MATH 238 Homework 6

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Contents

Homework for Section 4.4: 50 points

In Problems 27 - 36 solve the given initial-value problem.

Problem 4.4.30

Given: $y'' + 4y' + 4y = (3 + x)\, e^{-2x}; \quad y(0) = 2,\ y'(0) = 5$
Find: $y(x)$

SOLUTION: When we are solving for our general solution, we must first solve for our complimentary solution, using auxiliary equations, and solve for our paricular solution, using the method of undetermined coefficients.

Solving auxiliary equation for $y_c(x)$.

$$\begin{aligned}
&y'' + 4y' + 4y = 0\\
&m^2 + 4m + 4 = 0\\
&m = (m + 2)^2,\ m = -2, -2\\
\Longrightarrow &\boxed{y_c(x) = C_1 e^{-2x} + C_2 x e^{-2x}}
\end{aligned}$$

Method of undetermined coefficients for $y_p(x)$. Notice how there are multiple repeated roots for e^{--2x}.

$$
\begin{aligned}
\text{Assume: } y_p(x) &= x^2(Ax+B)e^{-2x} \\
y_p(x) &= (Ax^3+Bx^2)e^{-2x} \\
y_p'(x) &= (3Ax^2+2Bx-2Ax^3-2Bx^2)e^{-2x} \\
y_p'(x) &= (-2Ax^3+3Ax^2-2Bx^2+2Bx)e^{-2x} \\
y_p''(x) &= (-6Ax^2+6Ax-4Bx+2B+4Ax^3-6Ax^2+4Bx^2-4Bx)e^{-2x} \\
y_p''(x) &= (4Ax^3-12Ax^2+4Bx^2+6Ax-8Bx+2B)e^{-2x}
\end{aligned}
$$

Substitute $y_p(x)$ and solve for constants.

$$
\begin{aligned}
&y_p(x)''+4y_p(x)'+4y_p(x) = (3+x)\,e^{-2x} \\
&(4Ax^3-12Ax^2+4Bx^2+6Ax-8Bx+2B)\cancel{e^{-2x}}+4(-2Ax^3+3Ax^2-2Bx^2+2Bx)\cancel{e^{-2x}}+4(Ax^3+Bx^2)\cancel{e^{-2x}} = (3+x)\cancel{e^{-2x}} \\
&(\cancel{4Ax^3}\cancel{-12Ax^2}+\cancel{4Bx^2}+6Ax\cancel{-8Bx}+2B)+(\cancel{-8Ax^3}+\cancel{12Ax^2}\cancel{-8Bx^2}+\cancel{8Bx})+(\cancel{4Ax^3}+\cancel{4Bx^2}) = 3+x \\
&6Ax+2B = 3+x
\end{aligned}
$$

$$
\Longrightarrow \begin{cases} 6Ax = x \\ B = 3 \end{cases}
$$

$$
A = \frac{1}{6},\ B = 3
$$

$$
\Longrightarrow \boxed{y_p(x) = \left(\frac{1}{6}x^3+3x^2\right)e^{-2x}}
$$

Combine to create $y_g(x)$

$$
y_g(x) = y_c(x)+y_p(x)
$$

$$
\Longrightarrow \boxed{y_g(x) = C_1e^{-2x}+C_2xe^{-2x}+\left(\frac{1}{6}x^3+3x^2\right)e^{-2x}}
$$

Now we have to solve the IVP and obtain our constants.

Problem 4.4.34

Given: $\dfrac{d^2x}{dt^2}+\omega^2x = F_0\cos\gamma t; \quad x(0)=0,\ x'(0)=0$

Find: $y(x)$

Problem 4.4.42

In Problems 41 and **42** solve the given initial-value problem in which the input function $g(x)$ is discontinuous. [*Hint*: Solve each problem on two intervals, and then find a solution so that y and y' are continuous at $x=\pi/2$ (Problem 41) and $\boldsymbol{x=\pi}$ (**Problem 42**)]

Given: $y''+4y=g(x),\ y(0)=1,\ y'(0)=2,$ where

$$
g(x) = \begin{cases} 20 & 0\le x\le\pi \\ 0 & x\ge\pi \end{cases}
$$

Find: $y(x)$

Homework for Section 4.6: 50 points

In Problems 1 - 20 solve each differential equation by variation of parameters.

Problem 4.6.4

Given: $y'' + y = \sec\theta\tan\theta$
Find: $y_g(x)$

SOLUTION:

Problem 4.6.10

Given: $4y'' - y = e^{\frac{x}{2}} + 3$
Find: $y_g(x)$

SOLUTION: