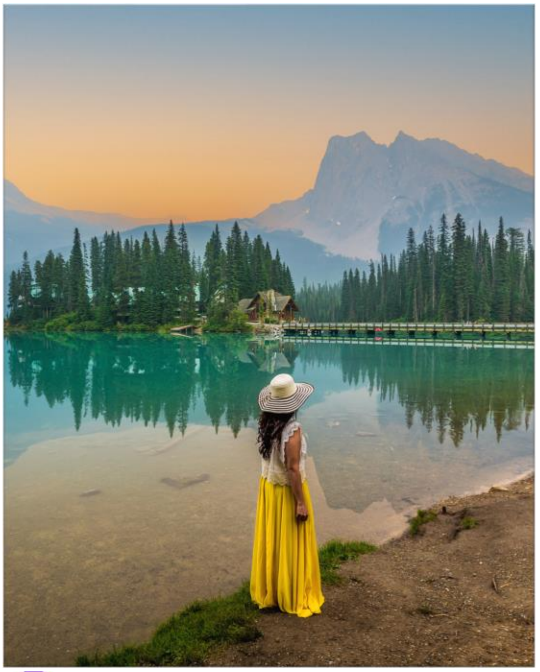


Full resolution photo on my Instagram @feenafoto

1

ECOR1043: Circuits

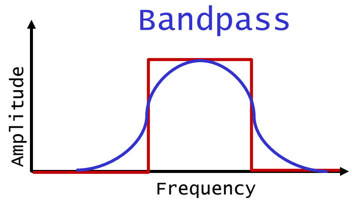
Passive Bandpass Filters

Capacitor and Inductor Based Bandpass Filters

4

Bandpass filters

- Bandpass of filter
  - Passes range of frequencies from lower cut-off frequency f_L to upper cut-off frequency f_H and significantly attenuates all other frequencies.



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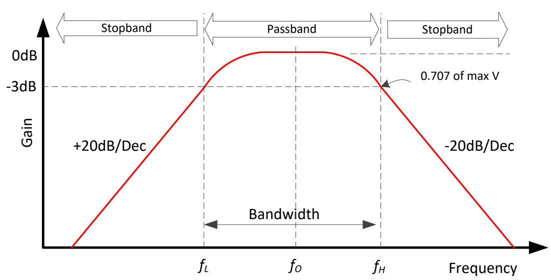
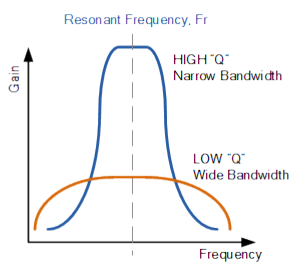
5

Bandpass filters

- Useful terms
  - **Bandwidth (BW):** width of the passband
  - **Lower cut-off frequency f_L:** Frequency, which separates passband and lower stopband.
  - **Upper cut-off frequency f_H :** Frequency, which separates passband and upper stopband.
  - **Center frequency f_o :** Frequency at the center of the passband where magnitude is max
  - **Quality factor Q:** ratio of the center frequency to the bandwidth which is an indication of the selectivity of a bandpass filter (how narrow the pass band is)
  - **Roll-off** : rate at which attenuation increases/decreases after/before the cut-off frequency





6

6

Cascaded Systems

- Any finite number of transfer functions blocks connected in series (cascade) can be algebraically combined by multiplication of the transfer functions

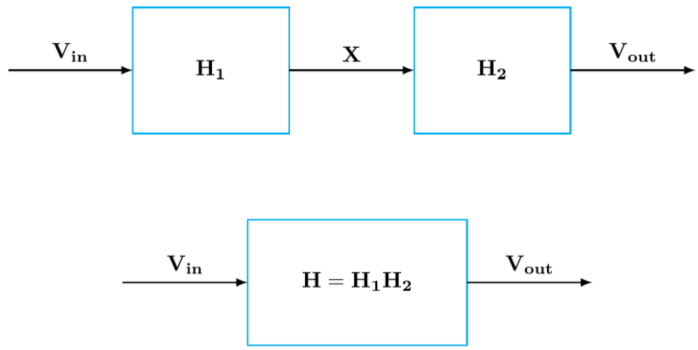
$$H_1 = \frac{X}{V_{in}}$$

$$H_2 = \frac{V_{out}}{X}$$

$$H = H_1 \times H_2$$

$$H = \frac{X}{V_{in}} \times \frac{V_{out}}{X}$$

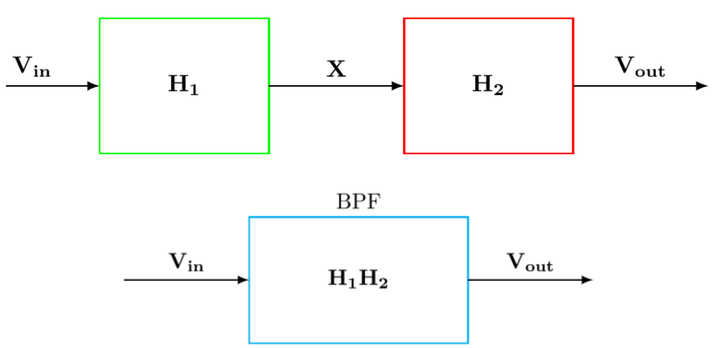
$$H = \frac{V_{out}}{V_{in}}$$

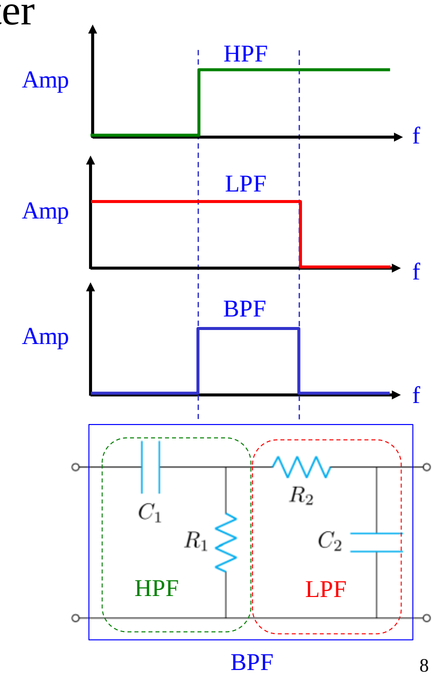


7

Cascaded Bandpass Filter

Bandpass filter can be formed by putting a lowpass and highpass filter in series (which is termed as cascading)



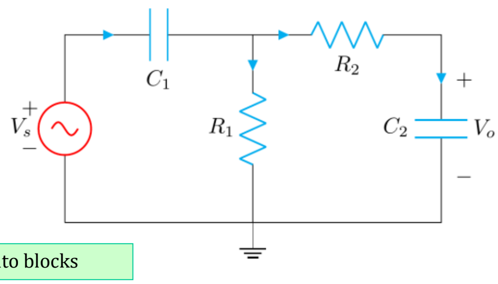


The high-pass filter sets the lower cut-off frequency, and the low-pass filter sets the upper cut-off frequency of such a filter.

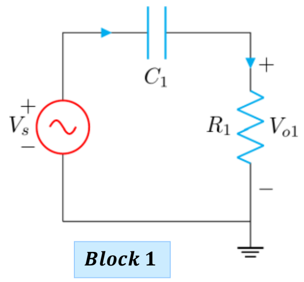
8

‘Cascaded’ RC Bandpass Filter

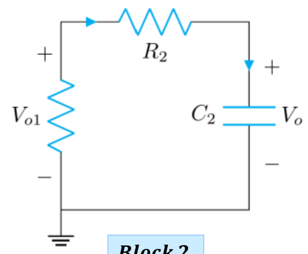
- Example 1a: Find the transfer function (V_0/V_s) of the circuit below



1) We can seperate the system into blocks



Block 1

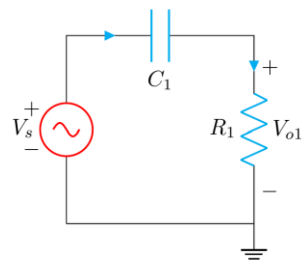


Block 2

‘Cascaded’ RC Bandpass Filter

- Example 1a: Find the transfer function (V_0/V_s)

2) We begin by analyzing block 1



$$\mathrm{H_1} = \frac{\mathrm{V_{01}}}{\mathrm{V_s}} = \left(\frac{R_1}{R_1 + \mathrm{Z}_{C_1}}\right)$$

Where $Z_{c_1} = \frac{1}{j\omega C_1}$

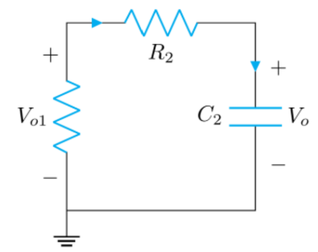
Magnitude

$$H_1 = \frac{V_{01}}{V_s} = \left(\frac{\boldsymbol{R_1}}{\sqrt{\boldsymbol{R_1^2}+\left(\frac{\boldsymbol{1}}{\boldsymbol{\omega C_1}}\right)^2}}\right)$$

Magnitude $H = \frac{R_1}{\sqrt{R_1^2 + X_{C_1}^2}}$ where $X_c = \frac{1}{\omega C_1}$

'Cascaded' RC Bandpass Filter

- Example 1a: Find the transfer function (V_0/V_s)



2) Now we look at block 2

$$\mathbf{H_2} = \frac{\mathbf{V_0}}{\mathbf{V_{01}}} = \left(\frac{\mathbf{Z_{c_2}}}{R_2 + \mathbf{Z_{c_2}}}\right)$$

Where $Z_{c_2} = \frac{1}{j\omega C_2}$

Magnitude

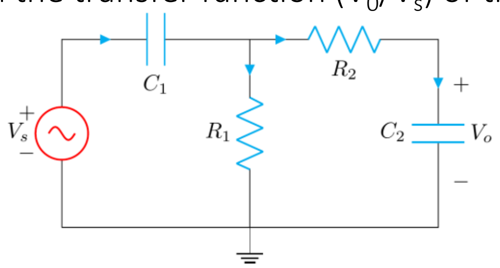
$$H_2 = \frac{V_0}{V_{01}} = \left(\frac{\frac{1}{\omega C_2}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_2}\right)^2}}\right)$$

Magnitude $H_2 = \frac{Xc_2}{\sqrt{R_2^2 + X_{c2}^2}}$ where $X_{c2} = \frac{1}{\omega C_2}$

11

'Cascaded' RC Bandpass Filter

- Example 1a: Find the transfer function (V_0/V_s) of the circuit below



3) Now we recombine our blocks into the original circuit

Transfer functions of two systems in series are multiplied to get the transfer function of the complete system.

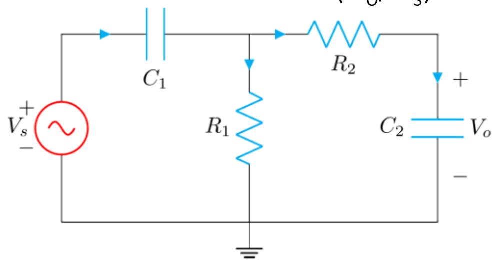
$$H = H_1 \times H_2$$

$$\boldsymbol{mag}\left\{\frac{\mathbf{V_0}}{\mathbf{V_s}}\right\} = \boldsymbol{mag}\left\{\frac{\mathbf{V_{01}}}{\mathbf{V_s}}\right\}\boldsymbol{mag}\left\{\frac{\mathbf{V_0}}{\mathbf{V_{01}}}\right\}$$

12

‘Cascaded’ RC Bandpass Filter

- Example 1a: Find the transfer function (V_0/V_s) of the circuit below



4) Next we substitute our values into the equation

$$H = H_1 \times H_2$$

$$H = \frac{V_0}{V_s} = \left(\frac{R_1}{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}\right)\left(\frac{\frac{1}{\omega C_2}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_2}\right)^2}}\right)$$

13

13

Cascaded Capacitor Based Filters

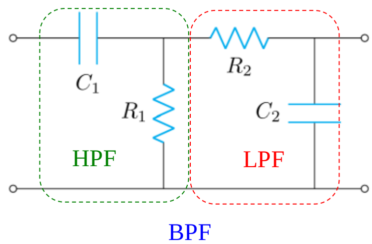
- Bandpass filter
  - Magnitude transfer function

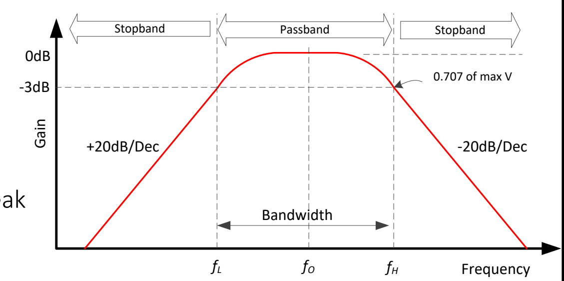
$$H = \frac{V_0}{V_s} = \left(\frac{R_1}{\sqrt{R_1^2 + \left(\frac{1}{\omega C_1}\right)^2}}\right)\left(\frac{\frac{1}{\omega C_2}}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_2}\right)^2}}\right)$$

  - Cut-off frequencies
    - $f_H = \frac{1}{2\pi R_2 C_2}$ and $f_L = \frac{1}{2\pi R_1 C_1}$
  - Bandwidth $(BW) = f_H - f_L$
  - Center frequency (resonant frequency)
    - Where the magnitude is at its maximum or peak value

$$f_o = \sqrt{f_L \times f_H}$$

    - Quality factor $Q = \frac{f_o}{BW}$



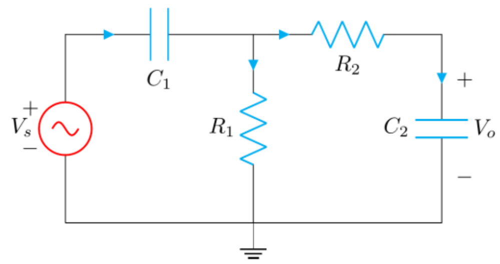


14

14

‘Cascaded’ RC Bandpass Filter

- Example 1b: Calculate f_L, f_o, f_H, BW, Q and the magnitude at f_o



Given:
C_1 = 10 uF
R_1 = 1 kΩ
C_2 = 1 nF
R_2 = 10 kΩ

15

‘Cascaded’ RC Bandpass Filter

- Example 1b: Calculate f_L, f_o, f_H, BW, Q and the magnitude at f_o

5) Calculate f_L

$$f_L = \frac{1}{2\pi R_1 C_1}$$

$$f_L = \frac{1}{2\pi \times (1 \times 10^3)(10 \times 10^{-6})}$$

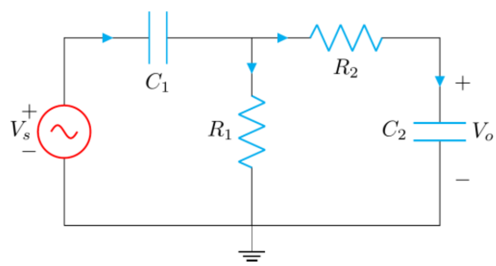
$$f_L = 15.915\ Hz$$

6) Calculate f_H

$$f_H = \frac{1}{2\pi R_2 C_2}$$

$$f_L = \frac{1}{2\pi \times (10 \times 10^3)(1 \times 10^{-9})}$$

$$f_H = 15.915\ kHz$$



Given:
C_1 = 10 uF
R_1 = 1 kΩ
C_2 = 1 nF
R_2 = 10 kΩ

16

'Cascaded' RC Bandpass Filter

- Example 1b: Calculate f_L, f_o, f_H, BW, Q and the magnitude at f_o

7) Calculate f_o

$$f_0 = \sqrt{f_L \times f_H}$$

$$f_H = 15.915\ kHz$$

$$f_L = 15.915\ Hz$$

Given:
C_1 = 10 uF
R_1 = 1 kΩ
C_2 = 1 nF
R_2 = 10 kΩ

$$f_0 = \sqrt{15.915 \times 15.915 \times 10^3}$$

$$f_o = 503.29\ Hz$$

8) Calculate bandwidth (BW)

$$BW = f_H - f_L$$

$$BW = 15.915 \times 10^3 - 15.915$$

$$BW = 15.90\ kHz$$

8) Calculate quality factor (Q)

$$Q = \frac{f_o}{BW}$$

$$Q = \frac{503.29}{15.90 \times 10^3}$$

$$Q = 0.032$$

17

17

'Cascaded' RC Bandpass Filter

- Example 1b: Calculate f_L, f_o, f_H, BW, Q and the magnitude at f_o

5) Calculate magnitude at f = 503.28 Hz

Given:
C_1 = 10 uF
R_1 = 1 kΩ
C_2 = 1 nF
R_2 = 10 kΩ

$$mag\left\{\frac{V_0}{V_s}\right\} = \left(\frac{\mathrm{R}_1}{\sqrt{R_1^2 + \left(\frac{1}{2\pi f C_1}\right)^2}}\right)\left(\frac{\frac{1}{2\pi f C_2}}{\sqrt{R_2^2 + \left(\frac{1}{2\pi f C_2}\right)^2}}\right)$$

$$mag\left\{\frac{V_0}{V_s}\right\} = (0.9995)\left(\frac{316235.38}{316393.45}\right) = 0.999$$

18

18

‘Cascaded’ RC Bandpass Filter

- Example 1c: Generate Bode plot using 0.3, 5, f_L, f_O, f_H, 70kHz, 800kHz

$$mag\left\{\frac{V_0}{V_s}\right\} = \left(\frac{R_1}{\sqrt{R_1^2 + \left(\frac{1}{2\pi f C_1}\right)^2}}\right)\left(\frac{\frac{1}{2\pi f C_2}}{\sqrt{R_2^2 + \left(\frac{1}{2\pi f C_2}\right)^2}}\right)$$

Given:
C_1 = 10 uF
R_1 = 1 kΩ
C_2 = 1 nF
R_2 = 10 kΩ

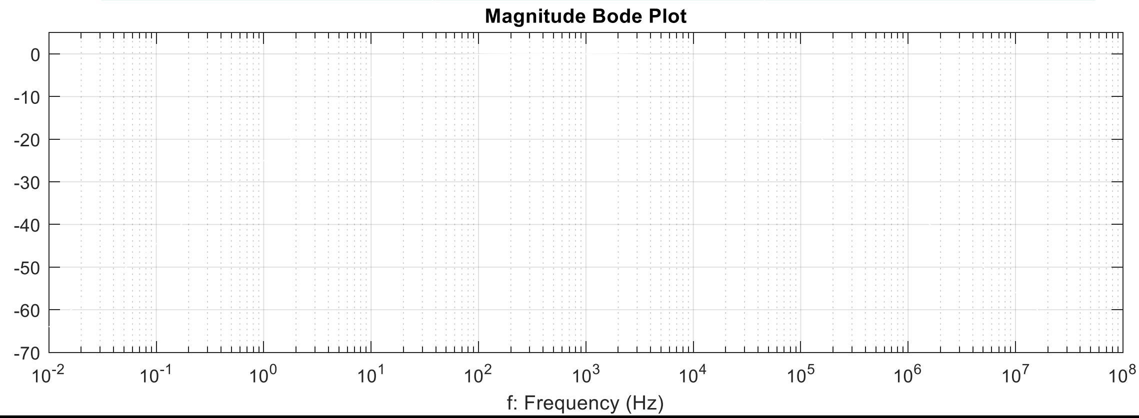
Frequency (Hz)	Amplitude [V/V]	Amplitude dB [$20\log_{10}$(V/V)]
0.3	0.0188	-34.4955
5	0.2997	-10.4658
15.915 (f_L)	$\boxed{0.7071}$	-3.0104
503.28	0.9990	-0.0087
15.915k (f_H)	$\boxed{0.7071}$	-3.0102
70k	0.2217	-13.0845
800k	0.0199	-34.0271

19

19

‘Cascaded’ RC Bandpass Filter

Frequency (Hz)	Amplitude [V/V]	Amplitude [dB – $20\log_{10}$(V/V)]
0.3	0.0188	-34.4955
5	0.2997	-10.4658
15.915	0.7071	-3.0104
503.28	0.9990	-0.0087
15.915k	0.7071	-3.0102
70k	0.2217	-13.0845
800k	0.0199	-34.0271

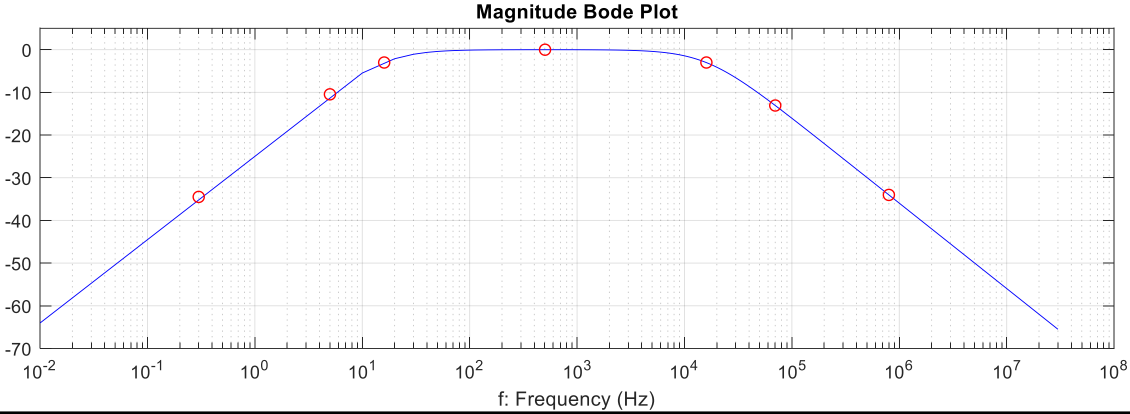


20

20

‘Cascaded’ RC Bandpass Filter

Frequency (Hz)	Amplitude [V/V]	Amplitude [dB – $20\log_{10}$(V/V)]
0.3	0.0188	-34.4955
5	0.2997	-10.4658
15.915	0.7071	-3.0104
503.28	0.9990	-0.0087
15.915k	0.7071	-3.0102
70k	0.2217	-13.0845
800k	0.0199	-34.0271

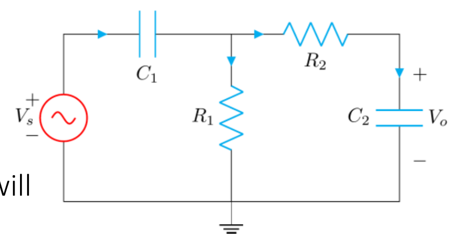


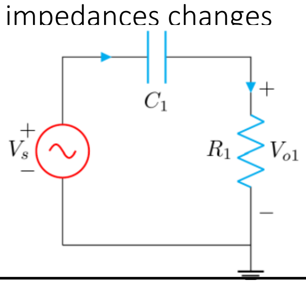
21

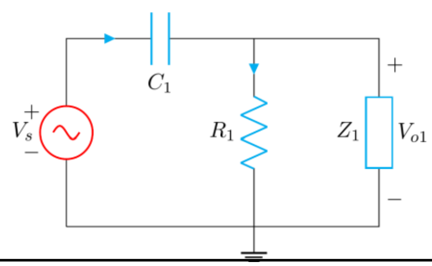
21

Cascaded Filters and Limitation

- Cascading filters makes analysis generally straightforward and implementation easier
- Simple cascading has a potential drawback:
  - We would like to analyze the filters as separate ‘blocks’ however they do interact, therefore we will introduce some error by simply multiplying their transfer functions
  - Second filter overloads the first as effective impedances changes





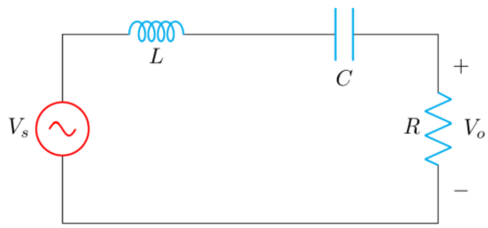


22

22

RLC Bandpass Filter

- The magnitude response of RC cascaded BPF filter is not the same as just multiplying the response of HPF to LPF
- This is because one circuit loads the other circuit, and the resulting transfer function is altered
- However, we can make a BPF using resistor, inductor and a capacitor which does not suffer from this type of magnitude loss.
- This kind of circuit is termed as RLC BPF



23

RLC Bandpass Filter

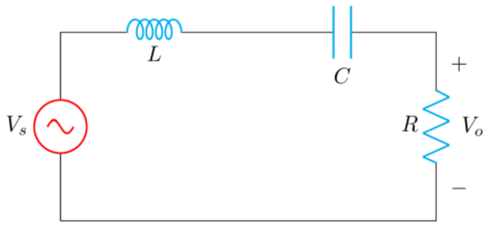
- Example 2a: Find the transfer function (V_0/V_s) of the circuit below

1) Use a voltage divider

$$\mathbf{V_0} = \mathbf{V_s}\left(\frac{R}{R + \frac{1}{j\omega C} + j\omega L}\right)$$

$$\frac{\mathbf{V_0}}{\mathbf{V_s}} = \left(\frac{R}{R - \frac{j}{\omega C} + j\omega L}\right)$$

$$\frac{\mathbf{V_0}}{\mathbf{V_s}} = \left(\frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}\right)$$



Magnitude

$$H = \frac{V_0}{V_s} = \left(\frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right)$$

24

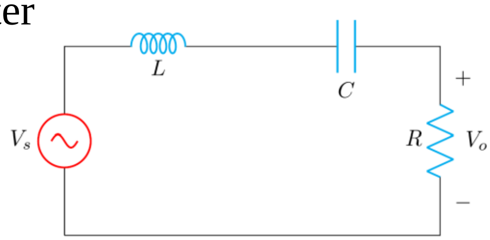
RLC Bandpass Filter

- RLC Bandpass filter
  - Transfer function

$$\mathbf{H} = \frac{\mathbf{V_0}}{\mathbf{V_s}} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R} + \boldsymbol{j}\left(\boldsymbol{\omega L} - \frac{\mathbf{1}}{\boldsymbol{\omega C}}\right)}\right)$$

Magnitude

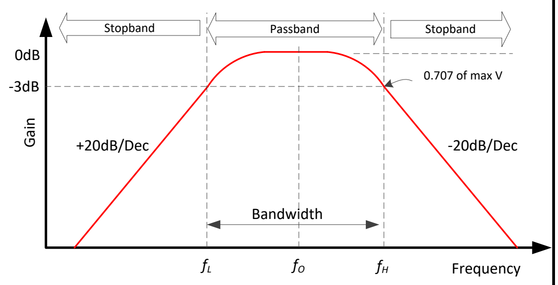
$$H = \frac{V_0}{V_s} = \left(\frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right)$$



  - Cut-off frequencies
    - $\omega_H = \left(\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) + \frac{R}{2L}$
    - $\omega_L = \left(\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) - \frac{R}{2L}$
  - Bandwidth $(BW) = f_H - f_L$
  - Center frequency (resonant frequency)

$$\omega_o = \frac{1}{\sqrt{LC}}$$

  - Quality factor $Q = \frac{f_o}{BW}$



25

RLC Bandpass Filter

- Example 2a: Calculate f_o , and the magnitude at f_o for the circuit

Given:
C = 10 uF
R = 1 kΩ
L = 2.5 mH

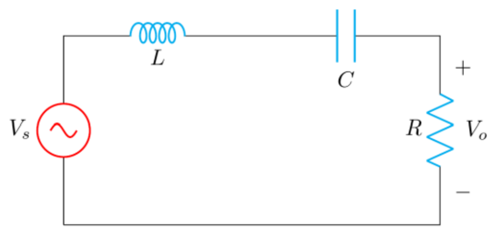
2) Calculate f_o

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{2.5 \times 10^{-3} \times 10 \times 10^{-6}}}$$

$$f_o = 1.007\ kHz$$



26

RLC Bandpass Filter

- Example 2a: Calculate f_o, and the magnitude at f_o for the circuit

Given:
C = 10 uF
R = 1 kΩ
L = 2.5 mH

3) Calculate the magnitude at f_o

$f_o = 1.007\ kHz$

$$\frac{\mathbf{V_0}}{\mathbf{V_s}} = \left(\frac{\boldsymbol{R}}{\boldsymbol{R} + \boldsymbol{j}\left(\boldsymbol{\omega L} - \frac{\mathbf{1}}{\boldsymbol{\omega C}}\right)}\right)$$

Magnitude

$$H = \frac{V_0}{V_s} = \left(\frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right)$$

$$H = \left(\frac{R}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}\right) = \left(\frac{1\ k\Omega}{\sqrt{(1\ k\Omega)^2 + (15.82 - 15.81)^2}}\right) \boxed{= 1}$$

27

RLC Bandpass Filter

- Example 2b: Generate Bode plot using 0.2, f_L,400, 3k, f_H, 5MHz

$$H = \left(\frac{R}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}\right)$$

Given:
C = 10 uF
R = 1 kΩ
L = 2.5 mH

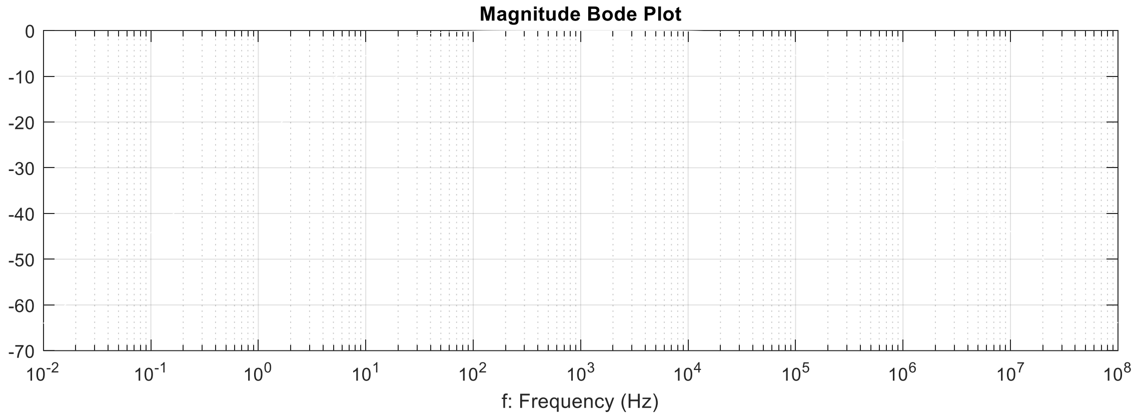
$f_o = 1.007\ kHz$

Frequency (Hz)	Amplitude [V/V]	Amplitude [dB – $20\log_{10}$(V/V)]
.2	0.0126	-38.0165
15.91 (f_L)	0.7071	-3.0103
400	0.9994	-0.0049
3k	0.9991	-0.0076
63.68k (f_H)	0.7071	-3.0103
5M	0.0127	-37.9025

28

RLC Bandpass Filter

Frequency (Hz)	Amplitude [V/V]	Amplitude dB [– $20\log_{10}$(V/V)]
.2	0.0126	-38.0165
15.91 (f_L)	0.7071	-3.0103
400	0.9994	-0.0049
3k	0.9991	-0.0076
63.68k (f_H)	0.7071	-3.0103
5M	0.0127	-37.9025

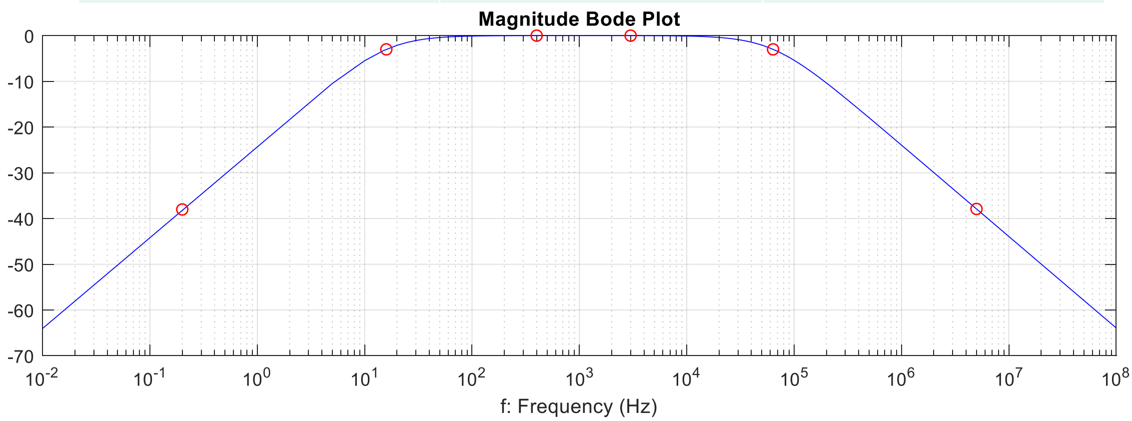


29

29

RLC Bandpass Filter

Frequency (Hz)	Amplitude [V/V]	Amplitude dB [– $20\log_{10}$(V/V)]
.2	0.0126	-38.0165
15.91 (f_L)	0.7071	-3.0103
400	0.9994	-0.0049
3k	0.9991	-0.0076
63.68k (f_H)	0.7071	-3.0103
5M	0.0127	-37.9025



30

30

Thank you

31

31