MATH 238 Homework #2

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Homework for Section 2.1

1. Use computer software to obtain a direction field for the given differential equation. Also sketch an approximate solution curve passing through each of the given points

$$\frac{dy}{dx} = e^x \sin(y)$$

   (a) $y(-1) = 4.5$
   (b) $y(-1) = -4.5$
   (c) $y(-1) = 1$
   (d) $y(-1) = -1$

   Give the equilibrium solutions as solution functions.

   SOLUTION:

$$(a) \quad \frac{dy}{dx} = e^x \sin(y)$$
$$e^x \sin(y) = 0$$
$$e^x \neq 0, \ \sin(y) = 0$$
$$\Longrightarrow \boxed{y = \pi n, 0, \ \in \mathbb{Z}} \text{ is the general solution.}$$

Equilibrium equations occur when $\frac{dy}{dx} = 0$ allowing us to the find the $y(x)$.

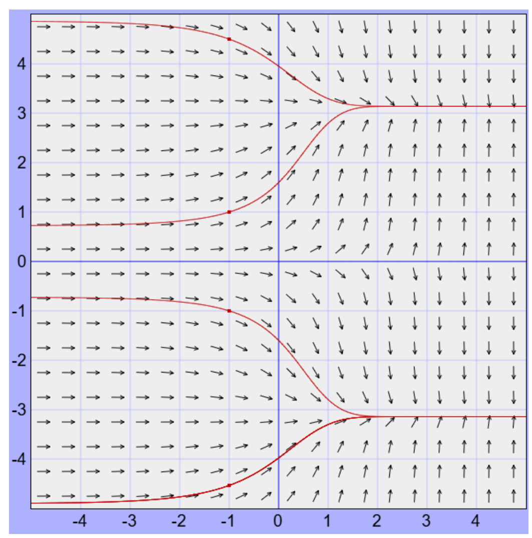
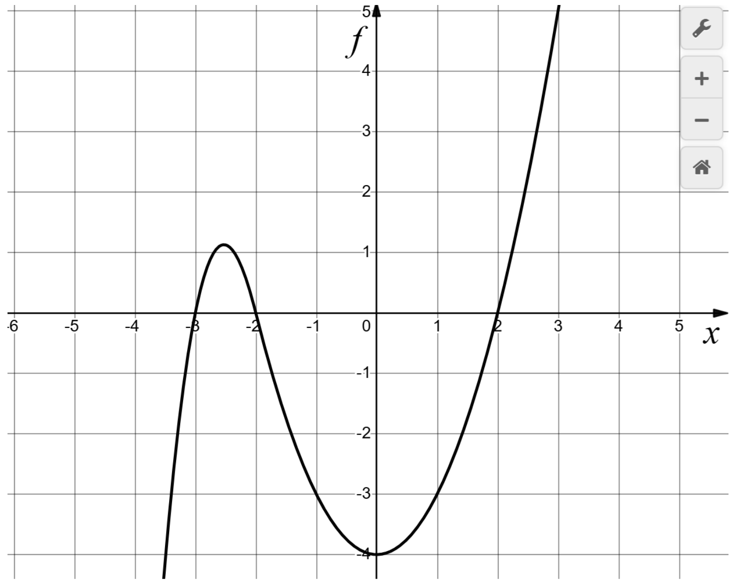
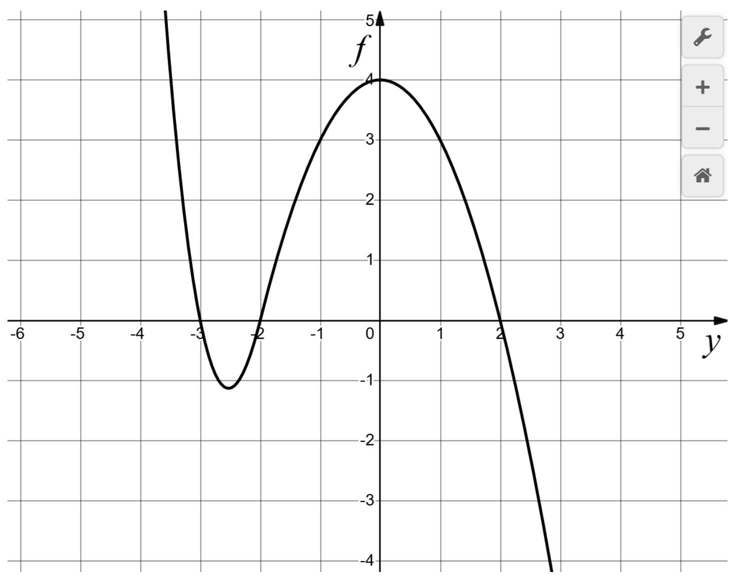


Figure 1: Direction field of $\frac{dy}{dx} = e^x \sin(y)$

2. The given figure represents the graph of $f(x)$ when $dy/dx = f(x)$. By hand, sketch a direction field over an appropriate grid and include it as an image.



3. The given figure represents the graph of $f(y)$ when $dy/dx = f(y)$. By hand, sketch a direction field over an appropriate grid and include it as an image.



4. Consider the autonomous first-order differential equation $dy/dx = 2y - y^2$ and the initial condition $y(0) = y_0$. By hand, sketch the graph of a typical solution $y(x)$ when y_0 has the given valuesand include them as images.

   (a) $y_0 < 0$

   (b) $0 < y_0 < 2$

   (c) $y_0 > 2$

Homework for Section 2.2

1. Solve the initial-value problem

$$\frac{dy}{x^2 + e^x} = \frac{y dx}{y^2 + 1}, y(0) = 1$$

2. Solve the initial-value problem

$$x + 3y^2\sqrt{x^1 + 1}\frac{dy}{dx} = 0, y(0) = 1$$

3. A glucose solution is administered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Thus a model for the concentration $C = C(t)$ of the glucose solution in the bloodstream is

$$\frac{dC}{dt} = r - kC$$

where k is a positive constant.

(a) Suppose that the concentration at time $t = 0$ is C_0. Determine the concentration at any time t by solving the differential equation.

(b) Assuming that $C0 < \frac{r}{k}$, find limit: $\lim t \to \infty C(t)$ and interpret your answer.