

# Homework 1: Introduction to Differential Equations

MATH 238: Differential Equations

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Instructions:** You have received a latex document. Write your answer in this document. Write your name and date in the space provided. Show all your work and clearly justify your answers. Recompile your work. This assignment is intended to introduce the basic ideas and language of differential equations and allow you to practice writing in Latex.

## Problems

### 1. What is a Differential Equation?

Which of the following equations are differential equations? Explain your reasoning.

- (a)  $y = 3x^2 + 1$
- (b)  $\frac{dy}{dx} = 5x^4$
- (c)  $\frac{d^2y}{dx^2} + y = 0$
- (d)  $x^2 + y^2 = 9$

### 2. Order of a Differential Equation

Determine the order of each differential equation below. Explain.

- (a)  $\frac{dy}{dx} = \sin x$
- (b)  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$
- (c)  $\left(\frac{d^3y}{dx^3}\right)^2 + y = 1$

### 3. Linear vs. Nonlinear

State whether each differential equation is **linear or nonlinear**. Justify your response.

- (a)  $\frac{dy}{dx} + 2y = e^x$
- (b)  $\frac{dy}{dx} = y^2$
- (c)  $x^2\frac{d^2y}{dx^2} + y = 0$

#### 4. Checking a Solution

Verify whether the given function is a solution of the differential equation.

- (a) Check whether  $y = x^2$  is a solution of

$$\frac{dy}{dx} = 2x$$

- (b) Check whether  $y = e^{-2x}$  is a solution of

$$\frac{dy}{dx} + 2y = 0$$

#### 5. Verifying solutions

Verify that the given function is a solution to the given differential equation.

(a)  $\frac{dy}{dx} + 4xy = 8x^3$ ,  $y = 2x^2 - 1 + c_1e^{-2x^2}$

(b)  $x^2 \frac{dy}{dx} + xy = 10\sin x$ ,  $y = \frac{5}{x} + \frac{10}{x} \int_1^x \frac{\sin t}{t} dt$

#### 6. General vs. Particular Solutions

- (a) Explain the difference between a **general solution** and a **particular solution** of a differential equation.

- (b) Identify which of the following is a general solution:

$$y = Ce^{3x}, \quad y = 5e^{3x}$$

#### 7. Initial Value Problem

Consider the differential equation

$$\frac{dy}{dx} = 3x^2$$

- (a) Find the general solution.

- (b) Find the particular solution that satisfies  $y(0) = 4$ .

#### 8. Modeling with Differential Equations

Suppose that the rate of change of a population  $P(t)$  is proportional to the size of the population.

- (a) Write a differential equation that models this situation.

- (b) Identify the dependent and independent variables.

#### 9. Slope Fields (Conceptual)

Without solving, describe what information a slope field provides about solutions to a differential equation.

*End of Assignment*