Full resolution photo on my Instagram @feenafoto

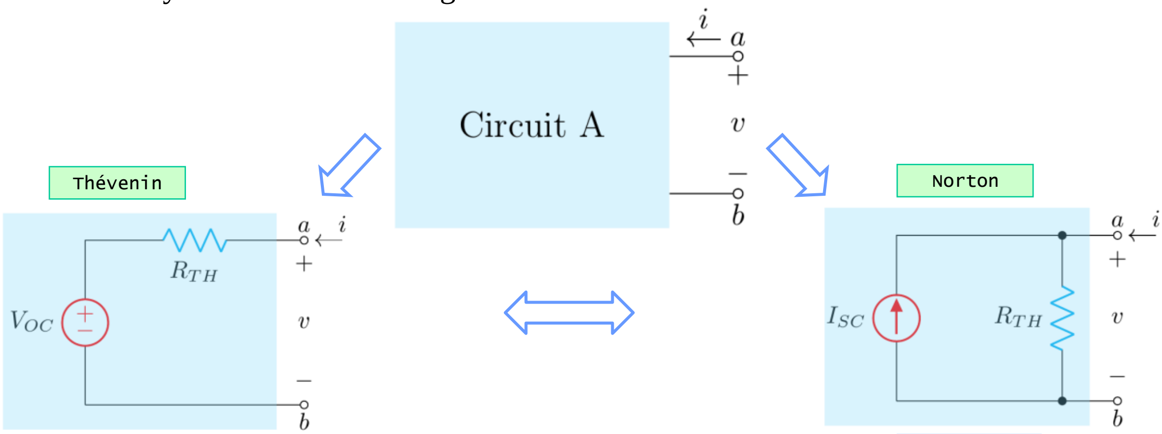
ECOR1043: Circuits

Additional Analysis Techniques

Source Transformation & Superposition

Source Transformation

- The Thévenin and Norton equivalent circuits both represent the same circuit
  - They have the same voltage-current characteristics



$v = i\,R_{TH} + V_{oc}$

$i = \dfrac{v}{R_{TH}} - i_{sc}$

Source Transformation

- We can equate the two representations $v = i\,R_{TH} + V_{oc}$ $\quad i = \dfrac{v}{R_{TH}} - i_{sc}$
- Solving for i from the Thévenin equivalent

$v = i\,R_{TH} + V_{oc}$ Thevenin

$$i = \frac{v}{R_{TH}} - \frac{V_{oc}}{R_{TH}}$$

- Substituting this i in the Norton equivalent equation

$i = \dfrac{v}{R_{TH}} - i_{sc}$ Norton

$$\frac{v}{R_{TH}} - \frac{V_{oc}}{R_{TH}} = \frac{v}{R_{TH}} - i_{sc}$$

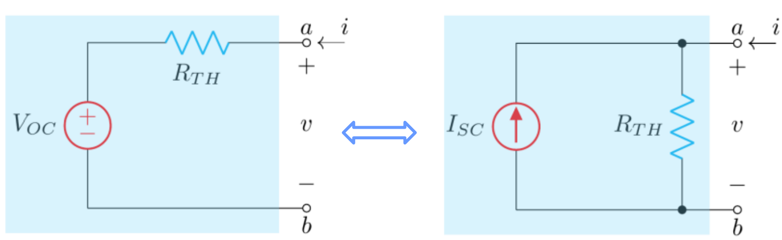
$$v - V_{OC} = v - i_{SC}R_{TH}$$

- Solving for V_{OC}:

$V_{oc} = i_{SC}R_{TH}$

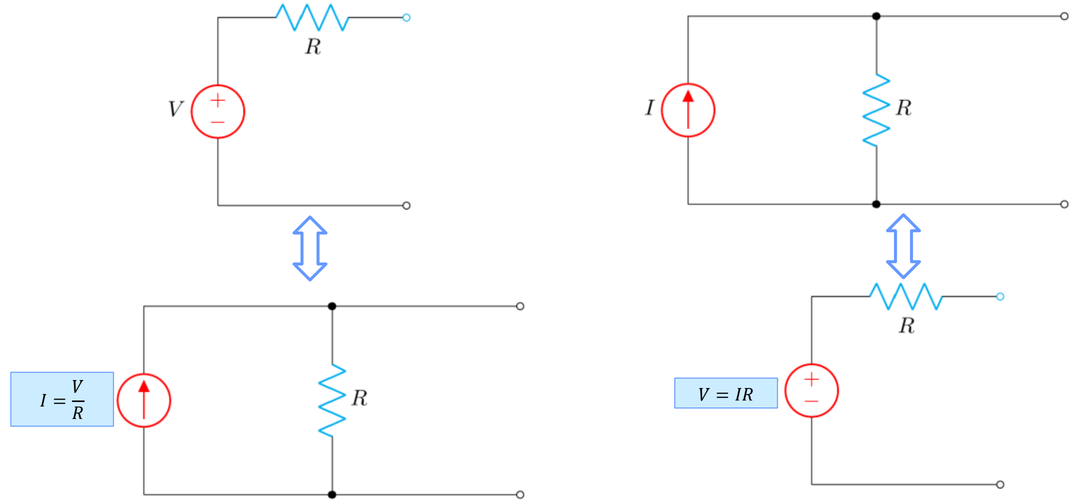
What this result means?

Does it remind you of anything?



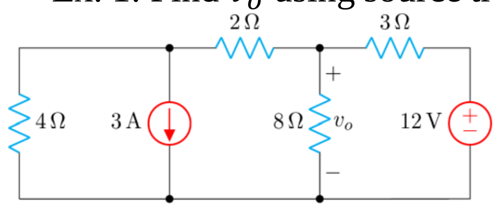
Source Transformation

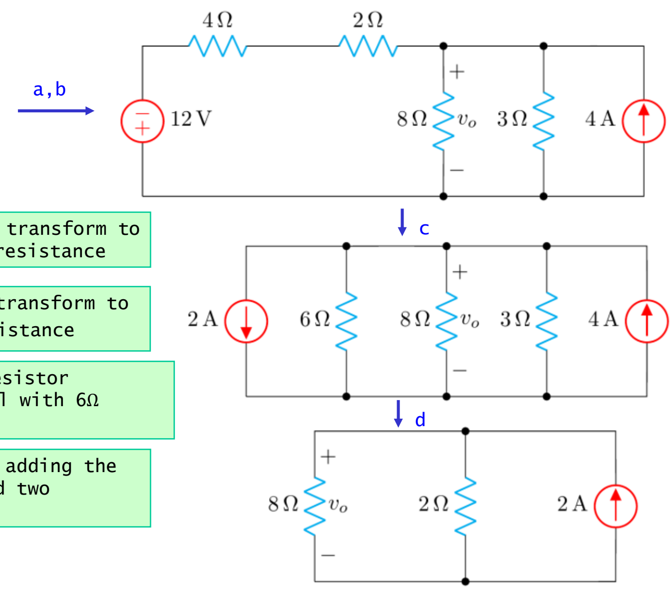
- Any voltage source in series with a resistance can be modeled as a current source in parallel with the same resistance and vice-versa



Source Transformation for Circuit Analysis

- Ex. 1: Find v_o using source transformation





a) 3A source in parallel with 4Ω resistance transform to 12V ($v = 3 \times 4 = 12V$) source in series with 4Ω resistance

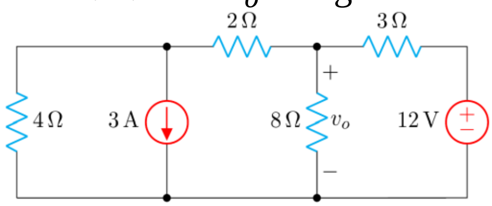
b) 12V source in series with 3Ω resistance transform to 4A ($i = \frac{12}{3} = 4A$) source in parallel with 3Ω resistance

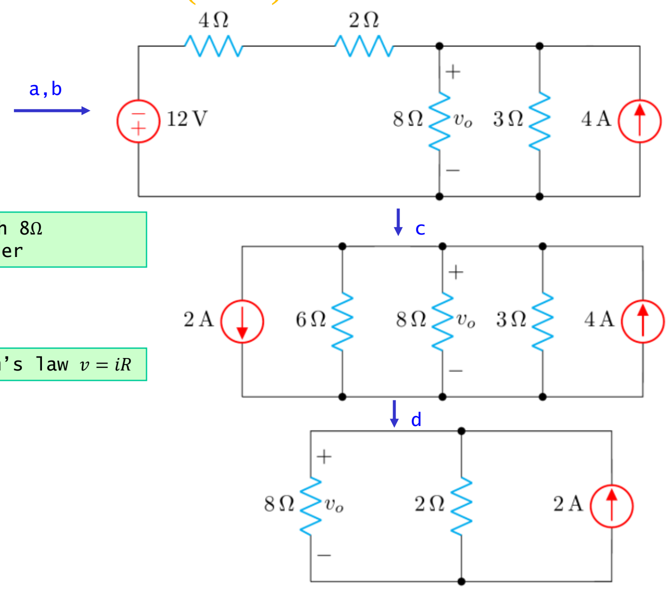
c) 12V source in series with ($4\Omega + 2\Omega = 6\Omega$) resistor transforms to 2A source ($i = 12/6$) in parallel with 6Ω resistor

d) Now we can make an equivalent circuit by adding the two parallel current sources (4A and 2A) and two parallel resisters 3Ω and 6Ω.

Source Transformation for Circuit Analysis

- Ex. 1: Find v_o using source transformation (cont.)





e) Now we can solve for the current i through 8Ω resistor using special case of current divider

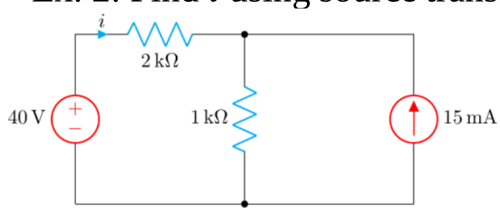
$$i = \frac{2}{8+2} \times 2 = 0.4A$$

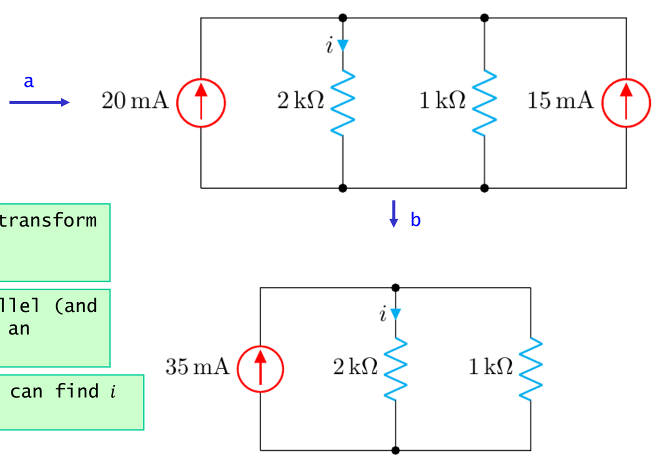
f) Thereafter, we can find voltage using Ohm's law $v = iR$

$$v_0 = 0.4 \times 8 = 3.2V$$

Source Transformation-Incorrect Application

- Ex. 2: Find i using source transformation





a) 40V source in series with 2kΩ resistance transform to 20mA current source in parallel with 2kΩ resistance $(I = 40/2k = 20mA)$

b) 20mA and 15mA current sources are in parallel (and same direction) so they can be added to form an equivalent current source of 35mA

c) Now we have a single node circuit, and we can find i using current divider

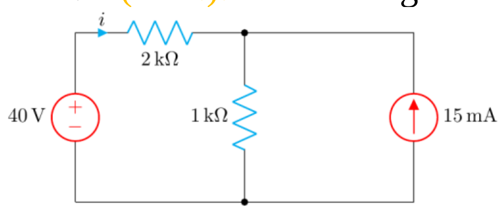
$$i = \frac{1k}{1k + 2k} \times 35m = 11.67mA$$

It seems ok but it is not correct. Use mesh analysis to find i and see if it matches with the above value

This is the result of transforming the component for which we needed to find the current

Source Transformation-Correct Application

- Ex. 2 (cont.): Find i using source transformation



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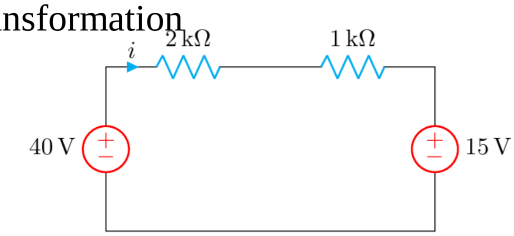


a) 15mA current source in parallel with 1kΩ resistor transform to 15V voltage source in series with 2kΩ resistance $V = IR$

b) Now can have a single loop circuits and we can find current i using KVL

$-40 + i \times 2k + i \times 1k + 15 = 0$

$-25 + i \times 3k = 0$

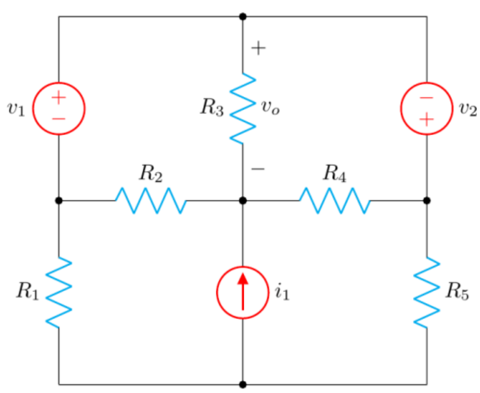
$i = 25/3k$

$i = 8.33mA$

This is the correct solution.

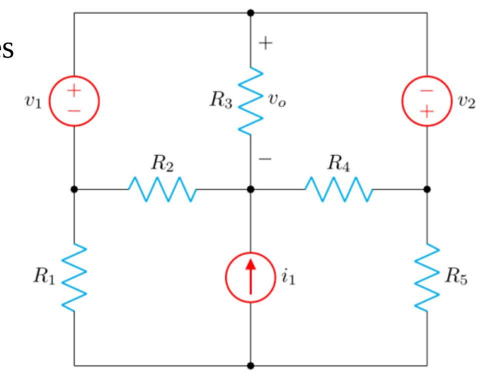
Superposition

- If a linear circuit has multiple inputs (sources), we can determine the response (the current or the voltage at any point) to each input individually and sum the responses to get the net response.



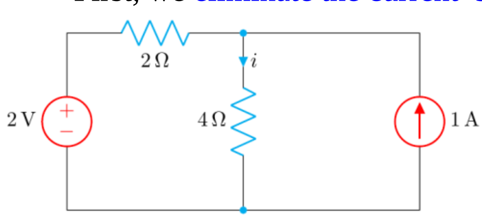
Superposition

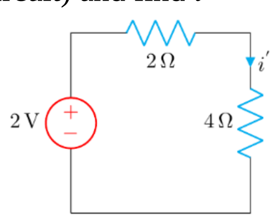
- Analyzing a circuit containing multiple sources using superposition:
  - Determine the output response to each source
    - Eliminate all other sources (short-circuit voltage sources, open-circuit current sources)
    - Analyze resulting circuit (using the techniques we have learned) to determine response to the one remaining source
  - Repeat the process for each source
  - Sum contributions of responses from all sources



Superposition

- Ex. 3: Determine the current *i* in the circuit using superposition
  - First, we eliminate the current source (open-circuit) and find i'





Solve for i' using a simple mesh

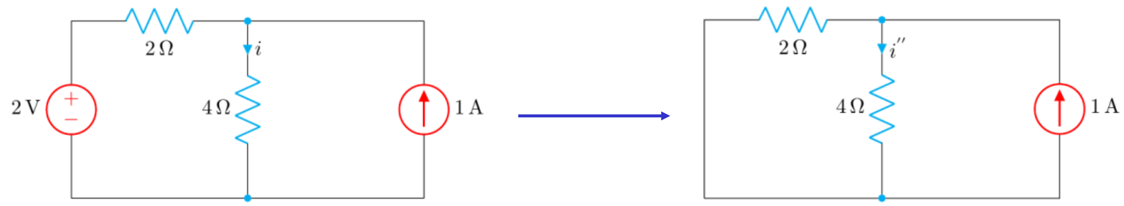
$$-2 + 2i' + 4i' = 0$$

$$-2 + 6i' = 0$$

$$i' = \frac{2}{6} = \frac{1}{3} A$$

Superposition

- Ex. 3 (cont.): Determine the current i in the circuit using superposition
  - Now, we eliminate the voltage source (short-circuit) and find i''

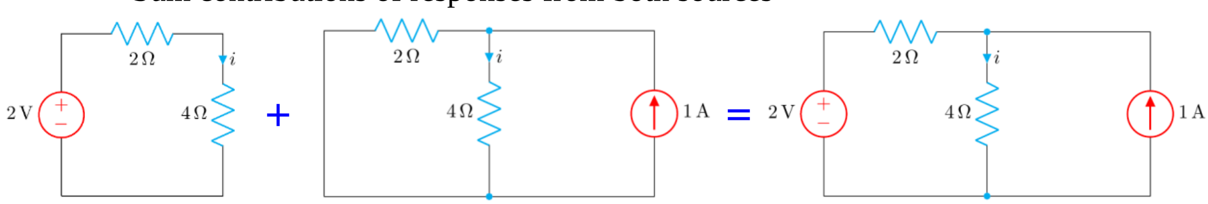


Solve for i'' using current divider

$$i'' = \frac{2\Omega}{2\Omega + 4\Omega} \times 1A = \frac{1}{3}A$$

Superposition

- Ex. 3 (cont.): Determine the current i in the circuit using superposition
  - Sum contributions of responses from both sources



$$i' = \frac{1}{3}A$$

$$i'' = \frac{1}{3}A$$

$i = ?$

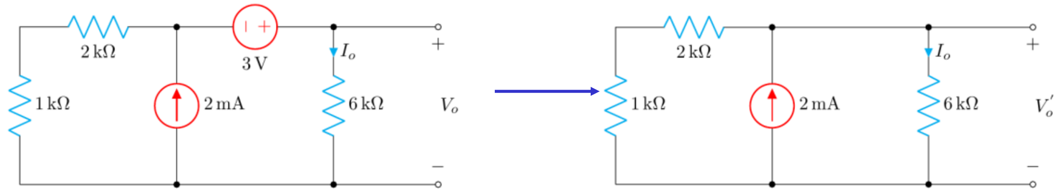
So the total current due both sources

$$i = i' + i'' = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}A$$

Homework: Try using Mesh/Node analysis or source transformation to double-check your answer

Superposition

- Ex. 4: Compute V_o using source superposition
  - First, we eliminate the voltage source (short-circuit) and find V_o'



Solve for I_o current division

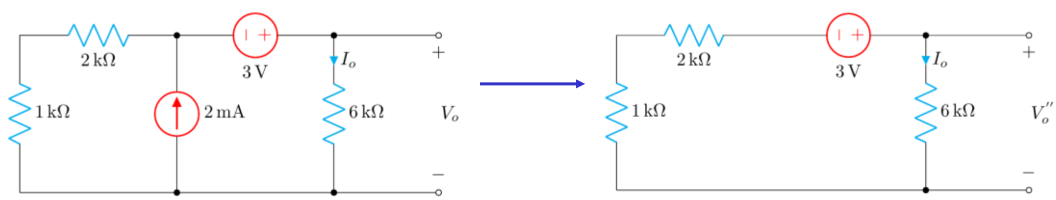
$$I_o = \frac{1k + 2k}{1k + 2k + 6k} \times (2 \times 10^{-3}) = \frac{2}{3} mA$$

Solve for V_o' Ohm's law

$$V_o' = I_o \times 6k = \frac{2}{3} mA \times 6k = 4V$$

Superposition

- Ex. 4 (cont.): Compute V_o using source superposition
  - Then, we eliminate the current source (open-circuit) and find V_o''

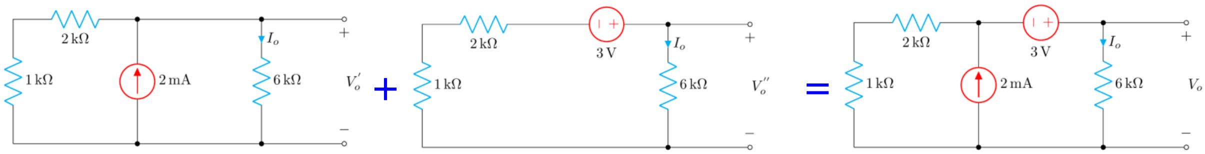


Find V_0'' Voltage Divider

$$V_0'' = \frac{6k}{1k + 2k + 6k} \times 3 = 2V$$

Superposition

- Ex. 4 (cont.): Compute V_o using source superposition
  - Sum contributions of responses from both sources



$V_o' = 4V$

$V_0^{"} = 2V$

$V_o = ?$

Therefore, the total output voltage V_0

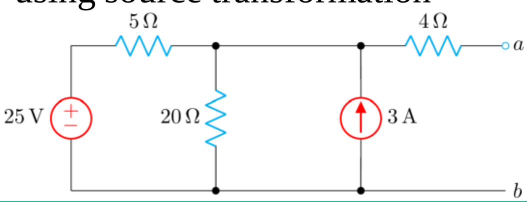
$V_0 = V_0' + V_0^{"} = 6\ V$

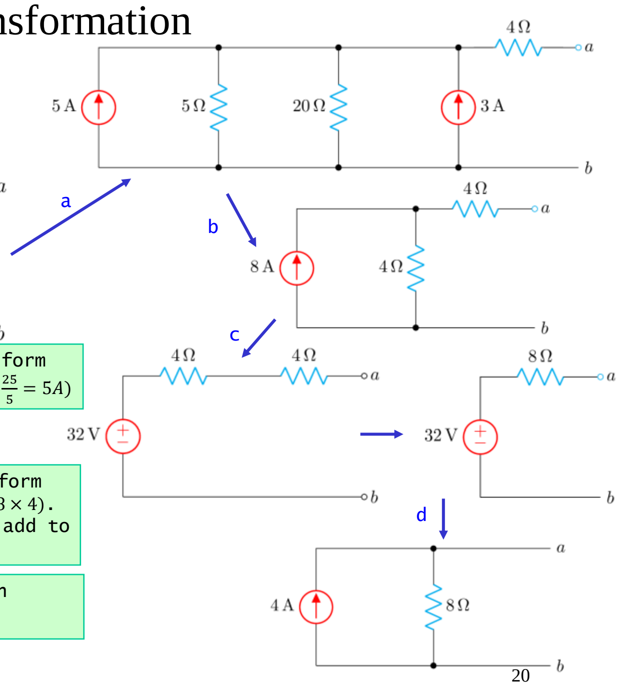
Homework: Try it using source transformation to double-check your results

Practice Problems

Source Transformation

- Prob 1: Find Thevenin equivalent using source transformation





a) 25V source in series with 5Ω resistance transform to 5A source in parallel with 5Ω resistance $(i = \frac{25}{5} = 5A)$

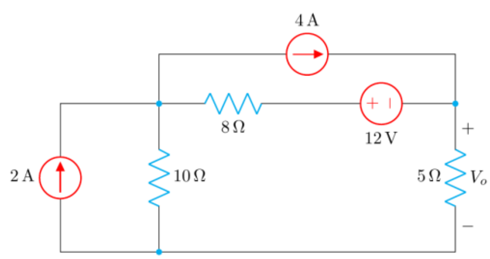
b) $5\Omega||20\Omega = 4\Omega$ and $5A||3A = 8A$.

c) 8A source in parallel with 4Ω resistor transform to 32V source in series with 4Ω resistance $(v = 8 \times 4)$. This 4Ω resistance and the other 4Ω resistance add to 8Ω. This gives us Thevenin equivalent circuit.

d) A simple source transformation give us Norton equivalent circuit as well $(i = \frac{32}{8} = 4A)$.

Superposition

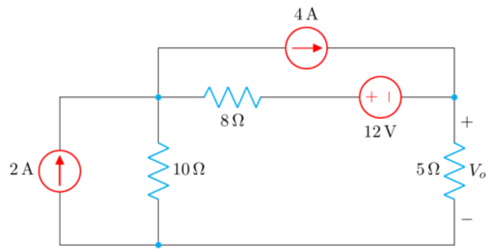
- Prob 2: Use superposition to find V_o in the circuit



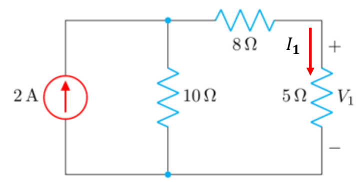
- Let $V_o = V_1 + V_2 + V_3$, where V_1, V_2 , and V_3 are due to the three sources.

Superposition

- Prob 2(cont.): Use superposition to find V_o in the circuit



First, we consider 2A current source only



Solve for I_1

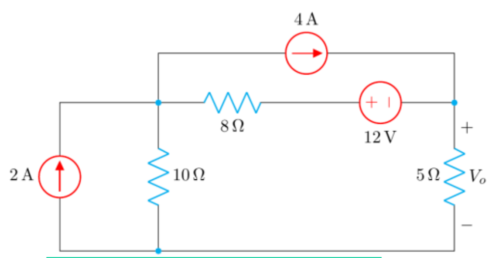
$I_1 = \frac{10}{10+8+5} \times 2 = 0.87A$

Solve for V_1

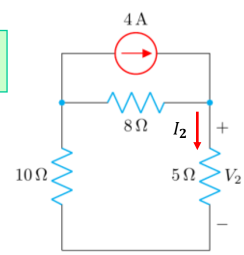
$V_1 = IR = 0.87 \times 5 = 4.35V$

Superposition

- Prob 2(cont.): Use superposition to find V_o in the circuit



Then we consider 4A current source only



Solve for I_2

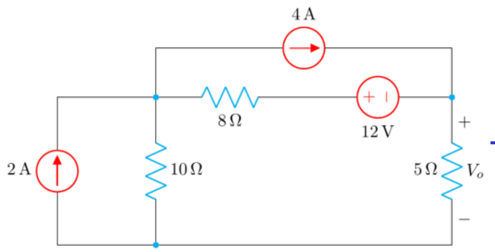
$I_2 = \frac{8}{8+5+10} \times 4 = 1.39A$

Solve for V_1

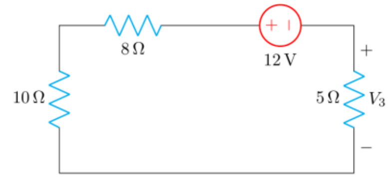
$V_2 = IR = 1.39 \times 5 = 6.96V$

Superposition

- Prob 2(cont.): Use superposition to find V_o in the circuit



Finally, we consider 12V voltage source only



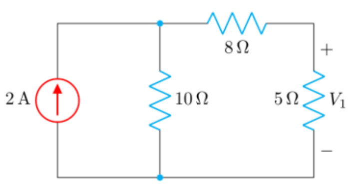
Solve for V_3

$$V_3 = \frac{5}{5+10+8} \times -12$$

$$V_3 = -2.61V$$

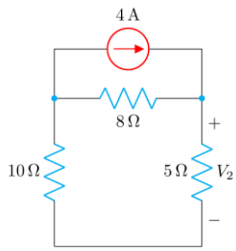
Superposition

- Prob 2(cont.): Use superposition to find V_o in the circuit



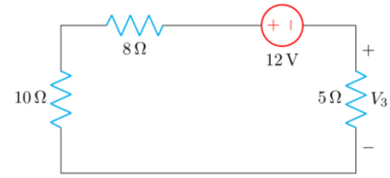
$V_1 = 4.35V$

+



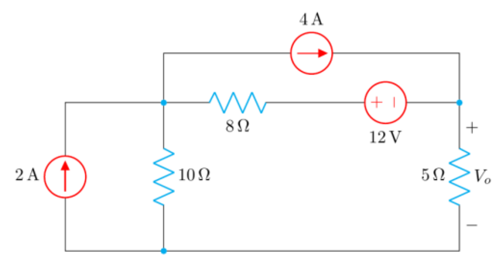
$V_2 = = 6.96V$

+



$V_3 = -2.61V$

=



So the total voltage due three sources

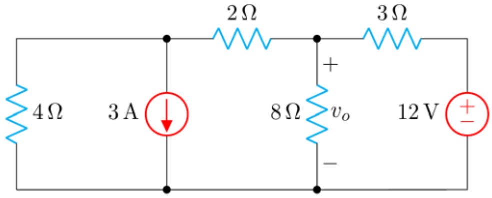
$$V_0 = V_1 + V_2 + V_3$$

$$V_0 = 4.35 + 6.96 - 2.61$$

$$V_0 = 8.70\,V$$

Source Transformation

- Prob 3: Ex. 1 Alternate way of finding v_o using source transform step c onwards



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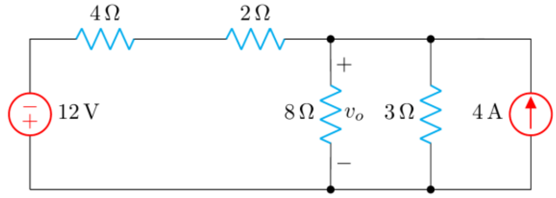

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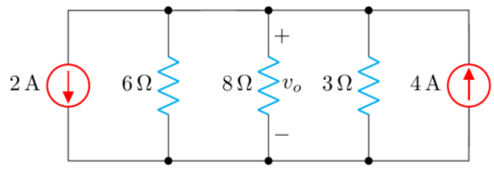


a) 3A source in parallel with 4Ω resistance transform to 12V source in series with 4Ω resistance ($v_{TH} = 3 \times 4 = 12V$)

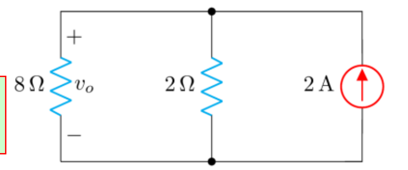
b) 12V source in series with 4Ω resistor transforms to 2A source ($i_{SC} = 12/6$) in parallel with 6Ω resistor

c) $3\Omega || 6\Omega = 2\Omega$. Since 8Ω and 2Ω resistors are in parallel, they have the same voltage v_0 across them, so I can combine them to one and find v_o using just Ohm's Law

$v_0 = R \times I$

$$v_0 = 8||2 \times 2A = \frac{8 \times 2}{8+2} \times 2 = 3.2V$$

You cannot transform the component for which you are trying to find voltage or current



Thank You!