Full resolution photo on my Instagram @feenafoto

1

ECOR1043: Circuits

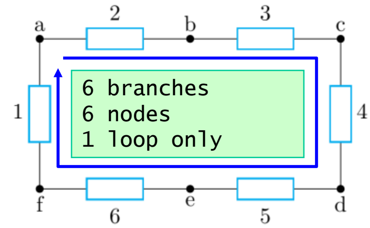
Resistive Circuits

Single-Loop Circuits

3

Single-Loop Circuits

- Background
  - Using KVL, KCL & Ohm's Law, we can write enough equations to analyze any linear circuit
  - We now start the study of systematic, and efficient, ways of using fundamental circuit laws
- Single Loop Circuit
  - Elements are in series
  - Elements are *in series* if they carry the same current
- Goal:
  - Start with the simplest one loop, once source circuit
  - Extend the results to multiple source & multiple resistors circuits



Voltage Divider: General Equation

- In a single loop circuit with multiple resistors and a voltage source, we can find the voltage across any resistor using Voltage Divider
- According to Voltage Divider, voltage v_{Ri} across any resistor R_i is given as:

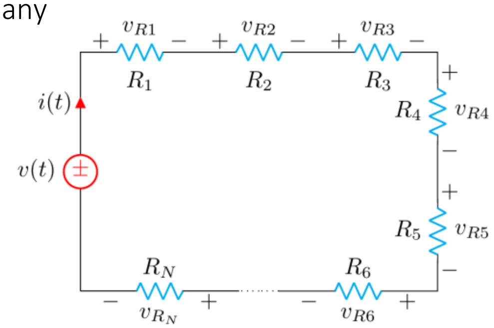
$$\boxed{v_{Ri} = \frac{R_i}{R_{eq}} v(t)} \longrightarrow \text{A}$$

- Where

$$\boxed{R_{eq} = R_1 + R_2 + \ldots + R_N}$$

$\boxed{v(t)\text{: Voltage source}}$

- In other words, in single loop circuits, voltage across any resistor is proportional to its value



Derivation of eq. A is given in Appendix-A at the end

Voltage Divider: General Equation

- Ex. 1: If $V_s = 12V$, find voltage drops across resistors

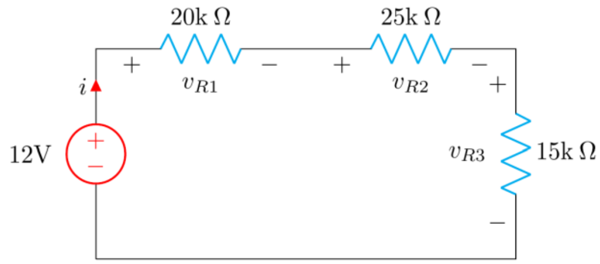
$$v_{Ri} = \frac{R_i}{R_{eq}} v(t)$$

$$v_{20k\Omega} = \frac{20k}{25k + 15k + 20k} \times 12$$

$$v_{20k\Omega} = \frac{20k}{60k} \times 12 = 4V$$

$$v_{25k\Omega} = \frac{25k}{60k} \times 12 = 5V$$

$$v_{15k\Omega} = \frac{15k}{60k} \times 12 = 3V$$



What is the sum of the voltages across resistors?

Voltage Divider: Example

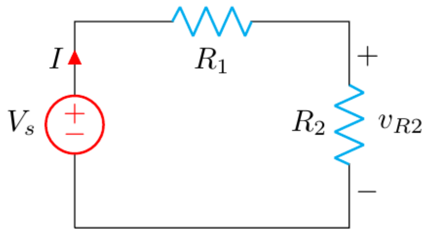
- Ex. 2: Find voltage across R_2, power dissipated in R_2, and the current I. Given: V_s = 9V, R_1= 90 kΩ, and R_2 = 30 kΩ

To find v_{R2}

$$v_{R2} = \frac{R_2}{R_1 + R_2} V_s$$

$$v_{R2} = \left(\frac{30\ k\Omega}{90\ k\Omega + 30\ k\Omega}\right) \times 9V$$

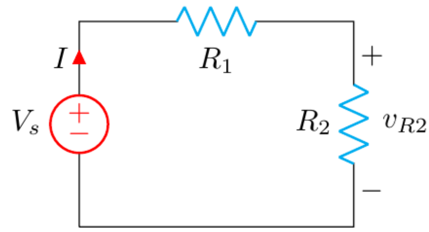
$$v_{R2} = 2.25\ V$$



Voltage Divider: Example

- Ex. 2 (cont.): Find voltage across R_2, power dissipated in R_2, and the current I. Given: V_s = 9V, R_1= 90 kΩ, and R_2 = 30 kΩ

$v_{R2} = 2.25\ V$



To find P_{R2}

$$P = \frac{v^2}{R}$$

$$P_{R2} = \frac{v_{R2}^2}{R_2}$$

$$P_{R2} = \frac{(2.25)^2}{30\ k\Omega}$$

$$P_{R2} = 0.169\ mW$$

To find I

$$V = IR$$

$$i_{R2} = \frac{v_{R2}}{R_2}$$

$$i_{R2} = \frac{2.25\ V}{30\ k\Omega}$$

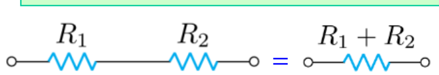
$$i_{R2} = 75\ \mu A$$

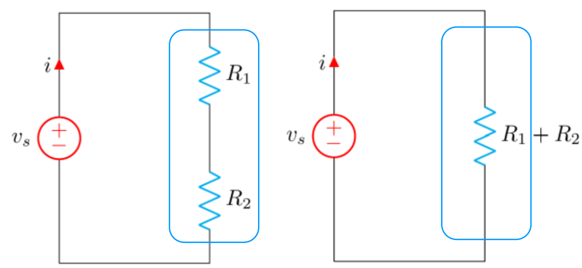
$$I = i_{R2} = 75\ \mu A$$

Equivalent Resistances: Series

- Equivalent Resistances
  - Simplifies our analysis by summing resistors

Series combination of resistors





  - The current flowing through the circuit sees the same resistance, whether it is a single large resistor or two smaller ones. They are called equivalent.

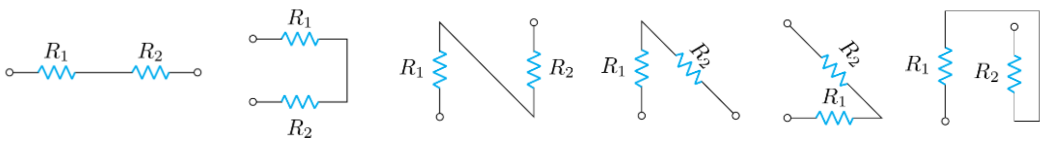
$$i = \frac{v_s}{R_1 + R_2}$$

Remember Ohm's Law:
$v = iR$

Equivalent Resistances: Series

- Resistors in series appear as a 'chain' of resistors, they provide only one path for the current to take

- Different orientations for series resistances:



For all the above diagrams, the resistors R_1 and R_2 are in series (as only one current flows through them)

Equivalent Resistance and Sources

- We can also use KVL to find the algebraic sum of voltage sources, or an equivalent voltage source
  - We start with KVL:

$$v_{R1} + V_2 - V_3 + v_{R2} + V_4 + V_5 - V_1 = 0$$

  - Next collect all voltage sources

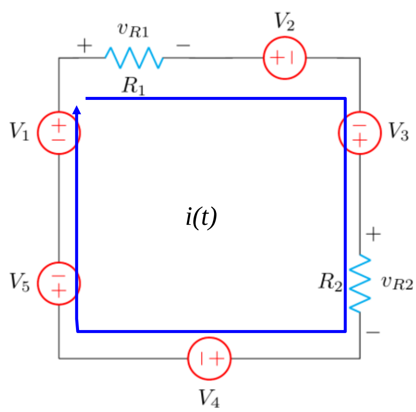
$$(-V_1 + V_2 - V_3 + V_4 + V_5) + v_{R1} + v_{R2} = 0$$

$$V_1 - V_2 + V_3 - V_4 - V_5 = v_{R1} + v_{R2}$$

$$V_{eq} = v_{R1} + v_{R2}$$

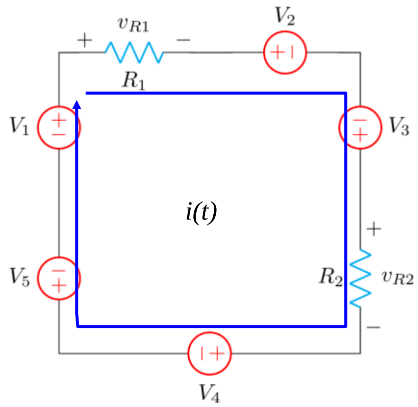
  - Where V_{eq} is the sum of voltage sources

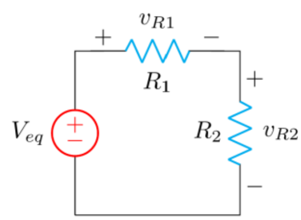
$$V_{eq} = (V_1 - V_2 + V_3 - V_4 - V_5)$$



Equivalent Resistance and Sources

- We can now draw an equivalent circuit for multi-source circuit.
- Note: You can switch the location of any component without affecting anything

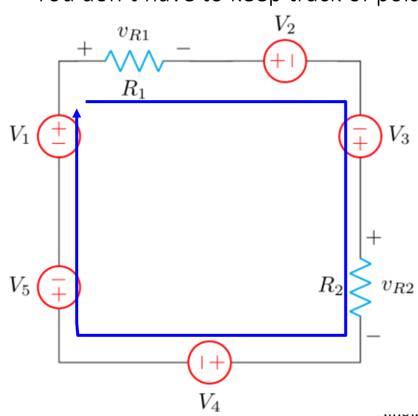




$$V_{eq} = v_{R1} + v_{R2}$$

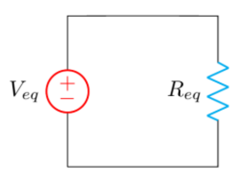
Equivalent Resistance and Sources

- Notes
  - Voltage sources in series add directly to form an equivalent voltage source
    - Keep track of voltage polarities
  - Resistances in series add directly to form an equivalent resistance
    - You don't have to keep track of polarity (as they only absorb energy)



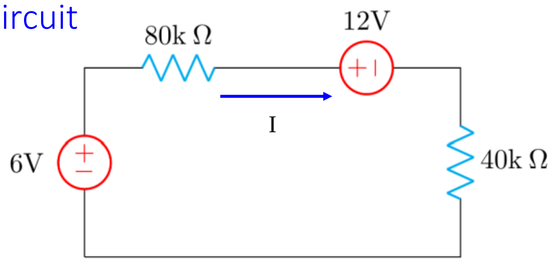
$$V_{eq} = (V_1 - V_2 + V_3 - V_4 - V_5)$$

$$R_{eq} = R_1 + R_2$$



Multiple-source/Resistor Networks

- Ex. 3: Find **current I** and **power** associated with each source by first reducing the circuit to **an equivalent circuit**
  - Step 1: Reduce to an equivalent circuit
  - Step 2: Apply KVL/Ohm's Law
  - Step 3: Solve for I
  - Knowing the one current we can go back to original circuit and compute powers



Multiple-source/Resistor Networks

- Ex. 3(cont.): Find **current I** and **power** associated with each source…

1. Reduce the circuit to an equivalent circuit

Find the current

2. Using KVL in equivalent circuit

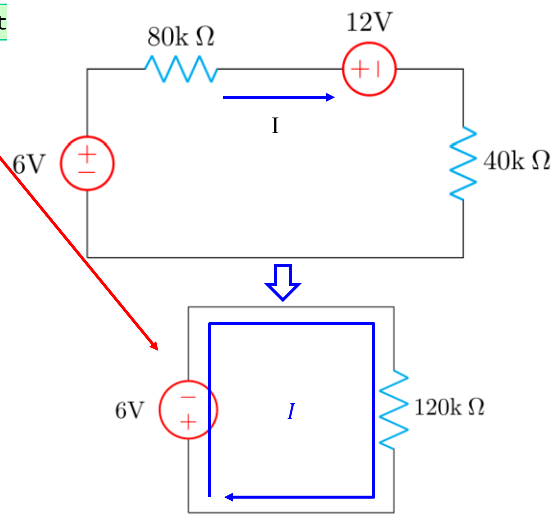
$$+6 + V_{120k} = 0$$

3. Using Ohm's Law V=IR, solve for I

$$+6 + I \times 120k = 0$$

$$I = -\frac{6}{120k} = -50u\ A$$

Which way the current is actually flowing?



Equivalent circuit

Multiple-source/Resistor Networks

- Ex. 3 (cont.): Find current I and power associated with each source…

Power associated with 6V source

$P_{6V} = -V \times I$

$P_{6V} = -6 \times -50u\ A$

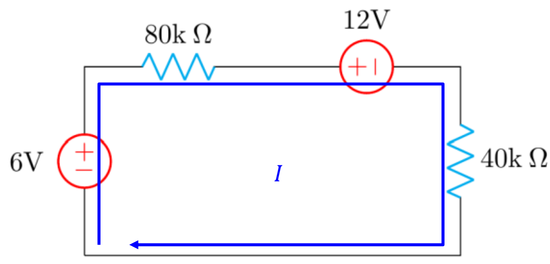
$P_{6V} = 0.30m\ W$

Power associated with 12V source

$P_{12V} = V \times I$

$P_{12V} = 12 \times -50u\ A$

$P_{12V} = -0.60m\ W$



You should calculate power absorbed by the resistors and confirm that total power delivered is equal to total power absorbed

Strategy: Single Loop Circuits

- STEP 1: Define a current *i(t).* We know from KCL that there is only one current for a single-loop circuit.
  - This current is assumed to be flowing either clockwise or counterclockwise around the loop.
- STEP 2: Apply KVL to the single-loop circuit in terms of *i(t)* and resistances (Ohm's Law $V = IR$)
- STEP 3: Solve the single KVL equation for the current *i(t).*
  - If *i(t)* is positive, the current is flowing in the direction assumed; if not, then the current is flowing in the opposite direction.
- STEP 4: Knowing the one current we can compute ALL the remaining voltages and powers provided we know the resistances of the components

Multiple-resistor/sources Networks

- Ex. 4 Find I, V_{bd}, and P_{30k}

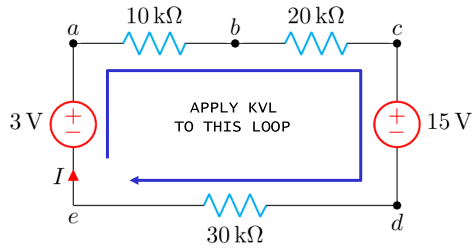
To find I

$-3\,V + V_{10k} + V_{20k} + 15\,V + V_{30k} = 0$

$-3\,V + I(10\;k\Omega) + I(20\;k\Omega) + 15\,V + I(30\;k\Omega) = 0$

$12V + I(60\;k\Omega) = 0$

$I = \dfrac{-12\text{V}}{60\text{k}\Omega} = -0.2\text{ mA}$



Step 1: Define a current I

Step 2: Apply KVL to the single-loop circuit in terms of I and resistances

Step 3: Solve the single KVL equation for the current I

Step 4: Knowing I, we can compute ALL the remaining voltages and powers

You can also solve it by re-drawing the equivalent circuit

Multiple-resistor/sources Networks

- Ex. 4 Find I, V_{bd}, and P_{30k} (cont.)

To find V_{bd} $\quad$ $I = -0.2mA$

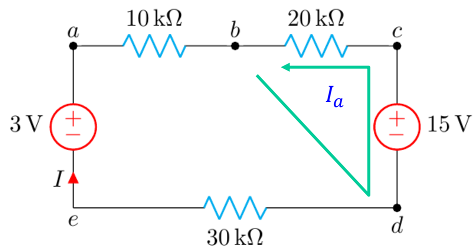
$V_{bd} - 15\,V + V_{20k} = 0$

$V_{bd} = 15\,V - V_{20k}$

$V_{bd} = 15\,V - (I_a \times 20k)$

$V_{bd} = 15\,V - (0.2mA \times 20k)$ $\quad$ Note $I_a = -I$

$V_{bd} = 11\,V$



You can also solve it by taking any other closed path
Do try it this at home :)

Multiple-resistor/sources Networks

- Ex. 4 Find I, V_{bd}, and P_{30k}

To find P_{30k}

$I = -0.2mA$

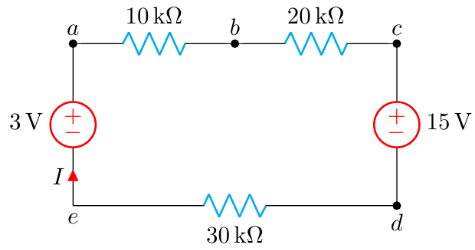
$P = I^2 \times R$

$P_{30k\Omega} = I^2 R_{30k\Omega}$

$P_{30k\Omega} = (-0.2 \times 10^{-3} A)^2 (30 \times 10^3\ \Omega)$

$P_{30k\Omega} = 1.2\ mW$

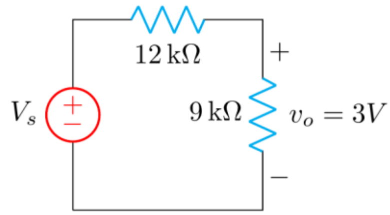
Homework: Find power associated with all the element and check your work by using law of conservation of energy



Practice Problems

Single-loop Circuits

- Prob. 1:
  - If $V_o = 3V$, find V_s



Ans $V_s = 7V$

Multiple-source/Resistor Networks

- Prob. 2: If $V_{ad} = 3V$, find V_s

We can apply a voltage divider, except that known and unknown have been reversed

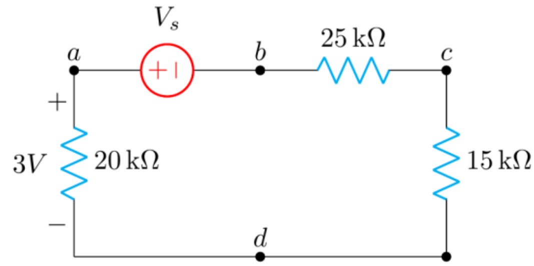
To find V_s

$$v_{R20k} = \frac{R_{20k}}{R_{eq}} V_s$$

$$V_s = v_{R20k} \frac{R_{eq}}{R_{20k}}$$

$$V_s = 3 \times \frac{25k + 15k + 20k}{20k}$$

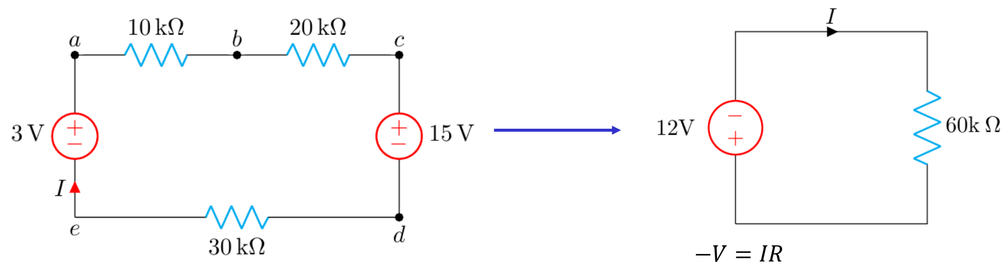
$$V_s = 3 \times \frac{60k}{20k} = 9V$$



We can also do it by first finding current I through 20kΩ (which is the one through the whole loop), then apply KVL to find Vs.

Multiple-resistor/sources Networks

- Prob. 3: In Ex 4, find I by drawing equivalent circuit



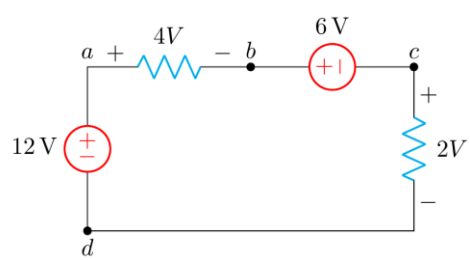
$$-V = IR$$

$$I = \frac{-12\ V}{60\ k\Omega} = -0.2mA$$

Can you figure out what is V_{bc}?

Single-loop Circuits

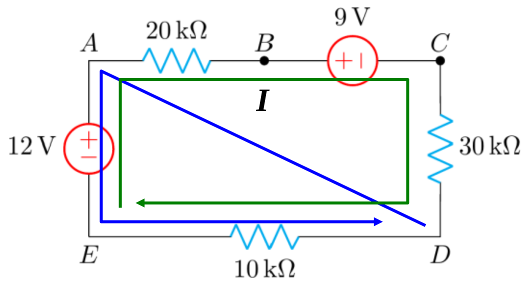
- Prob. 5:
  - Find V_{bd}



Ans $V_{bd} = 8V$

Single-loop Circuits

- Prob. 6:
  - Find indicated V_{DA}, V_{CD}, I_{DE}
  - For I_{DE}, Apply KVL for green loop



$-12 + 20k \times I + 9 + 30k \times I + 10k \times I = 0$

$60k \times I - 3 = 0$

$I = \frac{3}{60k} = 0.05\text{mA}$

$I_{DE} = I = \frac{3}{60k} = 0.05\text{mA}$

  - For V_{DA}, apply KVL for blue loop

$V_{DA} + 12 + 10k \times (-I) = 0$

$V_{DA} + 12 + 10k \times -0.05m = 0$

$V_{DA} + 12 - 0.5 = 0$

$V_{DA} = -11.5\text{V}$

  - For V_{CD}

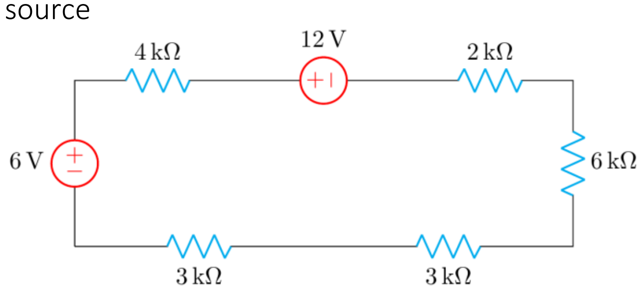
$V_{CD} = I \times 30k$

$V_{CD} = 0.05m \times 30k$

$V_{CD} = 1.5V$

Single-loop Circuits

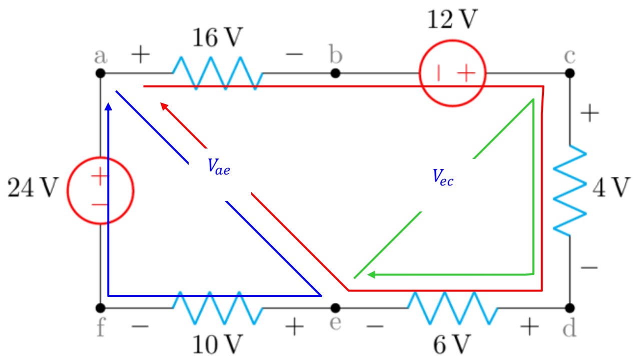
- Prob. 7:
  - Find the power supplied by each source



Ans
$P_{12V} = -4mW$ so supplied is 4mW
$P_{6V} = +2mW$ so supplied is -2mW

Single-loop Circuits

- Prob. 8 :
  - Find V_{ae} and V_{ec}



$$V_{ae} + 10 - 24 = 0$$

$$16 - 12 + 4 + 6 - V_{ae} = 0$$

`Both the above equations will yield`

$$V_{ae} = 14$$

`Given the choice, use the simplest loop`

$$V_{ec} + 4 + 6 = 0$$

$$V_{ec} = -10$$

Appendix-A

Derivation of General Case of Voltage Divider

Voltage Divider: General Equation

- We can derive a general equation for voltage divider for any number (N) of resistors in series (single loop):
  - Applying KVL we get:

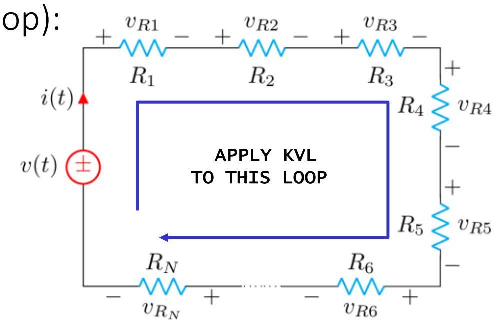
$$-v(t) + v_{R1} + v_{R2} + \ \dots + v_{RN} = 0$$

$$v(t) = v_{R1} + v_{R2} + \ \dots + v_{RN}$$

  - Substitute in Ohm's Law and factor out *i(t)*:

$$v(t) = i(t)R_1 + i(t)R_2 + \ \dots + i(t)R_N$$

$$v(t) = i(t)[R_1 + R_2 + \dots + R_N]$$ E



Voltage Divider: General Equation

- Find the equivalent series resistance for the loop:

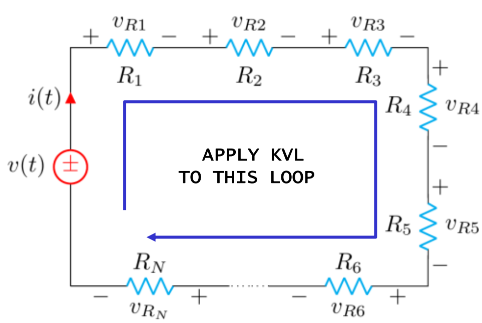
$$R_{eq} = R_1 + R_2 + \ \dots + R_N$$ F

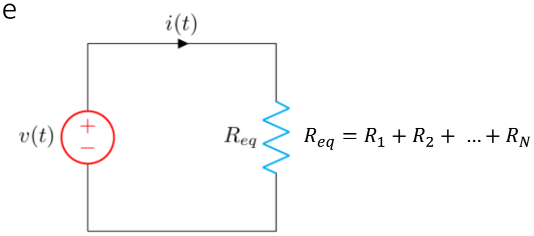
- Substitute Req (F) into our equation (E):

$$v(t) = i(t)R_{eq}$$ or $$i(t) = \frac{v(t)}{R_{eq}}$$

- The current through any one resistor can be found using Ohm's Law, for example R_1:

$$i_{R1} = i(t) = \frac{v_{R1}}{R_1}$$ H





Voltage Divider: General Equation

– Substitute $i(t)$ from equation H into equation G

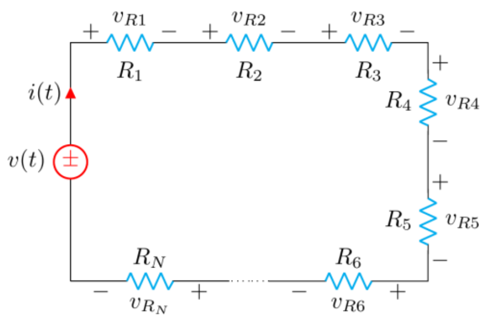
$$\frac{v_{R1}}{R_1} = \frac{v(t)}{R_{eq}}$$

– Rearranging to solve for V_{R1} gives:

$$v_{R1} = v(t)\frac{R_1}{R_{eq}}$$

– Therefore the general equation (for voltage across any resistor R_i) is:

$$v_{Ri} = \frac{R_i}{R_{eq}} v(t)$$

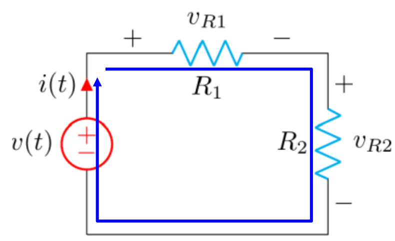


Appendix-B

Derivation of Special Case of Voltage Divider

Voltage Divider: Special Case

- Note: It is a single loop circuit, therefore only one current flows through it



  – Apply KVL to this circuit:

$$-v(t) + v_{R1} + v_{R2} = 0$$

$$v(t) = v_{R1} + v_{R2} \longrightarrow \text{A}$$

  – Also, using Ohm's Law:

$$v_{R1} = R_1 i(t) \longrightarrow \text{B}$$

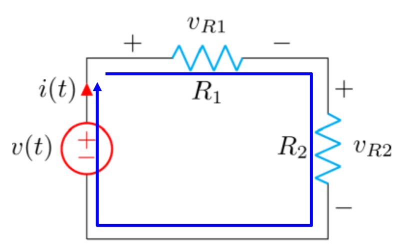
$$v_{R2} = R_2 i(t) \longrightarrow \text{C}$$

Voltage Divider: Special Case

– Now we Substitute B and C into A and solve for $i(t)$

$$v(t) = R_1 i(t) + R_2 i(t)$$

$$i(t) = \frac{v(t)}{R_1 + R_2} \longrightarrow \text{D}$$



– Now that we've solved for $i(t)$ we substitute D into B and C independently

$$v_{R1} = \frac{R_1}{R_1 + R_2} v(t)$$

$$v_{R2} = \frac{R_2}{R_1 + R_2} v(t)$$

Voltage υ(t) is divided between the resistors R_1 and R_2 in direct proportion to their resistances.

White Board

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36