Full resolution photo on my Instagram @feenafoto

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ECOR1043: Circuits

Multi-loop Analysis

Develop systematic techniques to determine all the currents in a circuit

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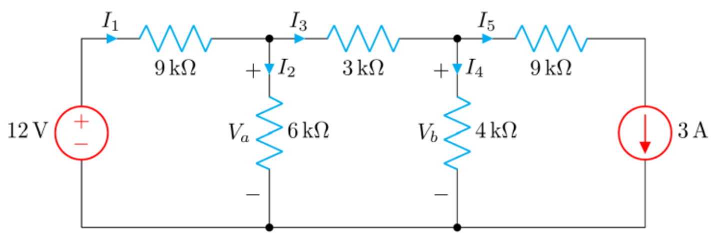
Multi-Loop Analysis

- A Multi-Loop circuit is a circuit with more loops than the basic one loop circuit we discussed
  - Due to the added loops, analysis becomes more complex
  - Therefore, we introduce a new type of analysis

- Multi-Loop Analysis: Mesh Analysis
  - This method uses the "Loop" equations of Kirchhoff's Voltage Law as well as Ohm's Law to find the various currents around the circuit.



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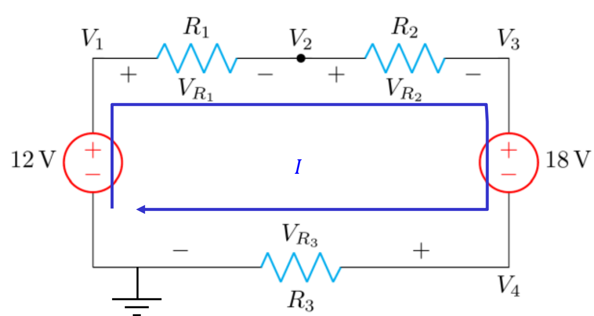
Nodal or Loop Analysis?

- What if we try to apply nodal analysis to this circuit?
  - There are 4 non-reference nodes
  - There is one node connected to the reference through a voltage source
  - We know that $V_1 = 12V$, so we need three equations to computer all node voltages
  - But there is only one current flowing through all components and if that current is determined all voltages can be computed with ohm's law
- So, we base our analysis on KVL instead



When a voltage source is connected to two non-reference nodes, nodal analysis is more involved

When a current source shared by two meshes, mesh analysis is more involved

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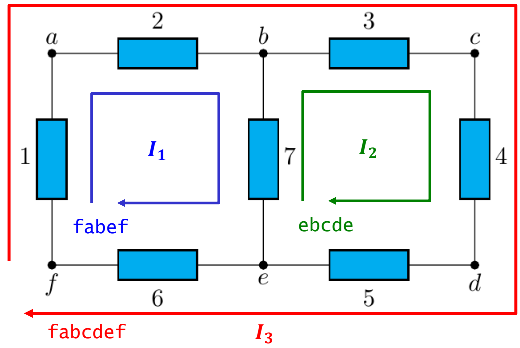
Definitions and Terms

- Definitions:
  - Loop: A closed path that does not go over any node twice
    - This circuit has three loops (red, blue , green)
  - Mesh: A loop that does not enclose any other loop. [fabef] and [ebcde] are meshes.
  - Loop Current: A (fictitious) current that is assumed to flow around a loop. I_1, I_2, and I_3 are loop currents.
  - Mesh Current: A loop current associated with a mesh. I_1, I_2 are mesh currents, I_3 is not
  - We can calculate currents through various component by using loop currents



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Multi-Loop Analysis

- In a circuit, the current through any component can be expressed in terms of the loop (fictitious) currents.
- Ex. 1: Express the currents I_{af}, I_{be}, and I_{bc} in terms of I_1, I_2, and I_3

I_{af}

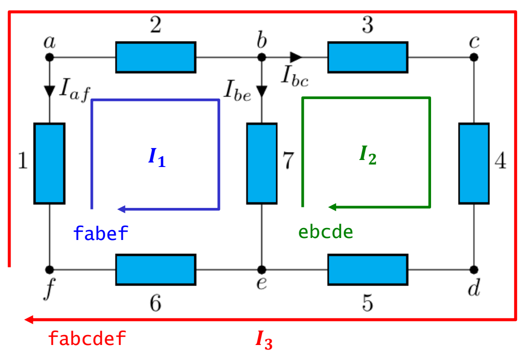
$I_{af} = -I_1 - I_3$

I_{be}

$I_{be} = I_1 - I_2$

I_{bc}

$I_{bc} = I_2 + I_3$



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Multi-Loop Analysis

- Just because a loop exists does not mean it is independent
  - In last slide, one of the loops is dependent
- Ex. 2: Express the currents I_{af}, I_{be}, and I_{bc} in terms of I_1, and I_3 only

I_{af}

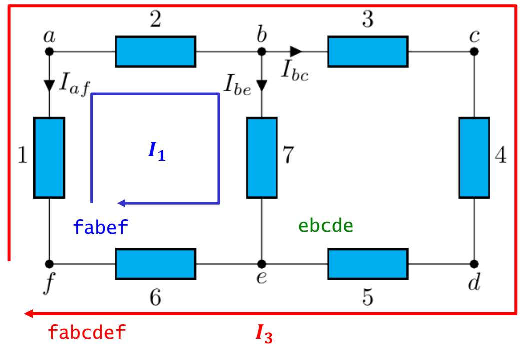
$I_{af} = -I_1 - I_3$

I_{be}

$I_{be} = I_1$

I_{bc}

$I_{bc} = I_3$



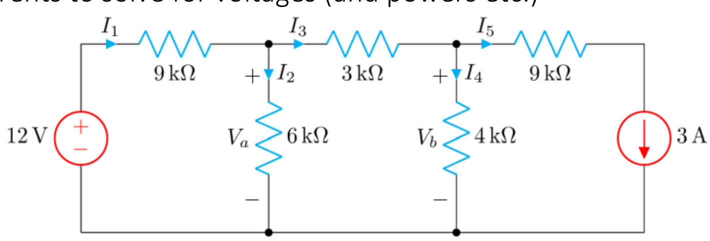
For every circuit there is a minimum number of loop currents that are necessary to compute every current in the circuit.

This is equal to the number of independent loops in that circuit:
(l=b-n+1)

Multi-Loop Analysis

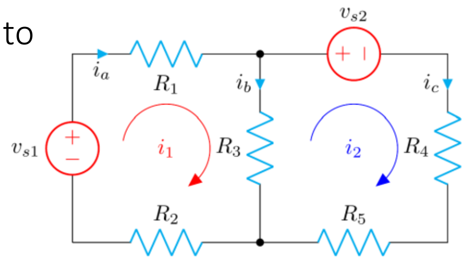
- Steps for Mesh Analysis:

0) Determine how many mesh/loops do we need
1) Identify all mesh's and assign mesh currents
2) Identify all known currents
3) For each loop with an unknown current, write a KVL equation
4) Replace voltages in terms of mesh currents
5) Solve for mesh currents
6) Solve for unknown currents as needed
7) Use currents to solve for voltages (and powers etc.)



Multi-Loop Analysis

- Ex. 3: Find the equations that can be used to solve for element currents i_a, i_b and i_c



0. Let's see how many independent loop/mesh we need.

$B = 7$
$N = 6$
$L = B - (N - 1)$
=7-(6-1)=2

1. Identify all mesh's and assign mesh currents

2. Identify all known currents

There is none ☹

3. For each loop with an unknown current, write a KVL equation

KVL on left mesh (Loop 1)

$v_1 + v_3 + v_2 - v_{s1} = 0$

KVL on right mesh (Loop 2)

$v_{s2} + v_4 + v_5 + v_3' = 0$

You may skip this step and directly go to next step

4. Replace voltages in terms of mesh currents

$i_1R_1 + (i_1 - i_2)R_3 + i_1R_2 - v_{s1} = 0$

$i_1R_1 + i_1R_3 - i_2R_3 + i_1R_2 - v_{s1} = 0$

$v_{s2} + i_2R_4 + i_2R_5 + (i_2 - i_1)\,R_3 = 0$

$v_{s2} + i_2R_4 + i_2R_5 + i_2R_3 - i_1R_3 = 0$

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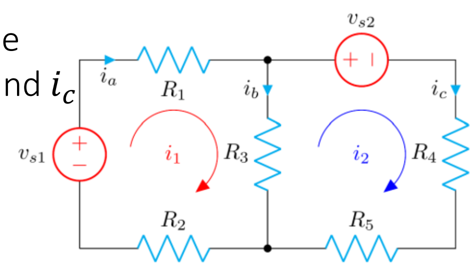
Multi-Loop Analysis

- Ex. 3(cont.): Find the equations that can be used to solve for element currents i_a, i_b and i_c



$i_1(R_1 + R_2 + R_3) - i_2R_3 = v_{s1}$

$-i_1R_3 + i_2(R_3 + R_4 + R_5) = -v_{s2}$

Two unknowns (i_1, i_2) and two equations, which we can solve using various methods

Step (cont.):
5. Solve for mesh currents

i_1, i_2

6. Solve for unknown currents as needed

Once mesh currents i_1 and i_2 are known, we can find the desired currents

$i_a = i_1$, $i_b = i_1 - i_2$ *and* $i_c = i_2$

7. Use currents to solve for voltage (and powers)

There is even a more direct way ☺

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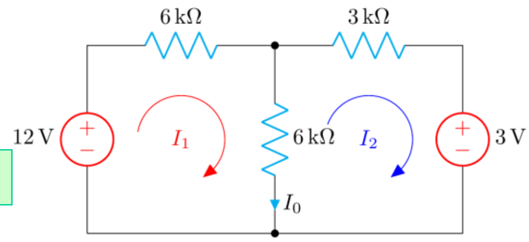
Multi-Loop Analysis

- Ex 4: Find I_o

1. First, assign mesh current I_1 and I_2

2. Identify all known currents   There is none ☹

3. For each loop with an unknown current, write a KVL directly replacing voltages in terms of mesh currents



1) KVL for loop 1:

$$-12V + 6k\Omega(I_1) + 6k\Omega(I_1 - I_2) = 0$$ Loop 1

2) KVL for loop 2:

$$6k\Omega(I_2 - I_1) + 3k\Omega(I_2) + 3V = 0$$ Loop 2

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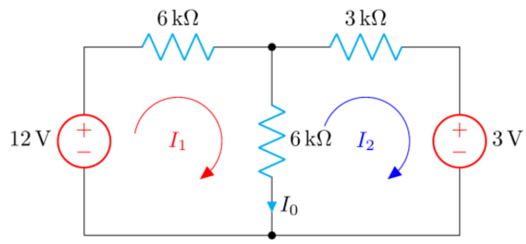
Multi-Loop Analysis

- Ex. 4 (cont.): Find I_o



Simplify KVL for loop 1

$$-12V + 6k\Omega(I_1) + 6k\Omega(I_1 - I_2) = 0$$

$$-12V + 6k\Omega(I_1) + 6k\Omega I_1 - 6k\Omega I_2 = 0$$

$$12k\Omega I_1 - 6k\Omega I_2 = 12V$$ A

Simplify KVL for loop 2

$$6k\Omega(I_2 - I_1) + 3k\Omega(I_2) + 3V = 0$$

$$6k\Omega I_2 - 6k\Omega I_1 + 3k\Omega(I_2) + 3V = 0$$

$$-6k\Omega I_1 + 9k\Omega I_2 = -3V$$ B

Let's try them in a more direct way

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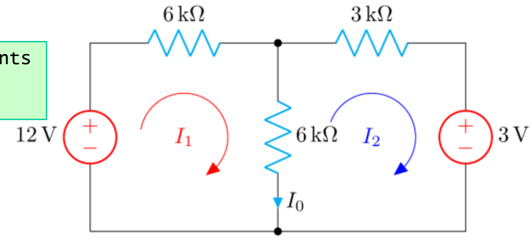
Multi-Loop Analysis

- Ex. 4 (cont.): Find I_o



$(12k\Omega)I_1 - (6k\Omega)I_2 = 12V$ A

$(-6k\Omega)I_1 + (9k\Omega)I_2 = -3V$ B

5. Solve for mesh currents (by using process of Elimination)

Multiply eq. B with 2 and add A and B

$(12k\Omega)I_1 - (6k\Omega)I_2 = 12V$ A

$(-12k\Omega)I_1 + (18k\Omega)I_2 = -6V$ B'

$12kI_2 = 6V$ Add A and B'

$I_2 = 0.5mA$

Substitute I_2 back in B

$(-6k\Omega)I_1 + (9k\Omega) \times 0.5m = -3V$

$(-6k\Omega)I_1 = -3V - 4.5V$

$$I_1 = \frac{-3V - 4.5V}{(-6k\Omega)}$$

$I_1 = 1.25mA$

So, mesh currents are

$I_1 = 1.25mA$ $I_2 = 0.5mA$

6. Solve for unknown currents as needed

We know I_1 and I_2 but we want to find I_0

$I_o = I_1 - I_2$

$I_o = 1.25mA - 0.5mA$

$I_o = 0.75mA$

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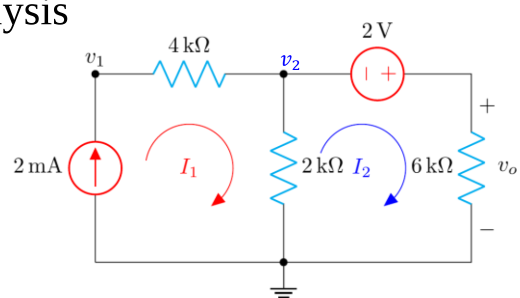
Multi-Loop Analysis

- Ex.5: Find both v_o and v_2



For v_o

1. First, assign mesh current I_1 and I_2

2. Identify all known currents

Current sources that are not shared by other meshes (or loops) serve to define a mesh (loop) current and reduce the number of required equations

Mesh 1 eq ← Mesh 1 current is constrained

$I_1 = 2mA$

Mesh 2 equation

$6kI_2 + 2k(I_2 - I_1) - 2 = 0$

$6kI_2 + 2kI_2 - 2kI_1 - 2 = 0$

$8kI_2 - 2kI_1 = 2$

Substitute I_1

$8kI_2 - 2k \times 2m = 2$

$8kI_2 - 4 = 2$

$8kI_2 = 6$

$$I_2 = \frac{6}{8k} = 0.75mA$$

Now we can find v_o

$v_o = 6k \times I_2 = 6k \times 0.75m = 4.5V$

Now we will find v_2

$v_2 = (I_1 - I_2) \times 2k$

$v_2 = (2m - 0.75m) \times 2k$

$v_2 = 1.25m \times 2k$

$v_2 = 2.5V$

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Practice Problems

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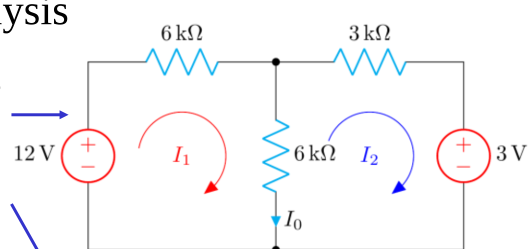
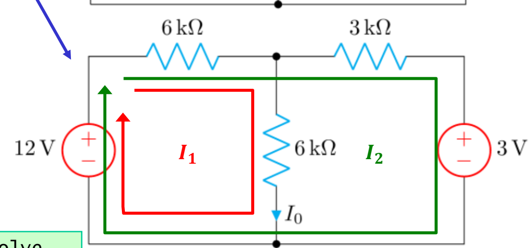
Multi-Loop Analysis

- Prob 1: Ex. 4 Alternative Solution: Find I_o

Earlier (ex.4) we used mesh currents as shown in top right figure

Alternatively, we can use different mesh/loop currents as shown in bottom right figure and get same answer





KVL @ I_1 $\quad -12 + 6k(I_1 + I_2) + 6kI_1 = 0$ $\quad$ A

KVL @ I_2 $\quad -12 + 6k(I_1 + I_2) + 3kI_2 + 3 = 0$ $\quad$ B

NOW $I_O = I_1$

This selection is better as we can find I_o directly

Rearrange

$12kI_1 + 6kI_2 = 12$ $\quad$ A'

$6kI_1 + 9kI_2 = 9$ $\quad$ B'

Use Process of Elimination to solve Multiply B' by 2 and subtract from A'

$-12kI_2 = -6 \Rightarrow I_2 = 0.5mA$

$I_1 = \frac{9-9k\times0.5m}{6k} \Rightarrow I_1 = 0.75mA$

Therefore

$I_o = I_1 = 0.75mA$ which is what we got before.

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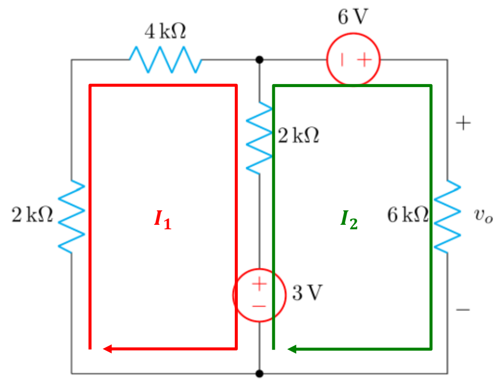
Multi-Loop Analysis

- Prob 2: Use mesh equations to find v_o



1. DRAW THE MESH CURRENTS

2. WRITE MESH EQUATIONS

MESH 1

$(2k + 4k + 2k)I_1 - 2kI_2 = -3[V]$

$8kI_1 - 2kI_2 = -3$

$8I_1 - 2I_2 = -3m$ A

MESH 2

$-2kI_1 + (2k + 6k)I_2 = (6V + 3V)$

$-2kI_1 + 8kI_2 = 9$

$-2I_1 + 8I_2 = 9m$ B

Multiply B by 4 and add to A

$30I_2 = 33m$

$I_2 = \frac{33}{30} mA$

Once I_2 is known we can find v_o

$v = 6k \times I_2 = \frac{33}{5} V$

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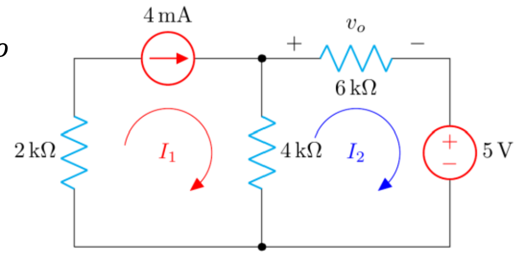
Multi-Loop Analysis

- Prob 3: Using multi-loop method, find v_o



MESH 1:

$I_1 = 4mA$

We actually need the current on the right mesh. Hence, use mesh analysis.

MESH 2:

$4k(I_2 - I_1) + 6kI_2 + 5 = 0$

$10kI_2 - 4kI_1 = -5$

$I_2 = \frac{-5 + 4k \times 4m}{10k}$

$I_2 = \frac{11}{10} mA$

After we know I_2 we can find out V_o

$V_O = 6kI_2 = 6k \times \frac{11}{10} m$

$V_o = 6.6V$

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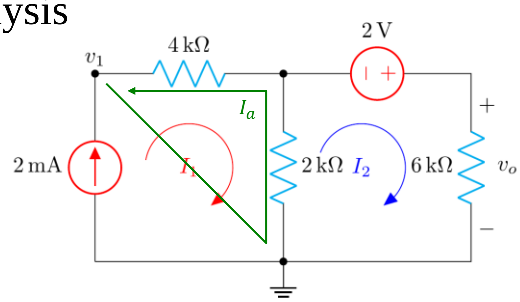
Multi-Loop Analysis

- Prob 4: In the circuit of Ex.5, find v_1:



For v_1

v_1 is basically the voltage between point a to reference ground (as there is no second subscript so we assume it to be ground)

To obtain v1, apply KVL to any closed path that includes v1. We choose green loop path, therefore

$v_1 + 2k(I_2 - I_1) + 4k \times (-I_1) = 0$

$I_1 = 2mA$ $\quad$ $I_2 = 0.75mA$

$v_1 + 2k(I_2 - I_1) + 4k \times (-2m) = 0$

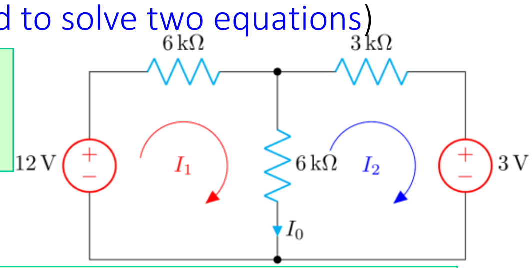
$v_1 + 2k(0.75m - 2m) - 8 = 0$

$v_1 - 2.5 - 8 = 0$

$v_1 = 10.5V$

Multi-Loop Analysis

- Prob. 5: Ex. 4 Find I_o (Alternate method to solve two equations)



$(12k\Omega)I_1 - (6k\Omega)I_2 = 12V$ A

$(-6k\Omega)I_1 + (9k\Omega)I_2 = -3V$ B

Solve them by process of Substitution (instead of Elimination as done before in Ex. 4

From A, isolate I_1

$$I_1 = \frac{12V + (6k\Omega)I_2}{(12k\Omega)}$$ A'

Substitute in B

$$(-6k\Omega) \times \frac{12V + (6k\Omega)I_2}{(12k\Omega)} + (9k\Omega)I_2 = -3V$$

$$\frac{-12V - (6k\Omega)I_2}{(2k\Omega)} + (9k\Omega)I_2 = -3V$$

$$-12V - (6k\Omega)I_2 + (18k\Omega)I_2 = -6V$$

$$(12k\Omega)I_2 = -6V + 12V$$

$$I_2 = \frac{6V}{12k\Omega}$$

$$I_2 = 0.5mA$$

Substitute I_2 in A' and solve for I_1

$$I_1 = \frac{12V + (6k\Omega)I_2}{(12k\Omega)}$$

$$I_1 = 1.25mA$$

We know I_1 and I_2 but we want to find I_0

$$I_o = I_1 - I_2$$

$$I_o = 1.25mA - 0.5mA$$

$$I_o = 0.75mA$$

White Board

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