Full resolution photo on my Instagram @feenafoto

1

ECOR1043: Circuits

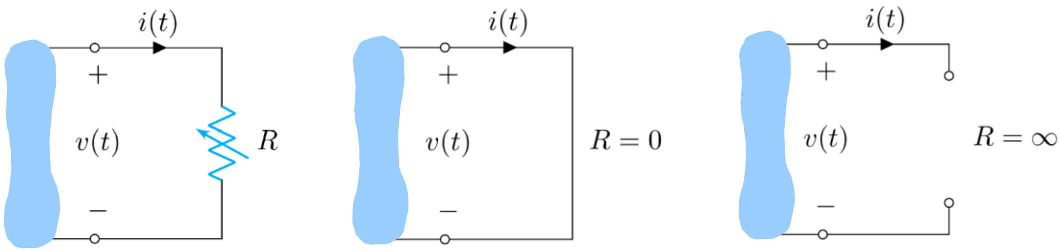
Additional Analysis Techniques

Thevenin's and Norton's Theorems

3

Reminder From Earlier

- Special Cases of Resistance

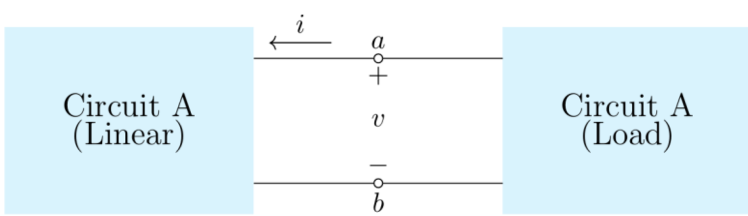


Variable Resistor | Short-Circuit | Open-Circuit

4

Thévenin and Norton's Theorems

- Why do we use them?
  - We want to replace a complicated circuit with a simple one, such that the 'load' cannot tell the difference

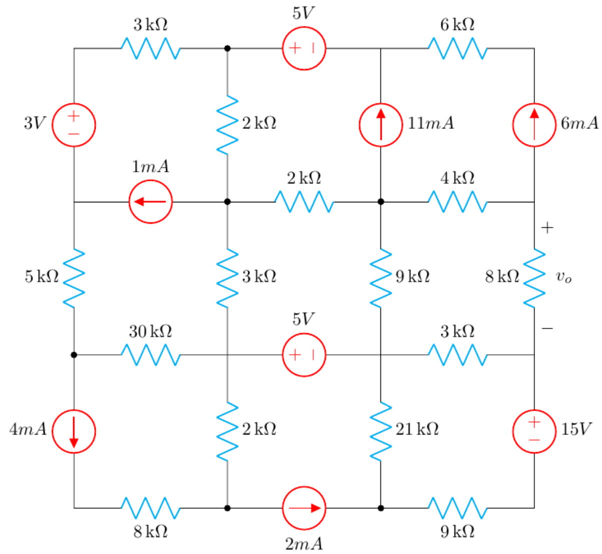


  - They make analysis easier when performing & evaluating load design
  - We replace circuit "A" with a simple circuit with the same voltage-current characteristics

5

Thévenin and Norton’s Theorems

- Why do we use them?



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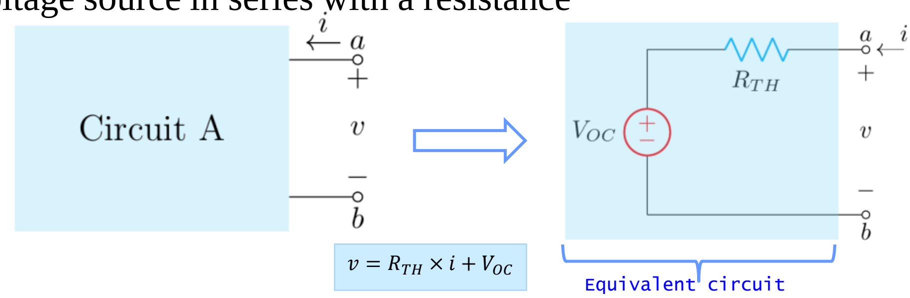
6

Thévenin Theorem

- Thévenin’s Theorem replaces the linear two-terminal circuit with a voltage source in series with a resistance



  - This is a general voltage-current relation for a linear, two-terminal network
  - V_{OC} is the terminal voltage if $i = 0$ (open-circuit voltage)
  - Thevenin voltage $V_{TH} = V_{OC}$
  - R_{TH} is the equivalent resistance seen at the terminals (the Thévenin resistance)

7

7

Creating the Thévenin equivalent circuit

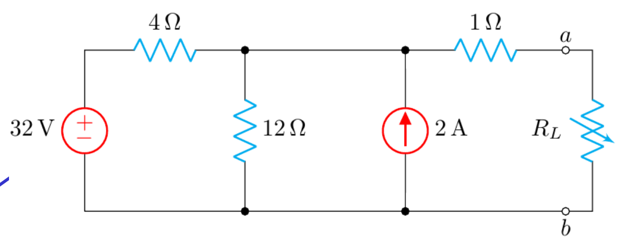
- Procedure to get the Thévenin equivalent
  1) Identify and isolate the circuit and terminals for which the Thévenin equivalent circuit is desired
  2) Eliminate the independent sources in the circuit (short-circuit voltage sources, open-circuit current sources) and determine the equivalent resistance R_{TH} of the circuit
  3) Re-activate the sources and determine the open-circuit voltage V_{OC} across the circuit terminals. This will be V_{TH} for the circuit.
  4) Place the Thévenin equivalent circuit into the original overall circuit and perform the desired analysis

8

8

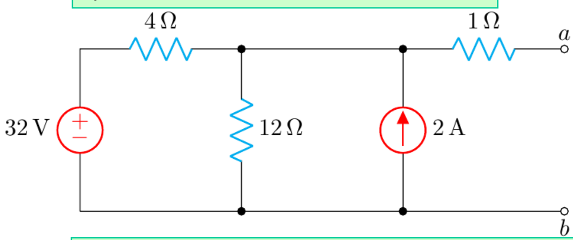
Thévenin's Theorems

- Ex. 1: Find the Thevenin equivalent circuit to the left of the terminals a-b



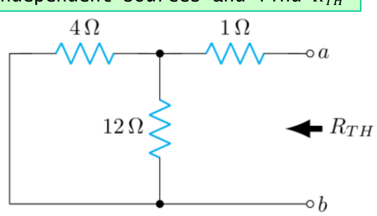
To find R_{TH}:

1) Isolate the circuit and terminals



2) Eliminate independent sources and find R_{TH}

Find R_{TH}



$R_{TH} = 4\Omega || 12\Omega + 1\Omega$

$R_{TH} = \dfrac{4\Omega \times 12\Omega}{4\Omega + 12\Omega} + 1\Omega$

$R_{TH} = \dfrac{48\Omega}{16\Omega} + 1\Omega$

$R_{TH} = 3\Omega + 1\Omega$

$R_{TH} = 4\Omega$

9

9

Thévenin's Theorems

- Ex. 1(cont.): Find the Thevenin equivalent circuit to the left of the terminals a-b

To Find V_{TH}:

$R_{TH} = 4\Omega$

3) Re-activate the sources and determine the open-circuit voltage V_{TH}

You can use any method of choice. I am using Nodal analysis

Apply KCL at node 1 (assuming currents exiting the node are '+'

$i_1 + i_2 - 2A = 0$

Replace currents using Ohm' law (V=IR)

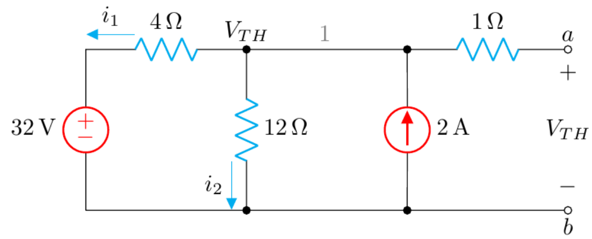
$(V_{TH} - 32)/4 + V_{TH}/12 - 2 = 0$

$3V_{TH} - 96 + V_{TH} - 24 = 0$

$4V_{TH} - 120 = 0$

$V_{TH} = 120/4$

$V_{TH} = 30\ V$



Homework: Use Mesh Analysis to find V_{TH}

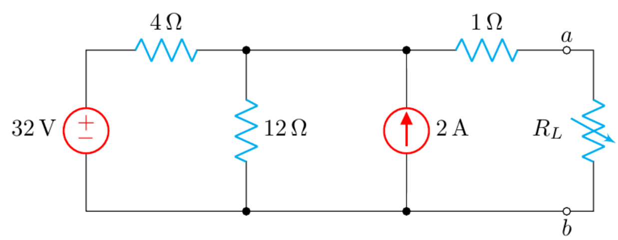
Thévenin's Theorems

- Ex. 1(cont.): Find the Thevenin equivalent circuit to the left of the terminals a-b

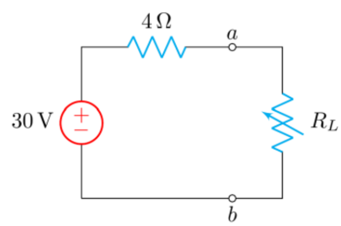
Finally:

4) Implement the Thévenin equivalent

$V_{TH} = 30V\ \&\ R_{TH} = 4\Omega$

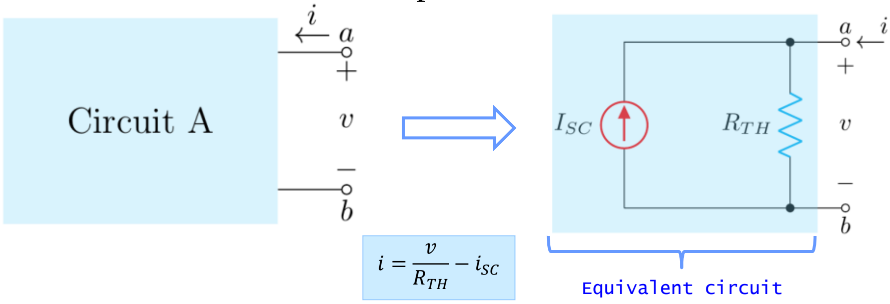


Thevenin equivalent circuit diagram:



Norton's Theorem

- Norton's Theorem: any linear two-terminal circuit can also be modeled as a current source in parallel with a resistor



  - This is a general voltage-current relation for a linear, two-terminal network
  - I_{SC} is the short-circuit current through the terminals

12

Norton's Theorem

- Procedure to get Norton equivalent
  1) Identify and isolate the circuit and terminals the circuit for which the Norton equivalent circuit is desired

  2) Eliminate sources and determine R_{TH} of the circuit

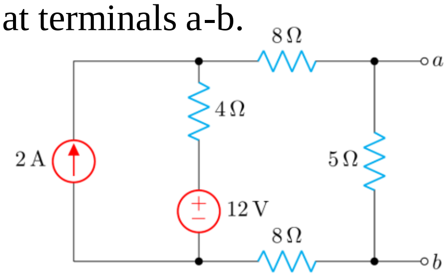
  3) Re-activate the sources, short-circuit the output terminals, and determine I_{SC}

  4) Place the Norton equivalent circuit into the original overall circuit and perform the desired analysis

13

Norton’s Theorem

- Ex. 2: Find the Norton equivalent circuit at terminals a-b.



To find R_{TH}:

1) Isolate the circuit and terminals

2) Eliminate independent sources and find R_{TH}

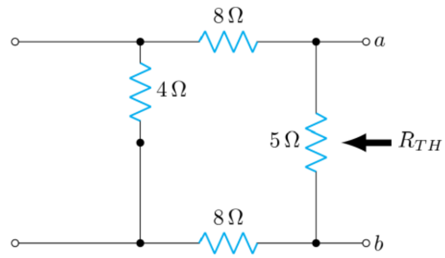
$R_{TH} = 5||(8+4+8)$

$R_{TH} = 5||(20)$

$R_{TH} = \frac{5 \times 20}{5+20}$

$R_{TH} = \frac{100}{25} = 4\Omega$

Eliminate the sources and find R_{TH}

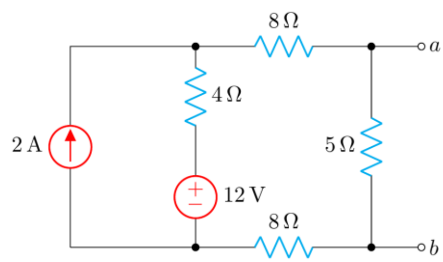


Norton’s Theorem

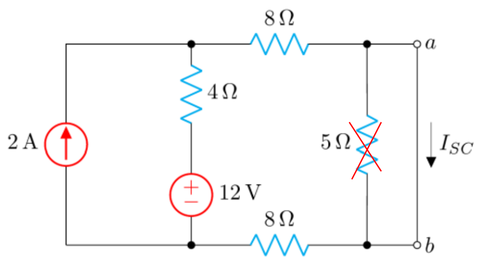
- Ex. 2(cont.): Find the Norton equivalent

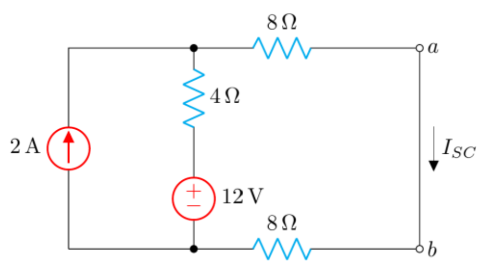
To Find I_{SC}: $R_{TH} = 4\Omega$

3) Re-activate the sources, short the output terminals, and determine I_{SC}



Short circuiting the terminal a-b by passes 5Ω resistor





Norton's Theorem

- Ex. 2(cont.): Find the Norton equivalent

To Find I_{SC}:

$R_{TH} = 4\Omega$

3) Re-activate the sources, short the output terminals, and determine I_{SC}

You can use any method of choice. I am using mesh analysis.

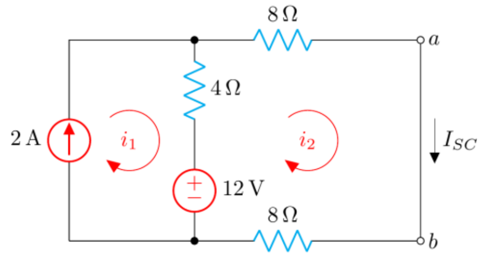
Mesh 1:

$i_1 = 2A$

Mesh 2:

$(4+8+8)i_2 - 4i_1 - 12V = 0$

$20i_2 - 4i_1 - 12V = 0$



Norton's Theorem

- Ex. 2(cont.): Find the Norton equivalent circuit at terminals a-b.

To Find V_{TH}:

3) Re-activate the sources, short the output terminals, and determine I_{SC}

$20i_2 - 4i_1 - 12 = 0$

$i_1 = 2A$

Solve for i_2

$20i_2 - 4 \times 2 - 12 = 0$

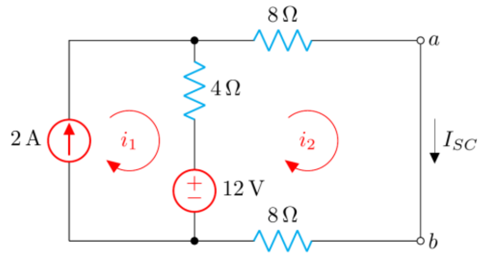
$20i_2 - 8 - 12 = 0$

$20i_2 = 20$

$i_2 = \frac{20}{20} = 1A$

Find I_{SC}

$i_{sc} = i_2 = 1A$



Homework: Use Nodal Analysis to find I_{SC}

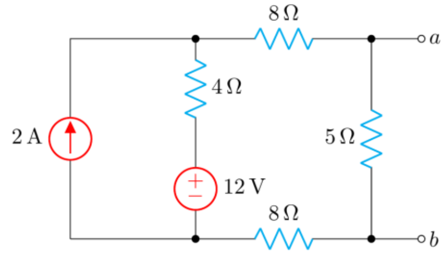
Norton's Theorem

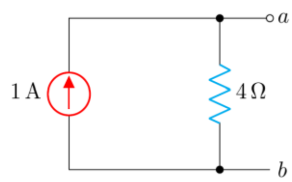
- Ex. 2 (cont.): Find the Norton equivalent circuit at terminals a-b.

Finally:

4) Implement the Norton equivalent

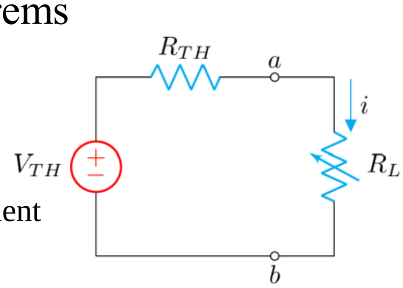
$i_{sc} = 1A$ | $R_{TH} = 4\Omega$





Thévenin's and Norton's Theorems

- Maximum Power Transfer
  - The Thévenin equivalent is useful in finding the maximum power transferred to a load
  - If the entire circuit is replaced by its Thévenin equivalent except for the load, the power delivered to the load is:



$$P = \frac{V^2}{R_L}$$

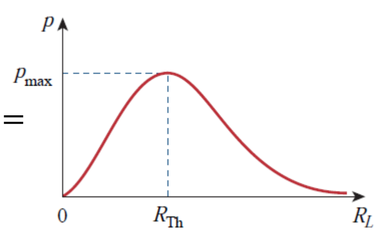
$$V = \frac{R_L}{R_{TH} + R_L} \times V_{TH}$$

$$P = \left(\frac{R_L}{R_{TH} + R_L} V_{TH}\right)^2 \frac{1}{R_L}$$

$$P = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 R_L$$

  - The maximum power dissipated P_{max} occurs when $R_{TH} = R_L$

$$P_{max} = \frac{V_{TH}^2}{4R_L}$$



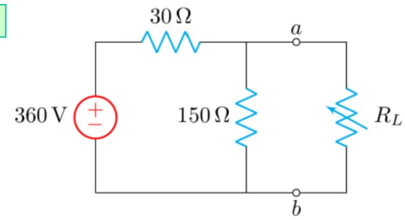
Thévenin's and Norton's Theorems

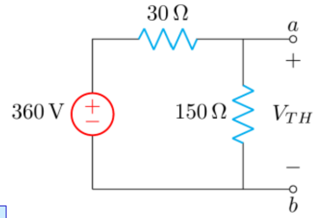
- Ex. 3: Find the Thévenin equivalent circuit at terminal ab, and P_{max}

V_{TH} can be computed using a voltage divider across the 150Ω resistor

$$V_{TH} = \frac{150\Omega}{150\Omega + 30\Omega} \times 360$$

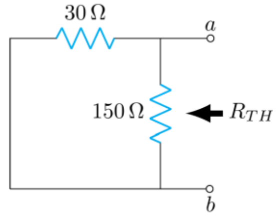
$$V_{TH} = 300V$$





R_{TH} is

$$R_{TH} = 30\Omega || 150\Omega = \frac{(150\Omega)(30\Omega)}{150\Omega + 30\Omega} = 25\Omega$$



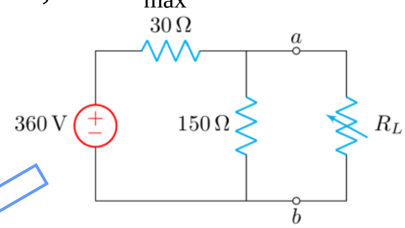
20

Thévenin's and Norton's Theorems

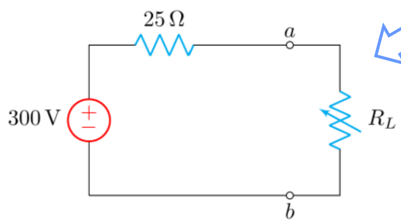
- Ex. 3 (cont.): Find the Thévenin equivalent circuit, and P_{max}

$$V_{TH} = 300V$$

$$R_{TH} = 25\Omega$$



So the Thevenin equivalent cct is



Max power transfer occurs when $R_L = R_{TH}$

$$P_{max} = \frac{V_{TH}^2}{4R_L}$$

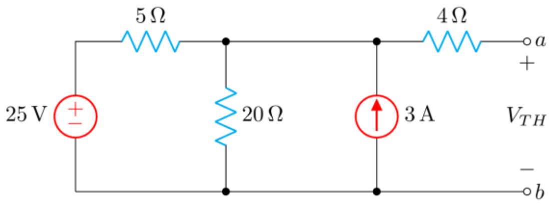
$$P_{max} = \frac{V_{TH}^2}{4R_L} = \frac{(300V)^2}{4(25\Omega)} = 900W$$

21

Practice Problems

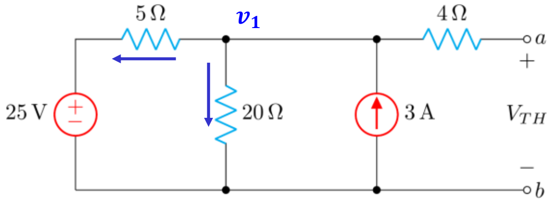
Thevenin's Theorems

- Prob 1: Find v_{th} and R_{th}

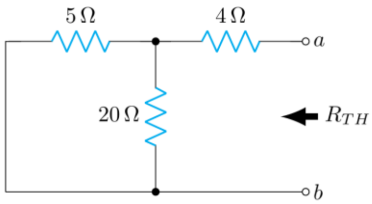


Thevenin's Theorems

- Prob 1(sol.): Find v_{th} and R_{th}



`1. Eliminate the sources and find` R_{TH}

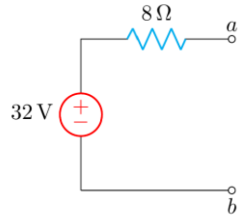


$$R_{ab} = R_{TH} = 4 + \frac{5 \times 20}{25} = 8\Omega$$

`2. To find` V_{TH} `, apply KCL at` v_1

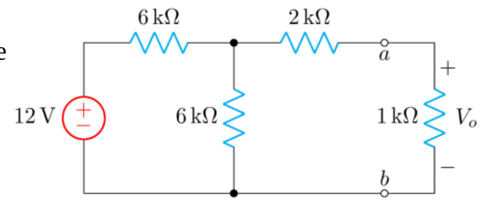
$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0$$
$$4v_1 - 100 + v_1 = 60$$
$$v_1 = \frac{160}{5}$$
$$v_{TH} = v_1 = 32V$$

`3. So Thevenin equivalent cct:`



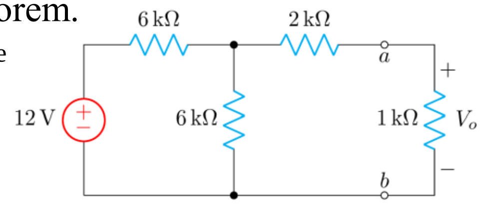
Thevenin's Theorems

- Prob 2: Find V_o using Thevenin's theorem.
  - Note that 1kΩ is the load resistor so you need to find the Thevenin's equivalent of the circuit to the left of it.

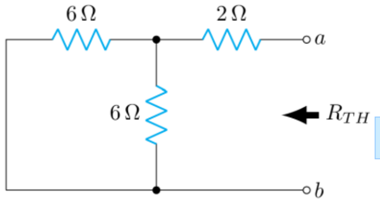


Thevenin's Theorems

- Prob 2(sol.): Find V_o using Thevenin's theorem.
  - Note that 1kΩ is the load resistor so you need to find the Thevenin's equivalent of the circuit to the left of it.

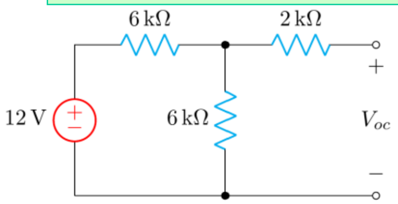


1. For R_{TH}



$=5k\Omega$

2. For V_{TH}

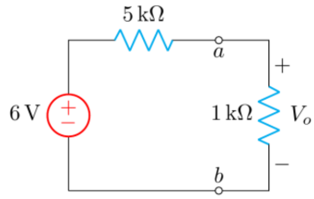


Using voltage divider

$$V_{Oc} = \frac{6k}{6k + 6k}(12V) = 6[V]$$

$$V_{TH} = V_{Oc} = 6[V]$$

3. Finally, find V_o using Thevenin equivalent circuit

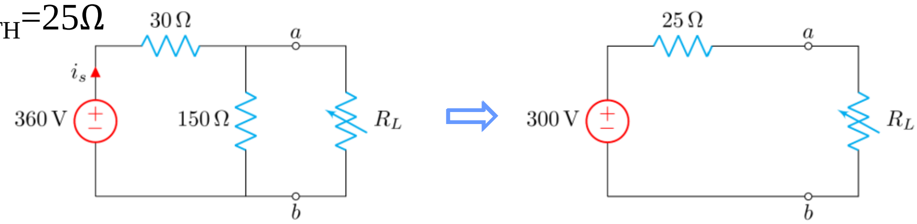


$$V_O = \frac{1k}{1k + 5k}(6V) = 1[V]$$

26

Thevenin's and Norton's Theorems

- Prob 3: (In Ex. 3) Find % of power delivered to the load if R_L = R_{TH}=25Ω

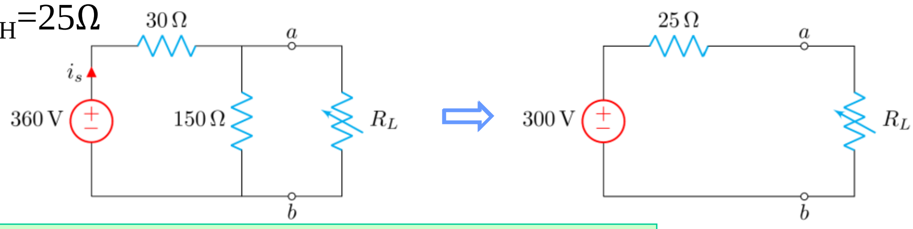


Thevenin equivalent

27

Thevenin's and Norton's Theorems

- Prob 3(sol): (In Ex. 3) Find % of power delivered to the load if R_L = R_{TH}=25Ω



Thevenin equivalent

To find out the % power delivered to the load, we first need to find the total power supplied by the source at $R_L = 25\Omega$. To do that we only need to find i_s, then we can use P=VI

When $R_L = 25\Omega$, the voltage V_{ab} can be found using voltage divider on equivalent circuit

$$V_{ab} = \frac{25\Omega}{25\Omega + 25\Omega} \times 300V = 150V$$

When $V_{ab} = 150$V, the current in the voltage source is also through the 30Ω resistor:

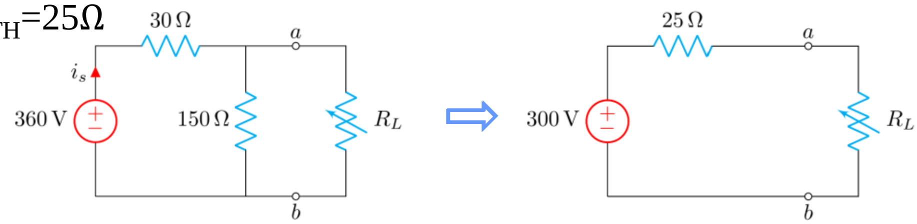
$$i_s = \frac{360V - 150V}{30\Omega} = 7\ A$$

So the source is delivering

$$P_s = -i_s \times 360V = -2520\,W$$

Thevenin's and Norton's Theorems

- Prob 3(sol): (In Ex. 3) Find % of power delivered to the load if R_L = R_{TH}=25Ω



Thevenin equivalent

The % power delivered to the load

$$= \frac{P_{max}}{P_s} \times 100$$

We found P_{max} earlier in Ex. 3

$$P_{max} = \frac{V_{TH}^2}{4R_L} = 900W$$

Therefore:

$$\frac{900W}{2520W} \times 100 = 35.71\%$$