

I_b

$$I_b = \frac{3k\Omega}{3k\Omega + 6k\Omega} \times 6mA = 2\ mA$$

$$I_b = 2\ mA$$

I_c

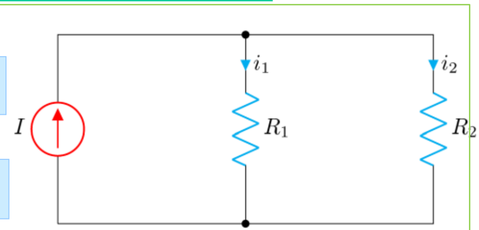
$$I_c = \frac{6k\Omega}{6k\Omega + 3k\Omega}(6mA)$$

$$I_c = 4\ mA$$

Recall current divider

$$i_1 = \frac{R_2}{R_1 + R_2} \times I$$

$$i_2 = \frac{R_1}{R_1 + R_2} \times I$$



14

14

Multi-node Analysis

- Ex. 4: Solve for V_1, V_2, and V_3

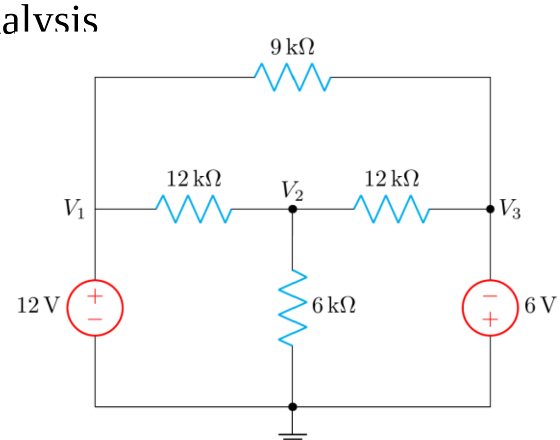
3 nodes plus the reference. In principle we need 3 equations.

Two nodes are connected to the reference through voltage sources. Hence, those node voltages are known implicitly.

$V_1 = 12\ V$

$V_3 = -6\ V$

Therefore, only one KCL equation is necessary (to V_2)



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15

Multi-node Analysis

- Ex. 4 (cont.): Solve for V_1, V_2, and V_3

$V_1 = 12\,V$ | $V_3 = -6\,V$ | $V_2 = ?\,V$

Assuming all currents are leaving, and currents leaving are positive

To find V_2, apply KCL at node V_2

$I_1 + I_2 + I_3 = 0$

Replace currents in terms of node voltages

$$\frac{V_2}{6k\Omega} + \frac{V_2 - V_3}{12k\Omega} + \frac{V_2 - V_1}{12k\Omega} = 0$$

Solve for V_2 (the only unknown)

$$\left(\frac{V_2}{6k\Omega} + \frac{V_2 - V_3}{12k\Omega} + \frac{V_2 - V_1}{12k\Omega}\right) \times 12k = 0 \times 12k$$

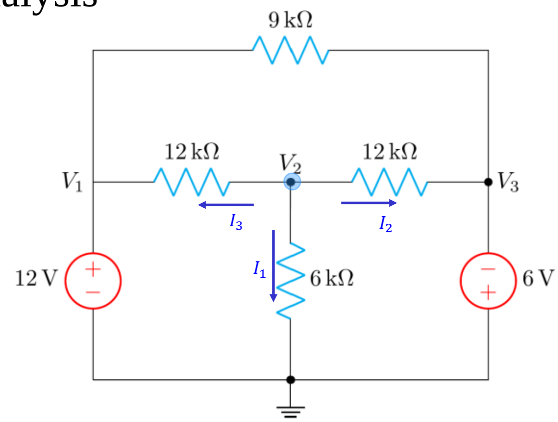
$$2V_2 + V_2 - V_3 + V_2 - V_1 = 0$$

$$4V_2 - V_3 - V_1 = 0$$

$$4V_2 - (-6) - 12 = 0$$

$$4V_2 + 6 - 12 = 0$$

$$V_2 = \frac{6}{4} = 1.5V$$



16

16

Practice Problems

17

17

Multi-node Analysis

- Prob. 1: Calculate the node voltages

Apply KCL and Ohm's law at node 1

$i_1 = i_2 + i_3$

$5 = (v_1 - v_2)/4 + (v_1 - 0)/2$

$20 = v_1 - v_2 + 2v_1$

$3v_1 - v_2 = 20 \longrightarrow A$

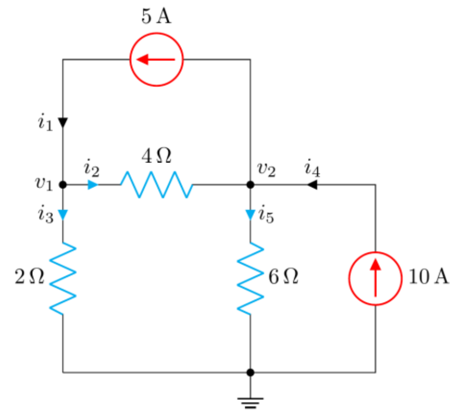
Apply KCL and Ohm's law at node 2

$i_2 + i_4 = i_1 + i_5$

$(v_1 - v_2)/4 + 10 = 5 + (v_2 - 0)/6$

$3v_1 - 3v_2 + 120 = 60 + 2v_2$

$3v_1 - 5v_2 = -60 \longrightarrow B$



Multi-node Analysis

- Prob. 1(cont.): Calculate the node voltages

We now have two equations and two unknowns

$3v_1 - v_2 = 20 \longrightarrow A$

$3v_1 - 5v_2 = -60 \longrightarrow B$

Subtract B from A

$4v_2 = 80$

$v_2 = 20V$

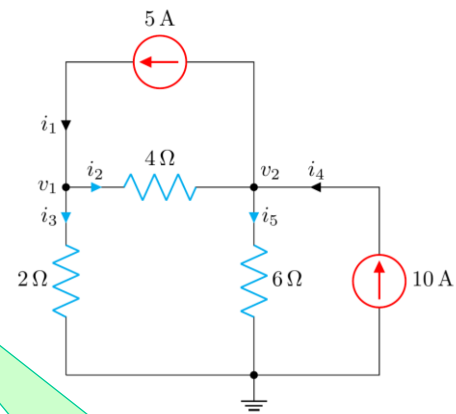
Substitute v_1 back in equation A

$3v_1 - v_2 = 20 \longrightarrow A$

$3v_1 - 20 = 20$

$3v_1 = 40$

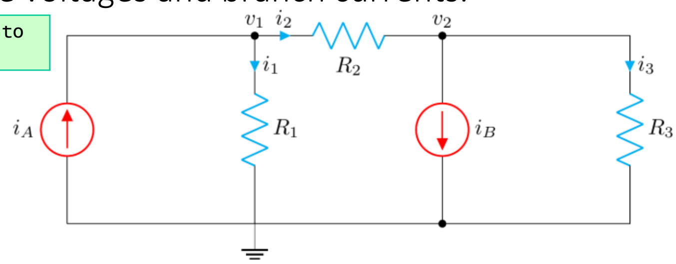
$v_1 = \frac{40}{3} = 13.33V$



You can use any method that you know to solve the two equations for two unknown voltages.
I am using method of elimination

Multi-node Analysis

- Prob. 2: Given: $i_A = 1mA$, $R_1 = 12k\Omega$, $R_2 = 6k\Omega$, $i_B = 4mA$ and $R_3 = 6k\Omega$. Determine all node voltages and branch currents.



Apply KCL at node 1 (assuming current into the node are $-ve$)

$$-i_A + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0$$

$$-1m + \frac{v_1}{12k} + \frac{v_1}{6k} - \frac{v_2}{6k} = 0$$

$$\frac{v_1}{4k} - \frac{v_2}{6k} = 1 \times 10^{-3} \longrightarrow \text{A}$$

Apply KCL at node 2 (assuming current into the node are $+ve$)

$$-i_B - \frac{v_2}{R_3} + \frac{v_1 - v_2}{R_2} = 0$$

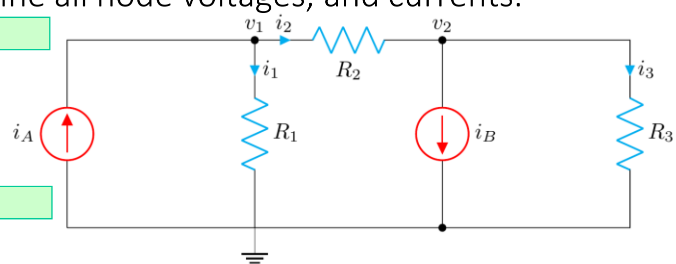
$$-4m - \frac{v_2}{6k} + \frac{v_1}{6k} - \frac{v_2}{6k} = 0$$

$$\frac{v_1}{6k} - \frac{v_2}{3k} = 4 \times 10^{-3} \longrightarrow \text{B}$$

Now we have two equations (A,B) and you can use any method that you know to solve the two equations for two unknown voltages.
I am using method of substitution on next page

Multi-node Analysis

- Prob. 2(cont.): Given: $i_A = 1mA$, $R_1 = 12k\Omega$, $R_2 = 6k\Omega$, $i_B = 4mA$ and $R_3 = 6k\Omega$. Determine all node voltages, and currents.



From A, get v_1

$$\frac{v_1}{4k} = \frac{v_2}{6k} + 1 \times 10^{-3}$$

$$v_1 = \frac{2v_2}{3} + 4 \longrightarrow \text{A'}$$

Substitute v_1 in B

$$\frac{1}{6k} \times \left(\frac{2v_2}{3} + 4\right) - \frac{v_2}{3k} = 4 \times 10^{-3}$$

$$\frac{2v_2}{3} + 4 - 2v_2 = 24$$

$$2v_2 + 12 - 6v_2 = 72$$

$$\boxed{v_2 = -15V}$$

Substitute v_2 in A'

$$v_1 = \frac{2 \times -15}{3} + 4 \qquad \boxed{v_1 = -6V}$$

Multi-node Analysis

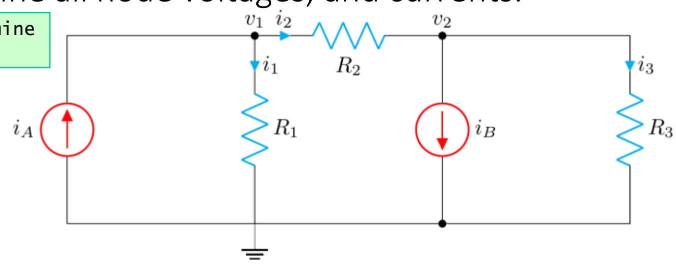
- Prob. 2(cont.): Given: $i_A = 1mA$, $R_1 = 12k\Omega$, $R_2 = 6k\Omega$, $i_B = 4mA$ and $R_3 = 6k\Omega$. Determine all node voltages, and currents.

Knowing the node voltages, we can determine all the currents using Ohm's law

$$i_1 = \frac{v_1}{R_1} = -\frac{6}{12k} = -0.5mA$$

$$i_2 = \frac{v_1 - v_2}{R_2} = \frac{-6+15}{6k} = \frac{9}{6k} = 1.5mA$$

$$i_3 = \frac{v_2}{R_3} = -\frac{15}{6k} = -2.5mA$$



22

22

White Board

23

23