Full resolution photo on my Instagram @feenafoto

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ECOR1043: Circuits

Basics of Frequency Response & Filters

Transfer functions, logarithm, decibels, Bode plots, filters

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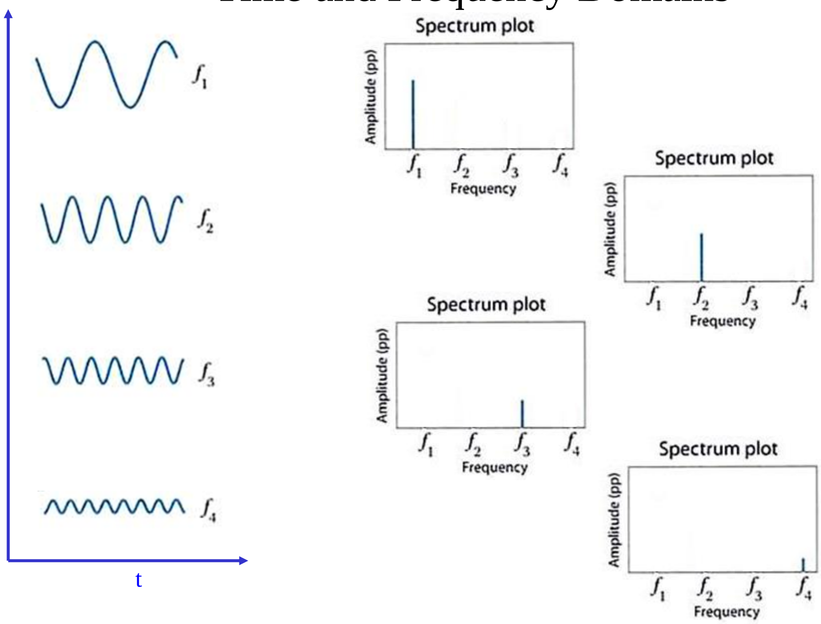
Time and Frequency Domains



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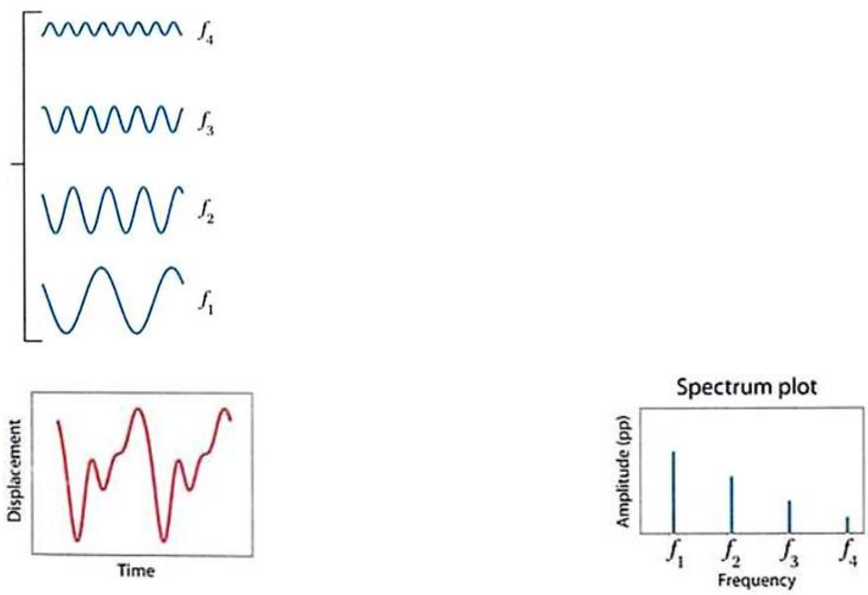
Time and Frequency Domains



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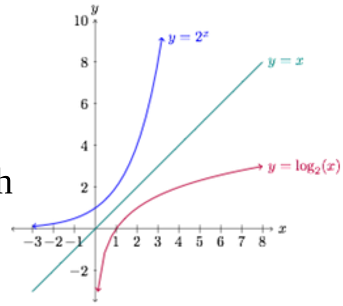
The Logarithm

- In mathematics, the logarithm is the inverse function to exponentiation.

Exponent 2

Exp: $10^2 = ? \Rightarrow 100$ Log: $10^? = 100 \Rightarrow 2$

Base 10



- The log of a given number x is the exponent to which another number, the base b, must be raised, to produce x . It is denoted by: $log_b(x)$
- Example 1: What is the logarithm of 1000 in base 10?

$10^x = 1000 \Rightarrow x = 3$

In other words, logarithms answers the question:
How many of one number ("base") do we multiply to get another number?

Mathematically speaking

$\log_{10}(1000) = 3$

The Logarithm

- Example 2:
  a) What is the logarithm of 16, with a base of 2?

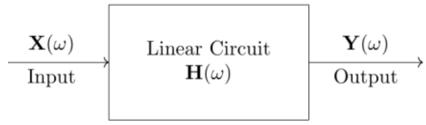
$log_2(16) = 4$

  a) What is the logarithm of 0.1, with a base of 10?

$log_{10}(0.1) = -1$

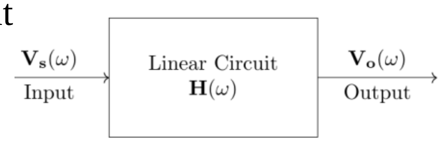
Frequency Response and Transfer Functions

- Frequency response – the variation in a circuit's behavior with change in signal frequency
- Transfer function $\mathbf{H}(\omega)$ - A frequency dependents ratio of output (response) to the input (source) of a circuit



$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

- Typically given as a ratio of voltages for our applications, comparing output voltage to input voltage



$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_s}(\omega)}$$

$$\mathbf{H} = \frac{\mathbf{V_o}}{\mathbf{V_s}}$$

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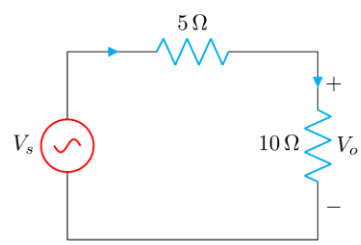
Transfer Functions - Concept

- Example 3: Find the transfer function ($H=V_0/V_s$) of the circuit below

1) Find V_0 in terms of V_s

$$V_0 = V_s\left(\frac{10\Omega}{5\,\Omega + 10\Omega}\right)$$

Using Voltage Divider



2) Solve for $\frac{V_0}{V_s}$

$$H = \frac{V_0}{V_s} = \left(\frac{10\Omega}{5\,\Omega + 10\Omega}\right) = \frac{2}{3}$$

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Transfer Functions

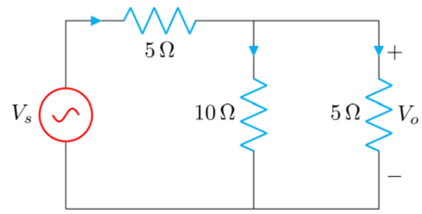
- Example 4: Find the transfer function ($H=V_0/V_s$) of the circuit below

1) Find V_0 in terms of V_s

You can also use voltage divider

$$\frac{V_s - V_0}{5\ \Omega} - \frac{V_0}{10\ \Omega} - \frac{V_0}{5\ \Omega} = 0$$

Using Nodal Analysis



2) Multipy both sides by 10 and rearrange to isolate V_0 and V_s

$$\left(\frac{V_s - V_0}{5\ \Omega} - \frac{V_0}{10\ \Omega} - \frac{V_0}{5\ \Omega}\right) \times 10 = 0 \times 10$$

$$2V_s - 2V_o - V_o - 2V_o = 0$$

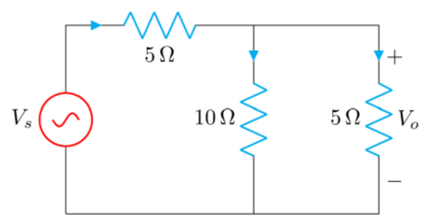
$$2V_s - 5V_o = 0$$

$$2V_s = 5V_0$$

Transfer Functions

- Example 4(cont.): Find the transfer function ($H=V_0/V_s$) of the circuit below

3) Find $H = \frac{V_0}{V_s}$



$$2V_s = 5V_0$$

$$\frac{V_0}{V_s} = \frac{2}{5}$$

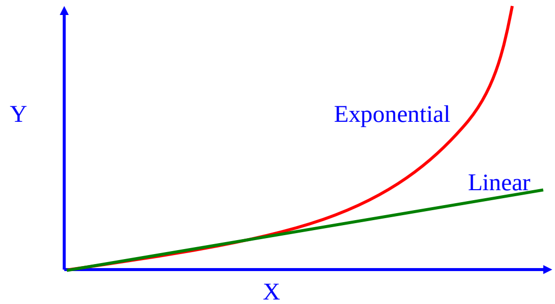
$$\frac{V_0}{V_s} = 0.4$$

$$H = \frac{V_0}{V_s} = 0.4$$

$$H = 0.4$$

Linear versus Decibel

- So far, we've calculated voltage in Volts and compared them that way, this is useful for comparing linear relationships, but what if we want to compare something non-linear or exponential?
- While a linear relationship can be plotted relatively easily, the exponential quickly becomes too large to reasonably plot for any large value of x



Linear versus Decibel

- To account for non-linear relationship, we introduce the units of dB
- The dB is a logarithmic way of describing a ratio. The ratio may be sound pressure, intensity, power, voltage, or other quantities
- If we're given a ratio of voltages e.g., output voltage/input voltage:

$$H = \frac{V_0}{V_s}$$

- We can calculate the equivalent in dB using the equation below:

$$\left(\frac{V_0}{V_s}\right)_{dB} = 20\log_{10}\left(\frac{V_0}{V_s}\right)$$

Calculating Decibels

- Example 5:
  a) What is the value of $\frac{V_o}{V_s}$ =1 in dB?

$$\left(\frac{V_0}{V_s}\right)_{dB} = 20log_{10}\left(\frac{V_0}{V_s}\right)$$

$$20log_{10}(1) = 0\ dB$$

  b) What is the value of $\frac{V_o}{V_s}$ =1000 in dB?

$$20log_{10}(1000) = 60\ dB$$

Bode Plots - Plotting In Decibels

- Once we have our values in dBs, we can plot our transfer function, this is called a Bode plot.
- Bode plot uses logarithmic axis for frequency and decibels for magnitude. This permits wider range of frequencies and magnitudes
- Let's generate Bode plot of the transfer function: $H = \frac{V_0}{V_s} = \omega^2$

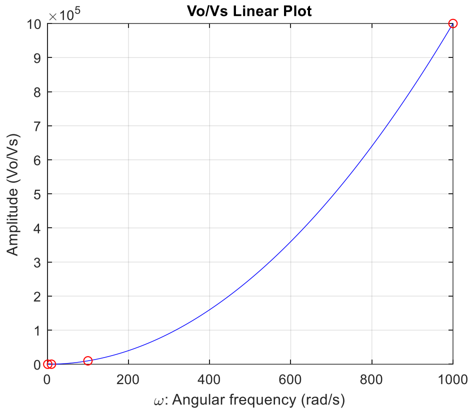
Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20log_{10}$(V/V)
1	1	0
10	100	40
100	10000	80
1000	1000000	120

Plotting In Decibels

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1	1	0
10	100	40
100	10000	80
1000	1000000	120

Just the usual plot

$$\frac{V_0}{V_s} = \omega^2$$



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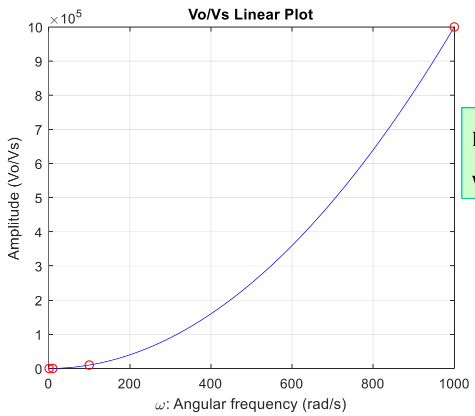
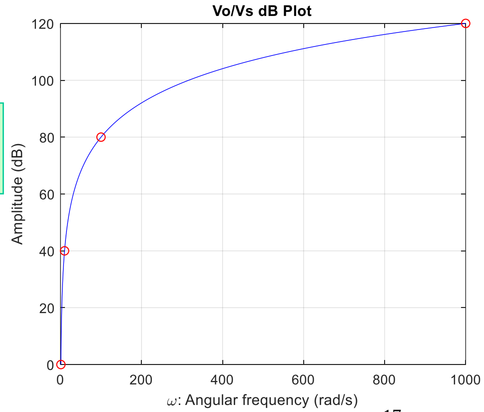
Plotting In Decibels

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1	1	0
10	100	40
100	10000	80
1000	1000000	120



$$H = \frac{V_0}{V_s} = \omega^2$$

Instead of the linear $\frac{V_0}{V_s}$ we plot the y − axis in dB



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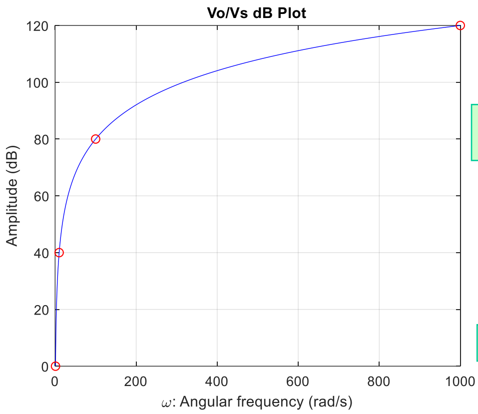
Bode Plot - Plotting In Decibels

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1	1	0
10	100	40
100	10000	80
1000	1000000	120

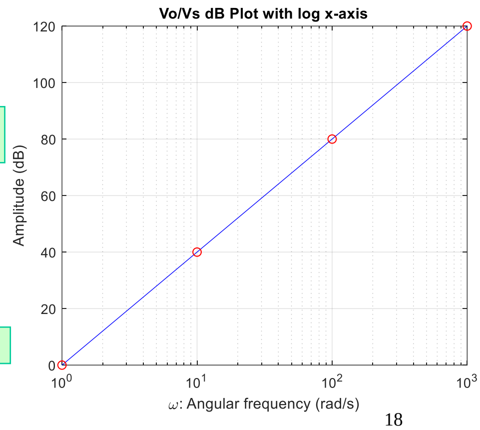


$$H = \frac{V_0}{V_s} = \omega^2$$

Now we change the x axis to a 'log' scale as well

⇨

So what is a 'log' scale?



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Bode Plot Example

- Example 6: Create a bode plot for the following transfer function. Use $\boldsymbol{\omega}$ = 1, 10, 30, 100, 1000 $H = \frac{V_0}{V_s} = \frac{1}{\omega^3}$

1) First we determine the Amplitude

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1		
10		
30		
100		
1000		

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Bode Plot Example

- Example 6: Create a bode plot for the following transfer function. Use $\boldsymbol{\omega}$ = 1, 10, 30, 100, 1000

$$H = \frac{V_0}{V_s} = \frac{1}{\omega^3}$$

2) Next we convert it to dB

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1	1	
10	0.001	
30	0.000037	
100	0.000001	
1000	0.000000001	

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Bode Plot Example

- Example 6: Create a bode plot for the following transfer function. Use $\boldsymbol{\omega}$ = 1, 10, 30, 100, 1000

$$H = \frac{V_0}{V_s} = \frac{1}{\omega^3}$$

$$\left(\frac{V_0}{V_s}\right)_{dB} = 20log_{10}\left(\frac{V_0}{V_s}\right)$$

3) Now we have our final values, we can plot them

Frequency (rad/s)	Amplitude [V/V]	Amplitude [dB]= $20\log_{10}$(V/V)
1	1	0
10	0.001	-60
30	0.000037	-88.63
100	0.000001	-120
1000	0.000000001	-180

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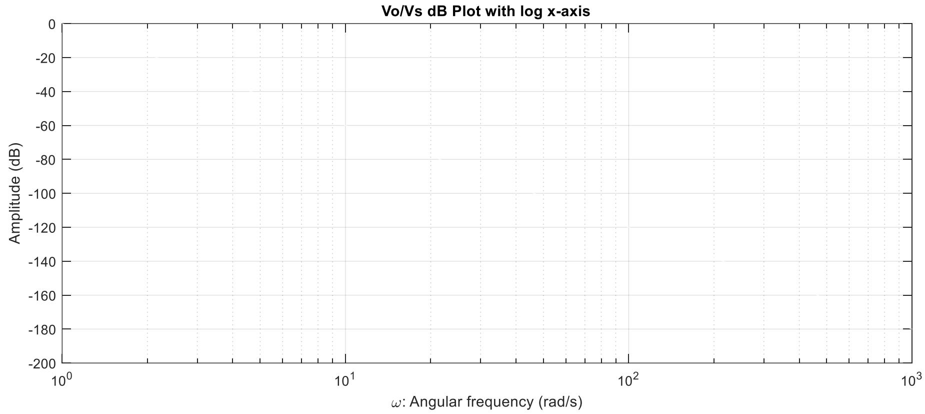
Bode Plot Example

Frequency (rad/s)	**Amplitude [V/V]**	**Amplitude [dB]= 20log$_{10}$(V/V)**
1	1	0
10	0.001	-60
30	0.000037	-88.63
100	0.000001	-120
1000	0.000000001	-180



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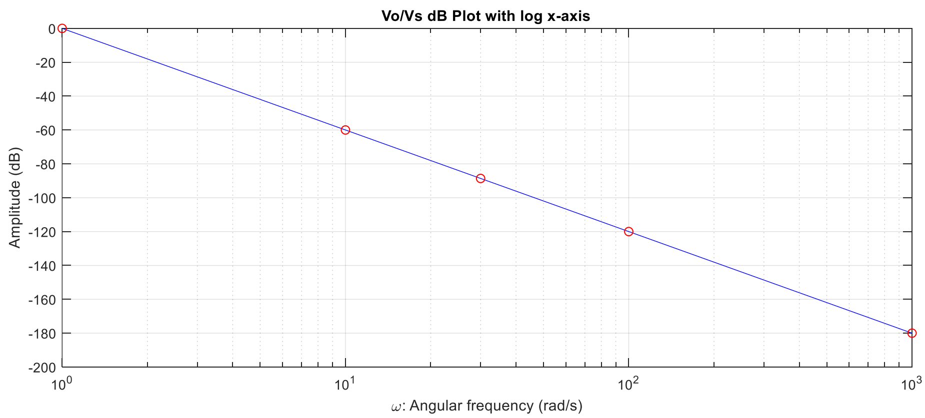
Bode Plot Example

- Example 6: Create a bode plot for the following transfer function. Use $\boldsymbol{\omega}$ = 1, 10, 30, 100, 1000

$$H = \frac{V_0}{V_s} = \frac{1}{\omega^3}$$



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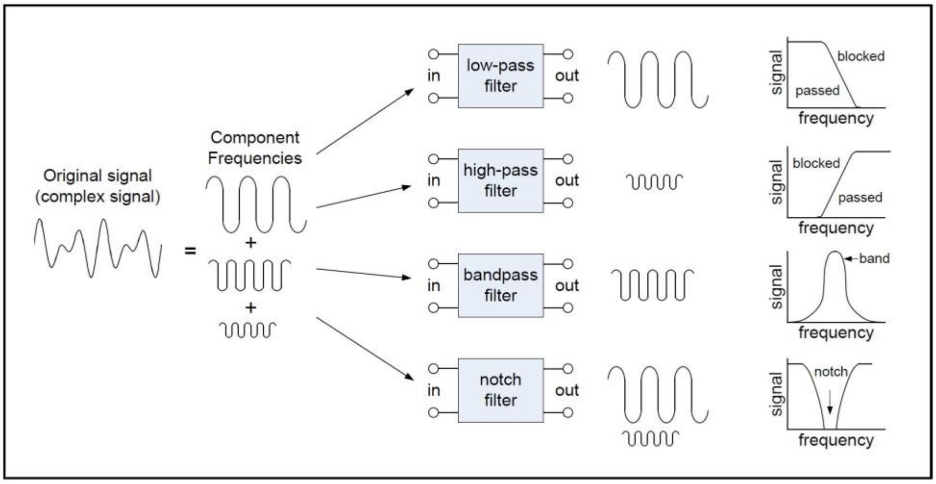
Filters

- Filter
  - A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.



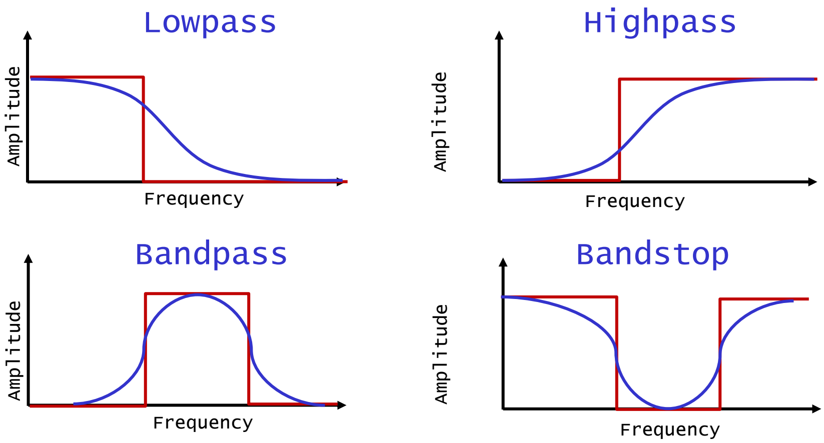
https://www.allaboutcircuits.com/technical-articles/an-introduction-to-filters/

Filters

- Categories of filters
  - ***A passive filter*** is one that contains only R, L, and C components.
    - It is not necessary that all three be present. L is often omitted (on purpose) from passive filter design because of the size and cost of inductors etc.
  - ***An active filter*** is one that, along with R, L, and C components, also contains an energy source, such as that derived from an operational amplifiers or transistors

Filters

- Types of filters
  - Ideal (red) vs. realistic (blue) responses



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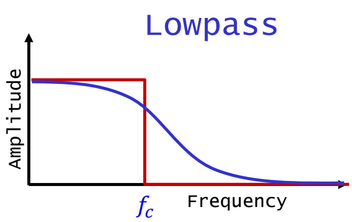
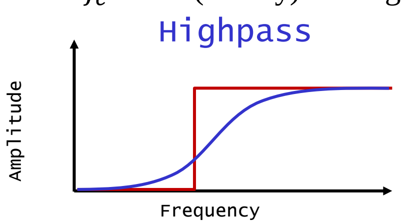
Types of filters

- Low-pass Filters
  - Passes frequencies from DC (0 Hz) to f_c and significantly attenuates all others



- High-pass of filters
  - Passes frequencies from f_c to ∞ (ideally) and significantly attenuates all others



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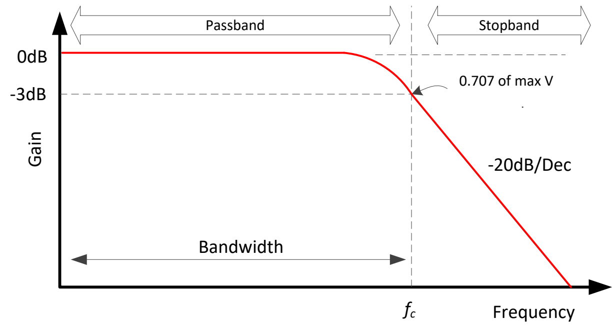
Bode plots-Filters

- Useful terms
  - **Transfer function:** ratio of output (response) to the input (source) of a circuit
  - **Passband**: the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than −3dB (70.7%) of attenuation)
  - **Stopband**: frequencies not in a circuit's passband are in its stopband
  - **cutoff frequency f_c:** (also called the critical frequency) separates passband and stopband
  - **Bandwidth (BW):** width of the passband
  - **Roll-off** : rate at which attenuation increases/decreases after/before the cut-off frequency



Thank You