

WHAT YOU NEED TO KNOW FOR EXAM 1  
AND THE HOMEWORK TONIGHT, 10/9

**DIFFERENTIATION RULES**

THE CONSTANT RULE :  $\frac{d}{dx}[C] = 0$ , for any real number  $C$  (1)

THE SUM RULE :  $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$  (2)

THE POWER RULE :  $\frac{d}{dx}[x^n] = nx^{n-1}$  (3)

THE PRODUCT RULE :  $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$  (4)

THE RECIPROCAL RULE :  $\frac{d}{dx}\left[\frac{1}{f(x)}\right] = -\frac{f'(x)}{[f(x)]^2}$  (5)

THE QUOTIENT RULE :  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$  (6)

**TRIGONOMETRIC IDENTITIES**

$$\tan x = \frac{\sin x}{\cos x} \quad (7)$$

$$\csc x = \frac{1}{\sin x} \quad (8)$$

$$\sec x = \frac{1}{\cos x} \quad (9)$$

$$\cot x = \frac{\cos x}{\sin x} \quad (10)$$

**DERIVATIVES OF THE SIX TRIG FUNCTIONS**

$$\frac{d}{dx}[\sin(x)] = \cos(x) \quad (11)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x) \quad (12)$$

$$\frac{d}{dx}[\tan(x)] = \sec^2(x) \quad (13)$$

$$\frac{d}{dx}[\sec(x)] = \sec(x) \tan(x) \quad (14)$$

$$\frac{d}{dx}[\csc(x)] = -\csc(x) \cot(x) \quad (15)$$

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x) \quad (16)$$

## HOMEWORK FOR THURSDAY NIGHT

Look at the derivatives of the trigonometric functions on the previous page. In the last two classes, we proved (11) and (12). Use that information to derive the (13), (14), (15), and (16). Do only this tonight, if you have not already done so. Another way to say this is to follow these instructions:

Use the basic trigonometric identities in (7), (8), (9), and (10) along with the facts that  $\frac{d}{dx}[\sin(x)] = \cos(x)$  and  $\frac{d}{dx}[\cos(x)] = -\sin(x)$  to find the derivatives of the following functions. (First, express them in terms of sine and cosine, then employ the correct differentiation rules.)

1.  $f(x) = \tan(x)$
2.  $f(x) = \cot(x)$
3.  $f(x) = \sec(x)$
4.  $f(x) = \csc(x)$

**MAKE AN IMPORTANT OBSERVATION ABOUT THE DERIVATIVES OF THE SIX TRIG FUNCTIONS:** Notice that the derivatives of the co-functions are all negative!

## PRACTICE PROBLEMS FOR EXAM 1

Differentiate the following functions. Use proper notation and be clear in your work, especially in the algebra steps that are required on some of the problems. This set of problems, 1-30, are intended to be quick but careful, and are simpler than the ones that follow. It is my advice to do all 30 of these in one sitting, timing yourself for 15 minutes (minimum) to 20/25 minutes(maximum), then check your work, see what you got correct/incorrect, and then take a break and return later to the next set. I will post handwritten answers in a separate document later to some of the later problems, but you have the answers to the first 30 in this document.

1.  $y = x$
2.  $y = x^2$
3.  $y = x^3$
4.  $y = 2x^3$
5.  $y = 2x^{-1}$
6.  $y = 2x^{-1/2}$
7.  $y = 2\sqrt{x}$

8.  $y = \frac{2}{\sqrt{x}}$
9.  $y = x^{\frac{1}{3}}$
10.  $y = 3x^{-\frac{1}{3}}$
11.  $f(x) = x^{\frac{2}{3}}$
12.  $f(x) = \frac{3}{2}x^{\frac{2}{3}}$
13.  $f(x) = 3x^{-\frac{2}{3}}$
14.  $f(x) = \frac{1}{x^6}$
15.  $f(x) = \frac{1}{6x^6}$
16.  $y = 6 - x$
17.  $y = x^2 - 3x + 23$
18.  $y = 4\pi$
19.  $y = 4\pi x$
20.  $y = 4\pi x^2$
21.  $y = \cos x$
22.  $f(x) = \sin x$
23.  $f(x) = \tan x$
24.  $f(x) = -\frac{1}{\sin x}$
25.  $y = -\frac{1}{\cos x}$
26.  $y = \cot x$
27.  $y = \sec x$
28.  $y = \csc x$
29.  $y = 7 \sin x$
30.  $y = -2 \cos x$

**Differentiate the following functions, using good notation. Show as much detail in your work as you can. COMMUNICATE well.**

31.  $f(x) = 2x \cos x$
32.  $f(x) = x^2 \sin x$
33.  $f(x) = (2 - x^2) \tan x$

$$34. f(x) = (4 - x - x^2) \sec x$$

$$35. f(x) = (x - 3x^3) \csc x$$

$$36. f(x) = 5x \cot x$$

$$37. y = \frac{2x}{3-x^2}$$

$$38. y = \frac{x^{-\frac{2}{3}}}{1-x}$$

$$39. y = \frac{1}{3-x}$$

$$40. y = 2x \sin x + \frac{1}{x+1}$$

$$41. f(x) = x^{\frac{1}{4}} \csc x - 3x^{\frac{4}{3}}$$

$$42. y = 1 - 4\sqrt{x} \sec x$$

$$43. y = \frac{\tan x}{1-x-x^2}$$

$$44. y = \frac{2}{x-\sin x}$$

$$45. f(x) = 6\sqrt[3]{x^2} \cos x$$

## STOP. CHECK ANSWERS.

You will read that you must use the internet or an app to check your answers here. Same with the other sections! Yes, I am making you use the calculus apps for help. You should be teaching me where those are and how to use them wisely.

**Differentiate the following functions.**

$$46. y = 2x \sec x$$

$$47. y = 4x^2 \cos x$$

$$48. y = \sin x \cos x$$

$$49. y = 3x^3 \sin x$$

$$50. y = 8x \tan x$$

$$51. y = -x^4 \csc x$$

$$52. y = 5x^3 \cot x$$

$$53. y = (3 - x^3) \cos x$$

$$54. y = \frac{\sin x - \cos x}{x^2}$$

$$55. y = \frac{x^2}{\sin x - \cos x}$$

$$56. y = \frac{x^2 - 3x - 6}{x^2}$$

$$57. y = \frac{1}{10-x}$$

$$58. y = 2x \sin x \cos x$$

$$59. y = 3x^2 \tan x \sin x$$

$$60. y = \frac{(7-x^3)(2x+2)}{3-2x^2}$$

**STOP. CHECK ANSWERS.**

**Differentiate the following functions.**

$$61. y = \sin x$$

$$62. y = 2 \cos x$$

$$63. y = 3 \tan x$$

$$64. y = 3 \sec x$$

$$65. y = 2 \csc x$$

$$66. y = \cot x$$

$$67. y = 3\pi^3 - 2x^3$$

$$68. f(x) = \frac{1-x^4}{3-2x}$$

$$69. y = \sqrt[5]{x^3}$$

$$70. y = x^{3/5}$$

# ANSWERS FOR 1-30

I use  $y'$  instead of  $\frac{dy}{dx}$  for my answers. Know that  $y'$  is not the best notation, but it is permissible. Be sure you are focusing on your writing. I know to some of you this sounds ridiculous, but I guarantee you it matters. Note the comments at the end if you wish to refine some prerequisite skills. **IMPORTANT:** *Seriously diagnose your weaknesses as you do this practice. Put them in words. Identify the areas of mathematics (be specific) in which you can tell you need to improve.*

- |                               |                                   |                             |
|-------------------------------|-----------------------------------|-----------------------------|
| 1. $y' = 1$                   | 11. $f'(x) = \frac{2}{3}x^{-1/3}$ | 21. $y' = -\sin x$          |
| 2. $y' = 2x$                  | 12. $f'(x) = x^{-1/3}$            | 22. $f'(x) = \cos x$        |
| 3. $y' = 3x^2$                | 13. $f'(x) = -2x^{-5/3}$          | 23. $f'(x) = \sec^2 x$      |
| 4. $y' = 6x^2$                | 14. $f'(x) = -\frac{6}{x^7}$      | 24. $f'(x) = \csc x \cot x$ |
| 5. $y' = -2x^{-2}$            | 15. $f'(x) = -\frac{1}{x^7}$      | 25. $y' = -\sec x \tan x$   |
| 6. $y' = -x^{-3/2}$           | 16. $y' = -1$                     | 26. $y' = -\csc^2 x$        |
| 7. $y' = \frac{1}{\sqrt{x}}$  | 17. $y' = 2x - 3$                 | 27. $y' = \sec x \tan x$    |
| 8. $y' = x^{-3/2}$            | 18. $y' = 0$                      | 28. $y' = -\csc x \cot x$   |
| 9. $y' = \frac{1}{3}x^{-2/3}$ | 19. $y' = 4\pi$                   | 29. $y' = 7 \cos x$         |
| 10. $y' = -x^{-4/3}$          | 20. $y' = 8\pi x$                 | 30. $y' = 2 \sin x$         |

**COMMENTS:(a)** Look at 6 and 8 and reason why those derivatives should be the same. **(b)** Look at problem 14. Rework that derivative in two ways: find the derivative using the power rule. Then do it again, but find the derivative using the reciprocal rule. (*what is the point of me asking you to do this?*)

Go to the next set of problems after a break from this first 30. Come back and do them later in a single sit-down, but be prepared to present your work as there will be some algebra involved, kind of similar to the homework.

# ANSWERS FOR 31-70

I am not giving the answers to these; instead, allow me to teach you how to look them up on your own using the internet. This time, you practice something important: *your decision-making skills about how you should write the functions.* I will suggest [www.wolframalpha.com](http://www.wolframalpha.com) as it is free (or ought to be), but the internet is crowded with apps for which you can use to cheat, check your work, study different solution techniques, get explanations, etc... **USE IT WISELY!** Technology *does* help, but only when you choose to use your independence and thinking brain first and make yourself stronger.

Send me questions if you have any questions on your quest to find the answers to the later problems. The ideal time to do this will be Sunday, between the hours of noon

and 5 pm. I will post handwritten solutions to some of the problems. Do not hesitate to request a solution to a problem if you are stuck and need it. Use the internet and/or the various calculus apps that explain things and contact me after that try. I will be on alert this Sunday during those hours.